Study of the two-body nonleptonic $B_c(2S)$ weak decays with the QCD factorization approach

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Inspired by the promising prospect of the $B_c(2S)$ meson at the coming HL-LHC experiments, the nonleptonic $B_c(2S)$ meson weak decays induced by both the b and c decays are investigated with the quantum chromodynamics (QCD) factorization approach. It is found that branching ratios for the color- and Cabibbo-Kobayashi-Maskawa (CKM)-favored $B_c(2S) \to B_s\rho$, $B_s\pi$ decays can reach up to $\mathcal{O}(10^{-9})$, which might be measurable. The $B_c(2S)$ decays into the final states containing one charmonium are highly suppressed by the CKM factors, and have significantly small branching ratios, $\mathcal{O}(10^{-12})$ and less, which might be outside the future measurement capability. This paper provides a ready reference for the future experimental study on the hadronic $B_c(2S)$ weak decays.

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I. INTRODUCTION

The B_c meson family is very unique within the quark model assignments. The B_c mesons are composed of two heavy quarks b and c with different flavors, and have nonzero bottomness (B) and charm (C) and electric charge (Q) quantum numbers, $B = C = Q = \pm 1$, no matter what configuration of the spin and relative orbital angular momentum of two quarks. The charged "self-tagging" B_c mesons should in principle be easily identified at experiments. However, up to now, only two established pseudoscalar B_c mesons with the same isospin and spin and parity quantum numbers $I(J^P) = 0(0^-)$ among the abundant spectrum have been clearly listed in the latest PDG [1], i.e., the ground S-wave spin-singlet $B_c(1S)$ meson and its first radial excited state marked as the $B_c(2S)$ meson.

Theoretically, besides the direct production through virtual γ^* and Z^* boson exchanges in e^+e^- annihilation and the indirect production in Z^0, $W^\pm$, H^0 boson decays and t quark decays [2–29], the B_c mesons can be produced principally in hadron-hadron collisions by both the gluon–gluon fusion mechanism and quark-antiquark annihilation [30–32]. The gluon-gluon fusion mechanism is dominant over quark-antiquark annihilation at the Tevatron and LHC [31]. Both the bottomness and charm quantum numbers should be strictly and individually conserved in the chromatic interactions. A $b\bar{b}$ quark pair plus a $c\bar{c}$ quark pair must be generated simultaneously, in which a b (or $\bar{b}$) quark and a $\bar{c}$ (c) quark should be extremely fortunate to bind together and finally hadronize into a color singlet B_c bound state. At the leading order in α_s, the production cross section in hadron-hadron collisions is proportional to α_s^4 and so significantly small and strongly dependent on the collision energy [30–33], where α_s is the coupling of the strong interactions. It was estimated that the inclusive production cross section of the $B_c(1S)$ meson at the LHC is at a level of 1 μb^{-1} for $\sqrt{s} \approx 14$ TeV, about one order of magnitude larger than that at the Tevatron [31–34]. This means that $\mathcal{O}(10^9)B_c(1S)$ mesons can be anticipated with 1 fb^{-1} of accumulated data at the LHC, and more than 10^{12} $B_c(1S)$ mesons will be available with a prospective integrated luminosity of 4 ab^{-1} at the HL-LHC during its exploitation period [35]. Correspondingly, there will be approximately $\mathcal{O}(10^{11})$ $B_c(2S)$ available at HL-LHC experiments, using the ratio of the $B_c(2S)$ to $B_c(1S)$ production cross sections at $\sqrt{s} = 13$ TeV measured by the CMS group, $R = \frac{\sigma(B_c(2S))}{\sigma(B_c(1S))}\mathcal{B}r(B_c(2S) \to B_c(1S)\pi^+\pi^-) = (3.47 \pm 0.63 \pm 0.33)\%$ [36]. A vast amount of promising data provides a solid experimental foundation and precious opportunity to investigate thoroughly the properties of the $B_c(1S)$ and $B_c(2S)$ mesons.

Experimentally, compared with the $B_c(1S)$ meson, the $B_c(2S)$ meson is still an immature and up-and-coming particle, who is first discovered in the $B_c(1S)\pi^+\pi^-$ invariant-mass spectrum by the ATLAS group in 2014 [37] and then identified by both the CMS [38] and LHCb [39] groups in

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TABLE I. The partial widths (Γ_i in unit of keV) and branching ratios ($\mathcal{B}r_i$ in unit of %) of the $B_c(2S)$ meson decay, where the excited axionlike $1P$ and $1P'$ particles are the weighted mixtures of the P-wave spin triplet $^3P_1$ and singlet $^1P_1$ states with the same spin-parity $J^P = 1^+$ quantum numbers; the vector $B_c^*(1S)$ meson is the ground S-wave spin triplet $^3S_1$ state with the spin-parity $J^P = 1^-$ quantum numbers; the total width of the $B_c(2S)$ meson is approximately the sum of four partial widths listed here.

		Ref. [40]		Ref. [41]		Ref. [42]		Ref. [43]		Ref. [44]		Ref. [45]	
Decay mode	Final states	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$
Hadronic	$B_c(1S)\pi\pi$	50	90.4	50	71.9	57	88.1	10.68	51.42	42	54.4	25	76.9
E1	$B_c(1P)\gamma$	0.0	0.0	6.4	9.2	1.3	2.0	7.11	34.23	0.0	0.0	2.8	8.6
E1	$B_c(1P')\gamma$	5.2	9.4	13.1	18.8	6.1	9.4	2.91	14.01	35	45.3	4.4	13.5
M1	$B_c^*(1S)\gamma$	0.1	0.2	0.06	0.1	0.3	0.5	0.07	0.34	0.25	0.3	0.3	1.0
Total		55.3	100	69.6	100	64.7	100	20.77	100	77.3	100	33	100

