

Twist-3 contributions to $\gamma\gamma \rightarrow \pi^0\pi^0, K_S^0K_S^0$ in k_T factorization

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We compute the cross sections for the two-photon processes $\gamma\gamma \rightarrow \pi^0\pi^0$ and $\gamma\gamma \rightarrow K_S^0K_S^0$ in k_T factorization, including the chirally enhanced two-parton twist-3 light cone distribution amplitudes. For these charge-suppressed neutral channels the twist-3 cross sections exceed the twist-2 ones by close to an order of magnitude in the intermediate-energy region, bringing the predictions much closer to the Belle data, the residual underestimate being plausibly attributable to higher-order QCD corrections. The calculation reproduces the measured angular distributions and the energy dependence of the charged channels and the neutral pion, though not the steeper fall of the neutral kaon. The neutral-to-charged ratios are the most discriminating observables. They depend strongly on energy in the data, whereas our calculation, like other approaches in the literature, yields a nearly flat ratio. Finally, in a phenomenological discussion, we combine our contribution with the soft handbag contribution and largely reproduce the observed energy dependence, suggesting that the hard and soft contributions are comparably important in the few-GeV region.

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I. INTRODUCTION

The exclusive two-photon annihilation into meson pairs at large invariant mass, $\gamma\gamma \rightarrow M\bar{M}$, has long served as one of the cleanest testing grounds for the application of perturbative quantum chromodynamics (pQCD) to exclusive hard processes. Because the initial state is purely electromagnetic, all nonperturbative dynamics resides in the meson light cone distribution amplitudes (LCDAs) of the final state, so the process probes the LCDAs essentially directly—without the additional nonperturbative complications—parton distribution functions, soft spectators, and initial-state radiation—that accompany an initial-state hadron, as in deep inelastic or hadron-hadron scattering. In the asymptotic regime $s, |t|, |u| \gg \Lambda_{\text{QCD}}^2$, Brodsky and Lepage [1,2] demonstrated that the amplitude factorizes into a perturbatively computable hard kernel for the partonic process $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$ convoluted with two single-meson twist-2 LCDAs, yielding the very sharp leading-twist (LT) prediction $d\sigma/d|\cos\theta^*| \sim \sin^{-4}\theta^*/Q^6$ for the charged pairs and a total cross section falling as $\sigma(Q) \sim Q^{-6}$. The neutral channels are suppressed for a reason common to both. A neutral pseudoscalar meson is

built from valence (anti)quarks of equal charge, so its charge $e_1 - e_2$ vanishes. Because the leading $\sin^{-4}\theta^*$ term of the Brodsky-Lepage amplitude scales as $(e_1 - e_2)^4$, it cancels identically for both neutral channels, which are then governed by the subleading $g(\theta^*)$ pieces, so that the rate is strongly suppressed and the angular distribution departs from the simple $\sin^{-4}\theta^*$ law [1,3,4]. Detailed LT calculations give a small ratio $\sigma(\pi^0\pi^0)/\sigma(\pi^+\pi^-) \simeq 0.04\text{--}0.07$ [3,5,6]. Over the past four decades these channels have been measured with steadily improving precision, from the early TPC/Two-Gamma [7] and ALEPH [8] experiments to the very high-statistics Belle program [4,6,9–16], which now spans the entire relevant range $0.6 \lesssim Q \lesssim 4.1$ GeV for the charged channels $\pi^+\pi^-, K^+K^-$ and for the neutral channels $\pi^0\pi^0, K_S^0K_S^0, \eta\pi^0$, and $\eta\eta$. Confronting these data with the pure LT predictions in the perturbative window $Q \sim 2\text{--}4$ GeV exposes a striking and systematic discrepancy. The measured cross sections exceed the LT results by roughly an order of magnitude for the charged pairs [1,9,17–19] and by even more for the charge-suppressed neutral pairs. The measured ratios sharpen the contrast: $\sigma(\pi^0\pi^0)/\sigma(\pi^+\pi^-)$ stabilizes around 0.3–0.5 [6,9,11], far above the LT expectation of 0.04–0.07, while $\sigma(K_S^0K_S^0)/\sigma(K^+K^-)$ falls steeply from ~ 0.13 to ~ 0.01 across the same energy interval [4]. Since the K^0 is charge-suppressed for exactly the same reason as the π^0 , the LT mechanism predicts a comparably small and nearly energy-independent neutral-kaon ratio, so that neither its magnitude at low Q nor its pronounced energy variation is reproduced. The energy

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power-laws extracted from the data, $\sigma(Q) \sim Q^{-n}$ with $n_{\pi^0\pi^0} \simeq 7-8$ and $n_{K_S^0 K_S^0} \simeq 10-11$ [6,14], the latter in particular being far steeper than the canonical $n = 6$. Taken together, the substantial underestimate of the cross sections across all channels indicates that the leading-twist amplitude does not capture the dominant contribution to these processes in the present energy window, and that subleading power corrections play a quantitatively decisive role rather than a perturbative one.

Several theoretical strategies have been put forward to bridge this gap. At the level of pure collinear pQCD, the next-to-leading-order (NLO) calculation of Duplanić and Nižić [20] yields sizable K -factors. At the renormalization scale $\mu_R^2 = Q^2$ the NLO term amounts to about 90% of the leading order at $Q = 4$ GeV (a K -factor of order 1.9), so that the perturbative series converges slowly across the Belle window. The NLO predictions nonetheless still lie below the measured cross sections, which suggests that in this regime important contributions arise from still higher orders—a point we return to in connection with our own results. Within the soft handbag mechanism developed by Diehl, Kroll, and Vogt [21–23], the hard subprocess $\gamma\gamma \rightarrow q\bar{q}$ is folded with a phenomenological $q\bar{q} \rightarrow M\bar{M}$ form factor. This approach reproduces the charged cross sections through phenomenological nonperturbative inputs fitted to the data, and predicts nearly energy-independent SU(3) ratios— $\sigma(\pi^0\pi^0)/\sigma(\pi^+\pi^-) \sim 1/2$, in fair agreement with the pion data, and $\sigma(K_S^0 K_S^0)/\sigma(K^+K^-) \approx 2/25$ in the SU(3) limit [21,22]. The latter is in tension with the pronounced fall of the measured ratio from ~ 0.13 to ~ 0.01 [4], which is not easily accommodated within the handbag picture even once SU(3)-breaking is included, and which suggests that further contributions are at play [4,5]. Below the perturbative window the amplitudes are dominated by hadronic resonances and have been studied in detail using dispersive and Roy-Steiner techniques [24–28] and in coupled-channel chiral models [29].

To compute the subleading contributions systematically we adopt the k_T -factorization (Li-Sterman) framework, originally developed for the pion electromagnetic form factor [30–32] and applied to two-photon meson-pair production in Refs. [17,18]. Supplemented with the chirally enhanced two-parton twist-3 LCDAs, it was applied in Refs. [19,33] to the Belle charged-channel data $\gamma\gamma \rightarrow \pi^+\pi^-, K^+K^-$, and we extend it here to the neutral channels. Retaining the parton transverse momentum $\mathbf{k}_\perp$, together with the Sudakov [30,31] and threshold [34–36] resummations, regulates the twist-3 end point region and makes the framework well suited to the intermediate-energy window $Q \sim 2-4$ GeV, where the smallest partonic virtualities approach Λ_{QCD}^2 and a purely collinear treatment is no longer reliable. The twist-3 terms are numerically decisive in this window because they are weighted by the chiral mass $\mu_M = m_M^2/(m_{q_1} + m_{q_2})$, $M \in \{\pi, K\}$, which by

the Gell-Mann-Oakes-Renner (GMOR) relation [37,38] is numerically as large as $\mu_M(2 \text{ GeV}) \simeq 2.6 \text{ GeV}$. The two-parton twist-3 contributions then enter relative to the leading-twist ones with the chiral-enhancement parameter $4\mu_M^2/Q^2$, which is of order unity across the window [39–41].

For the neutral channels the transverse-momentum and twist-3 effects do not merely compete with the leading twist but carry the cross section, since the charge suppression noted above removes the would-be dominant leading-twist amplitude. Ratios of the neutral to the charged cross sections are theoretically more robust than absolute rates, since the overall normalization and many input and systematic uncertainties largely cancel. It is therefore significant that the measured neutral-to-charged ratios vary so rapidly with energy. Specifically, the pion ratio turns upward at the upper end of the window, and the kaon ratio falls by an order of magnitude across it. Such a strong energy dependence of a quantity in which the leading effects largely cancel is a strong indication that the subleading contributions of higher twist, intrinsic transverse momentum, and higher-order QCD corrections are important in this regime. Their roles are distinct. Throughout this window the chiral enhancement renders the twist-3 contributions dominant, exceeding the leading-twist ones and adding constructively to them, while the transverse-momentum and Sudakov effects, which by themselves already modify the twist-2 cross sections substantially [33], act in the twist-3 sector as the joint regulator of the end point region. The higher-order QCD corrections are sizable in collinear factorization, as noted above, whereas for the pion electromagnetic form factors in k_T factorization the partial cancellation between the twist-2 and twist-3 NLO corrections leaves only a moderate net effect [42,43]. Even so, the strong and channel-dependent energy dependence of the ratios may signal the coexistence of several competing mechanisms, perturbative hard scattering alongside soft end point or handbag dynamics, in the intermediate-energy region.

The present paper addresses the natural next step, a complete k_T -factorization calculation, including the chirally enhanced two-parton twist-3 LCDAs, of the neutral channels $\gamma\gamma \rightarrow \pi^0\pi^0$ and $\gamma\gamma \rightarrow K_S^0 K_S^0$. These neutral channels are the most stringent discriminators among the competing dynamical mechanisms, whose predictions for the ratio $\sigma(\pi^0\pi^0)/\sigma(\pi^+\pi^-)$ quoted above differ by an order of magnitude, while the measured values lie in between. A first-principles perturbative calculation with consistent power corrections is therefore necessary to disentangle the perturbative from the soft content in this regime. Moreover, in contrast to the charged channels the neutral amplitudes carry a distinctive charge-flavor structure—the flavor superposition $\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ and the CP projection of the $K_S^0 K_S^0$ final state—which modifies the color-flavor combinatorics and the symmetry pattern of the four diagrammatic groups in nontrivial ways and propagates into

the helicity hard kernels, so that the neutral channels probe the twist-3 dynamics from an angle complementary to that of the charged channels studied in Ref. [33].

This paper is organized as follows. Section II sets up the theoretical framework, from the kinematics and the LCDAs of π^0 , K^0 and $\bar{K}^0$ to the k_T -factorization helicity amplitudes and the twist-3 hard kernels of the four diagrammatic groups, and the extension to $\gamma\gamma \rightarrow K_S^0 K_S^0$. Section III presents the numerical analysis. It first defines the cross sections and collects the nonperturbative inputs, then gives the twist-2 and twist-3 predictions for the cross sections, their energy dependence, the normalized angular distributions and the neutral-to-charged ratios in comparison with the Belle data, and closes with a discussion in which a soft handbag contribution is added coherently to the hard amplitude to account for the observed energy dependence of the ratios. Section IV gives a summary.

II. THEORETICAL FRAMEWORK

A. Kinematics and conventions

We work in the center-of-mass (CM) frame of the two-photon system, with the outgoing neutral meson $M^0(p_3)$ directed along the positive z -axis. We employ light cone coordinates $A^\mu = (A^+, A^-, \mathbf{A}_\perp)$ with $A^\pm \equiv A^0 \pm A^3$. In these coordinates the four external momenta of the process $\gamma(p_1, \lambda_1)\gamma(p_2, \lambda_2) \rightarrow M^0(p_3)\bar{M}^0(p_4)$ are

$$\begin{aligned} p_1^\mu &= (2\omega c^2, 2\omega s^2, \mathbf{p}_\perp), & p_2^\mu &= (2\omega s^2, 2\omega c^2, -\mathbf{p}_\perp), \\ p_3^\mu &= (2\omega, 0, \mathbf{0}_\perp), & p_4^\mu &= (0, 2\omega, \mathbf{0}_\perp), \end{aligned} \quad (1)$$

with the half-CM energy $\omega = Q/2$, $s \equiv \sin(\theta^*/2)$, $c \equiv \cos(\theta^*/2)$, and the transverse photon momentum $\mathbf{p}_\perp = (2\omega s c, 0)$. Here θ^* is the polar scattering angle of the meson $M^0(p_3)$ with respect to the incoming photon $\gamma(p_1)$, and we neglect the meson masses ($m_M^2 \ll Q^2$) in the external kinematics. The corresponding photon polarization vectors are

$$\begin{aligned} \epsilon^\mu(p_1, +) &= \frac{1}{\sqrt{2}}(2sc, -2sc, s^2 - c^2, -i), \\ \epsilon^\mu(p_1, -) &= \frac{1}{\sqrt{2}}(-2sc, 2sc, c^2 - s^2, -i), \\ \epsilon^\mu(p_2, +) &= \frac{1}{\sqrt{2}}(-2sc, 2sc, c^2 - s^2, -i), \\ \epsilon^\mu(p_2, -) &= \frac{1}{\sqrt{2}}(2sc, -2sc, s^2 - c^2, -i). \end{aligned} \quad (2)$$

For each outgoing meson, the constituent quark and antiquark are labeled by their longitudinal momentum fractions x , $\bar{x} = 1 - x$ in $M^0(p_3)$ and y , $\bar{y} = 1 - y$ in $\bar{M}^0(p_4)$, together with intrinsic transverse momenta $\mathbf{k}_{\perp 1}$ and $\mathbf{k}_{\perp 2}$

$$\begin{aligned} k_1^\mu &= (2x\omega, 0, \mathbf{k}_{\perp 1}), & k_2^\mu &= (2\bar{x}\omega, 0, -\mathbf{k}_{\perp 1}), \\ k_3^\mu &= (0, 2y\omega, \mathbf{k}_{\perp 2}), & k_4^\mu &= (0, 2\bar{y}\omega, -\mathbf{k}_{\perp 2}). \end{aligned} \quad (3)$$

The Mandelstam invariants reduce in this parametrization to $t = -Q^2 s^2$ and $u = -Q^2 c^2$, the squared center-of-mass energy being Q^2 (so that $t + u = -Q^2$). All ratios of hard scales we shall encounter are functions of (x, y, θ^*) alone, which substantially organizes the analytic structure of the hard kernels.

