

GUT-motivated noninvertible symmetry as a solution to the strong CP problem and the neutrino CP -violating phase

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The unsuppressed CP violation in QCD is a problem in the standard model. If we have some mechanism to guarantee real determinants of the quark mass matrices, the vanishing physical vacuum angle $\bar{\theta}$ indicates the CP invariance at the fundamental level. Thus, the small $\bar{\theta}$ is technically natural, since we have an enhanced CP symmetry in the limit of the vanishing $\bar{\theta} = 0$. In fact, it was proved that the vacuum angle is never renormalized up to the four-loop level once it is fixed at 0 value at some high energy scale. The purpose of this paper is to construct a model that guarantees the real determinants of the quark mass matrices assuming a noninvertible symmetry. In the present model, we have only one CP violating phase and hence all observable phases such as phases in the Cabibbo-Kobayashi-Maskawa matrix, in the neutrino oscillation and in the leptogenesis are related. It should be remarkable that the correct sign of the baryon-number asymmetry in the present Universe is obtained and a narrow region for the CP violating phase $\delta_{CP} \simeq 192^\circ\text{--}197^\circ$ is predicted.

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I. INTRODUCTION

The fundamental theory of particle physics may be CP invariant. For example, string theory as well as a certain type of higher dimensional theory has CP symmetry [1,2]. CP symmetry can be violated by moduli stabilization through compactifications. However, CP violating vacua are hardly realized by simple moduli stabilization. In particular, if the theory has other symmetries such as modular symmetries, CP invariant vacua are favorable. (See e.g. Refs. [3–8].) That leads to CP invariant four-dimensional low energy effective field theory at the compactification scale, where the QCD phase θ is vanishing and Yukawa couplings as well as other couplings are real.

It is very interesting if our Universe is indeed CP invariant in the fundamental theory at high energies. It means that the CP invariance must be spontaneously broken down at an intermediate energy scale, since we know the CP is violated in various meson decays at low energies. This fact stimulates us to consider the baryon-number asymmetry in the present Universe is dynamically created in the early Universe [9]. We may understand the small baryon asymmetry in the present Universe.

Furthermore, the above hypothesis would open a new window to solve the strong CP problem without the QCD axion [10]. The experimental constraint on the physical vacuum angle $\bar{\theta} \leq 10^{-10}$ [11] requires a strict constraint on the mass matrices for the quarks such as $\arg[\det M_u M_d] \leq 10^{-10}$ [11], where $\bar{\theta} = \theta + \arg[\det M_u M_d]$. If we have some mechanism to guarantee real determinants of the quark mass matrices, the vanishing physical vacuum angle $\bar{\theta}$ indicates the CP invariance $\theta = 0$ at the fundamental level. Thus, the small $\bar{\theta}$ is technically natural, since we have an enhanced CP invariance in the limit of the vanishing $\bar{\theta} = 0$. In fact, it was proved that the vacuum angle is never renormalized up to the four-loop level [12].

We need a proper form of quark mass matrices to realize $\arg[\det M_u M_d] \leq 10^{-10}$. We consider, in this paper, a noninvertible symmetry to control the quark mass matrices to

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satisfy the above constraint in the framework of the standard model (SM). In particular, we use $\mathbb{Z}_2$ gauging of $\mathbb{Z}_N$ symmetries [13–15]. That is a novel symmetry to lead interesting coupling selection rules. One can realize realistic quark and lepton mass textures [14,16–22] including axionless solutions to the strong CP problem [23,24].¹

By using a noninvertible symmetry, we propose a simple model for the axionless solution of the strong CP problem.² To simplify the charge assignment of the fermions, we take the representations of the SM fermions in the $SU(5)$ grand unification group. We will not, however, unify all of the gauge interactions in the SM. That is, we have a global $SU(5)$ symmetry in the limit of vanishing gauge coupling constants in the SM. As for $SU(2)$ doublet Higgs bosons, we consider they belong to representations like **5** and **45** in the $SU(5)$. We call this framework a GUT-inspired SM.³ In addition to the SM gauge groups, we introduce the $U(1)_{B-L}$ gauge symmetry to explain naturally the seesaw mechanism to generate the observed small neutrino masses [38–41].