2019. The properties of the $B_c(1S)$ meson has been extensively studied whether at experiment or theory. In contrast, our knowledge about the properties of the $B_c(2S)$ meson is woefully inadequate and urgently needs to be enhanced and enriched further. In the dictionary of the authoritative PDG [1], the averaged mass of the $B_c(2S)$ meson is $m = 6871.2 \pm 1.0$ MeV and lies below the open BD threshold. The favorite decays, predominantly by hadronic cascades and the electric dipole (E1) or magnetic dipole (M1) transitions, lead to the total width of the $B_c(2S)$ meson are generally less than one hundred keV [40–45], some of which are listed in Table I. It is easy to imagine that the reconstruction of demonstrable signals of the hadronic decay $B_c(2S) \to B_c(1S)\pi^+\pi^-$ at experiments should be very challenging, because $m_{B_c(2S)} - m_{B_c(1S)} - m_{2\pi} \approx 320$ MeV and the soft pions are submerged in noisy background. For the $B_c(2S)$ electromagnetic transitions, both the final $B_c(1P^{(\prime)})$ and $B_c^*(1S)$ states have not been identified definitely at experiments for the moment. Besides, the $B_c(2S)$ meson can still decay via the weak interactions.

The $B_c(2S)$ meson weak decays have not attracted necessary attention, and so there was very few research work on the $B_c(2S)$ weak decays. Theoretically, the weak decays of the $B_c(2S)$ meson are similar to those of the lowest $B_c(1S)$ meson that can decay only through the weak interactions within the standard model (SM) of elementary particles, and can be categorized into three classes [46–55]: (1) the b quark decay with the c quark as a spectator, (2) the c quark decay with the b quark as a spectator, and (3) the b and c quarks annihilation into a virtual $W^{\pm}$ boson. It is estimated that the branching ratio of the $B_c(2S)$ meson weak decays is tiny, about $1/\Gamma_{B_c(2S)}\tau_{B_c(1S)} \sim \mathcal{O}(10^{-8})$, where $\tau_{B_c(1S)}$ and $\Gamma_{B_c(2S)}$ are the lifetime of the $B_c(1S)$ meson and the total width of the $B_c(2S)$ meson, respectively. An observation of an abnormally large production rate of the $B_c(2S)$ meson weak decays would be a hint of new physics beyond SM. An advantage of weak decays compared with the strong and electromagnetic transitions is that the $B_c(2S)$ meson might be easily identified at experiments from its secondary particles having relatively larger momentum at the rest $B_c(2S)$ frame. Furthermore, there are much richer weak decay modes, which could be used to overconstrain the parameters obtained from the B and D meson decays, and check the applicability of various phenomenological models. Based on the potential prospects of the $B_c(2S)$ meson in the planed LH-LHC, it seems that there are some possibilities for experimental study on the $B_c(2S)$ meson weak decays.

In this paper, we will estimate the branching ratios for the nonleptonic two-body $B_c(2S)$ weak decays using the phenomenological QCD factorization (QCDF) approach [56–61], focusing on the $B_c(2S) \to \psi(nS)P$, $\eta_c(nS)P$ decays induced by the $b \to c$ quark transition belonging to the class (1), and the $B_c(2S) \to BP, BV$ decays induced by the c quark decays belonging to the class (2), where P and V denote the light ground pseudoscalar and vector mesons. The $B_c(2S)$ weak annihilation decays belonging to the class (3) are power suppressed with the QCDF approach. It is estimated that the dominant contribution to the lifetime of the $B_c(1S)$ meson is given by the c quark decays (≈70%) and the b quark decays (≈20%) [31]. The class (1) and (2) decays can be easily distinguishable on the diagrammatic level, and they do not interfere with each other in any appreciable way [48]. Our study will provide a ready and helpful reference for experimental discovery and investigation. The remaining parts of this paper are as follows. Section II delineates the theoretical framework for the nonleptonic $B_c(2S)$ weak decays. Section III is dedicated to presenting the numerical results and discussions. Section IV is a brief summary.

II. THEORETICAL FRAMEWORK

A. The effective Hamiltonian

For the $B_c(2S) \to BP$, BV decays induced by the c quark decays, the effective weak interaction Hamiltonian can be written as [50],

$$\mathcal{H}_{\rm eff}^c = \frac{G_F}{\sqrt{2}} \sum_{q_1,q_2} V_{cq_1}^* V_{uq_2} [C_1(\mu)O_1 + C_2(\mu)O_2] + \text{H.c.}, \tag{1}$$

$$O_1 = [\bar{q}_{1,\alpha}\gamma_\mu(1-\gamma_5)c_\alpha]\,[\bar{u}_\beta\gamma^\mu(1-\gamma_5)q_{2,\beta}], \quad (2)$$

$$O_2 = [\bar{q}_{1,\alpha}\gamma_\mu(1-\gamma_5)c_\beta]\,[\bar{u}_\beta\gamma^\mu(1-\gamma_5)q_{2,\alpha}], \quad (3)$$

where $G_F \approx 1.166 \times 10^{-5}$ GeV^{-2} [1] is the Fermi coupling constant of the weak interactions; $C_{1,2}$ is the Wilson coefficient; the local operator $O_{1,2}$ denotes the four-quark pointlike weak interactions; α and β are the $SU(3)$ color indices; V_{uq_1} and V_{cq_2} are the quark-mixing Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, with $q_{1,2} = d$ and s. Using the Wolfenstein parametrization, the CKM elements, up to $\mathcal{O}(\lambda^6)$, can be written as [1],

$$V_{ud} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 + \mathcal{O}(\lambda^6), \quad (4)$$

$$V_{us} = \lambda + \mathcal{O}(\lambda^6), \quad (5)$$

$$V_{ub} = A\lambda^3(\rho - i\eta), \quad (6)$$

$$V_{cd} = -\lambda + A^2\lambda^5\left[\frac{1}{2} - (\rho + i\eta)\right] + \mathcal{O}(\lambda^6) \quad (7)$$

$$V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 - \frac{1}{2}A^2\lambda^4 + \mathcal{O}(\lambda^6), \quad (8)$$

