

Gravitational waves from spectator scalar fields

Marcos A. G. Garcia^{*}

Departamento de Física Teórica, Instituto de Física, Universidad Nacional Autónoma de México, Ciudad de México 04510, Mexico

Sarunas Verner[†]

Institute for Fundamental Theory, Physics Department, University of Florida, Gainesville, Florida 32611, USA



(Received 19 June 2025; accepted 29 June 2026; published 23 July 2026)

We propose a mechanism for gravitational wave (GW) production sourced by spectator scalar fields during inflation. These fields, while not driving cosmic expansion, generate blue-tilted isocurvature fluctuations that naturally satisfy current cosmic microwave background (CMB) constraints at large scales while producing enhanced power spectra at smaller scales accessible to GW detectors. The resulting GW spectrum spans an exceptionally broad frequency range from 10^{-20} to 1 Hz, with amplitudes ranging from $\Omega_{\text{GW}} h^2 \sim 10^{-19}$ to 10^{-14} depending on the reheating temperature and spectator field mass. Observable signals across multiple detector bands are possible in a restricted region of parameter space, m_χ/H_I within the isocurvature- and ΔN_{eff} -allowed window and T_{reh} , such that the spectator either provides all of the dark matter (stable case) or decays near matter-radiation equality (unstable case). Our analysis reveals multiple complementary constraints on spectator field parameters. For a high reheating temperature of $T_{\text{reh}} > 10^{10}$ GeV, GW-induced limits on the effective number of relativistic species (ΔN_{eff}) require $m_\chi \gtrsim 0.59 H_I$, stronger than CMB isocurvature bounds alone ($m_\chi \gtrsim 0.54 H_I$). The nonobservation of primordial B-modes by *Planck* provides stronger constraints, $m_\chi \gtrsim 0.64 H_I$, with projected LiteBIRD sensitivity potentially reaching $m_\chi \gtrsim 0.67 H_I$. This mechanism enables a unique multimessenger probe of beyond the standard model physics during inflation, providing simultaneous constraints on inflationary dynamics, dark matter production, and reheating through current and next-generation GW experiments.

DOI: [10.1103/dxht-n25w](https://doi.org/10.1103/dxht-n25w)

I. INTRODUCTION

Cosmic inflation remains the leading theoretical framework addressing several fundamental problems in cosmology, including the horizon problem, the flatness problem, and the origin of large-scale structure [1–6]. Despite extensive observational efforts, primordial B-modes, expected imprints of tensor perturbations, have yet to be detected, but they remain a pivotal observational target for testing the inflationary paradigm [7,8]. Simple single-field slow-roll models of inflation not only resolve these fundamental issues but also predict an approximately scale-invariant curvature power spectrum, consistent with

current observational constraints from the cosmic microwave background (CMB) [9] and large-scale structure (LSS) surveys [10]. The curvature power spectrum measured by *Planck* gives $\Delta_\zeta^2(k_*) \simeq 2.1 \times 10^{-9}$ at the pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$ [9].

The existence of the Higgs scalar field in the standard model (SM) [11,12] strongly motivates the presence of other scalar fields, including spectator fields—fields present during inflation that do not drive the expansion and do not significantly influence its dynamics. Recent studies [13,14] have shown that large isocurvature fluctuations associated with spectator scalar fields can produce significant and potentially observable gravitational wave (GW) signals, while Ref. [15] demonstrated that stochastic effects arising from these fields can independently generate detectable GWs. In contrast to Ref. [15], where the isocurvature modes are solved using the superhorizon stochastic fluctuations, in our setup the spectator is a massive field with $m_{\text{eff}} \sim \mathcal{O}(H_I)$, whose fluctuations arise from gravitational particle production and are evolved using adiabatic regularization of the energy-momentum

^{*}Contact author: marcos.garcia@fisica.unam.mx

[†]Contact author: verner.s@ufl.edu

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

tensor. This leads to a distinct dependence of the small-scale spectrum on m_{eff} and the reheating history, and allows us to map the viable parameter space (masses, couplings, and reheating temperature) consistent with both CMB and GW constraints.

The fact that isocurvature perturbations can source gravitational waves at second order, either directly or after conversion into curvature perturbations, is by now well established and has been reviewed in detail in Refs. [13,14]. Our scenario belongs to the second class identified there: isocurvature fluctuations of a spectator sector are first transferred into the total curvature perturbation and subsequently source induced GWs. However, we extend the general mechanism and consider the contribution arising from the quantum perturbations and not just at a classical level. Related works have explored GW production via parametric resonance [16] and the connection to enhanced curvature perturbations and primordial black hole formation [17,18].

Current *Planck* constraints limit the isocurvature power spectrum contribution at the CMB pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$ to [9],

$$\frac{\Delta_S^2(k_*)}{\Delta_\zeta^2(k_*) + \Delta_S^2(k_*)} < 0.038 \text{ (95\% C.L.)}. \quad (1)$$

This implies for the isocurvature spectrum $\Delta_S^2(k_*) < 8.3 \times 10^{-11}$, indicating that these perturbations must be strongly suppressed at CMB scales. However, it is possible for isocurvature perturbations to have a larger amplitude at smaller scales beyond the reach of current CMB experiments. Current constraints from CMB and LSS primarily limit the curvature power spectrum across scales in the range $k \sim 10^{-4} - 1 \text{ Mpc}^{-1}$, while scales smaller than $k > 1 \text{ Mpc}^{-1}$ remain largely unconstrained.

In this paper, we propose a novel mechanism for GW production sourced by spectator scalar fields, incorporating the complete evolution of isocurvature fluctuations from inflation through reheating to the present day. Our analysis combines analytical approximations with numerical calculations to capture the full cosmological history and reveals that heavy spectator fields naturally produce blue-tilted isocurvature spectra with distinctive observational signatures spanning a broad frequency range from 10^{-20} Hz to 1 Hz . This mechanism provides a powerful probe of beyond the standard model physics applicable to various spectator scalar field scenarios, including dark matter (DM) candidates [19–28], curvaton models [29–32], axions [33–36], and supersymmetric theories [37,38]. As GW astronomy achieves unprecedented multiband sensitivity, our framework establishes spectator field fluctuations as a novel observational window into inflationary dynamics and the poorly understood reheating epoch, with the broad frequency coverage enabling multimessenger constraints across pulsar timing arrays (PTAs) [39–42], space-based

interferometers [43–45], and ground-based detectors [46–48].¹

The paper is organized as follows. In Sec. II, we introduce the general model and theoretical framework. In Sec. III, we describe the reheating dynamics. In Sec. IV, we discuss the evolution and decay of the spectator scalar field. In Sec. V, we analyze the associated curvature and isocurvature perturbations. In Sec. VI, we compute the gravitational wave signals sourced by these perturbations. Finally, we present our results and conclusions in Sec. VII.

II. FRAMEWORK

We adopt natural units, $k_B = \hbar = c = 1$, with metric signature $(-, +, +, +)$. The homogeneous and isotropic Friedmann-Robertson-Walker (FRW) metric is given by $ds^2 = a(\eta)^2(-d\eta^2 + \delta_{ij}dx^i dx^j)$, where $a(\eta)$ is the scale factor and $d\eta = dt/a$ denotes conformal time. The general action for our theory is

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R - \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) - \frac{1}{2} (\partial_\mu \chi)^2 - \frac{1}{2} m_\chi^2 \chi^2 - \frac{1}{2} \sigma \phi^2 \chi^2 \right], \quad (2)$$

where $g = \det g_{\mu\nu}$ is the determinant of the metric, $M_P = 1/\sqrt{8\pi G_N} \simeq 2.435 \times 10^{18} \text{ GeV}$ is the reduced Planck mass, and R represents the Ricci scalar. Here, ϕ is the inflaton field with potential $V(\phi)$, and χ is the spectator scalar field, with bare mass m_χ . The interaction term $\sigma \phi^2 \chi^2$ characterizes the coupling between the inflaton and the spectator field. When considering the perturbations, we decompose the scalar fields into homogeneous backgrounds and perturbations as

$$\phi(t, \mathbf{x}) = \bar{\phi}(t) + \delta\phi(t, \mathbf{x}), \quad \chi(t, \mathbf{x}) = \delta\chi(t, \mathbf{x}), \quad (3)$$

that is, $\bar{\chi}(t) = 0$ at the background level and $\delta\chi$ denotes the inhomogeneous spectator field. Starting in its vacuum state, χ is treated as a purely quantum field living on the homogeneous FRW background, with its fluctuations described by the canonically normalized mode functions $X_k(\eta)$ (see below and Appendix A for details).

Introducing the canonically normalized field $X \equiv a\chi$, and varying the action with respect to X , we obtain the equation of motion,

$$(\square - a^2 m_{\text{eff}}^2)X = 0, \quad m_{\text{eff}}^2 = m_\chi^2 + \sigma \phi^2 - \frac{1}{6}R. \quad (4)$$

¹See Refs. [49,50] for scenarios with spectator fields that provide tensor perturbations. Owing to their linear coupling to metric tensor perturbations, such fields can generate a stochastic background as large as $\Omega_{\text{GW}} \sim 10^{-7}$, comparable to that sourced by curvature perturbations.

The Ricci scalar is given by

$$R = 6(2H^2 + \dot{H}), \quad (5)$$

where H denotes the Hubble parameter. During slow-roll inflation one has $|\dot{H}| \ll H^2$ so that $R \simeq 12H_I^2 > 0$, where H_I is the inflationary scale. Therefore, the curvature contribution in m_{eff}^2 is negative, $-R/6 \simeq -2H_I^2$, which can lead to the tachyonic excitation of the field mode functions for small bare masses and large isocurvature fluctuations [25,51–53]. For the present work, in all numerical examples we require $m_{\text{eff}}^2 > 0$ and focus on the range $0.5H_I \lesssim m_{\text{eff}}$, where the spectator is sufficiently heavy to avoid the tachyonic instability, keeping its contribution to CMB-scale perturbations small, while still generating a strongly blue isocurvature spectrum on the small scales of interest.

We expand the field X in Fourier modes as

$$X(\eta, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}} [X_k(\eta)\hat{a}_{\mathbf{k}} + X_k^*(\eta)\hat{a}_{-\mathbf{k}}^\dagger], \quad (6)$$

where $\mathbf{k}$ is the comoving momentum, with $|\mathbf{k}| = k$, and $\hat{a}_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}^\dagger$ are the annihilation and creation operators, respectively, that satisfy the canonical commutation relations: $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta^{(3)}(\mathbf{k} - \mathbf{k}')$ and $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = [\hat{a}_{\mathbf{k}}^\dagger, \hat{a}_{\mathbf{k}'}^\dagger] = 0$. By substituting the Fourier-decomposed field from Eq. (6) into the equation of motion (4), we obtain the mode equation,

$$X_k'' + \omega_k^2 X_k = 0, \quad \text{with} \quad \omega_k^2 = k^2 + a^2 m_{\text{eff}}^2, \quad (7)$$

where primes denote derivatives with respect to conformal time.

Evaluating GW signals from spectator scalar fields requires careful treatment of their gravitational production during and after inflation. This process determines the energy density ratio between spectator and inflaton fields, ρ_χ/ρ_ϕ , which critically influences isocurvature power spectrum evolution as modes re-enter the horizon. We solve the mode equations with initial conditions set by the Bunch-Davies vacuum. In the early-time asymptotic limit, $\eta \rightarrow -\infty$, modes deep inside the horizon ($k \gg aH$) evolve in effectively flat Minkowski space with frequency $\omega_k = k$, motivating the vacuum state $\lim_{\eta \rightarrow -\infty} X_k(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta}$. The comoving energy density of gravitationally produced spectator scalar fields is computed as [24–26,54]²:

$$\rho_\chi a^3 = \int_{k_0}^\infty \frac{dk}{k} \mathcal{E}_k(\eta), \quad \text{with} \quad \mathcal{E}_k(\eta) = \frac{k^3}{2\pi^2} \frac{\omega_k(\eta)}{a(\eta)} n_k, \quad (8)$$

²For a detailed review on gravitational particle production, see Ref. [24].

where the occupation number is given by $n_k = \frac{1}{2\omega_k} |\omega_k X_k - iX_k'|^2$.³ We numerically evolve these mode equations to track the complete energy density evolution throughout cosmological history.⁴

For concreteness, we adopt the T-model inflationary potential [55]⁵:

$$V(\phi) = \lambda M_P^4 \left[\sqrt{6} \tanh\left(\frac{\phi}{\sqrt{6}M_P}\right) \right]^2, \quad (9)$$

where the coupling is normalized to *Planck* observations via $\lambda \simeq 3\pi^2 \Delta_\zeta^2(k_*)/N_*^2$. For $N_* = 55$ e-folds, this yields $\lambda \simeq 2 \times 10^{-11}$, spectral tilt $n_s \simeq 0.963$, and tensor-to-scalar ratio $r \simeq 0.004$, which is highly favored by current CMB measurements [9,56]. The parameter λ determines the inflaton mass at the potential minimum $V(0)$, with $m_\phi = \sqrt{2\lambda} M_P \simeq 1.6 \times 10^{13}$ GeV. The Hubble parameter at the horizon exit is given by $H_I = 1.5 \times 10^{13}$ GeV (with $\phi_* = 5.35 M_P$), and at the end of inflation, $H_{\text{end}} = 6.3 \times 10^{12}$ GeV.

III. REHEATING DYNAMICS

The inflationary phase terminates once the expansion is no longer accelerating and the inflaton ϕ departs from the slow-roll approximation. From that moment on, ϕ begins coherent oscillations about the minimum of its potential $V(\phi)$. The energy stored in these oscillations is gradually transferred to lighter fields, thereby reheating the Universe. If the inflaton couples (directly or through mediators) to SM degrees of freedom, this process efficiently populates a relativistic plasma that thermalizes and provides the initial conditions for standard big bang nucleosynthesis [57–59].

In this work, we model reheating as a perturbative decay of the homogeneous inflaton condensate reheating around a quadratic minimum of the inflaton potential. Under this assumption, the condensate remains coherent throughout reheating, and nonlinear effects such as broad parametric resonance of the decay products (preheating) can be

³This expression is computed in the late-time asymptotic limit. The complete time-dependent evolution is provided in Appendix A.

