Role of $a_0(1710)$ in the $D_s^+ \to \rho^+\phi, \rho^+\omega$ decays

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We look into the $D_s^+ \to \rho^+\phi$ and $D_s^+ \to \rho^+\omega$ weak decays recently measured by the BESIII collaboration, which proceed very differently in a first step of the weak decay. While the first reaction proceeds directly via external emission, the second one does not go via external nor internal emission, what prompted the experimental team to claim that it proceeds via W-annihilation. We show in this work that the unexpectedly large rate of the $D_s^+ \to \rho^+\omega$ has a different explanation since is it naturally obtained once final state interaction of the $\rho^+\phi, \rho^+\omega$ and $K^{*+}\bar{K}^{*0}$ channels is taken into consideration. The interaction of these channels produces the $a_0(1710)$ resonance, predicted long ago as a molecular state of these coupled channels, and only recently observed, and it is the presence of this resonance what makes the effect of the final state interaction very important in the weak decays and provides a natural explanation of the experimental decay rates at a quantitative level. We also take into account the $D_s^+ \to \rho^+\omega$ decay in P- and D-waves empirically, and show that a good reproduction of the experimental results is obtained in this case.

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I. INTRODUCTION

The study of the vector-vector (VV) interaction was initiated in the works of [1], with the interaction of $\rho\rho$, and [2] with the extension to the $SU(3)$ space. Many resonances are obtained dynamically generated from this interaction which is obtained in the framework of the local hidden gauge approach [3–6], exchanging vector mesons. The $f_0(1370)$ and $f_2(1270)$ resonances are obtained from the $\rho\rho$ interaction, while extending the theory to include the strange sector, many other resonances are obtained, as the $f_0(1710), f_2'(1525)$, and $K_2^*(1430)$. The decay channels into two pseudoscalars (PP) are considered in [1,2] through box diagrams. In [7] the PP channels are considered as coupled channels together with the VV ones for the $\rho\rho$ case, with results similar to those of [1].

In [2] predictions were made for a new a_0, scalar $I = 1$ resonance, with mass around 1777 MeV, that was recently vindicated by several experimental works, Ref. [8] in *BABAR*, Ref. [9] in BESIII, and Ref. [10] in LHCb. The resonance is already cataloged in the PDG as the $a_0(1710)$ [11]. Predictions for one a_0 state with 1450 MeV, in between the $a_0(980)$ and $a_0(1710)$, were done using Regge phenomenology in [12]. The masses and widths reported by the different experiments are shown in Table I.

As one can see, there are still some discrepancies about the precise value of the mass, but there is consensus about the existence of the resonance and the value of the width, between 100 and 150 MeV. In the Table we also show the prediction of Ref. [2] and the results of a recent independent calculation for the mass, using dispersion relations [14]. As one can see, there is agreement about the mass between these two independent calculations. Similar results, depending somewhat on the cut off chosen, are obtained in [15].

The experimental observation of the a_0 state had immediate theoretical repercussion. In the molecular picture of [2,14] the $a_0(1710)$ couples to the $K^*\bar{K}^*$ in $I = 1$ and to $\rho\phi, \rho\omega$. In [16] the $D_s^+ \to \pi^+ K^{*+} K^{*-}, \pi^+ K^{*0}\bar{K}^{*0}$ reactions were studied and the final state interaction of $K^*\bar{K}^*$ was studied by means of the $f_0(1710)$ and $a_0(1710)$ dynamically generated resonances. The vector-vector (VV) channels were allowed to make a transition to the $K^+K^-, K^0\bar{K}^0$ and with this mechanism a ratio of the decay rates of these two decay channels was found, in agreement with experiment. In the same work a suggestion was made for a reaction which would isolate the $a_0(1710)$ resonance, the $D_s^+ \to \pi^0 K^+ K_S$ reaction, making predictions for the branching ratio of the reaction. This reaction was later observed in [13] with results similar to those predicted in [16]. In Ref. [17] the authors retake the idea of [16] and

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TABLE I. Masses and widths for the $a_0(1710)$ (in units of MeV).

