

Spin polarized quasiparticle in an off-equilibrium medium

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We classify radiative corrections to spin polarization into three types: (i) spectral function, (ii) modified distribution function, and (iii) modified Kubo-Martin-Schwinger relation. The last two contributions modify the well-known free theory results. The first contribution is present with radiative correction only. We perform a detailed analysis on the first type for an energetic quark in a medium with hydrodynamic gradients, finding the radiative correction to spectral function leads to a polarized quasiparticle. The degeneracy of spin polarization induced by different hydrodynamic sources in free theory is lifted by radiative correction. We argue that the corrections of the last two types are parametrically suppressed with respect to the first type for the energetic quark we consider.

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I. INTRODUCTION AND SUMMARY

Spin polarization is one of the most important observables in particle physics. It offers a unique probe of the production mechanism and internal structure of particles. In relativistic heavy ion collisions, a novel mechanism of spin polarization due to initial orbital angular momentum in off-central collisions has been proposed [1]. It is expected to generate a spin polarization of baryon and spin alignment of vector meson [1,2]. An outstanding feature of spin polarization in heavy ion collision is the role of the quark-gluon plasma (QGP) medium undergoing rapid evolution, which provides a valuable probe to the spinning property of QGP. Over the past decade, a variety of spin polarization phenomena have been observed in heavy ion experiments [3]. One of the best understood examples among them is the global polarization of Λ hyperon, which characterizes the vorticity of QGP [4–9]. The local spin polarization of Λ hyperon and vector meson spin alignment allows us to study spin induced by more general off-equilibrium effects described by the hydrodynamic gradient [10–17] and fluctuation effect [18–21].

Understanding the massive precise data requires a theoretical framework that can treat spin degree of freedom (DOF) systematically. Kinetic theory formulated in terms of quasiparticles and their distribution function has been a

widely used framework in early studies of spin independent transport in QGP. A description of spin transports requires generalization to the spin dependent distribution function. This has been pursued actively in recent years, leading to the quantum kinetic theory (QKT); see [22] for a recent review. Spin polarization has been predicted in free theory based on QKT [10,23] and field theory [4,11,12]. The contributions can be recognized as being from gradient correction to the Kubo-Martin-Schwinger (KMS) relation and spin dependent distribution. The free theory prediction is expected to be corrected by the interaction effect.

In QKT, the interaction effect can be taken into account systematically from self-energy of the quasiparticles. The interaction effect can be separated into effects of collision and modified spectral function, which for massless DOF arise from two-loop and one-loop self-energy diagrams respectively [24]. Incorporation of collision to QKT has been made recently in [24–27], and phenomenological studies have been carried out in [28–33]. Despite the fact that two-loop self-energies are formally suppressed by g^4 , it is enhanced by a factor of $1/g^4$ from the large deviation of the steady state from the equilibrium state, leading to an overall $O(1)$ contribution to spin polarization [30,32–34].

One-loop self-energy has also been considered recently in [31,35,36].¹ In an inhomogeneous medium characterized by gradient of temperature and fluid velocity, the corresponding gradient of self-energy plays the role of the effective electromagnetic field, giving rise to contribution to spin polarization at $O(g^2\partial)$. In fact, there is yet another contribution to spin polarization at the same order. It arises from the $O(g^2\partial)$ contribution to the axial component of the

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¹See also [37] for related studies on spin averaged transports.

spectral function. In this case, the spectral function modified quasiparticle itself becomes spin polarized. Both contributions manifest themselves in a retarded correlator. The former comes from local equilibrium self-energy at $O(g^2)$ and gradient correction to the resummation equation, leading to a contribution at $O(g^2\partial)$ in the resummed retarded correlator. The latter comes from off-equilibrium self-energy at $O(g^2\partial)$. We consider the two contributions for massless quark in QGP with vorticity, shear, and temperature gradient. It turns out the former contribution is parameterically suppressed for an energetic quark in medium while the latter contribution lifts the degeneracy in spin polarization induced by vorticity, shear, and temperature gradient in free theory. This is in close analogy with the counterpart for external electromagnetic fields [38].

The paper is organized as follows: in Sec. II we first give a general account of possible radiative corrections to spin polarization in free theory. We then consider local equilibrium and off-equilibrium corrections to quark self-energy, which are used to extract the spectral function; Sec. III is devoted to the conclusion and outlook. Computational details are reserved in the two Appendixes.

II. RADIATIVE CORRECTION TO QUARK SELF-ENERGY AND SPIN POLARIZATION

We start by summarizing the prediction for spin polarization in free theory. For simplicity, we take chiral kinetic theory (CKT), which is the massless limit of QKT. The central quantity is the Wigner function defined as

$$S_{\alpha\beta}^<(X = \frac{x_1 + x_2}{2}, P) = \int d^4y e^{-ip \cdot y} \langle \bar{\psi}_\beta(x_2) \psi_\alpha(x_1) \rangle, \quad (1)$$

where ψ is the quark field and α, β are Dirac indices. In the absence of a background gluon field, the color structure is trivial and will be suppressed throughout the paper. $X = \frac{1}{2}(x_1 + x_2)$, $y = x_1 - x_2$ correspond to the center coordinate of the quark and the relative coordinate conjugate to the momentum. For our purposes, the expectation value is taken in a near equilibrium medium characterized by hydrodynamic gradients $\partial_X \equiv \partial$, for which the Wigner function can be solved in gradient expansion as

$$S^< = S^{<(0)} + S^{<(1)} + \dots \quad (2)$$

The explicit expressions of the first two orders are given by

$$S^{<(0)}(P) = \gamma_\mu \rho^\mu \tilde{f} \\ S^{<(1)}(P) = \gamma^5 \gamma_\mu \left[\delta \tilde{f} \rho^\mu + \frac{\epsilon^{\mu\nu\rho\sigma} \rho_\rho u_\sigma \partial_\nu \tilde{f}}{2P \cdot u} \right], \quad (3)$$

with $\rho_\mu = 2\pi\epsilon(P \cdot u)P_\mu \delta(P^2)$ being the free theory spectral function. Equation (3) contains two terms in the square bracket: the first term encodes modification of the

distribution function $\delta \tilde{f}$ in an off-equilibrium state. In case of vorticity, $\delta \tilde{f}$ is given by a spin-vorticity coupling shift of the distribution. The second term arises from the free spectral function and local equilibrium distribution, which can be viewed as a modification of the KMS relation at $O(\partial)$. The radiative corrections may occur in three types:

- (i) Spectral function. The spectral function is modified by self-energy in equilibrium and its off-equilibrium correction. It can contribute to the axial component of the Wigner function in the form $\delta S^{<(0)} = \gamma^\mu \gamma^5 \delta \rho_{\mu 5} \tilde{f}$ with $\gamma_\mu \gamma^5 \delta \rho_{\mu 5}$ being the axial component of the spectral function from radiative correction. This contribution is always proportional to equilibrium distribution.
- (ii) Modified distribution. Ignoring collisional effect, the distribution function is modified due to spin-vorticity coupling only. The radiative correction to the modified distribution function arises from local equilibrium self-energy and its derivatives [36].
- (iii) Modified KMS relation. By parity the local equilibrium self-energy contains vector component $\Sigma = \gamma^\mu \Sigma_\mu$, in which Σ_μ acts as a correction of momentum. A contribution to the axial component of the Wigner function is then expected in the form of the second term of $S^{<(1)}$, with $P_\mu \rightarrow \Sigma_\mu$. This contribution is always proportional to derivatives of the distribution function in contrast to the type (i) contribution.

The last two contributions modify the well-understood $S^{<(1)}$ in free theory. The first contribution is present with radiative correction only. We shall focus on the first one and study its impact on spin polarization, leaving radiative corrections to the other two types for future explorations.

