

Topological production of charmonia with event-shape engineering in pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8

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The production of heavy quarks (charm and beauty) in high-energy hadronic and nuclear collisions provides an excellent testing ground for the theory of strong interactions and validates models based on quantum chromodynamics (QCD). In this work, prompt and nonprompt production of J/ψ in pp collisions at $\sqrt{s} = 13$ TeV are studied as a function of the transverse sphericity using PYTHIA8. J/ψ is reconstructed via its electromagnetic decay to dielectrons and dimuons, in mid- and forward rapidity, respectively. Transverse sphericity, an event-shape observable, is used to distinguish hard-QCD events from the softer, isotropic ones. In PYTHIA8, the production of J/ψ can be influenced by the average number of multiple parton interactions ($\langle N_{\text{mpi}} \rangle$), owing to the underlying events, which have a dominant contribution to particle production at lower transverse momentum. Since transverse sphericity is correlated to $\langle N_{\text{mpi}} \rangle$, this can serve as an experimentally accessible tool for event selection to study the underlying QCD processes influencing the prompt and nonprompt J/ψ production. This study reveals the correlation between heavy-flavor production dynamics and topological event selection in pp collisions using PYTHIA8, whose relevance awaits experimental validation.

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I. INTRODUCTION

Experimental explorations at the world's most powerful particle accelerator facilities like the Large Hadron Collider (LHC) at CERN, Switzerland and the Relativistic Heavy Ion Collider (RHIC) at BNL, USA aim to study the formation of a deconfined state of hot and dense subnuclear matter called as quark-gluon plasma (QGP) and also to understand the fundamental processes of “hadronization,” which explain the formation of hadrons out of quarks [1]. During such violent collisions of nuclear matter, quarks and gluons are liberated as the relevant degrees of freedom for a very short duration ($\simeq 10^{-23}$ s). In this environment, heavy quarks (charm and beauty) are produced predominantly in the initial hard-partonic interactions, which can be reliably understood via perturbative quantum chromodynamics (pQCD). As heavy quarks are created early in the collision, their production provides an excellent testing ground for the theory of strong interaction, i.e., QCD. Within the framework of the QCD factorization theorem, the heavy quark production cross section is expressed as the convolution of

three terms: (i) the parton distribution functions, (ii) the parton hard-scattering cross sections, and (iii) the heavy-flavor fragmentation functions [2].

Among the various hadrons containing heavy quarks, quarkonia—bound states of a heavy quark and its antiquark—have been studied extensively, with particular emphasis on the lightest charmonium vector meson, $J/\psi(c\bar{c})$. Although the initial production of $c\bar{c}$ pairs from hard-partonic interactions is fairly described by pQCD, the subsequent formation of a color-neutral bound state involves soft-scale, nonperturbative processes that are still not fully understood and are typically addressed via phenomenological models [3].

The inclusive J/ψ production occurs via two distinct mechanisms, i.e., prompt and nonprompt. Prompt J/ψ is produced directly in the hard scattering or formed from the radiative decay of higher charmonium states. They reflect the early collision dynamics and serve as ideal probes to test strong-interaction models and study suppression/regeneration in the QGP. In contrast, nonprompt J/ψ are the weak decay products of beauty hadrons [4–6], which carry most of the momentum of their parent hadrons, thus providing insight into beauty production in elementary, hadronic, and nuclear collisions. Together, they offer complementary probes of quarkonium production mechanisms. These two production modes of J/ψ exhibit distinctive topological signatures: The decay vertex of nonprompt J/ψ is significantly displaced from the primary vertex compared to

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prompt J/ψ because of the longer lifetime of the weakly decaying parent beauty hadrons [7,8]. Exploiting these features allows experimental separation of prompt and nonprompt contributions towards inclusive production, which in turn provides crucial insights into hadronization mechanisms in both the charm and beauty sectors.

In experiments, the inclusive J/ψ measurements are performed by reconstructing J/ψ through its electromagnetic decays to dileptons, as the hadronic decays are mostly suppressed, which is described by the Okubo-Zweig-Iizuka rule. Among the experiments at the LHC, ALICE is capable of measuring the topological production of J/ψ at both midrapidity and forward rapidity regions [9]. The production of J/ψ at midrapidity ($|y| < 0.9$) is measured via the dielectron decay channel, i.e., $J/\psi \rightarrow e^+ + e^-$. In contrast, the production at the forward rapidity ($2.5 < y < 4$) is measured via the dimuon decay channel, i.e., $J/\psi \rightarrow \mu^+ + \mu^-$.

In Refs. [9–12], the ALICE Collaboration explores the production of self-normalized yield of inclusive J/ψ , as a function of the self-normalized charged-particle density at midrapidity for event selection based on multiplicity at both midrapidity and forward rapidity regions. Here, the collision energy dependence of the self-normalized yield of J/ψ is also explored. Such correlation studies of charmonium production with charged-particle multiplicity (N_{ch}) are important to explore the interplay between hard and soft mechanisms of J/ψ production, both at the partonic level and at hadronization [13]. One observes that the self-normalized yield of J/ψ at forward rapidity increases linearly as a function of the self-normalized charged-particle density at midrapidity when event classification is performed based on the multiplicity at midrapidity in pp collisions at $\sqrt{s} = 5.02$ TeV [11]. This follows the $x = y$ line. However, the self-normalized yield of J/ψ starts to deviate from the linear behavior with an increase in $\sqrt{s}$. The deviations from the linear behavior become stronger when the measurement of J/ψ is changed from the forward rapidity to midrapidity regions, irrespective of event classification based on midrapidity or forward rapidity multiplicity [10]. This observation underlines the fact that the production of J/ψ is strongly affected when one moves from the midrapidity to forward rapidity regions. The phenomenological studies suggest that this nonlinear behavior is partially contributed from the nonprompt production of J/ψ [13]. In addition, the latest ALICE experimental data on the multiplicity dependence of J/ψ where both N_{ch} and J/ψ yields are measured at forward rapidities, reiterate the finding of the steeper-than-linear increase of their correlation, attributing it to the autocorrelation effects [12]. Thus, it is necessary to make a systematic study of the prompt and nonprompt production of J/ψ as a function of pseudorapidity.

The pQCD-based event generators, such as PYTHIA8, are able to explain the heavy-flavor production up to a greater

extent, in addition to a number of heavy-ion-like observations in high multiplicity pp collisions [8,14–16]. This is possible thanks to the improved implementation of multipartonic interactions (MPIs), color reconnection (CR), and rope hadronization (RH) [14,15]. The strange and multi-strange hadron ratios showing strangeness enhancement, as well as the particle ratios carrying information about the radial flow, are well explained by PYTHIA8 with MPI, CR, and RH [14,15,17]. Even the increase in charmonium production with charged-particle multiplicity is well explained by PYTHIA8 when MPI is included [18–20]. In fact, it is identified that the J/ψ production in pp collisions is either directly linked to a strong hadronic activity, which increases particle production and biases the $dN_{\text{ch}}/d\eta$ distributions to higher values, or the scenario is that the influence of MPI extends to the domain of hard processes affecting the harder momentum scales relevant for quarkonia production [9]. Moreover, the production of high-momentum and heavier particles in the initial scattering restricts the energy available for MPI and, in turn, affects the particle produced through MPI and the event shape. However, estimating the number of MPI (N_{mpi}) in experiments is not trivial. This challenge could be overcome if event-shape observables proportional to N_{mpi} were employed, one such event classifier being the transverse sphericity S_0 [14]. In this context, the present work bridges this gap by utilizing an event-shape-based approach—through transverse sphericity—to investigate the dynamics of J/ψ production.