⁴We note that in computing gravitational particle production of the spectator, we treat χ as a quantum field on the homogeneous FRW background and neglect scalar metric perturbations in its mode equation. This is justified because the metric perturbations are small and their feedback on the particle production rate is a higher-order effect. The effect of scalar perturbations is fully included at the level of the total curvature perturbation ζ , which sources the induced tensor modes at second order following the standard formalism.

⁵The results presented in this paper are mostly general and can be applied to any plateau-like inflationary potential.

neglected.⁶ The homogeneous background is then governed by the coupled Boltzmann equations

$$\rho'_\phi + 3\mathcal{H}\rho_\phi = -a\Gamma_\phi\rho_\phi, \quad (10)$$

$$\rho'_R + 4\mathcal{H}\rho_R = a\Gamma_\phi\rho_\phi, \quad (11)$$

where ρ_ϕ and ρ_R denote the inflaton and radiation energy densities, respectively, Γ_ϕ is the inflaton decay rate, and $\mathcal{H} \equiv aH$ is the conformal Hubble parameter. The expansion rate is determined by the Friedmann constraint,

$$3\mathcal{H}^2 M_P^2 = a^2(\rho_\phi + \rho_R). \quad (12)$$

We define the end of reheating as the time t_{reh} at which the inflaton and radiation energy densities are equal, $\rho_\phi(t_{\text{reh}}) = \rho_R(t_{\text{reh}}) \equiv \rho_{\text{reh}}$. The associated reheating temperature T_{reh} is obtained from the usual relation

$$\rho_R(t_{\text{reh}}) = \frac{\pi^2}{30} g_{\text{reh}} T_{\text{reh}}^4, \quad (13)$$

where g_{reh} is the effective number of relativistic degrees of freedom at T_{reh} . For $T_{\text{reh}} > m_t$ we take $g_{\text{reh}} = 427/4$.

In the early stages of reheating, when $a_{\text{end}} < a \ll a_{\text{reh}}$, the radiation component is still subdominant and the inflaton behaves as nonrelativistic matter,

$$\rho_\phi(a) \simeq \rho_{\text{end}} \left(\frac{a}{a_{\text{end}}} \right)^{-3}, \quad (14)$$

where a_{end} and ρ_{end} denote the scale factor and total energy density at the end of inflation. Inserting this matterlike scaling into Eqs. (11) and (12) yields a useful analytic approximation for the build-up of the radiation bath during reheating [63],

$$\rho_R(a) \simeq \frac{2}{5} \sqrt{3M_P^2 \Gamma_\phi^2 \rho_{\text{end}}} \left(\frac{a_{\text{end}}}{a} \right)^4 \left[\left(\frac{a}{a_{\text{end}}} \right)^{5/2} - 1 \right]. \quad (15)$$

At early times the bracket in Eq. (15) captures the growth of ρ_R driven by inflaton decays. At later times the radiation density approaches the standard $\rho_R \propto a^{-3/2}$ scaling driven by inflaton decays, and eventually transitions to free-radiation scaling $\rho_R \propto a^{-4}$ once ρ_ϕ has been sufficiently depleted.

Equation (15) also allows one to read off characteristic scales of the thermal history. The maximum temperature T_{max} of the primordial plasma occurs at $a_{\text{max}} \simeq (8/3)^{2/5} a_{\text{end}}$

and is approximately

$$T_{\text{max}} \simeq \frac{5}{3} \left(\frac{3}{8} \right)^{8/5} \sqrt{3M_P^2 \Gamma_\phi^2 \rho_{\text{end}}}, \quad (16)$$

while the reheating temperature can be expressed in terms of the decay rate as

$$T_{\text{reh}} \simeq \left(\frac{72\Gamma_\phi^2 M_P^2}{5\pi^2 g_{\text{reh}}} \right)^{1/4}, \quad (17)$$

reached when the scale factor has grown to

$$a_{\text{reh}} \simeq \left(\frac{25}{12} \frac{\rho_{\text{end}}}{\Gamma_\phi^2 M_P^2} \right)^{1/3} a_{\text{end}}. \quad (18)$$

These relations provide a convenient parametrization of the reheating era in terms of the microscopic parameter Γ_ϕ (or, equivalently, T_{reh}) and the inflationary initial condition ρ_{end} . In our numerical analysis, we solve Eqs. (10)–(12) without further approximation and treat Γ_ϕ as a free parameter controlling the duration of reheating and the subsequent expansion history. This background evolution defines the time-dependent FRW geometry in which the spectator field is gravitationally produced and subsequently evolves. We illustrate the background evolution in Fig. 1, which shows the energy densities of the decaying inflaton, ρ_ϕ , the visible-sector radiation, ρ_R , and the gravitationally produced spectator field, ρ_χ , for a benchmark reheating temperature $T_{\text{reh}} = 10^{13}$ GeV (see the discussion below).

IV. SPECTATOR FIELD DYNAMICS AND DECAY

Once the background evolution is fixed, the spectator scalar χ is treated as a purely quantum field with a vanishing homogeneous expectation value. Its excitation is entirely due to gravitational particle production in the time-dependent FRW geometry. To describe this process, we introduce the canonically normalized mode functions X_k and solve their equation of motion with Bunch-Davies initial conditions deep inside the horizon during inflation. The resulting $X_k(\eta)$ are then used in the renormalized energy density expression, Eq. (8), which we evolve numerically from the inflationary phase, through reheating, and well into the subsequent radiation-dominated era. This procedure captures the nonperturbative production of χ quanta and their subsequent redshifting over the entire evolution.

The numerical solutions exhibit several distinct regimes. During inflation, the effective mass m_{eff} and the Hubble rate vary slowly, and the spectator energy density is nearly constant in time. In this regime, one must use the full expression, Eq. (8), to accurately evaluate ρ_χ . After inflation ends and the inflaton begins to oscillate, the Hubble parameter decreases. Once H drops below m_{eff} the

⁶For the T-model potential in Eq. (9), this regime is realized, for instance, if reheating proceeds via two-body decays into fermions with Yukawa couplings $y \lesssim 10^{-5}$ [60], or through multibody decays in no-scale scenarios, as discussed in Refs. [61,62].

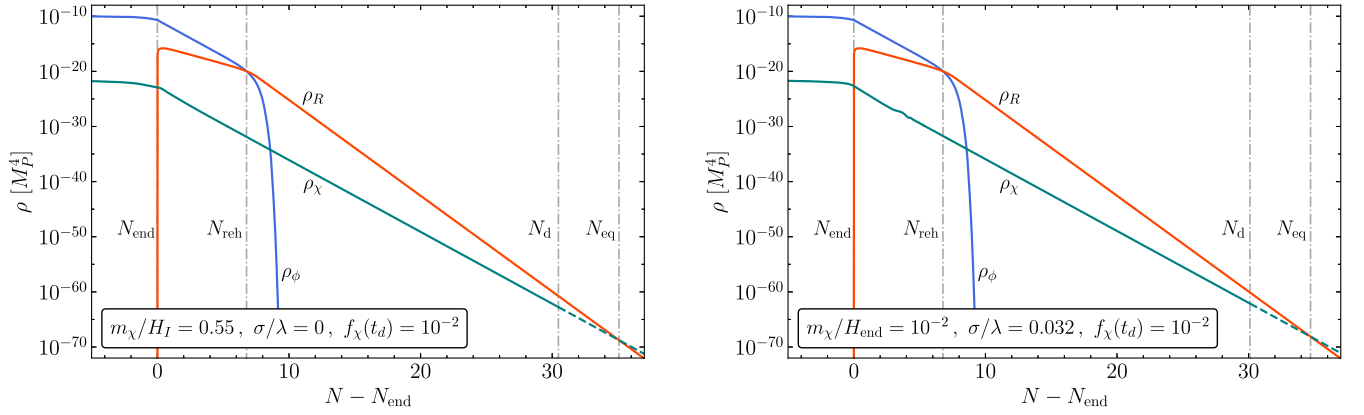


FIG. 1. Evolution of the inflaton energy density ρ_ϕ , visible-sector radiation ρ_R , and the spectator density ρ_χ as functions of the number of e -folds N , for a benchmark with reheating temperature $T_{\text{reh}} = 10^{13}$ GeV. The solid green curve tracks ρ_χ in the unstable (curvatonlike) scenario, where the spectator is taken to decay into radiation once its fractional contribution reaches $f_\chi(t_d) = \rho_\chi(t_d)/\rho_R(t_d) = 10^{-2}$. The dashed green curve shows the corresponding evolution in the stable (dark matter) case with no decay.

relevant modes become adiabatic, and the produced χ quanta behave as a nonrelativistic gas. The comoving number density approaches a constant, $n_\chi a^3 \simeq \text{const.}$, so that the spectator energy density subsequently scales as

$$\rho_\chi \propto a^{-3}, \quad (19)$$

while the radiation bath sourced by inflaton decays redshifts as $\rho_R \propto a^{-4}$ after reheating. The energy density ratio $f_\chi \equiv \rho_\chi/\rho_R$, therefore, grows with the scale factor during radiation domination. Deep inside the horizon ($k/a \gg H$) and once $H \ll m_{\text{eff}}$, when the background curvature is negligible and the expansion is slow compared to the oscillation frequency, gravitational particle production effectively ceases and the comoving number density $n_\chi a^3$ becomes constant.

In the following, we consider two limiting scenarios for the late-time fate of the spectator. In the *unstable* (curvatonlike) case, we assume that χ decays into visible radiation. Rather than specifying a particular microphysical coupling, we parametrize the decay by an effective width Γ_χ and implement it as a sudden conversion of ρ_χ into radiation at a decay time t_d defined by $H(t_d) = \Gamma_\chi$. The key quantity controlling the impact of the decay is the energy fraction $f_\chi(t_d) = \rho_\chi(t_d)/\rho_R(t_d)$. For generic choices of Γ_χ , $f_\chi(t_d)$ can range from $\mathcal{O}(10^{-2})$ (a small entropy injection) to $\mathcal{O}(1)$ (a curvatonlike scenario). In Fig. 1, we illustrate a benchmark in which we impose decay when $f_\chi(t_d) = 10^{-2}$. The solid green curve follows ρ_χ up to t_d , after which its energy is added to ρ_R , while the dashed green curve shows the same evolution in the absence of decay.

In the *stable* (dark matter) case, we set $\Gamma_\chi = 0$ and evolve the spectator energy density without decay, allowing ρ_χ to remain as an additional nonrelativistic component. The

normalization of ρ_χ in this scenario is fixed by the gravitational production process described above. In our parameter scans, we require that the relic abundance generated in this way matches the observed dark matter density at matter-radiation equality. The two cases, unstable and stable, will lead to different induced gravitational wave signals, which we analyze in detail in the following sections.

V. CURVATURE AND ISOCURVATURE POWER SPECTRA

We analyze metric perturbations in the Newtonian gauge with line element

$$ds^2 = a^2(\eta)[-(1 + 2\Phi)d\eta^2 + [(1 - 2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j], \quad (20)$$

where Φ represents scalar perturbations and h_{ij} denotes tensor perturbations satisfying the transverse-traceless conditions $h_i^i = \partial_i h^{ij} = 0$. Primordial fluctuation dynamics are governed by two gauge-invariant quantities: the curvature perturbation $\zeta = -\Phi - \mathcal{H}\delta\rho/\bar{\rho}'$ on uniform-density hypersurfaces, and the isocurvature perturbation

$$S \equiv 3(\zeta_\chi - \zeta_R) = \frac{\delta\rho_\chi}{\bar{\rho}_\chi + \bar{p}_\chi} - \frac{\delta\rho_R}{\bar{\rho}_R + \bar{p}_R}, \quad (21)$$

measuring entropy fluctuations between spectator field and radiation components.⁷ Here, $\delta\rho$ is the total density perturbation, and $\bar{\rho}'$ is the conformal time derivative of the background energy density. For spectator scalar field χ with vanishing background value $\bar{\chi} = 0$, the effective mass

⁷We assume the inflaton decays into radiation, thus $\zeta_\phi = \zeta_R$.

$m_{\text{eff}}^2 = m_\chi^2 + \sigma\phi^2 - \frac{1}{6}R$ determines the amplitude of quantum fluctuations during inflation. When $m_{\text{eff}} \lesssim H_I$, super-horizon modes are efficiently excited, generating substantial isocurvature perturbations that persist after inflation ends. The statistical properties of these isocurvature fluctuations are captured by the dimensionless power spectrum, $\Delta_S^2 \equiv \frac{k^3}{2\pi^2} \mathcal{P}_S(\eta, k)$, where $\mathcal{P}_S$ is defined through the two-point correlation function $\langle \mathcal{S}(\eta, \mathbf{k}) \mathcal{S}(\eta, \mathbf{k}') \rangle = \delta^{(3)}(\mathbf{k} + \mathbf{k}') \mathcal{P}_S(\eta, k)$.

After expressing energy density fluctuations in terms of the canonically normalized field $X = a\chi$, we can derive an expression for the exact isocurvature spectrum, valid at all times,

$$\Delta_S^2(\eta, k) = \frac{1}{\bar{\rho}_\chi^2(\eta)} \frac{k^3}{a^8 (2\pi)^5} \int d^3p \{ |X'_p|^2 |X'_q|^2 + a^4 m_{\text{eff}}^4 |X_p|^2 |X_q|^2 + a^2 m_{\text{eff}}^2 [(X_p X_q^{*}) (X_q X_p^{*}) + \text{H.c.}] \}, \quad (22)$$

where $q = |\mathbf{p} - \mathbf{k}|$. In the late-time asymptotic limit,⁸ an analytical approximation to this expression can be constructed, based on the de Sitter solution of the mode function equations in (7),

$$\Delta_S^2(k) \simeq \Lambda^2 \left(\frac{k}{k_{\text{end}}} \right)^{2\Lambda} \ln \left(\frac{k}{a_{\text{end}} H_I} \right) \left(\frac{H_{\text{end}}}{H_I} \right)^{2\Lambda}, \quad (23)$$

where $\Lambda = 2m_{\text{eff}}^2/3H_I^2$ and $k_{\text{end}} = a_{\text{end}} H_{\text{end}}$, the mode that leaves the horizon at the end of inflation [53,64] (see Appendix B for more details on the derivation of the isocurvature spectrum).⁹

We emphasize that throughout this work, we focus on the regime $m_{\text{eff}} \sim \mathcal{O}(H_I)$, which yields a strongly blue-tilted isocurvature spectrum, see Fig. 5. For much lighter spectators with $m_{\text{eff}} \ll H_I$, the field behaves as a curvaton: its fluctuations are nearly scale invariant, and if it later decays into SM radiation it can dominate the adiabatic curvature perturbation. This regime is tightly constrained by CMB measurements of A_s and n_s and has been extensively studied in the literature. We, therefore, do not consider $m_{\text{eff}} \ll H_I$, which is primarily excluded by isocurvature perturbation constraints. In all benchmarks shown, the spectator contribution to Δ_ζ^2 at the Planck pivot scale remains well below the observed value.