A. Modified spectral function

The spectral function is defined in a similar way as the Wigner function:

$$\rho_{\alpha\beta}(X, P) = \int d^4y e^{iP \cdot y} \langle \psi_\alpha(x_1) \bar{\psi}_\beta(x_2) + \bar{\psi}_\beta(x_2) \psi_\alpha(x_1) \rangle, \quad (4)$$

again with $X = \frac{1}{2}(x_1 + x_2)$, $y = x_1 - x_2$. By introducing the *ra* basis in which the fields are related to those in the Schwinger-Keldysh contour,

$$\psi_r = \frac{1}{2}(\psi_1 + \psi_2), \quad \psi_a = \psi_1 - \psi_2, \quad (5)$$

and similarly for $\bar{\psi}$, we immediately have the following identities²:

²In our convention, $S^< = -S_{12}$.

$$\begin{aligned}
S_{ra} &= \frac{1}{2}(S_{11} - S_{22} - S_{12} + S_{21}), \\
S_{ar} &= \frac{1}{2}(S_{11} - S_{22} + S_{12} - S_{21}), \\
S_{rr} &= \frac{1}{4}(S_{11} + S_{22} + S_{12} + S_{21}), \\
S_{aa} &= S_{11} + S_{22} - S_{12} - S_{21},
\end{aligned} \tag{6}$$

where the two-point function with indices 11, 22, 12, and 21 correspond to time-ordered, antitime-ordered, lesser, and greater functions respectively. One can define the retarded function based on (6) as

$$\begin{aligned}
S_{ra,\alpha\beta}(X, P) &= \int d^4y e^{iP \cdot y} \theta(y_0) \langle \psi_\alpha(x_1) \bar{\psi}_\beta(x_2) \\
&\quad + \bar{\psi}_\beta(x_2) \psi_\alpha(x_1) \rangle.
\end{aligned} \tag{7}$$

Using the representation of the Heaviside function

$$\theta(y_0) = i \int \frac{dk_0}{2\pi} \frac{e^{-ik_0 y_0}}{k_0 + i\eta}, \tag{8}$$

we easily obtain

$$S_{ra}(X, P) = \int \frac{dp'_0}{2\pi} \frac{i\rho(X, p'_0, \mathbf{p})}{p_0 - p'_0 + i\eta}. \tag{9}$$

If ρ were real, we could simply take the real part of both sides to obtain

$$\rho(P) = 2\text{Re}[S_{ra}(P)]. \tag{10}$$

With ρ being a matrix in Dirac space, the reality condition is replaced by $\gamma^0 \rho(P)^\dagger \gamma^0 = \rho(P)$, which indeed holds at the operator level from the definition (4).³ It follows that (10) holds with the real part defined as $\text{Re}A = \frac{1}{2}(A + \gamma^0 A^\dagger \gamma^0)$. Importantly, the relation (10) holds in the off-equilibrium state as well. It suggests that we perform the self-energy calculations in the ra basis [39].

As remarked upon earlier, equilibrium self-energy and its off-equilibrium correction will lead to radiative correction to spectral function. Below we derive their contribution to the retarded function based on the following Kadanoff-Baym equation [24]:

$$i\partial_x S_R(x, y) - \int d^4z \Sigma_R(x, z) S_R(z, y) = -\delta(x - y). \tag{11}$$

³To confirm the reality condition at the expectation value level, we still need the Hermitian condition in the density matrix, i.e., the state is invariant under time reversal. We assume this holds for the off-equilibrium state considered in this paper.

Applying the Wigner transform on both sides and performing gradient expansion, we obtain

$$\begin{aligned}
\frac{i}{2} \partial S_R(X, P) + P S_R(X, P) - \left(\Sigma_R(X, P) S_R(X, P) \right. \\
\left. + \frac{i}{2} \{ \Sigma_R(X, P), S_R(X, P) \}_{\text{PB}} \right) = -1,
\end{aligned} \tag{12}$$

where $\{\}_{\text{PB}}$ is the Poisson bracket defined as

$$\{A, B\}_{\text{PB}} = \partial_P A \cdot \partial_X B - \partial_X A \cdot \partial_P B. \tag{13}$$

$S_R = iS_{ra}$, and Σ_R is the retarded self-energy to be expressed in the ra basis later.

Equation (12) is solved in Appendix A in a double expansion of the gradient and size of the self-energy (counted as g^2 from one-loop calculations). Up to $O(g^2 \partial)$, the solution is given by

$$\begin{aligned}
S_R &= S_R^{(0)} + S_R^{(1)} + \dots, \\
S_R^{(0)} &= -\frac{1}{P} - \frac{1}{P} \Sigma_R \frac{1}{P}, \\
S_R^{(1)} &= \gamma^5 \gamma^\beta P^\nu T^{\mu\lambda} \epsilon_{\beta\lambda\mu\nu} \frac{-1}{(P^2)^2},
\end{aligned} \tag{14}$$

where the superscript denotes the order in ∂_X and $T_{\mu\lambda} = \partial_\mu \Sigma_R^\lambda$. The solution has been obtained by assuming Σ_R to be the local equilibrium self-energy. It is parity invariant thus contains the vector component only $\Sigma_R = \gamma_\lambda \Sigma_R^\lambda$. It depends on the coordinate through local temperature and fluid velocity giving rise to nonvanishing $T_{\mu\lambda}$. We will also consider the off-equilibrium correction to self-energy below, which we denote as $\delta\Sigma_R$. It is not difficult to see $\delta\Sigma_R$ leads to the following change to $S_R^{(1)}$:

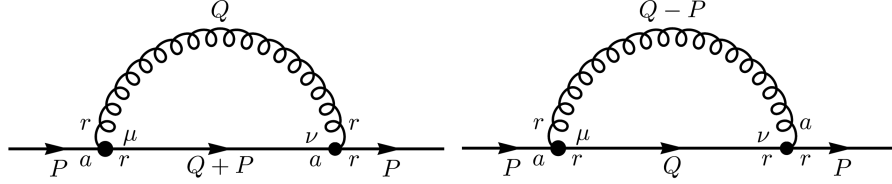
$$S_R^{(1)} = -\frac{1}{P} \delta\Sigma_R \frac{1}{P} + \gamma^5 \gamma^\beta P^\nu T^{\mu\lambda} \epsilon_{\beta\lambda\mu\nu} \frac{-1}{(P^2)^2}. \tag{15}$$

The formal solution cannot be used on the free particle $P^2 = 0$. In vacuum, the $i\epsilon$ prescription is needed to regularize the expression. In the medium, the self-energy Σ_R shifts the dispersion relation by a small amount. In the regime of $P \gg T$ we consider below, the quasiparticle gains a small damping rate. Using $S_R = iS_{ra}$ and (10), we obtain

$$\rho = 2\text{Im}[S_R]. \tag{16}$$

B. Self-energy in equilibrium and off-equilibrium correction

The retarded self-energies Σ_R and $\delta\Sigma_R$ are related to their counterpart in the ra basis as $\Sigma_R = -i\Sigma_{ar}$ and similarly for $\delta\Sigma_R$. We start with the evaluation of the equilibrium one. The diagrams in the ra basis are shown in Fig. 1.

FIG. 1. The retarded self-energies of fermions written in ra basis describing the equilibrium case.

The equilibrium quark and gluon propagators are given by

$$\begin{aligned} S_{ra}(Q) &= \frac{i\cancel{Q}}{(q_0 + i\eta)^2 - \vec{q}^2}, \\ S_{ar}(Q) &= \frac{i\cancel{Q}}{(q_0 - i\eta)^2 - \vec{q}^2}, \\ S_{rr}(Q) &= -2\pi\epsilon(q_0)\delta(Q^2)\cancel{Q}f(q_0), \end{aligned} \quad (17)$$

and

$$\begin{aligned} D_{\mu\nu}^{ra,AB} &= \frac{i\delta_{AB}}{(q_0 + i\eta)^2 - \vec{q}^2} \left(P_{\mu\nu}^T + \frac{Q^2 u_\mu u_\nu}{q^2} \right), \\ D_{\mu\nu}^{ar,AB} &= \frac{i\delta_{AB}}{(q_0 - i\eta)^2 - \vec{q}^2} \left(P_{\mu\nu}^T + \frac{Q^2 u_\mu u_\nu}{q^2} \right), \\ D_{\mu\nu}^{rr,AB} &= 2\pi\delta_{AB}\epsilon(q_0)\delta(Q^2)P_{\mu\nu}^T f(q_0), \end{aligned} \quad (18)$$

respectively. We have used the Coulomb gauge for the gluon propagator with $P_{\mu\nu}^T$ being the transverse projector, which contains a spatial component only, $P_{ij}^T = \delta_{ij} - \hat{q}_i \hat{q}_j$. The vertices in the ra basis are easily derived easily as follows⁴:

$$\begin{aligned} g\bar{\psi}_1\gamma^\mu\psi_1 t_A A_{\mu,1}^A - g\bar{\psi}_2\gamma^\mu\psi_2 t_A A_{\mu,2}^A \\ = g\bar{\psi}_r\gamma^\mu\psi_r t_A A_{\mu,a}^A + g\bar{\psi}_r\gamma^\mu\psi_a t_A A_{\mu,r}^A + g\bar{\psi}_a\gamma^\mu\psi_r t_A A_{\mu,r}^A \\ - \frac{1}{4}g\bar{\psi}_a\gamma^\mu\psi_a t_A A_{\mu,a}^A. \end{aligned} \quad (19)$$

We will consider an energetic quark with $P \gg T$ and $p_0 > 0$. It follows that the right diagram with an on shell quark in the loop is exponentially suppressed. We stress that the regime we consider is opposite to that of a hard thermal loop (HTL), in which both diagrams contribute. The explicit representation of Σ_{ar} is given by