In small collision systems, event activity is often characterized using the charged-particle multiplicity measured at the midrapidity or forward rapidity regions. While this approach is straightforward, it can suffer from autocorrelation biases, as the same particles may simultaneously define the event class and contribute to the observables being measured [14,15]. Such autocorrelations can artificially enhance apparent effects, making it difficult to disentangle genuine physics effects from methodological artifacts.¹ Although forward-rapidity-based multiplicity estimators reduce these biases compared to midrapidity selections, they do not eliminate them entirely. Event-shape observables offer a more differential and less biased way, compared to event classifiers based solely on multiplicity, to characterize the underlying event [22]. They therefore provide a useful alternative for studying the interplay between soft and hard particle production with reduced sensitivity to autocorrelation effects [12]. S_0 is one such event-shape classifier, which enables us to segregate hard jettylike events from soft-processes-dominated isotropic events based on the geometry of particle production in each event, the classification strength of which is retained from pp to Pb-Pb collisions [23–28]. Interestingly, the classification strength

¹For detailed discussions on the effects of autocorrelation, see Refs. [12,13,21].

of transverse sphericity changes weakly with a change in rapidity [29], which makes transverse sphericity a suitable event classifier to study the production of prompt and nonprompt J/ψ in the midrapidity and forward rapidity regions. Moreover, in addition to multiplicity-based studies, classifying collisions based on S_0 gives us the added benefit of analyzing various mechanisms of particle production or hadronization for charm and beauty quarks [22]. Thus, this paper explores the transverse sphericity and multiplicity-dependent production of prompt and nonprompt J/ψ at the midrapidity and forward rapidity regions in pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8 through several observables believed to be sensitive to the underlying production mechanisms and autocorrelation bias effects. J/ψ production is studied via the electronic decay channel at midrapidity and the muonic decay channel at the forward rapidity regions. We focus on the observables such as partonic modification factor Q_{pp} , mean transverse momentum $\langle p_T \rangle$, nonprompt J/ψ fraction f_B , and self-normalized yield of prompt and nonprompt J/ψ .

The paper is organized as follows: In Sec. I, we provide a brief introduction and motivation for the study. Section II discusses the event generation and methodology. In Sec. III, we present our results and discuss the same. Finally, the findings of the paper are summarized in Sec. IV, with a brief future outlook.

II. EVENT GENERATION AND METHODOLOGY

In this section, we briefly discuss the event generation using PYTHIA8, along with the tunes used in this study. A brief discussion on the event-shape selection using transverse sphericity is also made.

A. PYTHIA8

The pQCD-based Monte Carlo event generators such as PYTHIA8 [30,31] describe many features of relativistic pp collisions at the RHIC and LHC energies. PYTHIA8 has several models which incorporate soft- and hard-partonic interactions, initial and final state parton showers [32,33], MPIs [34], beam remnants [35], string fragmentation [36,37], and hadronic decays [36,38]. Here, along with MPI-based interactions, $2 \rightarrow 2$ hard processes are implemented, which leads to the production of charm and beauty hadrons. In this study, we have used the 4C tune of PYTHIA8 [33] (version 8.308)² to generate the minimum bias pp collisions at $\sqrt{s} = 13$ TeV, which include the inelastic and nondiffractive components (HardQCD:all = on) of the total collision cross section. In addition, a p_T cutoff of $p_T > 0.5$ GeV/c (using PhaseSpace:pTHatMinDiverge) is set to avoid the divergence of QCD processes in the limit $p_T \rightarrow 0$.

²The results for the 4C tune are compared with CR-BLC2 [39] in Appendix B.

Along with the CR and MPI, we have enabled all possible charmonia and bottomonia production processes available in PYTHIA using “Charmonium:all = on” and “Bottomonium:all = on.” J/ψ is allowed to decay into opposite charge pairs of electrons and muons individually to have better statistics. For both electronic and muonic decay channels of J/ψ , we produce ten billion minimum bias events with the tunes mentioned above in pp collisions at $\sqrt{s} = 13$ TeV. A detailed description of the event generation, along with the identification of prompt and nonprompt J/ψ , can be found in Ref. [8].

In PYTHIA8, the production of heavy quark (Q) and antiquark ($\bar{Q}$) pairs is implemented through the pQCD scattering processes which include the pair creation through gluon fusion ($gg \rightarrow Q\bar{Q}$) and light quark(q)-antiquark($\bar{q}$) pair annihilation ($q\bar{q} \rightarrow Q\bar{Q}$) [40]. In addition to this, the production of heavy quarks can be possible through the gluon splitting of parton showers, i.e., $g \rightarrow Q\bar{Q}$. At sufficiently large energy transfer, the heavy quarks also contribute to the parton distribution function, leading to the production of heavy quarks via flavor excitation ($gQ \rightarrow gQ$) [13]. Similarly, the quarkonia ($c\bar{c}$ or $b\bar{b}$) production in PYTHIA8 can also be implemented via several mechanisms. The leading-order nonrelativistic QCD (NRQCD) channels of $Q\bar{Q}$ production via color-singlet and color-octet preresonant states are included in the pQCD processes [13,40]. The cluster collapse mechanism also contributes to quarkonia production when a heavy quark gets connected to a corresponding heavy antiquark close in phase space to form a quarkonium bound state during the hadronization stage [41]. If charmonia ($c\bar{c}$) is produced from the weak decay of a beauty hadron, it is referred to as nonprompt charmonium production.

B. Transverse sphericity

Transverse sphericity (S_0) is an event-shape observable that characterizes the geometry of an event based on the azimuthal distribution of the particles produced. It is defined for a unit vector $\hat{n}(n_T, 0)$ in the transverse plane, which minimizes the sum of cross products in the bracket as shown below [14,17,42]:

$$S_0 = \frac{\pi^2}{4} \min_{\hat{n}} \left(\frac{\sum_{i=1}^{N_{ch}} |\vec{p}_{T_i} \times \hat{n}|}{\sum_{i=1}^{N_{ch}} |\vec{p}_{T_i}|} \right)^2. \quad (1)$$

Here, $\vec{p}_{T_i}$ stands for the transverse momentum vector of the i th charged particle in an event, and the iteration to find the suitable $\hat{n}$ runs over all charged particles (N_{ch}). The multiplication factor $\pi^2/4$ in Eq. (1) is to normalize the quantity S_0 to lie between 0 and 1. Only the charged tracks within $|\eta| < 0.8$ with $p_T > 0.15$ GeV/c are considered for the calculation of S_0 . In this study, S_0 is constructed by setting $|\vec{p}_{T_i}| = 1.0$ to reduce the charge-to-neutral bias, where a minimum constraint of ten charged particles at

TABLE I. Transverse sphericity cuts for the jetty and isotropic events for different multiplicity classes in pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8 (Tune 4C). The errors for the mean charged-particle multiplicity density are too small ($<1\%$); hence, they are not tabulated.

VOM percentile	$\langle dN_{\text{ch}}/d\eta \rangle_{ \eta <0.5}$	S_0 range	
		Jetty	Isotropic
0–1	20.17	0–0.646	0.844–1
1–5	12.47	0–0.576	0.812–1
5–10	7.81	0–0.528	0.784–1
10–20	5.27	0–0.504	0.770–1
20–40	3.34	0–0.486	0.760–1
40–100	2.39	0–0.468	0.750–1
0–100	3.74	0–0.525	0.785–1

$|\eta| < 0.8$ and $p_T > 0.15$ GeV/ c is applied for each collision so that the description of unweighted sphericity is meaningful [17]. The lower limit $S_0 \rightarrow 0$ corresponds to jetty events containing back-to-back jets, which are consequences of hard scatterings. On the other hand, $S_0 \rightarrow 1$ depicts isotropic events where particles are produced uniformly in the azimuthal plane, resulting from softer interactions. Thus, qualitatively, transverse sphericity S_0 disentangles pQCD-dominated hard processes from soft-QCD processes, helping with understanding the underlying particle production mechanism.³ In this work, 20% of events having the lowest and highest values of S_0 are, respectively, referred to as jetty and isotropic events. The cuts in S_0 for jetty and isotropic events for various VOM multiplicity classes, which are estimated in the forward pseudorapidity ranges $2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$, are shown in Table I. A comparison of the sphericity distribution from PYTHIA8 results (Tune 4C and CR-BLC2)⁴ to that of the experimental data from the ALICE Collaboration for the highest VOM multiplicity class (0%–1%) is presented in Fig. 7 (Appendix A), where a good qualitative and quantitative similarity is observed between them.

