

Uncertainties in the neutrino fog: Studying the isospin-violating dark matter case

Laura Duque^{✉,*}, J. M. Lamprea^{✉,†} and O. G. Miranda^{✉,‡}

*Departamento de Física,
Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional Apartado Postal 14-740,
07000 Ciudad de México, México*



(Received 13 January 2026; accepted 9 July 2026; published 3 August 2026)

First observation of solar neutrinos through coherent elastic neutrino-nucleus scattering by dark matter (DM) direct detection experiments makes the study of the neutrino fog of the most relevance. This irreducible neutrino background depends on the target material as well as other experimental parameters. Recently, the dependence of the neutrino fog on the DM models under consideration has also been remarked on. In this work, we study the case of isospin-violating dark matter (IVDM) models, discussing specific examples of DM models and making a detailed analysis of the implications of IVDM on the neutrino fog. We also explore the conditions under which this background can be mitigated for specific DM models.

DOI: [10.1103/f3w6-f8sp](https://doi.org/10.1103/f3w6-f8sp)

I. INTRODUCTION

Dark matter (DM) is one of the most significant and well-established mysteries in modern physics, with weakly interacting massive particles (WIMPs) among the most promising candidates. WIMPs naturally appear in many theories beyond the Standard Model and also naturally freeze-out in the early Universe, with an expected relic abundance matching the observed DM density. This cosmological coincidence is behind the motivation for a wide variety of experimental searches aimed at directly detecting WIMPs. DM direct detectors are sensitive to the tiny nuclear recoils that are expected if a WIMP from the Galactic halo scatters off a target nucleus. They are usually located deep underground to shield them from cosmic-ray backgrounds. DM direct detection (DD) experiments have shown an accelerated progress both in the size of the target as well as in detection techniques. They have increased from kilogram-scale detectors to current multiton-scale detectors, with liquid noble gas time projection chambers being the preferred technique, as seen in XENON1T [1], LUX [2], and PandaX [3]. The next generation of

experiments, including LZ [4] and XENONnT [5], will improve the sensitivity in the parameter space.

As DM DD experiments grow in size and improve the low-energy thresholds, they become sensitive to the irreducible background of astrophysical neutrinos. More specifically, Boron and hep solar neutrinos can interact with the detector. These neutrinos could scatter off electrons [Elastic neutrino-electron scattering (E ν Es)], but the most important background for WIMP searches arises from coherent elastic neutrino-nucleus scattering (CE ν NS). This is a process happening at low momentum transfer in which neutrinos scatter off an entire nucleus, having a large cross section that scales with the square of the number of neutrons [6]. This process has been theorized for a long time [7] and was confirmed experimentally by the COHERENT collaboration [8]. The CE ν NS interaction produces a low-energy nuclear recoil, which is the exact signal that DM DD experiments are designed to detect.

The CE ν NS background imposes a fundamental limitation on DD experiments, which is often referred to as the “neutrino floor” [9–11]. In the neutrino floor region, the expected neutrino background almost perfectly imitates the expected WIMP signal. For example, ⁸B solar neutrinos produce a recoil spectrum nearly identical to that of a 6 GeV WIMP with a DM-nucleon cross section of $5 \times 10^{-45} \text{ cm}^2$, while atmospheric neutrinos mimic a 100 GeV WIMP with a DM-nucleon cross section of $2.5 \times 10^{-49} \text{ cm}^2$ (See Sec. III).¹ However, the neutrino floor does not represent

*Contact author: laura.duque@cinvestav.mx

†Contact author: jorge.lamprea@cinvestav.mx

‡Contact author: omar.miranda@cinvestav.mx

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

¹The analog neutrino fog for DM-electron scattering, the reader can look at Refs. [12,13], while for the Migdal effect, please see Refs. [14,15].

a hard “floor,” but rather a limit that can change depending on the target material and the DM parameters. That is why the term “neutrino fog” [16] is more appropriate, since the limit is not an absolute barrier, but rather a transitional region where identifying a recoil as a DM signal becomes ambiguous due to the irreducible neutrino background. This “neutrino fog” has been reached experimentally in recent years, first in 2024 by the PandaX-4T [17] and XENONnT [18] collaborations, who reported the first indications of ^8B neutrinos at 2.64σ and 2.73σ significance, respectively. After these initial indications, the LZ experiment [19] succeeded in making a measurement with a statistical significance of 4.5σ in late 2025. This recent CE ν NS measurement in DD experiments makes it more urgent to have a clear understanding of the neutrino fog shape and its robustness against different DM models.

On the other hand, to compute the expected DM signal in DD experiments, we usually rely on the hypothesis that the DM candidate interacts with equal strength with the protons and neutrons of the target material. Such a hypothesis has important consequences on the expectations for the neutrino fog; therefore, it is important to question how robust the isospin conservation hypothesis is. Although this could be a reasonable first guess, there is no reason why DM must be blind to the different isospins of these nucleons, and some works have analyzed their impact in DD experiments [20–22]. Actually, in the Standard Model (SM), this statement does not hold, and, for instance, the neutrino coupling to protons in CE ν NS is much more smaller than the one to neutrons. Although there is no strong reason why DM should present the same isospin violation as in the Standard Model, there is also no reason why they should have isospin conservation.

Based on this idea, we have studied in this article the impact of isospin-violating dark matter (IVDM) on predictions for the neutrino fog. IVDM proposes that DM interacts with protons (f_p) and neutrons (f_n) with different strengths. This framework relaxes the standard assumption of DM models where these couplings are identical. Considering IVDM introduces an asymmetry in the DM couplings to protons and neutrons and may induce a destructive interference for a given atomic nucleus. This change in the neutron-to-proton coupling ($f_n \neq f_p$) can significantly reduce the expected event rate for a given target material. In a previous work, the “xenophobic” scenario, typically characterized by $f_n/f_p \approx -0.7$, has been explored [23], motivated by the destructive interference that made xenon negative searches consistent with the DAMA/LIBRA and CoGeNT signals. Furthermore, it is worth noting that the neutrino fog has been analyzed in the context of light mediators [24], and the effects on this limit have also been explored for an IVDM model, specifically a Z' model [25]. In our analysis, however, we use the profile likelihood ratio test approach, which offers a more robust

analysis of the neutrino fog. Unlike the isocurve method, which treats the neutrino fog as a fixed limit, this method integrates all the uncertainties as nuisance parameters, converting the neutrino fog into a dynamic limit that evolves depending on how much the DM signal overlaps with the background neutrinos. Additionally, other models are explored in the following sections.

This article is organized as follows. Before describing the statistical procedure for computing the neutrino fog, we first discuss, in Sec. II, the technical details for the computation of DM events that may be expected in a DM DD experiment. We devote Sec. III to describe the corresponding computations for the neutrino case. Afterward, we proceed with the detailed explanation for the calculation of the neutrino fog in Sec. IV. The results for our case of interest, the IVDM, are discussed in Sec. V. Finally, we draw our conclusions in Sec. VI.