The reliability of the perturbative treatment is controlled not by Q^2 alone but by the smaller momentum transfers $|t|$ and $|u|$, which set the virtualities of the internal quark and gluon propagators. Since $\min(|t|, |u|) = Q^2 \min(s^2, c^2)$ becomes parametrically small near the forward and backward directions, the hard-scattering picture inevitably degrades at large scattering angles, where (i) the smallest propagator virtuality drops toward Λ_{QCD}^2 and the running coupling is probed in the nonperturbative regime, (ii) the soft end point regions of the parton momentum fractions become enhanced, and (iii) the soft handbag/overlap mechanism competes with the leading hard-scattering contribution [18,21]. To stay within the domain where the perturbative calculation is trustworthy—and to match the kinematic range over which the Belle Collaboration extracts the cross sections for theory comparison—we restrict the scattering angle throughout this work to $|\cos \theta^*| < 0.6$ [6,14], for which $\min(|t|, |u|) > 0.2Q^2$ remains safely above the soft scale across the Belle energy window.

At the parton level the amplitude is built from the lowest-order subprocess $\gamma(p_1)\gamma(p_2) \rightarrow q(k_1)\bar{q}(k_2)q(k_3)\bar{q}(k_4)$, in which the two photons couple to the quark lines and a single gluon is exchanged to bind the two $q\bar{q}$ pairs into the outgoing mesons. The contributing diagrams fall into four topological groups a, b, c, d shown in Fig. 1, distinguished by the virtuality of the internal gluon: the gluon is attached to a single quark line in groups a and b (the direct topology) and is exchanged between the two quark lines in groups c and d (the cross-quark topology). These topologies set the analytic structure of the hard kernels evaluated in Sec. II D.

B. Light cone distribution amplitudes

The nonperturbative content of each outgoing meson is encoded in its light cone distribution amplitudes (LCDAs), defined by gauge-invariant nonlocal bilinear quark-antiquark matrix elements between the meson state and the vacuum. At twist-3 accuracy three two-parton LCDAs enter—one twist-2 (axial-vector projection) and two twist-3 (pseudoscalar and pseudotensor projections). The three-parton twist-3 LCDA is suppressed by $\eta_{3M} = f_{3M}/(f_{M\mu\mu}) \sim 10^{-2}$ and is neglected throughout this work [33,39–41]. The convergence of the twist expansion beyond this accuracy has been examined for the closely related pion and kaon electromagnetic form factors, where

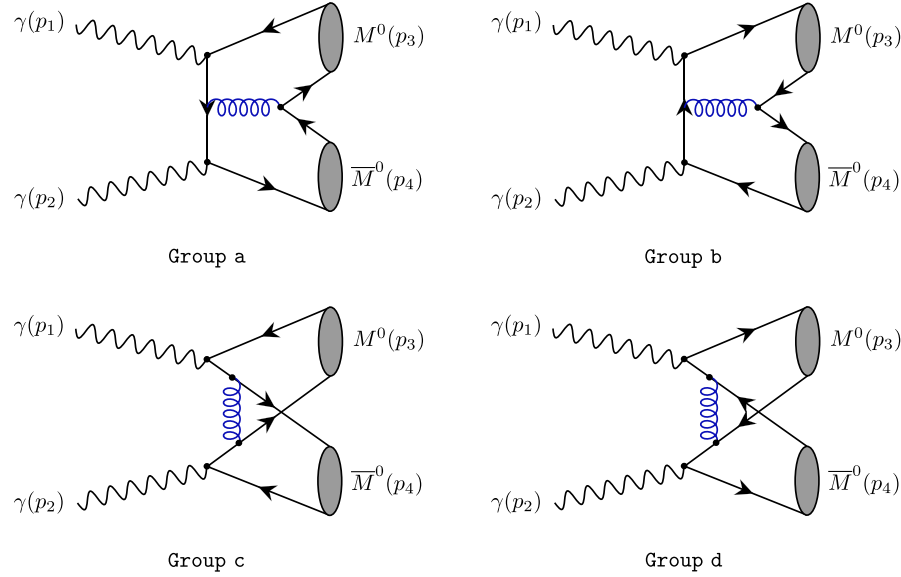


FIG. 1. The four basic Feynman diagrams contributing to the partonic amplitude $\gamma(p_1)\gamma(p_2) \rightarrow q(k_1)\bar{q}(k_2)q(k_3)\bar{q}(k_4)$ for $\gamma\gamma \rightarrow M^0(p_3)\bar{M}^0(p_4)$. The total numbers of diagrams in groups *a*, *b*, *c*, and *d* are 6, 6, 4, 4, respectively, and the grouping is dictated by the virtuality of the internal gluon propagator.

recent k_T -factorization analyses including the higher-twist contributions from both the two-particle and three-particle configurations find a satisfactory convergence beyond twist 3 [44–46], the three-particle twist-3 terms being suppressed by η_{3M} and the twist-4 corrections carrying a further $1/Q^2$ power suppression without chiral enhancement. Given the similarity between those convolutions and the present helicity amplitudes, the two-parton twist-2 and twist-3 contributions retained here are expected to capture

the dominant part of the twist expansion in the intermediate-energy region.

By isospin symmetry the neutral and charged pions share the same decay constant, $f_{\pi^0} = f_{\pi^\pm} \equiv f_\pi$, and the same set of LCDAs, and we therefore define the π^0 LCDAs through the isospin-symmetric matrix elements normalized directly to the physical decay constant f_π . With $z \equiv z_1 - z_2$ a lightlike separation, the relevant matrix elements read [39,41,47–49]

$$\begin{aligned} \langle \pi^0(p) | \bar{q}(z_1) \gamma_\mu \gamma_5 q(z_2) | 0 \rangle &= -i f_\pi p_\mu \int_0^1 dx e^{i(xp \cdot z_1 + \bar{x}p \cdot z_2)} \phi_\pi^\pi(x, \mu), \\ \langle \pi^0(p) | \bar{q}(z_1) i \gamma_5 q(z_2) | 0 \rangle &= f_\pi \mu_\pi \int_0^1 dx e^{i(xp \cdot z_1 + \bar{x}p \cdot z_2)} \phi_\pi^p(x, \mu), \\ \langle \pi^0(p) | \bar{q}(z_1) i \gamma_5 \sigma_{\mu\nu} q(z_2) | 0 \rangle &= -f_\pi \mu_\pi (p_\mu z_\nu - p_\nu z_\mu) \int_0^1 dx e^{i(xp \cdot z_1 + \bar{x}p \cdot z_2)} \frac{\phi_\pi^\sigma(x, \mu)}{6}, \end{aligned} \quad (4)$$

where $f_\pi = 130.2(8)$ MeV [50] and $\bar{q}\Gamma q = (\bar{u}\Gamma u - \bar{d}\Gamma d)/\sqrt{2}$ denotes the isospin-projected light-quark bilinear appropriate to the π^0 state. The superscript on each LCDA labels the two-parton amplitude by the Dirac structure of its defining matrix element rather than by a meson: ϕ_M^π is the leading twist-2 (axial-vector) LCDA, while ϕ_M^p and ϕ_M^σ are the twist-3 pseudoscalar and pseudotensor LCDAs. The same convention is used for the kaon below, so that, for instance, ϕ_K^π denotes the twist-2 kaon LCDA. The resolution of this isospin structure into the individual u - and d -quark contributions, together with the photon electric charges, is carried out at the level of the

hard-scattering amplitude in Sec. II D. The chirally enhanced mass parameter

$$\mu_\pi(\mu) = \frac{m_\pi^2}{m_u(\mu) + m_d(\mu)} = -\frac{2\langle \bar{q}q \rangle(\mu)}{f_\pi^2} + \mathcal{O}(m_q) \quad (5)$$

is, by the isospin symmetry of the matrix elements (4), common to both flavor components of the π^0 , and in the isospin-symmetric limit it is fixed by the GMOR relation $f_\pi^2 m_\pi^2 = -2(m_u + m_d)\langle \bar{q}q \rangle$ [37,38], where $\langle \bar{q}q \rangle$ is the single-flavor quark condensate. Although μ_π multiplies a formally power-suppressed twist-3 contribution, the smallness of the current-quark mass ($m_u + m_d$) in its

denominator makes it numerically large, $\mu_\pi \simeq 2.6$ GeV at $\mu = 2$ GeV, comparable to the hard scale $Q \sim 2\text{--}4$ GeV of the Belle window. This chiral enhancement, introduced in Sec. I, is why the two-parton twist-3 sector contributes significantly in the intermediate-energy region.

The shape of each LCDA is organized by the conformal expansion. At the next-to-leading conformal spin [39,40,47,48], the twist-2 LCDA of a pseudoscalar meson M is

$$\phi_M^\pi(x, \mu) = 6x\bar{x} \left[1 + \sum_{n=1}^{\infty} a_n^M(\mu) C_n^{3/2}(2x-1) \right], \quad (6)$$

with $C_n^{3/2}$ the Gegenbauer polynomials. For the isospin-symmetric pion, G -parity enforces $a_{2n+1}^\pi = 0$, and we retain a_2^π and a_4^π . The two-parton twist-3 LCDAs of the pion, including the meson-mass corrections of Refs. [39,40], are

$$\begin{aligned} \phi_\pi^p(x, \mu) = & 1 + [30\eta_{3\pi}(\mu) - \frac{5}{2}\rho_\pi^2(\mu)] C_2^{1/2}(2x-1) \\ & - \left[3\eta_{3\pi}(\mu)\omega_{3\pi}(\mu) + \frac{27}{20}\rho_\pi^2(\mu) + \frac{81}{10}\rho_\pi^2(\mu)a_2^\pi(\mu) \right] C_4^{1/2}(2x-1), \end{aligned} \quad (7)$$

$$\phi_\pi^\sigma(x, \mu) = 6x\bar{x} \left\{ 1 + \left[5\eta_{3\pi}(\mu) - \frac{1}{2}\eta_{3\pi}(\mu)\omega_{3\pi}(\mu) - \frac{7}{20}\rho_\pi^2(\mu) - \frac{3}{5}\rho_\pi^2(\mu)a_2^\pi(\mu) \right] C_2^{3/2}(2x-1) \right\}, \quad (8)$$

where the Legendre polynomials are $C_2^{1/2}(\xi) = \frac{1}{2}(3\xi^2 - 1)$ and $C_4^{1/2}(\xi) = \frac{1}{8}(35\xi^4 - 30\xi^2 + 3)$, the three-particle coupling is $\eta_{3\pi} = f_{3\pi}/(f_\pi\mu_\pi)$, $\omega_{3\pi}$ is its gluon-momentum-fraction parameter, and $\rho_\pi^2 = (m_u + m_d)^2/m_\pi^2$ is the meson-mass correction parameter, which is numerically tiny but kept for consistency because at twist-3 it formally enters at the same order as $f_{3\pi}$ [39].

The neutral-kaon LCDAs are defined analogously. Adopting the standard convention in which the strange quark sits in the q_2 position [40], the LCDAs of $\bar{K}^0 = s\bar{d}$ coincide, in the exact-isospin limit, with the conventionally defined LCDAs of the $K^- = s\bar{u}$ meson. The relevant matrix elements read [40,41]

$$\begin{aligned} \langle \bar{K}^0(p) | \bar{s}(z_1) \gamma_\mu \gamma_5 d(z_2) | 0 \rangle &= -if_K p_\mu \int_0^1 dx e^{i(xp \cdot z_1 + \bar{x}p \cdot z_2)} \phi_K^\pi(x, \mu), \\ \langle \bar{K}^0(p) | \bar{s}(z_1) i\gamma_5 d(z_2) | 0 \rangle &= f_K \mu_K \int_0^1 dx e^{i(xp \cdot z_1 + \bar{x}p \cdot z_2)} \phi_K^p(x, \mu), \\ \langle \bar{K}^0(p) | \bar{s}(z_1) i\gamma_5 \sigma_{\mu\nu} d(z_2) | 0 \rangle &= -f_K \mu_K (p_\mu z_\nu - p_\nu z_\mu) \int_0^1 dx e^{i(xp \cdot z_1 + \bar{x}p \cdot z_2)} \frac{\phi_K^\sigma(x, \mu)}{6}, \end{aligned} \quad (9)$$

with the kaon decay constant $f_K = 155.7(3)$ MeV [50] and the chirally enhanced mass parameter $\mu_K = m_{K^0}^2/(m_d + m_s)$. The LCDAs of the strangeness-conjugate state $K^0 = d\bar{s}$ follow from charge conjugation, which acts on the longitudinal momentum fraction as $x \leftrightarrow \bar{x}$,

$$\phi_{K^0}^i(x, \mu) = \phi_{\bar{K}^0}^i(\bar{x}, \mu), \quad i \in \{\pi, p, \sigma\}, \quad (10)$$

with the odd Gegenbauer moments and the $SU(3)$ -breaking twist-3 parameters changing sign accordingly [40]. The relation (10) is the origin of the argument $(1-x)$ carried by the K^0 leg in the helicity amplitude (42) below. In the twist-2 conformal expansion (6) the kaon retains, besides a_2^K , a nonzero a_1^K generated by $SU(3)$ -flavor breaking.