III. RESULTS AND DISCUSSION

This section explains the results we obtained as a function of the transverse sphericity (S_0) for prompt and nonprompt J/ψ produced in pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8. Here, J/ψ is reconstructed via the dileptonic decay channels $J/\psi \rightarrow e^+ + e^-$ at midrapidity ($|y| < 0.9$) and $J/\psi \rightarrow \mu^+ + \mu^-$ in the forward rapidity ($2.5 < y < 4$). From this point onward, J/ψ in the midrapidity and forward rapidity, respectively, implies the reconstruction using the $J/\psi \rightarrow e^+ + e^-$ channel and $J/\psi \rightarrow \mu^+ + \mu^-$ channel.

³Elaborate discussion on transverse sphericity can be found in Ref. [14].

⁴See Appendix B for details on CR-BLC2 PYTHIA8 settings.

Shown in Fig. 1 are the S_0 dependence of event-normalized p_T spectra of prompt and nonprompt J/ψ produced in pp collisions simulated using PYTHIA8 at $\sqrt{s} = 13$ TeV for the (0%–100%) VOM class. The p_T spectra of J/ψ , reconstructed via dielectron and dimuon channels are shown in the left and right plots of Fig. 1, respectively. We observe that the yield of nonprompt J/ψ is almost 10 times smaller than that of prompt J/ψ in the low- p_T regime, while this difference shrinks as we approach higher values of p_T for both the rapidity regions, as the nonprompt J/ψ grows harder [13,43]. Toward the low- p_T region, the contributions of prompt J/ψ come primarily from the charmonia produced in the leading-order semihard processes, i.e., $q\bar{q} \rightarrow C$ and $g\bar{g} \rightarrow C$ (where C refers to final state charmonium) [20]. Here, the probability of beauty-hadron production is significantly small due to its large mass compared to charmonium states. Further, in Ref. [13], the authors show that at the high- p_T regime, the particle production is largely dominated by the hard scatterings, where the production of beauty hadrons is comparable to the charmonium states. Thus, the hardening of the p_T spectrum of nonprompt J/ψ as compared to prompt J/ψ is a consequence of the competing effects of two different production mechanisms of charmonium states and beauty hadrons in different regions of transverse momentum. We note that these trends are in line with the results obtained from experiments [44–46].

We also see a considerable dependence of transverse sphericity on the p_T -dependent yield of topological production of J/ψ in both rapidity regions, as shown in Fig. 1. Here, the production of both prompt and nonprompt J/ψ is observed to be favored more in isotropic events as compared to the jetty events. In Refs. [14,15,29], it is explored that the transverse sphericity possesses a large correlation with N_{mpi} . Additionally, it is also concluded in Ref. [20] that the production of inclusive J/ψ , which is mostly prompt in nature, is favored in events having large N_{mpi} . In fact, the yield of inclusive J/ψ increases linearly with N_{mpi} [20]. The observation of a higher(lower) prompt J/ψ yield in isotropic(jetty) events, which corresponds to events having large(small) N_{mpi} as compared to S_0 -integrated events, is a testimony of the observations shown in Ref. [20]. Analogously, it can also be argued that the events having large N_{mpi} favor the b -hadron production, as compared to events having small N_{mpi} , which results in a larger yield of nonprompt J/ψ for isotropic events, as compared to the jetty events [13].

A clearer comparison of yields between different S_0 classes is presented in the bottom panel of Fig. 1, where the ratio of p_T spectra for isotropic and jetty events to that of the S_0 -integrated events is shown for both prompt and nonprompt J/ψ . One can see that the relative prompt J/ψ production in isotropic events increases with p_T till 10 GeV/ c to fall gradually thereafter, indicating the hardness of the p_T spectra of prompt J/ψ in isotropic events in

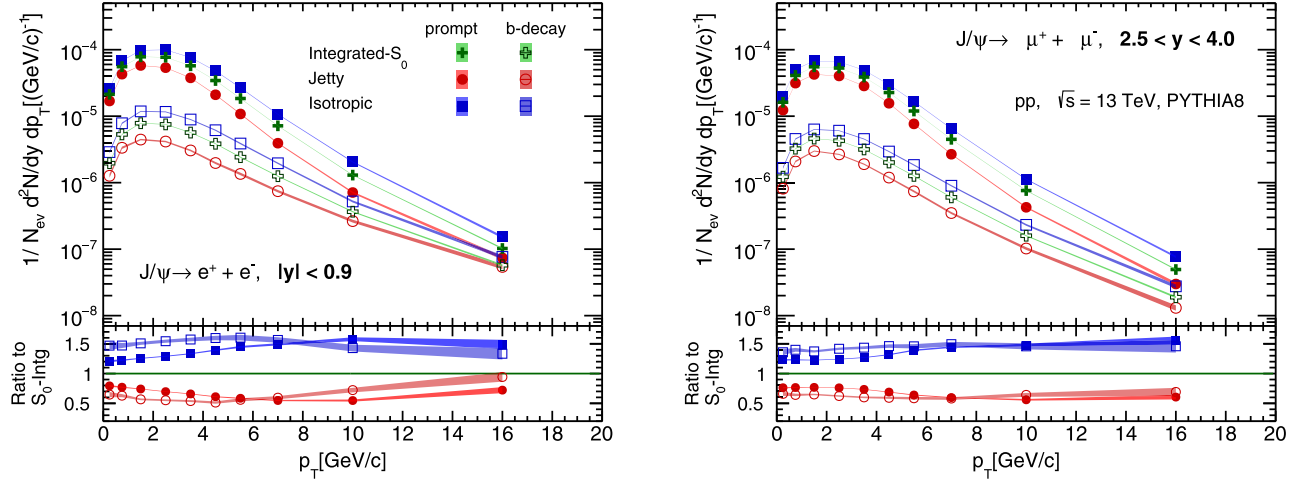


FIG. 1. Event-normalized transverse momentum (p_T) spectra of prompt and nonprompt J/ψ in S_0 -integrated, jetty, and isotropic events from pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8 for dielectron (left) and dimuon (right) production channels at $|y| < 0.9$ and $2.5 < y < 4$ rapidity ranges, respectively, for the (0%–100%) VOM class. The ratio of p_T spectra of the jetty and isotropic to S_0 -integrated events is shown in the bottom panel. Error bands incorporate the estimated statistical uncertainties.

comparison to S_0 -integrated events. In contrast, it decreases with p_T for jetty events to rise after $p_T \approx 10$ GeV/ c , showing the softness of p_T spectra in jetty events as compared to S_0 -integrated events. In Ref. [13], the authors show that a non-negligible amount of prompt J/ψ are formed through the cluster collapse mechanism, which is largely enhanced for the CR-on scenario for the events with large N_{mpi} . Thus, the hardness and softness of p_T spectra for the isotropic and jetty events, respectively, for the prompt J/ψ case are a consequence of the effects from color reconnection, where the charm quarks from independent partonic scatterings combine to form charmonium states having a large transverse momentum [13]. The effect of color reconnection is enhanced for the events having large N_{mpi} , consequently, the isotropic event, leading to the observed hardening of prompt J/ψ spectra. At the same time, the yield in both isotropic and jetty events relative to S_0 -integrated events of nonprompt J/ψ shows only modest dependence on p_T till $p_T \approx 6$ GeV/ c in both isotropic and jetty events. However, after $p_T > 6$ GeV/ c , the relative nonprompt J/ψ production abruptly decreases for isotropic events and steeply increases for jetty events to converge with the S_0 -integrated yield. The weaker hardening of nonprompt J/ψ spectra as compared to the prompt case is because of the large mass of the beauty hadrons, which are produced in the initial hard scatterings, which remain unaffected by CR [20]. The effects of hardening (and softening) of p_T spectra of prompt and nonprompt J/ψ are diminished at the forward rapidity regions because the effects of MPI are found to be smaller when observations are made in forward rapidity regions, as compared to midrapidity, where the particle yield is significantly higher [29].