⁸We use the “late-time asymptotic limit” [25,53] to denote the regime in which both $m_{\text{eff}} \gg H$ and $k/a \gg H$ for the modes of interest, so that the expansion is slow compared to the oscillation frequency and the mode functions approach their Minkowski form.

⁹An alternative analytical approximation is given by $\Delta_S^2(k) \simeq \frac{4}{\pi} \Gamma(2 - 2\Lambda) \sin(\pi\Lambda) \left(\frac{k}{k_{\text{end}}} \right)^{2\Lambda}$ [15,27,53,64–66], though this form does not fully capture the transition from inflation to the matter-dominated epoch.

Our analysis reveals a key feature: the isocurvature spectrum exhibits exponential sensitivity to the ratio m_{eff}/H_I . Small increases in this ratio dramatically suppress isocurvature perturbations at large scales, producing a strongly blue-tilted spectrum that naturally satisfies CMB constraints at observable scales ($k_* = 0.05 \text{ Mpc}^{-1}$) while maintaining significant power at smaller scales where GW production occurs.

After inflation, as the Universe transitions to radiation domination, the total curvature perturbation receives contributions from both radiation (originating from inflaton decay) and the spectator field,

$$\zeta = \frac{4\bar{\rho}_R \zeta_R + 3\bar{\rho}_\chi \zeta_\chi}{4\bar{\rho}_R + 3\bar{\rho}_\chi}. \quad (24)$$

Here, ζ_R and ζ_χ denote the individual curvature perturbations associated with the radiation and spectator components, respectively, defined by $\zeta_i \equiv -\Phi - H\delta\rho_i/\dot{\rho}_i$ for $i = R, \chi$. Before reheating, ζ_ϕ describes the curvature perturbation of the inflaton condensate. After reheating, we identify ζ_R with the curvature perturbation of the radiation fluid sourced by the inflaton decay products and use this as the adiabatic component. Using the isocurvature perturbation $\mathcal{S} = 3(\zeta_\chi - \zeta_R)$, we can express the total curvature perturbation as

$$\zeta = \zeta_R + \frac{\bar{\rho}_\chi}{4\bar{\rho}_R + 3\bar{\rho}_\chi} \mathcal{S} = \zeta_R + \frac{f_\chi}{4 + 3f_\chi} \mathcal{S}, \quad (25)$$

where $f_\chi = \rho_\chi/\rho_R$ is the energy density ratio. Since radiation and matterlike spectator fields scale differently with expansion ($\rho_R \propto a^{-4}$, $\rho_\chi \propto a^{-3}$), this ratio increases as $f_\chi \propto a$ until the spectator field decays, creating a scale-dependent modulation of the curvature power spectrum. Assuming statistical independence between inflaton and spectator field fluctuations, the total curvature power spectrum becomes

$$\Delta_\zeta^2 = \Delta_{\zeta_R}^2 + \left(\frac{\bar{\rho}_\chi}{4\bar{\rho}_R + 3\bar{\rho}_\chi} \right)^2 \Delta_S^2, \quad (26)$$

where $\Delta_\zeta^2(k) = \frac{k^3}{2\pi^2} \mathcal{P}_\zeta(k)$ and $\langle \zeta(\mathbf{k}) \zeta(\mathbf{k}') \rangle \equiv \delta^{(3)}(\mathbf{k} + \mathbf{k}') \mathcal{P}_\zeta(k)$, with $\Delta_{\zeta_R}^2(k)$ and $\Delta_S^2(k)$ defined analogously. For modes re-entering the horizon at different epochs, this density ratio creates a scale-dependent enhancement. A mode with wave number k re-enters when $k = aH$, allowing us to relate $f_\chi(t)$ to scale k through $f_\chi(t) = f_\chi(t_d) \left(\frac{k_d}{k} \right)$, where k_d is the wave number of the mode re-entering at the spectator field decay time t_d .

This leads to a distinctive piecewise curvature power spectrum [15],

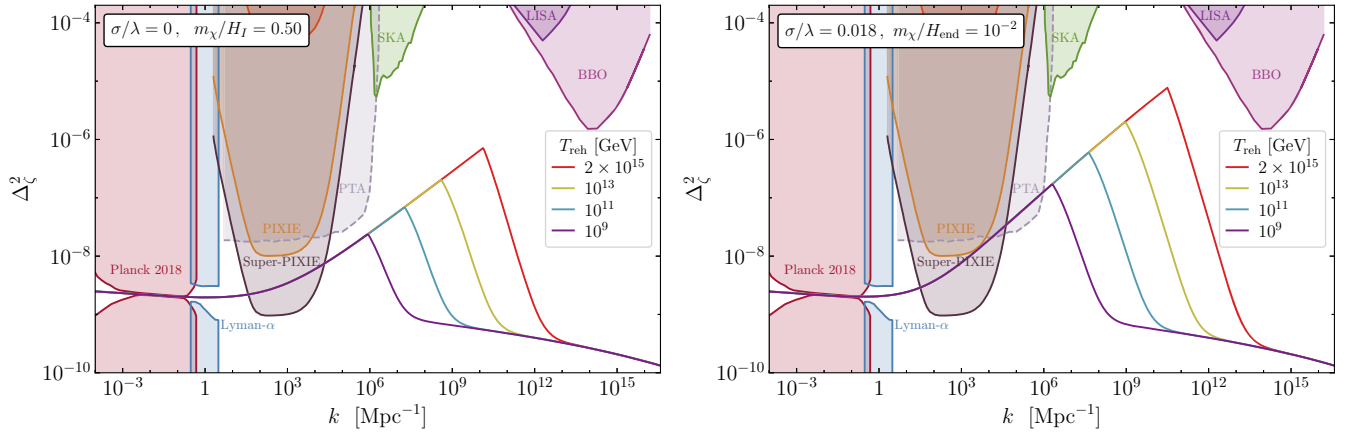


FIG. 2. Curvature power spectra for selected spectator field masses and couplings. Each panel shows four reheating temperatures, with the highest corresponding to instantaneous reheating, $a_{\text{reh}} = a_{\text{end}}$. The spectrum exhibits a characteristic peak at k_d (corresponding to spectator field decay), with red-tilted suppression at smaller scales where $f_\chi \ll 1$ and $\Delta_\zeta^2 \simeq (k_d/k)^2 \Delta_S^2/16$. Background curves indicate current and projected sensitivities: CMB spectral distortion experiments (PIXIE and Super-PIXIE [67,68]), PTA constraints on enhanced dark matter substructure [69], and GW observatories (SKA [42,70,71], LISA [43,72], and BBO [44,73]). Here, we fix the spectator decay rate Γ_χ and vary the inflaton decay rate Γ_ϕ , so that each curve corresponds to a different reheating temperature T_{reh} while the decay time t_d (and hence k_d) is held fixed.

$$\Delta_\zeta^2(k) = \begin{cases} \Delta_{\zeta_R}^2(k) + \left(\frac{f_\chi(t_d)}{4+3f_\chi(t_d)}\right)^2 \Delta_S^2(k), & k < k_d, \\ \Delta_{\zeta_R}^2(k) + \left(\frac{f_\chi(t_d)(k_d/k)}{4+3f_\chi(t_d)(k_d/k)}\right)^2 \Delta_S^2(k), & k > k_d. \end{cases} \quad (27)$$

The resulting spectrum exhibits a characteristic peak at k_d , with position depending on reheating temperature, followed by a red-tilted region at smaller scales. In the examples shown here, we fix Γ_χ (and hence t_d and k_d) and vary the inflaton decay rate Γ_ϕ , or equivalently the reheating temperature T_{reh} . Changing T_{reh} modifies the expansion history between the end of inflation and t_d , thereby shifting the position and height of the peak in k -space. Figure 2 illustrates this dependence of $\Delta_\zeta^2(k)$ on T_{reh} for fixed spectator parameters. These enhanced scalar perturbations directly source GWs through nonlinear interaction, as we demonstrate in the following section. The complete isocurvature power spectrum for various spectator field masses and couplings σ is presented in Appendix B.

VI. GRAVITATIONAL WAVES FROM ISOCURVATURE PERTURBATIONS

Enhanced scalar perturbations from spectator fields source secondary GWs through nonlinear gravitational interactions. The tensor perturbation equation in Fourier space is

$$h''_\lambda(\eta, \mathbf{k}) + 2\mathcal{H}h'_\lambda(\eta, \mathbf{k}) + k^2 h_\lambda(\eta, \mathbf{k}) = S_\lambda(\eta, \mathbf{k}), \quad (28)$$

where h_λ represents the two polarization modes of GWs, and

$$h_{ij}(\eta, \mathbf{x}) = \sum_\lambda \int \frac{d^3k}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}} \varepsilon_{ij}^\lambda(\mathbf{k}) h_\lambda(\eta, \mathbf{k}), \quad (29)$$

where $\varepsilon_{ij}^\lambda(\mathbf{k})$ is the corresponding polarization tensors. The source term directly couples to the isocurvature perturbations through the gauge-invariant Poisson equation $\nabla^2 \Phi = \delta\rho_\chi/2M_P^2$, with $S_\lambda(\eta, \mathbf{k}) \sim \int d^3p p^2 |\mathbf{k} - \mathbf{p}|^2 \Phi(\eta, \mathbf{p}) \Phi(\eta, \mathbf{k} - \mathbf{p}) \sim \int d^3p p \delta\rho_\chi(\eta, \mathbf{p}) \delta\rho_\chi(\eta, \mathbf{k} - \mathbf{p})$. The gravitational wave power spectrum is defined as $\langle h_\lambda(\eta, \mathbf{k}) h_{\lambda'}(\eta, \mathbf{k}') \rangle = \delta_{\lambda\lambda'} \delta^{(3)}(\mathbf{k} + \mathbf{k}') \mathcal{P}_\lambda(\eta, k)$ with dimensionless spectrum $\Delta_\lambda^2(\eta, k) = k^3 \mathcal{P}_\lambda(\eta, k)/2\pi^2$. The present-day gravitational wave energy density is

$$\Omega_{\text{GW}} h^2 = \frac{1}{12} \left(\frac{k}{a_0 H_0} \right)^2 \Delta_h^2, \quad (30)$$

where a_0 is the present-day scale factor, $H_0 = 100h \text{ km/s/Mpc}$ is the present-day Hubble parameter, and $\Delta_h^2 = \sum_\lambda \Delta_\lambda^2$ represents the total power across both polarization states.

One can then show that if one imposes statistical homogeneity and isotropy on the perturbation $\delta\rho_\chi$ that arises in the source term S_λ , it leads to $\Delta_h^2 \sim \langle h_\lambda h_\lambda \rangle \sim \langle S_\lambda S_\lambda \rangle \sim \langle \delta\rho_\chi \delta\rho_\chi \delta\rho_\chi \delta\rho_\chi \rangle \sim \langle \delta\rho_\chi \delta\rho_\chi \rangle^2$, where we assume that there is no significant presence of primordial non-Gaussianities as perturbative corrections to the Gaussian statistics [74]. Therefore, we derive

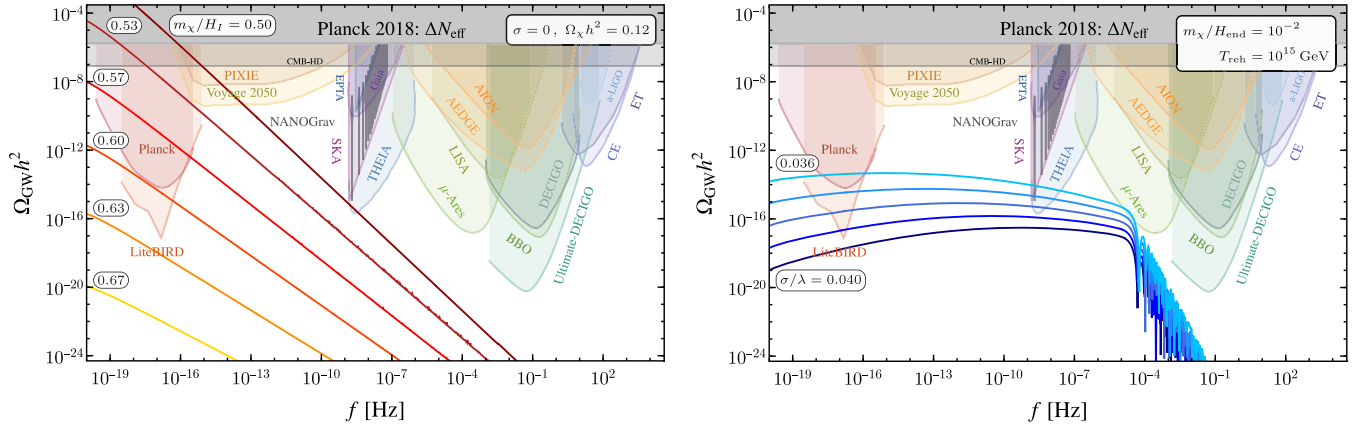


FIG. 3. Gravitational wave spectra for two spectator field scenarios. Left: stable spectator field constituting dark matter through purely gravitational production. The labeled values $m_\chi/H_I = \{0.50, 0.53, 0.57, 0.60, 0.63, 0.67\}$ correspond to the colored dots on the $\Omega_\chi = \Omega_{\text{DM}}$ teal curve of Fig. 4. Right: unstable spectator field decaying near matter-radiation equality in a high reheating scenario ($T_{\text{reh}} \approx 10^{15}$ GeV). Background curves show current and projected sensitivities for CMB experiments [68,75–79], PTAs [39–42], and space/ground-based interferometers [43–48,80–83].

$$\begin{aligned} \Delta_h^2 = \mathcal{C} \times k^2 \int_0^\infty dp p \int_{|k-p|}^{k+p} & \\ \times dq q \frac{(k^4 - 2k^2(p^2 + q^2) + (p^2 - q^2)^2)}{p^7 q^7} & \\ \times \Delta_S^2(p) \Delta_S^2(q). \end{aligned} \quad (31)$$

This expression directly relates the GW spectrum to the isocurvature power spectrum, creating a distinctive observational signature of spectator field dynamics. The complete derivation and the numerical factor $\mathcal{C}$ are provided in Appendix C.