$$V_{cb} = A\lambda^2 + \mathcal{O}(\lambda^6). \quad (9)$$

The latest numerical values of the Wolfenstein parameters given by PDG [1] are,

$$A = 0.826^{+0.016}_{-0.015}, \quad (10)$$

$$\lambda = 0.22501 \pm 0.00068, \quad (11)$$

$$\bar{\rho} = 0.1591 \pm 0.0094, \quad (12)$$

$$\bar{\eta} = 0.3523^{+0.0073}_{-0.0071}. \quad (13)$$

It should be pointed out that (1) both the penguin and annihilation contributions being proportional to the CKM factor $V_{ub}V_{cb}^* \sim \mathcal{O}(\lambda^5)$ are highly suppressed relative to those from the operator $O_{1,2}$ being proportional to $V_{ud}V_{cs}^* \sim \mathcal{O}(1)$ or $V_{ud}V_{cd}^* \sim \mathcal{O}(\lambda)$ or $V_{us}V_{cs}^* \sim \mathcal{O}(\lambda)$ or $V_{us}V_{cd}^* \sim \mathcal{O}(\lambda^2)$. Hence, the penguin and annihilation contributions to branching ratios can be safely neglected here for the moment. (2) The Wilson coefficients C_i summarize the physics contributions from scale higher than μ and can be calculated with the perturbation theory. It is generally recognized that the Wilson coefficients C_i have nothing to do with the hadronization process of quarks, and so can be factorized from specific hadron processes, i.e., they are generally universal and process-independent. Their values including the next-to-leading order (NLO) corrections can be evaluated with the renormalization group equation [62]. The numerical values of Wilson coefficients for the c and b quark decays can be found in numerous references, such as Refs. [49–51].

For the $B_c(2S) \to \psi(1S, 2S)P$, $\eta_c(1S, 2S)P$ decays induced by the $b \to c$ quark transition, the expression of the low-energy effective Hamiltonian is written as [49],

$$\mathcal{H}^b_{\rm eff} = \frac{G_F}{\sqrt{2}}\sum_{q=d,s} V_{cb}V_{uq}^*[C_1(\mu)O_1' + C_2(\mu)O_2'] + \text{H.c.}, \quad (14)$$

$$O_1' = [\bar{c}_\alpha\gamma_\mu(1-\gamma_5)b_\alpha]\,[\bar{q}_\beta\gamma^\mu(1-\gamma_5)u_\beta], \quad (15)$$

$$O_2' = [\bar{c}_\alpha\gamma_\mu(1-\gamma_5)b_\beta]\,[\bar{q}_\beta\gamma^\mu(1-\gamma_5)u_\alpha]. \quad (16)$$

Note that because the valence quarks in both final states differ from each other, there is no contributions from penguin and annihilation in Eq. (14).

Taking the $B_c(2S) \to \psi\pi$ decay as an example, the decay amplitude can be written as,

$$\begin{aligned}\mathcal{A} &= \langle\psi\pi|\mathcal{H}_{\rm eff}|B_c(2S)\rangle \\ &= \frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*\sum_{i=1}^{2} C_i\langle\psi\pi|O_i'|B_c(2S)\rangle, \end{aligned} \quad (17)$$

where $\langle\psi\pi|O_i'|B_c(2S)\rangle$ is usually called as the hadronic matrix element (HME). HME describes the transitions from the local effective operator at the quark level to the initial and final states at the hadron level. The participation of the strong interactions make it very complicated to calculate HME, which including the perturbative and nonperturbative contributions. To obtain the decay amplitudes, the remaining and most complex task is to accurately evaluate HMEs.

B. Hadronic matrix elements

The QCDF approach [56–61] is one of the recently developed QCD-improved phenomenological models to deal with HME, based on the exclusive hard scattering approach [63] and the power series expansion in the heavy quark limit. At the leading power of inverse heavy quark mass, the master QCDF formula for HME is generally expressed as a convolution integral form, incorporating both the perturbative (hard scattering kernel) and nonperturbative (hadronic wave function) parts; specifically, for the case with a heavy recoiled meson, HME is written as [57],

$$\langle\psi\pi|O_i'|B_c(2S)\rangle = F^{B_c(2S)\to\psi}\int_0^1 dz T(z)\phi_\pi(z), \quad (18)$$

where both the transition form factor $F^{B_c(2S)\to\psi}$ and the light-cone distribution amplitude ϕ_π of the emitted pion are nonperturbative but universal parameters at the hadron level, and can be obtained from other nonperturbative method or extracted from data. The scattering kernel T arising from hard

gluon exchange among quarks are perturbatively calculable. By including the QCD radiative corrections to HME, part of the strong phases can be retrieved for CP asymmetries, and the renormalization scale dependence of HME can be recuperated to cancel the scale dependence of the Wilson coefficients and obtain a physical decay amplitude.

The decay amplitude in Eq. (17) with the QCDF approach can be written as,

$$\mathcal{A} = \sqrt{2} G_F V_{cb} V_{ud}^* A_0^{B_c(2S)\to\psi} f_\pi m_\psi (\epsilon_\psi \cdot p_\pi) a_1, \tag{19}$$

where $A_0^{B_c(2S)\to\psi}$ is the form factor, and its definition is given by Eq. (31). The decay constant f_π is defined by Eq. (26), and it can also be obtained from the normalization condition,

$$\int_0^1 dz \phi_\pi(z) = f_\pi, \tag{20}$$

with the leading twist distribution amplitude [64,65],

$$\phi_\pi(z) = f_\pi 6 z\bar{z} \sum_{n=0} a_n^\pi C_n^{3/2}(z-\bar{z}), \tag{21}$$

where z denotes the longitudinal momentum fractions of valence quark; $\bar{z} = 1 - z$; the expansion coefficient coefficient a_n^π of the Gegenbauer polynomials $C_n^{3/2}$ is usually called as the Gegenbauer moment, which is a nonperturbative shape parameter. The QCDF coefficients a_i including the NLO QCD radiative contributions are written as,