II. DARK MATTER EXPECTED SIGNAL

Before discussing the IVDM effects on the neutrino fog predictions, we must discuss the typical signal that is searched for in DD experiments. DD experiments search for a DM particle interacting with the nuclei in the detector, assuming that there is a very weak interaction with the detector material. In this section, we show the signal produced by a WIMP χ with mass m_χ scattered off a nucleus target of mass $m_N = m_A/N_A$, where m_A is the molar mass and N_A is the Avogadro number in units of mol^{-1} . The differential rate R_{DM} of DM events per recoil energy E_r is given by [26]

$$\frac{dR_{\text{DM}}}{dE_r} = \epsilon \frac{\sigma_0 \rho_0}{2m_\chi \mu_N^2} \int_{v>v_{\min}} \frac{f(\mathbf{v})}{v} d^3v, \quad (1)$$

where ϵ is the detector exposure in units of ton-year, $\rho_0 = 0.3 \text{ GeV c}^{-2} \text{ cm}^{-3}$, the local DM mass density, $\mu_N = \frac{m_N + m_\chi}{m_\chi m_N}$ is the nucleus-DM reduced mass, $f(\mathbf{v})$ the DM velocity distribution, and σ_0 , the cross section, which is assumed to be independent of the velocity, $\mathbf{v}$, and the momentum transfer, q . The integral in the above equation is computed over all the velocities greater than the minimum WIMP velocity, $v_{\min}$, needed to achieve a nuclear recoil with energy E_r ,

$$v_{\min} = \sqrt{\frac{m_N E_r}{2\mu_N}}. \quad (2)$$

The cross section, σ_0 , is defined as the velocity-independent and thermal averaged WIMP-nucleus cross section, which is composed of the spin-independent (SI) and the spin-dependent (SD) parts as

$$\sigma_0 = \left(\sigma_0^{\text{SI}} F_{\text{SI}}^2(q) + \sigma_0^{\text{SD}} F_{\text{SD}}^2(q) \right), \quad (3)$$

where $F_{\text{SI},\text{SD}}$ are the SI and SD nuclear form factors, respectively. For simplicity, we will consider that the proton and neutron form factors are equal. Moreover, in our case study, we will focus on the SI part of this cross section and denote $F_{\text{SI}}(q) = F(q)$. Then, in the case where the WIMP scatters off a nucleus of Z protons and $A - Z$ neutrons, the SI cross section takes the form

$$\sigma_0^{\text{SI}} = \frac{4\mu_n^2}{\pi} [Zf_p + (A - Z)f_n]^2, \quad (4)$$

where f_p and f_n are the proton and neutron couplings to WIMP, and μ_n the nucleon-WIMP reduced mass.

The above mentioned nuclear form factor, $F^2(q)$, describes the loss of coherence as the momentum transfer q increases. For extremely low momentum transfer, $q \ll 1/R$, there is no sensitivity to the nuclear structure and $F(q) = 1$. When q increases, the WIMP begins to perceive the spatial distribution of nucleons inside the nucleus, causing the individual scattering amplitudes to no longer add in phase. For the nuclear form factor, in this work, we have considered the Klein-Nystrand (KN) parametrization [27],

$$F_{\text{KN}}(q) = 3 \frac{j_1(qR_A)}{qR_A} [1 + (qa_k)^2]^{-1}. \quad (5)$$

In this expression, j_1 is the first-order spherical Bessel function, R_A is the nuclear radius, which is often

approximated by $R_A = 1.2A^{1/3}$ fm [28], while $a_k = 0.7$ fm is a parameter related to the nuclear skin thickness.

Regarding the DM velocity distribution in our Galactic halo, we assumed it to follow a truncated Maxwellian distribution (the standard halo model [29,30]) with σ_v the velocity dispersion, $v_0 = \sqrt{2/3}\sigma_v = 220$ km/s the local circular velocity of the Earth with respect to the Galactic Center, and $v_{\text{esc}} = 544$ km/s the DM escape velocity. In the Galactic Center frame of reference, this distribution takes the form

$$f(\mathbf{v}) = \begin{cases} \frac{1}{N_{\text{esc}}} \left(\frac{3}{2\pi\sigma_v^2} \right)^{3/2} e^{-3\mathbf{v}^2/2\sigma_v^2}, & \text{for } |\mathbf{v}| < v_{\text{esc}} \\ 0, & \text{otherwise,} \end{cases} \quad (6)$$

and the normalization

$$N_{\text{esc}} = \text{erf}(z) - \frac{2}{\sqrt{\pi}} z e^{-z^2}, \quad (7)$$

where

$$z := v_{\text{esc}}/v_0. \quad (8)$$

The integral of the inverse of the velocity distribution $\eta(v_{\text{min}}) = \int_{v > v_{\text{min}}} \frac{f(\mathbf{v})}{v} d^3v$ averaged over the solar cycle and boosted to the laboratory frame of reference, is analytically calculated as

$$\eta(v_{\text{min}}) = \begin{cases} \frac{1}{v_{\text{obs}}}, & \text{for } z < y, x < |y - z| \\ \frac{1}{2N_{\text{esc}}v_{\text{obs}}} \left[\text{erf}(x + y) - \text{erf}(x - y) - \frac{4ye^{-z^2}}{\sqrt{\pi}} \right], & \text{for } z > y, x < |y - z| \\ \frac{1}{2N_{\text{esc}}v_{\text{obs}}} \left[\text{erf}(z) - \text{erf}(x - y) - \frac{2}{\sqrt{\pi}} e^{-z^2} (y + z - x) \right], & \text{for } |y - z| < x < y + z \\ 0, & \text{for } y + z < x \end{cases} \quad (9)$$

where

$$x := v_{\text{min}}/v_0, \quad y := v_{\text{obs}}/v_0, \quad (10)$$

and $v_{\text{obs}} = 232$ km/s [29,30] is the averaged laboratory velocity with respect to the DM rest frame of reference.

With this information, we can compute the expected number of WIMP events in the i th energy bin in the detector

$$N_{\chi}^i(m_{\chi}, \sigma_{\chi-n}) = \int_{E_i}^{E_{i+1}} \frac{dR_{\text{DM}}}{dE_r} dE_r. \quad (11)$$

This expected number of WIMP events, for a given DM candidate mass and cross section, will be used in Sec. IV to calculate the neutrino fog. Besides, the expected number of neutrino CE ν NS events, which is the subject of the next Section, will also be needed for such a computation.

III. COHERENT ELASTIC NEUTRINO-NUCLEUS SCATTERING EXPECTED SIGNAL

Neutrinos represent the principal irreducible background in the DD searches for DM. They interact with the detector through CE ν NS, producing recoils that mimic the expected signal from a WIMP in the detector (see Fig. 1). The