⁴We have absorbed i^2 into the definition of self-energy following [39].

$$\begin{aligned} \frac{\Sigma_{ar}}{g^2 C_F} &= \int_Q \gamma^\nu S^{ra}(P-Q) \gamma^\mu D_{\nu\mu}^{rr}(Q) \\ &= \int_Q \gamma^\nu \frac{i\gamma^\rho (P-Q)_\rho}{(P-Q)^2 + i\epsilon(p_0 - q_0)\eta} \gamma^\mu \left[P_{\mu\nu}^T + \frac{Q^2}{q^2} u_\mu u_\nu \right] \\ &\quad \times 2\pi\epsilon(q_0)\delta(Q^2)f(q_0) \\ &\simeq \int_Q \gamma^\nu \frac{i\gamma^\rho P_\rho}{P^2 - 2P \cdot Q + i\eta} \gamma^\mu P_{\mu\nu}^T 2\pi\epsilon(q_0)\delta(Q^2)f(q_0), \end{aligned} \quad (20)$$

where the Casimir comes from the color sum $C_F = t_A t_A = \frac{N_c^2 - 1}{2N_c}$. For the on shell gluon, we may write $P_{\mu\nu}^T = -\eta_{\mu\nu} + \frac{Q_\mu u_\nu + Q_\nu u_\mu}{q_0} - \frac{Q_\mu Q_\nu}{q_0^2}$ and simplify the product of gamma matrices as

$$\gamma^\nu P \gamma^\mu P_{\mu\nu}^T = 2p_0 \gamma^0 - \frac{2}{q_0^2} q_j \gamma^i q_i q_j. \quad (21)$$

By rotational invariance, the relevant phase space integrals can be parametrized as

$$\begin{aligned} \int_Q \frac{q_i q_j}{P^2 - 2P \cdot Q + i\eta} \frac{f(q_0)}{q_0^2} 2\pi\epsilon(q_0)\delta(Q^2) &= A\delta_{ij} + B\hat{p}_i \hat{p}_j, \\ \int_Q \frac{1}{P^2 - 2P \cdot Q + i\eta} f(q_0) 2\pi\epsilon(q_0)\delta(Q^2) &= C. \end{aligned} \quad (22)$$

Contracting the first integral with δ_{ij} , we easily find $C = 3A + B$. Thus

$$\begin{aligned} \frac{\Sigma_{ar}}{g^2 C_F} &= 2ip_0\gamma^0(3A + B) - 2ip_i\gamma^i(A + B) \\ &= 2i\vec{P}(A + B) + 4ip_0\gamma^0 A. \end{aligned} \quad (23)$$

The coefficients A and B are evaluated in Appendix B close to the mass shell with $P^2 \ll p/\beta$, for which we have

$$\begin{aligned} 3A + B &= \frac{1}{2(2\pi)^2} \frac{\pi(\pi + 2i \ln \frac{p\beta(-1+a)}{2})}{4p\beta}, \\ A + B &= \frac{1}{2(2\pi)^2} \frac{\pi(\pi + 4ia + 2i \ln \frac{p\beta(-1+a)}{2})}{4p\beta}, \end{aligned} \quad (24)$$

with $a = p_0/p + i\eta$. The results are obtained by keeping the leading order terms in expansions in both $a - 1$ and $(p\beta)^{-1}$. It is instructive to compare our results with those in the HTL regime: firstly using (23) and (24), we easily obtain the dispersion $p_0 = p(1 + 8A)$ up to $O(g^2)$ with $A = \frac{1}{2(2\pi)^2} \frac{-i\pi}{2p\beta}$. This indicates a finite damping rate in contrast to a vanishing damping rate in the HTL case at $O(g^2)$; secondly Σ_R is complex for general p_0 in our case while the counterpart in the HTL case is real except for Landau damping cut at spacelike momentum. The origin of the branch cut is also Landau damping, but the resulting branch cut lies at $a < 1$ due to different kinematics. The branch cut appears when both particles in the loop go on shell, i.e., $(P - Q)^2 = Q^2 = 0$, so that $P^2 = 2P \cdot Q$. Note that we have restricted ourselves to $(a - 1)p \ll 1/\beta \sim q$. The condition reduces to $2P \cdot Q \simeq 0$. It occurs at $a = \hat{q} \cdot \hat{p}$, which agrees with $a < 1$.

We then turn to off-equilibrium correction to self-energy. At one loop, the relevant diagrams are shown in Fig. 2,

which are the same as the diagrams in Fig. 1, except with the equilibrium propagators replaced with the off-equilibrium counterpart. Given that we aim at $\delta\Sigma_{ar} \sim O(g^2\partial)$ and the vertices contributing g^2 already, we only need to include $O(\partial)$ correction in the off-equilibrium propagators. These are given by the existing CKT framework. Note that in CKT, the spectral function is not corrected at $O(\partial)$, and by the KMS relation (10), we deduce that $\delta S_{ra} = 0$. This can also be seen from (14) and (15). The same conclusion holds for gluons $\delta D_{ra,AB}^{\mu\nu} = 0$. The only $O(\partial)$ correction appears in S_{rr} and $D_{rr,AB}^{\mu\nu}$. Below we relate S_{rr} to the Wigner function known from CKT as an example.

Let us return to (6). The lesser function is just the Wigner function, and the greater function is deduced as follows: $\rho = S_{21} - S_{12}$. Again the spectral function is an unmodified $\delta\rho = 0$ at $O(\partial)$. We then find $\delta S_{21} = \delta S_{12}$. Furthermore, from the identity $S_{aa} = 0$, we obtain $\delta S_{rr} = \frac{1}{2}(\delta S_{12} + \delta S_{21}) = \delta S_{12}$. We collect the explicit expressions for quark and gluon propagators from CKT [10,40,41],

$$\begin{aligned}\delta S_{rr}(Q) &= -2\pi\epsilon(q_0)\delta(Q^2) \left[\frac{\epsilon^{\mu\nu\alpha\beta} Q_\alpha u_\beta}{2q_0} \partial_\nu \tilde{f} + Q^\mu \frac{Q^\rho \omega_\rho}{q_0} \beta \frac{\partial}{\partial(\beta q_0)} \tilde{f} \right] \gamma^5 \gamma_\mu, \\ \delta D_{\nu\mu}^{rr,AB} &= 2\pi\delta_{AB}\epsilon(q_0)\delta(Q^2) \left[\frac{iP_{\nu\alpha}^T Q^\alpha P_\mu^{T,\beta}}{2q_0^2} \partial_\beta f - (\nu \leftrightarrow \mu) - i \frac{\epsilon_{\nu\mu\alpha\beta} Q^\alpha u^\beta}{q_0^2} Q^\rho \omega_\rho \beta \frac{\partial}{\partial(\beta q_0)} f \right],\end{aligned}\quad (25)$$

where $\omega_\rho = \frac{1}{2}\epsilon_{\rho\nu\alpha\beta} u^\nu \partial^\alpha u^\beta$ is the fluid vorticity.

We are ready to calculate the off-equilibrium correction to self-energy. For the same reason as its equilibrium counterpart, the diagram with on-shell quark in the loop is exponentially suppressed and not kept. The explicit representation of the other diagram in Fig. 2 is given by

$$\begin{aligned}\frac{\delta\Sigma_{ar}}{g^2 C_F} &= \int_Q \gamma^\nu S^{ar}(-P - Q) \gamma^\mu \delta D_{\nu\mu}^{rr}(Q) \\ &= \int_Q \frac{\gamma^\nu i\gamma^\rho (P + Q)_\rho \gamma^\mu}{(P + Q)^2 + i\epsilon(p_0 + q_0)\eta} \left[\frac{iP_{\nu\alpha}^T Q^\alpha P_\mu^{T,\beta}}{2q_0^2} \partial_\beta f - (\nu \leftrightarrow \mu) - i \frac{\epsilon_{\nu\mu\alpha\beta} Q^\alpha u^\beta}{q_0^2} Q^\rho \omega_\rho \beta f' \right] 2\pi\epsilon(q_0)\delta(Q^2) \\ &= \int_Q \frac{\gamma^\nu i\gamma^\rho P_\rho \gamma^\mu}{P^2 + 2P \cdot Q + i\eta} \left[\frac{iP_{\nu\alpha}^T Q^\alpha P_\mu^{T,\beta}}{2q_0^2} \partial_\beta f - (\nu \leftrightarrow \mu) - i \frac{\epsilon_{\nu\mu\alpha\beta} Q^\alpha u^\beta}{q_0^2} Q^\rho \omega_\rho \beta f' \right] 2\pi\epsilon(q_0)\delta(Q^2).\end{aligned}\quad (26)$$