VII. RESULTS AND DISCUSSION

Before describing the spectra in detail we note that, as anticipated on general grounds, scalar-induced GWs from a stable spectator are generically small because $\rho_\chi/\rho_{\text{tot}} \ll 1$ throughout radiation domination. An observable signal requires the peak of the matter perturbation to sit at scales not far above k_{eq} , which restricts m_{eff}/H_I to the narrow window allowed by isocurvature and ΔN_{eff} constraints, and additionally requires T_{reh} tuned so that $\Omega_\chi h^2 = 0.12$. The benchmarks shown below should, therefore, be read as illustrating the best-case observable scenarios rather than generic predictions.

We demonstrate that heavy spectator fields with effective masses near the Hubble scale ($m_{\text{eff}} \sim H_I$) naturally generate blue-tilted isocurvature spectra with spectral indices $\Lambda \simeq 2m_{\text{eff}}^2/3H_I^2$ (typically $\Lambda \simeq 0.25\text{--}0.50$), producing observable GWs across frequencies from 10^{-20} to 1 Hz. Our analysis combines analytical approximations with numerical evolution of spectator field dynamics, employing normal-ordered energy density operators and regularized

momentum integrals to capture the complete gravitational production mechanism.

For each benchmark shown below in Fig. 3, the underlying spectator energy fraction can be read off Fig. 1 (for the representative cases shown there), the corresponding isocurvature spectrum from Fig. 5, and, for the $\sigma = 0$ stable case, the position relative to the isocurvature, ΔN_{eff} , r , and overclosure bounds from Fig. 4.

Two distinct phenomenological regimes emerge depending on reheating temperature T_{reh} and spectator field evolution.

For dark matter production scenarios ($T_{\text{reh}} \lesssim 10$ TeV), stable spectator fields produce suppressed amplitudes with scaling $\propto T_{\text{reh}}^{-2/3}$ and

$$\Omega_{\text{GW}} h^2 \propto \begin{cases} k^{4\Lambda-2} \ln^2(k/H_I), & k \ll a_{\text{reh}} H_{\text{reh}}, \\ k^{4\Lambda-6} \ln^2(k/H_I), & k \gg a_{\text{reh}} H_{\text{reh}}. \end{cases} \quad (32)$$

In curvatonlike scenarios with high reheating temperatures ($T_{\text{reh}} \geq 10^7$ GeV), spectator fields decay before matter-radiation equality, leading to

$$\Omega_{\text{GW}} h^2 \propto \begin{cases} k^{4\Lambda-2} \ln^2(k/H_I), & k \ll a_d H_d, \\ k^{4\Lambda-4} \ln^2(k/H_I), & k \gg a_d H_d, \end{cases} \quad (33)$$

with GW amplitude scaling $\propto T_{\text{reh}}^{4/3}$. Both scenarios share identical large-scale behavior but differ at small scales, where gravitational production shows stronger suppression due to the absence of spectator field decay enhancement.

The GW spectra exhibit rich phenomenology depending on the inflaton-spectator coupling. For purely gravitational production ($\sigma = 0$), spectra are universally red-tilted for potentially detectable amplitudes. For dark matter production scenarios, the left panel of Fig. 3 demonstrates that

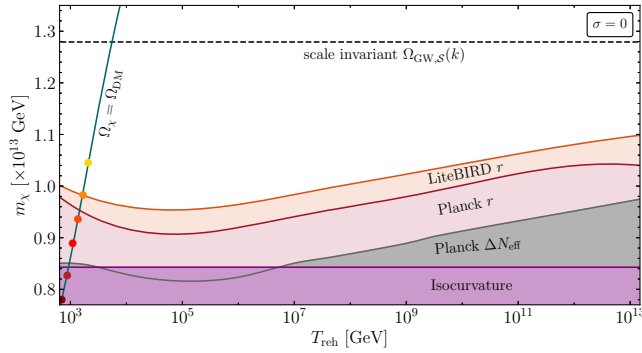


FIG. 4. Parameter space for purely gravitational production of heavy spectator fields [$\sigma = 0$, $f_\chi(t_d) = 1$], with inflationary Hubble rate $H_I = 1.5 \times 10^{13}$ GeV fixed by the T-model normalization. The colored dots correspond to benchmark values from the left panel of Fig. 3. Excluded regions: CMB isocurvature bounds (purple, $m_\chi \lesssim 0.54H_I \simeq 8.4 \times 10^{12}$ GeV), ΔN_{eff} from GW backgrounds (gray), nonobservation of primordial B -modes by Planck/BICEP2/Keck (red), and projected LiteBIRD sensitivity (orange). The teal curve marks $\Omega_\chi h^2 = 0.12$ with overproduction (underproduction) to the right (left). The viable region for an observable stable-spectator GW signal lies on the teal curve within the white wedge.

purely gravitational production can account for the observed relic abundance $\Omega_{\text{DM}} h^2 = 0.12$ while generating detectable GW signals. Spectator masses $m_\chi/H_I = \{0.50, 0.53, 0.57, 0.60, 0.63, 0.67\}$ correspond to reheating temperatures $T_{\text{reh}} = \{700, 876, 1096, 1360, 1680, 2071\}$ GeV, respectively. This dark matter boundary (teal curve in Fig. 4) separates parameter regions producing over- or under-abundance relative to observations.

Nonzero coupling introduces additional complexity: small couplings ($\sigma/\lambda \lesssim 0.036$) maintain red-tilted spectra, intermediate values ($0.036 \lesssim \sigma/\lambda \lesssim 0.04$) produce peaked spectra with maxima around $f \simeq 10^{-5}$ Hz, while larger couplings ($\sigma/\lambda \geq 0.04$) generate blue-tilted infrared behavior for $m_\chi = 10^{-2}H_{\text{end}}$. The right panel of Fig. 3 shows that for instantaneous reheating at $T_{\text{reh}} = 10^{15}$ GeV and $m_\chi/H_{\text{end}} = 10^{-2}$, the coupling range $0.036 \lesssim \sigma/\lambda \lesssim 0.040$ produces correlated signatures across CMB experiments, PTAs, and space-based interferometers.

Parameter space constraints from multiple observations are summarized in Fig. 4. CMB isocurvature bounds exclude spectator masses $m_\chi \lesssim 0.54H_I \simeq 8.4 \times 10^{12}$ GeV, while our GW analysis reveals stronger constraints through $\Delta N_{\text{eff}} \lesssim 0.28$ bounds that exclude combinations of small masses and high reheating temperatures [75], requiring $m_\chi \gtrsim 0.59H_I \simeq 9.1 \times 10^{12}$ GeV for $T_{\text{reh}} \simeq 10^{10}$ GeV. The nonobservation of primordial B -modes by Planck/BICEP2/Keck [75,84] further constrains $m_\chi \gtrsim 0.64H_I \simeq 10^{13}$ GeV except at very low reheating temperatures. Future CMB observatories like LiteBIRD [78] will probe remaining viable parameter space, with projected sensitivities

reaching $m_\chi \gtrsim 0.67H_I \simeq 1.1 \times 10^{13}$ GeV. We note that $\Omega_{\text{GW}} h^2 \propto (H_I/M_p)^4$ at fixed m_χ/H_I , σ , and T_{reh} . Furthermore, for fixed m_χ/H_I and T_{reh} , increasing σ shifts the constraint contours of Fig. 4 toward smaller m_χ .

Our analysis reveals that spectator field isocurvature perturbations provide a novel probe of the elusive reheating epoch, as GW amplitudes depend strongly on reheating temperature. Future detection or nondetection could yield the first direct observational constraints on postinflationary thermalization. The broad frequency coverage, spanning 1 nHz (accessible to SKA [42], NANOGrav [39], and EPTA [40,41]) through 1–10 Hz (e.g., μ -Ares [83], AION [80], and AEDGE [81]) to 1 kHz (Einstein Telescope [47] and Cosmic Explorer [48]), makes multi-band GW astronomy essential for characterizing these signatures.

Spectator fields bridge multiple frontiers in fundamental physics, connecting inflation with dark matter production while probing nonstandard thermal histories and potential beyond-standard-model coupling structures. We identify parameter space “sweet spots” (e.g., $m_\chi/H_I \simeq 0.6$ – 0.7 for purely gravitational production with $\sigma = 0$) where signals satisfy current constraints yet remain detectable by future experiments, providing concrete observational targets.

Our results show that significant induced GWs from spectator fields require (i) effective masses $m_{\text{eff}}/H_I \gtrsim 0.5$, (ii) sufficiently large gravitational production to increase the energy density of the spectator scalar field, and (iii) specific choices of reheating and decay rates. For example, the dark matter benchmark in Fig. 3 (left) relies on low reheating temperatures, whereas the benchmark in Fig. 3 (right) assumes an unstable spectator that decays near matter-radiation equality, corresponding to a very small ratio Γ_χ/m_χ . These assumptions are chosen to maximize the induced GW signal and illustrate what is in principle possible, rather than representing the typical behavior of generic spectator models. As next-generation experiments achieve unprecedented sensitivity, from CMB-HD targeting $\Delta N_{\text{eff}} = 0.014$ and LiteBIRD probing $r > 0.001$ [78] to BBO reaching $\Omega_{\text{GW}} h^2 \simeq 10^{-16} - 10^{-17}$ [44], these signals could provide unique insights into beyond the standard model physics at 10^{13} – 10^{15} GeV energy scales, dark matter production mechanisms, and the thermal history between inflation and big bang nucleosynthesis. The blue-tilted spectra naturally avoid problematic infrared divergences that plague inflation scenarios, while our combined analytical-numerical methodology establishes a robust framework for interpreting data from this emerging multi-messenger window into the primordial universe.

ACKNOWLEDGMENTS

The authors would like to thank Andrew Long for a detailed discussion on the isocurvature power spectrum, and also thank Jeff Dror, David Kaiser, Rocky Kolb,

Soubhik Kumar, Evan McDonough, and Vincent Vennin for helpful conversations. The work of S. V. was supported in part by DOE grant No. DE-SC0022148 at the University of Florida. M. G. was supported by the DGAPA-PAPIIT grants No. IA103123 and No. IA100525 at UNAM, and the SECIHTI “Ciencia de Frontera” grant No. CF-2023-I-17.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: REGULARIZATION AND RENORMALIZATION OF ENERGY DENSITY

This appendix details the regularization and renormalization procedure for the spectator field energy density $\rho_\chi(\eta)$, addressing ultraviolet divergences that arise from high-momentum field modes in the curved spacetime background. The treatment requires both systematic regularization of divergent integrals and renormalization through normal ordering with respect to the appropriate vacuum state.

We begin with the stress-energy tensor for the spectator field χ , derived from the action via $\sqrt{-g}T_{\mu\nu}^{(\chi)} = 2\delta(\sqrt{-g}\mathcal{L}_\chi)/\delta g^{\mu\nu}$ [85],

$$T_{\mu\nu}^{(\chi)} = \nabla_\mu \chi \nabla_\nu \chi - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} (\nabla_\rho \chi) (\nabla_\sigma \chi) + \frac{1}{2} g_{\mu\nu} (m_\chi^2 + \sigma\phi^2) \chi^2. \quad (\text{A1})$$

In the FRW background, the 00 component becomes

$$T_{00}^{(\chi)}(\eta, \mathbf{x}) = \frac{1}{2a^2} [\dot{\chi}^2 + (\nabla\chi)^2 + a^2(m_\chi^2 + \sigma\phi^2)\chi^2]. \quad (\text{A2})$$

Using the field redefinition $X = a\chi$ and the relation $\chi' = a^{-1}(X' - \mathcal{H}X)$, we obtain

$$T_{00}^{(\chi)}(\eta, \mathbf{x}) = \frac{1}{2a^4} \left[(X')^2 + (\nabla X)^2 - 2\mathcal{H}XX' + \left(\mathcal{H}^2 - \frac{a^2 R}{6} \right) X^2 + a^2 m_{\text{eff}}^2 X^2 \right], \quad (\text{A3})$$

where $\mathcal{H} = a'/a$ is the conformal Hubble parameter and R is the Ricci scalar.

We expand the rescaled field X in Fourier modes,

$$X(\eta, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}} [X_k(\eta) \hat{a}_k + X_k^*(\eta) \hat{a}_{-k}^\dagger], \quad (\text{A4})$$

where $\hat{a}_k$ and $\hat{a}_k^\dagger$ satisfy canonical commutation relations,

$$[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta^{(3)}(\mathbf{k} - \mathbf{k}'), \quad \text{and} \quad [\hat{a}_k, \hat{a}_{k'}] = [\hat{a}_k^\dagger, \hat{a}_{k'}^\dagger] = 0. \quad (\text{A5})$$

The mode functions $X_k(\eta)$ evolve according to

$$X_k''(\eta) + \omega_k^2(\eta) X_k(\eta) = 0, \quad (\text{A6})$$

with $\omega_k^2(\eta) = k^2 + a^2(\eta) m_{\text{eff}}^2(\eta)$ and Wronskian normalization $X_k X_k'^* - X_k^* X_k' = i$.