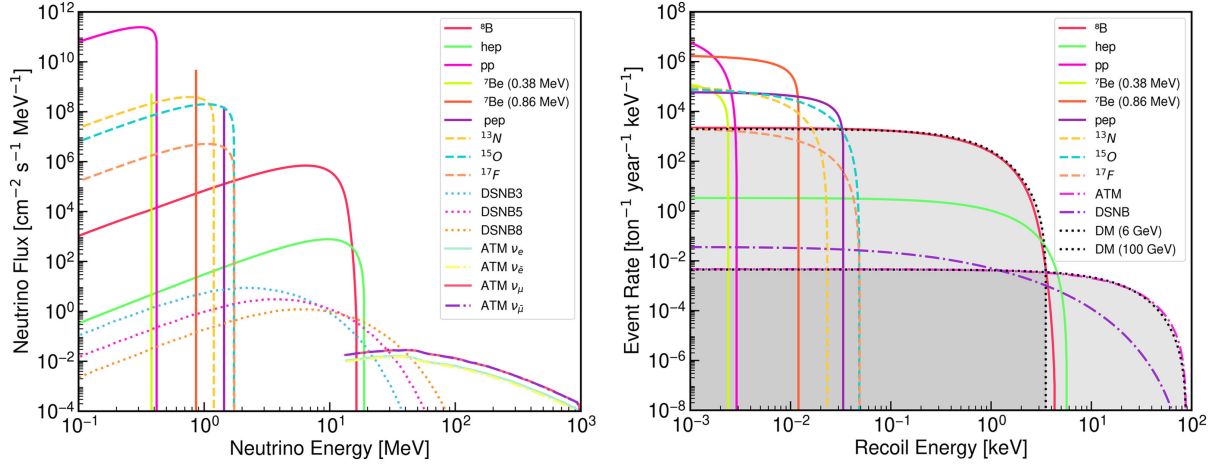


FIG. 1. Left: Neutrino energy spectra for solar, atmospheric (Atm), and the DSNB [31]. Right: Xenon recoil spectra showing the irreducible neutrino backgrounds (colored) compared to potential WIMP signals (black dotted). The recoil spectra of a WIMP with mass 6 GeV and cross section $\sigma_{\chi-n} = 5 \times 10^{-45} \text{ cm}^2$ is shown to overlap with ^8B neutrinos, while a $m_\chi = 100 \text{ GeV}$ and $\sigma_{\chi-n} = 2.5 \times 10^{-49} \text{ cm}^2$ WIMP overlaps with atmospheric neutrinos.

coherence in this process is guaranteed if the momentum transfer, q , is small, making the neutrino wavelength ($\lambda \sim 1/q$) larger than the radius of the nucleus target, R_A .

The differential event rate for CE ν NS arises from the convolution of the CE ν NS differential cross section with the neutrino flux,

$$\frac{dR_\nu}{dE_r} = \epsilon \frac{N_A}{m_A} \int_{E_\nu^{\min}}^{E_\nu^{\max}} \frac{d\Phi}{dE_\nu} \frac{d\sigma}{dE_r} dE_\nu, \quad (12)$$

where the minimum neutrino energy to generate a nuclear recoil with energy E_r is $E_\nu^{\min} = \sqrt{m_N E_r / 2}$; ϵ is the exposure in units of ton-year; m_A is the molar mass of the target nucleus, and the differential cross section is given by [32]

$$\frac{d\sigma}{dE_r}(E_r, E_\nu) = \frac{G_F^2 m_N}{2\pi} Q_W^2 F^2(q) \left(2 - \frac{m_N E_r}{E_\nu^2} \right). \quad (13)$$

Here, G_F is the Fermi constant and Q_W is the vector weak charge, defined as

$$Q_W^2 = (Zg_p^V + Ng_n^V)^2, \quad (14)$$

where Z indicates the number of protons in the nucleus, and $N = A - Z$ the number of neutrons. The vector couplings of the proton and neutron are $g_p^V = 1/2 - 2\sin^2\theta_W$ and $g_n^V = -1/2$, respectively. It is worth mentioning that, considering the value of the weak mixing angle at low energies, $\sin^2\theta_W = 0.2368$ [33,34], the proton vector coupling is very small; therefore, as it is well known, the weak charge is dominated by the neutron contribution, making CE ν NS especially sensitive to the number of neutrons in the nucleus.

Finally, the expected number of neutrino events (N_ν^i) within a specific energy bin, i , is calculated by integrating the differential recoil rate, given in Eq. (12), over the energy limits for each bin

$$N_\nu^i(\Phi) = \int_{E_i}^{E_{i+1}} \frac{dR_\nu}{dE_r} dE_r, \quad (15)$$

where we have considered the contribution of all possible neutrino fluxes $\Phi = (\phi_1, \dots, \phi_{n_\nu})$ reaching the detector,

TABLE I. Summary of the normalization factors and their associated uncertainties for the principal neutrino backgrounds. The sources include solar neutrinos from the pp chain (pp, pep, ^7Be , ^8B , hep) and the CNO cycle (^{13}N , ^{15}O , ^{17}F), as well as atmospheric (Atm) neutrinos and the DSNB [31]. The normalization factors are given in units of $\text{cm}^{-2} \text{ s}^{-1}$.

	Normalization ($\text{cm}^{-2} \text{ s}^{-1}$)	Unc.		Normalization ($\text{cm}^{-2} \text{ s}^{-1}$)	Unc.
pp	5.98×10^{10}	0.6%	^{13}N	2.78×10^8	15%
pep	1.44×10^8	1%	^{15}O	2.05×10^8	17%
^7Be (0.861 MeV)	4.35×10^9	3%	^{17}F	5.29×10^6	20%
^7Be (0.383 MeV)	4.84×10^8	3%	DSNB	86	50%
^8B	5.25×10^6	4%	Atm	10.5	20%
hep	7.98×10^3	30%			

which are displayed in Table I. Such neutrino fluxes include solar neutrinos created from thermonuclear reactions in the core of the Sun [primarily through the proton-proton (pp) chain and the Carbon–nitrogen–oxygen (CNO) cycle]; this process produces a spectrum of different components, from low-energy pp neutrinos to high-energy ^8B neutrinos. Other sources include the atmospheric neutrinos, which are created when high-energy cosmic rays interact with the atmosphere, resulting in a cascade of particles, including a flux of muon and electron neutrinos, and finally the Diffuse Supernova neutrino background (DSNB) [31], which is comprised of the isotropic glow of neutrinos from all past core-collapse supernovae. In Fig. 1, we have shown the energy spectra for all of these flux components.

IV. NEUTRINO FOG COMPUTATION

With neutrino and DM expected rates already computed, we can turn now our attention to the computation of the expectations for the neutrino fog. There are several ways to define the neutrino fog limit. We derive our neutrino fog definition following the already known discovery limits procedure first developed in [9–11]. This is defined as the limit in the cross-section versus DM mass, σ_d , at which a given experiment has a 90% probability of detecting a DM particle with mass, m_χ , and with a scattering cross section $\sigma_{n-\chi} > \sigma_d$ at 3σ CL.

In order to construct our discovery limit, we define the likelihood function as the product of the Poissonian distribution of all the bins ($N_{\text{bin}} = 150$) multiplied by the Gaussian distribution, $G(\phi^j)$, of each neutrino flux normalization $\Phi = (\phi_1, \dots, \phi_{n_\nu})$ in Table I,

$$L(m_\chi, \sigma_{\chi-n}, \Phi) = \prod_{i=1}^{N_{\text{bin}}} \mathcal{P}(N_{\text{obs}}^i | N_{\text{exp}}^i) \times \prod_{j=1}^{n_\nu} G(\phi^j). \quad (16)$$

We have considered $N_{\text{bin}} = 150$ with a threshold energy of $E_r^1 = E_{\text{th}} = 10^{-3}$ eV and a maximum recoil energy, $E_r^{150} = 1$ MeV. Let us point out that the present statistical treatment is very elaborate and helpful as we can, for instance, derive the limit in the same way in both low- and high-event-number scenarios.