In experiments, it is observed that the self-normalized yield of charged particles for $p_T > 4$ GeV/ c shows a

stronger than linear increase with an increase in charged-particle multiplicity density in the midrapidity region [22]. These observations are also made for strange [47] and charm [9–12] hadrons at high- p_T regions. With PYTHIA8, the stronger-than-linear increase of the self-normalized particle yield of prompt and nonprompt J/ψ is shown in Fig. 5, the discussion of which will follow later. As discussed in Sec. I, these effects mostly come as a consequence of autocorrelation bias. Also, event classifiers based on multiplicity can lead to contributions from multijet scatterings in the final state [48,49]. On the other hand, event classifiers based on event shapes, such as transverse sphericity, can probe N_{mpi} ⁵ and the contributions from multijet topologies can be effectively reduced [12]. Since these effects have p_T dependence, one needs to make a comparison of transverse momentum spectra by taking out the contributions from the final state multiplicity.

To exclusively understand the modification in the p_T spectral shape for prompt and nonprompt J/ψ for isotropic and jetty events, we study the partonic modification factor Q_{pp} whose formulation is inspired by the nuclear modification factor R_{AA} [50]. Through Q_{pp} , we aim to make a qualitative comparison of the event-normalized p_T -differential yield of prompt and nonprompt J/ψ from the isotropic and jetty events to that of S_0 -integrated events, where the yield is scaled by the average number of prompt (or nonprompt) J/ψ in that particular sphericity class. Consequently, Q_{pp} is defined as follows [29,51]:

⁵ S_0 and N_{ch} have a fair degree of correlation with N_{mpi} , implying that events having higher $\langle N_{\text{mpi}} \rangle$ produce higher final state multiplicity involving softer isotropic particle production [14].

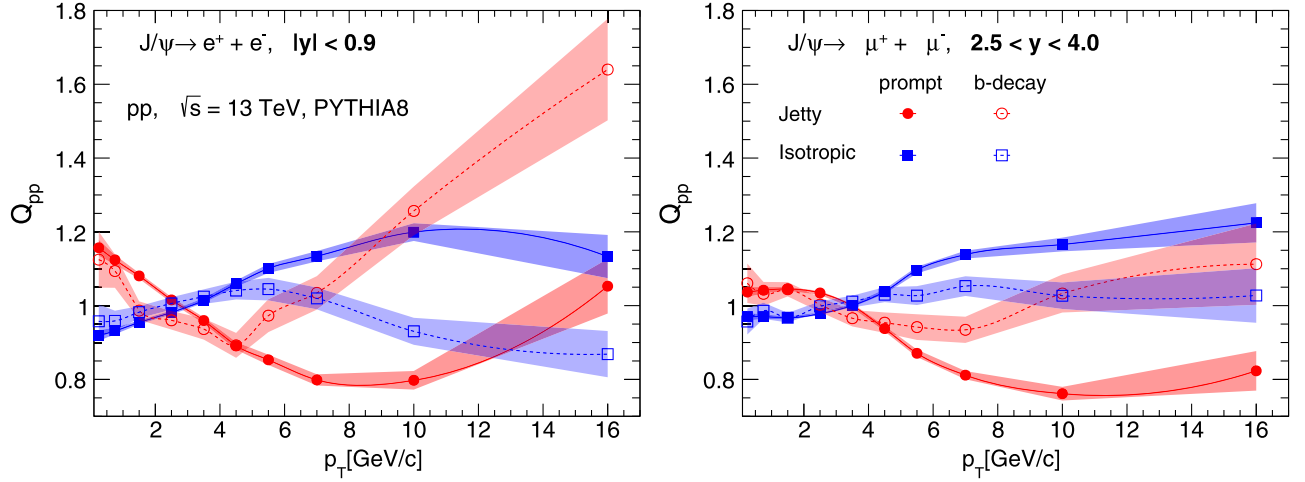


FIG. 2. Q_{pp} as a function of p_T for prompt and nonprompt J/ψ in the mid- (left) and forward (right) rapidities for jetty and isotropic events in pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8. Error bands incorporate the estimated statistical uncertainties.

$$Q_{pp} = \frac{d^2 N_{J/\psi}^{S_0} / \langle N_{J/\psi}^{S_0} \rangle dy dp_T}{d^2 N_{J/\psi}^{S_0-int} / \langle N_{J/\psi}^{S_0-int} \rangle dy dp_T}. \quad (2)$$

Here, $\langle N_{J/\psi}^{S_0} \rangle$ and $\langle N_{J/\psi}^{S_0-int} \rangle$ are the average number of prompt or nonprompt J/ψ in the corresponding sphericity event class and S_0 -integrated events, respectively.

Figure 2 shows the transverse momentum dependence of the partonic modification factor defined in Eq. (2) for prompt and nonprompt J/ψ . The left and right panels show the results for the prompt and nonprompt J/ψ , at the mid- and forward rapidity regions, respectively. At the midrapidity region, the trend for both prompt and nonprompt J/ψ is such that it first increases (decreases) with an increase in p_T for the isotropic (jetty) case and then starts to fall (rise) toward higher p_T . However, Q_{pp} attains the peak (valley) at a much earlier p_T (~ 5 GeV/c) in the case of nonprompt J/ψ than for the prompt J/ψ . As discussed earlier, the p_T spectra of both prompt and nonprompt J/ψ show different levels of hardening for different classes of sphericity, which is reflected in the positions of the peaks and valleys in the Q_{pp} spectra of prompt and nonprompt J/ψ . Further, the observed deviations from unity at high p_T for the nonprompt J/ψ measured at the midrapidity region for both the isotropic and jetty events are interesting. As noted in Ref. [13], the beauty-hadron production is often accompanied by a high-energy jet in the opposite direction, which makes the events pencil-like. On the contrary, when the events are isotropic, which corresponds to a high N_{mpi} , most of the energy is used up in softer particle production, with little beauty hadrons being produced in the initial scatterings [13]. This shows that the events where the beauty hadrons are produced acquire a unique shape, which is picked up by transverse sphericity. In addition, small autocorrelation bias effects may contribute here as $S_0 \propto N_{ch}$, which can be avoided when measurements are done at different rapidity regions.

Hence, the measurements made at forward rapidity (right plot) are crucial, as negligible autocorrelation bias is expected in this case. Here, Q_{pp} varies smoothly with increase in p_T . Q_{pp} , which starts slightly above (below) 1 for jetty (isotropic) events at low p_T , suddenly starts to decrease (increase) after 3 GeV/c for both prompt and nonprompt J/ψ . Q_{pp} from isotropic and jetty events has its first crossing at around 3–4 GeV/c for both prompt and nonprompt J/ψ and the second crossing at ~ 10 GeV/c for the nonprompt J/ψ case. However, unlike in midrapidity, here at forward rapidity, the nonprompt J/ψ shows no deviations from unity. This is a testimony that the production of beauty hadrons at forward rapidity would not modify the event shape measured through the charged particles in the midrapidity region. Moreover, the p_T dependence of Q_{pp} is found to be in alignment within uncertainties when the study is repeated for the CR-BLC2 PYTHIA model for both the dileptonic decay channels of J/ψ (particularly prompt J/ψ).

