

Broad axion resonances: Clockwork ALPs at hadron collidersSrimoy Bhattacharya^{*}*School of Physics and Institute for Collider Particle Physics, University of the Witwatersrand, Johannesburg, Wits 2050, South Africa*Debajyoti Choudhury[†]*Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India*Suvam Maharana[‡]*Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India
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Scenarios with multiple pseudoscalars are interesting as they usually tend to provide a framework to naturally realize a light axion with a large decay constant which has rich applications in cosmology, especially in the context of inflation and light dark matter physics. On the other hand, from a particle physics perspective, this facilitates a solution to the strong CP problem with a low Peccei-Quinn symmetry breaking scale. One such realization is afforded within the framework of the *clockwork* mechanism where the axion can have suppressed couplings with the gluons or photons while its companion axionlike particles (ALPs) have relatively unsuppressed couplings, thereby facilitating detectability. We study a minimal clockwork model for the QCD axion invoking a Kim–Shifman–Vainshtein–Zakharov (KSVZ)-like setup and examine the visibility of its unique multi-ALP (a_n) signature at the LHC, the most sensitive channel being $pp \rightarrow a_n (+\text{additional jets})$ followed by $a_n \rightarrow \gamma\gamma$. In congruence with the astrophysical and cosmological bounds for the axion, a striking feature emerges for the case of light ALPs ($m \sim \mathcal{O}(10 \text{ GeV})$) wherein the mass splittings among the former are so small that the signal profile mimics that of a single broad resonance, or an *axion iceberg*. The scenario is found to be imminently testable by the end of LHC's Run 3 phase for an integrated luminosity of $\sim 300 \text{ fb}^{-1}$. A larger average ALP mass, on the other hand, results in multiple closely spaced peaks with a characteristic signal profile, and would be expected to be seen at the forthcoming HL-LHC. Possible additional signals are also listed.

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The strong CP problem is, perhaps, one of the strongest motivations to look for new dynamics beyond the Standard Model (SM) of particle physics. Of particular interest has

been establishing the existence of the light pseudoscalar particle—popularly called an axion or, more specifically, the QCD axion—predicted by the Peccei-Quinn (PQ) mechanism [1–3] which is, arguably, the simplest and the most elegant solution¹ to the problem proposed to date.

Some theoretical and cosmological issues such as the quality problem and domain wall formation do question the

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¹There exist interesting alternatives to the PQ scenario, most notable among them being solutions based on a massless up quark [4–8] and those invoking spontaneous CP violation (*a la* the Nelson-Barr mechanism) [9,10]. However, the $m_u = 0$ solution within the standard theory currently stands in tension with the lattice QCD result of the topological mass contribution [11]. Similarly, models based on the Nelson-Barr mechanism are also subject to several theoretical challenges [12].

mechanism's general viability, although there have been several noteworthy developments towards mitigating these issues [13–16]. Nevertheless, the axion solution offers an interesting and important domain to explore phenomenologically, more so because the axion can potentially also play the role of an ultralight dark matter candidate [17,18].

The axion is typically expected to have a very large decay constant [$f \gtrsim \mathcal{O}(10^{10} \text{ GeV})$]—with the lower limit depending on its mass—so as to be consistent with the current limits (drawn from astrophysical and cosmological observations as well as from direct search experiments) on its effective couplings with photons (as also electrons and nucleons) [19,20]. In other words, this essentially mandates that the PQ symmetry breaking scale (f_{PQ}) must be hierarchically larger than the electroweak (EW) scale. Furthermore, the QCD axion mass typically scales inversely with the decay constant and, hence, a large f also implies a very light mass for the axion. The extremely large value of f_{PQ} (and, the attendant smallness of the axion mass), thus, introduces a new hierarchy problem.

In addition, the tiny mass of the minimal QCD axion and its extremely feeble couplings render it nearly invisible to most of the current and future experimental probes, especially in the context of high-energy experiments. Consequently, recent collider searches have tended to concentrate, instead, either on axionlike particles (ALPs) that have no role in the strong CP issue [21–23], or on heavy QCD axions obtained by way of introducing new or extended confining sectors [24–29]. The latter case, although interesting in its own right, typically does not capture the cosmological attributes of a light axion, i.e., a role as a possible dark matter candidate and/or as the inflaton field in the so-called *natural inflation* scenario [30,31]. In view of this, a question naturally arises as to whether the discovery of a heavy axion would necessarily negate the existence of the light QCD axion. This aspect assumes particular significance as there are hints of several local excesses in searches for such scalar/pseudoscalar resonances [32–37] without these being robust enough to merit a discovery claim.

Scenarios with multiple aligned axions [38,39] could, in principle, address these issues without assuming a large hierarchy in the individual decay constants and yet allowing for a quasiflat direction in the field space, thereby leading to an effective decay constant $f_a \gg f_{\text{PQ}}$ for the QCD axion. Reference [40] sought to exploit this with f_{PQ} close to the EW scale and ALPs with masses $\gtrsim m_{\text{EW}}$.

In this work, we seek to present a theoretically better-motivated model that also leads to a very *distinctive* multi-ALP spectrum. With ALP masses $\lesssim m_{\text{EW}}$, this exposition would not only be complementary to the scenarios of Refs. [40,41] but would also confront features in existent data at the LHC [32–37]. Furthermore, the light QCD axion inherent to the scenario, apart from being a viable dark matter (DM) candidate, would likely be testable at the current and/or next generation of dedicated experiments. To this end, we focus on the clockwork (CW) paradigm,

originally proposed in Refs. [39,42–44], as it automatically generates hierarchical couplings or mass scales through the localization of the lightest particle in the theory space lattice defined by N fields that interact with each other through nearest-neighbor couplings with a strength characterized by an $\mathcal{O}(1)$ parameter q . The massive modes, on the other hand, are delocalized over the entire lattice.