The kaon twist-3 LCDAs are richer in structure because $SU(3)$ -flavor breaking lifts the G -parity selection rules that forbid odd-conformal-spin terms in the pion case. Two new dimensionless quark-mass parameters,

$$\rho_K^+ = \frac{(m_s + m_q)^2}{m_K^2}, \quad \rho_K^- = \frac{m_s^2 - m_q^2}{m_K^2}, \quad m_q = \frac{1}{2}(m_u + m_d), \quad (11)$$

encode the symmetric and antisymmetric (G -parity violating) parts of the quark-mass-induced corrections. In the Ball-Braun-Lenz (BBL) parametrization [40] the two-parton twist-3 LCDAs of $\bar{K}^0$ at next-to-leading conformal spin read [40] [Eqs. (3.25) and (3.26)]

$$\begin{aligned}
\phi_K^p(x, \mu) = & 1 + 3\rho_K^+(1 + 6a_2^K - 9\rho_K^- a_1^K) + \left[\frac{27}{2}\rho_K^+ a_1^K - \frac{\rho_K^-}{2}(3 + 27a_2^K) \right] C_1^{1/2}(2x - 1) \\
& + (30\eta_{3K} + 15\rho_K^+ a_2^K - 3\rho_K^- a_1^K) C_2^{1/2}(2x - 1) + \left(10\eta_{3K}\omega_{3K} - \frac{9}{2}\rho_K^- a_2^K \right) C_3^{1/2}(2x - 1) \\
& - 3\eta_{3K}\omega_{3K} C_4^{1/2}(2x - 1) + \frac{3}{2}(\rho_K^+ + \rho_K^-)(1 - 3a_1^K + 6a_2^K) \ln x \\
& + \frac{3}{2}(\rho_K^+ - \rho_K^-)(1 + 3a_1^K + 6a_2^K) \ln \bar{x},
\end{aligned} \tag{12}$$

$$\begin{aligned}
\phi_K^\sigma(x, \mu) = & 6x\bar{x} \left\{ 1 + \frac{3}{2}\rho_K^+ + 15\rho_K^+ a_2^K - \frac{15}{2}\rho_K^- a_1^K \right. \\
& + \left[3\rho_K^+ a_1^K - \frac{15}{2}\rho_K^- a_2^K \right] C_1^{3/2}(2x - 1) + \left[5\eta_{3K} - \frac{1}{2}\eta_{3K}\omega_{3K} + \frac{3}{2}\rho_K^+ a_2^K \right] C_2^{3/2}(2x - 1) \\
& + \left. \eta_{3K}\omega_{3K} C_3^{3/2}(2x - 1) \right\} + 9x\bar{x}(\rho_K^+ + \rho_K^-)(1 - 3a_1^K + 6a_2^K) \ln x \\
& + 9x\bar{x}(\rho_K^+ - \rho_K^-)(1 + 3a_1^K + 6a_2^K) \ln \bar{x},
\end{aligned} \tag{13}$$

with $\eta_{3K} = f_{3K}/(f_K\mu_K)$ and ω_{3K} defined in analogy with the pion case. As compared with the pion expansions (7) and (8), the kaon LCDAs contain three new structures. The first consists of the odd-conformal-spin Gegenbauer terms $C_1^{1/2,3/2}$ and $C_3^{1/2,3/2}$, generated by the G -parity-violating combinations $\rho_K^- a_{1,2}^K$ and ω_{3K} entering with a_1^K . The second consists of the end point logarithms $\ln x$ and $\ln \bar{x}$, which are intrinsic to the kaon and absent for the pion (the G -parity-symmetric limit $\rho_K^\pm \rightarrow 0$ removes them). The third consists of the explicit quark-mass corrections proportional to $\rho_K^\pm$. Following the conventions of Ref. [40], the constant offset $3\rho_K^+(1 + 6a_2^K - 9\rho_K^- a_1^K)$ in ϕ_K^p combines with the $\ln x, \ln \bar{x}$ terms so as to preserve the canonical normalization $\int_0^1 dx \phi_K^p(x, \mu) = 1$ to the order considered, whereas no such cancellation operates in ϕ_K^σ , whose normalization $\int_0^1 dx \phi_K^\sigma(x, \mu)$ is shifted away from unity by an amount of order $\rho_K^\pm \sim 4 \times 10^{-2}$ [40]. The three-particle twist-3 distribution amplitude (DA) of the kaon takes the

standard form $\phi_{3\sigma}^K(\alpha_i, \mu) = 360 \alpha_1 \alpha_2 \alpha_3^2 [1 + \lambda_{3K}(\alpha_1 - \alpha_2) + \frac{1}{2}\omega_{3K}(7\alpha_3 - 3)]$ [40]; it brings in the new G -parity-violating parameter λ_{3K} (also vanishing in the chiral $SU(3)$ limit) but is power-suppressed by $f_{3K}/f_K \sim 10^{-2}$ and is dropped in the present analysis.

All Gegenbauer moments and twist-3 parameters above depend on the factorization scale. We evolve them at leading order with the Efremov-Radyushkin-Brodsky-Lepage and renormalization-group equations of Refs. [2,40,51,52]. For the kaon, $SU(3)$ breaking induces the mixing of the twist-3 parameters with the mass-twist-2 combinations given in Ref. [40], which we adopt. The numerical values of all input parameters, and the discussion of their uncertainties, are collected in Sec. III A.

C. Light cone wave functions and helicity amplitudes

Fourier-transforming Eq. (4) to momentum space and including the transverse-momentum dependence yields the standard twist-3 spinor projector of the pion [33,41,49,53]

$$M_{\alpha\beta}^\pi = \frac{if_\pi}{4} \left\{ \not{p}\gamma_5\Psi_\pi^\pi - \mu_\pi\gamma_5 \left[\Psi_\pi^p - i\sigma_{\mu\nu} \frac{p^\mu \bar{p}^\nu \Psi_\pi^{\sigma'}}{p \cdot \bar{p}} \frac{1}{6} + i\sigma_{\mu\nu} p^\mu \frac{\Psi_\pi^\sigma}{6} \frac{\partial}{\partial k_{\perp\nu}} \right] \right\}_{\alpha\beta}, \tag{14}$$

where $\bar{p}^\mu = (p^0, -\mathbf{p})$ is the conjugate lightlike vector, $\Psi_\pi^{\sigma'} \equiv \partial\Psi_\pi^\sigma/\partial x$, and $\Psi_\pi^i(x, \mathbf{k}_\perp)$ are the (momentum-space) light cone wave functions whose collinear parts coincide with $\phi_\pi^i(x)$ defined above ($i \in \{\pi, p, \sigma\}$). We assume that the three wave functions factorize into a longitudinal LCDA times a common transverse-momentum profile

$$\Psi_\pi^i(x, \mathbf{k}_\perp) = \phi_\pi^i(x) \Sigma_\pi(x, \mathbf{k}_\perp), \quad i \in \{\pi, p, \sigma\}, \tag{15}$$

where the transverse profile is taken in the Brodsky-Huang-Lepage (BHL) form [32,33,54–56]

$$\Sigma_\pi(x, \mathbf{k}_\perp) = \frac{16\pi^2\beta_\pi^2}{x\bar{x}} \exp \left[-\frac{\beta_\pi^2 \mathbf{k}_\perp^2}{x\bar{x}} \right]. \tag{16}$$

We stress that the BHL ansatz is not a fully factorized Gaussian in $\mathbf{k}_\perp$ and x , because its harmonic-oscillator

origin couples the transverse and longitudinal degrees of freedom through the $x\bar{x}$ factor in the exponent, so that the width of the $\mathbf{k}_\perp$ distribution shrinks toward the end points $x \rightarrow 0, 1$. This nonfactorized x - $\mathbf{k}_\perp$ correlation is essential for the correct soft end point behavior and distinguishes the BHL form from a naive $\phi(x) \exp(-\mathbf{k}_\perp^2/2\langle\mathbf{k}_\perp^2\rangle)$ ansatz. The oscillator parameter β_π is fixed by the constraint on the intrinsic average transverse momentum $\langle\mathbf{k}_\perp^2\rangle_\pi$,

$$\langle\mathbf{k}_\perp^2\rangle_\pi = \frac{1}{2\beta_\pi^2} \frac{\int dx |\phi_\pi(x)|^2}{\int dx |\phi_\pi(x)|^2 / [x\bar{x}]}. \quad (17)$$

In the k_T -factorization theorem the convolution is carried out in the conjugate impact-parameter ($\mathbf{b}$) space, where $\mathbf{b}$ is the transverse separation between the quark and the antiquark inside the meson (conjugate to $\mathbf{k}_\perp$), and it sets the factorization scale $\mu_F = 1/b$ at which the soft and hard physics are separated. Fourier-transforming the BHL

profile (16) to $\mathbf{b}$ -space yields the clean exponential damping factor (the hat denotes $\mathbf{b}$ -space quantities throughout)

$$\hat{\Sigma}_\pi(x, \mathbf{b}) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \Sigma_\pi(x, \mathbf{k}_\perp) e^{-i\mathbf{k}_\perp \cdot \mathbf{b}} = 4\pi \exp\left[-\frac{x\bar{x}\mathbf{b}^2}{4\beta_\pi^2}\right], \quad (18)$$

which together with the Sudakov factor specified below ensures the perturbative convergence of the convolution in the soft end point region. The corresponding $\mathbf{b}$ -space wave functions are $\hat{\Psi}_\pi^i(x, \mathbf{b}) = \phi_\pi^i(x) \hat{\Sigma}_\pi(x, \mathbf{b})$. The kaon wave functions $\Psi_K^i(x, \mathbf{k}_\perp)$ take the same form with $\beta_\pi \rightarrow \beta_K$ and $\phi_\pi^i \rightarrow \phi_K^i$.

With these wave functions in hand, the k_T -factorization helicity amplitudes for $\gamma\gamma \rightarrow \pi^0\pi^0$ follow [30,31,34,35,57] as a convolution of the two soft wave functions and the hard kernels in the joint $(x, y; \mathbf{b}_1, \mathbf{b}_2)$ space

$$\begin{aligned} \mathcal{M}_{\lambda_1\lambda_2}(\gamma\gamma \rightarrow \pi^0\pi^0) &= \int dx dy \int \frac{d^2\mathbf{b}_1}{4\pi} \frac{d^2\mathbf{b}_2}{4\pi} \sum_{i,j \in \{\pi,p,\sigma,\sigma'\}} \hat{\Psi}_\pi^j(y, \mathbf{b}_2, \mu_F) \hat{T}_{ij}^{\lambda_1\lambda_2}(x, y, Q, \theta^*, \mathbf{b}_1, \mathbf{b}_2, \mu_R) \\ &\times \hat{\Psi}_\pi^i(x, \mathbf{b}_1, \mu_F) S_i(x) S_j(y) \exp[-S(x, y, \mathbf{b}_1, \mathbf{b}_2, Q, \mu_F, \mu_R)], \end{aligned} \quad (19)$$

with $\hat{\Psi}_\pi^{\sigma'} \equiv \partial \hat{\Psi}_\pi^\sigma / \partial x$. The sum over the two flavor components of the $\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ state, together with the photon electric charges, is contained in the overall prefactor of the hard kernel $\hat{T}_{ij}^{\lambda_1\lambda_2}$ (see Sec. IID), so that $\mathcal{M}$ already represents the full $\pi^0\pi^0$ amplitude.

Before discussing the resummation factors that appear in Eq. (19), it is useful to specify the two physical scales that enter the convolution. The first is the factorization scale $\mu_F = 1/b$, naturally identified with the inverse transverse separation of the quark-antiquark pair, and the second is the renormalization scale $\mu_R = t_n$, taken as the largest characteristic invariant of the internal gluon exchange in the n th diagrammatic group (defined in Sec. IID below). This choice of μ_R minimizes the higher-order QCD corrections in the standard manner [31,53]. The strong coupling is evaluated at one loop, matched across the charm and bottom thresholds to the world average $\alpha_s(M_Z) = 0.1180$ [50], which corresponds to $\Lambda_{\text{QCD}}^{(N_f=4)} \simeq 0.12$ GeV. The same scale enters the Sudakov resummation, so that the hard coupling and the Sudakov factor are mutually consistent. These two scales control, respectively, the Sudakov and the threshold resummation factors of Eq. (19), which we now specify in turn.