	Mass [MeV]	Γ [MeV]
PDG [11]	1713 ± 19	107 ± 15
LHCb [10]	$1736 \pm 10 \pm 12$	$134 \pm 17 \pm 61$
BESIII [13]	$1817 \pm 8 \pm 20$	$97 \pm 22 \pm 15$
BABAR [8]	$1704 \pm 5 \pm 2$	$110 \pm 15 \pm 11$
Geng *et al.* [2]	1777	148
Du *et al.* [14]	1750–1790	No calculated width

show that the consideration of the $f_0(1710)$ and $a_0(1710)$ resonances produces mass distributions for $K^0_S K^0_S$ and $\pi^+ K^S_0$ invariant mass distributions in agreement with experiment. In Ref. [18] the $D_s^+ \to a_0^+(1710) \to \pi^0 K^+ K^0_S$ reaction is studied and, in addition to $a_0(1710)$, the contributions of K^* and $a_0(980)$ are also taken into account. It is found that the experimental measurements on the $K^+K^0_S$, π^0K^+, and $\pi^0K^0_S$ invariant mass distributions can be well reproduced, which supports the molecular, mostly $K^*\bar{K}^*$, nature of the scalar $a_0(1710)$ resonance. In [19] the $J/\psi \to \phi K\bar{K}$ decay is studied, looking for differences in the production rates of K^+K^- or $K^0\bar{K}^0$ in the region of 1700–1800 MeV, where the two resonances $f_0(1710)$ and $a_0(1710)$ appear, and predictions are made for mass distributions. In [20] the $D_s^+ \to K^0_S K^+\pi^0$ decay is again reanalyzed, and the contributions of the resonances $a_0^+(980), a_0^+(1710)$ in S-wave and $\bar{K}^{*0}(892), K^{*+}(892)$ in P-wave are considered, and agreement with experimental data is also shown. In Ref. [21] the authors advocate for the measurement of land-shapes in $a_0(1700/1800) \to K^*\bar{K}^*, \rho\phi, \rho\omega$ decays as a method to further learn about these VV dynamically generated resonances. In Ref. [22] the authors investigate the $K^-p \to a_0^+(1710)$ reaction as a means of learning more about this a_0 resonance, evaluating cross sections and suggesting the optimal energies where this resonance plays a relevant role. In [23] the authors suggest to look for the $a_0(1710)$ resonance in the $\eta_c \to \bar{K}^0K^+\pi^-$ reaction. In Ref. [24] the molecular picture is again retaken and predictions are made for the $J/\psi \to \bar{K}^0K^+\rho^-$ reaction, stressing the role played by the $a_0(1710)$ resonance. In [25] the $a_0(1710) - f_0(1710)$ mixing effect in the $D_s^+ \to K^0_S K^0_S \pi+$ decay is studied in detail considering the experimental mass distributions. A perspective on these and other possible reactions concerning the $a_0(1710)$ resonance is provided by the paper [26].

Very recently the role of the $f_0(1710)$ and $a_0(1710)$ resonances in the $D^0 \to \rho^0\phi$, $\omega\phi$ decays was established. These two decays proceed in a direct mode via internal emission with equal rates. Yet, the experimental branching ratio for the $\rho^0\phi$ mode is twice as big as that for the $\omega\phi$ mode [11,27]. By considering the contribution of the $f_0(1710)$ and $a_0(1710)$ resonances in these decays, a ratio for the branching fractions consistent with experiment was found [28].