To realize a QCD axion within such a construction, we consider a model with a clockwork sector comprising $N + 1$ complex scalars $\Phi_j (j = 0, \dots, N)$, each charged under a unique $U(1)$ symmetry. Inspired by the KSVZ model [45,46], we introduce a pair of new, $SU(2)_L$ singlet, colored Weyl fermions ($\Psi_{L,R}$), chiral under the N th group $U(1)_N$, which couple to the N th complex scalar. With only a single species of new colored fermions, the putative domain wall problem, on account of a spontaneous breaking of the discrete symmetry present in the QCD-induced potential for the axion, is well under control, and such a KSVZ-like model is free of the problems that often beset, say, a DFSZ-like [47,48] scenario.

A spontaneous breaking of the full symmetry $[U(1)^{N+1}]$ at a scale f (an analog of f_{PQ}) in the clockwork sector, along with an explicit breaking of the symmetry to a single factor $U(1)_{\text{CW}}$ due to the nearest-neighbor interactions characterized by an effective mass scale $m \ll f$, leads to one massless Nambu-Goldstone Boson (NGB) and N pseudo-NGBs in the spectrum. The chiral anomaly, then, induces a coupling of the light NGB to the gluonic topological operator ($\tilde{G}G$) with an exponentially large effective decay constant $\sim q^N f$, thanks to the localization mechanism. Below the confinement scale, this leads to a small mass for the light NGB through nonperturbative effects. Therefore, for $q > 1$ and a sizable number of CW fields, the light NGB can be identified with the QCD axion even for the fundamental SSB scale f being as low as a few TeVs. The pNGBs, on the other hand, act as ALPs with masses of the order $\sim mq$ (thereby creating a mass gap) and couple to gluons with an effective decay constant comparable to the scale f . The nonzero hypercharge of $\Psi_{L,R}$ not only allows them to decay, but also engenders loop-induced couplings of the ALPs to a pair of photons and Z 's. At a hadron collider (current or future), ALPs are dominantly produced through gluon-gluon fusion, and their most prominent, yet unique, signature would be the occurrence of multiple closely spaced resonance peaks in the $\gamma\gamma$, ZZ and $Z\gamma$ channels. The closeness in the ALP masses emerges naturally in the model as a consequence of the clockwork construction wherein the mass splitting varies as $\sim 2m/N$, (with $N \gtrsim 10$ for adequate enhancement of the axion decay constant). Understandably, for lighter ALPs ($m \lesssim 100 \text{ GeV}$), the mass splittings would be small, and in certain cases so much so that the currently operating detectors at the LHC might not be able to completely resolve the individual peaks, leading to the appearance of the entire ALP spectrum as a single broad resonance.

We argue that both signals (multiple closely spaced narrow peaks or a single anomalously broad peak) are unique to the clockwork paradigm and can be generated neither by a single, phenomenologically viable ALP candidate, nor by a generic model of multiple ALPs without invoking an *ad hoc* fine-tuning in the parameter space. Additionally, such signatures are complementary to the possibility of observing *periodic* (in invariant mass) signals in the case of relatively heavier ALPs, e.g., in the spirit of the analyses carried out in the context of the gravity model in Refs. [49–51]. Driven by this notion, we assume benchmark points with ALP masses at or below the EW scale and investigate their detection prospects in the diphoton channel at the LHC. The most interesting outcome is found in the case of the mass parameter m being small, where although the individual resonances are barely visible over the background, the significance of the conjoint signal profile, composed of multiple overlapping peaks, is projected to reach the discovery threshold even as early as by the end of the LHC’s Run-3 phase. An analysis of the diphoton final state, which is arguably the cleanest channel to probe the ALPs, is also motivated by the number of dedicated collider searches for light scalar resonances, near and below the EW scale, performed by the ATLAS [32,52] and CMS [33] Collaborations in the recent years. Thus, our study also appeals to the case for further experimental probes of light scalar or pseudoscalar particles in the diphoton channel, scanning not only for one but multiple closely spaced candidates or even in the form of a broad resonance.

Note that although the introduction of a clockwork sector does not alter or improve upon the basic features of the PQ mechanism, it does provide ways to mitigate certain issues present in the minimal QCD axion models, which in turn makes this avenue even more desirable to explore. For one, the possibility of a low scale f_{PQ} helps improve the *quality* of the axion solution against putative quantum gravity effects² [40,53]. Second, the geometric

²Since the degree of the global symmetry breaking depends on the exact manifestation of quantum gravity effects near the Planck scale, a robust estimate of the quality problem in our model, given the complexity of the global symmetry involved, is difficult to obtain. Nevertheless, under the assumption that the leading QG effect arises as a dim-5 operator of the form $(g/M_{\text{Pl}})(|\Phi_j|^2)^2\Phi_j$ in the IR, the required fine-tuning for an acceptable PQ solution can be characterized in terms of an upper limit on the coupling $|g|$, which, in a model with $f_{\text{PQ}} \sim \mathcal{O}(1 \text{ TeV})$, can be estimated to be $|g| \lesssim 10^{-12}/N^4$. For example, $N \sim 30$ implies $|g| \lesssim 10^{-18}$. Although this seems a very stringent upper limit in the absolute sense, it is nevertheless many orders of magnitude larger than what is required in the minimal models with $f_{\text{PQ}} \sim 10^{12} \text{ GeV}$, namely $|g| \lesssim 10^{-55}$ [53]. Thus, even with the aforementioned ambiguities, it can be qualitatively argued that the clockwork scenario offers some respite from the severe quality issues of the typical axion models, if not a solution. The severity may be relaxed further with the imposition of additional discrete symmetries in the UV in order to restrict lower dimensional operators. However, we desist from doing so.

progression of the light axion’s distribution over the lattice can be exploited to obtain an enhanced coupling with photons, independent of the coupling to the gluons, thereby opening a window in the parameter space that falls outside of the usual QCD band on the photon-coupling vs axion-mass plane. This aspect was studied in detail in Ref. [41].

The structure of the paper is as follows. In Sec. II, we define a minimal clockwork model for a KSVZ-like QCD axion and demonstrate how it addresses the strong *CP* problem. We identify, in Sec. III, four benchmark points of the model suitable for probing the ALPs through the gluon-fusion channel at the LHC. In Sec. IV, we discuss the corresponding signal and background profiles for $\sqrt{s} = 13 \text{ TeV}$ and an integrated luminosity $\mathcal{L} = 138 \text{ fb}^{-1}$. Section V, on the other hand, briefly describes the implications of the vectorlike quark and the heavy radial scalars present in the model. Finally, we summarize the results and conclude in Sec. VI.