This paper is organized as follows. In Sec. II, we briefly review $\mathbb{Z}_2$ gauging of $\mathbb{Z}_N$ symmetries. In Sec. III, we show our model. In Sec. IV, we study phenomenological aspects of our model. Section V is devoted to our conclusion.

II. NONINVERTIBLE SELECTION RULES

In this section, we review the noninvertible selection rules for matter fields. It was known that selection rules of twisted and/or untwisted strings in heterotic string theory on Abelian and non-Abelian orbifolds become noninvertible. Since they are labeled by conjugacy groups of the space group action [42–45], the selection rules are not understood in a grouplike manner [46–49]. A similar phenomenon can be seen in the D-brane system [13,15]. In the framework of type IIB string theory on toroidal orbifolds with magnetic fluxes, the fusion rule of momentum operators on orbifolds has a noninvertible structure, e.g., Fibonacci and Ising rules, depending on the value of magnetic fluxes [13].

¹See, e.g., Refs. [25–28], for reviews on noninvertible symmetries, and Refs. [29–37], for other applications of noninvertible symmetries in particle physics.

²As for the normal symmetry like $U(1)$ or $\mathbb{Z}_N$, we can not have the desired three zero texture due to the stronger constraints, as shown in Section II in Ref. [10].

³The doublet-triplet splitting problem for the Higgs sector is indeed present in this framework. However, this issue is generic to GUT-based constructions and is not specific to our model. As is commonly done in the literature, we assume that this splitting is achieved by fine-tuning, and we do not attempt to address this well-known problem in the present work. Since the full GUT symmetry is not gauged, the three gauge coupling constants are not required to unify at a high-energy scale. Consequently, there is no conflict with current experimental observations. For the same reason, the model does not predict the existence of magnetic monopoles.

Following Refs. [13,14], we introduce the noninvertible selection rule for matter fields. First, let us start from $\mathbb{Z}_N$ symmetry under which matter fields ϕ_i transform as

$$\phi_i \rightarrow g^{k_i} \phi_i. \quad (2.1)$$

Here, $g = e^{2\pi i/N}$ denotes the generator of $\mathbb{Z}_N$ and k_i corresponds to $\mathbb{Z}_N$ charges. Note that the selection rule of two matter fields ϕ_i and ϕ_j obeys the grouplike form $g^{k_i} g^{k_j} = g^{k_i+k_j}$.

Next, we focus on the automorphism of $\mathbb{Z}_N$, i.e., $\mathbb{Z}_2$,

$$r^2 = e, \quad r g^k r^{-1} = g^{-k}, \quad (2.2)$$

where r and e , respectively, denote the $\mathbb{Z}_2$ generator and the identity. Among the conjugacy classes of $D_N \cong \mathbb{Z}_N \rtimes \mathbb{Z}_2$, we pick up the $\mathbb{Z}_2$ invariant one, i.e.,

$$[g^{(k)}] = \{r^n g^k r^{-n} | n = 0, 1\} = \{g^k, g^{-k}\}, \quad (2.3)$$

obeying the following multiplication rule:

$$[g^{(k_1)}] \otimes [g^{(k_2)}] = [g^{(k_1+k_2)}] + [g^{(k_1-k_2)}]. \quad (2.4)$$

This selection rule is different from the grouplike one, and in what follows, we call $\tilde{\mathbb{Z}}_N$ for a $\mathbb{Z}_2$ gauging of $\mathbb{Z}_N \rtimes \mathbb{Z}_2$. Note that the matter fields labeled by the above class $[g^{(k)}]$ correspond to the $\mathbb{Z}_2$ invariant mode,

$$\Phi_j = \phi_j + \phi_{N-j}. \quad (2.5)$$

This $\mathbb{Z}_2$ symmetry is nothing but a geometric $\mathbb{Z}_2$ twist in the context of type IIB magnetized D-brane models on $T^2/\mathbb{Z}_2$.