$$a_1 = C_1^{\mathrm{NLO}} + \frac{1}{N_c} C_2^{\mathrm{NLO}} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c} C_2^{\mathrm{LO}} V, \tag{22}$$

$$a_2 = C_2^{\mathrm{NLO}} + \frac{1}{N_c} C_1^{\mathrm{NLO}} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c} C_1^{\mathrm{LO}} V, \tag{23}$$

where the analytic expression of vertex correction factor V will no longer be displayed here. The details of the vertex factor V and decay amplitudes have already been addressed in Ref. [49] for the $B_c(2S) \to \psi P$, $\eta_c P$ decays, and Ref. [51] for the $B_c(2S) \to BP$, BV decays. It has been proven that the NLO QCDF coefficients a_i are independent of the renormalization scale at the order of α_s [49,61]. The QCDF coefficients a_i beyond NLO can be found in Ref. [66] for the $b \to c$ transition, and Ref. [67] for the $b \to q$ transition. As stated in Ref. [67], the coefficients a_1 is rather stable against radiative corrections. For a rough estimate to branching ratio for the $B_c(2S)$ weak decay, the NLO QCDF coefficients a_i are basically sufficient for the requirements.

C. Decay constants and form factors

In the above QCDF formula for HME, Eq. (18), at the leading order in α_s expansion, the scattering kernel $T(z) = 1$. With the normalization condition Eq. (21), HME are parametrized as the product of form factors and decay constants. One will return to the simple "naive factorization" (NF) scenario, i.e., HME in Eq. (18) can be divided into two HMEs of color-singlet current operators, for example,

$$\langle \psi\pi | O_1' | B_c(2S)\rangle = \langle \pi | \bar{d}_\beta \gamma^\mu (1-\gamma_5) u_\beta | 0\rangle \times \langle \psi | \bar{c}_\alpha \gamma_\mu (1-\gamma_5) b_\alpha | B_c(2S)\rangle. \tag{24}$$

HME for creating a meson from the vacuum is parametrized by,

$$\langle P(p)|V_\mu|0\rangle = 0, \tag{25}$$

$$\langle P(p)|A_\mu|0\rangle = -i f_P p_\mu, \tag{26}$$

$$\langle V(p,\epsilon)|V_\mu|0\rangle = f_V m_V \epsilon_{V,\mu}^*, \tag{27}$$

$$\langle V(p,\epsilon)|A_\mu|0\rangle = 0, \tag{28}$$

where f_P and f_V are the decay constants of pseudoscalar P and vector V mesons, respectively; m_V and ϵ_V denote the mass and polarization of vector meson, respectively.

For the mixing of physical pseudoscalar η and η' meson, we adopt the quark-flavor basis description proposed in Ref. [68], and neglect the contributions from possible gluonium and $c\bar{c}$ compositions, i.e.,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \tag{29}$$

where $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$; the mixing angle $\phi = (39.3 \pm 1.0)^\circ$ [68].

Here, we will adopt a convention for hadronic transition form factors defined in Ref. [69].

$$\langle X(p_2)|\bar{q}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha | B_c(2S)(p_1)\rangle = \frac{m_{B_c(2S)}^2 - m_X^2}{q^2} q^\mu F_0^{B_c(2S)\to X}(q^2) + \left(p_1^\mu + p_2^\mu - \frac{m_{B_c(2S)}^2 - m_X^2}{q^2} q^\mu \right) F_1^{B_c(2S)\to X}(q^2), \tag{30}$$

$$\langle\psi(p_2,\epsilon)|\bar{q}_\alpha\gamma^\mu(1-\gamma_5)b_\alpha|B_c(2S)(p_1)\rangle = \epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}_\psi p_1^\alpha p_2^\beta \frac{2V^{B_c(2S)\to\psi}(q^2)}{m_{B_c(2S)}+m_\psi} + i\frac{2m_\psi(\epsilon^*_\psi\cdot q)}{q^2}q_\mu A_0^{B_c(2S)\to\psi}(q^2)$$
$$+ i\epsilon^*_{\psi,\mu}(m_{B_c(2S)}+m_\psi)A_1^{B_c(2S)\to\psi}(q^2) - i\frac{(\epsilon^*_\psi\cdot q)}{m_{B_c(2S)}+m_\psi}(p_1+p_2)_\mu A_2^{B_c(2S)\to\psi}(q^2)$$
$$- i\frac{2m_\psi(\epsilon^*_{\psi,\mu}\cdot q)}{q^2}q_\mu A_3^{B_c(2S)\to\psi}(q^2), \tag{31}$$

where X denotes the pseudoscalar $B_{u,d,s}$, $\eta_c(1S)$ and $\eta_c(2S)$ mesons; ψ denotes the vector $\psi(1S)$ and $\psi(2S)$ mesons; the momentum transfer $q = p_1 - p_2$; ϵ^*_ψ denotes the polarization vector of the ψ meson; $F_{0,1}^{B_c(2S)\to X}$, $V^{B_c(2S)\to\psi}$ and $A_{0,1,2,3}^{B_c(2S)\to\psi}$ are the transition form factors. At the large recoil limit, $q^2 = 0$, there is

$$F_0^{B_c\to X}(0) = F_1^{B_c\to X}(0), \tag{32}$$

$$A_0^{B_c\to\psi}(0) = A_3^{B_c\to\psi}(0). \tag{33}$$

Currently, there is no theoretical calculation for these form factors. We will use the Wirbel-Stech-Bauer approach [69] to give a rough estimation of the form factor. The form factors at the pole $q^2 = 0$ are defined as the convolution of wave functions of both the initial and the recoiled mesons [69], i.e.,

$$F_0^{B_c(2S)\to X}(0) = \int d\vec{k}_\perp \int_0^1 dx \Phi_X(\vec{k}_\perp, x, 0, 0)\Phi_{B_c(2S)}(\vec{k}_\perp, x, 0, 0), \tag{34}$$