The Poissonian distribution of the number of events expected N_{exp} , given the observed ones N_{obs} , is written as

$$\mathcal{P}(N_{\text{obs}} | N_{\text{exp}}) = \frac{(N_{\text{exp}})^{N_{\text{obs}}} e^{-N_{\text{exp}}}}{N_{\text{obs}}!}. \quad (17)$$

The profile likelihood ratio consists of the confrontation of two hypotheses. The background hypothesis H_0 considers only the number of neutrino events [Eq. (15)] as background, while the test hypothesis H_1 considers the neutrinos and DM events [Eq. (11)]. Thus, in the background hypothesis, the number of expected and observed events in the bin i is

$$H_0: N_{\text{obs}}^i = N_\nu^i + N_\chi^i, \quad N_{\text{exp}}^i = N_\nu^i, \quad (18)$$

while the test hypothesis

$$H_1: N_{\text{obs}}^i = N_\nu^i + N_\chi^i, \quad N_{\text{exp}}^i = N_\nu^i + N_\chi^i. \quad (19)$$

In order to test the background-only hypothesis against the alternative hypothesis, we use the profile likelihood ratio

$$\lambda(0) = \frac{L_0(m_\chi, \sigma_{\chi-n} = 0, \hat{\Phi})}{L_1(m_\chi, \hat{\sigma}_{\chi-n}, \hat{\Phi})}, \quad (20)$$

where the values of $\hat{\Phi}$ maximize the constrained Likelihood, L_0 , while for the unconstrained likelihood, L_1 , we have used the results of [9,35]. In this sense, we utilize the Asimov dataset, where one representative dataset gives the maximum unconstrained likelihood at the mean values of Φ .

Finally, the test statistic q_0 , in this case, is defined as

$$q_0 = \begin{cases} -2 \log \lambda(0), & \hat{\sigma}_{\chi-n} > 0 \\ 0, & \hat{\sigma}_{\chi-n} \leq 0 \end{cases} \quad (21)$$

where large values of q_0 prefer the background hypothesis H_0 over the alternative one H_1 .

The p value is

$$p_0 = \int_{q_0^{\text{obs}}}^{\infty} f(q_0 | H_0) dq_0, \quad (22)$$

where $f(q_0 | H_0)$ is the probability distribution of the test statistic under the background-only hypothesis. From Wilk's theorem [9], q_0^{obs} asymptotically behaves like a χ^2 distribution with only one degree of freedom. Thus, the significance is related as $Z = \sqrt{q_0^{\text{obs}}}$, where $Z \geq 3$ gives the 90% CL discovery limit.

With all the computations described up to here, we are now able to compute the neutrino fog both standard and nonstandard scenarios. But before starting such a discussion it is important to notice that different systematic uncertainties can also play an important role in the determination of the neutrino fog. Our results in the next Section assume a good knowledge of such systematic effects in the experimental set ups that we have considered.

In order to illustrate the relevance of systematics in our analysis, we have chosen two different systematic uncertainties as example, they are the quenching factor uncertainties and the binning. Other systematics of relevance, such as energy resolution and especially flux normalization should also be treated with care in any analysis. Coming back to the case of quenching factor uncertainties we have utilized the parametrization of the Migdal effect in the

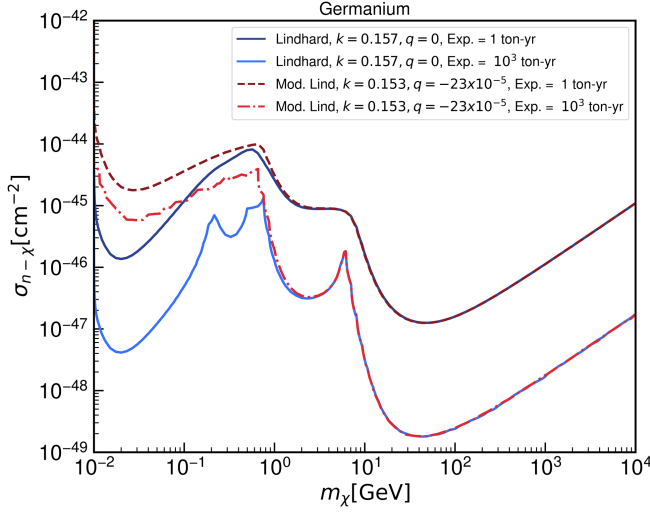


FIG. 2. Modification of the neutrino fog in the WIMP-nucleon SI cross section, $\sigma_{n-\chi}$, versus WIMP mass, m_χ when we consider the impact of the Migdal effect in the Lindhard model in the case of isospin conservation $f_n/f_p = 1$. The solid lines represent the expected neutrino fog for a germanium-type detector with an exposure of one ton-year, in dark blue, and one kton-year, in blue, in the Lindhard model [37], while the dashed lines describes the correction due to the Migdal effect, parametrized as in [36] in dark red and red for the same exposures. As expected, only the low-mass region has a drastic change.

Lindhard model [36] for a germanium based detector. We can notice in Fig. (2), that the Migdal effect can affect the low recoil energy quenching prediction $E_r \lesssim 5 \text{ keV}_{nr}$ and therefore the low-mass regime, $m_\chi \lesssim 1 \text{ GeV}$. Such effect would change the prediction for the quenching factor at low energies, shifting the threshold energy prediction as well as the number of events for the low-energy nuclear recoils.

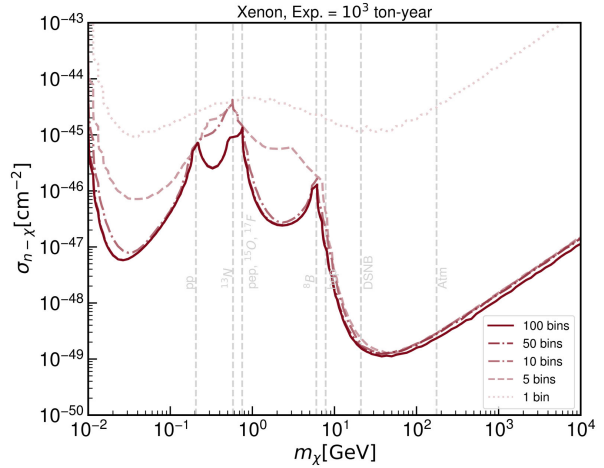
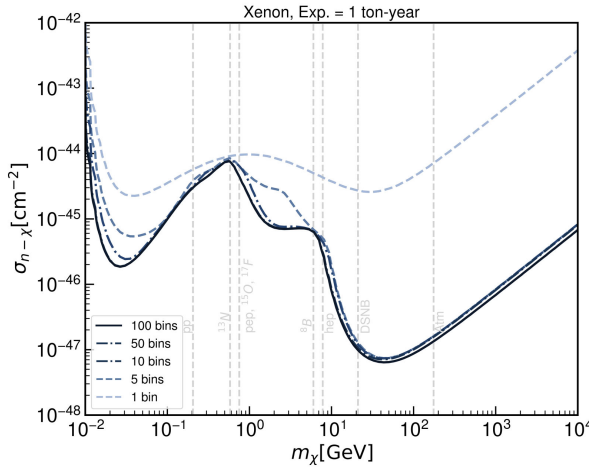


FIG. 3. Modification of the neutrino fog in the WIMP-nucleon SI cross-section, $\sigma_{n-\chi}$, versus WIMP mass, m_χ with the number of bins used in the analysis for a xenon-based detector in an isospin-conserving case ($f_n/f_p = 1$). On the left one ton-year exposure and on the right 1 kton-year exposure.

Regarding the binning, we are considering that the DM experiments are capable of having a large number of bins, otherwise the resolution to DM signal would be severely diminished as illustrated in Fig. (3) where we illustrate how an experiment with a binning smaller to 50 bins will lose resolution over the broad DM mass range: from 10^{-2} to 10^4 GeV .

After this discussion on the needed elements to compute the neutrino fog, we can finally turn our attention into our case of interest: how the IVDM models shift the neutrino fog.

V. ISOSPIN VIOLATING DARK MATTER MODELS

After describing the indispensable technical details for our computations, we focus on two IVDM realizations that illustrate how a particular DM model may cause a strong influence on the neutrino fog prediction.