II. THE MODEL

We begin by constructing a clockwork realization of the minimal QCD axion, adopting a KSVZ-like [45,46] scenario. In the ensuing, we first detail the structure and spectrum of the clockwork sector and then proceed towards describing how the KSVZ setup can be implemented in the context of the clockwork mechanism.

A. Clockwork scalars

We define the CW sector in terms of $N + 1$ complex scalars Φ_j , each charged under a global Abelian group $U(1)_j$ described by the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{CW}} &= \mathcal{L}_1 + \mathcal{L}_2, \\ \mathcal{L}_1 &\equiv \sum_{j=0}^N \left[(\partial_\mu \Phi_j^\dagger)(\partial^\mu \Phi_j) - \lambda \left(\Phi_j^\dagger \Phi_j - \frac{f^2}{2} \right)^2 \right], \\ \mathcal{L}_2 &\equiv \lambda' \Lambda^{3-q} \sum_{j=0}^{N-1} \Phi_j^\dagger \Phi_{j+1}^q + \text{H.c.}, \end{aligned} \quad (1)$$

where λ, λ' are dimensionless real constants and $\Lambda (\ll f)$ is a characteristic scale associated with $\mathcal{L}_2$ (the exponent q being, as yet, unspecified). To understand better the pieces $\mathcal{L}_{1,2}$, let us examine them in isolation. In the absence of $\mathcal{L}_2$ (i.e., in the limit $\lambda' \rightarrow 0$), the Lagrangian has a $U(1)^{N+1}$ global symmetry, the spontaneous breaking of which, at the scale f (through $\langle \Phi_j^\dagger \Phi_j \rangle = f^2/2$ for all j) would lead to $(N + 1)$ Goldstone bosons π_j , with the complex scalars being representable as

$$\Phi_j = \frac{1}{\sqrt{2}} (\phi_j + f) e^{i\pi_j/f}, \quad (2)$$

where ϕ_j are the corresponding massive scalars. Note that this breaking mechanism still retains the additional global discrete symmetry $\Phi_j \leftrightarrow \Phi_k$ inherent in $\mathcal{L}_1$.

The large global symmetry, discrete or continuous (and the attendant plethora of Goldstones in the broken phase), is neither well motivated nor desirable. Clearly, it can be broken by arbitrary terms expressing interactions between the scalars. On the other hand, at least a single $U(1)$ needs to be present³ so that its spontaneous breaking could lead to the axion. The interaction Lagrangian $\mathcal{L}_2$ serves exactly this purpose.⁴ Representing perhaps the simplest set of operators which preserve a single $U(1)$ while exhibiting the clockwork mechanism, it can be understood in terms of nearest-neighbor interaction terms⁵ that exhibit locality in a theory space and, hence, the lattice defined by the fields Φ_j (each specifying a *site* in the theory space) can be regarded as the fifth dimension.⁶ As for the exponent q in $\mathcal{L}_2$, a noninteger value would imply compositeness, thereby adding a further layer of complications. The clockwork mechanism, which will be employed in the following discussions, necessitates $q > 1$. On the other hand, for $q > 3$ one would obtain nonrenormalizable operators in $\mathcal{L}_{CW}$ implying that the explicit symmetry breaking takes place due to new dynamics operational at the heavy scale $\Lambda \gg f$ and beyond. Furthermore, $q > 3$ would also tend to destabilize the vacuum. To avoid such issues and for simplicity we consider a renormalizable theory with $1 < q \leq 3$.

³As previously mentioned, an exact global symmetry is unrealistic because even in the absence of new symmetry breaking effects one would expect the global $U(1)$ to be explicitly broken by quantum gravity effects. The Planck suppressed symmetry violating terms thus generated can, in general, upset the PQ solution through corrections to the QCD axion potential to be discussed later [53]. While the small f_{PQ} in our scenario already alleviates this so-called *quality problem*, it may be further assuaged by ensuring that the Planck suppressed operators in the theory appear only at a sufficiently high mass dimension, e.g., by considering the CW scalars to be composites of entities belonging to new confining sector(s) [54,55].

⁴The requirement $\Lambda \ll f$ at best introduces a *little* hierarchy and for $q = 2$, as we would adopt, it represents only a soft breaking of the symmetry.

⁵In general, one could also have non-nearest-neighbor interactions while keeping the CW symmetry intact, an example being a term of the form $\Phi_j^\dagger \Phi_{j+1}^{q/p} \dots \Phi_{j+p}^{q/p}$. With reference to Eq. (5), such a term adds, to the average mass scale of the pNGBs, a contribution $m_a^{(p)} \sim m(q^p/p)$ [56]. For the case $q = 2$ (which would be our primary choice as a benchmark for the collider analysis to be discussed) renormalizability necessitates $p = 1, 2$ for which the overall pNGB mass scale changes only slightly with $m_a \sim \sqrt{2}m_a^{(1)}$. The case for $q = 3$ is a little different with $m_a \sim 3m_a^{(1)}$, although the qualitative aspects of our results and conclusions in this work would be applicable just as well.

⁶A continuum theory that reproduces this theory on discretization, though, requires the introduction of certain additional features and we desist from discussing it.