$$A_0^{B_c(2S)\to\psi}(0) = \int d\vec{k}_\perp \int_0^1 dx \Phi_\psi(\vec{k}_\perp, x, 1, 0)\sigma_z\Phi_{B_c(2S)}(\vec{k}_\perp, x, 0, 0), \tag{35}$$

where σ_z is the Pauli matrix acting on the spin indices of the decaying quark; the variables $\vec{k}_\perp$, x, J, J_z of the wave function $\Phi(\vec{k}_\perp, x, J, J_z)$ denote respectively the transverse momentum and the fraction of the longitudinal momentum carried by the nonspectator quark, the total spin and its z-component. Both the $B_c(2S)$ meson and charmonium consist of two heavy flavors. The motion of the valence quarks in these mesons should be nonrelativistic. For the $B_c(2S)$ and charmonium bound states, the radial solution of the Schrödinger equation with an isotropic harmonic oscillator potential by separation of the angular variables, after the Fourier transformation from coordinate space to momentum domain, is written as [70–74],

$$\Phi_{B_c(2S)}(\vec{k}_\perp, x) = A\left\{\frac{\vec{k}_\perp^2 + \bar{x}m_c^2 + xm_b^2}{6\alpha_1^2 x\bar{x}} - 1\right\}\exp\left\{-\frac{\vec{k}_\perp^2 + \bar{x}m_c^2 + xm_b^2}{8\alpha_1^2 x\bar{x}}\right\}, \tag{36}$$

$$\Phi_{\eta_c(1S)}(\vec{k}_\perp, x) = \Phi_{\psi(1S)}(\vec{k}_\perp, x) = B\exp\left\{-\frac{\vec{k}_\perp^2 + m_c^2}{8\alpha_2^2 x\bar{x}}\right\}, \tag{37}$$

$$\Phi_{\eta_c(2S)}(\vec{k}_\perp, x) = \Phi_{\psi(2S)}(\vec{k}_\perp, x) = C\left\{\frac{\vec{k}_\perp^2 + m_c^2}{6\alpha_2^2 x\bar{x}} - 1\right\}\exp\left\{-\frac{\vec{k}_\perp^2 + m_c^2}{8\alpha_2^2 x\bar{x}}\right\}, \tag{38}$$

where the mean value of square of transverse momentum is $\alpha_1^2 = \mu\omega$ with the reduced mass $\mu = \frac{m_b m_c}{m_b+m_c}$ and the characteristic frequency $\omega \approx 0.50 \pm 0.05$ GeV [40–45,53,75–84]; and $\alpha_2 = m_c\alpha_s$ for the charmonium [73,74]. The parameters of A, B, C are the normalization coefficients. For the $B_{u,d,s}$ meson, we will adopt the Gaussian type proposed in [85], after the Fourier transformation,

$$\Phi_{B_{u,d,s}}(\vec{k}_\perp, x) = Dx^2\bar{x}^2\exp\left\{-\frac{\vec{k}_\perp^2 + x^2m_B^2}{2\alpha_3^2}\right\}, \tag{39}$$

where D is the normalization constant; $\alpha_3 = 0.45 \pm 0.05$ GeV for $B_{u,d}$ meson and 0.55 ± 0.05 GeV for B_s meson [53,86–88].

TABLE II. The numerical values of the input parameters, where their central values are regarded as the default inputs unless otherwise specified.

Mass of particle (in units of MeV) [1]			
$m_{B_u} = 5279.41(7)$	$m_{\eta_c(1S)} = 2984.1(4)$	$m_{\pi^\pm} = 139.57$	$m_\rho = 775.26(23)$
$m_{B_d} = 5279.72(8)$	$m_{\psi(1S)} = 3096.900(6)$	$m_{\pi^0} = 134.98$	$m_\omega = 782.66(13)$
$m_{B_s} = 5366.93(10)$	$m_{\eta_c(2S)} = 3637.7(9)$	$m_\eta = 547.862(17)$	$m_\phi = 1019.461(16)$
$m_{B_c(2S)} = 6871.2(1.0)$	$m_{\psi(2S)} = 3686.097(11)$	$m_{\eta'} = 957.78(6)$	$m_{K^{*\pm}} = 895.5(8)$
$m_b = 4183(7)$	$m_{K^\pm} = 493.677(15)$	$m_{K^0} = 497.611(13)$	$m_{K^{*0}} = 895.55(20)$
Decay constant (in units of MeV)			
$f_\pi = 130.2(1.2)$ [1]	$f_K = 155.7(3)$ [1]	$f_{\eta_q} = 1.07(2)f_\pi$ [68]	$f_{\eta_s} = 1.34(6)f_\pi$ [68]
$f_\rho = 216(3)$ [65]	$f_\omega = 187(5)$ [65]	$f_\phi = 215(5)$ [65]	$f_{K^*} = 220(5)$ [65]
Gegenbauer moments at the scale $\mu = 1$ GeV [64,65]			
$a_1^{\bar{K}} = -a_1^K = 0.06(3)$	$a_2^{\bar{K}} = a_2^K = 0.25(15)$	$a_2^\pi = 0.25(15)$	
$a_1^{\bar{K}^*} = -a_1^{K^*} = 0.03(2)$	$a_2^{\bar{K}^*} = a_2^{K^*} = 0.11(9)$	$a_2^\rho = a_2^\omega = 0.15(7)$	$a_2^\phi = 0.18(8)$

According to the above definition and conventions, and using the input parameters listed in Table II, we obtain the numerical values of form factors at the pole $q^2 = 0$,

$$F_0^{B_c(2S)\to B_{u,d}}(0) = 0.397, \tag{40}$$

$$F_0^{B_c(2S)\to B_s}(0) = 0.426, \tag{41}$$

$$F_0^{B_c(2S)\to \eta_c(1S)}(0) = A_0^{B_c(2S)\to \psi(1S)}(0) = 0.270, \tag{42}$$

$$F_0^{B_c(2S)\to \eta_c(2S)}(0) = A_0^{B_c(2S)\to \psi(2S)}(0) = 0.260, \tag{43}$$

which will be used to provide an order-of-magnitude estimation of the branching ratio of the $B_c(2S)$ meson weak decay.