The present work addresses the production of $\rho^+\phi$ and $\rho^+\omega$ in D_s^+ decay which has been recently measured by the BESIII collaboration [29]. The reactions are interesting because the $\rho^+\phi$ mode appears immediately via external emission, the strongest of the weak decay modes [30], while the $\rho^+\omega$ mode is not produced directly neither via external nor internal emission. The BESIII team in [29] interpreted this fact suggesting that the $\rho^+\omega$ decay mode proceeds via W-annihilation. This would provide a W-annihilation decay mode with an exceptionally large decay rate since experimentally it is found in [29] that the $\rho^+\omega$ decay rate is only a factor of four smaller than that of the $\rho^+\phi$ mode. This is indeed surprising since the strength of the weak decays decreases in the order [30]: external emission, internal emission, W-exchange, W-annihilation, horizontal W-loop and vertical W-loop, and only from external emission to internal emission one already expects an order of magnitude decrease in the decay rates. However, we shall see that one does not need to invoke W-annihilation. We can obtain the $\rho^+\omega$ decay by means of external plus internal emission of $\rho^+\phi$ and $K^*\bar{K}^*$ together with final state interaction. It is important to recall from the beginning that in order to have a given final state in the weak decay one does not need to produce this state in a first step. It is sufficient to produce another channel which is coupled to the final one by means of strong interaction, and then allow the transition from one channel to the other by means of the strong interaction rescattering. This is not the first time that this occurs: Indeed, in [31] the $D_s^+ \to a_0^+(980)\pi^0$ and $D_s^+ \to a^0(980)\pi^+$ decays leading to $D_s^+ \to \pi^+\pi^0\eta$ were reported and the mechanism was branded as a clean example of W-annihilation, with a rate which is one order of magnitude bigger than the typical W-annihilation rates. This, however, was explained by means of a mechanism of internal emission via the $D_s^+ \to K^+K^-\pi^+, D_s^+ \to K^0\bar{K}^0\pi^+$ followed by final state interaction of $K\bar{K} \to \pi\eta$ [32], which is driven by the $a_0(980)$ resonance, dynamically generated from the interaction of the $K\bar{K}$ and $\pi\eta$ channels [33] (see related works on the subject in [34–36]). Another example is found in the $D_s^+ \to \pi^+\pi^+\pi^-\eta$ reaction, measured for the first time in [37] by the BESIII collaboration. One surprising aspect of the analysis of [37] is the claim that the $D_s^+ \to a_0^+(980)\rho^0$ decay mode proceeds via weak annihilation, with a rate about one order of magnitude bigger than ordinary weak-annihilation processes. Once again it was shown in the detailed theoretical work of [38] that this decay mode was unnecessary, and the 6 invariant mass distributions were well described starting from external and internal emission mechanisms followed by final state interaction.

In the present work we shall also see that starting from the external and internal emission decay mechanisms of the $D_s^+ \to \rho^+\phi$ and $K^*\bar{K}^*$, respectively, together with final state interaction of VV, we shall be able to obtain the ratio of $D_s^+ \to \rho^+\omega$ to $D_s^+ \to \rho^+\phi$ decay rates compatible with

the experimental observation. Since the VV channels are in isospin $I = 1$, the interaction of these channels generate the $a_0(1710)$ and the $D_s^+ \to \rho^+\omega$ decay is directly tied to this resonance. In other words, it is the presence of the $a_0(1710)$ that makes the $D_s^+ \to \rho^+\omega$ decay possible and with large strength, without having to invoke the W-annihilation mode.

II. FORMALISM

The BESIII experiment of [29] reports

$$\begin{aligned}\mathrm{BR}(D_s^+ \to \rho^+\phi) &= (3.98 \pm 0.33 \pm 0.21) \times 10^{-2},\\ \mathrm{BR}(D_s^+ \to \rho^+\omega) &= (0.99 \pm 0.08 \pm 0.07) \times 10^{-2}.\end{aligned} \tag{1}$$

Summing errors in quadrature, we obtain a ratio

$$R_B = \frac{\mathrm{BR}(D_s^+ \to \rho^+\phi)}{\mathrm{BR}(D_s^+ \to \rho^+\omega)} = [3.5 \sim 4.6]. \tag{2}$$

The $\rho^+\phi$ production proceeds, as shown in [29] by means of external emission as shown in Fig. 1.

The $s\bar{s}$ produced in Fig. 1 gives rise to the ϕ, but not to the ω which is $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$. The mechanism of internal emission shown in Fig. 2 also does not directly produce

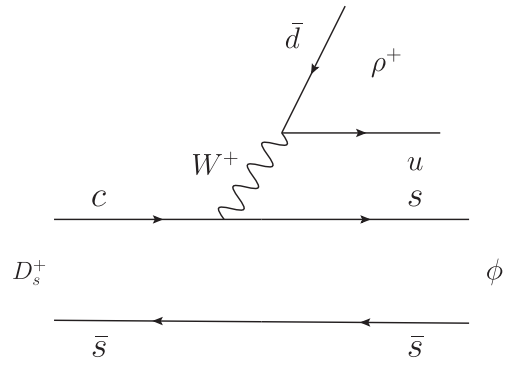


FIG. 1. Mechanism for $D_s^+ \to \rho^+\phi$ from external emission.