To understand the contribution of J/ψ produced from the decay of b -hadrons to the inclusive J/ψ production, the fraction of nonprompt-to-inclusive J/ψ (f_B) is studied as a function of the transverse momentum and transverse sphericity in the (0%–100%) VOM class, both at the midrapidity and forward rapidity regions, as shown in Fig. 3. A qualitative similarity of our results with the ALICE and LHCb data could be observed from the comparison presented in Fig. 8 in Appendix A. Similar to the experimental observations at midrapidity in Ref. [44], the nonprompt J/ψ fraction stays independent of p_T for $p_T \leq 4$ GeV/c after which f_B increases steadily with an increase in p_T . This relative increase in the number of nonprompt J/ψ to inclusive J/ψ with an increase in p_T is consistent with the observations of the hardening of the p_T spectra of nonprompt J/ψ with respect to prompt J/ψ , as shown in Fig. 1, for both rapidity regions. In addition, f_B shows

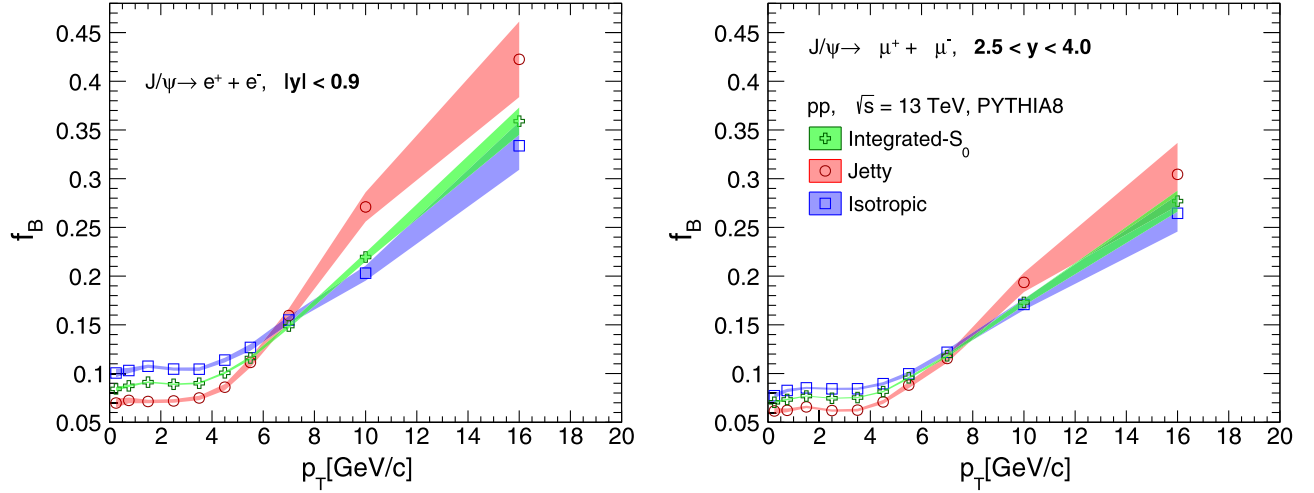


FIG. 3. The fraction of nonprompt J/ψ produced as a function of p_T in midrapidity (left) and forward rapidity (right) for S_0 -integrated, jetty, and isotropic events from pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8. Error bands incorporate the estimated statistical uncertainties.

significant transverse sphericity dependence. Here, at the midrapidity region, for $p_T \leq 6$ GeV/c, isotropic events show a larger value of f_B . Interestingly, this trend is reversed for $p_T > 6$ GeV/c, where jetty events lead f_B . This transverse sphericity dependence at the low- p_T region reflects the effects of MPI on the production of beauty hadrons. In this low- p_T regime, a larger fraction of nonprompt J/ψ is produced in the isotropic events (large N_{mpi}) as compared to the jetty events (having small N_{mpi}). This is, hence, affirmation that, in PYTHIA8, MPI activity can affect the beauty-hadron production [13]. However, at high- p_T regions, a large fraction of beauty hadrons are produced from the initial hard scattering itself. These hard scatterings are significantly large in jetty events by their very nature, which leads to the enhanced value of f_B at large values of p_T . A similar observation is also made at the forward rapidity region. Here, however, the crossing of f_B for jetty and isotropic events happens at $p_T \approx 8$ GeV/c. In addition, we find f_B to be smaller in forward rapidity as compared to the midrapidity region for all S_0 classes. Note that these differences based on rapidity also qualitatively match the ALICE and LHCb experimental data provided in Fig. 8 (Appendix A). A comparison with the CR-BLC2 configuration of PYTHIA8 indicates that the qualitative trend and the segregation power of S_0 are valid irrespective of model differences, though an average reduction of $\sim 40\%$ relative to Tune 4C is observed in both the dielectron and dimuon channels over the studied p_T range.

Figure 4 shows the mean transverse momentum ($\langle p_T \rangle$) as a function of the average charged-particle multiplicity at midrapidity ($\langle dN_{\text{ch}}/d\eta \rangle_{|\eta| < 0.5}$) for prompt and nonprompt J/ψ in the three S_0 classes for measurements at midrapidity (left plot) and forward rapidity (right plot) in pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8. $\langle p_T \rangle$ of both prompt and nonprompt J/ψ is seen to rise with multiplicity, where

midrapidity measurements achieve greater magnitudes of $\langle p_T \rangle$ than those in forward rapidity. This observation of rapidity dependence of $\langle p_T \rangle$ is consistent with the results of light flavor hadrons, as shown in Ref. [29]. For both pseudorapidity regions, one finds a larger $\langle p_T \rangle$ for the nonprompt J/ψ as compared to the prompt case. Since the nonprompt J/ψ carries most of the transverse momentum of its parent beauty hadrons, $\langle p_T \rangle$ of nonprompt J/ψ is the reflection of $\langle p_T \rangle$ of the decaying beauty hadrons, which is evidently larger than the $\langle p_T \rangle$ of prompt J/ψ . This is because the beauty hadrons that contribute to nonprompt J/ψ possess a larger mass compared to the charmonium states. In other words, Fig. 4 reflects the mass ordering of $\langle p_T \rangle$, which is consistent with the mass ordering of $\langle p_T \rangle$ for different hadrons in the light-flavor sector [50]. There is also a notable S_0 dependence for the enhancement of $\langle p_T \rangle$ with increase in $\langle dN_{\text{ch}}/d\eta \rangle_{|\eta| < 0.5}$, especially for the J/ψ reconstructed at midrapidity. At low multiplicity density, $\langle p_T \rangle$ of prompt and nonprompt J/ψ from jetty events has a slight dominance in magnitude over the J/ψ produced from isotropic events. This is because the jetty events of the lowest multiplicity class are strongly dominated by the hard scatterings; the hadrons produced in such interactions have large transverse momenta, consequently possessing a large value of $\langle p_T \rangle$ as compared to isotropic events in a similar multiplicity class. This difference is clearer in the case of midrapidity nonprompt J/ψ , the production source of which is primarily the initial hard scatterings. Here, at higher multiplicities ($\langle dN_{\text{ch}}/d\eta \rangle_{|\eta| < 0.5} \gtrsim 12$), the distinction based on sphericity diminishes for nonprompt J/ψ , while prompt J/ψ from isotropic events possesses slightly higher $\langle p_T \rangle$ than their counterparts from jetty events. This indirectly signifies the role of MPI in charm hadron production in contrast to beauty hadrons. Further, one