We begin with the Sudakov factor. The exponential $\exp[-S]$ resums to all orders the double logarithms $\alpha_s \ln^2(xQb)$ generated by the overlap of soft and collinear gluon emission off the massless quark lines, together with

the single logarithm associated with the renormalization-group evolution between the scales $1/b$ and μ_R . In the next-to-leading-logarithm approximation [17,31]

$$\begin{aligned} S(x, y, Q, b, t) &= s(x, b, Q) + s(\bar{x}, b, Q) + s(y, b, Q) \\ &+ s(\bar{y}, b, Q) - \frac{2}{\beta_0} \ln \frac{\ln(t/\Lambda_{\text{QCD}})}{\ln[1/(b\Lambda_{\text{QCD}})]}, \end{aligned} \quad (20)$$

with the universal function $s(\xi, b, Q)$ originally derived by Botts and Sterman [30] and subsequently refined to next-to-leading-logarithmic accuracy [31,32], its explicit form being given in those references and in our earlier work [33]. The factor $\exp[-S]$ falls off rapidly at large b and effectively suppresses the nonperturbative regime $b \gtrsim 1/\Lambda_{\text{QCD}}$, providing a dynamically generated infrared cutoff that obviates *ad hoc* prescriptions of the type used in collinear analyses [18,32].

The Sudakov damping alone is not, however, sufficient at twist-3 accuracy. Because the asymptotic pseudoscalar DA $\phi_p^{\text{as}} = 1$ does not vanish at the partonic end points, the two-parton twist-3 contributions generate additional logarithmic end point enhancements $\alpha_s \ln^2 x$. These are resummed into a jet function $S_i(x)$ via the threshold resummation [34–36]

$$S_t(x, Q) = \frac{2^{1+2c}\Gamma\left(\frac{3}{2} + c\right)}{\sqrt{\pi}\Gamma(1+c)} [x\bar{x}]^c, \\ c(Q) = \min\{0.04Q^2 - 0.51Q + 1.87, 1\}, \quad (21)$$

where the parabolic parametrization of the parameter c is taken from the Li-Mishima fit of the pion transition form factor [58]. Because $S_t(x) \rightarrow 0$ as $x \rightarrow 0, 1$, the remaining

soft-end point contributions are damped sufficiently to keep the perturbative calculation self-consistent down to $Q \sim 2$ GeV.

With the Sudakov and the threshold resummation factors specified, we can now reduce the two-impact-parameter convolution (19) to a single b -integration. Starting from the Fourier representation of the hard kernel in $\mathbf{b}$ -space

$$\hat{T}_{ij}^{\lambda_1\lambda_2}(x, y, Q, \theta^*, \mathbf{b}_1, \mathbf{b}_2, \mu_R) = \int \frac{d^2\mathbf{k}_{\perp 1}}{(2\pi)^2} \frac{d^2\mathbf{k}_{\perp 2}}{(2\pi)^2} T_{ij}^{\lambda_1\lambda_2}(x, y, Q, \theta^*, \mathbf{k}_{\perp 1}, \mathbf{k}_{\perp 2}, \mu_R) e^{-i\mathbf{k}_{\perp 1} \cdot \mathbf{b}_1 - i\mathbf{k}_{\perp 2} \cdot \mathbf{b}_2}. \quad (22)$$

The parity invariance of the underlying $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$ amplitude implies

$$\hat{T}_{ij}^{++} = \hat{T}_{ij}^{--}, \quad \hat{T}_{ij}^{+-} = \hat{T}_{ij}^{-+}, \quad (23)$$

so that only two independent helicity components must be retained. Within the standard small- x kinematic hierarchy [59,60]

$$Q^2 \gg xQ^2, \bar{x}Q^2, yQ^2, \bar{y}Q^2 \gg \bar{x}yQ^2, x\bar{y}Q^2, \mathbf{k}_{\perp 1}^2, \mathbf{k}_{\perp 2}^2,$$

the transverse-momentum dependence is dropped from the quark propagators, where it is subleading, and retained only in the internal gluon propagator, where it regulates the end point singularity. The hard kernel then localizes in $\mathbf{b}$ -space at $\mathbf{b}_1 + \mathbf{b}_2 = 0$, so that

$$\hat{T}_{ij}^{\lambda_1\lambda_2}(x, y, Q, \theta^*, \mathbf{b}_1, \mathbf{b}_2, \mu_R) \propto \delta^{(2)}(\mathbf{b}_1 + \mathbf{b}_2). \quad (24)$$

After performing the trivial azimuthal angle integration the helicity amplitude takes the compact one- b form

$$\mathcal{M}_{\lambda_1\lambda_2}(\gamma\gamma \rightarrow \pi^0\pi^0) = \int_0^1 dx \int_0^1 dy \int_0^\infty \frac{b db}{(4\pi)^2} \sum_{i,j \in \{\pi, p, \sigma, \sigma'\}} \sum_{n \in \{a, b, c, d\}} \hat{\Psi}_\pi^j(y, b, 1/b) \tilde{T}_{nij}^{\lambda_1\lambda_2}(x, y, Q, \theta^*, b, t_n) \\ \times \hat{\Psi}_\pi^i(x, b, 1/b) S_t(x) S_t(y) \exp[-S(x, y, Q, b, t_n)], \quad (25)$$

where $\tilde{T}_{nij}^{\lambda_1\lambda_2}$ are the partial hard kernels associated with the four diagrammatic groups $n = a, b, c, d$ displayed in Fig. 1. Each group corresponds to a topologically distinct virtuality of the internal gluon line, and the remaining diagrams of each group are generated by permutations of the gluon and quark lines. The upper b -integration limit is taken at $b_{\max} \sim 4.5$ GeV $^{-1}$ in the numerical analysis to avoid the α_s Landau pole, while the Sudakov damping (20) makes the integrand fall well below this scale.

D. Hard kernels for $\gamma\gamma \rightarrow \pi^0\pi^0$

For each group $n \in \{a, b, c, d\}$ the partial hard kernel factorizes as

$$\tilde{T}_{nij}^{\lambda_1\lambda_2}(x, y, Q, \theta^*, b, t_n) \propto \kappa_m [\mathcal{R}_{n,ij}^{\lambda_1\lambda_2}(x, y, \theta^*)] F(s_n, b) B_{n,ij}(pb), \quad (26)$$

where $\mathcal{R}_{n,ij}^{\lambda_1\lambda_2}$ is a rational function of the partonic fractions and the angular variables s, c alone (coming from the Dirac trace), and the prefactors $\kappa_{1,2}$ collect the electromagnetic and strong couplings together with the charge-flavor

structure of the π^0 . The latter follows from $\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$, since the two photons couple to a common quark line of flavor q and the coherent sum over the u and d components—weighted by the squared flavor-projection $(1/\sqrt{2})^2$ and the quark charges—yields the charge-flavor factor $(e_u^2 + e_d^2)/2$ that is common to all four diagrammatic groups. Hence the electric charge e_q is already absorbed into the prefactors, which read

$$\kappa_1 = \frac{e_u^2 + e_d^2}{2} \pi^2 \alpha_{\text{em}} \alpha_s(t_n) f_\pi^2, \quad \kappa_2 = \frac{\mu_\pi^2}{\omega^2} \kappa_1, \quad (27)$$

with $e_u = +\frac{2}{3}$ and $e_d = -\frac{1}{3}$. The factor $\kappa_2/\kappa_1 = \mu_\pi^2/\omega^2 = 4\mu_\pi^2/Q^2$ is the chiral-enhancement parameter discussed in Sec. I. In the Belle window $Q \sim 2\text{--}4$ GeV this ratio is of order 1.8–7, so the twist-3 corrections are not parametrically suppressed. In particular, the chiral masses μ_π and μ_K are of order the hard scale in this window, which is the physical reason why the two-parton twist-3 contributions can become comparable to or even exceed the leading-twist ones. The allowed range of μ_M is therefore directly tied to the importance of the twist-3 sector and is discussed in

Sec. III A. The four group-specific invariants of the internal gluon are

$$\begin{aligned} s_a &= 4\bar{x}y\omega^2, & s_b &= 4x\bar{y}\omega^2, \\ s_c &= 4(x - c^2)(\bar{y} - s^2)\omega^2, & s_d &= 4(\bar{x} - c^2)(y - s^2)\omega^2, \end{aligned} \quad (28)$$

and the corresponding renormalization scales

$$\begin{aligned} t_a &= \max\{2\omega\sqrt{\bar{x}y}, 1/b\}, & t_b &= \max\{2\omega\sqrt{x\bar{y}}, 1/b\}, \\ t_c &= \max\left\{2\omega\sqrt{c^2\bar{x}\bar{y} + s^2xy}, 1/b\right\}, & t_d &= \max\left\{2\omega\sqrt{s^2\bar{x}\bar{y} + c^2xy}, 1/b\right\}. \end{aligned} \quad (29)$$

Note that s_a, s_b are sign-definite (always positive in the physical region), whereas s_c, s_d change sign as a function of x, y, θ^* , signalling the timelike/spacelike character of the exchanged gluon in the cross-quark diagrams. The Fourier-transformed propagator function reads accordingly

$$F(s_n, b) = \begin{cases} 2\pi H_0^{(1)}(\sqrt{s_n}b), & s_n > 0, \\ -4iK_0(\sqrt{-s_n}b), & s_n < 0, \end{cases} \quad (30)$$

with $H_0^{(1)}$ the Hankel function of the first kind and K_0 the modified Bessel function of the second kind. The Bessel factor $B_{n,ij}(pb)$, with $p = Qsc$, is $J_0(pb)$, $J_1(pb)$ or $J_2(pb)$ depending on the helicity and spin structure of the particular diagram (see the explicit expressions below). Its physical origin is the Fourier transform of the additional photon-side transverse momentum carried by the cross-quark exchange diagrams of groups c and d —it is therefore intrinsically a transverse-momentum effect that is absent in the collinear LT calculation.

We now list the explicit kernels for the independent groups a and c . Those of groups b and d follow from the discrete symmetries given below. The twist-2 hard kernels, built from the $\phi^\pi \otimes \phi^\pi$ convolution with the chiral-enhancement-free prefactor κ_1 , read [19,33]

$$\begin{aligned} \tilde{T}_{a\pi\pi}^{++} &= \frac{4}{9}\kappa_1 \frac{xy + \bar{x}\bar{y}}{s^2c^2x\bar{y}} F(s_a, b), \\ \tilde{T}_{a\pi\pi}^{+-} &= \frac{4}{9}\kappa_1 \left[4 + \frac{s^2(\bar{x}y + x\bar{y})}{c^2x\bar{y}} + \frac{c^2(\bar{x}y + x\bar{y})}{s^2x\bar{y}} \right] F(s_a, b), \\ \tilde{T}_{c\pi\pi}^{++} &= \frac{4}{9}\kappa_1 \left[\frac{xy + \bar{x}\bar{y}}{s^2xy} + \frac{xy + \bar{x}\bar{y}}{c^2\bar{x}\bar{y}} \right] J_0(pb) F(s_c, b), \\ \tilde{T}_{c\pi\pi}^{+-} &= -\frac{4}{9}\kappa_1 \left[4 - \frac{s^2(\bar{x}y + x\bar{y})}{c^2\bar{x}\bar{y}} - \frac{c^2(\bar{x}y + x\bar{y})}{s^2xy} \right] J_0(pb) F(s_c, b). \end{aligned} \quad (31)$$

The twist-3 hard kernels are most naturally classified by the pair of LCDA components (i, j) with $i, j \in \{p, \sigma, \sigma'\}$ that they multiply. At twist-3 accuracy and in the asymptotic conformal expansion there are nine independent

kernels for the helicity-conserving $(++)$ amplitude $-(p, p), (p, \sigma'), (\sigma', p), (p, \sigma), (\sigma, p), (\sigma', \sigma'), (\sigma', \sigma), (\sigma, \sigma')$ —plus an additional (σ, σ) structure that contributes only to the helicity-flipping $(+-)$ amplitude (it carries $J_2(pb)$, which vanishes by parity in the $(++)$ projection). The (p, p) and (p, σ') kernels of group a read

$$\begin{aligned} \tilde{T}_{app}^{++} &= -\frac{2}{9}\kappa_2 \left[2 - \frac{1 - \bar{x}y}{s^2x\bar{y}} - \frac{1 - \bar{x}y}{c^2x\bar{y}} \right] F(s_a, b), \\ \tilde{T}_{app}^{+-} &= \frac{2}{9}\kappa_2 \left[\frac{1}{s^2c^2} - \frac{2(1 - x\bar{y})}{\bar{x}y} \right] F(s_a, b), \\ \tilde{T}_{ap\sigma'}^{++} &= \frac{1}{27}\kappa_2 \left[\frac{1}{s^2c^2} - \frac{2}{x} \right] F(s_a, b), \\ \tilde{T}_{ap\sigma'}^{+-} &= \frac{1}{27}\kappa_2 \left[\frac{\bar{x}^2}{s^2c^2x\bar{x}} - \frac{2}{x\bar{x}} \right] F(s_a, b). \end{aligned} \quad (32)$$