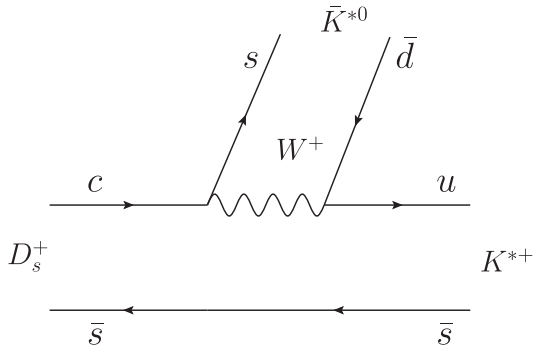


FIG. 2. Mechanism for D_s^+ decay based on internal emission.

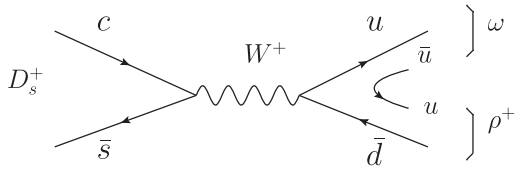


FIG. 3. Mechanism for $D_s^+ \to \rho^+\omega$ based on W-annihilation as suggested in [29].

$\rho^+\omega$, but two strange mesons. This reaction leads the BESIII Collaboration to claim that this production mode proceeds via W-annihilation as shown in Fig. 3. However, as discussed in the Introduction, and according to general values of weak decays [30], one expects the ratio of this mechanism versus external emission to be suppressed by at least two orders of magnitude, in clear disagreement with the band of values of Eq. (2). In view of this, and recalling the finding of Refs. [32,38], where experimental claims of W-annihilation [31,37] were dismissed in favor of a final state interaction mechanism, we look now at the role played by final state interaction in the $D_s^+ \to \rho^+\phi$ and $D_s^+ \to \rho^+\omega$ decays.

As shown in [2,7,14,15] the VV interaction in $I = 1$ is driven by the channels $K^*\bar{K}^*, \rho^+\phi$, and $\rho^+\omega$, where the $a_0(1710)$ is dynamically generated. The amplitude behaves as

$$\begin{aligned} t_{ij} &= \tilde{t}_{ij} P^{(0)};\\ \tilde{t}_{ij} &= \frac{g_i g_j}{s - s_R}, \qquad i, j = K^*\bar{K}^*, \rho^+\phi, \rho^+\omega, \end{aligned} \tag{3}$$

close to the pole, where g_i are the couplings of the resonance to the coupled channels, which we reproduce in Table II extracted from the work of [2]. $P^{(0)}$ is the spin projection in $J = 0$ of the two vectors [1]

$$P^{(0)} = \frac{1}{3}(\vec{\epsilon}_1\vec{\epsilon}_2)(\vec{\epsilon}_3\vec{\epsilon}_4) \tag{4}$$

with $\vec{\epsilon}_i$ the vector polarization of the vector meson and the indices corresponding to $1 + 2 \to 3 + 4$. The couplings are complex due to the existence of the $\rho\omega$ decay channel and mass distributions through their widths, of the ρ and K^*. Note that the width of $a_0(1710)$, using the mass of Ref. [2], for decay to $\rho\omega$ ($\Gamma = \frac{1}{8\pi}\frac{1}{M_{a_0}^2}|g_{\rho\omega}|^2 q_{\mathrm{on},\rho\omega}$) is about 100 MeV, the rest of the width of Table I coming from decay to $\rho\phi$, $K^*\bar{K}^*$ when the ρ and K^* mass distributions are considered.