Within the IVDM framework, DM couples differently to protons (f_p) and neutrons (f_n), departing from the assumption in standard WIMP scenarios. Allowing for unequal proton and neutron couplings introduces an isospin asymmetry that can lead to destructive interference. As a result, deviations from $f_p = f_n$ can substantially suppress the expected scattering rate for specific detector materials. Previous studies have examined the so-called xenophobic case, $f_n/f_p \approx -0.7$, in which destructive interference weakens xenon-based constraints and helps reconcile null xenon results with the DAMA/LIBRA and CoGeNT observations [23], which are in clear tension with more recent data from COSINE-100 [38].

There are several DM models in which isospin violation arises naturally, for instance, some supersymmetric realizations [39–41], Z' -DM portal models [42], and the two Higgs doublet DM portal [43,44], among others [45,46]. We have focused on two specific realizations of IVDM: the

Scotogenic and the effective Z portal DM model. The first is a popular model in which the DM phenomenology and the origin of the small neutrino masses are related. Moreover, the Scotogenic scheme involves a family of models that are of the most relevance for DM and neutrino phenomenology. For this reason, several extensions of this model have been proposed recently, remarking on its importance nowadays. The latter, although it is a simpler realization, is ruled out by data; still, the origin of the isospin violation is straightforward, thus this model gives a clear example to show the effects of this asymmetry in the neutrino fog.

A. Scotogenic model

The Scotogenic model [47,48] is a minimal extension of the SM. In addition to the SM particle content, an extra inert scalar doublet with hypercharge $Y = 1/2$, $\eta = (\eta^+, \eta_0 + i\eta_A)^T$, and $(i = 1, 2, 3)$ generation of Right-handed (RH) neutrinos N_i are added. These RH neutrinos have a Majorana mass term M_i . The new fields transform under a global symmetry $\mathbb{Z}_2$, while the SM model particles transform trivially. The relevant part of the Lagrangian is given by

$$\mathcal{L} \subset -\frac{M_i}{2} \bar{N}_i^c P_R N_i + h_{ai} \bar{\ell}_\alpha \eta^\dagger P_R N_i - V + \text{h.c.}, \quad (23)$$

where $P_R = \frac{1}{2}(1 + \gamma_5)$ is the right projector, h_{ai} are the Yukawa couplings, and V is the scalar potential.

The Scotogenic model has scalar or fermionic DM candidates depending on the mass spectra. For scalar candidates, the DM phenomenology is similar to that of the IDM [49,50], where the η_0 and η_A couple in the same way to neutrons and protons at tree level; therefore, no isospin violating couplings exist. On the other hand, fermionic DM, N_i , has no interaction with quarks at tree level; the model is leptophilic. However, at the one-loop level, there is a coupling between the quarks and the lightest RH neutrino N_1 mediated by the photon, the Z boson, and the Higgs [51]. From these interactions, the ones mediated by the photon and the Z boson are SD, as they take the form as effective operator: $\bar{N}_1 \gamma^\mu \gamma_5 N_1 \bar{q} \gamma_\mu \gamma_5 q$. Lastly, the interaction via the Higgs, although it generates a SI interaction, couples to the u- and d quarks via a Yukawa interaction; therefore, the isospin violation is proportional to the mass difference, therefore it becomes negligible.

However, as pointed out in [52], it is possible to realize an SI interaction that violates isospin at the one-loop level in the case where the RH neutrinos N_1 and N_2 are almost degenerate. This consists of the inelastic scattering of N_1 into N_2 off the nucleus through the photon; the Feynman diagrams for this process are shown in Fig. 4. In this case, the SI inelastic scattering cross section occurs through the effective Lagrangian

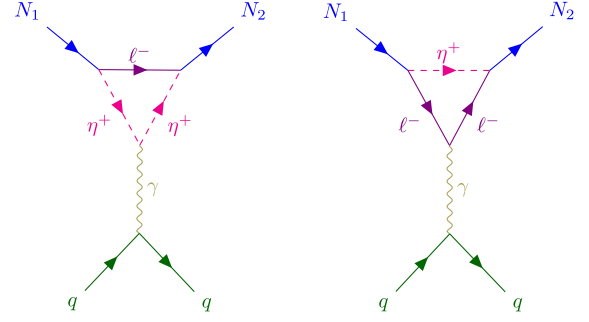


FIG. 4. Feynman diagrams for the SI inelastic scattering process of DM (N_1) off a nucleus at the lowest loop level in the Scotogenic model. This process is mediated via a photon and has been calculated in [52].

$$\mathcal{L}_{\text{eff}} = ia_{12} \bar{N}_2 \gamma^\mu N_1 \partial^\nu F_{\nu\mu} + i \left(\frac{\mu_{12}}{2} \right) \bar{N}_2 \sigma^{\mu\nu} N_1 F_{\mu\nu} + ic_{12} \bar{N}_2 \gamma^\mu N_1 A_\mu \quad (24)$$

where the functions are

$$a_{12} = -\sum_a \frac{\text{Im}(h_{a2}^* h_{a1}) e}{2(4\pi) M_\eta^2} I_a \left(\frac{M_1^2}{M_\eta^2}, \frac{m_a^2}{M_\eta^2} \right), \quad (25)$$

$$\mu_{12} = -\sum_a \frac{\text{Im}(h_{a2}^* h_{a1}) e}{2(4\pi) M_\eta^2} 2M_1 I_m \left(\frac{M_1^2}{M_\eta^2}, \frac{m_a^2}{M_\eta^2} \right), \quad (26)$$

$$c_{12} = \sum_a \frac{\text{Im}(h_{a2}^* h_{a1}) e}{2(4\pi) M_\eta^2} q^2 I_c \left(\frac{M_1^2}{M_\eta^2}, \frac{m_a^2}{M_\eta^2} \right), \quad (27)$$

with q the momentum transfer and $I_{a,m,c}$ the loop functions defined in [52].

Here, $f_n = 0$, because, in this case, the DM candidate N_1 couples only to u and d -type quarks through the photon, then the asymmetry is $f_n/f_p = 0$. Other choices in the parameter regions could give rise to different leading contributions to the SI cross section. However, we have used this, somehow fine-tuned realization of the model, to exemplify the limit case where DM couples only to protons in the nucleus. In contrast, in the model in the next Subsection, the effective Z portal DM, the contribution to the SI cross section is mainly due to a DM-neutron coupling.

B. Effective Z portal DM

The effective Z portal DM model is a simple extension of the SM in which the Z boson serves as the mediator between the dark and visible sectors. This framework is considered a minimal setup, as the Z boson and the Higgs boson are the only particles within the SM that can serve as mediators between these sectors. In this model, a DM Dirac fermion particle, χ , couples to the Z boson, which then

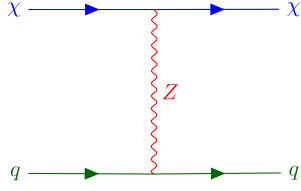


FIG. 5. Feynman diagram for the DM (χ) -quark (q) elastic scattering in the effective Z-portal DM model.

interacts with the fermions in the SM, f . The general Lagrangian that describes these interactions is given by [53],

$$\mathcal{L} = \frac{g}{4\cos\theta_W} (\bar{\chi}\gamma^\mu (V_\chi - A_\chi\gamma^5)\chi Z_\mu + \bar{f}\gamma^\mu (V_f - A_f\gamma^5)f Z_\mu). \quad (28)$$

Here, g is the weak coupling constant, θ_W is the weak mixing angle, and $V_{\chi,f}$ and $A_{\chi,f}$ represent the vectorial and axial couplings for the DM (χ) and SM fermions (f), respectively.