When $N = 4$ and 5 , there are three independent classes. In the following discussion, we analyze the case with $N = 5$, i.e., $\tilde{\mathbb{Z}}_5$, where there exist three classes $\{[g^0], [g^1], [g^2]\}$ obeying

$$\begin{aligned} [g^0] \otimes [g^0] &= [g^0], & [g^0] \otimes [g^1] &= [g^1] \otimes [g^0] = [g^1], \\ [g^0] \otimes [g^2] &= [g^2] \otimes [g^0] = [g^2], & [g^1] \otimes [g^1] &= [g^0] + [g^2], \\ [g^1] \otimes [g^2] &= [g^2] \otimes [g^1] = [g^1] + [g^2], & [g^2] \otimes [g^2] &= [g^0] + [g^1]. \end{aligned} \quad (2.6)$$

These three independent classes can be assigned to three generations of fermions. For example, when three generations of all left-handed and right-handed fermion fields correspond to these three classes, $\{[g^0], [g^1], [g^2]\}$, one can realize the Yukawa matrix,

$$\begin{pmatrix} \checkmark & 0 & 0 \\ 0 & \checkmark & 0 \\ 0 & 0 & \checkmark \end{pmatrix}, \quad (2.7)$$

for the Higgs field corresponding to $[g^0]$,

$$\begin{pmatrix} 0 & \checkmark & 0 \\ \checkmark & 0 & \checkmark \\ 0 & \checkmark & \checkmark \end{pmatrix}, \quad (2.8)$$

for the Higgs field with $[g^1]$,

$$\begin{pmatrix} 0 & 0 & \checkmark \\ 0 & \checkmark & \checkmark \\ \checkmark & \checkmark & 0 \end{pmatrix}, \quad (2.9)$$

for the Higgs field with $[g^2]$, where the check symbol $\checkmark$ denotes allowed coupling coefficient. If we combine the first one and the third one, we can obtain the Yukawa matrix, where only the (2,2) entry is nonvanishing, and the others are vanishing.

III. MODEL

In this section, we first construct a GUT-inspired SM model with a noninvertible symmetry for providing an axionless solution to the strong CP problem. The textures derived from $\tilde{\mathbb{Z}}_5$ are interesting as shown in the previous section, but those are not sufficient for our purpose of solving the strong CP problem. The selection rule due to $\tilde{\mathbb{Z}}_4$ is not either. We extend simply our selection rules to $\tilde{\mathbb{Z}}_5^{(1)} \times \tilde{\mathbb{Z}}_5^{(2)}$. Other extensions such as $\tilde{\mathbb{Z}}_5 \times \tilde{\mathbb{Z}}_4$ and ones including noninvertible selection rules in Ref. [50] are possible, but we focus on $\tilde{\mathbb{Z}}_5^{(1)} \times \tilde{\mathbb{Z}}_5^{(2)}$.⁴ We would study other extensions elsewhere. We consider a simple flavor structure such that three generations of all left-handed and right-handed fermions are assigned to the three classes, $[g^0]$, $[g^1]$, and $[g^2]$ in both $\tilde{\mathbb{Z}}_5^{(1)}$ and $\tilde{\mathbb{Z}}_5^{(2)}$. Thus the flavor structure is simple and consistent with $SO(10)$ GUT. Other flavor assignments would be studied elsewhere. The model is very similar to the model in Ref. [24]. Charges and class assignments of fermion and scalar fields are shown in Tables I and II.

We introduce the SM Higgs boson called H_1 . Then, we have the following Yukawa couplings:

$$f_u TTH_1^\dagger + f_d T\bar{F}H_1, \quad (3.1)$$

with

⁴We can derive $\tilde{\mathbb{Z}}_5$ in $T^2/\mathbb{Z}_2$ magnetized orbifold models [13] as well as heterotic string theory on T^2/D_5 [49]. The selection rule, $\tilde{\mathbb{Z}}_5^{(1)} \times \tilde{\mathbb{Z}}_5^{(2)}$, can be realized, e.g., in magnetized orbifold models on $(T^2 \times T^2)/\mathbb{Z}_2$ and $(T^2/\mathbb{Z}_2)^2$, although our approach is rather bottom up. At any rate, assuming $\tilde{\mathbb{Z}}_5^{(1)} \times \tilde{\mathbb{Z}}_5^{(2)}$ is consistent in four-dimensional field theory.

TABLE I. Charges and class assignments of matter fields under $SU(5) \times U(1)_{B-L}$ and $\tilde{\mathbb{Z}}_5^{(1)} \times \tilde{\mathbb{Z}}_5^{(2)}$ symmetries.