Note that since we have the phase convention for isospin multiplets, $(K^{*+}, K^{*0}), (\bar{K}^{*0}, -K^{*-}), (-\rho^+, \rho^0, \rho^-)$, the couplings for the charge states in Table II change sign for $\rho^+\phi, \rho^+\omega$ compared to those in the isospin basis. As we can see in Fig. 1 we produce $\rho^+\phi$ by external emission, to which we associated a weight $A\vec{\epsilon}_1\vec{\epsilon}_2$ in

TABLE II. Couplings of the $a_0(1710)$ to $K^*\bar{K}^*, \rho^+\phi, \rho^+\omega$ for the $I = 1$ states and charged states (in units of MeV).

	$K^*\bar{K}^*$	$\rho\phi$	$\rho\omega$
g_i	$7525 - i1529$	$4998 - i1872$	$-4042 + i1391$
	$K^{*+}\bar{K}^{*0}$	$\rho^+\phi$	$\rho^+\omega$
	$7525 - i1529$	$-4998 + i1872$	$4042 - i1391$

S-wave and in Fig. 2 we produce the vectors $\bar{K}^{*0}K^{*+}$ by internal emission, to which, we associate a weight $B\vec{\epsilon}_1\vec{\epsilon}_2, B = \beta A$. We expect, based on the color counting, which favors external emission by a factor of the order of N_c, that $\beta \sim 1/3$. Considering the final state interaction, the mechanism for $\rho^+\phi$ and $\rho^+\omega$ are shown in Fig. 4.

Analytically we will have now for $D_s^+ \to \rho^+\phi$ and $D_s^+ \to \rho^+\omega$,

$$t_{\rho^+\phi} = \tilde{t}_{\rho^+\phi}\vec{\epsilon}_{\rho^+}\vec{\epsilon}_{\phi}; \qquad t_{\rho^+\omega} = \tilde{t}_{\rho^+\omega}\vec{\epsilon}_{\rho^+}\vec{\epsilon}_{\omega}, \tag{5}$$

with

$$\tilde{t}_{\rho^+\phi} = A(1 + G_{\rho^+\phi}(m_{D_s^+})\tilde{t}_{\rho^+\phi,\rho^+\phi}(m_{D_s^+}) + \beta G_{K^*\bar{K}^*}(m_{D_s^+})\tilde{t}_{K^*\bar{K}^*,\rho^+\phi}(m_{D_s^+})), \tag{6}$$

$$\tilde{t}_{\rho^+\omega} = A(G_{\rho^+\phi}(m_{D_s^+})\tilde{t}_{\rho^+\phi,\rho^+\omega}(m_{D_s^+}) + \beta G_{K^*\bar{K}^*}(m_{D_s^+})\tilde{t}_{K^*\bar{K}^*,\rho^+\omega}(m_{D_s^+})), \tag{7}$$

with the $\tilde{t}_{ij}$ matrices given by

$$\tilde{t}_{\rho^+\phi,\rho^+\phi} = \frac{g^2_{\rho^+\phi}}{m^2_{D_s^+} - M_R^2 + iM_R\Gamma_R};$$
$$\tilde{t}_{\rho^+\phi,\rho^+\omega} = \frac{g_{\rho^+\phi}g_{\rho^+\omega}}{m^2_{D_s^+} - M_R^2 + iM_R\Gamma_R};$$
$$\tilde{t}_{K^*\bar{K}^*,\rho^+\phi} = \frac{g_{K^*\bar{K}^*}g_{\rho^+\phi}}{m^2_{D_s^+} - M_R^2 + iM_R\Gamma_R};$$
$$\tilde{t}_{K^*\bar{K}^*,\rho^+\omega} = \frac{g_{K^*\bar{K}^*}g_{\rho^+\omega}}{m^2_{D_s^+} - M_R^2 + iM_R\Gamma_R}, \tag{8}$$

with the couplings from Table II. The G functions from Eqs. (6) and (7) are the meson-meson loop function which are regularized with a cutoff, given by

$$G = \int_{|\vec{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3}\frac{w_1(q)+w_2(q)}{2w_1(q)w_2(q)}\frac{1}{s-(w_1(q)+w_2(q))^2+i\epsilon}, \tag{9}$$

with $w_i = \sqrt{\vec{q}^2 + m_i^2}$ and m_i the mass of the meson, and following Refs. [2,28] we take $q_{\max} = 960$ MeV.