The (σ', σ') kernels of group a are

$$\begin{aligned} \tilde{T}_{a\sigma'\sigma'}^{++} &= -\frac{1}{162}\kappa_2 \left[\frac{s^2(1 - \bar{x}y)}{c^2x\bar{y}} + \frac{c^2(1 - \bar{x}y)}{s^2x\bar{y}} \right] F(s_a, b), \\ \tilde{T}_{a\sigma'\sigma'}^{+-} &= -\frac{1}{162}\kappa_2 \left[\frac{s^2(1 + \bar{x}y)}{c^2x\bar{y}} + \frac{c^2(1 + \bar{x}y)}{s^2x\bar{y}} \right] F(s_a, b). \end{aligned} \quad (33)$$

The corresponding (p, p) and (p, σ') kernels of group c read

$$\begin{aligned} \tilde{T}_{cpp}^{++} &= -\frac{2}{9}\kappa_2 \left[2 + \frac{1 - \bar{x}\bar{y}}{s^2xy} + \frac{1 - xy}{c^2\bar{x}\bar{y}} \right] J_0(pb) F(s_c, b), \\ \tilde{T}_{cpp}^{+-} &= -\frac{2}{9}\kappa_2 \left[\frac{1}{s^2c^2} - 2 \right] J_0(pb) F(s_c, b), \\ \tilde{T}_{cp\sigma'}^{++} &= \frac{1}{27}\kappa_2 \left[\frac{s^2 - c^2}{s^2c^2} - \frac{x - \bar{x}}{x\bar{x}} \right] J_0(pb) F(s_c, b), \\ \tilde{T}_{cp\sigma'}^{+-} &= \frac{1}{27}\kappa_2 \left[\frac{s^2x^2 - c^2\bar{x}^2}{s^2c^2x\bar{x}} + \frac{x - \bar{x}}{x\bar{x}} \right] J_0(pb) F(s_c, b). \end{aligned} \quad (34)$$

The remaining $(p, \sigma), (\sigma, \sigma), (\sigma, \sigma')$ and (σ', σ') kernels appear only in groups c, d , and carry the Bessel factors $J_1(pb)$ and $J_2(pb)$ generated by the $\partial/\partial\mathbf{k}_{\perp\nu}$ derivative term

of the projector (14) acting through the gluon transverse momentum. The (p, σ) and (σ, σ) kernels read

$$\begin{aligned}\tilde{T}_{p\sigma}^{++} &= -\frac{1}{27}\kappa_2 \frac{\omega b(s^2 x + c^2 \bar{x})}{s c x \bar{x}} J_1(p b) F(s_c, b), \\ \tilde{T}_{p\sigma}^{+-} &= -\frac{1}{27}\kappa_2 \frac{\omega b(s^2 \bar{x} + c^2 x + 1)}{s c x \bar{x}} J_1(p b) F(s_c, b), \\ \tilde{T}_{\sigma\sigma}^{+-} &= -\frac{1}{81}\kappa_2 \left[\frac{\omega^2 b^2}{x y} + \frac{\omega^2 b^2}{\bar{x} \bar{y}} \right] J_2(p b) F(s_c, b),\end{aligned}\quad (35)$$

while the (σ, σ') and (σ', σ') kernels are

$$\begin{aligned}\tilde{T}_{\sigma\sigma'}^{++} &= \frac{1}{162}\kappa_2 \left[\frac{\omega b s}{c \bar{y}} - \frac{\omega b c}{s y} \right] J_1(p b) F(s_c, b), \\ \tilde{T}_{\sigma\sigma'}^{+-} &= \frac{1}{162}\kappa_2 \left[\frac{\omega b s(1+x)}{c \bar{x} \bar{y}} - \frac{\omega b c(1+\bar{x})}{s x y} \right] J_1(p b) F(s_c, b), \\ \tilde{T}_{\sigma'\sigma'}^{++} &= -\frac{1}{162}\kappa_2 \left[\frac{s^2(1-xy)}{c^2 \bar{x} \bar{y}} + \frac{c^2(1-\bar{x}\bar{y})}{s^2 x y} \right] J_0(p b) F(s_c, b), \\ \tilde{T}_{\sigma'\sigma'}^{+-} &= -\frac{1}{162}\kappa_2 \left[\frac{s^2(1+xy)}{c^2 \bar{x} \bar{y}} + \frac{c^2(1+\bar{x}\bar{y})}{s^2 x y} \right] J_0(p b) F(s_c, b).\end{aligned}\quad (36)$$

The kernels of groups b and d are obtained from those of groups a and c , respectively, by discrete operations of the kinematic variables that reflect the exchange symmetries of the partonic amplitude

$$\begin{aligned}\tilde{T}_{bii}^{\lambda_1\lambda_2} &= \tilde{T}_{aii}^{\lambda_1\lambda_2}(s \leftrightarrow c, x \leftrightarrow y), \quad \tilde{T}_{b\sigma'p}^{\lambda_1\lambda_2} = \tilde{T}_{a\sigma'p}^{\lambda_1\lambda_2}(s \leftrightarrow c, x \leftrightarrow y), \\ \tilde{T}_{dii}^{\lambda_1\lambda_2} &= \tilde{T}_{cii}^{\lambda_1\lambda_2}(x \leftrightarrow \bar{x}, y \leftrightarrow \bar{y}), \quad \tilde{T}_{d\sigma\sigma'}^{\lambda_1\lambda_2} = \tilde{T}_{c\sigma\sigma'}^{\lambda_1\lambda_2}(x \leftrightarrow \bar{x}, y \leftrightarrow \bar{y}), \\ \tilde{T}_{d\sigma\sigma'}^{\lambda_1\lambda_2} &= -\tilde{T}_{c\sigma\sigma'}^{\lambda_1\lambda_2}(x \leftrightarrow \bar{x}, y \leftrightarrow \bar{y}), \quad \tilde{T}_{d\sigma\sigma'}^{\lambda_1\lambda_2} = -\tilde{T}_{c\sigma\sigma'}^{\lambda_1\lambda_2}(x \leftrightarrow \bar{x}, y \leftrightarrow \bar{y}).\end{aligned}\quad (37)$$

In addition, the Bose symmetry of the identical-meson final state and the parity of the LCDA convolutions imply

$$\begin{aligned}\tilde{T}_{n\sigma p}^{\lambda_1\lambda_2} &= \tilde{T}_{n\bar{p}\bar{\sigma}}^{\lambda_1\lambda_2}(s \leftrightarrow c, x \leftrightarrow \bar{y}), \\ \tilde{T}_{n\sigma'p}^{\lambda_1\lambda_2} &= -\tilde{T}_{n\bar{p}\bar{\sigma}'}^{\lambda_1\lambda_2}(s \leftrightarrow c, x \leftrightarrow \bar{y}), \\ \tilde{T}_{n\sigma'\sigma}^{\lambda_1\lambda_2} &= -\tilde{T}_{n\bar{\sigma}\bar{\sigma}'}^{\lambda_1\lambda_2}(s \leftrightarrow c, x \leftrightarrow \bar{y}).\end{aligned}\quad (38)$$

Here the $a \leftrightarrow b$ relation implements the exchange of the two final-state mesons ($x \leftrightarrow y$ and $s \leftrightarrow c$, corresponding to $\theta^* \rightarrow \pi - \theta^*$), the $c \leftrightarrow d$ relation the quark-antiquark interchange inside each meson ($x \rightarrow \bar{x}$, $y \rightarrow \bar{y}$, with the relative signs set by the parity of the corresponding bilinear projectors), and the Bose symmetry (38) links the cross-LCDA kernels to the direct ones within each group. Together these relations reduce the number of independent kernels from the formal 40 to 8 for group a (or c), which

also serves as an internal consistency check on the calculation.

The kernels exhibit several physical features. In the chiral limit the photon couples to a $q\bar{q}$ pair of opposite helicities, so the twist-2 amplitude (31) favors the helicity-conserving $(++)$ configuration, whose universal denominator $s^2 c^2 = \sin^2(\theta^*/2) \cos^2(\theta^*/2)$ vanishes only at $\theta^* \rightarrow 0, \pi$, so that within the Belle range $|\cos \theta^*| < 0.6$ [6,14] the $(++)$ kernel is moderately enhanced over the $(+-)$ one. The chirality-flipping twist-3 projectors of Eq. (14) populate both helicity channels at the same order in μ_M/Q , with the (σ, σ) kernel $\tilde{T}_{\sigma\sigma}^{+-} \propto J_2(p b)$ feeding the helicity-flip amplitude alone—the quark-mass-induced chirality flip being the mechanism by which twist-3 reaches helicity channels forbidden by massless QCD at leading twist. The Bessel factors $J_1(p b), J_2(p b)$ in the (p, σ) , (σ, σ) and (σ, σ') kernels of groups c, d arise from the $\partial/\partial \mathbf{k}_\perp$ term of the projector (14) acting through the photon-side transverse momentum $p b = Q s c b$, and are genuine transverse-momentum effects with no collinear analog, their end point enhancements being cured at large b by the Gaussian damping (18) and the Sudakov factor. Finally, since $s_a, s_b > 0$ throughout the physical region while s_c, s_d change sign, groups a, b involve only the Hankel piece of Eq. (30) (timelike gluon exchange) whereas the cross-quark groups c, d carry both $H_0^{(1)}$ and K_0 contributions, reflecting the two color-flow topologies of the $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$ amplitude.

A noteworthy consequence is that the two-parton twist-3 contribution can exceed the twist-2 one in the intermediate- Q window. The twist-3 kernels are folded with the asymptotic constant $\phi_p^{\text{as}} = 1$, which does not vanish at the end points and renders the convolution power-enhanced near $x = 0, 1$, in contrast to the twist-2 case folded with $\phi_2 \sim 6x\bar{x}$. Although the twist-3 prefactor $\kappa_2/\kappa_1 = 4\mu_\pi^2/Q^2$ is formally $1/Q^2$ -suppressed, the chiral enhancement $\mu_\pi \sim 2.6$ GeV together with this end point behavior makes the twist-3 contribution exceed the twist-2 one for $Q \lesssim 3.5$ GeV, as noted by Gorsky [61] and quantified for the charged channels in Ref. [33].

E. Extension to $\gamma\gamma \rightarrow K_S^0 K_S^0$

The short-lived neutral kaon K_S^0 is to an excellent approximation [neglecting the CP -violating mixing parameter $|\epsilon_K| \sim 2.2 \times 10^{-3}$, whose effect on the cross section is of $\mathcal{O}(10^{-6})$] the CP -even combination of the strangeness eigenstates,

$$|K_S^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle), \quad |K_L^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle), \quad (39)$$

in the phase convention $CP|K^0\rangle = -|\bar{K}^0\rangle$. Because strangeness is conserved by the strong and electromagnetic interactions, the only nonvanishing $\gamma\gamma$ -induced amplitudes in the strangeness basis are $\mathcal{M}(\gamma\gamma \rightarrow K^0 \bar{K}^0)$ and

$\mathcal{M}(\gamma\gamma \rightarrow \bar{K}^0 K^0)$, and these are related by C -invariance of QCD + QED to the same Bose-symmetric amplitude [62,63]. Projecting on the $K_S^0 K_S^0$ final state (a $C = P = +1$, $J = \text{even}$ state, as required by Yang's theorem [64] for two real photons) and using the vanishing of the strangeness-violating amplitudes $\mathcal{M}(\gamma\gamma \rightarrow K^0 K^0) = \mathcal{M}(\gamma\gamma \rightarrow \bar{K}^0 \bar{K}^0) = 0$, one finds

$$\begin{aligned} \mathcal{M}_{\lambda_1 \lambda_2}(\gamma\gamma \rightarrow K_S^0(p_3) K_S^0(p_4)) \\ = \mathcal{M}_{\lambda_1 \lambda_2}(\gamma\gamma \rightarrow K^0(p_3) \bar{K}^0(p_4)). \end{aligned} \quad (40)$$

The differential cross section then carries an additional $1/2!$ identical-particle phase-space factor for the two K_S^0 final mesons (which is absent in the $K^0 \bar{K}^0$ case where the two mesons are distinguishable by strangeness)

$$\frac{d\sigma(\gamma\gamma \rightarrow K_S^0 K_S^0)}{d|\cos\theta^*|} = \frac{1}{2} \frac{d\sigma(\gamma\gamma \rightarrow K^0 \bar{K}^0)}{d|\cos\theta^*|}, \quad (41)$$

the overall factor $1/2$ being a pure quantum-mechanical basis-rotation effect, independent of any dynamical input [14,33].