	T	$\bar{F}$	$\bar{N}$
$SU(5) \times U(1)_{B-L}$	$(\mathbf{10}, -1)$	$(\bar{\mathbf{5}}, 3)$	$(\mathbf{1}, -5)$
$\tilde{\mathbb{Z}}_5^{(1)}$	$([g^0], [g^1], [g^2])$	$([g^0], [g^1], [g^2])$	$([g^0], [g^1], [g^2])$
$\tilde{\mathbb{Z}}_5^{(2)}$	$([g^0], [g^1], [g^2])$	$([g^0], [g^1], [g^2])$	$([g^0], [g^1], [g^2])$

$$f_{u,d} = \begin{pmatrix} 0 & \checkmark & 0 \\ \checkmark & 0 & \checkmark \\ 0 & \checkmark & \checkmark \end{pmatrix}. \quad (3.2)$$

We can confirm that both mass matrices, M_u and M_d are four-zero textures as,

$$M_{u,d}^0 = \begin{pmatrix} 0 & a_{u,d} & 0 \\ a'_{u,d} & 0 & c_{u,d} \\ 0 & c'_{u,d} & d_{u,d} \end{pmatrix}, \quad (3.3)$$

where $a_{u,d}$, $a'_{u,d}$, $c_{u,d}$, $c'_{u,d}$, and $d_{u,d}$ are all real constants. Note that one cannot derive these mass textures from conventional group-based symmetries.

We now introduce a complex scalar boson η whose vacuum-expectation value (vev), $\langle \eta \rangle$, is complex, which induces the spontaneous CP violation. This boson is a singlet under all the gauge groups, but it has nontrivial charges of the noninvertible symmetries as shown in Table II. The possible renormalizable potential for it is given by

$$V_\eta = -\mu^2 \eta \eta^\dagger - \mu'^2 (\eta^2 + \eta^{\dagger 2}) + \xi (\eta \eta^\dagger)^2 + \xi' (\eta^4 + \eta^{\dagger 4}) + \xi'_{33} (\eta^2 + \eta^{\dagger 2}) \eta \eta^\dagger. \quad (3.4)$$

We see that the vev $\langle \eta \rangle$ has a complex phase and its absolute value is basically given by the scale of its mass. In this paper, we consider the vev $|\langle \eta \rangle| = O(10^{10}-10^{12})$ GeV and consider the inflation scale H_{inf} to be smaller than $|\langle \eta \rangle|$ to avoid the domain wall formation. (If we introduce a higher dimensional operator such as $\eta^5 + \eta^{\dagger 5}$ we do not have the domain wall problem.)

We show a mechanism that induces a coupling between the η boson and the SM fermion proposed in Ref. [10].

TABLE II. Charges and class assignments of scalar fields under $SU(5) \times U(1)_{B-L}$ and $\tilde{\mathbb{Z}}_5^{(1)} \times \tilde{\mathbb{Z}}_5^{(2)}$ symmetries.

	H_1	H_2	Φ	η
$SU(5) \times U(1)_{B-L}$	$(\bar{\mathbf{5}}, -2)$	$(\bar{\mathbf{5}}, -2) + (\mathbf{45}, -2)$	$(\mathbf{1}, 10)$	$(\mathbf{1}, 0)$
$\tilde{\mathbb{Z}}_5^{(1)}$	$[g^1]$	$[g^2]$	$[g^0]$	$[g^2]$
$\tilde{\mathbb{Z}}_5^{(2)}$	$[g^1]$	$[g^0]$	$[g^0]$	$[g^1]$