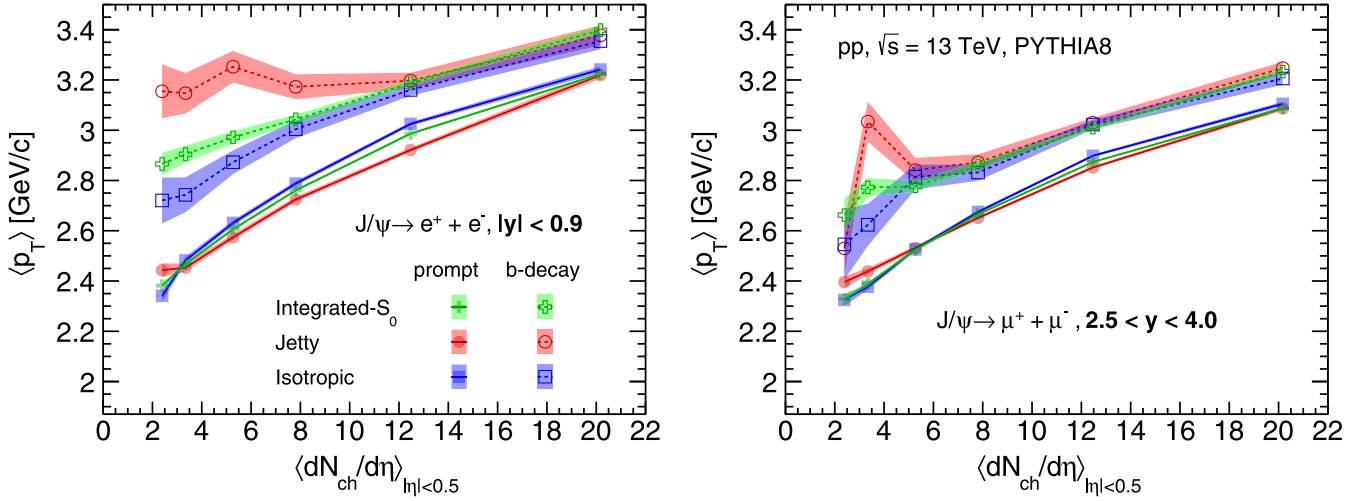


FIG. 4. Mean transverse momentum ($\langle p_T \rangle$) as a function of the midrapidity charged-particle multiplicity density for prompt and nonprompt J/ψ in the mid- (left) and forward (right) rapidity regions for S_0 -integrated, jetty, and isotropic events for pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8. Error bands incorporate the estimated statistical uncertainties.

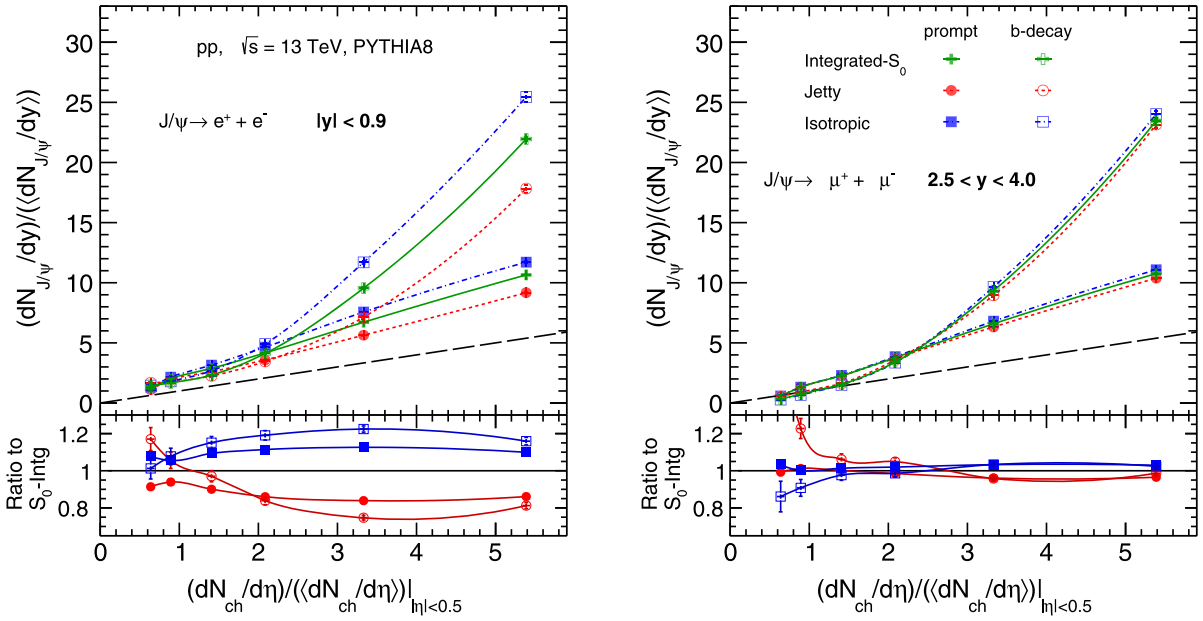


FIG. 5. Normalized p_T -integrated prompt and nonprompt J/ψ yield as a function of the normalized charged-particle multiplicity density at midrapidity region ($|\eta| < 0.5$) with V0 multiplicity selection for S_0 -integrated, jetty, and isotropic events from pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8, for dielectron (left) and dimuon (right) production channels at $|y| < 0.9$ and $2.5 < y < 4$ rapidity ranges, respectively. The dashed black line guides $(dN_{J/\psi}/dy)/(\langle dN_{J/\psi}/dy \rangle) = (dN_{ch}/d\eta)/(\langle dN_{ch}/d\eta \rangle)$. Estimated statistical uncertainties are well within the marker sizes.

finds that the sphericity dependence of the $\langle p_T \rangle$ of prompt and nonprompt J/ψ is diminished at the forward rapidity.

The upper panels in Fig. 5 show the self-normalized yield of prompt and nonprompt J/ψ as a function of the self-normalized charged-particle multiplicity density at midpseudorapidity ($|\eta| < 0.5$) for S_0 -integrated, jetty, and isotropic events in pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8. The left and right panels show the results at the mid- and forward rapidities, respectively. The normalized

J/ψ yield for both prompt and nonprompt cases increases with normalized charged-particle density for measurements at both mid- and forward rapidities and for all classes of transverse sphericity. However, in both rapidity regions, for the S_0 -integrated case, one finds that the increment rate of nonprompt J/ψ is stronger as compared to the prompt case with an increase in self-normalized charged-particle density. This is because the beauty hadrons are usually formed in initial hard scattering and are accompanied by

high-energy jets, which can further fragment to produce hadrons, which can increase the charged-particle multiplicity in the final state [13]. Thus, the stronger-than-linear increase in nonprompt J/ψ production can be attributed to an autocorrelation bias [12,13]. As the particle multiplicity selection is performed in the forward rapidity region, one finds a comparatively smaller autocorrelation bias in the left panel, where both prompt and nonprompt J/ψ are measured in the midrapidity region, in contrast to the right panel, where the J/ψ is measured in the forward rapidity region.

Furthermore, the effect of this autocorrelation bias can be further explored with transverse sphericity. In Ref. [13], the influence of N_{mpi} is explored for both inclusive and nonprompt J/ψ . Here, the effects of N_{mpi} are similar for both inclusive and nonprompt J/ψ , except for extreme N_{mpi} events, where the yield of nonprompt J/ψ saturates. Interestingly, in the left panel of Fig. 5, we find that the yield of both prompt and nonprompt J/ψ is enhanced for the isotropic events having large N_{mpi} , while a lower self-normalized yield is observed for the jetty events with a small N_{mpi} value. This is a contradiction to the above-mentioned dependence of nonprompt J/ψ production with N_{mpi} , and can be attributed to self-correlation bias caused by measuring transverse sphericity in a similar rapidity region to the J/ψ . Although this bias is present in both prompt and nonprompt J/ψ measured at midrapidity, its production mechanism enhances the effects for the nonprompt case, which is clearly visible in the lower left panel of Fig. 5. In contrast, the dependence of transverse sphericity is small on both prompt and nonprompt J/ψ measured at the forward rapidity, where the S_0 -induced bias is small. Additionally, the efficiency of S_0 in categorizing J/ψ production topologically is found to be maintained when the results are calculated using the CR-BLC2 settings of PYTHIA8. Moreover, the increase of the self-normalized yield of prompt and nonprompt J/ψ with respect to multiplicity from the CR-BLC2 model is observed to very closely follow the trend from Tune 4C of PYTHIA8, such that the magnitude of the former is $\sim(10\text{--}15)\%$ ($\sim(15\text{--}25)\%$) less than the latter in the highest (lowest) multiplicity class considered.