We note that the square bracket is antisymmetric in $\mu\nu$, so that there are only axial components in $\delta\Sigma_{ar}$ by using the identity

$$\gamma^\mu \gamma^\nu \gamma^\rho = \gamma^\mu \eta^{\nu\rho} - \gamma^\nu \eta^{\mu\rho} + \gamma^\rho \eta^{\mu\nu} - i\epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma_\sigma. \quad (27)$$

Explicitly we have

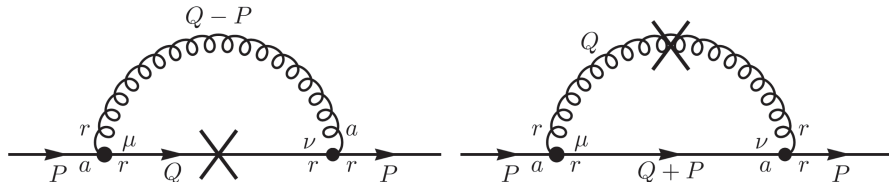


FIG. 2. Self-energy of the fermions with the correction from off equilibrium. The crosses in the diagrams denote the counterparts.

$$\begin{aligned}
\frac{\delta\Sigma_{ar}}{g^2C_F} = & \int_Q \frac{\epsilon^{ijk}\gamma^5\gamma_0 p_j}{P^2 + 2P \cdot Q + i\eta} \left[\frac{iq_i}{q_0^2} (\beta q_l \partial_k u_l + q_0 \partial_k \beta) f' \right] 2\pi\epsilon(q_0)\delta(Q^2) \\
& - \int_Q \frac{2i\gamma^5\gamma_0(-p_l)}{P^2 + 2P \cdot Q + i\eta} \frac{q_l}{q_0^2} (-q_m \omega^m) \beta f' 2\pi\epsilon(q_0)\delta(Q^2) \\
& + \int_Q \frac{-\epsilon^{ijk}\gamma^5\gamma_k p_0}{P^2 + 2P \cdot Q + i\eta} \left[\frac{iq_i}{q_0^2} (\beta q_l \partial_j u_l + q_0 \partial_j \beta) f' \right] 2\pi\epsilon(q_0)\delta(Q^2) \\
& + \int_Q \frac{2\gamma^5\gamma_k p_0}{P^2 + 2P \cdot Q + i\eta} \frac{iq_k}{q_0^2} (-q_m \omega^m) \beta f' 2\pi\epsilon(q_0)\delta(Q^2).
\end{aligned} \tag{28}$$

Following the same method as in the equilibrium case, we parametrize the relevant tensor integrals as

$$\begin{aligned}
\int_Q \frac{q_i q_j}{P^2 + 2P \cdot Q + i\eta} \frac{f'(q_0)}{q_0^2} \epsilon(q_0)\delta(Q^2) &= \delta A \delta_{ij} + \delta B \hat{p}_i \hat{p}_j, \\
\int_Q \frac{q_i}{P^2 + 2P \cdot Q + i\eta} \frac{f'(q_0)}{q_0} \epsilon(q_0)\delta(Q^2) &= \delta C \hat{p}_i.
\end{aligned} \tag{29}$$

In terms of the coefficients, we can express $\delta\Sigma_{ar}$ as

$$\begin{aligned}
\frac{\delta\Sigma_{ar}}{g^2C_F} = & 2i\gamma^5\gamma_0\omega^m p_m \beta (-2\delta A - \delta B) + i\gamma^5\gamma_k \omega^k \beta p (-3\delta A) + i\epsilon^{ijk}\gamma^5\gamma_k \hat{p}_i \hat{p}_l \partial_j u_l \beta p (-\delta B) \\
& + i\epsilon^{ijk}\gamma^5\gamma_k \hat{p}_i \partial_j \beta p (-\delta C) + i\gamma^5\gamma_k \hat{p}_k \hat{p}_m \omega^m \beta p (-2\delta B).
\end{aligned} \tag{30}$$

Using the relation $\partial_j u_l = \sigma_{jl} - \epsilon^{jlk}\omega^k$,⁵ we arrive at

$$\begin{aligned}
\frac{\delta\Sigma_{ar}}{g^2C_F} = & 2i\gamma^5\gamma_0\omega^m p_m \beta (-2\delta A - \delta B) + i\gamma^5\gamma_k \omega^k \beta p (-4\delta A - \delta B) + i\epsilon^{ijk}\gamma^5\gamma_k \hat{p}_i \hat{p}_l \sigma_{jl} \beta p (-\delta B) \\
& + i\epsilon^{ijk}\gamma^5\gamma_k p_i \partial_j \beta (-\delta C) + i\gamma^5\gamma_k \hat{p}_k p_m \omega^m \beta (-\delta B) \\
= & 2i\gamma^5\gamma_0\omega^m p_m \beta (-2\delta A - \delta B) + i\gamma^5\gamma_k \omega_{\parallel}^k \beta p (-4\delta A - 2\delta B) + i\gamma^5\gamma_k \omega_{\perp}^k \beta p (-4\delta A - \delta B) \\
& + i\epsilon^{ijk}\gamma^5\gamma_k \hat{p}_i \hat{p}_l \sigma_{jl} \beta p (-\delta B) + i\epsilon^{ijk}\gamma^5\gamma_k p_i \partial_j \beta (-\delta C).
\end{aligned} \tag{31}$$

In the last step, we have defined $\omega_{\parallel}^k = \omega^i \hat{p}_i \hat{p}^k$, $\omega_{\perp}^k = \omega^k - \omega_{\parallel}^k$. $\delta\Sigma_{ar}$ contains axial components only and can be decomposed as

$$\begin{aligned}
\delta\Sigma_{ar} = & \gamma^5\gamma^\mu \mathcal{A}_\mu, \\
\mathcal{A}^0 = & i\omega^i p_i \beta (-4\delta A - 2\delta B), \\
\mathcal{A}^k = & i\omega_{\parallel}^k \beta p (-4\delta A - 2\delta B) + \omega_{\perp}^k \beta p (-4\delta A - \delta B) + \epsilon^{ijk} \hat{p}_i \hat{p}_l \sigma_{jl} \beta p (-\delta B) + \epsilon^{ijk} p_i \partial_j \beta (-\delta C).
\end{aligned} \tag{32}$$

A notable property is that $P_\mu \mathcal{A}^\mu = 0$. The coefficients A , B , and C have been evaluated in Appendix B with the following results to leading order in T/p :

⁵We have left out a possible contribution from acceleration and $\partial \cdot u$. The former is not allowed in our approach, and the latter does not couple to spin.

$$\begin{aligned}
3\delta A + \delta B &= \frac{1}{4(2\pi)^2} \left(-\frac{2i\pi}{(a^2-1)p^2\beta^2} + \frac{2C_b + i\pi C_a - 2C_a \ln \frac{p\beta(-1+a)}{2}}{2p\beta} \right), \\
\delta A + \delta B &= \frac{1}{4(2\pi)^2} \left(-\frac{2i\pi}{3(a^2-1)p^2\beta^2} + \frac{-4C_a + 2C_b + i\pi C_a - 2C_a \ln \frac{p\beta(-1+a)}{2}}{2p\beta} \right), \\
\delta C &= \frac{1}{4(2\pi)^2} \left(\frac{-4C_a + 2C_b + i\pi C_a - 2C_a \ln \frac{p\beta(-1+a)}{2}}{2p\beta} \right).
\end{aligned} \tag{33}$$

Now we are ready to plug (23) and (32) into (14). We first look at the first term of (14), which can be simplified using $P \cdot \mathcal{A} = 0$ as

$$\delta S_R^{(0)} = -\frac{1}{p} \delta \Sigma_R \frac{1}{p} = \frac{1}{p^2} \delta \Sigma_R. \tag{34}$$

To compare with known contribution in free theory, which is localized at $p_0 = p$, we integrate over p_0 . Since the explicit results (33) have been obtained assuming $(a-1)p \ll 1/\beta$, we integrate over $-\Lambda + p, \Lambda + p$ with $\Lambda \lesssim 1/\beta$ to obtain the following integrals needed for our purpose:

$$\begin{aligned}
\int_{p-\Lambda}^{p+\Lambda} dp_0 \frac{1}{p^2} \ln \frac{p_0 - p + i\eta}{p} &\simeq \int_{p-\Lambda}^{p+\Lambda} dp_0 \frac{1}{2p} \frac{1}{p_0 - p + i\eta} \ln \frac{p_0 - p + i\eta}{p} = \frac{1}{2p} \left(\frac{\pi^2}{2} + i\pi \ln(p\beta) \right), \\
\int_{p-\Lambda}^{p+\Lambda} dp_0 \frac{1}{p^2} &\simeq \int_{p-\Lambda}^{p+\Lambda} dp_0 \frac{1}{2p} \frac{1}{p_0 - p + i\eta} = \frac{1}{2p} (-i\pi), \\
\int_{p-\Lambda}^{p+\Lambda} dp_0 \frac{i}{(p^2)^2} &\simeq \int_{p-\Lambda}^{p+\Lambda} dp_0 \frac{1}{(2p)^2} \frac{i}{(p_0 - p + i\eta)^2} = \frac{1}{(2p)^2} \left(\frac{2}{\Lambda} \right).
\end{aligned} \tag{35}$$