To handle ultraviolet divergences, we employ adiabatic regularization based on the Wentzel-Kramers-Brillouin expansion. We introduce the adiabatic basis functions

$$f_k(\eta) \equiv \frac{e^{-i \int^\eta \omega_k d\eta'}}{\sqrt{2\omega_k}}, \quad (\text{A7})$$

and express the mode functions through Bogoliubov coefficients,

$$X_k(\eta) = \alpha_k(\eta) f_k(\eta) + \beta_k(\eta) f_k^*(\eta), \quad (\text{A8})$$

with the corresponding derivative

$$X_k'(\eta) = -i\omega_k [\alpha_k(\eta) f_k(\eta) - \beta_k(\eta) f_k^*(\eta)], \quad (\text{A9})$$

where $|\alpha_k|^2 - |\beta_k|^2 = 1$ preserves canonical commutation relations. We define time-dependent ladder operators through the Bogoliubov transformation,

$$\begin{aligned} A_k(\eta) &= \alpha_k(\eta) \hat{a}_k + \beta_k^*(\eta) \hat{a}_{-k}^\dagger, \\ A_{-k}^\dagger(\eta) &= \beta_k(\eta) \hat{a}_k + \alpha_k^*(\eta) \hat{a}_{-k}^\dagger. \end{aligned} \quad (\text{A10})$$

The adiabatic vacuum $|0\rangle$ is defined by $A_k|0\rangle = 0$ for all $\mathbf{k}$. The vacuum is normalized such that $\langle 0|0\rangle = 1$.

The inverse Bogoliubov transformation allows us to express the original operators in terms of the adiabatic ladder operators,

$$\begin{aligned} \hat{a}_k &= \alpha_k^* A_k - \beta_k^* A_{-k}^\dagger, \\ \hat{a}_{-k} &= \alpha_k^* A_{-k} - \beta_k^* A_k^\dagger. \end{aligned} \quad (\text{A11})$$

Substituting these relations into the field expansion, we can express the rescaled field and its derivative directly in terms of the adiabatic basis,

$$\begin{aligned} X(\eta, \mathbf{x}) &= \int \frac{d^3 k}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}} [f_k(\eta) A_k + f_k^*(\eta) A_{-k}^\dagger], \\ X'(\eta, \mathbf{x}) &= -i \int \frac{d^3 k}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}} \omega_k [f_k(\eta) A_k - f_k^*(\eta) A_{-k}^\dagger]. \end{aligned} \quad (\text{A12})$$

Using the Bogoliubov transformation with rotated ladder operators A_k , $A_{-k}^\dagger$, A'_k , and $A'_{-k}^\dagger$, the 00 component of the stress-energy tensor for the spectator field can be expressed as

$$T_{00}^{(\chi)}(\eta, \mathbf{x}) = \frac{1}{2a^4} \int \frac{d^3k d^3k'}{(2\pi)^3} [\mathcal{B}_1(k, k') A_k A_{k'} + \mathcal{B}_2(k, k') A_k A_{-k'}^\dagger + \mathcal{B}_3(k, k') A_{-k}^\dagger A_{k'} + \mathcal{B}_4(k, k') A_{-k}^\dagger A_{-k'}^\dagger]. \quad (\text{A13})$$

The coefficient functions $\mathcal{B}_i(k, k')$ encode the field dynamics and curvature coupling effects,

$$\mathcal{B}_1(k, k') = \left[(i\omega_k + \mathcal{H})(i\omega_{k'} + \mathcal{H}) - kk' + a^2 m_{\text{eff}}^2 - \frac{a^2 R}{6} \right] f_k(\eta) f_{k'}(\eta), \quad (\text{A14})$$

$$\mathcal{B}_2(k, k') = \left[(-i\omega_k - \mathcal{H})(i\omega_{k'} - \mathcal{H}) - kk' + a^2 m_{\text{eff}}^2 - \frac{a^2 R}{6} \right] f_k(\eta) f_{k'}^*(\eta), \quad (\text{A15})$$

$$\mathcal{B}_3(k, k') = \left[(i\omega_k - \mathcal{H})(-i\omega_{k'} - \mathcal{H}) - kk' + a^2 m_{\text{eff}}^2 - \frac{a^2 R}{6} \right] f_k^*(\eta) f_{k'}(\eta), \quad (\text{A16})$$

$$\mathcal{B}_4(k, k') = \left[(i\omega_k - \mathcal{H})(i\omega_{k'} - \mathcal{H}) - kk' + a^2 m_{\text{eff}}^2 - \frac{a^2 R}{6} \right] f_k^*(\eta) f_{k'}^*(\eta). \quad (\text{A17})$$

This expression contains ultraviolet divergences that arise from high-momentum mode contributions where the adiabatic approximation breaks down. Renormalization through normal ordering with respect to the adiabatic vacuum state is required to extract finite results [24,51]. The procedure involves expressing the rotated operators A_k and $A_{-k}^\dagger$ in terms of the original operators $\hat{a}_k$ and $\hat{a}_{-k}^\dagger$ using the inverse Bogoliubov transformation from Eq. (A10). The resulting normal-ordered expressions are as follows:

$$:a_k a_{k'} := a_k a_{k'} + \alpha_k^* \beta_k^* \delta^{(3)}(\mathbf{k} + \mathbf{k}'), \quad (\text{A18})$$

$$:a_k a_{-k'}^\dagger := a_{-k'}^\dagger a_k - |\beta_k|^2 \delta^{(3)}(\mathbf{k} + \mathbf{k}'), \quad (\text{A19})$$

$$:a_{-k'}^\dagger a_k := a_{-k'}^\dagger a_k - |\beta_k|^2 \delta^{(3)}(\mathbf{k} + \mathbf{k}'), \quad (\text{A20})$$

$$:a_{-k'}^\dagger a_k^\dagger := a_{-k'}^\dagger a_k^\dagger + \alpha_k \beta_k \delta^{(3)}(\mathbf{k} + \mathbf{k}'). \quad (\text{A21})$$

After normal ordering with respect to the adiabatic vacuum state and taking the vacuum expectation value, the renormalized energy density becomes

$$\rho_\chi(\eta) = \langle :T_{00}^{(\chi)}(\eta, \mathbf{x}): \rangle = \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} [\mathcal{C}_1(k, -k) \alpha_k^* \beta_k^* - \mathcal{C}_2(k, -k) |\beta_k|^2 - \mathcal{C}_3(k, -k) |\beta_k|^2 + \mathcal{C}_4(k, -k) \alpha_k \beta_k], \quad (\text{A22})$$

where the $\mathcal{C}_i(k)$ coefficients arise from the normal ordering transformation and are linear combinations of the $\mathcal{B}_i(k, k')$ terms, given by

$$\begin{aligned} \mathcal{C}_1(k, k') &= \mathcal{B}_1(k, k') \alpha_k \alpha_{k'} + \mathcal{B}_2(k, k') \alpha_k \beta_{k'} + \mathcal{B}_3(k, k') \alpha_{k'} \beta_k + \mathcal{B}_4(k, k') \beta_k \beta_{k'}, \\ \mathcal{C}_2(k, k') &= \mathcal{B}_1(k, k') \alpha_k \beta_{k'}^* + \mathcal{B}_2(k, k') \alpha_k \alpha_{k'}^* + \mathcal{B}_3(k, k') \beta_k \beta_{k'}^* + \mathcal{B}_4(k, k') \alpha_{k'}^* \beta_k, \\ \mathcal{C}_3(k, k') &= \mathcal{B}_1(k, k') \alpha_{k'} \beta_k^* + \mathcal{B}_2(k, k') \beta_{k'} \beta_k^* + \mathcal{B}_3(k, k') \alpha_k^* \alpha_{k'} + \mathcal{B}_4(k, k') \alpha_k^* \beta_{k'}, \\ \mathcal{C}_4(k, k') &= \mathcal{B}_1(k, k') \beta_k^* \beta_{k'}^* + \mathcal{B}_2(k, k') \alpha_{k'}^* \beta_k^* + \mathcal{B}_3(k, k') \alpha_k^* \beta_{k'}^* + \mathcal{B}_4(k, k') \alpha_k^* \alpha_{k'}^*. \end{aligned} \quad (\text{A23})$$

Using the explicit forms of the Bogoliubov coefficients,

$$\alpha_k = \frac{\omega_k^* X_k + i X_k'}{2\omega_k f_k}, \quad \beta_k = \frac{\omega_k X_k - i X_k'}{2\omega_k f_k^*}, \quad (\text{A24})$$

the normal-ordered energy density can be expressed directly in terms of the mode functions,

$$\rho_\chi(\eta) = \frac{1}{2a^2} \int \frac{d^3k}{(2\pi)^3} \left[|\omega_k X_k - iX'_k|^2 + \left(\mathcal{H}^2 - \frac{a^2 R}{6} \right) |X_k|^2 - \mathcal{H}(X_k X'_k + X'_k X_k^*) - \frac{1}{2\omega_k} \left(\mathcal{H}^2 - \frac{a^2 R}{6} \right) \right]. \quad (\text{A25})$$

APPENDIX B: ISOCURVATURE COMPUTATION

This section provides the detailed derivation of the isocurvature power spectrum used in the main text. The isocurvature power spectrum is computed from the two-point correlation function of energy density fluctuations,

$$\langle \delta\rho_\chi(\eta, \mathbf{x}) \delta\rho_\chi(\eta, \mathbf{y}) \rangle = \langle : \rho_\chi(\eta, \mathbf{x}) : : \rho_\chi(\eta, \mathbf{y}) : \rangle - \bar{\rho}_\chi^2(\eta). \quad (\text{B1})$$

The normal-ordered product of energy density operators can be expressed as

$$\begin{aligned} : \rho_\chi(\eta, \mathbf{x}) : : \rho_\chi(\eta, \mathbf{y}) : &:= : T_{00}^{(\chi)}(\eta, \mathbf{x}) : : T_{00}^{(\chi)}(\eta, \mathbf{y}) : \\ &:= \frac{1}{4a^8} \int \frac{d^3k d^3k'}{(2\pi)^3} [C_1(k, k') : a_k a_{k'} : + C_2(k, k') : a_k a_{-k'}^\dagger : + C_3(k, k') : a_{-k}^\dagger a_{k'} : + C_4(k, k') : a_{-k}^\dagger a_{-k'}^\dagger :] \\ &\quad \times \int \frac{d^3q d^3q'}{(2\pi)^3} [C_1(q, q') : a_q a_{q'} : + C_2(q, q') : a_q a_{-q'}^\dagger : + C_3(q, q') : a_{-q}^\dagger a_{q'} : + C_4(q, q') : a_{-q}^\dagger a_{-q'}^\dagger :], \end{aligned} \quad (\text{B2})$$

where the coefficients $C_i(k, k')$ are given by Eq. (A23). When taking the vacuum expectation value, most terms vanish due to the normal ordering, except for specific combinations that create particle pairs. The surviving terms correspond to contractions between creation and annihilation operators,

$$\langle C_1(k, k') C_4(q, q') a_k a_{k'} a_{-q}^\dagger a_{-q'}^\dagger \rangle = C_1(k, k') C_4(q, q') [\delta^{(3)}(\mathbf{k} + \mathbf{q}) \delta^{(3)}(\mathbf{k}' + \mathbf{q}') + \delta^{(3)}(\mathbf{k} + \mathbf{q}') \delta^{(3)}(\mathbf{k}' + \mathbf{q})]. \quad (\text{B3})$$

Performing the integrals with respect to q and q' , we will be left with the contribution that is proportional to $C_1(k, k') C_4(-k, -k') + C_1(k, k') C_4(-k', -k)$. Therefore, we find that the full isocurvature expression is given by¹⁰

$$\begin{aligned} \Delta_S^2(\eta, k) &= \frac{1}{\bar{\rho}_\chi^2(\eta)} \frac{1}{2a^8} \frac{k^3}{2\pi^2} \int d^3\mathbf{r} \int \frac{d^3k'}{(2\pi)^3} \int \frac{d^3k''}{(2\pi)^3} \times e^{i(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) \cdot \mathbf{r}} \\ &\quad \times \left(X_{k'} X_{k''} \left(a^2 m_{\text{eff}}^2 - k' k'' + \left(\mathcal{H}^2 - \frac{a^2 R}{6} \right) \right) + X'_{k'} X'_{k''} - \mathcal{H}(X_{k''} X'_{k'} + X_{k'} X'_{k''}) \right) \\ &\quad \times \left(X_k^* X_{k''}^* \left(a^2 m_{\text{eff}}^2 - k' k'' + \left(\mathcal{H}^2 - \frac{a^2 R}{6} \right) \right) + X_{k'}^* X_{k''}^* - \mathcal{H}(X_{k''}^* X_{k'}^* + X_k^* X_{k''}^*) \right). \end{aligned} \quad (\text{B4})$$

After performing integrals and relabeling $k' \rightarrow p$, we find

$$\begin{aligned} \Delta_S^2(\eta, k) &= \frac{1}{\bar{\rho}_\chi^2(\eta)} \frac{1}{2a^8} \frac{k^3}{2\pi^2} \int \frac{d^3p}{(2\pi)^3} \left(X_p X_q \left(a^2 m_{\text{eff}}^2 - pq + \left(\mathcal{H}^2 - \frac{a^2 R}{6} \right) \right) + X'_p X'_q - \mathcal{H}(X'_p X_q + X_p X'_q) \right) \\ &\quad \times \left(X_p^* X_q^* \left(a^2 m_{\text{eff}}^2 - pq + \left(\mathcal{H}^2 - \frac{a^2 R}{6} \right) \right) + X_p^{I*} X_q^{I*} - \mathcal{H}(X_p^{I*} X_q^* + X_p^* X_q^{I*}) \right), \end{aligned} \quad (\text{B5})$$

where $q = |\mathbf{p} - \mathbf{k}|$. In the late-time asymptotic limit where the spacetime approaches Minkowski ($\mathcal{H} \rightarrow 0$, $R \rightarrow 0$) and for modes with $pq \ll a^2 m_{\text{eff}}^2$, the isocurvature power spectrum simplifies to

¹⁰Here, we relabeled $k \rightarrow k'$ and $k' \rightarrow k''$.