We first introduce a heavy Higgs boson H_2 and couplings with H_1 and η given by

$$V_{H-\eta} = \mu_{12}(H_1^\dagger H_2 \langle \eta \rangle + \text{H.c.}) + \mu_{21}(H_1^\dagger H_2 \langle \eta^\dagger \rangle + \text{H.c.}), \quad (3.5)$$

where μ_{12} and μ_{21} are dimension one real parameters, which we will take of the order of $|\langle \eta \rangle|$. The heavy Higgs H_2 has Yukawa couplings with the SM fermions, T and $\bar{F}$, and its noninvertible charges are chosen so that its Yukawa couplings are localized [10] in both matrices $M_{u,d}$ through the following term:

$$f'_u T T H_2^\dagger + f'_d T \bar{F} H_2. \quad (3.6)$$

Then, we have the following Yukawa terms:

$$f'_{u,d} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \checkmark & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (3.7)$$

where $f'_{u,d}$ are real. As a consequence, we have an effective Yukawa coupling of the SM Higgs H_1 after the integration of the heavy Higgs H_2 , which are localized in the (2, 2) element as

$$M'_{u,d} = \begin{pmatrix} 0 & a_{u,d} & 0 \\ a'_{u,d} & b_{u,d} e^{i\phi_{u,d}} & c_{u,d} \\ 0 & c'_{u,d} & d_{u,d} \end{pmatrix}, \quad (3.8)$$

where $a_{u,d}, a'_{u,d}, b_{u,d}, c_{u,d}, c'_{u,d}$, and $d_{u,d}$ are real. We can write

$$\begin{aligned} b_u e^{i\phi_u} &= \frac{f'_u}{M_2^2} (\mu_{12} \langle H_1^\dagger \rangle \langle \eta \rangle + \mu_{21} \langle H_1^\dagger \rangle \langle \eta^\dagger \rangle), \\ b_d e^{i\phi_d} &= \frac{f'_d}{M_2^2} (\mu_{12} \langle H_1 \rangle \langle \eta^\dagger \rangle + \mu_{21} \langle H_1 \rangle \langle \eta \rangle), \end{aligned} \quad (3.9)$$

where $b_{u,d}$ are absolute values of the right-hand sides and $e^{i\phi_{u,d}}$ are their phases. Note that $\langle H_1 \rangle$ is real, while $\langle \eta \rangle$ is complex. Remarkably, we find $\phi_d = -\phi_u$. We denote $\phi = \phi_d = -\phi_u$, where the phase ϕ is nonvanishing unless $\mu_{12} = \mu_{21}$. Here, M_2 is the mass of the heavy Higgs H_2 and we consider it to be of the order of $|\langle \eta \rangle|$.

A crucial point is that both matrices have complex phases and we can explain the CP violation in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Furthermore, the determinants of both matrices are real and hence we can solve the strong CP problem as pointed out in Refs. [10,23,24]. Note that we consider that the present Lagrangian with the noninvertible selection rule holds at some high-energy scale M and we take it $M = 10^{16}$ GeV. The predictions at the electroweak scale have been obtained

by taking into account the renormalization effects from the weak to the high energy scale. Indeed, we have fitted the model parameters by the data of the masses and mixing matrix at $M = 10^{16}$ GeV, which were given by using the renormalization effects in some works, for example Ref. [51] in our paper. We have also checked that we do not have any visible changes for the low energy predictions even if we change the scale M by a few digits.

We ignore, throughout this paper, the cutoff suppressed higher order terms in the Lagrangian, since such terms are strongly dependent on the dynamics of the physics at the cutoff scale and also of the nature of the η .

We add the $U(1)_{B-L}$ gauge symmetry. This symmetry naturally requires three right-handed neutrinos $N_{Ri} i = 1-3$ to cancel the gauge anomalies. Here, the N_R is defined as a charge conjugate state of the original $\bar{N}$. Its spontaneous breaking generates heavy Majorana masses for the N_{Ri} , which produce the observed small neutrino masses through the seesaw mechanism. We introduce a new scalar boson Φ whose vev gives the Majorana masses via the Yukawa coupling $\Phi^\dagger N_{Ri} N_{Ri}$. The quantum numbers of the Φ and the N_{Ri} are shown in Tables I and II. The mass matrix for the N_{Ri} is a real diagonal matrix by our assignment of classes shown in Tables I and II. Notice that the CP -violating scalar η never couples to $N_{Ri} N_{Rj}$, since it does not have the $U(1)_{B-L}$ charge.

IV. PHENOMENOLOGY

We are ready to discuss the phenomenology of our model.