In the case of a pure SI interaction, we take the axial part of the interaction as negligible, A_χ , then the DM-nucleon interaction is through the t -channel exchange of a Z boson, as shown in the Feynman diagram in Fig. 5. Then, we are able to calculate the ratio of asymmetry, as the ratio of the neutron-DM to proton-DM couplings at tree level, giving

$$\frac{f_n}{f_p} = \frac{2V_d + V_u}{V_d + 2V_u} = -\frac{1}{1 - 4\sin^2\theta_W} \approx -22.18, \quad (29)$$

where the vector quark-Z couplings are given by $V_u = \frac{1}{2} - (\frac{4}{3})\sin^2\theta_W$ and $V_d = -\frac{1}{2} + (\frac{2}{3})\sin^2\theta_W$ and the weak mixing angle (in the $\overline{\text{MS}}$ scheme at low energies) is taken as $\sin^2\theta_W = 0.23873 \pm 0.00005$ [33,34].

The effective Z-portal DM model provides a direct example of Isospin violation in which the coupling ratio is fixed and determined by the weak mixing angle. Since our goal is only to consider the effect of isospin violation on the neutrino fog, an effective Z-portal model allows us to demonstrate the impact of a large isospin violation without necessarily affecting the standard neutrino background. The effect of adding a Z' mediator to the neutrino fog has been studied in [25].

C. Neutrino fog in IVDM models

Motivated by the above discussed IVDM models, we can compute the expected neutrino fog region in terms of the ratio f_n/f_p , to have an expectation for a whole family of IVDM models, including the two models described above. To perform such a computation, we start from Eq. (4), where the DM event rate is proportional to the square of the sum of the contributions from protons and neutrons in the nucleus. This expression can be rewritten as

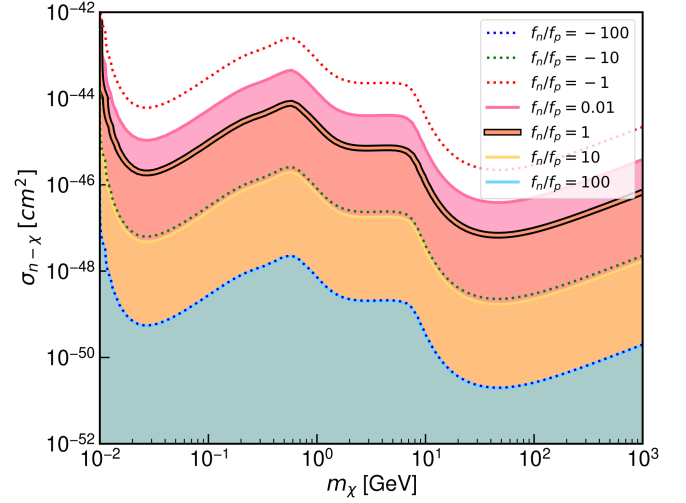


FIG. 6. Discovery limit for a xenon-based detector plotted in the WIMP mass (m_χ) versus a WIMP-nucleon SI cross section ($\sigma_{n-\chi}$) plane for different values of the ratio f_n/f_p . The standard isospin-conserving scenario, $f_n/f_p = 1$, is indicated by the thick black line within the peach-colored region.

$$R_{\text{DM}} \propto \left[Z + \left(\frac{f_n}{f_p} \right) (A - Z) \right]^2. \quad (30)$$

The value and sign of f_n/f_p can change the balance of the contribution of each of the nucleons, or determine whether these contributions add up or cancel each other out, as shown in Fig. 6. The sign will determine the type of interference. When the sign is positive, the contributions of protons and neutrons add up to each other. This results in constructive interference, increasing the DM event rate, improving the sensitivity of potential detectors and pushing the discovery limit to smaller cross-sections. On the other hand, a negative sign generates destructive interference. In this case, the contributions of protons and neutrons cancel each other, suppressing DM events. This reduces the detector sensitivity and shifts the discovery limit to larger cross sections. For those elements with one isotope that dominates the abundance, there is a specific negative ratio value that generates perfect cancellation. In the case of elements with multiple isotopes, complete destructive interference cannot be achieved for all isotopes simultaneously, so only partial cancellation can be generated. For xenon, the value of the ratio between the neutron and proton coupling strengths that causes maximum suppression is $f_n/f_p \approx -0.7$ [54]. Figure 6 shows the case for $f_n/f_p = -1$, and although this value does not yield maximum cancellation for xenon, it is the closest to -0.7 among the scenarios presented, leading to a higher neutrino fog.

The value of the ratio f_n/f_p determines which part of the nucleus dominates the interaction. A large ratio would imply neutron dominance, which can be realized in several

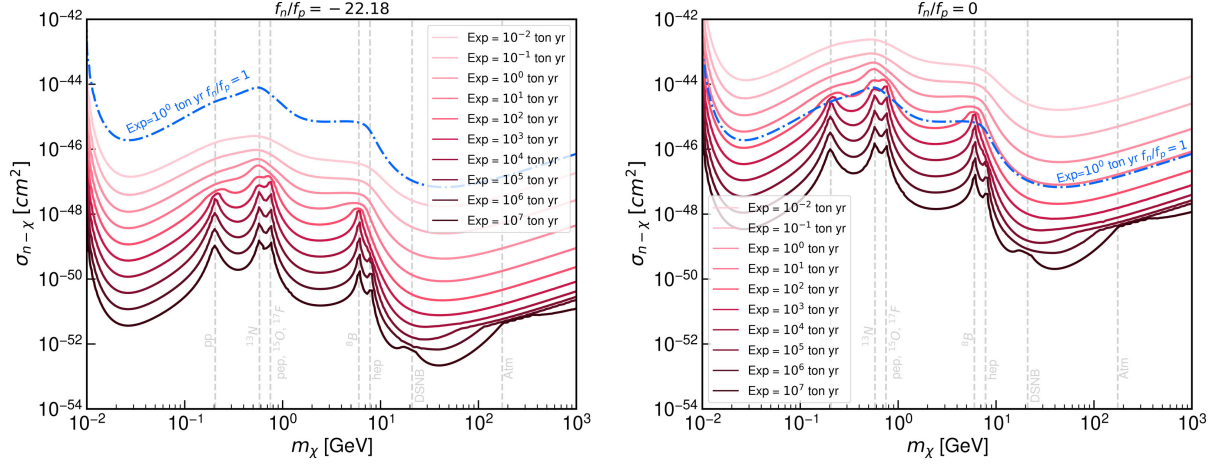


FIG. 7. Evolution of the neutrino fog in the WIMP-nucleon SI cross section, $\sigma_{n-\chi}$, versus WIMP mass, m_χ , plane for a xenon-based detector. The solid curves represent discovery limits for experimental exposures ranging from 10^{-2} ton-year (light pink) to 10^7 ton-year (dark maroon). The panels compare two isospin-violating scenarios: the Z-portal DM model with $f_n/f_p = -22.18$ (left) and the Scotogenic model realization with $f_n/f_p = 0$ (right). For reference, the blue dashdotted line indicates the projected limit for the standard isospin-conserving case ($f_n/f_p = 1$) at one ton-year exposure.

IVDM models without a fine-tuning. In this case, the sign of the ratio is not particularly relevant, since this term is squared. As illustration we show that both ratios, -100 and 100 , produce a large number of events, making the detector very sensitive. On the contrary, if the magnitude of the ratio is small, both neutrons and protons contribute similarly. They can combine perfectly, as in the standard case for a ratio of 1, or they can perfectly cancel each other out.