It is worth noting that the first two integrals are independent of Λ , which indicates a localized contribution. On the contrary the third integral does depend on Λ . Interestingly, the integrand of the latter is actually exact [see discussion below (B12)]. This allows us to set $\Lambda \sim p \gg 1/\beta$, leading to a result that is suppressed by $p\beta$ compared to other contributions. We drop the corresponding contribution below. Using $\delta \Sigma_R = -i\delta \Sigma_{ar}$ and (35), we find terms logarithmic in $p\beta$ cancel and finally arrive at

$$\begin{aligned}
\int dp_0 \delta S_R^{(0)} &= \gamma^5 \gamma^\mu \mathcal{B}_\mu, \\
\text{Im}[\mathcal{B}^0] &= \frac{g^2 C_F}{4(2\pi)^2} \omega^i \hat{p}_i \frac{\pi}{2p} (2C_b + (-2 + \ln 4)C_a), \\
\text{Im}[\mathcal{B}^k] &= \frac{g^2 C_F}{4(2\pi)^2} \frac{\pi}{2p} \left[\omega_\parallel^k (2C_b + (-2 + \ln 4)C_a) + \frac{2C_b + (2 + \ln 4)C_a}{2} \omega_\perp^k \right. \\
&\quad \left. + \frac{2C_b + (-6 + \ln 4)C_a}{2} \epsilon^{ijk} \hat{p}_i \hat{p}_l \sigma_{jl} + \frac{2C_b + (-4 + \ln 4)C_a}{2} \epsilon^{ijk} \hat{p}_i \frac{\partial_j \beta}{\beta} \right].
\end{aligned} \tag{36}$$

We then turn to the second term of (15), which is written explicitly as

$$S_R^{(1)} = \gamma^5 \gamma_\beta P_\nu \partial_\mu \Sigma_\lambda^R \frac{-1}{(p^2)^2} \epsilon^{\beta\lambda\mu\nu}, \tag{37}$$

with

$$\Sigma_R^\lambda = (\Sigma_R^0, \Sigma_R^i) = \Sigma_R^i \hat{p}_i \frac{1}{p} P^\lambda + \Delta \Sigma u^\lambda, \tag{38}$$

with $\Delta \Sigma = \Sigma_R^0 - \Sigma_R^i \hat{p}_i \frac{p_0}{p}$. Note that the first term in (38) does not contribute to (37), and the second term gives

$$S_R^{(1)} = \gamma^5 \gamma_\beta P_\nu \partial_\mu (\Delta \Sigma u_\lambda) \frac{-1}{(P^2)^2} \epsilon^{\beta\lambda\mu\nu}. \quad (39)$$

The coordinate dependence of $\Delta \Sigma$ comes from local fluid velocity and temperature. The former determines components of momentum as $p_0 = P \cdot u$ and $p = (p_0^2 - P^2)^{1/2}$. Thus

$$\begin{aligned} \partial_\mu \Delta \Sigma &= \frac{\partial \Delta \Sigma}{\partial p_0} \partial_\mu p_0 + \frac{\partial \Delta \Sigma}{\partial p} \partial_\mu p + \frac{\partial \Delta \Sigma}{\partial \beta} \partial_\mu \beta \\ &= \left(\frac{\partial \Delta \Sigma}{\partial p_0} + \frac{\partial \Delta \Sigma}{\partial p} \frac{p_0}{p} \right) \partial_\mu p_0 - \Delta \Sigma \frac{\partial_\mu \beta}{\beta} \\ &= \Delta \Sigma_p^\partial P^\alpha \partial_\mu u_\alpha - \Delta \Sigma \frac{\partial_\mu \beta}{\beta}, \end{aligned} \quad (40)$$

where in the second line we have used the fact that $\Delta \Sigma \propto \beta^{-1}$ based on (24). Specializing to the static off-equilibrium state, we can write out $S_R^{(1)}$ explicitly as

$$\begin{aligned} S_R^{(1)} &= \left[2\gamma^5 (\gamma_0 p_i + \gamma_i p_0) \Delta \Sigma \omega^i \right. \\ &\quad + \gamma^5 \gamma_i \Delta \Sigma_p^\partial (p_l p_j \sigma_{kl} \epsilon^{ijk} - p_i p_m \omega^m + p^2 \omega^i) \\ &\quad \left. - \gamma^5 \gamma_i p_j \frac{\partial_k \beta}{\beta} \Delta \Sigma \epsilon^{ijk} \right] \frac{1}{(P^2)^2}. \end{aligned} \quad (41)$$

The explicit expressions for $\Delta \Sigma$ and $\Delta \Sigma_p^\partial$ are worked out as

$$\begin{aligned} \Delta \Sigma &= \frac{2a}{(2\pi)^2 \beta} \left(-\frac{i\pi}{2} \right), \\ \Delta \Sigma_p^\partial &= \frac{1}{p} \frac{\partial \Delta \Sigma}{\partial a} (1 - a^2) = \frac{2(1 - a^2)}{(2\pi)^2 p \beta} \left(-\frac{i\pi}{2} \right). \end{aligned} \quad (42)$$

An important property is that both $\Delta \Sigma$ and $\Delta \Sigma_p^\partial$ are regular as $a \rightarrow 1$, so that here we can simply use $a = p_0/p$ to proceed. The following integrals are needed in the evaluation of $S_R^{(1)}$:

$$\begin{aligned} \int_{p-\Lambda}^{p+\Lambda} dp_0 \frac{a^2 - 1}{(P^2)^2} &\simeq \frac{1}{p^2} \int_{p-\Lambda}^{p+\Lambda} dp_0 \frac{1}{P^2} = \frac{1}{p^2} \frac{1}{2p} (-i\pi), \\ \int_{p-\Lambda}^{p+\Lambda} dp_0 \frac{a}{(P^2)^2} &\simeq \frac{1}{p} \int_{p-\Lambda}^{p+\Lambda} dp_0 \frac{p_0}{(P^2)^2} \sim \frac{1}{p^2 \Lambda}. \end{aligned} \quad (43)$$

In the last line we have again set $\Lambda \sim p$ because $\Delta \Sigma$ is exact like the off-equilibrium counterpart. Using (41) and the integrals above, we find the corresponding contribution to $\int dp_0 S_R^{(1)}$ is parametrically suppressed compared to (36). The reason can be traced in different temperature dependence of the equilibrium and off-equilibrium self-energies: the former is linear in temperature while the latter is dominated by a temperature independent contribution in

the regime $p\beta \gg 1$ we choose to work. It follows that the equilibrium contribution to the spectral function is suppressed.

In summary, the radiative correction to the spectral function is given by

$$\begin{aligned} \int dp_0 \delta \rho(P) &= 2 \text{Im} [\delta S_R^{(0)} + S_R^{(1)}] \\ &= \frac{g^2 C_F}{2(2\pi)^2} \frac{\pi}{2p} \gamma^5 \gamma_i \left[\omega_\parallel^i (2C_b + (-2 + \ln 4)C_a) \right. \\ &\quad + \frac{2C_b + (2 + \ln 4)C_a}{2} \omega_\perp^i \\ &\quad + \frac{2C_b + (-6 + \ln 4)C_a}{2} \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl} \\ &\quad \left. + \frac{2C_b + (-4 + \ln 4)C_a}{2} \epsilon^{ijk} \hat{p}_j \frac{\partial_k \beta}{\beta} \right]. \end{aligned} \quad (44)$$

The fact that the spectral function is modified by a hydrodynamic gradient indicates that the particle itself is polarized. Using the equilibrium KMS relation, we obtain the following contribution to the spin polarization:

$$\begin{aligned} \int dp_0 \delta S^<(P) &= \int dp_0 \delta \rho(P) f(p_0) \\ &= \frac{g^2 C_F}{2(2\pi)^2} \frac{\pi}{2p} \gamma^5 \gamma_i \left[0.95 \omega_\parallel^i + 1.48 \omega_\perp^i \right. \\ &\quad \left. - 0.52 \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl} - 0.02 \epsilon^{ijk} \hat{p}_j \frac{\partial_k \beta}{\beta} \right] \tilde{f}(p), \end{aligned} \quad (45)$$

where we have used $C_a = 0.5$ and $C_b \simeq 0.63$ from Appendix B. This is to be compared with the tree-level contribution from the same sources (3),

$$\begin{aligned} \int dp_0 S_{(0)}^<(P) &= -\gamma^5 \gamma_i \frac{2\pi}{2} \left(\omega^i + \epsilon^{ijk} \hat{p}_k \frac{\partial_j \beta}{\beta} + \epsilon^{ijk} \hat{p}_k \hat{p}_l \sigma_{lj} \right) \\ &\quad \times \beta \frac{\partial}{\partial(\beta p)} \tilde{f}(p) \\ &\simeq \gamma^5 \gamma_i \frac{2\pi}{2} \left(\omega^i + \epsilon^{ijk} \hat{p}_k \frac{\partial_j \beta}{\beta} + \epsilon^{ijk} \hat{p}_k \hat{p}_l \sigma_{lj} \right) \\ &\quad \times \beta \tilde{f}(p), \end{aligned} \quad (46)$$

where we have used $\frac{\partial}{\partial(\beta p)} \tilde{f}(p) \simeq -\tilde{f}(p)$ in the regime $p\beta \gg 1$.