With the introduction of $\mathcal{L}_2$, the erstwhile discrete symmetry is completely broken, whereas the $U(1)^{N+1}$ symmetry breaks explicitly to one combination $U(1)_{CW}$ corresponding to the generator,

$$\mathcal{Q}_{CW} = \sum_j \frac{\mathcal{Q}_j}{q^j}, \quad (3)$$

with $\mathcal{Q}_j$ being the generators (charges) corresponding to the individual $U(1)_j$'s. Consequently, only one combination of the π_j 's would now be a true Goldstone boson, while all the rest would gain masses much smaller than f . In other words, the latter are now only pNGBs. To be specific, the full spin-0 Lagrangian, in the broken phase, can be written as

$$\begin{aligned} \mathcal{L}_\Phi^{Full} = & \sum_{j=0}^N \left[\frac{1}{2} \partial_\mu \phi_j \partial^\mu \phi_j - \frac{1}{4} \lambda (\phi_j + f)^4 \right] + 2^{(1-q)/2} \lambda' \Lambda^{3-q} \\ & \times \sum_{j=0}^{N-1} (\phi_j + f)(\phi_{j+1} + f)^q \cos \frac{\pi_j - q\pi_{j+1}}{f}. \end{aligned} \quad (4)$$

The mass eigenvalues of the pseudoscalar system are given by

$$m_n^2 = \begin{cases} 0 & n = 0 \\ m^2 \left[1 + q^2 - 2q \cos \frac{n\pi}{N+1} \right] & n \neq 0 \end{cases}, \quad (5)$$

where $m^2 \equiv 2^{(1-q)/2} \lambda' \Lambda^{3-q} f^{q-1}$ (thus, $m_n^2 \ll f^2$ as was expected). The transformation relation between the unphysical basis (π_j) and the physical basis (a_n) is specified by a matrix C (viz. $a_n \equiv \sum_{j=0}^N C_{nj} \pi_j$) with elements

$$\begin{aligned} C_{0j} &= \mathcal{N}_0 q^{-j}, \\ C_{nj} &= \mathcal{N}_n \left[q \sin \frac{jn\pi}{N+1} - \sin \frac{(j+1)n\pi}{N+1} \right]. \end{aligned} \quad (6)$$

Here $\mathcal{N}_0$ and $\mathcal{N}_n$ are the normalization factors

$$\mathcal{N}_0 = \sqrt{\frac{q^2 - 1}{q^2 - q^{-2N}}}, \quad \mathcal{N}_{n>0} = \frac{m}{m_n} \sqrt{\frac{2}{N+1}}. \quad (7)$$

It is evident from Eq. (6) that while the heavy pseudoscalars are nearly delocalized over the entire lattice, the massless mode—which will subsequently assume the role of the QCD axion (a_0)—is localized towards the $j = 0$ site, i.e., it has an exponentially suppressed overlap with π_N by virtue of the clockwork mechanism. Interestingly, this feature will eventually ensure that the light axion has a hierarchically

large decay constant⁷ as compared to the heavy axions in the spectrum.

B. A KSVZ axion

Now that we have a massless pseudoscalar in the clockwork spectrum, we would like to promote it to the status of a realistic QCD axion. This is most easily carried out by embedding the clockwork sector in any one of the usual QCD axion models. Owing to the singlet nature of the CW scalars, the KSVZ construction is the simplest one. To this end, we introduce one generation of Weyl fermions Ψ_L , Ψ_R with the following nontrivial charges under the SM gauge group and the N th (global) Abelian group⁸ of the CW sector [$G_{\text{SM}} \times U(1)_N$]:

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_N$
Φ_N	1	1	0	ξ
Ψ_L	3	1	Y_Ψ	ξ_L
Ψ_R	3	1	Y_Ψ	ξ_R

with the restriction that $\xi = \xi_L - \xi_R$. This is reminiscent of the chiral PQ charges typically assigned to the new (i.e., non-SM) fermions in KSVZ models. The SM fields are assumed to be uncharged under the new $U(1)$'s. For $\xi_L \neq \xi_R$, the new singlet quark cannot have a bare mass term. On the other hand, for the aforementioned choice of $\xi = \xi_L - \xi_R$, the new fermion may now interact with the CW sector through a Yukawa term localized at the N th site of the lattice. However, with just this interaction, the new quark would be absolutely stable and, hence, phenomenologically untenable. The situation can be remedied only if it couples with the SM fermions which, for our charge assignments, can be most simply ensured when $\xi_R = 0$ (and, thus, $\xi_L = \xi$) along with an appropriate choice of Y_Ψ . To be specific, we choose $Y_\Psi = 2/3$, thereby allowing Ψ to mix with the up-type quarks. The complete set of Yukawa interactions is, thus, given by

$$\mathcal{L}_\Psi \supset -\lambda_\Psi \Phi_N \bar{\Psi}_L \Psi_R - \sum_{\alpha=1}^3 y_\Psi^{(\alpha)} \bar{Q}_L^{(\alpha)} \tilde{H} \Psi_R - \sum_{\alpha=1}^3 y_\Psi'^{(\alpha)} \Phi_N \bar{\Psi}_L u_R^{(\alpha)} + \text{H.c.}, \quad (8)$$

where $Q_L^{(\alpha)}$ are the SM quark doublets, $u_R^{(\alpha)}$ the up-type singlets, and H the SM Higgs doublet. As for the

⁷Note that the usage of the terminology “decay constant” here is different from that encountered in the context of processes such as $\pi^\pm \rightarrow l^\pm \nu$, where $\pi^\pm$ are the charged QCD pions.

⁸The chiral configuration only at the N site ensures that the color anomaly is localized at a single site which enables us to exploit the CW mechanism to generate a large effective decay constant for the axion, as we will see shortly. On the other hand, while it is possible to have the $\Psi_{L,R}$ charged nonchirally under all the additional $U(1)$ s, this only adds a layer of complexity without any qualitative changes in the phenomenology.

dimensionless couplings λ_Ψ , $y_\Psi^{(\alpha)}$, and $y_\Psi'^{(\alpha)}$, the first determines the mass of the Ψ while the other two determine its decay rate. For the sake of simplicity, we assume that $\Psi_{L,R}$ couple to only the third-generation quarks, thereby precluding sizable flavor-changing neutral currents involving the first two generations.

In addition, the CW sector can also couple with the SM Higgs through⁹

$$\mathcal{L}_{\Phi-H} = -\lambda_{\Phi H} \sum_{j=0}^N \Phi_j^\dagger \Phi_j H^\dagger H. \quad (9)$$

We will come back to reviewing the dynamics of the heavy radial scalars and the quarks, along with the flavor constraints on the Yukawa couplings, in a later section. Alluding to the core objective of the study, we first describe the low-energy physics of the pseudoscalars in the discussions below.