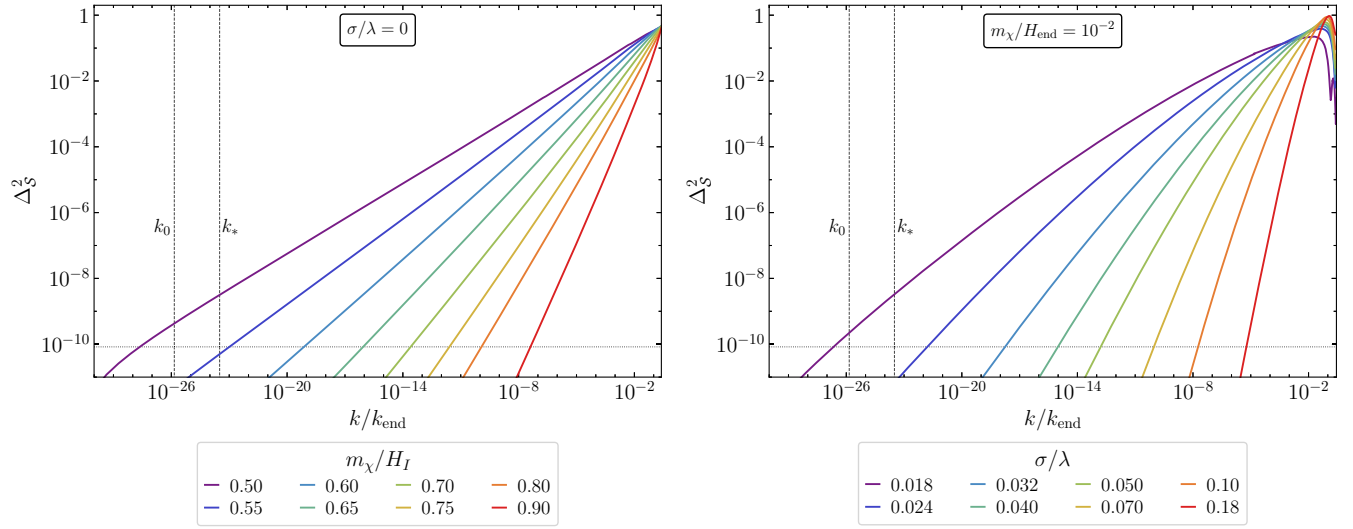


FIG. 5. Isocurvature power spectrum for the spectator field χ for pure minimal gravitational production (left), and for a fixed mass with varying inflaton coupling (right). Each mass parameter (left panel) and coupling strength (right panel) is represented by a different color. Here, $N_{\text{tot}} = 76.5$ e -folds of inflation. The vertical dashed lines show the positions of the present horizon scale ($k_0 = 1.4 \times 10^{-26} k_{\text{end}}$) and the *Planck* pivot scale ($k_* = 3.2 \times 10^{-24} k_{\text{end}}$). The horizontal dotted line represents the current upper bound on isocurvature perturbations from *Planck*, $\Delta_S^2 \simeq 8.3 \times 10^{-11}$ at the *Planck* pivot scale.

$$\Delta_S^2(\eta, k) = \frac{1}{\bar{\rho}_\chi^2(\eta)} \frac{k^3}{a^8 (2\pi)^5} \int d^3p [|X'_p|^2 |X'_q|^2 + a^4 m_{\chi, \text{eff}}^4 |X_p|^2 |X_q|^2 + a^2 m_{\chi, \text{eff}}^2 (X_p X_q'^* + \text{H.c.})], \quad (\text{B6})$$

which matches previous results [25,53]. This asymptotic expression provides a good approximation for estimating the isocurvature power spectrum at CMB scales. However, for a complete analysis across all scales, particularly at small scales relevant for gravitational wave signatures, we employ the full expression (B5) and evolve it numerically throughout the cosmological history.

Figure 5 shows the numerical isocurvature power spectrum for both purely gravitational production ($\sigma = 0$) and scenarios with nonzero inflaton-spectator coupling ($\sigma \neq 0$).

APPENDIX C: GRAVITATIONAL WAVE SPECTRUM

This appendix details how scalar perturbations from spectator fields source secondary gravitational waves at second order in perturbation theory [6,86,87]. We review the general formalism and examine how the evolving isocurvature power spectrum influences the scalar potential Φ , which subsequently sources tensor perturbations h_{ij} .

Working in the Newtonian gauge with the perturbed line element from Eq. (20), the equation of motion for secondary gravitational waves sourced by scalar perturbations is [87]

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2 h_{ij}}{a^2} = a^{-2} P_{ij}^{ab} \left\{ 4\partial_a \Phi \partial_b \Phi + \frac{2}{M_P^2} \partial_a \delta\chi \partial_b \delta\chi \right\}, \quad (\text{C1})$$

where P_{ij}^{ab} is the transverse-traceless projection operator and $\delta\chi$ represents spectator field fluctuations. The inflaton perturbation $\delta\phi$ is negligible compared to the spectator field contribution since $\rho_\phi \gg \rho_\chi$ during the relevant epoch.

We work in the Newtonian gauge for scalar perturbations, with the line element defined in Eq. (20). The equation of motion for secondary GWs sourced by scalar perturbations is given by [87]

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2 h_{ij}}{a^2} = a^{-2} P_{ij}^{ab} \left\{ 4\partial_a \Phi \partial_b \Phi + \frac{2}{M_P^2} \partial_a \delta\chi \partial_b \delta\chi \right\}, \quad (\text{C2})$$

where P_{ij}^{ab} is the projection operator, and $\delta\chi$ represents the dark matter fluctuations. Here, we neglected the $\delta\phi$ contribution since it is negligible compared to χ at the background level. Converting to conformal time using $dt = a d\eta$, the equation becomes

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = P_{ij}^{ab} \left\{ 4\partial_a \Phi \partial_b \Phi + \frac{2}{M_p^2} \partial_a \chi \partial_b \chi \right\}, \quad (\text{C3})$$

where primes denote derivatives with respect to conformal time η . The gravitational wave source consists of two distinct contributions:

Scalar potential term ($4\partial_a \Phi \partial_b \Phi$). This arises from the nonlinear evolution of curvature perturbations enhanced by isocurvature fluctuations. The scalar potential Φ is related to density perturbations through the Poisson equation, creating a direct link between the isocurvature power spectrum and gravitational wave production.

Direct field fluctuations ($\frac{2}{M_p^2} \partial_a \chi \partial_b \chi$). This term represents direct coupling of spectator field gradients to the gravitational wave equation. However, this contribution typically generates very high-frequency signals that fall outside the sensitivity range of current gravitational wave detectors and would require specialized resonant cavity experiments for detection.

For the present analysis, we focus exclusively on gravitational waves induced by the scalar potential term, which dominates in the frequency ranges accessible to pulsar timing arrays, space-based interferometers, and ground-based detectors. The computation of the resulting gravitational wave spectrum is detailed in the main text. To compute the tensor perturbations h_{ij} and resulting gravitational wave spectra, we transform to Fourier space and employ Green's function methods. The energy density fluctuations are decomposed as

$$\delta\rho_\chi(\eta, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}} \delta\rho_\chi(\eta, \mathbf{k}). \quad (\text{C4})$$

Using the gauge-invariant Poisson equation $\nabla^2 \Phi = \delta\rho_\chi/(2M_p^2)$ and projecting onto the transverse-traceless polarization tensors $\epsilon_{ij}^\lambda(\mathbf{k})$, the gravitational wave equation becomes

$$h''_{\mathbf{k},\lambda}(\eta) + 2\mathcal{H}h'_{\mathbf{k},\lambda}(\eta) + k^2 h_{\mathbf{k},\lambda}(\eta) = S_\lambda(\eta, \mathbf{k}), \quad (\text{C5})$$

where the source term $S_\lambda(\eta, \mathbf{k})$ is given by

$$S_\lambda(\eta, \mathbf{k}) = - \int \frac{d^3 p}{(2\pi)^3} \frac{d^4}{M_p^4} \frac{\epsilon_\lambda^{ab} p_a q_b}{p^2 q^2} \delta\rho_\chi(\eta, \mathbf{p}) \delta\rho_\chi(\eta, \mathbf{q}). \quad (\text{C6})$$

To solve the inhomogeneous differential Eq. (C5), we construct the retarded Green's function from two linearly independent solutions $h_1(\eta, k)$ and $h_2(\eta, k)$ of the homogeneous equation,

$$\mathcal{G}_k(\eta, \tilde{\eta}) = \frac{1}{W(h_1, h_2, \tilde{\eta})} [h_1(\eta)h_2(\tilde{\eta}) - h_1(\tilde{\eta})h_2(\eta)], \quad (\text{C7})$$

where the Green's function satisfies

$$\mathcal{G}_k''(\eta, \tilde{\eta}) + 2\mathcal{H}\mathcal{G}_k'(\eta, \tilde{\eta}) + k^2 \mathcal{G}_k(\eta, \tilde{\eta}) = \delta(\eta - \tilde{\eta}). \quad (\text{C8})$$

The Wronskian determinant ensures the proper normalization,

$$W(h_1, h_2, \tilde{\eta}) = h_1'(\tilde{\eta})h_2(\tilde{\eta}) - h_1(\tilde{\eta})h_2'(\tilde{\eta}). \quad (\text{C9})$$

With vanishing initial conditions $h_\lambda(\eta_i, \mathbf{k}) = h'_\lambda(\eta_i, \mathbf{k}) = 0$ at some early time η_i before significant source contribution, the particular solution is obtained through convolution,

$$h_\lambda(\eta, \mathbf{k}) = \int_{\eta_i}^{\eta} d\tilde{\eta} \mathcal{G}_k(\eta, \tilde{\eta}) S_\lambda(\tilde{\eta}, \mathbf{k}). \quad (\text{C10})$$

This formulation provides the foundation for our numerical computation of the gravitational wave amplitude $h_\lambda(\eta, \mathbf{k})$, from which we derive the energy density spectrum $\Omega_{\text{GW}}(k)$ presented in the main text.

Next, we follow the methodology developed in Refs. [74,88–90]. We assume statistical homogeneity and isotropy of the spectator field density perturbations $\delta\rho_\chi$ that appear in the gravitational wave source term (C6). With vanishing mean $\langle \delta\rho_\chi \rangle = 0$, the two-point correlation function of induced gravitational waves can be expressed as

$$\langle h_\lambda(\eta, \mathbf{k}_1) h_\lambda(\eta, \mathbf{k}_2) \rangle \sim \int \frac{d^3 q_1}{(2\pi)^{3/2}} \frac{d^3 q_2}{(2\pi)^{3/2}} \langle \delta\rho_\chi(\mathbf{q}_1) \delta\rho_\chi(\mathbf{k}_1 - \mathbf{q}_1) \delta\rho_\chi(\mathbf{q}_2) \delta\rho_\chi(\mathbf{k}_2 - \mathbf{q}_2) \rangle. \quad (\text{C11})$$

This expression involves a four-point correlation function of density perturbations, which contains both Gaussian and non-Gaussian contributions. The four-point correlation function can be decomposed into disconnected and connected parts,

$$\langle \delta\rho_\chi(\mathbf{k}_1) \delta\rho_\chi(\mathbf{k}_2) \delta\rho_\chi(\mathbf{k}_3) \delta\rho_\chi(\mathbf{k}_4) \rangle = \langle \delta\rho_\chi(\mathbf{k}_1) \delta\rho_\chi(\mathbf{k}_2) \delta\rho_\chi(\mathbf{k}_3) \delta\rho_\chi(\mathbf{k}_4) \rangle_d + \langle \delta\rho_\chi(\mathbf{k}_1) \delta\rho_\chi(\mathbf{k}_2) \delta\rho_\chi(\mathbf{k}_3) \delta\rho_\chi(\mathbf{k}_4) \rangle_c, \quad (\text{C12})$$

where the disconnected part represents the Gaussian contribution and the connected part captures non-Gaussian effects. These contributions are given by

$$\langle \delta\rho_\chi(\mathbf{k}_1)\delta\rho_\chi(\mathbf{k}_2)\delta\rho_\chi(\mathbf{k}_3)\delta\rho_\chi(\mathbf{k}_4) \rangle_c = \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)\mathcal{T}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4), \quad (\text{C13})$$

and

$$\begin{aligned} \langle \delta\rho_\chi(\mathbf{k}_1)\delta\rho_\chi(\mathbf{k}_2)\delta\rho_\chi(\mathbf{k}_3)\delta\rho_\chi(\mathbf{k}_4) \rangle_d &= \langle \delta\rho_\chi(\mathbf{k}_1)\delta\rho_\chi(\mathbf{k}_2) \rangle \langle \delta\rho_\chi(\mathbf{k}_3)\delta\rho_\chi(\mathbf{k}_4) \rangle \\ &+ \langle \delta\rho_\chi(\mathbf{k}_2)\delta\rho_\chi(\mathbf{k}_3) \rangle \langle \delta\rho_\chi(\mathbf{k}_4)\delta\rho_\chi(\mathbf{k}_1) \rangle + \langle \delta\rho_\chi(\mathbf{k}_1)\delta\rho_\chi(\mathbf{k}_3) \rangle \langle \delta\rho_\chi(\mathbf{k}_2)\delta\rho_\chi(\mathbf{k}_4) \rangle. \end{aligned} \quad (\text{C14})$$

Here, $\mathcal{T}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$ is the connected trispectrum, and the two-point function is defined as

$$\langle \delta\rho_\chi(\mathbf{k}_1)\delta\rho_\chi(\mathbf{k}_2) \rangle = \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2)\bar{\rho}_\chi^2\mathcal{P}_S(\mathbf{k}_1). \quad (\text{C15})$$

For spectator field models with weak primordial non-Gaussianities, we treat the connected contribution as a small perturbative correction to the dominant Gaussian statistics. The gravitational wave power spectrum accordingly splits as

$$\mathcal{P}_\lambda(k) = \mathcal{P}_\lambda(k)|_d + \mathcal{P}_\lambda(k)|_c, \quad (\text{C16})$$

where the Gaussian piece arises from products of two-point functions (disconnected diagrams) and the non-Gaussian piece encodes contributions from the primordial trispectrum (connected diagrams). Since the connected contribution is typically subdominant for the parameter ranges of interest, we focus on computing the Gaussian contribution. A comprehensive analysis including non-Gaussian effects is provided in Ref. [74].