III. CONCLUSION AND OUTLOOK

We have classified the general off-equilibrium contribution to spin polarization into three types including modifications to the (i) spectral function, (ii) distribution function, and (iii) KMS relation. The last two types exist already in free theory, while the first type is visible with radiative correction only. As an explicit example, we have

calculated radiative correction to quark spectral function in an off-equilibrium medium characterized by hydrodynamic gradients. The correction at $O(g^2\partial)$ arises from both local equilibrium self-energy at $O(g^2)$ combined with gradient correction to the resummation equation and off-equilibrium self-energy correction at $O(g^2\partial)$. For an energetic quark in the medium, we have found the former contribution to be parametrically suppressed. The latter contribution leads to a quasiparticle with a polarized spectral function.

Clearly, a complete contribution to spin polarization also involves a radiative correction to type (ii) and (iii). Correction of type (ii) comes from spin-vorticity coupling only. Correction of type (iii) would involve gradient correction to the Kadanoff-Baym equation. They have been worked out recently in the hard thermal loop approximation in [31,36]. Generically, these contributions come from local equilibrium self-energy Σ . We expect that they lead to parametrically similar contributions to the subdominant contribution to type (i) from $S_R^{(1)}$. It implies the dominant contribution from off-equilibrium correction to self-energy might remain unchanged. We leave more complete studies for future exploration.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: SOLVING THE KADANOFF-BAYM EQUATION

In this appendix, we solve (12) order by order in gradient

$$S_R = S_R^{(0)} + S_R^{(1)} + \dots, \quad (\text{A1})$$

with the superscript denoting the order. Each order solution is further expanded in self-energy. $S_R^{(0)}$ is solved as

$$P S_R^{(0)} - \Sigma_R S_R^{(0)} = -1 \Rightarrow S_R^{(0)} = -\frac{1}{P} - \frac{1}{P} \Sigma_R \frac{1}{P} + \dots \quad (\text{A2})$$

$S_R^{(1)}$ satisfies the following equation:

$$P S_R^{(1)} - \Sigma_R S_R^{(1)} = -\frac{i}{2} \partial S_R^{(0)} + \frac{i}{2} \{\Sigma_R, S_R^{(0)}\}. \quad (\text{A3})$$

We shall find the solution of $S_R^{(1)}$ only to the lowest order in Σ_R , which is counted as $O(g^2)$. To this end, we may drop the self-energy term on the lhs and keep terms up to $O(g^2\partial)$ on the rhs. We simplify the rhs as

$$\begin{aligned} \frac{i}{2} \partial \left(\frac{1}{P} \Sigma_R \frac{1}{P} \right) - \frac{i}{2} \partial \Sigma_R \cdot \partial_P \left(\frac{-1}{P} \right) &= \frac{i}{2} \partial \Sigma_R \frac{1}{P^2} - \frac{i}{2} \partial \left[\frac{1}{P}, \Sigma_R \right] \frac{1}{P} + \frac{i}{2} \Sigma_R \overleftarrow{\partial} - i \partial_\lambda \Sigma_R P \frac{P^\lambda}{(P^2)^2} \\ &= i \partial \cdot \Sigma_R \frac{1}{P^2} + i \partial \Sigma_\lambda^R (P P^\lambda - \gamma^\lambda P^2) \frac{1}{(P^2)^2} - i \partial_\lambda \Sigma_R P \frac{P^\lambda}{(P^2)^2} \\ &= -\sigma^{\mu\lambda} \partial_\mu \Sigma_\lambda^R \frac{1}{P^2} + i \gamma^\mu (\partial_\mu \Sigma_\lambda^R (\lambda \leftrightarrow \mu)) P \frac{P^\lambda}{(P^2)^2} \\ &= -\sigma^{\mu\nu} T_{\mu\nu} \frac{1}{P^2} + 2 \sigma^{\mu\nu} P_\nu T_{\mu\lambda} \frac{P^\lambda}{(P^2)^2}, \end{aligned} \quad (\text{A4})$$

where we have used the product $\gamma^\mu \gamma^\nu = \eta^{\mu\nu} - i \sigma^{\mu\nu}$ and defined $T_{\mu\lambda} = \partial_{[\mu} \Sigma_{\lambda]}^R$.

Motivated by the solution in CKT without radiative correction, we solve (A3) with the following ansatz: $S_R^{(1)} = \gamma^5 \gamma^\nu \mathcal{A}_\nu$, using the relation

$$\begin{aligned} P \gamma^5 \gamma^\nu \mathcal{A}_\nu &= -\gamma^5 P \cdot \mathcal{A} + i \gamma^5 \sigma^{\mu\nu} P_\mu \mathcal{A}_\nu \\ &= -\gamma^5 P \cdot \mathcal{A} - \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \sigma_{\alpha\beta} P_\mu \mathcal{A}_\nu. \end{aligned} \quad (\text{A5})$$

We proceed by dropping the $P \cdot \mathcal{A}$ term above, whose validity will be verified below. Extracting coefficients of $\sigma^{\mu\nu}$ from (A3), we obtain

$$\begin{aligned} P_\alpha \mathcal{A}_\beta - (\alpha \leftrightarrow \beta) &= \left(-T^{\mu\nu} \frac{1}{P^2} + 2 P^\nu T^{\mu\lambda} \frac{P_\lambda}{(P^2)^2} \right) \epsilon_{\mu\nu\alpha\beta} \\ &= -T^{\mu\nu} \frac{1}{P^2} \epsilon_{\mu\nu\alpha\beta} \\ &\quad + 2 P^\nu T^{\mu\lambda} \frac{-1}{(P^2)^2} (P_\nu \epsilon_{\alpha\beta\lambda\mu} + P_\alpha \epsilon_{\beta\lambda\mu\nu} + P_\beta \epsilon_{\lambda\mu\nu\alpha}) \\ &= P^\nu T^{\mu\lambda} \frac{-1}{(P^2)^2} (P_\alpha \epsilon_{\beta\lambda\mu\nu} + P_\beta \epsilon_{\lambda\mu\nu\alpha}). \end{aligned} \quad (\text{A6})$$

It is solved by $\mathcal{A}_\beta = P^\nu T^{\mu\lambda} \frac{-1}{(P^2)^2} \epsilon_{\beta\lambda\mu\nu}$. It indeed satisfies $P \cdot \mathcal{A} = 0$, justifying dropping the corresponding term above.

APPENDIX B: EVALUATION OF MOMENTUM INTEGRALS

We first consider the following tensor integral for equilibrium self-energy:

$$\int_Q \frac{q_i q_j}{P^2 - 2P \cdot Q + i\eta} \frac{f(\beta q_0)}{q_0^2} 2\pi\epsilon(q_0) \delta(Q^2) = A\delta_{ij} + B\hat{p}_i \hat{p}_j. \quad (\text{B1})$$

Contracting the above with δ^{ij} and $\hat{p}_i \hat{p}_j$ respectively, we find

$$\begin{aligned} 3A + B &= \int_Q \frac{q^2}{P^2 - 2P \cdot Q + i\eta} \frac{f(\beta q_0)}{q_0^2} 2\pi\epsilon(q_0) \delta(Q^2), \\ A + B &= \int_Q \frac{(q \cdot \hat{p})^2}{P^2 - 2P \cdot Q + i\eta} \frac{f(\beta q_0)}{q_0^2} 2\pi\epsilon(q_0) \delta(Q^2). \end{aligned} \quad (\text{B2})$$

We shall illustrate the evaluation of the first integral. The same method applies to the second integral as well. We first perform the integration of q_0 using $\delta(Q^2)$ to obtain

$$\begin{aligned} \int \frac{d^3 q}{(2\pi)^3} &\left(\frac{1}{P^2 - 2(p_0 q - \vec{p} \cdot \vec{q}) + i\eta} \frac{f(\beta q)}{2q} \right. \\ &\left. - \frac{1}{P^2 - 2(-p_0 q - \vec{p} \cdot \vec{q}) + i\eta} \frac{f(-\beta q)}{2q} \right). \end{aligned} \quad (\text{B3})$$