After SSB in the clockwork sector, we have

$$\mathcal{L}_\Psi \supset -\frac{1}{\sqrt{2}} \lambda_\Psi (\phi_N + f) e^{i\xi\pi_N/f} \bar{\Psi}_L \Psi_R - y_\Psi \bar{Q}_L^{(3)} \tilde{H} \Psi_R - \frac{1}{\sqrt{2}} y_\Psi' (\phi_N + f) e^{i\xi\pi_N/f} \bar{\Psi}_L u_R^{(3)} + \text{H.c.} \quad (10)$$

A rephasing of the new fermions with respect to the Goldstone field π_N leads to the following interaction terms with the SM gauge bosons via the chiral anomaly [57] (see Appendix A for a discussion)

$$\mathcal{L}_{\pi VV} = -g_{\pi GG} \pi_N G^{A\mu\nu} \tilde{G}_{\mu\nu}^A - g_{\pi BB} \pi_N B^{\mu\nu} \tilde{B}_{\mu\nu}, \quad (11)$$

with the coefficients given by

$$|g_{\pi GG}| = \frac{g_s^2}{32\pi^2 f_{\text{eff}}}, \quad |g_{\pi BB}| = \frac{2N_c g'^2 Y_\Psi^2}{32\pi^2 f_{\text{eff}}}, \quad (12)$$

and an effective scale defined through $f_{\text{eff}} \equiv f/|\xi|$. Note that while f continues to determine the masses of the Ψ and the Φ_N , it is f_{eff} that encapsulates the pNGB decay constants.

1. The Peccei-Quinn mechanism

In order for the light pseudoscalar to be the quintessential QCD axion, it must offer a solution to the strong CP problem. Although Ref. [41] already has a brief discussion in this context, we outline here, for completeness, how the clockwork Lagrangian can be consistent with the usual Peccei-Quinn mechanism. The foremost requirement for

⁹For simplicity, we assume uniform value of the coupling $\lambda_{\Phi-H}$ for all j . Relaxing this only adds layers of complications without any qualitative change.

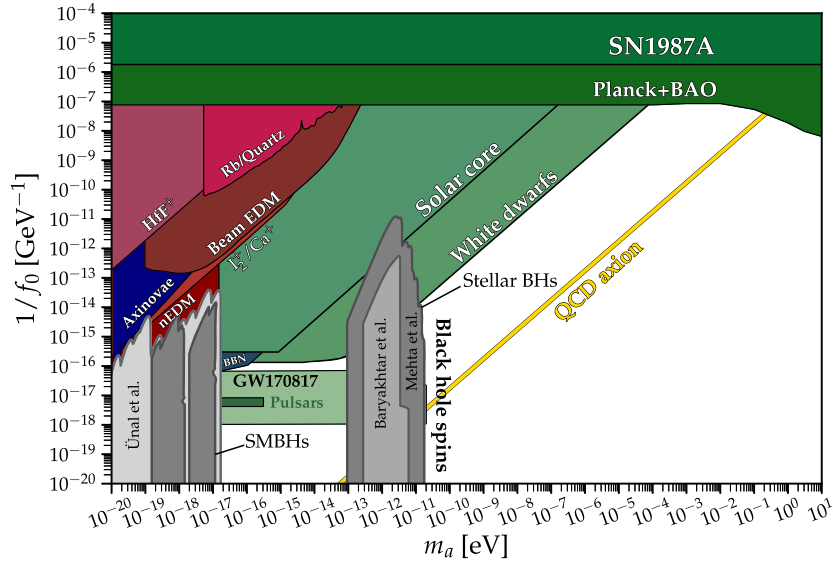


FIG. 1. Limits on the effective decay constant f_0 of the light QCD axion vs its mass. Plot adapted from Ref. [20].

the mechanism is the presence of a global symmetry (at least an accidental one) in the theory. In our case, this symmetry would correspond to the residual clockwork symmetry described in the preceding section [see Eq. (3)]. Now, in Eq. (11), we can perform a shift in the π_N field so as to exactly cancel the CP violating topological term $-\tilde{\theta}(\alpha_S/8\pi)\tilde{G}^{\mu\nu}G_{\mu\nu}$. This, however, is a true cancellation, and a potential solution to the strong CP problem, only when π_N does not acquire a nonzero vacuum expectation value (VEV) after employing the constant shift. It is important to note that the explicit breaking of the $[U(1)]^{N+1}$ symmetry in the clockwork sector, courtesy of the nearest-neighbor interactions, does not generate a nonzero VEV for the pseudoscalar fields thanks to the residual shift symmetry $U(1)_{CW}$ [58]. The only other source from where π_N may acquire a nontrivial potential are nonperturbative effects below the QCD confinement scale, leading to an effective potential of the form [59,60]

$$V(\pi_N) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\pi_N}{2f_{\text{eff}}}\right)} \quad (13)$$

where f_π and m_π are the pion decay constant and mass, respectively, and $m_{u,d}$ are the masses of the light SM quarks u and d . This shows that π_N does not, indeed, acquire a nonzero VEV.¹⁰ Expanding the potential in powers of π_N , the quadratic term engenders a mass correction to π_N

¹⁰Naively, the potential would have a degenerate set of minima at $\langle \pi_N \rangle = 2n\pi f_{\text{eff}}$ for integral values of n . However, such nonzero VEVs are unphysical as they can always be shifted to zero by the residual symmetry transformation $\pi_N \rightarrow \pi_N + 2n\pi f_{\text{eff}}$ as a result of the 2π periodicity of the potential.

$$\Delta m_{\pi_N}^2 = \frac{f_\pi^2 m_\pi^2}{f_{\text{eff}}^2} \frac{m_u m_d}{(m_u + m_d)^2}. \quad (14)$$