The $K^0 \bar{K}^0$ amplitude itself is computed from the same generic k_T -factorization formula (25), but with the two outgoing mesons identified as $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$. The Fourier-transformed light cone wave functions in the convolution are constructed from the BBL conformal expansion of $\phi_K^i(x, \mu)$ and the kaon-specific intrinsic transverse-momentum scale β_K . Because the LCDAs of K^0 and $\bar{K}^0$ are related by $x \leftrightarrow \bar{x}$, the kaon amplitude reads

$$\begin{aligned} \mathcal{M}_{\lambda_1 \lambda_2}(\gamma\gamma \rightarrow K^0 \bar{K}^0) = \int_0^1 dx \int_0^1 dy \int_0^{b_{\max}} \frac{bdb}{(4\pi)^2} \sum_{i,j} \sum_{n=a,b,c,d} \hat{\Psi}_K^j(y, b) \tilde{T}_{K,nij}^{\lambda_1 \lambda_2}(x, y, Q, \theta^*, b, t_n) \\ \times \hat{\Psi}_K^i(\bar{x}, b) S_t(x) S_t(y) \exp[-S(x, y, Q, b, t_n)]. \end{aligned} \quad (42)$$

The hard kernels for the kaon share the rational structure and the numerical color factors of the pion ones, and only the charge-flavor prefactor differs. They are therefore obtained from the pion kernels of Sec. IID by the simple replacement $\kappa_m \rightarrow \kappa_m^K$ in every group $n = a, b, c, d$,

$$\tilde{T}_{K,nij}^{\lambda_1 \lambda_2} = \tilde{T}_{nij}^{\lambda_1 \lambda_2}(\kappa_m \rightarrow \kappa_m^K), \quad \kappa_1^K = e_s^2 \pi^2 \alpha_{\text{em}} \alpha_s(t_n) f_K^2, \quad \kappa_2^K = \frac{\mu_K^2}{\omega^2} \kappa_1^K, \quad (43)$$

where the kaon charge-flavor factor is $e_s^2 = \frac{1}{9}$. Two features distinguish the kaon prefactor from the pion one. First, f_K is here the physical neutral-kaon decay constant (and not $f_K/\sqrt{2}$), since $K^0 = d\bar{s}$ is a flavor eigenstate rather than a flavor superposition, so that no $(1/\sqrt{2})^2$ projection weight appears. Second, because $e_d = e_s = -\frac{1}{3}$, the photon couples with equal strength to the two valence flavors of the kaon, so that all four diagrammatic groups carry the common charge factor $e_d^2 = e_s^2 = e_d e_s = \frac{1}{9}$, in place of the pion weight $(e_u^2 + e_d^2)/2$.

III. NUMERICAL ANALYSIS

A. Cross sections and input parameters

Averaging over the initial photon polarizations, the differential cross section for $\gamma\gamma \rightarrow \pi^0 \pi^0$ reads

$$\frac{d\sigma(\gamma\gamma \rightarrow \pi^0 \pi^0)}{d|\cos\theta^*|} = \frac{1}{64\pi Q^2} \frac{1}{4} \sum_{\lambda_1, \lambda_2 = \pm} |\mathcal{M}_{\lambda_1 \lambda_2}(\gamma\gamma \rightarrow \pi^0 \pi^0)|^2, \quad (44)$$

where $\mathcal{M}_{\lambda_1 \lambda_2}$ is the full amplitude of Eq. (25), whose charge-flavor prefactor $(e_u^2 + e_d^2)/2$ already incorporates

the coherent sum over the $u\bar{u}$ and $d\bar{d}$ components of the π^0 , while leaving the chirally enhanced twist-3 corrections intact. The differential cross section for $\gamma\gamma \rightarrow K_S^0 K_S^0$, obtained by combining Eq. (41) and the $K^0 \bar{K}^0$ amplitude (42), reads

$$\frac{d\sigma(\gamma\gamma \rightarrow K_S^0 K_S^0)}{d|\cos\theta^*|} = \frac{1}{64\pi Q^2} \frac{1}{4} \sum_{\lambda_1, \lambda_2 = \pm} |\mathcal{M}_{\lambda_1 \lambda_2}(\gamma\gamma \rightarrow K^0 \bar{K}^0)|^2. \quad (45)$$

Following the Belle analysis [6,14], and to remain within the region where the perturbative treatment is reliable, we compare with the partial cross section obtained by integrating the differential cross section over the central angular range $|\cos\theta^*| < 0.6$,

$$\sigma_0(\gamma\gamma \rightarrow M^0 \bar{M}^0) = \int_{|\cos\theta^*| < 0.6} \frac{d\sigma(\gamma\gamma \rightarrow M^0 \bar{M}^0)}{d|\cos\theta^*|} d|\cos\theta^*|. \quad (46)$$

This restriction keeps the minimum parton virtuality $\min(|t|, |u|) > 0.2Q^2$ safely above the soft scale (Sec. II A), preserving the validity of the perturbative

calculation, and at the same time matches the angular range over which the Belle Collaboration extracts the cross sections.

In our numerical evaluation the nonperturbative inputs—the meson decay constants, the twist-2 Gegenbauer moments $\{a_n^M\}$, the two-parton twist-3 parameters $\{\mu_M, \eta_{3M}, \omega_{3M}, \dots\}$, and the intrinsic transverse-momentum scales β_M —are set as follows.

The decay constants are taken at their physical values [50], $f_\pi = 130.2(8)$ MeV and $f_K = 155.7(3)$ MeV. The strong coupling is evaluated with the one-loop running formula, using the world-average $\alpha_s(M_Z) = 0.1180$ [50] as the single input and matching across the heavy-quark thresholds as described in Sec. II C. For the twist-2 LCDAs we use, at the reference scale $\mu_0 = 1$ GeV,

$$\begin{aligned} a_2^\pi &= 0.20 \pm 0.07, & a_4^\pi &= 0.10 \pm 0.10, \\ a_1^K &= 0.06 \pm 0.03, & a_2^K &= 0.25 \pm 0.15, \end{aligned} \quad (47)$$

which represent a conservative average of the most recent lattice-QCD [65] and light cone sum-rule (LCSR) [66–68] determinations. The relatively large uncertainty on a_2^π reflects the residual tension between the lattice value ($a_2^\pi \sim 0.12$) and the larger LCSR/dispersive value ($a_2^\pi \sim 0.27$), and we will display both extremes as a measure of the DA-shape uncertainty. The nonzero a_1^K encodes the $SU(3)$ -flavor breaking generated by the heavier strange quark, in the convention where the strange quark occupies the q_2 position [40].

The two-parton twist-3 sector is governed by the chiral mass and the three-particle parameters. Following the BBL QCD sum-rule analysis [39,40] we take, at $\mu = 2$ GeV,

$$\eta_{3\pi} = 0.0127 \pm 0.004, \quad \omega_{3\pi} = -1.1 \pm 0.5, \quad (48)$$

and, for the kaon, the corresponding $SU(3)$ -broken values

$$\begin{aligned} \eta_{3K} &= 0.0114 \pm 0.004, & \omega_{3K} &= -0.9 \pm 0.5, \\ \lambda_{3K} &= 1.45 \pm 0.35, \end{aligned} \quad (49)$$

where λ_{3K} is the G -parity-violating three-particle parameter generated by the strange-light quark-mass difference, which vanishes in the pion case. For the neutral kaon it enters with the opposite sign, $\lambda_{3K}^{K^0} = -\lambda_{3K}$, relative to the charged-kaon convention of Ref. [40]. The chiral masses are fixed by Eq. (5), together with the precisely known meson masses $m_{\pi^0} = 134.977$ MeV and $m_{K^0} = 497.611$ MeV. For the pion we input the well-known quark-mass ratio $R \equiv m_s/m_{ud}$, with $m_{ud} \equiv (m_u + m_d)/2$, together with m_s rather than the individual m_u, m_d , since R is determined to much higher precision than the single light-quark masses on the lattice [38,69]. In this scheme

$$\mu_\pi(\mu) = \frac{m_\pi^2 R}{2m_s(\mu)}, \quad (50)$$

which, using the FLAG 2024 averages $R = 27.227 \pm 0.081$ and $m_s(2 \text{ GeV}) = 93.46 \pm 0.58$ MeV [69], gives $\mu_\pi(2 \text{ GeV}) = 2.654 \pm 0.018$ GeV. For the kaon the relevant denominator is $m_d + m_s$ rather than $m_u + m_d$, so the quark-mass ratio R does not directly enter and we instead use the FLAG 2024 quark masses $m_d = 4.70(5)$ MeV and $m_s = 93.46(58)$ MeV in Eq. (5), which gives $\mu_K(2 \text{ GeV}) = 2.523 \pm 0.015$ GeV. Collected

$$\begin{aligned} \mu_\pi(2 \text{ GeV}) &= 2.654 \pm 0.018 \text{ GeV}, \\ \mu_K(2 \text{ GeV}) &= 2.523 \pm 0.015 \text{ GeV}. \end{aligned} \quad (51)$$

These inputs are consistent with the well-known relations of chiral perturbation theory (ChPT). At leading order in the chiral expansion the pion and the kaon share a common chiral mass, $\mu_\pi = \mu_K = -2\langle\bar{q}q\rangle/f_\pi^2$ [cf. Eq. (5)], so the ratio μ_K/μ_π equals unity in this limit. Our values give $\mu_K/\mu_\pi \simeq 0.95$, and the quark-mass ratio R quoted above lies close to its leading-order ChPT estimate $2m_K^2/m_\pi^2 - 1 \simeq 26.2$. These few-percent deviations are precisely the $SU(3)$ -breaking corrections established in the ChPT analysis of the light-quark mass ratios [38], which confirms that the phenomenological chiral masses used here are fully compatible with ChPT.

As emphasized in Sec. II D, the size of μ_M directly controls the importance of the two-parton twist-3 sector through the ratio $\kappa_2/\kappa_1 = 4\mu_M^2/Q^2$, which for $\mu_M \simeq 2.5$ – 2.7 GeV is of order unity throughout the Belle window $Q = 2$ – 4 GeV, so the twist-3 contributions are numerically as important as—and at the lower end larger than—the leading-twist ones. We note that older direct QCD sum-rule extractions favored a somewhat broader range $\mu_\pi \simeq 1.7$ – 2.0 GeV at $\mu = 1$ GeV (equivalently ~ 2.2 – 2.8 GeV at 2 GeV after evolution). We therefore treat μ_M also as a sensitivity parameter when assessing the theoretical uncertainty of the twist-3 prediction.

Finally, the intrinsic transverse-momentum scales entering the BHL wave function (16) are fixed, for the pion, by the $\pi^0 \rightarrow \gamma\gamma$ constraint, $\langle\mathbf{k}_\perp^2\rangle_\pi^{1/2} = 0.35$ GeV [33,54,55], and, for the kaon, by the recent intrinsic transverse-momentum analysis of the pseudoscalar electromagnetic form factors [45,46], which yields an appreciably larger $\langle\mathbf{k}_\perp^2\rangle_K^{1/2} \simeq 0.55$ GeV. All inputs are evolved to the running factorization scale $\mu_F = 1/b$ of the convolution by the leading-order equations of Sec. II B.

B. Numerical results

Figure 2 shows our twist-2 and twist-3 predictions for the partial cross sections $\sigma_0(\gamma\gamma \rightarrow \pi^0\pi^0)$ and $\sigma_0(\gamma\gamma \rightarrow K_S^0 K_S^0)$, integrated over $|\cos\theta^*| < 0.6$, together with the Belle data [4,6,11,14]. Two features stand out. First, throughout the

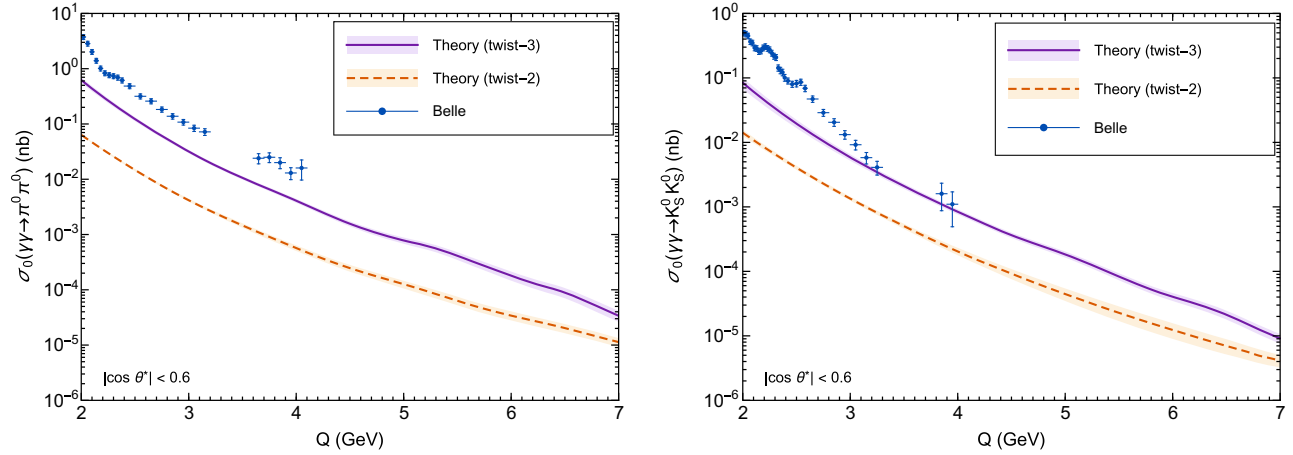


FIG. 2. Partial cross sections σ_0 ($|\cos \theta^*| < 0.6$) for $\gamma\gamma \rightarrow \pi^0\pi^0$ (left) and $\gamma\gamma \rightarrow K_S^0 K_S^0$ (right): twist-2 (dashed) and twist-3 (solid) predictions with their DA-input uncertainty bands, compared with the Belle data [4,6,11,14].