III. NUMERICAL RESULTS AND DISCUSSION

In the rest frame of $B_c(2S)$, branching ratio for the nonleptonic two-body $B_c(2S)$ weak decays is written as,

$$\mathcal{B}r = \frac{p_{\rm cm}}{8\pi m^2_{B_c(2S)}\Gamma_{B_c(2S)}}|\mathcal{A}|^2, \tag{44}$$

where $p_{\rm cm}$ is the common momentum of final particles; $\Gamma_{B_c(2S)}$ is the total width of the $B_c(2S)$ meson. The experimental data on $\Gamma_{B_c(2S)}$ is unavailable at present. According to the numbers in Table I obtained with various theoretical models, we will use $\Gamma_{B_c(2S)} = 55$ keV for the moment. The similar amplitudes $\mathcal{A}$ for the $B_c(2S) \to BP$, BV decays can be found in Refs. [50,51], and the $B_c(2S) \to \psi P$, $\eta_c P$ decays in Ref. [49], with the replacement of symbol B_c by $B_c(2S)$.

The numerical results on branching ratios for the $B_c(2S) \to BP$, BV decays are displayed in Table III, and the $B_c(2S) \to \eta_c(1S, 2S)P, \psi(1S, 2S)P$ decays in Table IV. Our comments are as follows.

(1) According to the QCDF coefficients $a_{1,2}$ and the CKM factors of the decay amplitudes, there is a clear hierarchy among the branching ratios for the $B_c(2S)$ weak decays, which can be categorized into I-a, I-b, I-c, I-d, II-a, II-b, II-c cases listed in Tables III and IV, i.e.,

$$\mathcal{B}r \sim \begin{cases} \mathcal{O}(10^{-9}), & \text{for the I-a case;} \\ \mathcal{O}(10^{-10}), & \text{for the I-b, II-a case} \\ \mathcal{O}(10^{-11} \sim 10^{-12}), & \text{for the II-b case;} \\ \mathcal{O}(10^{-12}), & \text{for the I-c case;} \\ \mathcal{O}(10^{-13}), & \text{for the I-d, II-c case.} \end{cases} \tag{45}$$

(2) The $B_c(2S) \to B_s\pi$, $B_s\rho$ decays belonging to the I-a case have branching ratio $\sim\mathcal{O}(10^{-9})$. The $B_c(2S) \to B_s K^{(*)}$, $B_d\pi$, $B_d\rho$ decays belonging to the I-b case and the $B_c(2S) \to B_u\bar{K}^{(*)}$ decays belonging to the II-a case have branching ratio $\sim\mathcal{O}(10^{-10})$. It is expected to have hundreds or dozens of the $B_c(2S)$ weak decay events belonging to the I-a, I-b and II-a cases with the potential $\mathcal{O}(10^{11})B_c(2S)$ data in the coming HL-LHC experiment, which might be observed and used to identify the $B_c(2S)$ mesons. Interestingly, the $B_c(2S) \to B_s\pi$, $B_s\rho$ decays have the top priority in searching for the nonleptonic $B_c(2S)$ weak decays at the HL-LHC experiment, where the characteristic charged $\pi^\pm$ and $\rho^\pm$ particles with a definite momentum $\sim$1.33 GeV in the rest $B_c(2S)$ frame should be easily and unambiguously identified by various fine-resolution subdetectors with high detection efficiency.

(3) The charmonium particles have often and widely been used to tag and reconstruct the weak decay events of the ground $B_c(1S)$ meson at experiments, such as the semileptonic $B_c(1S) \to J/\psi(1S)\ell\nu_\ell$ decays and hadronic $B_c(1S) \to \psi(nS)\pi$, $\psi(nS)\pi\pi$, $\psi(nS)\pi\pi\pi$, $\psi(nS)K\pi\pi$, $\psi(nS)KK\pi$, $J/\psi D$, $\chi_{cJ}\pi$, ... decays ($n = 1$ and 2) [1]. However, the numbers in Table IV show that branching ratios for the $B_c(2S) \to \eta_c(1S, 2S)P$, $\psi(1S, 2S)P$ decays

TABLE III. The numerical results on CP-averaged branching ratios for the $B_c(2S) \to BP, BV$ decays, where the theoretical uncertainties come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.2)m_c$, decay constants, and Gegenbauer moments, respectively.