Going to the physical basis of the CW axions, this generates a small mass for the lightest state, i.e., the KSVZ QCD axion, given by

$$m_{a_0} \simeq \frac{f_\pi m_\pi}{f_{\text{eff}} q^N} \frac{\sqrt{m_u m_d}}{(m_u + m_d)}, \quad (15)$$

and also leads to mass corrections for the heavy modes, albeit orders of magnitude smaller than $m_{n>0}$. Clearly, the light axion has a mass that is exponentially suppressed by the factor q^N as a result of the clockwork localization. Consequently, one need not assume a very large scale f in this scenario to accommodate a QCD axion (see Fig. 1). For example, with $q = 2$ and $N \gtrsim 15$ one obtains sub-eV masses for the a_0 even for $f \lesssim 1$ TeV (equivalently, f_{eff} in the few hundred GeVs range). From Eq. (11), it is apparent that such an additional suppression also appears in the axion's coupling with gluons and the electroweak bosons above the confinement scale in the form of the effective decay constant $f_0 = \mathcal{N}_0^{-1} q^N f_{\text{eff}} \approx q^N f_{\text{eff}}$. Therefore, with a nominal choice of values for the CW parameters q and N , and a PQ-esque scale f_{eff} which is not too far from the EW scale, we can have a viable QCD axion that is well within the current experimental bounds on f_0 with respect to the light axion mass [20,61]. These constraints are summarized in Fig. 1 where the yellow line reflects the mass vs f_0 relation as given in Eq. (15) for a QCD axion. The plot shows the limits from various experimental upper bounds on the electric dipole moment of the neutron and the electron [62–64] as well as from atomic and molecular transition experiments [65,66]. On the other hand, the observational limits shown include

those obtained from axion star decays (assuming postinflationary SSB and axion star formation) [67], Big bang nucleosynthesis (constraining axion DM) [68], black hole spins [69–71], pulsars and solar core [72], binary neutron star gravitational wave [73], cosmic microwave background (CMB) (Planck) and baryon acoustic oscillation [74], SN1987a [75,76], and white dwarfs [77]. Additionally, one also finds constraints from neutron star cooling as discussed in Refs. [78,79].

It is perhaps prudent to remark at this juncture about the possible nature and implications of the topological defects that may arise in the multi-axion configuration of the clockwork sector. The spontaneous breaking of the $U(1)^{N+1}$ symmetry at a temperature $T \sim f \gg \Lambda_{\text{QCD}}$ (post-inflation and reheating) is expected to lead to the formation of $N+1$ cosmic strings bound to one another through domain walls [80–82]. Such a string-wall network would, in principle, tend to radiate light axions as well as ALPs (as also gravitational waves [81,82]) during the course of its cosmological evolution towards the epoch of the QCD phase transition.¹¹ In a canonical framework, these emissions are estimated to scale as f^2 and, therefore, constitute a large fraction of the axion abundance in the usual QCD axion cosmology where $f \sim f_0 \gtrsim 10^9$ GeV. In the present context, though, the clockwork mechanism generates an exponential separation between the SSB scale f and the effective decay constant of the light axion f_0 , and such contributions to the axion abundance are exceedingly suppressed for $f \sim \mathcal{O}(\text{TeV})$. At the epoch of the QCD phase transition, the induced potential of Eq. (13) defines a unique vacuum for the system associated with a domain wall number $N_{DW} = 1$ —a direct consequence of invoking only a single generation of the extra $SU(2)$ singlet colored fermion Ψ . Similar to the case of a single axion [83–85], such a potential destabilizes the string-wall network and leads to its eventual collapse,¹² with its energy being released primarily in the form of axions and ALPs. This contribution to the axion abundance, too, can be estimated to be insignificant, with the axion emission being suppressed by the factor $\sim (m_{a_0}/m)^3$ [80].

While a certain abundance of the axion as dark matter is expected to be generated via radiation and collapse of the topological defects in the clockwork system, a robust estimate of the same can only be obtained from a dedicated numerical simulation which is extremely complicated to perform for a multi-axion setup, especially for $N \gg \mathcal{O}(1)$. In the absence of such analyses, we admit to a reasonable assertion, based on the aforementioned arguments, that for

the SSB scale of interest $f \sim \mathcal{O}(\text{TeV}) \ll f_0$ the dominant contribution to the axion DM abundance emanates from the misalignment mechanism [87], namely,

$$\Omega h^2 \simeq 0.2 \left(\frac{f_0}{10^{11} \text{ GeV}} \right)^{7/6} \left(\frac{\theta_0^2}{\pi^2/3} \right) \quad (16)$$

where θ_0 is the average initial misalignment angle of the QCD axion. This implies that for the natural choice of $\theta_0 \sim \mathcal{O}(1)$ in Eq. (16), the constraint from overclosure demands $f_0 \lesssim 10^{11}$ GeV. The experimental and observational limits demand, on the other hand, $f_0 \gtrsim 10^9$ GeV. The allowed window is then wide enough and comparable with the one usually encountered for the traditional QCD axion. Indeed, this is why it is also the target region for several cavity haloscope experiments such as ADMX, CAPP, HAYSTAC, QUAX, etc. (see, e.g., Fig. 90.5 of the PDG review on axions [19]).

In the spirit of these arguments, we would be assuming, in the discussions to follow, benchmarks that are consistent with the above mentioned bounds on f_0 .

C. Axion physics above the QCD confinement scale

Concerned with the physics observable at high-energy colliders, it suffices to consider the theory at energy scales below the EW symmetry breaking scale but significantly above the QCD confinement scale. At these scales, the axion Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{\pi v v} = & -g_{\pi g g} \pi_N G^{A\mu\nu} \tilde{G}_{\mu\nu}^A - g_{\pi \gamma \gamma} \pi_N F^{\mu\nu} \tilde{F}_{\mu\nu} \\ & - g_{\pi \gamma Z} \pi_N F^{\mu\nu} \tilde{Z}_{\mu\nu} - g_{\pi Z Z} \pi_N Z^{\mu\nu} \tilde{Z}_{\mu\nu}. \end{aligned} \quad (17)$$