Belle window the twist-3 cross section lies well above the twist-2 one—by up to an order of magnitude near $Q = 2$ GeV—which is the direct numerical manifestation of the chiral enhancement discussed in Secs. I and II D. Two distinct effects combine to produce an enhancement of this size. The first is the chiral mass itself, through which the twist-3 amplitude enters with the factor $\kappa_2/\kappa_1 = 4\mu_M^2/Q^2$. Because μ_M is an order parameter of chiral symmetry breaking of order 1 GeV rather than a current quark mass, this ratio is of order unity—about 1.8–7 across $Q = 2$ –4 GeV—so the nominal $1/Q$ power suppression of the twist-3 term is numerically defeated. The second is the end point behavior of the convolution, in which the asymptotic twist-3 pseudoscalar amplitude $\phi_p^{\text{as}} = 1$ does not vanish at the partonic end points, where the hard kernels are largest. The twist-3 integrand therefore receives a further enhancement that the end point-suppressed twist-2 amplitude does not. Squaring the amplitude turns these two $O(1)$ factors into the order-of-magnitude gap seen in Fig. 2. The same pattern—a twist-3 cross section exceeding the twist-2 one by a factor ranging from several to an order of magnitude—was found for the charged channels $\gamma\gamma \rightarrow \pi^+\pi^-, K^+K^-$ in the k_T -factorization analysis of Ref. [33], so the hierarchy is a generic feature of these reactions rather than a special property of the neutral final states. For the neutral channels, however, it carries additional weight, because the leading-twist amplitude is itself charge-suppressed (Sec. I). The chirally enhanced twist-3 and transverse-momentum contributions are therefore not merely the dominant perturbative piece but are indispensable for any quantitative understanding of the measured cross sections. Their inclusion moves the prediction much closer to the data than the leading-twist result alone.

Second, even at twist-3 the predicted cross section remains below the measurements, by a factor of roughly three across most of the window. This residual deficit is

common to both neutral channels and, as discussed below, is plausibly attributable to higher-order QCD corrections.

To make the comparison quantitative we extract the effective power-law index n defined by $\sigma_0(Q) \sim Q^{-n}$, fitting our computed cross sections over $Q = 2$ –4 GeV at $|\cos \theta^*| < 0.6$. The results for all four channels, at twist-2 and twist-3, are collected in Table I together with the Belle measurements.

Three points emerge from Table I. First, the twist-3 indices lie systematically above the twist-2 ones, the chiral $1/Q^2$ prefactor and the steeper end point-dominated convolution making the twist-3 cross section fall faster with energy. Second, the twist-3 indices are mutually similar and reproduce the measured values for three of the four channels—the charged-channel results $n = 7.87 \pm 0.07$ and $n = 7.26 \pm 0.11$ match the Belle values $7.9 \pm 0.4 \pm 1.5$ and $7.3 \pm 0.3 \pm 1.5$, and the neutral-pion

TABLE I. Effective power-law indices n , defined by $\sigma_0(Q) \sim Q^{-n}$ and obtained by a fit over $Q = 2$ –4 GeV at $|\cos \theta^*| < 0.6$, for the twist-2 and twist-3 calculations, compared with the Belle measurements. The quoted theoretical errors are obtained by propagating the distribution-amplitude input uncertainties through the power-law fit. All Belle indices refer to the same angular range $|\cos \theta^*| < 0.6$, and are fitted over the high- Q continuum regions of the respective experiments (excluding the charmonium window 3.3–3.6 GeV). Our index is only weakly sensitive to the fit window—it changes by $\lesssim 0.2$ between $Q = 2$ –4 and $Q = 3$ –4 GeV—and, since the normalized angular distribution is essentially energy independent (Fig. 3), also to the angular cut, so the comparison is meaningful.

Channel	$n^{\text{tw}2}$	$n^{\text{tw}3}$	n^{Belle}
$\pi^0\pi^0$	6.83 ± 0.16	7.25 ± 0.14	$6.9 \pm 0.6 \pm 0.7$ [6]
$K_S^0 K_S^0$	6.14 ± 0.03	6.66 ± 0.23	$10.5 \pm 0.6 \pm 0.5$ [4,14]
$\pi^+\pi^-$	5.54 ± 0.22	7.87 ± 0.07	$7.9 \pm 0.4 \pm 1.5$ [9]
K^+K^-	4.76 ± 0.48	7.26 ± 0.11	$7.3 \pm 0.3 \pm 1.5$ [9]

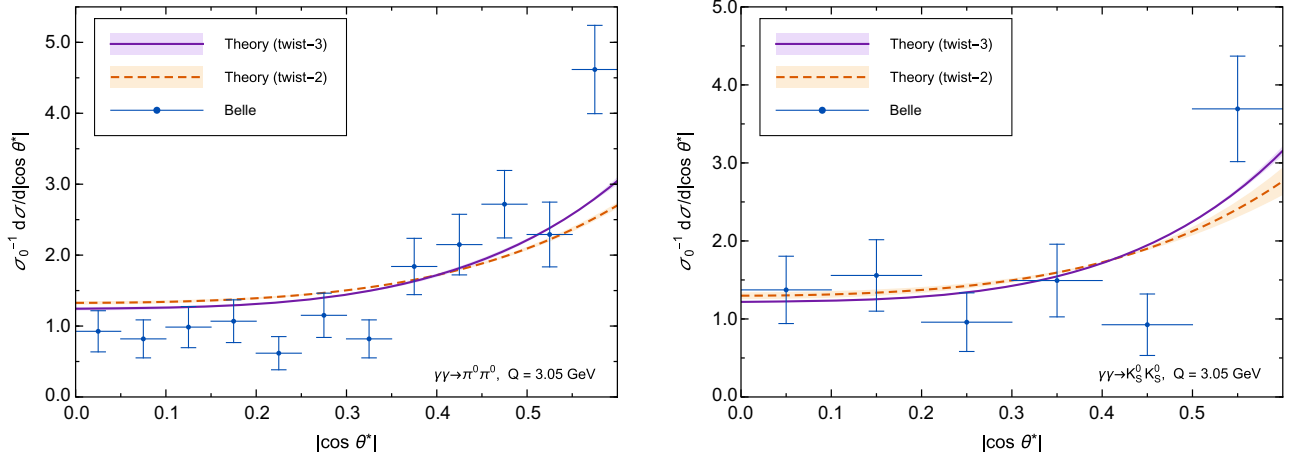


FIG. 3. Normalized differential cross sections $\sigma_0^{-1} d\sigma/d|\cos\theta^*|$ at $Q = 3.05$ GeV for $\gamma\gamma \rightarrow \pi^0\pi^0$ (left) and $\gamma\gamma \rightarrow K_S^0 K_S^0$ (right), at twist-2 and twist-3, compared with the Belle data [6,14].

result $n = 7.25 \pm 0.14$ agrees, within errors, with the value $6.9 \pm 0.6 \pm 0.7$ measured at the same angular cut $|\cos\theta^*| < 0.6$. All are consistent with the k_T -factorization study of Ref. [33], the small differences arising from our updated input parameters. The energy dependence of these three channels is thus set primarily by the common hard-scattering structure. Third, against this common pattern the neutral kaon stands out—the measured $n_{K_S^0 K_S^0} = 10.5 \pm 0.6 \pm 0.5$ is far steeper than any of the hard predictions, including our $n^{\text{tw}3} = 6.66 \pm 0.23$. This strong extra steepening cannot arise from the hard mechanism alone and already points to an additional, energy-dependent contribution—a conclusion reinforced by the ratio analysis below.

The deficit identified above does not by itself invalidate the angular dynamics of the hard kernels. To test that dynamics independently of the overall scale we consider the normalized differential cross section $\sigma_0^{-1} d\sigma/d|\cos\theta^*|$, in which the common prefactors—the chiral mass, the decay constants, and the bulk of the DA-shape dependence—cancel between numerator and denominator. This ratio also removes the dominant overall-normalization systematic of the Belle measurement, so that the shape comparison is far cleaner than the absolute one.

Figure 3 shows the normalized distributions at $Q = 3.05$ GeV. The predicted shapes agree well with the Belle data over the full measured range $|\cos\theta^*| < 0.6$, rising toward larger $|\cos\theta^*|$ as dictated by the $s^2 c^2$ denominator of the helicity-conserving kernel modulated by the charge-suppressed angular function of the neutral mesons. The twist-2 and twist-3 shapes are very similar, showing that the chiral enhancement rescales the magnitude of the cross section without strongly distorting its angular profile. This agreement confirms that the helicity structure and the angular content of the hard kernels are correctly captured, and it localizes the theory—experiment tension in

the overall normalization rather than in the angular dynamics of the reaction.

A still more incisive test is provided by the neutral-to-charged ratios

$$\mathcal{R}_\pi(Q) = \frac{\sigma_0(\pi^0\pi^0)}{\sigma_0(\pi^+\pi^-)}, \quad \mathcal{R}_K(Q) = \frac{\sigma_0(K_S^0 K_S^0)}{\sigma_0(K^+ K^-)}, \quad (52)$$

in which the chiral mass, the decay constants, the higher-order K -factor and most of the DA-shape uncertainty cancel to a large extent, leaving a quantity controlled almost entirely by the charge-flavor structure and by the relative dynamics of the neutral and charged amplitudes. Figure 4 compares the twist-3 prediction with the Belle data. The hard mechanism yields ratios that rise smoothly with energy— $\mathcal{R}_\pi$ from about 0.5 at $Q = 2$ GeV to about 0.8 at $Q = 4$ GeV, and $\mathcal{R}_K$ from about 0.07 to about 0.10—lying above the data over most of the window. It is instructive to set this result against the two limiting mechanisms discussed in Sec. I. Collinear leading-twist pQCD predicts much smaller and nearly energy-independent ratios, $\mathcal{R}_\pi \simeq 0.04\text{--}0.07$ and a comparably small $\mathcal{R}_K$, whereas the soft handbag predicts $\mathcal{R}_\pi \simeq 0.5$ and $\mathcal{R}_K \approx 2/25$, both again almost flat in energy. The data behave differently from all three: $\mathcal{R}_\pi$ decreases from its low- Q value to a minimum near $Q \simeq 3.0$ GeV and then rises again, while $\mathcal{R}_K$ falls steeply by an order of magnitude across the window. None of the calculations—collinear leading twist, soft handbag, or the present hard k_T -factorization result—reproduces this pattern on its own. The monotonic rise of the hard ratios in particular is the direct counterpart of the too-shallow power-law indices of Table I.

That a quantity in which the leading dynamics largely cancels should vary so strongly, and so differently in the two channels, suggests that more than one mechanism is simultaneously important in the few-GeV region, the observed energy dependence then arising from the interplay

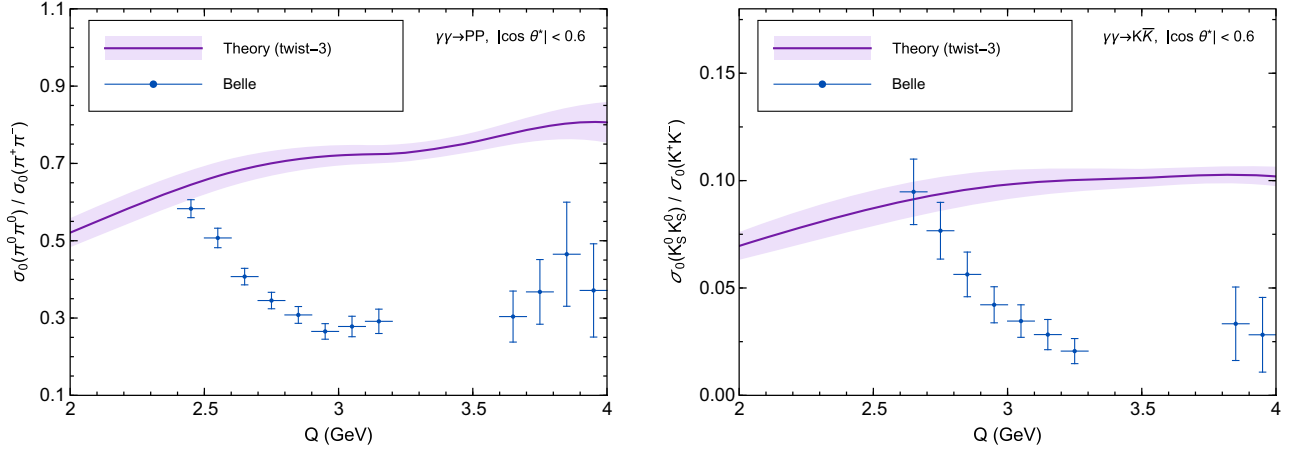


FIG. 4. Cross section ratios $\mathcal{R}_\pi$ (left) and $\mathcal{R}_K$ (right) from the hard twist-3 calculation (band) compared with the Belle data [4,6,11].

of these contributions rather than from any single one. Such a coexistence is in fact expected on kinematic grounds, as noted in Sec. II A, throughout the window $Q \sim 2\text{--}4$ GeV the minimum parton virtuality $\min(|t|, |u|)$ drops to the order of Λ_{QCD}^2 near the edge of the angular acceptance, so that soft end point and handbag configurations are not parametrically separated from the hard scattering and can feed the same amplitude coherently. The inability of the purely hard calculation to describe the ratios is therefore not merely a numerical shortfall but a physical indication that a soft contribution should be included alongside the hard one. We pursue this quantitatively in the following discussion.