	Decay amplitude ($\mathcal{A}$)			
Final states	Coefficient	CKM factor	Case	Branching ratio ($\mathcal{B}r$)
$B_s^0\pi^+$	a_1	$V_{cs}^*V_{ud} \sim \mathcal{O}(1)$	I-a	$(1.93^{+0.00+0.18+0.01}_{-0.00-0.10-0.01}) \times 10^{-9}$
$B_s^0\rho^+$	a_1	$V_{cs}^*V_{ud} \sim \mathcal{O}(1)$	I-a	$(3.34^{+0.00+0.31+0.10}_{-0.00-0.17-0.10}) \times 10^{-9}$
$B_s^0K^+$	a_1	$V_{cs}^*V_{us} \sim \mathcal{O}(\lambda)$	I-b	$(1.40^{+0.01+0.13+0.02}_{-0.01-0.07-0.02}) \times 10^{-10}$
$B_s^0K^{*+}$	a_1	$V_{cs}^*V_{us} \sim \mathcal{O}(\lambda)$	I-b	$(1.53^{+0.01+0.14+0.07}_{-0.01-0.08-0.07}) \times 10^{-10}$
$B_d^0\pi^+$	a_1	$V_{cd}^*V_{ud} \sim \mathcal{O}(\lambda)$	I-b	$(1.04^{+0.01+0.10+0.01}_{-0.01-0.05-0.01}) \times 10^{-10}$
$B_d^0\rho^+$	a_1	$V_{cd}^*V_{ud} \sim \mathcal{O}(\lambda)$	I-b	$(1.90^{+0.01+0.18+0.06}_{-0.01-0.10-0.05}) \times 10^{-10}$
$B_d^0K^+$	a_1	$V_{cd}^*V_{us} \sim \mathcal{O}(\lambda^2)$	I-c	$(7.60^{+0.09+0.72+0.10}_{-0.09-0.40-0.10}) \times 10^{-12}$
$B_d^0K^{*+}$	a_1	$V_{cd}^*V_{us} \sim \mathcal{O}(\lambda^2)$	I-c	$(8.97^{+0.11+0.84+0.43}_{-0.11-0.47-0.42}) \times 10^{-12}$
$B_u^+\bar{K}^0$	a_2	$V_{cs}^*V_{ud} \sim \mathcal{O}(1)$	II-a	$(2.85^{+0.00+1.60+0.07}_{-0.00-0.79-0.07}) \times 10^{-10}$
$B_u^+\bar{K}^{*0}$	a_2	$V_{cs}^*V_{ud} \sim \mathcal{O}(1)$	II-a	$(3.33^{+0.00+1.87+0.19}_{-0.00-0.92-0.18}) \times 10^{-10}$
$B_u^+\pi^0$	a_2	$V_{cd}^*V_{ud} \sim \mathcal{O}(\lambda)$	II-b	$(5.61^{+0.03+3.15+0.09}_{-0.03-1.55-0.09}) \times 10^{-12}$
$B_u^+\rho^0$	a_2	$V_{cd}^*V_{ud} \sim \mathcal{O}(\lambda)$	II-b	$(1.02^{+0.01+0.57+0.04}_{-0.01-0.28-0.03}) \times 10^{-11}$
$B_u^+\omega$	a_2	$V_{cd}^*V_{ud} \sim \mathcal{O}(\lambda)$	II-b	$(7.55^{+0.04+4.24+0.46}_{-0.04-2.09-0.44}) \times 10^{-12}$
$B_u^+\eta$	a_2	$V_{cs}^*V_{us}$, $V_{cd}^*V_{ud}$	II-b	$(2.17^{+0.01+1.22+0.19}_{-0.01-0.60-0.18}) \times 10^{-11}$
$B_u^+\eta'$	a_2	$V_{cs}^*V_{us}$, $V_{cd}^*V_{ud}$	II-b	$(2.79^{+0.02+1.56+0.64}_{-0.02-0.77-0.56}) \times 10^{-12}$
$B_u^+\phi$	a_2	$V_{cs}^*V_{us} \sim \mathcal{O}(\lambda)$	II-b	$(1.37^{+0.01+0.77+0.00}_{-0.01-0.38-0.00}) \times 10^{-11}$
$B_u^+K^0$	a_2	$V_{cd}^*V_{us} \sim \mathcal{O}(\lambda^2)$	II-c	$(8.21^{+0.10+4.61+0.21}_{-0.10-2.27-0.21}) \times 10^{-13}$
$B_u^+K^{*0}$	a_2	$V_{cd}^*V_{us} \sim \mathcal{O}(\lambda^2)$	II-c	$(9.53^{+0.12+5.35+0.54}_{-0.12-2.63-0.52}) \times 10^{-13}$

are significantly small, about $\mathcal{O}(10^{-12})$ for $P = \pi$ and $\mathcal{O}(10^{-13})$ for $P = K$. These $B_c(2S)$ weak decays seem to be exceedingly hard to measure, even at the future HL-LHC experiments. An observation of the nonleptonic $B_c(2S)$ weak decays into the final states containing one charmonium meson might be a hint of new physics beyond SM.

(4) Due to the relation of decay constants, $f_\rho > f_\pi$ and $f_{K^*} > f_K$, there are some hierarchy relations among the branching ratios, such as,

$$\mathcal{B}r(B_c(2S) \to B_q^0\rho) > \mathcal{B}r(B_c(2S) \to B_q^0\pi), \quad (46)$$

$$\mathcal{B}r(B_c(2S) \to B_q^0K^*) > \mathcal{B}r(B_c(2S) \to B_q^0K), \quad (47)$$

for the same subscript $q = u$, d and s. Because of the hierarchy of volume of phase space resulting from the mass relation, $m_{\eta_c(1S)} < m_{\psi(1S)} < m_{\eta_c(2S)} < m_{\psi(2S)}$, there is an approximate relationship among the branching ratios,

TABLE IV. The numerical results on CP-averaged branching ratios for the $B_c(2S) \to \eta_c(1S, 2S)P, \psi(1S, 2S)P$ decays, where the theoretical uncertainties come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.2)m_b$, decay constants, and Gegenbauer moments, respectively.