The coefficients in this case are given by

$$\begin{aligned} g_{\pi g g} &= \frac{\alpha_s}{8\pi f_{\text{eff}}}, & g_{\pi \gamma Z} &= \frac{-4N_c s_w^2 \alpha_{\text{EM}} Y_{\Psi}^2}{8\pi f_{\text{eff}} s_w c_w}, \\ g_{\pi \gamma \gamma} &= \frac{2N_c \alpha_{\text{EM}} Y_{\Psi}^2}{8\pi f_{\text{eff}}}, & g_{\pi Z Z} &= \frac{2N_c s_w^4 \alpha_{\text{EM}} Y_{\Psi}^2}{8\pi f_{\text{eff}} s_w^2 c_w^2}, \end{aligned}$$

where $s_w \equiv \sin \theta_w$ and $c_w \equiv \cos \theta_w$ with θ_w being the Weinberg angle. For the mass eigenstates, the couplings are simply scaled by the mixing parameters, viz.

$$g_{agg}^{(n)} = g_{gg}(f_{\text{eff}} \rightarrow f_n) \quad (18)$$

where $f_n \equiv f_{\text{eff}}/C_{nN}$ denotes the effective decay constant of the n th physical axion. Using Eq. (6), it is then readily apparent that the decay constants of the heavy pseudoscalars are hierarchically smaller than that of the light axion. For a SSB scale $f \sim \mathcal{O}(\text{TeV})$, this engenders significantly enhanced couplings of the heavy axions with gluons and the electroweak bosons as compared with those associated with typical light ALP candidates. This enhancement, in

¹¹The ALPs, owing to their relatively small decay constant, decay promptly once they are produced.

¹²Thus, apart from the obvious simplicity, the choice of a single species of the singlet heavy quark possesses the important feature of alleviating the domain wall problem [86] typical to axion models.

principle, can facilitate resonant production of the heavy CW axions at hadron colliders, which is going to be the main subject of discussion in the ensuing sections.

III. CLOCKWORK AXIONS AT COLLIDERS

As is already described above, the (light) axion, when compatible with the constraints from astrophysical and low-energy experiments [88], is nearly invisible at high-energy collider experiments. What seems promising, instead, is the prospect of probing the heavy pseudoscalars at the LHC and its forthcoming high-luminosity upgrade (HL-LHC). In this case, the production would be dominated by the gluon-fusion channel with the hadronic cross section given by

$$\sigma(pp \rightarrow a_n) = K_\sigma^0 \frac{d\mathcal{L}_{gg}}{d\hat{s}} \bigg|_{\hat{s}=m_n^2} \frac{\pi^2}{8m_n} \hat{\Gamma}_{gg}^{(n)}, \quad (19)$$

where $d\mathcal{L}_{gg}/d\hat{s}$ is the gluon-gluon luminosity and the K factor K_σ^0 encapsulates the higher order QCD corrections. Using the MSTW2008nnlo68 parton densities, Ref. [89] estimates $K_\sigma^0 \approx 3.7$. (including the full next-to-next-to leading order (NNLO) and approximate N³LO corrections [90–93]) over the pseudoscalar mass range 40–125 GeV. In the absence of a full computation of the QCD corrections, we make the reasonable extrapolation that $K_\sigma^0 \sim 3.7$ for the mass range 10–125 GeV. For ALP masses beyond 150 GeV we assume a more conservative $K_\sigma^0 = 2.5$ [93].

The leading (two-body) decay channels and the corresponding widths for the ALPs are

$$\begin{aligned} \Gamma(a_n \rightarrow gg) &\equiv K_{gg} \hat{\Gamma}_{gg}^{(n)} = K_{gg} |C_{nNg}(x_n)|^2 \frac{\alpha_S^2}{32\pi^3} \frac{m_n^3}{f_{\text{eff}}^2} \\ \Gamma(a_n \rightarrow \gamma\gamma) &\equiv \Gamma_{\gamma\gamma}^{(n)} = 9Y_\Psi^4 |C_{nNg}(x_n)|^2 \frac{\alpha_{\text{EM}}^2}{64\pi^3} \frac{m_n^3}{f_{\text{eff}}^2} \\ \Gamma(a_n \rightarrow Z\gamma) &\equiv \Gamma_{Z\gamma}^{(n)} = 9Y_\Psi^4 |C_{nNg}(x_n)|^2 \frac{\alpha_{\text{EM}}^2 \tan^2 \theta_w}{8\pi^3} \frac{(m_n^2 - m_Z^2)^3}{m_n^3 f_{\text{eff}}^2} \\ \Gamma(a_n \rightarrow ZZ) &\equiv \Gamma_{ZZ}^{(n)} = 9Y_\Psi^4 |C_{nNg}(x_n)|^2 \frac{\alpha_{\text{EM}}^2 \tan^4 \theta_w}{64\pi^3} \frac{(m_n^2 - 4m_Z^2)^{3/2}}{f_{\text{eff}}^2}, \end{aligned} \quad (20)$$

where $x_n \equiv 4m_\Psi^2/m_n^2$ and the loop-integral $g(x)$ is defined as

$$g(x) \equiv \begin{cases} x[\sin^{-1}(x^{-1/2})]^2 & x \geq 1 \\ \frac{-x}{4} \left[\ln \frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - i\pi \right]^2 & x < 1. \end{cases} \quad (21)$$

In the expression for $\Gamma(a_n \rightarrow gg)$, the QCD correction amounts to $K_{gg} = 2.1$ [94]. Clearly, the ALPs decay dominantly to two gluons followed by decays to photons (and to $Z\gamma$ and ZZ depending on kinematic feasibility). It is easy to infer—within the narrow width approximation—that the dijet cross section would exceed the diphoton one by a factor of¹³ $[2/(9Y_\Psi^4)]K_{gg}(\alpha_S^2/\alpha_{\text{EM}}^2)$. However, as far as detection at hadron colliders is concerned, the diphoton final state, understandably, offers better sensitivity as compared to the dijet channel. We will, therefore, consider only the processes $pp \rightarrow a_n \rightarrow \gamma\gamma$ for the case in hand.