The decay widths are given by

$$\Gamma_i = \frac{1}{8\pi}\frac{1}{m^2_{D_s^+}}3|\tilde{t}_i|^2 q_{\text{on},i}, \tag{10}$$

with the factor 3 coming from the sum of the final vector polarizations, and $q_{\text{on},i}$, given by

$$q_{\text{on},i} = \frac{\lambda^{1/2}(m^2_{D_s^+}, m^2_{1,i}, m^2_{2,i})}{2m_{D_s^+}}; \qquad i = \rho^+\phi, \rho^+\omega. \tag{11}$$

Equation (8) are Breit-Wigner forms for the amplitude, which are justified for masses of $m_{D_s^+}$ not far away from the resonance mass M_R. Actually, the difference of mass is

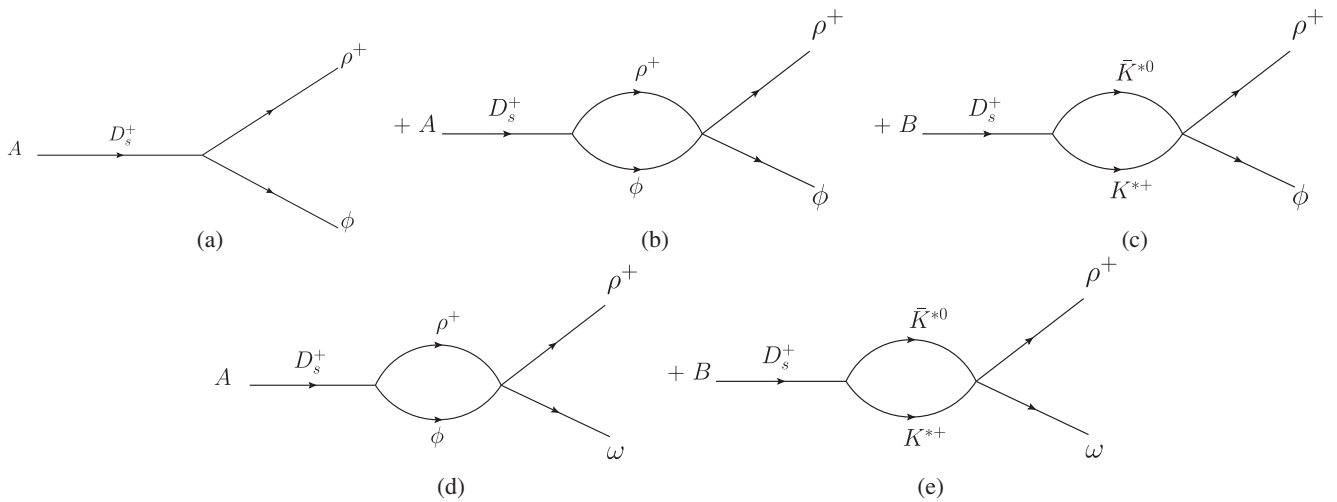


FIG. 4. Diagrams involving final state. (a),(b),(c) for $\rho^+\phi$ production, (d),(e) for $\rho^+\omega$ production.

200 MeV, which is not large compared with the resonance width of 148 MeV of Table I.

III. CONSIDERATION OF D-WAVE IN THE $D_s^+ \to \rho^+\omega$ DECAY

In the BESIII experiment [29], a large fraction of about 52% of the $D_s^+ \to \rho^+\omega$ decay rate was found in D-wave while the S-wave part accounted by about 30% and there is also about 25% of P-wave production. However, in $\rho^+\phi$ decay there is no D-wave reported and the fraction of P-wave is small.

We discuss here the way the P- and D-waves $D_s^+ \to \rho^+\omega$ decay would appear in our framework. The $\rho^+\phi$ mass is 178 MeV below the D_s^+ mass and the $\rho^+\omega$ mass is 418 MeV below the D_s^+ mass. With this available energy for the decay, the P- and D-waves decays in these channel are likely to happen, particularly in the $D_s^+ \to \rho^+\omega$ case, where the excess mass is so large. Once again, we can rely on the mechanisms of Fig. 4, starting from the $\rho^+\phi$ and $\bar{K}^{*0}K^{*+}$ states, to produce $\rho^+\phi$ and $\rho^+\omega$ at the end, however, in P- and D-waves, for the two vectors, which implies $J = 1, 2$ respectively to conserve total angular momentum. We do not have a theory for this production and the interaction of these channels, and we resort to use experimental data to take this into account. Hence, we take advantage of the experimental findings of the P- and D-waves and S-wave branching ratios, and given the incoherent sum of S- and D-wave or D-wave, we simply multiply the S-wave production contribution of $D_s^+ \to \rho^+\omega$ by the appropriate factor to include the P- and D-waves production.