A. The CKM matrix

The mass matrices $M'_{u,d}$ for the up-type and the down-type quarks are given by Eq. (3.8). As the usual convention, we define mass matrices as $\bar{u}_L M_u u_R$ and $\bar{d}_L M_d d_R$. Thus, we have $M_{u,d} = (M'_{u,d})^*$, which are

$$M_d = \begin{pmatrix} 0 & a_d & 0 \\ a'_d & b_d e^{-i\phi} & c_d \\ 0 & c'_d & d_d \end{pmatrix}, \quad M_u = \begin{pmatrix} 0 & a_u & 0 \\ a_u & b_u e^{i\phi} & c_u \\ 0 & c_u & d_u \end{pmatrix}. \quad (4.1)$$

We also consider the quark sector and the lepton sector together as $SU(5)$ multiplets. That is, we consider three $SU(5)$ multiplets $\bar{F} = \mathbf{5}^*$ and $T = \mathbf{10}$. Then, the up-type quark mass matrix is symmetric, that is $a_u = a'_u$ and $c_u = c'_u$ in Eq. (3.8). From the data on the masses and the observed CKM matrix elements [51], we determine all parameters in the above matrices.

The numerical values of parameters in Eq. (3.8) are obtained as shown in Table III.

The number of parameters in the up- and down-type quark mass matrices is 11, but the observable parameters

TABLE III. Allowed range of parameters in Eq. (3.8), where 2σ error bars of the quark masses and the CKM angles and CP phase are taken in Refs. [51,52].

$a_d/d_d \times 10^2$	$a'_d/d_d \times 10^2$	$b_d/d_d \times 10^2$	$c_d/d_d \times 10^2$	c'_d/d_d
0.71–0.84	0.48–0.58	5.1–6.3	5.0–6.1	0.89–1.13
$a_u/d_u \times 10^4$	$b_u/d_u \times 10^3$	$c_u/d_u \times 10^2$	$\phi^{[^\circ]}$	
0.78–1.04	2.4–3.2	1.2–1.6	32–42	

are the six quark masses and the four CKM matrix elements. This is a reason why the determined parameters have relatively large error bars.

B. The charged lepton mass matrix

We consider the charged lepton mass matrix. Here, we have assumed that the Higgs bosons H_1 and H_2 belong to $\mathbf{5}^*$ of the $SU(5)_{\text{GUT}}$. We see immediately the wrong mass relation, that is, $m_e = m_d$ and $m_\mu = m_s$. Thus, we consider that the heavy Higgs boson H_2 is not purely the $\mathbf{5}^*$ but a mixture with $\mathbf{45}$ of the $SU(5)$. Then we have one real free parameter k_e and the mass matrix is given by

$$M_e = \begin{pmatrix} 0 & a' & 0 \\ a & k_e b e^{-i\phi} & c' \\ 0 & c & d \end{pmatrix}, \quad (4.2)$$

where we multiply the (2, 2) element by a factor k_e . The correct mass ratios of the electron and the muon to the tau are obtained by setting $k_e = 3$. It is surprising that the two independent precise mass ratios are explained by choosing one free parameter k_e . See Refs. [53,54] for details. Note that our obtained mass texture is different from those in Refs. [24,53,54].⁵ In addition, while Refs. [53,54] realize such textures through geometric mechanisms in extradimensional constructions, our approach generates a new class of textures from noninvertible selection rules. Furthermore, Ref. [24] has a difficulty to be embedded into the GUT and hence the lepton mixing and the baryon asymmetry were not discussed in Ref. [24], where we must assume many extra symmetries such as $\mathbb{Z}_3$ and $\mathbb{Z}_2$.

C. The CP violation in the neutrino oscillations

As shown above, the mass matrix for the heavy Majorana right-handed neutrinos M_{N_R} is a real diagonal matrix, that is, $M_{N_R} = \text{diag}(M_1, M_2, M_3)$. And the Dirac type mass matrix M_D is given by

$$M_D = \begin{pmatrix} 0 & a'_\nu & 0 \\ a_\nu & b_\nu e^{i\phi} & c'_\nu \\ 0 & c_\nu & d_\nu \end{pmatrix}, \quad (4.3)$$

where we note that M_D is obtained by two terms of $f_\nu F N \bar{H}_1^\dagger + f'_\nu F N \bar{H}_2^\dagger$. The light neutrino mass matrix M_ν is given (via the seesaw mechanism) by,