We use $f(-\beta q) = -1 - f(\beta q)$ and note that the -1 gives rise to vacuum self-energy, which will be eliminated by spacetime derivatives, so that we can drop the -1 . We then perform angular integration to obtain

$$\begin{aligned} \int \frac{dq}{2(2\pi)^2} \frac{1}{2pq} &\left(\ln \frac{p^2(a^2 - 1) - 2pq(a - 1)}{p^2(a^2 - 1) - 2pq(a + 1)} \right. \\ &\left. + \ln \frac{p^2(a^2 - 1) + 2pq(a + 1)}{p^2(a^2 - 1) + 2pq(a - 1)} \right) \frac{f(\beta q)}{q}, \end{aligned} \quad (\text{B4})$$

with $a = p_0/p$ and $i\eta$ replaced by an infinitesimal positive imaginary part of a . The q integration cannot be performed analytically. To proceed, we restrict ourselves to the regime $(a - 1)p \ll 1/\beta \ll p$, or equivalently and $P^2 \ll p/\beta \ll p^2$, that is an energetic particle close to the mass shell. This allows us to split the integration domain into two parts separated by a floating point q^* , with $(a - 1)p \ll q^* \ll 1/\beta$. For $0 < q < q^*$, we use $f(q) \simeq \frac{1}{\beta q}$ to have

$$\begin{aligned} \int_0^{q^*} dq \frac{1}{2pq} &\left(\ln \frac{P^2 - 2pq(a - 1) + i\eta}{P^2 - 2(a + 1)pq + i\eta} \right. \\ &\left. + \ln \frac{P^2 + 2pq(a + 1) + i\eta}{P^2 + 2(a - 1)pq + i\eta} \right) \frac{q^2}{-q\beta q} \\ &\simeq \frac{1}{4p\beta} \left(\pi^2 + 2i\pi \ln \frac{(-1 + a)p}{2q^*} \right). \end{aligned} \quad (\text{B5})$$

The lhs can be integrated analytically with a lengthy expression not shown here. The rhs is obtained by keeping the leading term in the expansion in $a(p - 1)/q^*$ and then keeping the leading term in the expansion in q^*/p . For $q^* < q < \infty$, we apply the same approximation to the integrand of (B4) excluding the $f(\beta q)$ factor to obtain

$$\begin{aligned} \frac{q^2}{2pq} &\left(\ln \frac{p^2(a^2 - 1) - 2pq(a - 1)}{p^2(a^2 - 1) - 2pq(a + 1)} \right. \\ &\left. + \ln \frac{p^2(a^2 - 1) + 2pq(a + 1)}{p^2(a^2 - 1) + 2pq(a - 1)} \right) \frac{1}{2q} \\ &\simeq \frac{\ln(1 - a) - \ln(-1 + a)}{2p} = -\frac{i\pi}{2p}. \end{aligned} \quad (\text{B6})$$

An infinitesimal positive imaginary part of a has been invoked in the last equality. We then convolute the above with $f(q)$ and expand in βq^* to obtain

$$-\frac{i\pi}{2p} \int_{q^*}^{\infty} f(\beta q) \simeq \frac{i\pi \ln(q^* \beta)}{2p \beta}. \quad (\text{B7})$$

Adding up (B5) and (B7), we find the dependencies on the floating scale q^* cancel to give

$$3A + B = \frac{1}{2(2\pi)^2} \frac{\pi(\pi + 2i \ln \frac{p\beta(-1+a)}{2})}{4p\beta}. \quad (\text{B8})$$

Repeating the same procedure as above, we obtain for the second integral

$$A + B = \frac{1}{2(2\pi)^2} \frac{\pi(\pi + 4i + 2i \ln \frac{p\beta(-1+a)}{2})}{4p\beta}. \quad (\text{B9})$$

Now we turn to integrals for off-equilibrium self-energy. Contracting the first tensor integral of (29) by δ^{ij} and $\hat{p}_i \hat{p}_j$ and contracting the second vector integral by $\hat{p}_i$, we obtain

$$\begin{aligned} 3\delta A + \delta B &= \int_Q \frac{q^2}{P^2 + 2P \cdot Q i\eta} \frac{f'(\beta q_0)}{2q_0^2} \epsilon(q_0) \delta(Q^2), \\ \delta A + \delta B &= \int_Q \frac{(\dot{q}p)^2}{P^2 + 2P \cdot Q i\eta} \frac{f'(\beta q_0)}{2q_0^2} \epsilon(q_0) \delta(Q^2), \\ \delta C &= \int_Q \frac{\dot{q}p}{P^2 + 2P \cdot Q i\eta} \frac{f'(\beta q_0)}{2q_0^2} \epsilon(q_0) \delta(Q^2). \end{aligned} \quad (\text{B10})$$

We shall illustrate the evaluation of the first integral. As before, we integrate over q_0 using the Dirac delta function and then integrate over the angular variables to obtain

$$\frac{1}{2pq} \left(-\ln \frac{p^2(a^2-1) - 2pq(a-1)}{p^2(a^2-1) - 2pq(a+1)} + \ln \frac{p^2(a^2-1) + 2pq(a+1)}{p^2(a^2-1) + 2pq(a-1)} \right) q f'(q). \quad (\text{B11})$$

We have used $f'(-\beta q) = f'(\beta q)$. Note that there is no vacuum contribution in off-equilibrium self-energy. Following the equilibrium case, we split the integration domain into two parts with a floating scale q^* . The part below q^* is obtained by the approximation $f'(\beta q) \simeq -\frac{1}{(\beta q)^2}$ as

$$\begin{aligned} & \int_0^{q^*} dq \frac{1}{2pq} \left(-\ln \frac{p^2(a^2-1) - 2pq(a-1)}{p^2(a^2-1) - 2pq(a+1)} + \ln \frac{p^2(a^2-1) + 2pq(a+1)}{p^2(a^2-1) + 2pq(a-1)} \right) q \left(-\frac{1}{(\beta q)^2} \right) \\ &= F(q^*) - F(0) \\ &\simeq -\frac{2i\pi}{(a^2-1)p^2\beta^2} + \frac{2 + i\pi - 2\ln \frac{p(-1+a)}{2q^*}}{2pq^*\beta^2}, \end{aligned} \quad (\text{B12})$$

where F is the integrated function. $F(0) = \frac{2i\pi}{(a^2-1)p^2\beta^2}$ is exact. $F(q^*)$ is truncated to leading order expansions in $(a-1)$ and q^*/p . The part above q^* is evaluated using the following approximation for the integrand excluding the $f'(\beta q)$ factor:

$$\begin{aligned} & \frac{1}{2pq} \left(-\ln \frac{p^2(a^2-1) - 2pq(a-1)}{p^2(a^2-1) - 2pq(a+1)} + \ln \frac{p^2(a^2-1) + 2pq(a+1)}{p^2(a^2-1) + 2pq(a-1)} \right) q \\ &\simeq \frac{i\pi + 2\ln \frac{2q}{p(-1+a)}}{2p}. \end{aligned} \quad (\text{B13})$$

The convolution with $f'(\beta q)$ is performed as follows:

$$\begin{aligned} & \int_{q^*}^{\infty} dq \frac{i\pi + 2\ln \frac{2q}{p(-1+a)}}{2p} f'(\beta q) \\ &= \frac{i\pi + 2\ln \frac{2}{p\beta(-1+a)}}{2p\beta} \int_{x_0}^{\infty} dx f'(x) + \frac{1}{p\beta} \int_{x_0}^{\infty} f'(x) \ln x, \end{aligned} \quad (\text{B14})$$

where $x = q\beta$ and $x_0 = q^*\beta$. Since $x_0 \ll 1$, the x integrals can be expanded as

$$\begin{aligned} & \int_{x_0}^{\infty} dx f'(x) = -\frac{1}{x_0} + C_a, \\ & \int_{x_0}^{\infty} dx f'(x) \ln x = -\frac{1}{x_0} (1 + \ln x_0) + C_b, \end{aligned} \quad (\text{B15})$$

with $C_a = 0.5$ and $C_b \simeq 0.630$, which are calculated numerically. Combining (B12) and (B14), we again find the expected cancellation on the dependencies on q^* :

$$\begin{aligned} 3\delta A + \delta B &= \frac{1}{4(2\pi)^2} \left(-\frac{2i\pi}{(a^2-1)p^2\beta^2} + \frac{2C_b + i\pi C_a - 2C_a \ln \frac{p\beta(-1+a)}{2}}{2p\beta} \right). \end{aligned} \quad (\text{B16})$$

The other two integrals can be calculated similarly to give

$$\begin{aligned} \delta A + \delta B &= \frac{1}{4(2\pi)^2} \left(-\frac{2i\pi}{3(a^2-1)p^2\beta^2} + \frac{-4C_a + 2C_b + i\pi C_a - 2C_a \ln \frac{p\beta(-1+a)}{2}}{2p\beta} \right), \\ \delta C &= \frac{1}{4(2\pi)^2} \left(\frac{-4C_a + 2C_b + i\pi C_a - 2C_a \ln \frac{p\beta(-1+a)}{2}}{2p\beta} \right). \end{aligned} \quad (\text{B17})$$

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- [1] Zuo-Tang Liang and Xin-Nian Wang, Globally polarized quark-gluon plasma in non-central A + A collisions, *Phys. Rev. Lett.* **94**, 102301 (2005); **96**, 039901(E) (2006).
[2] Zuo-Tang Liang and Xin-Nian Wang, Spin alignment of vector mesons in non-central A + A collisions, *Phys. Lett. B* **629**, 20 (2005).