The Gaussian approximation requires evaluating the disconnected part of the four-point correlation function appearing in Eq. (C11),

$$\begin{aligned} \langle \delta\rho_\chi(\mathbf{q}_1)\delta\rho_\chi(\mathbf{k}_1 - \mathbf{q}_1)\delta\rho_\chi(\mathbf{q}_2)\delta\rho_\chi(\mathbf{k}_2 - \mathbf{q}_2) \rangle_d &= \langle \delta\rho_\chi(\mathbf{q}_1)\delta\rho_\chi(\mathbf{k}_1 - \mathbf{q}_1) \rangle \langle \delta\rho_\chi(\mathbf{q}_2)\delta\rho_\chi(\mathbf{k}_2 - \mathbf{q}_2) \rangle \\ &+ \langle \delta\rho_\chi(\mathbf{k}_1 - \mathbf{q}_1)\delta\rho_\chi(\mathbf{q}_2) \rangle \langle \delta\rho_\chi(\mathbf{k}_2 - \mathbf{q}_2)\delta\rho_\chi(\mathbf{q}_1) \rangle \\ &+ \langle \delta\rho_\chi(\mathbf{q}_1)\delta\rho_\chi(\mathbf{q}_2) \rangle \langle \delta\rho_\chi(\mathbf{k}_1 - \mathbf{q}_1)\delta\rho_\chi(\mathbf{k}_2 - \mathbf{q}_2) \rangle. \end{aligned} \quad (\text{C17})$$

When substituting the explicit form of the two-point functions, certain terms generate contributions proportional to $\delta^{(3)}(\mathbf{k}_1)$ and $\delta^{(3)}(\mathbf{k}_2)$. These zero-momentum modes correspond to unphysical infrared divergences that must be regularized. Since we are interested in gravitational waves with finite momentum $k_1, k_2 \neq 0$, these terms can be consistently dropped. After this regularization procedure, the physical contribution to the disconnected four-point function becomes

$$\langle \delta\rho_\chi(\mathbf{q}_1)\delta\rho_\chi(\mathbf{k}_1 - \mathbf{q}_1)\delta\rho_\chi(\mathbf{q}_2)\delta\rho_\chi(\mathbf{k}_2 - \mathbf{q}_2) \rangle_d = \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2)[\delta^{(3)}(\mathbf{q}_1 + \mathbf{q}_2) + \delta^{(3)}(\mathbf{k}_1 + \mathbf{q}_2 - \mathbf{q}_1)]\bar{\rho}_\chi^4\mathcal{P}_S(\mathbf{q}_1)\mathcal{P}_S(|\mathbf{k}_1 - \mathbf{q}_1|). \quad (\text{C18})$$

The gravitational wave two-point correlation function is obtained by convolving the source correlations with the appropriate Green's functions,

$$\langle h_\lambda(\eta, \mathbf{k}_1)h_\lambda(\eta, \mathbf{k}_2) \rangle = \int_0^\eta d\eta_1 \int_0^\eta d\eta_2 \mathcal{G}_\mathbf{k}(\eta, \eta_1)\mathcal{G}_\mathbf{k}(\eta, \eta_2)\langle S_\lambda(\eta_1, \mathbf{k}_1)S_\lambda(\eta_2, \mathbf{k}_2) \rangle, \quad (\text{C19})$$

where the source two-point function is given by

$$\begin{aligned} \langle S_\lambda(\eta_1, \mathbf{k}_1)S_\lambda(\eta_2, \mathbf{k}_2) \rangle &= \frac{a(\eta_1)^4}{M_P^4} \frac{a(\eta_2)^4}{M_P^4} \int \frac{d^3q_1}{(2\pi)^{3/2}} \frac{d^3q_2}{(2\pi)^{3/2}} \frac{Q_\lambda(\mathbf{k}_1, \mathbf{q}_1)Q_\lambda(\mathbf{k}_2, \mathbf{q}_2)}{q_1^2|\mathbf{k}_1 - \mathbf{q}_1|^2q_2^2|\mathbf{k}_2 - \mathbf{q}_2|^2} \\ &\times \mathcal{T}_S(\eta_1)\mathcal{T}_S(\eta_2)\langle \delta\rho_\chi(\mathbf{q}_1)\delta\rho_\chi(|\mathbf{k}_1 - \mathbf{q}_1|)\delta\rho_\chi(\mathbf{q}_2)\delta\rho_\chi(|\mathbf{k}_2 - \mathbf{q}_2|) \rangle. \end{aligned} \quad (\text{C20})$$

Here, we used the property that we found from the numerical analysis,

$$\Delta_S^2(\eta, k) = \mathcal{T}_S(\eta)\Delta_S^2(k), \quad (k < aH), \quad (\text{C21})$$

where $\mathcal{T}_S(\eta)$ represents a time-dependent transfer function. The gravitational wave polarization structure is encoded in the projection factor,

$$Q_\lambda(\mathbf{k}, \mathbf{q}) \equiv \varepsilon_{ij}^\lambda(\mathbf{k}) q_i q_j = -\varepsilon_{ij}^\lambda(\mathbf{k})(\mathbf{k} - \mathbf{q})_i q_j, \quad (\text{C22})$$

where $\varepsilon_{ij}^\lambda(\mathbf{k})$ are the transverse-traceless polarization tensors satisfying $\varepsilon_{ij}^\lambda(\mathbf{k}) k_i = 0$. For computational convenience, we choose coordinates with $\hat{\mathbf{k}} = \hat{\mathbf{z}}$ and parametrize $\mathbf{q} = q(\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$. The projection factors for the two polarization states become

$$\begin{aligned} Q_+(\mathbf{k}, \mathbf{q}) &= \frac{q^2}{\sqrt{2}} \sin^2\theta \cos(2\phi), \\ Q_\times(\mathbf{k}, \mathbf{q}) &= \frac{q^2}{\sqrt{2}} \sin^2\theta \sin(2\phi). \end{aligned} \quad (\text{C23})$$

The projection factors possess useful symmetry properties that simplify the momentum integrals. Because $\varepsilon_\lambda(\mathbf{k})$ is orthogonal to $\mathbf{k}$, we find

$$Q_\lambda(\mathbf{k}, \mathbf{q}) = Q_\lambda(\mathbf{k}, \mathbf{q} + c\mathbf{k}), \quad (\text{C24})$$

for any constant c . $Q_\lambda(\mathbf{k}, \mathbf{q})$ is symmetric under $\mathbf{k} \rightarrow -\mathbf{k}$ and $\mathbf{q} \rightarrow -\mathbf{q}$,

$$Q_\lambda(\mathbf{k}, \mathbf{q}) = Q_\lambda(-\mathbf{k}, \mathbf{q}) = Q_\lambda(\mathbf{k}, -\mathbf{q}) = Q_\lambda(-\mathbf{k}, -\mathbf{q}). \quad (\text{C25})$$

The GW power spectrum can be expressed as

$$\langle h_\lambda(\eta, \mathbf{k}) h_{\lambda'}(\eta, \mathbf{k}') \rangle = \delta_{\lambda\lambda'} \delta^{(3)}(\mathbf{k} + \mathbf{k}') \mathcal{P}_\lambda(\eta, k), \quad (\text{C26})$$

where the dimensionless power spectrum is given by

$$\Delta_\lambda^2(\eta, k) = \frac{k^3}{2\pi^2} \mathcal{P}_\lambda(\eta, k). \quad (\text{C27})$$

The gravitational wave energy density is conventionally expressed as a fraction of the critical density $\rho_c = 3H^2 M_P^2$,

$$\Omega_{\text{GW}}(k) \equiv \frac{1}{3M_P^2 H^2} \frac{d\rho_{\text{GW}}}{d \ln k}, \quad (\text{C28})$$

where ρ_{GW} is the gravitational wave energy density. The relationship between the dimensionless power spectrum and the energy density parameter is

$$\Omega_{\text{GW}}(k) = \frac{1}{12} \left(\frac{k}{a(\eta)H(\eta)} \right)^2 \sum_\lambda \Delta_\lambda^2. \quad (\text{C29})$$

For comparison with observational constraints, we evaluate the spectrum at the present epoch. Using the present-day Hubble parameter $H_0 = 100h \text{ kms}^{-1} \text{ Mpc}^{-1}$ and scale factor a_0 , the observable gravitational wave energy density becomes

$$\Omega_{\text{GW},0}(k) h^2 = \frac{1}{12} \left(\frac{k}{a_0 H_0} \right)^2 \Delta_h^2, \quad (\text{C30})$$

where $\sum_\lambda \Delta_\lambda^2 = \Delta_h^2$ is the total GW power spectrum across both polarization states.

The complete dimensionless gravitational wave power spectrum can be expressed in factorized form as

$$\Delta_h^2(k, N) = 2 \left[\frac{1}{8} \int^N dN' \mathcal{G}_\mathbf{k}(N, N') \mathcal{T}_S(N') \left(\frac{a(N')H(N')}{k} \right)^2 \left(\frac{\rho_\chi(N')}{H^2(N')M_P^2} \right)^2 \right]^2 g(k) \equiv \mathcal{C}(k, N) \times g(k), \quad (\text{C31})$$

where

$$g(k) = k^2 \int_0^\infty dp p \int_{|k-p|}^{k+p} dq q \frac{(k^4 - 2k^2(p^2 + q^2) + (p^2 - q^2)^2)^2}{p^7 q^7} \Delta_S^2(p) \Delta_S^2(q). \quad (\text{C32})$$

The gravitational wave energy density spectrum becomes

$$\Omega_{\text{GW},S} = \frac{1}{12} \left(\frac{k}{aH} \right)^2 \Delta_h^2(k, N) \equiv \mathcal{I}(k, N) g(k). \quad (\text{C33})$$

This factorization separates the time-dependent gravitational wave growth from the momentum-space structure determined by the isocurvature spectrum. The calculation is performed using the number of e -folds N as the time

variable, which provides computational advantages during the inflationary and postinflationary epochs where the scale factor evolves exponentially. Correspondingly, $\mathcal{G}_\mathbf{k}(N, N')$ represents the Green's function expressed in e -fold time rather than conformal time. The normalization of $g(k)$ includes appropriate factors of k to ensure dimensional consistency, specifically, the k^2 prefactor in Eq. (C32) renders $g(k)$ dimensionless while preserving the correct scaling behavior of the gravitational wave spectrum.