$$M_\nu = \begin{pmatrix} A'^2 & A' B e^{i\phi} & A' C \\ A' B e^{i\phi} & A^2 + C'^2 + B^2 e^{2i\phi} & B C e^{i\phi} + C' D \\ A' C & B C e^{i\phi} + C' D & D^2 + C^2 \end{pmatrix}, \quad (4.4)$$

where the parameters are defined by $A = a_\nu/\sqrt{M_1}$, $A' = a'_\nu/\sqrt{M_2}$, $B = b_\nu/\sqrt{M_2}$, $C = c_\nu/\sqrt{M_2}$, $C' = c'_\nu/\sqrt{M_3}$, and $D = d_\nu/\sqrt{M_3}$.

D. Predictions of the CP phase and the neutrinoless double beta decay

Since the charged lepton mass matrix and neutrino mass matrix are given in Eqs. (4.2) and (4.3), we can discuss the CP violation of neutrinos and the effective mass in the neutrinoless double beta decays $m_{\beta\beta}$ by inputting data of leptons in Ref. [55]. Then, we obtain the allowed ranges of parameters by scanning them in the real space with $|D| \geq |A|, |A'|, |B|, |C|, |C'|$ in order to realize normal hierarchy (NH) of neutrino masses.

Then, we predict the CP -violating phase δ_{CP} in the neutrino oscillation and the mass parameter for the neutrinoless double beta decay $m_{\beta\beta}$ by using the phase ϕ determined from the CKM matrix. The predicted δ_{CP} versus $m_{\beta\beta}$ are shown in Fig. 1. Since the real parameters are allowed in both plus and minus signs, δ_{CP} is allowed in two regions 180° apart. Since one region is in the middle of the experimental allowed region with 1σ , we exclude another region.

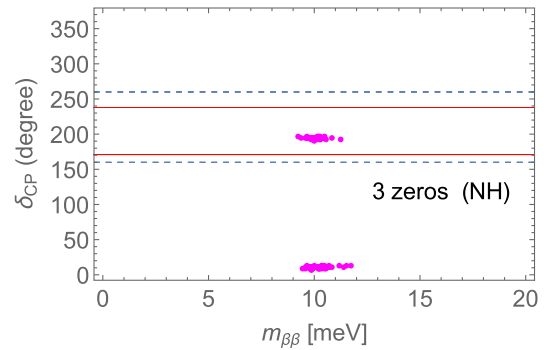


FIG. 1. Predicted δ_{CP} versus $m_{\beta\beta}$ for NH. The region between the horizontal red (blue dashed) lines denotes $1(2)\sigma$ allowed one of δ_{CP} in NuFIT 6.0 [55].

⁵In Ref. [24], the lepton sector was not studied, but we have studied the lepton sector and the baryon asymmetry.

TABLE IV. Allowed regions of parameters in M_ν .

$ A/D $	$ A'/D $	$ B/D $	$ C/D $	$ C'/D $
0.474–0.498	0.464–0.495	0.093–0.104	0.290–0.335	0–0.053

Thus, the predicted range of δ_{CP} is,

$$\delta_{CP} \simeq 192^\circ - 197^\circ. \quad (4.5)$$

It is remarkable that the prediction is within 1σ range of in NuFIT 6.0 [55].

The predicted $m_{\beta\beta}$ is,

$$m_{\beta\beta} \simeq 9 - 11 \text{ [meV]}, \quad (4.6)$$

which will be tested in future experiments. It is noted that $m_{\beta\beta}$ is almost proportional to $\sum m_i$. The predicted range of $\sum m_i$ is 69–74 meV.

These predictions may be compared with previous ones in Ref. [53], where the down-type quark mass matrix is the same as the one in Eq. (4.1), but the up-type quark mass matrix is diagonal,

$$\delta_{CP} \simeq 192^\circ - 195^\circ, \quad m_{\beta\beta} \simeq 8 - 11 \text{ [meV]}. \quad (4.7)$$

Our predictions of the present model in Eqs. (4.5) and (4.6) are a little bit different from the previous ones of Eq. (4.7). This is because the texture of the up-type quark mass matrix there is real diagonal, and the CKM matrix is given only by the down-type quark mass matrix.

We present the magnitude of neutrino parameters in Table IV, where the magnitudes of parameters are almost of the same order except for C' .