Since the D-wave production of $\rho^+\omega$ is sizable, it is interesting to see that simply based on phase space arguments we would expect to have a reduction of the $\rho^+\phi$ versus $\rho^+\omega$ production of the order of $(q_{\text{on},\rho^+\phi}/q_{\text{on},\rho^+\omega})^5 \simeq 1/7.6$, where $q_{\text{on},\rho^+\phi}, q_{\text{on},\rho^+\omega}$ refer to the momentum of the ρ in each case. This can justify why the D-wave decay into $\rho^+\phi$ is not reported experimentally, and we will also neglect it.

IV. RESULTS

In order to get the ratio of R_B of Eq. (2), we only have one parameter free, which is the value of β, since the factor A cancels in the ratio, and we would expect that the value of β is around $1/3$ or reasonably smaller than 1. Only if this is the case can we claim that we find a sensible solution to the problem. Note that if the final state interaction (FSI) had a small effect, we would not get any solution to this problem, even with unreasonable values of β. This is not the case, and the final state interaction is quite effective. To show that we first see what we get for $\Gamma_{\rho^+\phi}$ with or without final state interaction, comparing the final $\Gamma_{\rho^+\phi}$ with the one obtained with just the tree level [term 1 in Eq. (6)]. By taking $\beta = 1/3$ and $\beta = 0$, we find

$$\frac{\Gamma_{\rho^+\phi}(\beta = 1/3)}{\Gamma_{\rho^+\phi}(\text{tree level})} = 0.64;$$
$$\frac{\Gamma_{\rho^+\phi}(\beta = 0)}{\Gamma_{\rho^+\phi}(\text{tree level})} = 0.35. \quad (12)$$

The results of Eq. (12) are obtained with the inputs of [2] shown in Tables I and II. Using other masses and widths from Table I the results are qualitatively similar. What we see is that just with the effect of the FSI in external emission ($\beta = 0$), there is a reduction of the width by about a factor of three. The extra effect of the FSI due to internal emission ($\beta = 1/3$) softens the global effects of the FSI, reducing the width by about a factor of $\frac{2}{3}$. This indicates that the contribution of the internal emission is not negligible and that it has opposite sign to that of the external emission due to the signs of the couplings in Table II. The sizable strength of the internal emission mechanism is due to the nearly double strength of the $K^*\bar{K}^*$ couplings with respect to that of $\rho^+\phi$. What we can see it that the FSI is very important and it is tied to relevant information on the VV interaction in this particular case.

Once the important effect of the final state interaction is proved, we proceed to see its effect in the ratio R_B of Eq. (2). We aim at obtaining $R_B = 4$ in the middle of the experimental bracket of Eq. (2). Then we tune the value of β for different cases to obtain that value and we show the results in Table III. We show the results when we use only S-wave and when we consider the additional P- and D-waves for the $\rho^+\omega$ channel.

We have taken as an example the mass and width of [2], the one of LHCb [10], and the one of the BESIII [13], respectively, where discrepancies in the values of mass and width still persist. It is interesting to see the result that we obtain. When we consider only the S- wave for the $D_s^+ \to \rho^+\phi, \rho^+\omega$ decays we obtain a reasonable solution for R_B, with values of β between 0.05 and 0.26 depending on the mass provided by each approach. This is not a bad solution but relies upon values of β smaller than the expected $\beta \simeq \frac{1}{N_C} = 0.33$ value provided by the color reduction of the internal emission versus external emission. However, a more quantitative evaluation requires the consideration of the weak $D_s^+ \to \rho\omega$ decay in $L = 1$, $L = 2$. This is easily implemented technically from the former mechanism. Taking the experimental branching ratios we should increase the S-wave $\rho^+\omega$ decay by a factor 2.56 to incorporate the D-and P-wave decays. This can be empirically done by multiplying $\tilde{t}_{\rho^+\omega}$ of Eq. (7) by the factor

TABLE III. Different values of β needed to obtain $R_B = 4$.