	Decay amplitude ($\mathcal{A}$)			
Final states	Coefficient	CKM factor	Case	Branching ratio ($\mathcal{B}r$)
$\eta_c(1S)\pi^+$	a_1	$V_{cb}V_{ud}^* \sim \mathcal{O}(\lambda^2)$	I-c	$(9.90^{+0.48+0.14+0.03}_{-0.47-0.11-0.03}) \times 10^{-12}$
$\eta_c(2S)\pi^+$	a_1	$V_{cb}V_{ud}^* \sim \mathcal{O}(\lambda^2)$	I-c	$(6.40^{+0.31+0.10+0.02}_{-0.30-0.07-0.02}) \times 10^{-12}$
$\psi(1S)\pi^+$	a_1	$V_{cb}V_{ud}^* \sim \mathcal{O}(\lambda^2)$	I-c	$(9.36^{+0.46+0.14+0.03}_{-0.44-0.11-0.03}) \times 10^{-12}$
$\psi(2S)\pi^+$	a_1	$V_{cb}V_{ud}^* \sim \mathcal{O}(\lambda^2)$	I-c	$(6.19^{+0.30+0.09+0.02}_{-0.29-0.07-0.02}) \times 10^{-12}$
$\eta_c(1S)K^+$	a_1	$V_{cb}V_{us}^* \sim \mathcal{O}(\lambda^3)$	I-d	$(7.49^{+0.42+0.11+0.07}_{-0.40-0.09-0.07}) \times 10^{-13}$
$\eta_c(2S)K^+$	a_1	$V_{cb}V_{us}^* \sim \mathcal{O}(\lambda^3)$	I-d	$(4.83^{+0.27+0.07+0.04}_{-0.26-0.06-0.04}) \times 10^{-13}$
$\psi(1S)K^+$	a_1	$V_{cb}V_{us}^* \sim \mathcal{O}(\lambda^3)$	I-d	$(6.95^{+0.39+0.11+0.06}_{-0.37-0.08-0.06}) \times 10^{-13}$
$\psi(2S)K^+$	a_1	$V_{cb}V_{us}^* \sim \mathcal{O}(\lambda^3)$	I-d	$(4.56^{+0.25+0.07+0.04}_{-0.24-0.05-0.04}) \times 10^{-13}$

$$\begin{aligned}&\mathcal{B}r(B_c(2S)\to\eta_c(1S)P)\\&\geq\mathcal{B}r(B_c(2S)\to\psi(1S)P)\\&\geq\mathcal{B}r(B_c(2S)\to\eta_c(2S)P)\\&\geq\mathcal{B}r(B_c(2S)\to\psi(2S)P),\end{aligned}\tag{48}$$

for the same pseudoscalar $P=\pi$ and K meson.

(5) Although it has been proofed that the QCDF coefficients $a_{1,2}$ in Eqs. (22) and (23) are the renormalization scale independent at the order of α_s [49,61], the main theoretical uncertainties of branching ratios for the $B_c(2S)\to B_qP$, B_qV decays come from the choice of renormalization scale, which are expected to be reduced by considering the contributions of the further higher α_s order in the future. For the $B_c(2S)\to\eta_c(1S,2S)P$, $\psi(1S,2S)P$ decays, the theoretical uncertainties of branching ratios come mainly from the CKM factors. In addition, we would like to point out that in practice, one should not be too serious about the branching ratios in Tables III and IV, because at least the total decay width of the $B_c(2S)$ meson has yet not be determined experimentally. These numbers in Tables III and IV are just an order-of-magnitude estimation of the branching ratios. In order to reduce the uncertainties, a common way is to introduce the ratio of branching ratios. Here, we will give some ratios of branching ratios to be tested in the future experiments, for example,

$$\begin{aligned}&\frac{\mathcal{B}r(B_c(2S)\to B_s^0K^*)}{\mathcal{B}r(B_c(2S)\to B_s^0\rho)}\approx\frac{\mathcal{B}r(B_c(2S)\to B_d^0K^*)}{\mathcal{B}r(B_c(2S)\to B_d^0\rho)}\\&\approx|V_{us}|^2\frac{f_{K^*}^2}{f_\rho^2}\approx0.05,\end{aligned}\tag{49}$$

$$\begin{aligned}&\frac{\mathcal{B}r(B_c(2S)\to B_s^0K)}{\mathcal{B}r(B_c(2S)\to B_s^0\pi)}\approx\frac{\mathcal{B}r(B_c(2S)\to B_d^0K)}{\mathcal{B}r(B_c(2S)\to B_d^0\pi)}\\&\approx\frac{\mathcal{B}r(B_c(2S)\to\eta_c(1S)K)}{\mathcal{B}r(B_c(2S)\to\eta_c(1S)\pi)}\approx\frac{\mathcal{B}r(B_c(2S)\to\eta_c(2S)K)}{\mathcal{B}r(B_c(2S)\to\eta_c(2S)\pi)}\\&\approx\frac{\mathcal{B}r(B_c(2S)\to\psi(1S)K)}{\mathcal{B}r(B_c(2S)\to\psi(1S)\pi)}\approx\frac{\mathcal{B}r(B_c(2S)\to\psi(2S)K)}{\mathcal{B}r(B_c(2S)\to\psi(2S)\pi)}\\&\approx|V_{us}|^2\frac{f_K^2}{f_\pi^2}\approx0.07.\end{aligned}\tag{50}$$

IV. SUMMARY

The excited $B_c(2S)$ meson has been discovered and confirmed experimentally for a few years. The $B_c(2S)$ meson decays dominantly via the hadronic cascades and electromagnetic transitions. The $B_c(2S)$ meson decays via the weak interaction are also allowable in SM, but with a tiny share. The identification of the $B_c(2S)$ meson faces enormous challenges and many difficulties at experiments. It is expected that there is a promising prospect of $\mathcal{O}(10^{11})B_c(2S)$ meson available at the coming HL-LHC experiments. Motivated by the huge statistical $B_c(2S)$ data, the potential of searching for the $B_c(2S)$ meson weak decays is investigated with the popular QCDF approach. Using the form factors obtained with the Wirbel-Stech-Bauer approach, the branching ratios for both the $B_c(2S)\to B_qP$, B_qV decays induced by the $c\to q$ quark transition and the $B_c(2S)\to\eta_c(1S,2S)P$, $\psi(1S,2S)P$ decays induced by the $b\to c$ quark transition are estimated theoretically. It is found that branching ratios of the hadronic $B_c(2S)$ weak decays into final states containing one bottomed meson can reach up to $\mathcal{O}(10^{-9})$ for the I-a case, and $\mathcal{O}(10^{-10})$ for the II-a and I-b cases, and might be measurable. The $B_c(2S)$ weak decays into final states containing one charmonium are severely suppressed by the CKM factors, and have significantly small branching ratios, $\mathcal{O}(10^{-12})$ and less, which might be outside the future experimental detectability. It is hoped that our estimation on the hadronic $B_c(2S)$ weak decays can provide a ready reference for the future experimental study.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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