- [1] Alan H. Guth, The inflationary universe: A possible solution to the horizon and flatness problems, *Phys. Rev. D* **23**, 347 (1981).
- [2] Alexei A. Starobinsky, A new type of isotropic cosmological models without singularity, *Phys. Lett.* **91B**, 99 (1980).
- [3] Andrei D. Linde, A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, *Phys. Lett.* **108B**, 389 (1982).
- [4] Andreas Albrecht and Paul J. Steinhardt, Cosmology for grand unified theories with radiatively induced symmetry breaking, *Phys. Rev. Lett.* **48**, 1220 (1982).
- [5] Andrei D. Linde, Chaotic inflation, *Phys. Lett.* **129B**, 177 (1983).
- [6] Daniel Baumann, Paul J. Steinhardt, Keitaro Takahashi, and Kiyotomo Ichiki, Gravitational wave spectrum induced by primordial scalar perturbations, *Phys. Rev. D* **76**, 084019 (2007).
- [7] Marc Kamionkowski, Arthur Kosowsky, and Albert Stebbins, A probe of primordial gravity waves and vorticity, *Phys. Rev. Lett.* **78**, 2058 (1997).
- [8] Uros Seljak and Matias Zaldarriaga, Signature of gravity waves in polarization of the microwave background, *Phys. Rev. Lett.* **78**, 2054 (1997).
- [9] Y. Akrami *et al.* (Planck Collaboration), Planck 2018 results. X. Constraints on inflation, *Astron. Astrophys.* **641**, A10 (2020).
- [10] Simeon Bird, Hiranya V. Peiris, Matteo Viel, and Licia Verde, Minimally parametric power spectrum reconstruction from the Lyman-Alpha forest, *Mon. Not. R. Astron. Soc.* **413**, 1717 (2011).
- [11] Matti Herranen, Tommi Markkanen, Sami Nurmi, and Arttu Rajantie, Spacetime curvature and the Higgs stability during inflation, *Phys. Rev. Lett.* **113**, 211102 (2014).
- [12] J. R. Espinosa, G. F. Giudice, and A. Riotto, Cosmological implications of the Higgs mass measurement, *J. Cosmol. Astropart. Phys.* **05** (2008) 002.
- [13] Guillem Domènech, Samuel Passaglia, and Sébastien Renaux-Petel, Gravitational waves from dark matter isocurvature, *J. Cosmol. Astropart. Phys.* **03** (2022) 023.
- [14] Guillem Domènech, Cosmological gravitational waves from isocurvature fluctuations, *AAPPS Bull.* **34**, 4 (2024).
- [15] Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, and Lian-Tao Wang, Gravitational waves from stochastic scalar fluctuations, *Phys. Rev. D* **109**, 083519 (2024).
- [16] Yanou Cui, Pankaj Saha, and Evangelos I. Sfakianakis, Gravitational wave symphony from oscillating spectator scalar fields, *Phys. Rev. Lett.* **133**, 021004 (2024).
- [17] Bernard Carr, Tommi Tenkanen, and Ville Vaskonen, Primordial black holes from inflaton and spectator field perturbations in a matter-dominated era, *Phys. Rev. D* **96**, 063507 (2017).
- [18] Bernard Carr and Florian Kuhnel, Primordial black holes as dark matter: Recent developments, *Annu. Rev. Nucl. Part. Sci.* **70**, 355 (2020).
- [19] Daniel J. H. Chung, Edward W. Kolb, and Antonio Riotto, Superheavy dark matter, *Phys. Rev. D* **59**, 023501 (1998).
- [20] Daniel J. H. Chung, Edward W. Kolb, and Antonio Riotto, Production of massive particles during reheating, *Phys. Rev. D* **60**, 063504 (1999).
- [21] P. J. E. Peebles and A. Vilenkin, Noninteracting dark matter, *Phys. Rev. D* **60**, 103506 (1999).
- [22] Christian T. Byrnes and David Wands, Curvature and isocurvature perturbations from two-field inflation in a slow-roll expansion, *Phys. Rev. D* **74**, 043529 (2006).
- [23] Tommi Markkanen, Arttu Rajantie, and Tommi Tenkanen, Spectator dark matter, *Phys. Rev. D* **98**, 123532 (2018).
- [24] Edward W. Kolb and Andrew J. Long, Cosmological gravitational particle production and its implications for cosmological relics, *Rev. Mod. Phys.* **96**, 045005 (2024).
- [25] Siyang Ling and Andrew J. Long, Superheavy scalar dark matter from gravitational particle production in α -attractor models of inflation, *Phys. Rev. D* **103**, 103532 (2021).
- [26] Marcos A. G. Garcia, Mathias Pierre, and Sarunas Verner, Scalar dark matter production from preheating and structure formation constraints, *Phys. Rev. D* **107**, 043530 (2023).
- [27] Marcos A. G. Garcia, Wenqi Ke, Yann Mambrini, Keith A. Olive, and Sarunas Verner, Scalar field fluctuations and the production of dark matter, *J. Cosmol. Astropart. Phys.* **08** (2025) 039.
- [28] Ayan Chakraborty, Subhasis Maiti, and Debaprasad Maity, Probing a nonminimal coupling through superhorizon instability and secondary gravitational waves, *Phys. Rev. D* **111**, 083505 (2025).
- [29] Andrei D. Linde and Viatcheslav F. Mukhanov, Non-Gaussian isocurvature perturbations from inflation, *Phys. Rev. D* **56**, R535 (1997).
- [30] Kari Enqvist and Martin S. Sloth, Adiabatic CMB perturbations in pre—big bang string cosmology, *Nucl. Phys. B* **626**, 395 (2002).
- [31] Takeo Moroi and Tomo Takahashi, Effects of cosmological moduli fields on cosmic microwave background, *Phys. Lett. B* **522**, 215 (2001); **539**, 303(E) (2002).
- [32] David H. Lyth and David Wands, Generating the curvature perturbation without an inflaton, *Phys. Lett. B* **524**, 5 (2002).
- [33] R. D. Peccei and Helen R. Quinn, Constraints imposed by CP conservation in the presence of instantons, *Phys. Rev. D* **16**, 1791 (1977).
- [34] R. D. Peccei and Helen R. Quinn, CP conservation in the presence of instantons, *Phys. Rev. Lett.* **38**, 1440 (1977).
- [35] Yunjia Bao, JiJi Fan, and Lingfeng Li, Opening up a window on the postinflationary QCD axion, *Phys. Rev. Lett.* **130**, 241001 (2023).
- [36] Xingang Chen, JiJi Fan, and Lingfeng Li, New inflationary probes of axion dark matter, *J. High Energy Phys.* **12** (2023) 197.
- [37] Stephen P. Martin, A supersymmetry primer, *Adv. Ser. Dir. High Energy Phys.* **18**, 1 (1998).
- [38] Daniel Z. Freedman and Antoine Van Proeyen, *Supergravity* (Cambridge University Press, Cambridge, England, 2012).
- [39] Adeela Afzal *et al.* (NANOGrav Collaboration), The NANOGrav 15 yr data set: Search for signals from new physics, *Astrophys. J. Lett.* **951**, L11 (2023); **971**, L27(E) (2024).
- [40] L. Lentati *et al.* (EPTA Collaboration), European Pulsar Timing Array limits on an isotropic stochastic gravitational-wave background, *Mon. Not. R. Astron. Soc.* **453**, 2576 (2015).

- [41] Stanislav Babak *et al.* (EPTA Collaboration), European Pulsar Timing Array limits on continuous gravitational waves from individual supermassive black hole binaries, *Mon. Not. R. Astron. Soc.* **455**, 1665 (2016).
- [42] A. Weltman *et al.*, Fundamental physics with the square kilometre array, *Pub. Astron. Soc. Aust.* **37**, e002 (2020).
- [43] Pau Amaro-Seoane *et al.* (LISA Collaboration), Laser interferometer space antenna, [arXiv:1702.00786](#).
- [44] Jeff Crowder and Neil J. Cornish, Beyond LISA: Exploring future gravitational wave missions, *Phys. Rev. D* **72**, 083005 (2005).
- [45] Kent Yagi and Naoki Seto, Detector configuration of DECIGO/BBO and identification of cosmological neutron-star binaries, *Phys. Rev. D* **83**, 044011 (2011); **95**, 109901(E) (2017).
- [46] Rich Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Open data from the first and second observing runs of Advanced LIGO and Advanced Virgo, *SoftwareX* **13**, 100658 (2021).
- [47] M. Punturo *et al.*, The Einstein Telescope: A third-generation gravitational wave observatory, *Classical Quantum Gravity* **27**, 194002 (2010).
- [48] David Reitze *et al.*, Cosmic explorer: The U.S. Contribution to gravitational-wave astronomy beyond LIGO, *Bull. Am. Astron. Soc.* **51**, 035 (2019).
- [49] Mohammad Ali Gorji and Misao Sasaki, Primordial-tensor-induced stochastic gravitational waves, *Phys. Lett. B* **846**, 138236 (2023).
- [50] Mohammad Ali Gorji, Misao Sasaki, and Teruaki Suyama, Extra-tensor-induced origin for the PTA signal: No primordial black hole production, *Phys. Lett. B* **846**, 138214 (2023).
- [51] Daniel J. H. Chung, Edward W. Kolb, Antonio Riotto, and Leonardo Senatore, Isocurvature constraints on gravitationally produced superheavy dark matter, *Phys. Rev. D* **72**, 023511 (2005).
- [52] Daniel J. H. Chung and Hojin Yoo, Isocurvature perturbations and Non-Gaussianity of gravitationally produced nonthermal dark matter, *Phys. Rev. D* **87**, 023516 (2013).
- [53] Marcos A. G. Garcia, Mathias Pierre, and Sarunas Verner, Isocurvature constraints on scalar dark matter production from the inflaton, [arXiv:2303.07359](#).
- [54] Lev Kofman, Andrei D. Linde, and Alexei A. Starobinsky, Towards the theory of reheating after inflation, *Phys. Rev. D* **56**, 3258 (1997).
- [55] Renata Kallosh and Andrei Linde, Non-minimal inflationary attractors, *J. Cosmol. Astropart. Phys.* **10** (2013) 033.
- [56] John Ellis, Marcos A. G. Garcia, Dimitri V. Nanopoulos, Keith A. Olive, and Sarunas Verner, BICEP/Keck constraints on attractor models of inflation and reheating, [arXiv:2112.04466](#).
- [57] M. Kawasaki, Kazunori Kohri, and Naoshi Sugiyama, MeV scale reheating temperature and thermalization of neutrino background, *Phys. Rev. D* **62**, 023506 (2000).
- [58] P. F. de Salas, M. Lattanzi, G. Mangano, G. Miele, S. Pastor, and O. Pisanti, Bounds on very low reheating scenarios after Planck, *Phys. Rev. D* **92**, 123534 (2015).
- [59] Takuya Hasegawa, Nagisa Hiroshima, Kazunori Kohri, Rasmus S. L. Hansen, Thomas Tram, and Steen Hannestad, MeV-scale reheating temperature and thermalization of oscillating neutrinos by radiative and hadronic decays of massive particles, *J. Cosmol. Astropart. Phys.* **12** (2019) 012.
- [60] Marcos A. G. Garcia, Kunio Kaneta, Yann Mambrini, Keith A. Olive, and Sarunas Verner, Freeze-in from preheating, *J. Cosmol. Astropart. Phys.* **03** (2022) 016.
- [61] John Ellis, Marcos A. G. Garcia, Dimitri V. Nanopoulos, and Keith A. Olive, Phenomenological aspects of no-scale inflation models, *J. Cosmol. Astropart. Phys.* **10** (2015) 003.
- [62] John Ellis, Marcos A. G. Garcia, Natsumi Nagata, Nanopoulos Dimitri V., Keith A. Olive, and Sarunas Verner, Building models of inflation in no-scale supergravity, *Int. J. Mod. Phys. D* **29**, 2030011 (2020).
- [63] Marcos A. G. Garcia, Kunio Kaneta, Yann Mambrini, and Keith A. Olive, Reheating and post-inflationary production of dark matter, *Phys. Rev. D* **101**, 123507 (2020).
- [64] Mathias Pierre, Scalar dark matter production from the inflaton, in *Rencontres de Blois 2023* (2023), [arXiv:2309.08305](#).
- [65] Tommi Markkanen, Arttu Rajantie, Stephen Stopyra, and Tommi Tenkanen, Scalar correlation functions in de Sitter space from the stochastic spectral expansion, *J. Cosmol. Astropart. Phys.* **08** (2019) 001.
- [66] Gongjun Choi, Marcos A. G. Garcia, Wenqi Ke, Yann Mambrini, Keith A. Olive, and Sarunas Verner, Inflaton production of scalar dark matter through fluctuations and scattering, *Phys. Rev. D* **110**, 083512 (2024).
- [67] J. Chluba *et al.*, Spectral distortions of the CMB as a probe of inflation, Recombination, structure formation and particle physics: Astro2020 Science White Paper, *Bull. Am. Astron. Soc.* **51**, 184 (2019).
- [68] A. Kogut, M. H. Abitbol, J. Chluba, J. Delabrouille, D. Fixsen, J. C. Hill, S. P. Patil, and A. Rotti, CMB spectral distortions: Status and prospects, *Bull. Am. Astron. Soc.* **51**, 113 (2019).
- [69] Vincent S. H. Lee, Andrea Mitridate, Tanner Trickle, and Kathryn M. Zurek, Probing small-scale power spectra with pulsar timing arrays, *J. High Energy Phys.* **06** (2021) 028.
- [70] Chris L. Carilli and S. Rawlings, Science with the square kilometer array: Motivation, key science projects, standards and assumptions, *New Astron. Rev.* **48**, 979 (2004).
- [71] Gemma Janssen *et al.*, Gravitational wave astronomy with the SKA, *Proc. Sci. AASKA14* (2015) 037 [[arXiv:1501.00127](#)].
- [72] John Baker *et al.*, The laser interferometer space antenna: Unveiling the millihertz gravitational wave sky, [arXiv:1907.06482](#).
- [73] Vincent Corbin and Neil J. Cornish, Detecting the cosmic gravitational wave background with the big bang observer, *Classical Quantum Gravity* **23**, 2435 (2006).
- [74] Sebastian Garcia-Saenz, Lucas Pinol, Sébastien Renaux-Petel, and Denis Werth, No-go theorem for scalar-trispectrum-induced gravitational waves, *J. Cosmol. Astropart. Phys.* **03** (2023) 057.
- [75] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* **641**, A6 (2020); **652**, C4(E) (2021).
- [76] Kevork Abazajian *et al.*, CMB-S4 science case, reference design, and project plan, [arXiv:1907.04473](#).

- [77] Simone Aiola *et al.* (CMB-HD Collaboration), Snowmass2021 CMB-HD white paper, [arXiv:2203.05728](#).
- [78] M. Hazumi *et al.*, LiteBIRD: A satellite for the studies of B-Mode polarization and inflation from cosmic background radiation detection, *J. Low Temp. Phys.* **194**, 443 (2019).
- [79] Marcelo Alvarez *et al.*, PICO: Probe of inflation and cosmic origins, *Bull. Am. Astron. Soc.* **51**, 194 (2019).
- [80] Leonardo Badurina, Oliver Buchmueller, John Ellis, Marek Lewicki, Christopher McCabe, and Ville Vaskonen, Prospective sensitivities of atom interferometers to gravitational waves and ultralight dark matter, *Phil. Trans. A. Math. Phys. Eng. Sci.* **380**, 20210060 (2021).
- [81] Yousef Abou El-Neaj *et al.* (AEDGE Collaboration), AEDGE: Atomic experiment for dark matter and gravity exploration in space, *Eur. Phys. J. Quantum Technol.* **7**, 6 (2020).
- [82] Juan Garcia-Bellido, Hitoshi Murayama, and Graham White, Exploring the early universe with Gaia and Theia, *J. Cosmol. Astropart. Phys.* **12** (2021) 023.
- [83] Alberto Sesana *et al.*, Unveiling the gravitational universe at μ -Hz frequencies, *Exp. Astron.* **51**, 1333 (2021).
- [84] P. A. R. Ade *et al.* (BICEP, Keck Collaborations), Improved constraints on primordial gravitational waves using Planck, WMAP, and BICEP/Keck observations through the 2018 observing season, *Phys. Rev. Lett.* **127**, 151301 (2021).
- [85] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge Monographs on Mathematical Physics (Cambridge Univ. Press, Cambridge, England, 1984).
- [86] Kishore N. Ananda, Chris Clarkson, and David Wands, The cosmological gravitational wave background from primordial density perturbations, *Phys. Rev. D* **75**, 123518 (2007).
- [87] Guillem Domènech, Scalar induced gravitational waves review, *Universe* **7**, 398 (2021).
- [88] Peter Adshead, Kaloian D. Lozanov, and Zachary J. Weiner, Non-Gaussianity and the induced gravitational wave background, *J. Cosmol. Astropart. Phys.* **10** (2021) 080.
- [89] Caner Unal, Imprints of primordial non-Gaussianity on gravitational wave spectrum, *Phys. Rev. D* **99**, 041301 (2019).
- [90] Vicente Atal and Guillem Domènech, Probing non-Gaussianities with the high frequency tail of induced gravitational waves, *J. Cosmol. Astropart. Phys.* **06** (2021) 001; **10** (2023) E01.