E. The Universe's baryon asymmetry

The decays of the heavy right-handed neutrinos (RHNs), N_{Ri} , generate the lepton asymmetry, which is converted to the baryon-number asymmetry in the Universe [56]. The dominant asymmetry is generated by the decay of the lightest RHN and thus we have to identify the lightest RHN to calculate the baryon asymmetry. We show that the CP phase relevant to the generation of the baryon asymmetry is directly related to the low-energy CP phase in the CKM and Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mass matrices.

Let us assume that the N_{R1} is the lightest RHN. The induced lepton asymmetry ϵ_1 is estimated as [57,58]

$$\begin{aligned} \epsilon_1 &= \frac{\Gamma(N_{R1} \rightarrow LH) - \Gamma(N_{R1} \rightarrow \bar{L} \bar{H})}{\Gamma(N_{R1} \rightarrow LH) + \Gamma(N_{R1} \rightarrow \bar{L} \bar{H})} \\ &\simeq -\frac{1}{8\pi} \sum_{j \neq 1}^3 \frac{\text{Im}[(Y_D^\dagger Y_D)_{1j}^2]}{(Y_D^\dagger Y_D)_{11}} F^{V+S} \left(\frac{M_j^2}{M_1^2} \right), \end{aligned} \quad (4.8)$$

where $Y_D = M_D/v$, v being the vacuum expectation value of the standard Higgs, and

$$\begin{aligned} F^{V+S}(x) &= \sqrt{x} \left[(x+1) \ln \left(\frac{x+1}{x} \right) - 1 + \frac{1}{x-1} \right] \\ &\xrightarrow{[x \rightarrow \infty]} \frac{3}{2} \frac{1}{\sqrt{x}}. \end{aligned} \quad (4.9)$$

The baryon-number asymmetry is given by

$$Y_B \simeq -\frac{28}{79} \kappa \frac{\epsilon_1}{g^*}, \quad (4.10)$$

where κ is a dilution factor, which involves the integration of the full set of Boltzmann equations, and $g^* = 106.75$ is taken in SM.

Since only the (2, 2) element of M_D has the complex phase ϕ , the nonvanishing complex phase appears in $(Y_D^\dagger Y_D)_{12}^2$ as,

$$\text{Im}[(Y_D^\dagger Y_D)_{12}^2] = \frac{1}{v^4} a_\nu^2 b_\nu^2 \sin 2\phi, \quad (4.11)$$

which is positive because of $\phi = 32^\circ - 42^\circ$ as seen in Table III and hence the ϵ_1 is negative.

Now, we see that our model predicts the right sign of the baryon-number asymmetry if the N_{R1} is the lightest RHN. It is emphasized that the CP -violating phase for creating the baryon asymmetry is exactly the same as the CP -violating phase ϕ , which is determined from the observed CKM phase.

V. CONCLUSIONS

We have constructed a three-zero texture model for the quark/lepton mass matrices imposing a noninvertible $\tilde{Z}_5^{(1)} \times \tilde{Z}_5^{(2)}$ symmetry. The determinants of all quark mass matrices are real and hence the observed smallness of the physical $\bar{\theta}$ means we have a complete CP invariance at the fundamental level.⁶

Our model construction is based on a $SU(5)$ -GUT inspired SM and the inclusion of the charged lepton and

⁶The three zero textures are broken by quantum corrections. It happens at the two loop level as pointed out in Ref. [23]. However, owing to the small Yukawa coupling constants, it is sufficiently small to solve the strong CP problem.

the neutrino sectors is almost straightforward. This simple model has been successful in explaining naturally the correct sign of the baryon-number asymmetry in the present Universe. It also provides us with the predictions on the CP -violating phase δ_{CP} and on a key parameter $m_{\beta\beta}$ as shown in Eqs. (4.5) and (4.6).

A hope is to test the above predictions in future experiments. However, we should notice here that the predictions in the neutrino sector depend on the charge assignment of the noninvertible symmetry for the right-handed neutrinos, N_{Ri} and the Higgs boson Φ . If the predictions on δ_{CP} and on $m_{\beta\beta}$ in the present paper turn out to be excluded in future experiments, we will come back to the issue.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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