To visualize the impact of the two scenarios presented above, Fig. 7 shows the neutrino fog for a xenon detector and evaluates the effect of changes in the ratio predicted by each model. The solid curves indicate the discovery limits as experimental exposure increases. The left panel corresponds to the effective Z-portal DM model ($f_n/f_p = -22.18$), in which the large negative ratio enhances the DM event rate due to the neutron rich composition of the xenon nucleus, enabling the experiment to test significantly lower cross sections before the experiment reaches sensitivity to neutrinos, shifting the neutrino fog to smaller cross sections than the standard case of isospin conservation ($f_n/f_p = 1$) presented as a reference. For instance, at a fixed mass of 100 GeV and an exposure of 1 ton-year, the projected discovery limit in the Z-portal model improves by a factor of 159 relative to the isospin-conserving case. On the other hand, the right panel shows the Scotogenic model realization with $f_n/f_p = 0$, which couples only to protons at one-loop level; this decoupling from neutrons can suppress the DM event rate, thereby shifting the neutrino fog to larger cross sections: at the same fixed mass of 100 GeV and one ton-year exposure, the projected discovery limit is lowered by a factor of 0.17 compared to the isospin conserving case.

Finally, Fig. 8 highlights the important role of the target material composition in establishing the neutrino fog in the

different isospin-violation scenarios we have presented. This plot shows a comparison between a ^{131}Xe target and a ^{72}Ge target. Xenon generally produces a neutrino fog prediction at smaller cross-sections than germanium, this result is to be expected because the DM event rate scales according to Eq. (30). In the standard isospin-conserving scenario the cross section scales with the square of the number of nucleons (A^2), which naturally favors the

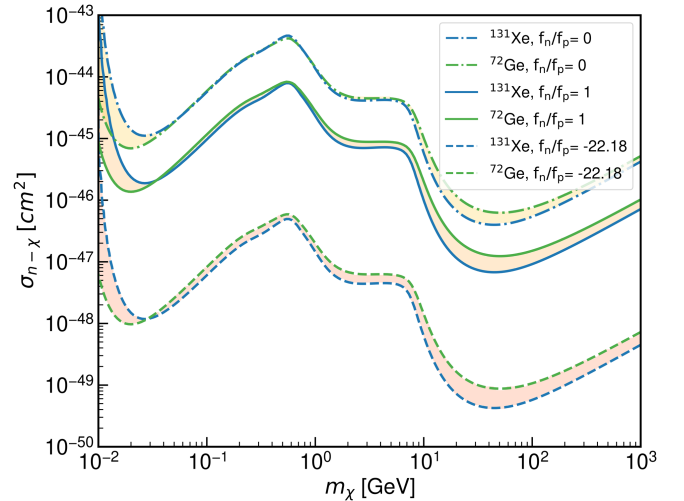


FIG. 8. Comparison of the neutrino fog for two target materials in the WIMP-nucleon SI cross-section ($\sigma_{n-\chi}$) versus WIMP mass (m_χ) plane: ^{131}Xe (blue lines) and ^{72}Ge (green lines). The different line styles correspond to three different f_n/f_p ratios: $f_n/f_p = 0$ (dash-dotted lines), the standard isospin-conserving case $f_n/f_p = 1$ (solid lines), and $f_n/f_p = -22.18$ (dashed lines). The plot shows that the relative sensitivity between the two target materials depends strongly on the specific f_n/f_p ratio.

heavier xenon nucleus. In the presented Scotogenic model realization with $f_n/f_p = 0$, the discovery limits for both targets improve relative to the standard scenario, and the difference between the discovery limits of xenon and germanium decreases, resulting in comparative rates of $R_{\text{DM}} \propto 3000$ for Xe and $R_{\text{DM}} \propto 1000$ for Ge. Meanwhile, the Z-portal DM model exhibits an effective coupling that strongly couples to neutrons. Since xenon contains a higher number of neutrons ($A - Z$), it benefits from this scaling, resulting in a higher event rate ($R_{\text{DM}} \propto 2.73 \times 10^6$ for Xe compared to $R_{\text{DM}} \propto 7.31 \times 10^5$ for Ge). As a result, the neutrino fog for xenon shifts to much smaller cross sections compared to germanium, creating a wider gap between the sensitivity of these materials.

Therefore, these results highlight the importance of complementary targets in future DD experiments. As shown, in scenarios such as the Z-portal DM model, materials with a higher neutron content, such as xenon, exhibit a neutrino fog at lower cross sections than that for germanium. This material-dependent response implies that rather than relying on a single material as a detector, an approach combining isotopic compositions and different materials may be crucial for fully exploring the parameter space.

VI. CONCLUSIONS

In this work, we have studied the impact of IVDM models, such as the Scotogenic and effective Z-portal DM models, on the projected sensitivity of DM DD experiments, focusing on variations in the coupling ratio f_n/f_p . The results presented illustrate the variable character of the neutrino fog and show that it shifts significantly depending on the underlying particle physics. For the concrete models we use as examples, we have shown that the effective Z-portal DM model, which has a strong coupling to

neutrons, produces a downward shift in the neutrino fog, thereby improving experimental sensitivity. The Scotogenic model, on the other hand, being a model that decouples from neutrons, results in a higher neutrino fog, requiring high exposure values in order to be tested.

Contrary to isospin-violating scenarios considered to reconcile conflicting experimental data (e.g., the xenophobic case), the coupling ratios studied here arise naturally from the proposed DM models, demonstrating that significant shifts in the neutrino fog can be direct predictions of physically motivated extensions of the SM.

Finally, we have shown the impact of the IVDM models on the neutrino fog for different target materials. We have compared the xenon and germanium cases, showing that differences in sensitivity between the materials depend on the model under study. These results then highlight that a multitarget approach will be essential not only for discovery but for determining the precise coupling properties of the DM.

ACKNOWLEDGMENTS

We would like to thank Luis Jorge Flores for his time and valuable discussion. O. G. M. and J. M. L. would like to thank Sistema Nacional de Investigadoras e Investigadores (SNI). J. M. L. is funded by the Secihti Estancias posdoctorales por México 2023 grant.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

-
- [1] E. Aprile, J. Aalbers, F. Agostini, M. Alfonsi, L. Althueser, F. D. Amaro, V. C. Antochi, E. Angelino, F. Arneodo, D. Barge *et al.* (XENON Collaboration), *Phys. Rev. Lett.* **123**, 251801 (2019).
 - [2] D. Akerib, H. Araújo, X. Bai, A. Bailey, J. Balajthy, P. Beltrame, E. Bernard, A. Bernstein, T. Biesiadzinski, E. Boulton *et al.*, *Phys. Rev. Lett.* **116**, 161301 (2016).
 - [3] Z. Bo, W. Chen, X. Chen, Y. Chen, Z. Cheng, X. Cui, Y. Fan, D. Fang, Z. Gao, L. Geng *et al.* (PandaX Collaboration), *Phys. Rev. Lett.* **134**, 011805 (2025).
 - [4] J. Aalbers, D. S. Akerib, C. W. Akerlof, A. K. Al Musalhi, F. Alder, A. Alqahtani, S. K. Alsum, C. S. Amarasinghe, A. Ames, T. J. Anderson *et al.* (LUX-ZEPLIN Collaboration), *Phys. Rev. Lett.* **131**, 041002 (2023).
 - [5] E. Aprile, J. Aalbers, K. Abe, S. A. Maouloud, L. Althueser, B. Andrieu, E. Angelino, D. A. Martin, S. R. Armbruster, F. Arneodo *et al.*, *Phys. Rev. Lett.* **135**, 221003 (2025).
 - [6] M. Abdullah *et al.* *arXiv:2203.07361*.
 - [7] D. Z. Freedman, *Phys. Rev. D* **9**, 1389 (1974).
 - [8] D. Akimov, J. B. Albert, P. An, C. Awe, P. S. Barbeau, B. Becker, V. Belov, A. Brown, A. Bolozdynya, B. Cabrera-Palmer *et al.*, *Science* **357**, 1123 (2017).
 - [9] G. Cowan, K. Cranmer, E. Gross, and O. Vitells, *Eur. Phys. J. C* **71**, 1554 (2011); **73**, 2501(E) (2013).
 - [10] J. Billard, E. Figueroa-Feliciano, and L. Strigari, *Phys. Rev. D* **89**, 023524 (2014).
 - [11] J. Billard, F. Mayet, and D. Santos, *Phys. Rev. D* **85**, 035006 (2012).