- [3] L. Adamczyk *et al.*, Global Λ hyperon polarization in nuclear collisions: Evidence for the most vortical fluid, *Nature (London)* **548**, 62 (2017).
[4] F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, Relativistic distribution function for particles with spin at local thermodynamical equilibrium, *Ann. Phys. (Amsterdam)* **338**, 32 (2013).

- [5] Ren-hong Fang, Long-gang Pang, Qun Wang, and Xin-nian Wang, Polarization of massive fermions in a vortical fluid, *Phys. Rev. C* **94**, 024904 (2016).
- [6] Hui Li, Long-Gang Pang, Qun Wang, and Xiao-Liang Xia, Global Λ polarization in heavy-ion collisions from a transport model, *Phys. Rev. C* **96**, 054908 (2017).
- [7] De-Xian Wei, Wei-Tian Deng, and Xu-Guang Huang, Thermal vorticity and spin polarization in heavy-ion collisions, *Phys. Rev. C* **99**, 014905 (2019).
- [8] Shuai Y.F. Liu, Yifeng Sun, and Che Ming Ko, Spin polarizations in a covariant angular-momentum-conserved chiral transport model, *Phys. Rev. Lett.* **125**, 062301 (2020).
- [9] F. Becattini and Iu. Karpenko, Collective longitudinal polarization in relativistic heavy-ion collisions at very high energy, *Phys. Rev. Lett.* **120**, 012302 (2018).
- [10] Yoshimasa Hidaka, Shi Pu, and Di-Lun Yang, Nonlinear responses of chiral fluids from kinetic theory, *Phys. Rev. D* **97**, 016004 (2018).
- [11] Shuai Y.F. Liu and Yi Yin, Spin polarization induced by the hydrodynamic gradients, *J. High Energy Phys.* **07** (2021) 188.
- [12] F. Becattini, M. Buzzegoli, and A. Palermo, Spin-thermal shear coupling in a relativistic fluid, *Phys. Lett. B* **820**, 136519 (2021).
- [13] Baochi Fu, Shuai Y.F. Liu, Longgang Pang, Huichao Song, and Yi Yin, Shear-induced spin polarization in heavy-ion collisions, *Phys. Rev. Lett.* **127**, 142301 (2021).
- [14] F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, Local polarization and isothermal local equilibrium in relativistic heavy ion collisions, *Phys. Rev. Lett.* **127**, 272302 (2021).
- [15] Cong Yi, Shi Pu, and Di-Lun Yang, Reexamination of local spin polarization beyond global equilibrium in relativistic heavy ion collisions, *Phys. Rev. C* **104**, 064901 (2021).
- [16] Feng Li and Shuai Y. F. Liu, Tensor polarization and spectral properties of vector meson in QCD medium, *arXiv:2206.11890*.
- [17] David Wagner, Nora Weickgenannt, and Enrico Speranza, Generating tensor polarization from shear stress, *Phys. Rev. Res.* **5**, 013187 (2023).
- [18] Xin-Li Sheng, Lucia Oliva, Zuo-Tang Liang, Qun Wang, and Xin-Nian Wang, Spin alignment of vector mesons in heavy-ion collisions, *Phys. Rev. Lett.* **131**, 042304 (2023).
- [19] Xin-Li Sheng, Lucia Oliva, Zuo-Tang Liang, Qun Wang, and Xin-Nian Wang, Relativistic spin dynamics for vector mesons, *Phys. Rev. D* **109**, 036004 (2024).
- [20] Berndt Müller and Di-Lun Yang, Anomalous spin polarization from turbulent color fields, *Phys. Rev. D* **105**, L011901 (2022); **106**, 039904(E) (2022).
- [21] Jian-Hua Gao, Helicity polarization in relativistic heavy ion collisions, *Phys. Rev. D* **104**, 076016 (2021).
- [22] Yoshimasa Hidaka, Shi Pu, Qun Wang, and Di-Lun Yang, Foundations and applications of quantum kinetic theory, *Prog. Part. Nucl. Phys.* **127**, 103989 (2022).
- [23] Yoshimasa Hidaka, Shi Pu, and Di-Lun Yang, Relativistic chiral kinetic theory from quantum field theories, *Phys. Rev. D* **95**, 091901 (2017).
- [24] Shu Lin, Quantum kinetic theory for quantum electrodynamics, *Phys. Rev. D* **105**, 076017 (2022).
- [25] Di-Lun Yang, Koichi Hattori, and Yoshimasa Hidaka, Effective quantum kinetic theory for spin transport of fermions with collisional effects, *J. High Energy Phys.* **07** (2020) 070.
- [26] Nora Weickgenannt, Enrico Speranza, Xin-li Sheng, Qun Wang, and Dirk H. Rischke, Derivation of the nonlocal collision term in the relativistic Boltzmann equation for massive spin-1/2 particles from quantum field theory, *Phys. Rev. D* **104**, 016022 (2021).
- [27] Xin-Li Sheng, Nora Weickgenannt, Enrico Speranza, Dirk H. Rischke, and Qun Wang, From Kadanoff-Baym to Boltzmann equations for massive spin-1/2 fermions, *Phys. Rev. D* **104**, 016029 (2021).
- [28] Shuo Fang, Shi Pu, and Di-Lun Yang, Quantum kinetic theory for dynamical spin polarization from QED-type interaction, *Phys. Rev. D* **106**, 016002 (2022).
- [29] Ziyue Wang, Spin evolution of massive fermion in QED plasma, *Phys. Rev. D* **106**, 076011 (2022).
- [30] Shu Lin and Ziyue Wang, Shear induced polarization: Collisional contributions, *J. High Energy Phys.* **12** (2022) 030.
- [31] Shuo Fang, Shi Pu, and Di-Lun Yang, Spin polarization and spin alignment from quantum kinetic theory with self-energy corrections, *Phys. Rev. D* **109**, 034034 (2024).
- [32] Shuo Fang and Shi Pu, Collisional corrections to spin polarization from quantum kinetic theory using Chapman-Enskog expansion, *Phys. Rev. D* **111**, 034015 (2025).
- [33] Shu Lin and Ziyue Wang, Steady state, displacement current, and spin polarization for massless fermion in a shear flow, *Phys. Rev. D* **111**, 034032 (2025).
- [34] Ziyue Wang and Shu Lin, Spin polarization for massive fermion in a shear flow: Complete results at $O(\partial)$, *Phys. Rev. D* **111**, 056023 (2025).
- [35] Naoki Yamamoto and Di-Lun Yang, Chiral kinetic theory with self-energy corrections and neutrino spin Hall effect, *Phys. Rev. D* **109**, 056010 (2024).
- [36] Shuo Fang, Shi Pu, and Di-Lun Yang, Radiative corrections on vortical spin polarization in hot QCD matter, *Phys. Rev. D* **112**, 014038 (2025).
- [37] Jacopo Ghiglieri, Guy D. Moore, and Derek Teaney, QCD shear viscosity at (almost) NLO, *J. High Energy Phys.* **03** (2018) 179.
- [38] Shu Lin and Jiayuan Tian, In-medium electromagnetic form factors and spin polarizations, *Eur. Phys. J. Plus* **139**, 109 (2024).
- [39] Lihua Dong and Shu Lin, Polarization dynamics in the paramagnet of a charged quark-gluon plasma, *Phys. Rev. D* **110**, 016004 (2024).
- [40] Xu-Guang Huang, Pavel Mitkin, Andrey V. Sadofyev, and Enrico Speranza, Zilch vortical effect, Berry phase, and kinetic theory, *J. High Energy Phys.* **10** (2020) 117.
- [41] Koichi Hattori, Yoshimasa Hidaka, Naoki Yamamoto, and Di-Lun Yang, Wigner functions and quantum kinetic theory of polarized photons, *J. High Energy Phys.* **02** (2021) 001.