IV. SUMMARY

In this work, we study the transverse sphericity dependence of prompt and nonprompt J/ψ production in pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8. For the first time, the topological production of J/ψ is studied with an event shape. The study shows the effect of bias and the effects of MPI by studying the topological production of J/ψ in different pseudorapidity regions. We find that the nonprompt J/ψ possesses a harder transverse momentum spectrum as compared to the prompt J/ψ . The prompt J/ψ spectra are harder for the isotropic events as compared to the jetty events. The hardness of the p_T spectra for the isotropic events, which may arise as a consequence of multijet production environments and events with significantly

large underlying event activity, in contrast to jetty events where the production of J/ψ is mostly dominated by dijet like events, is noteworthy. This suggests that, although the J/ψ is produced in the initial stages, underlying MPI plays a crucial role in shaping the p_T spectra. The effect extends to the beauty-hadron production. A similar observation is made in Ref. [52] for $\Upsilon(nS)$ production.

Moreover, the nonprompt production fraction for the jetty events dominated by the dijet structure of particle production increases with p_T and becomes higher than that of isotropic events. This identifies the dominance of the production of beauty-hadron fraction in dijet events in contrast to events with multijet environments. With enough statistics in Run 3 of LHC data taking, this property of transverse sphericity can be exploited to study the production of charmonia in jets and offer valuable insights into the underlying hadronization mechanisms from pp to Pb-Pb collisions. This study provides a crucial understanding of charm and beauty-hadron production through the study of prompt and nonprompt J/ψ with respect to different event shapes, including dijet and multijet environments, where transverse sphericity acts as a probe.

Further, the self-normalized yield of J/ψ in the forward rapidity as a function of the self-normalized charged-particle multiplicity is affected by a higher autocorrelation bias as compared to the measurement of J/ψ in midrapidity, as the multiplicity selection is performed in the forward rapidity regions. In contrast, this autocorrelation bias is enhanced in the midrapidity region for transverse sphericity selection, which is defined in the midrapidity region. This warns the scientific community to be careful while dealing with particle production at different rapidity regions and while making physics conclusions out of it.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: COMPARISON WITH EXPERIMENTAL DATA

In Fig. 6, we compare the production cross section of inclusive, prompt, and nonprompt J/ψ from PYTHIA8

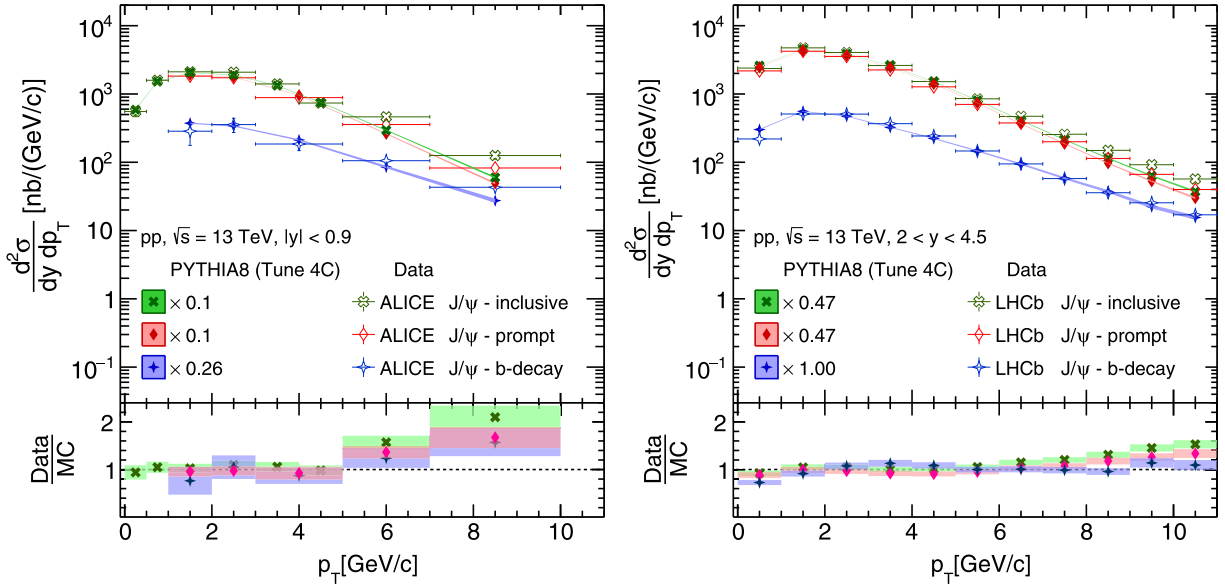


FIG. 6. Comparison of inclusive, prompt, and nonprompt J/ψ production cross section from PYTHIA8 (Tune 4C) with ALICE data [44,53] at midrapidity (left) and with LHCb data [54] at forward rapidity (right) for pp collisions at $\sqrt{s} = 13$ TeV. The bottom panel shows the ratio of experimental data to the PYTHIA8 Tune 4C results.

simulation of pp collisions at $\sqrt{s} = 13$ TeV with the corresponding data from ALICE [44,53] and LHCb [54] for the midrapidity J/ψ (left plot) and forward rapidity J/ψ (right plot), respectively. Similar to the observation in Ref. [8] and consistent with Fig. 1, the production yield of J/ψ from b -hadron decays is almost 10 times smaller than the prompt J/ψ for the lower- p_T values, while this difference gets reduced at higher values of p_T . In the case of J/ψ reconstructed at midrapidity, the results from PYTHIA8

overestimate the experimental ALICE data. Thus, scaling factors of 0.1, 0.1, and 0.26 are applied, respectively, to the PYTHIA8 data of inclusive, prompt, and nonprompt J/ψ . Similarly, for the J/ψ measurements at forward rapidity, we multiply the PYTHIA8 results of inclusive and prompt J/ψ by 0.47 to match the experimental data from LHCb. We see that the results from PYTHIA8 follow the experimental results fairly well till $p_T < 6$ TeV in both mid- and forward rapidities, to deviate thereafter for higher values of p_T .

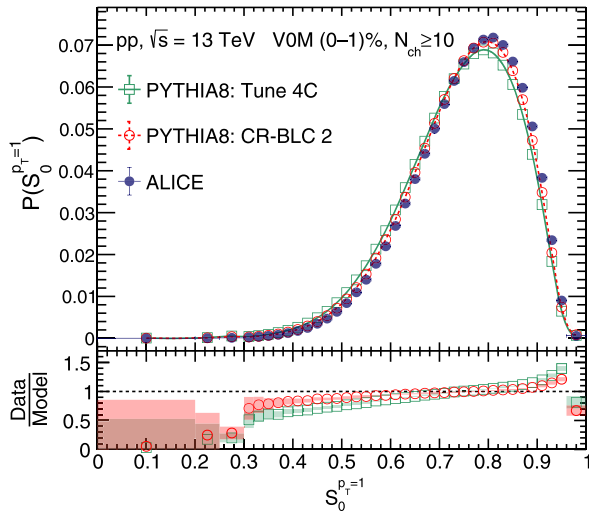


FIG. 7. Event-normalized $S_0^{p_T=1}$ distribution for the 0%–1% V0M multiplicity class for pp collisions at $\sqrt{s} = 13$ TeV from PYTHIA8 (Tune 4C and CR-BLC2 settings) and the experimental data from ALICE [17] (upper panel). Ratio in the lower panel compares ALICE experimental data to PYTHIA8 results.