C. Discussion

The simplest concrete realization of the soft sector identified above is the handbag mechanism of Diehl, Kroll, and Vogt [21,22], in which the two photons couple to a single quark line and the meson pair is formed through a soft annihilation form factor. A characteristic feature of this mechanism is that it feeds only the opposite-helicity $(+-)$ amplitude, leaving the helicity-conserving $(++)$ part untouched, so that it interferes coherently with the hard $(+-)$ amplitude. We add it at the amplitude level

$$\mathcal{M}^{++} = \mathcal{M}_{\text{hard}}^{++}, \quad \mathcal{M}^{+-} = \mathcal{M}_{\text{hard}}^{+-} + \mathcal{A}_{\text{soft}}^{+-},$$

$$\mathcal{A}_{\text{soft}}^{+-}(Q^2, \theta^*) = -\frac{16\pi\alpha}{\sin^2\theta^*} R_{2M}(Q^2), \quad (53)$$

where R_{2M} is the soft annihilation form factor of the relevant channel, carrying a slowly varying modulus and an energy-dependent strong phase. The hard amplitude carries its own phase, generated by the Sudakov and threshold-resummation factors and the parton-level convolution. We do not introduce any further relative phase between the two amplitudes, so that the soft-to-hard phase difference is absorbed into the phase of R_{2M} . Under exact

$SU(3)$ flavor symmetry the meson-pair form factors of the different channels are related by the same charge-flavor algebra as the hard amplitude [22]. Writing the modulus and phase explicitly

$$R_{\pi^0\pi^0} = R_{\pi^+\pi^-} \equiv R_{2\pi} = |R_{2\pi}| e^{i\delta_\pi(Q)},$$

$$R_{K^+K^-} \equiv R_{2K} = |R_{2K}| e^{i\delta_K(Q)}, \quad R_{K^0\bar{K}^0} = \frac{2}{5} |R_{2K}| e^{i\delta_K(Q)}, \quad (54)$$

the factor $2/5$ in the neutral-kaon form factor reflecting the smaller charge of its valence d quark relative to the u quark of the charged kaon. Exact $SU(3)$ symmetry would in addition set $R_{2\pi} = R_{2K}$ [21]. We retain the equality of the moduli, taking a common value

$$Q^2 |R_{2\pi}| = Q^2 |R_{2K}| \simeq \text{const}, \quad (55)$$

but, to allow for $SU(3)$ -breaking effects, keep the two phases independent, parametrizing each as

$$\delta_M(Q) = \delta_0^M + \delta_1^M(Q - 3 \text{ GeV}), \quad M = \pi, K. \quad (56)$$

Fitting the coherent soft-hard model to the two measured ratios then yields a common modulus

$$Q^2 |R_{2\pi}| = Q^2 |R_{2K}| = (0.36 \pm 0.11) \text{ GeV}^2, \quad (57)$$

together with the phases

$$\delta_0^\pi = (246 \pm 2)^\circ, \quad \delta_1^\pi = (54 \pm 6)^\circ/\text{GeV},$$

$$\delta_0^K = (247 \pm 4)^\circ, \quad \delta_1^K = (99 \pm 13)^\circ/\text{GeV}. \quad (58)$$

Figure 5 shows that with these parameters the model reproduces the decrease of $\mathcal{R}_\pi$ toward its minimum and the steep fall of $\mathcal{R}_K$.

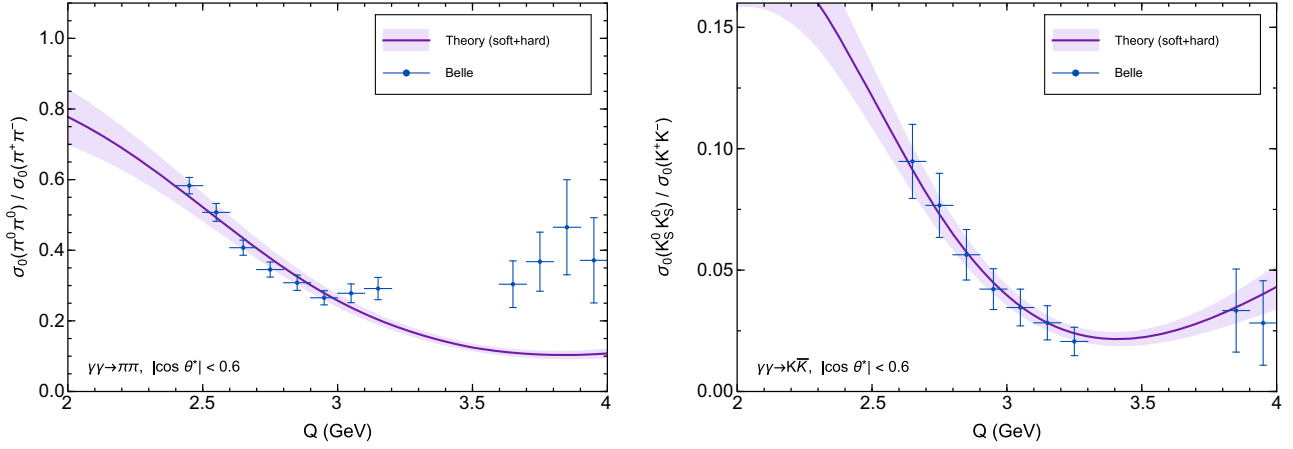


FIG. 5. Cross section ratios $\mathcal{R}_\pi$ (left) and $\mathcal{R}_K$ (right) in the coherent soft-hard model (band) compared with the Belle data [4,6,11].

Three features of this extraction carry physical information. First, the modulus we obtain is appreciably smaller than the value found in the original handbag analysis: Diehl *et al.* fit $Q^2|R_{2\pi}| = 0.75 \pm 0.07 \text{ GeV}^2$ and $Q^2|R_{2K}| = 0.64 \pm 0.04 \text{ GeV}^2$ to the two-photon data [21], about twice our common $Q^2|R_{2M}| = 0.36 \text{ GeV}^2$. Their two values are close to each other, consistent with the common modulus we adopt. The difference in overall size has a clear origin. In the handbag analysis the perturbative hard contribution is evaluated at leading twist, where it is far smaller than the measured cross section, and is therefore neglected, the entire cross section being assigned to the soft form factor. Our calculation instead retains the chirally enhanced twist-3 hard amplitude, which in the intermediate-energy window is of comparable size to the soft one and interferes with it coherently. A correspondingly smaller soft form factor then suffices to describe the data, so that the reduced $Q^2|R_{2M}|$ is a direct consequence of the soft and hard mechanisms sharing the cross section on an almost equal footing in this regime.

Second, the two phases have nearly identical constant parts, $\delta_0^\pi \simeq \delta_0^K \simeq 246^\circ$. Because the hard amplitude carries a nearly Q -independent phase, this constant offset fixes the overall soft-to-hard phase difference, and its near-equality for the pion and the kaon is consistent with $SU(3)$ symmetry, under which that phase difference is flavor independent.

Third, the energy slopes of the two phases differ markedly, $\delta_1^K \simeq 2\delta_1^\pi$. Since the moduli are taken equal, this is where the $SU(3)$ -breaking of the soft sector resides—the strange quark makes the kaon form factor phase rotate about twice as fast with energy as the pion one. The magnitude of these slopes is comparable to the strong, energy-dependent variation of the soft-sector phase found by Diehl and Kroll, whose relative form-factor phase rotates at roughly $50\text{--}75^\circ/\text{GeV}$ in the region $Q \simeq 3 \text{ GeV}$ where the bulk of

the data lie [22].¹ A strictly common phase, $\delta_\pi = \delta_K$, cannot reproduce the two ratios simultaneously, so the independent slopes are required by the data.

Physically, the rapid energy variation of the ratios is a coherence effect. As noted above, each mechanism on its own—collinear leading twist, the soft handbag, or our hard k_T -factorization result—gives a ratio that is almost flat across $Q = 2\text{--}4 \text{ GeV}$. In this window, however, the soft and hard (+−) amplitudes are of comparable magnitude, and their coherent sum, with a relative phase that varies with energy, changes rapidly. Only by retaining both contributions does the calculation reproduce the energy dependence of the ratios observed in the data. This coexistence of competing soft and hard dynamics, neither of which dominates, is what makes the few-GeV region both interesting and difficult.

Two caveats temper this picture. First, the soft-hard model is introduced phenomenologically, with the modulus and phase of R_{2M} fitted to the ratio data rather than computed from first principles, and the present treatment should be regarded as a physically motivated discussion rather than a complete dynamical account. Second, and more importantly, the soft contribution reshapes the ratios through the (+−) channel but does not cure the absolute-normalization deficit. The predicted absolute cross sections remain about a factor of three below the data even after the soft contribution is included. This residual gap is most plausibly attributed to the large, slowly converging higher-order QCD corrections discussed in Sec. I—the NLO

¹Diehl and Kroll [22] parametrize the relative phase between the valence and nonvalence soft form factors $R_{2\pi}^u$ and $R_{2\pi}^s$ as $\rho(s) = \pi[1 + \tanh(\kappa/(s - s_c))]$, with $\kappa \simeq 0.63 \text{ GeV}^2$ and $s_c \simeq 6 \text{ GeV}^2$, which tends to 180° at high energy. The range quoted here is the local slope $d\rho/dQ$ near $Q \simeq 3 \text{ GeV}$ ($s \simeq 9\text{--}10 \text{ GeV}^2$), which decreases at higher energy. This relative phase is not identical to our soft-to-hard phase δ_M , but its rate of energy variation provides the natural point of comparison.

K -factor of order 1.9 found by Duplanić and Nižić [20]—together with possible soft-overlap and higher-Fock contributions beyond the scope of the present leading-order twist-3 analysis. In this respect the pion electromagnetic form factors provide a useful benchmark within the same framework. Their NLO corrections have been computed in k_T factorization up to the two-parton twist-3 level for both spacelike [42] and timelike [43] momenta, and the twist-2 and twist-3 NLO corrections partially cancel each other, leaving a moderate net correction. Two features of the present process, however, caution against transferring this pattern directly. The two-meson final state involves two hard convolutions rather than one, and the collinear NLO analysis of $\gamma\gamma \rightarrow M^+M^-$ indeed finds much larger corrections than in the form-factor case. Furthermore, in the charge-suppressed neutral channels the twist-3 tower dominates the amplitude, so the compensation between the twist-2 and twist-3 NLO terms cannot operate in the same way, and the net correction is expected to be governed by the twist-3 NLO correction alone. An explicit NLO k_T -factorization computation of these two-photon processes, while beyond the scope of the present work, is therefore the natural next step toward a quantitative description of the absolute cross sections.

IV. SUMMARY

We have computed the two-photon production of the neutral pseudoscalar pairs $\gamma\gamma \rightarrow \pi^0\pi^0$ and $\gamma\gamma \rightarrow K_S^0 K_S^0$ in the k_T -factorization framework, including for the first time the chirally enhanced two-parton twist-3 distribution amplitudes for these channels. The calculation sets up the LCDAs of π^0 , K^0 and $\bar{K}^0$, the twist-3 spinor projectors and light cone wave functions, and the helicity hard kernels of the four diagrammatic groups, with the $K_S^0 K_S^0$ channel obtained from $K^0 \bar{K}^0$ by the appropriate CP projection.

The central physical message is that for these charge-suppressed channels the subleading dynamics is not a correction but the leading effect. The cancellation of the $(e_1 - e_2)^4$ term removes the dominant leading-twist amplitude, so that the cross sections are carried by the intrinsic transverse-momentum and chirally enhanced twist-3 contributions. Numerically the twist-3 cross section exceeds the twist-2 one throughout the Belle window and brings the prediction much closer to the data. The calculation reproduces the angular shapes and, through the effective power-law index $n_{\pi^0\pi^0}^{\text{tw}3} = 7.25$, the gross energy dependence of the pion channel. It does not, however, reproduce the very steep fall of the neutral-kaon cross section, for which we obtain

$n_{K_S^0 K_S^0}^{\text{tw}3} = 6.66$ against the measured $n \simeq 10$ –11, nor the pronounced energy dependence of the measured cross section ratios.

These ratio observables, in which the leading dynamics largely cancels, turn out to be the most discriminating. Their strong, channel-dependent energy variation points to the coexistence of competing mechanisms in the few-GeV region, and we have shown that adding the soft handbag amplitude coherently to the hard one—with a common form factor modulus and a phase that rises with energy by roughly 50–100° per GeV—reproduces the decrease of the pion ratio toward its minimum and the steep fall of the kaon ratio. In this picture the soft and hard amplitudes are of comparable size in the opposite-helicity channel, so that both contribute appreciably and neither can be neglected. This interplay of hard and soft dynamics is the central qualitative feature of the intermediate-energy regime.

Two issues remain open. First, even with the soft contribution the absolute cross sections stay about a factor of three below the data, a residual gap plausibly attributable to higher-order QCD corrections—the known NLO K -factor being sizable—supplemented possibly by soft-overlap and higher-Fock contributions. Second, the soft handbag form factor is here fitted to the ratio data rather than derived, so its modulus and phase await a first-principles understanding. Progress on both fronts—a next-to-leading-order k_T -factorization treatment of the absolute normalization and a dynamical calculation of the soft form factor—together with more precise data on the neutral channels at the upper end of the Belle range and at Belle II, would sharpen the separation of the hard and soft components and provide a stringent test of the picture advanced here.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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