- [12] B. Carew, A. R. Caddell, T. N. Maity, and C. A. J. O'Hare, *Phys. Rev. D* **109**, 083016 (2024).
- [13] R. Essig, M. Sholapurkar, and T.-T. Yu, *Phys. Rev. D* **97**, 095029 (2018).
- [14] T. N. Maity, *Phys. Rev. D* **111**, 123020 (2025).
- [15] G. Herrera, *J. High Energy Phys.* **05** (2024) 288.
- [16] C. A. J. O'Hare, *Phys. Rev. Lett.* **127**, 251802 (2021).
- [17] Z. Bo, W. Chen, X. Chen, Y. Chen, Z. Cheng, X. Cui, Y. Fan, D. Fang, Z. Gao, L. Geng *et al.* (PandaX Collaboration), *Phys. Rev. Lett.* **133**, 191001 (2024).
- [18] E. Aprile, J. Aalbers, K. Abe, S. Ahmed Maouloud, L. Althueser, B. Andrieu, E. Angelino, D. Antón Martín, F. Arneodo, L. Baudis *et al.* (XENON Collaboration), *Phys. Rev. Lett.* **133**, 191002 (2024).
- [19] D. S. Akerib, A. K. A. Musalhi, F. Alder, B. J. Almquist, C. S. Amarasinghe, A. Ames, T. J. Anderson, N. Angelides, H. M. Araújo, J. E. Armstrong *et al.*, *arXiv*: 2512.08065.
- [20] R. Catena and P. Gondolo, *J. Cosmol. Astropart. Phys.* **08** (2015) 022.
- [21] A. Brenner, G. Herrera, A. Ibarra, S. Kang, S. Scopel, and G. Tomar, *J. Cosmol. Astropart. Phys.* **06** (2022) 026.
- [22] N. P. Avis Kozar, P. Scott, and A. C. Vincent, *J. Cosmol. Astropart. Phys.* **02** (2025) 007.
- [23] J. L. Feng, J. Kumar, D. Marfatia, and D. Sanford, *Phys. Lett. B* **703**, 124 (2011).
- [24] V. D. Romeri, A. Majumdar, D. K. Papoulias, and R. Srivastava, *J. Cosmol. Astropart. Phys.* **05** (2026) 093.
- [25] V. M. Lozano, S. Pramanik, S. Sadhukhan, and A. Terrones, *arXiv*:2508.05787.
- [26] J. D. Lewin and P. F. Smith, *Astropart. Phys.* **6**, 87 (1996).
- [27] D. K. Papoulias, T. S. Kosmas, and Y. Kuno, *Front. Phys.* **7**, 191 (2019).
- [28] S. R. Klein and J. Nystrand, *Phys. Rev. C* **60**, 014903 (1999).
- [29] C. Savage, K. Freese, and P. Gondolo, *Phys. Rev. D* **74**, 043531 (2006).
- [30] K. Freese, M. Lisanti, and C. Savage, *Rev. Mod. Phys.* **85**, 1561 (2013).
- [31] D. Baxter, I. M. Bloch, E. Bodnia, X. Chen, J. Conrad, P. Di Gangi, J. E. Y. Dobson, D. Durnford, S. J. Haselschwardt, A. Kaboth *et al.*, *Eur. Phys. J. C* **81** (2021).
- [32] V. De Romeri, O. G. Miranda, D. K. Papoulias, G. Sanchez Garcia, M. Tórtola, and J. W. F. Valle, *J. High Energy Phys.* **04** (2023) 035.
- [33] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [34] J. Erler and R. Ferro-Hernández, *J. High Energy Phys.* **03** (2018) 196.
- [35] D. Aristizabal Sierra, V. De Romeri, L. J. Flores, and D. K. Papoulias, *J. Cosmol. Astropart. Phys.* **01** (2022) 055.
- [36] P. Sorensen, *Phys. Rev. D* **91**, 083509 (2015).
- [37] J. Lindhard, V. Nielsen, M. Scharff, and P. V. Thomsen, *Mat. Fys. Medd. Dan. Vid. Selsk* **33**, 10 (1963).
- [38] N. Carlin *et al.* (COSINE-100 Collaboration), *Sci. Adv.* **11**, adv6503 (2025).
- [39] Z. Kang, T. Li, T. Liu, C. Tong, and J. M. Yang, *J. Cosmol. Astropart. Phys.* **01** (2011) 028.
- [40] V. M. Lozano, M. Peiró, and P. Soler, *J. High Energy Phys.* **04** (2015) 175.
- [41] J. L. Feng, J. Kumar, D. Marfatia, and D. Sanford, *arXiv*:1307.1758.
- [42] M. T. Frandsen, F. Kahlhoefer, S. Sarkar, and K. Schmidt-Hoberg, *J. High Energy Phys.* **09** (2011) 128.
- [43] A. Drozd, B. Grzadkowski, J. F. Gunion, and Y. Jiang, *J. Cosmol. Astropart. Phys.* **10** (2016) 040.
- [44] C.-F. Chang, X.-G. He, and J. Tandean, *J. High Energy Phys.* **04** (2017) 107.
- [45] X.-G. He and J. Tandean, *J. High Energy Phys.* **12** (2016) 074.
- [46] X.-G. He, B. Ren, and J. Tandean, *Phys. Rev. D* **85**, 093019 (2012).
- [47] E. Ma, *Phys. Rev. D* **73**, 077301 (2006).
- [48] Z.-j. Tao, *Phys. Rev. D* **54**, 5693 (1996).
- [49] L. Lopez Honorez, E. Nezri, J. F. Oliver, and M. H. G. Tytgat, *J. Cosmol. Astropart. Phys.* **02** (2007) 028.
- [50] L. Lopez Honorez and C. E. Yaguna, *J. Cosmol. Astropart. Phys.* **01** (2011) 002.
- [51] A. Ibarra, C. E. Yaguna, and O. Zapata, *Phys. Rev. D* **93**, 035012 (2016).
- [52] D. Schmidt, T. Schwetz, and T. Toma, *Phys. Rev. D* **85**, 073009 (2012).
- [53] G. Arcadi, Y. Mambrini, and F. Richard, *J. Cosmol. Astropart. Phys.* **03** (2015) 018.
- [54] J. L. Feng, J. Kumar, and D. Sanford, *Phys. Rev. D* **88**, 015021 (2013).