	Geng *et al.* [2]	LHCb [10]	BESIII [13]
β	0.15	0.05	0.26
β (with $P + D$-wave)	0.31	0.24	0.39

$(2.56)^{1/2} = 1.6$. With the new change implemented, we see what we get now in Table III. Then, when the P- and D-waves $D_s^+ \to \rho^+\omega$ decays are considered, we obtain a solution using values of β which are all around 0.3 [0.24-0.39], providing a much more acceptable solution.

V. CONCLUSIONS

The $D_s^+ \to \rho^+\phi$ and $D_s^+ \to \rho^+\omega$ weak decays proceed directly through very different topologies. While the first reaction proceeds via external emission, the second one does not go in a first step through external nor internal emission, which are the two modes with largest strength. This realization lead the BESIII team to claim that this decay mode proceeds via W-annihilation. Then one might expect that the $D_s^+ \to \rho^+\omega$ decay rate is suppressed by at least two orders of magnitude with respect to that of the $D_s^+ \to \rho^+\phi$ one. Yet, this is not the case and the observed suppression is only of a factor about 4. We have then exploited the effect of final state interaction of the pair of vectors originated in the first step of the decay. The external emission directly produces $\rho^+\phi$, while the internal emission produces directly $K^{*+}\bar{K}^{*0}$. The studies of the vector-vector interaction done in several works show that in isospin $I = 1$, one has the interacting channels $\rho\phi, \rho\omega$ and $K^*\bar{K}^*$, and the interaction produces an a_0 resonance in the region of 1780 MeV. The resonance was found experimentally later by the *BABAR*, BESIII and LHCb collaborations and the masses are between 1710 and 1817 MeV, with the theoretical mass in the middle. The original $\rho^+\phi$ or $K^{*+}\bar{K}^{*0}$ channels make transition to the final $\rho^+\phi$ and $\rho^+\omega$, which is led by the dynamically generated $a_0(1710)$ resonance. What we find is that the final state interaction is very important in these processes, and can reduce the original $D_s^+ \to \rho^+\phi$ decays rate appreciably, while it provides a path for the $D_s^+ \to \rho^+\omega$ decay to occur through original production of $\rho^+\phi$ and $K^{*+}\bar{K}^{*0}$ followed by transition to $\rho^+\omega$. The $a_0(1710)$ is produced by the interaction of the coupled channels in S-wave and $J = 0$. We also noted that there are appreciable mass difference between the D_s^+ and the $\rho^+\phi$ and $\rho^+\omega$, which is very large in the case of $\rho^+\omega$ production. It is likely that the reaction can proceed via P- or D-wave. Actually, the D-wave production of the $D_s^+ \to \rho^+\phi, \rho^+\omega$ decays could be important, but for reasons of phase space it only shows up in the $D_s^+ \to \rho^+\omega$ decay. The $D_s^+ \to \rho^+\phi$ decay in D-wave is suppressed by nearly an order of magnitude compared to $D_s^+ \to \rho^+\omega$. This can justify why a sizable D-wave $\rho^+\omega$ component is found in the experimental analysis, while no sign of it is seen for the $\rho^+\phi$ decay. We have taken into account this D-wave component as well as another P-wave fraction also coming from the experimental analysis, and all together we find a very reasonable explanation of the relative $\rho^+\phi$ versus $\rho^+\omega$ production, involving a contribution of internal emission in agreement with the color suppression of the internal emission mode.

We have evaluated the ratio of the two branching ratios and find that, with reasonable values of the relative weight of internal to external emission of the order of $1/N_c$, one can obtain the experimental values of that ratio. We, thus, demonstrate that the final state interaction of the vector mesons can explain the experimental findings and there is no need to invoke an exceptionally large W-annihilation mechanism. At the same time we show in one more example the role of the lately observed $a_0(1710)$ resonance, which is turning into a key ingredient to explain unexpected observables in many new experiments.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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