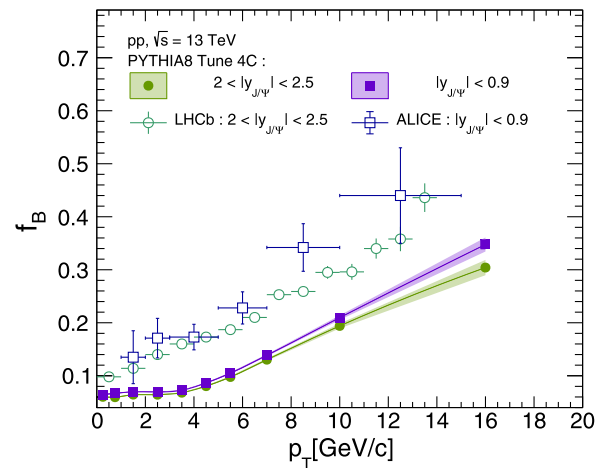


FIG. 8. Comparison of the fraction of nonprompt J/ψ produced as a function of p_T for minimum-bias events from pp collisions at $\sqrt{s} = 13$ TeV from PYTHIA8 (Tune 4C) with ALICE (midrapidity) and LHCb (forward rapidity) data [44,54].

APPENDIX B: MODEL COMPARISONS

To assess the robustness of the results against modeling uncertainties, the analysis was repeated using the PYTHIA CR-BLC2 color reconnection model built upon the Monash 2013 tune (hereafter referred to as CR-BLC2). This model is known to introduce beyond-leading-color effects, including junction formation and time-dependent reconnections, which differ significantly from the simplified color reconnection scheme used in Tune 4C [39,55]. As can be seen in the upper row of Fig. 9, the J/ψ yield with respect to p_T for the S_0 -integrated events from pp collisions at $\sqrt{s} = 13$ TeV for the CR-BLC2 model is (10–20)% lower relative to Tune 4C, for prompt J/ψ , and for both J/ψ decay channels. This difference increases as we shift our focus to nonprompt J/ψ . Although both configurations employ PYTHIA8 as the event generator, they incorporate fundamentally different treatments of nonperturbative QCD dynamics, especially in the hadronization stage. In

particular, the inclusion of junction formation and time-dilated color reconnections in the CR-BLC2 model leads to a redistribution of color flux and energy during the hadronization stage, which can affect the neutralization of heavy quark pairs and, consequently, the formation probability of bound charmonium states [39]. This yield reduction is more visible for the nonprompt J/ψ , possibly due to the enhanced sensitivity of beauty-hadron hadronization to junction-rich color reconnection effects. Such effects are absent in the simpler string-length-minimization-based color reconnection model employed in Tune 4C [39].

However, the actual important factor is the qualitative similarity one observes between the trends of p_T spectra from both PYTHIA settings. The lower plot in Fig. 9 shows the ratios of p_T spectra from jetty (or isotropic) events to that of S_0 -integrated events, for prompt and nonprompt J/ψ , for both decay channels of J/ψ from the PYTHIA CR-BLC2 model. The trends can be seen once again to be similar

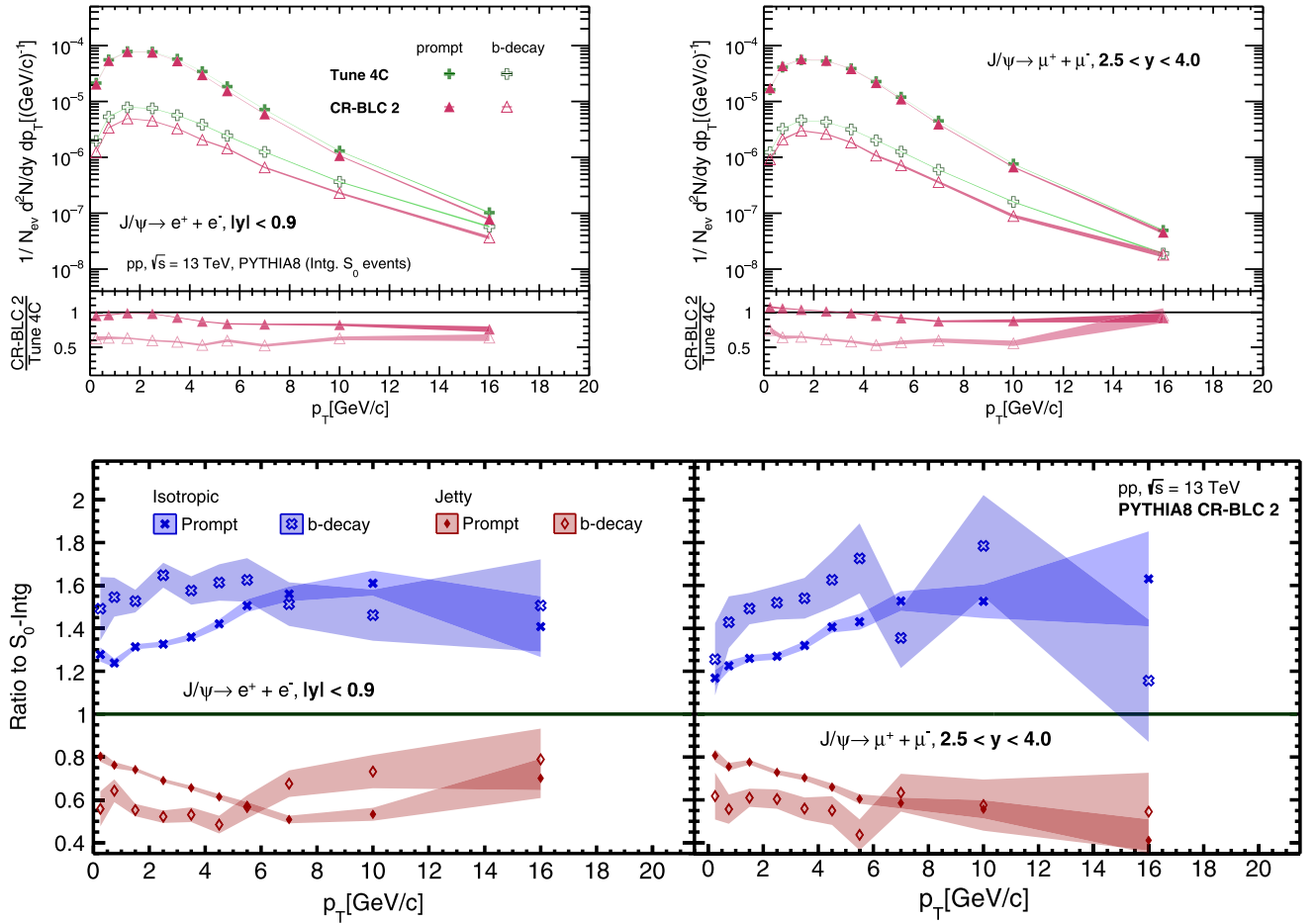


FIG. 9. Comparison of the p_T spectra of prompt and nonprompt J/ψ between Tune 4C and CR-BLC2 settings of PYTHIA8 for pp collisions at $\sqrt{s} = 13$ TeV for the integrated sphericity events is shown in the upper plots. The ratio of p_T spectra of the jetty and isotropic to S_0 -integrated events for prompt and nonprompt J/ψ for the PYTHIA8 CR-BLC2 setting is shown in the bottom panel. Error bands incorporate the estimated statistical uncertainties.

to the lower panel of Fig. 1, which corresponds to PYTHIA8 Tune 4C, which is the default setting employed for event generation throughout this work. Apart from these, the trends of all of the major observables studied using PYTHIA8

Tune 4C are seen to be qualitatively replicated by the CR-BLC2 model. The S_0 , p_T , and self-normalized $dN_{ch}/d\eta$ dependence of all these observables are observed to be well preserved despite the PYTHIA tuning/setting differences.

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