The diphoton final state has been extensively studied in the literature in the context of spin-0 and spin-2 resonances, with dedicated searches by the ATLAS and CMS Collaborations spanning almost the entire range of masses accessible with current sensitivities. With measurements

thus far reporting, at best, only marginal excess of events over the SM background, the most recent analyses from ATLAS [32,95,96] and CMS [33,97] place the strongest limits to date on the $pp \rightarrow \gamma\gamma$ fiducial cross section for spin-0 resonances over a wide mass bracket ranging from nearly 10 GeV to a few TeVs. Being optimistic that some new physics indeed exists beyond the SM, this broadly implies two possibilities—first, that the energy scale of new physics lies beyond the reach of the LHC and, second, that new dynamics exist within the LHC’s energy reach, albeit with feeble interaction strengths with the SM so as to be inaccessible with the current luminosity reach. In the following, we choose the latter viewpoint and explore viable scenarios where the diphoton cross sections for the CW pseudoscalar spectrum could be very close to the current exclusion limits at the LHC obtained from the Run-2 data, such that a sizable enhancement in the signal significance can be achieved by the end of the ongoing Run-3 phase. This is interesting because even if a 5σ discovery looks improbable at the LHC, such enhancements could hint at a potential discovery at the HL-LHC taking into account the projected integrated luminosity of $\sim 3000 \text{ fb}^{-1}$, i.e., nearly 20 times the luminosity achieved by the end of LHC Run 2 ($\sim 138 \text{ fb}^{-1}$). We consider for our study three benchmark points categorized by the ALP

¹³Note that $K_{\gamma\gamma} \approx 1$.

masses¹⁴ in the range 10–30 GeV, 35–105 GeV (with two subcategories in this case) and 150–450 GeV, respectively, and have as guiding references the ATLAS analyses carried out in Refs. [32,95,96].

For each of the benchmark points, we choose $\xi = 3$ and $q = 2$ (as the middle point of the theoretically allowed range) and also assume $Y_\Psi = 2/3$ so that the Ψ may mix with the top quark. The other particulars are as follows:

Benchmark I: We assume the parameter values $N = 28$, $m = 10$ GeV and $f = 1000$ GeV in the CW sector. The resulting light axion in this case has a decay constant $f_0 \approx 9 \times 10^{10}$ GeV and a mass $m_{a_0} \approx 6.4 \times 10^{-5}$ eV. The corresponding ALP masses span the range ~ 10 –30 GeV. This particular choice of the CW parameters, especially the sizable number of particles N , is also motivated by the fact that it is compatible with the light axion being a DM candidate¹⁵ (with an acceptable relic density and consistency with cosmological observations)—as discussed in Sec. II B 1—for $\mathcal{O}(1)$ values of the misalignment angle.¹⁶ With these choices in place, we can determine the diphoton cross section in the narrow-width approximation. Figure 2 (top) shows the cross sections for each of the ALPs (integrated over the full final state phase space) computed for a center of mass energy $\sqrt{s} = 13$ TeV using the NNPDF2.3LO PDF set [101]. Note that the cross section peaks for an intermediate mode of the spectrum, the exact identity of which depends on the periodic variation of the couplings due to the clockwork mixing as well as the energy dependence of the parton-parton luminosity. Figure 2 (bottom), on the other hand, shows Δm_n (the mass splitting between consecutive CW modes) along the full spectrum, with the average mass splitting being given by $\Delta m \sim 2m/N$. It is interesting to note that, with the consecutive differences $\Delta m_n \lesssim 1$ GeV over the entire extent of the ALP spectrum, the splittings are comparable with the prevailing detector resolution. Consequently, the individual resonances may not be entirely resolved and the events corresponding to the full spectrum (subject to the assumed bin size) may even appear as a single broad resonance¹⁷ within an envelope of mass width $\sim 2m$ in the diphoton invariant mass distribution.

Benchmark II(a): The parameter configuration in the CW sector for this case is— $m = 35$ GeV, $N = 28$ and $f = 1000$ GeV. The light axion has the same decay constant and mass as in benchmark I and, thus, is still a potential DM candidate, while the ALP masses now range from 35 to 105 GeV. The corresponding diphoton cross

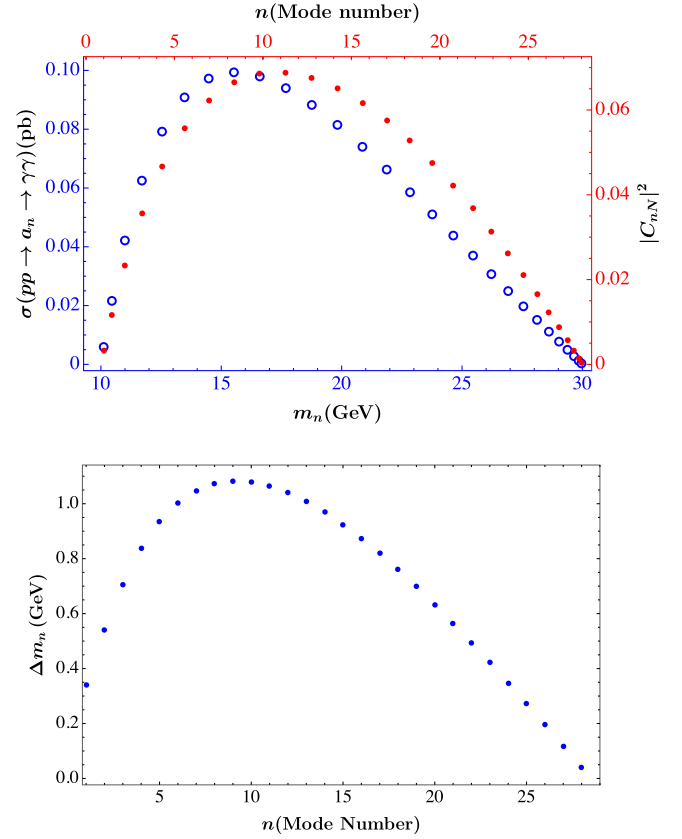


FIG. 2. Top: masses and couplings for individual ALPs and corresponding diphoton cross sections for benchmark I. The vertical axis on the right shows the extent of the field π_N contained in the mass eigenstates a_n . Bottom: mass splittings between consecutive ALPs for benchmark I.

sections are shown in Fig. 3. Because of an increase in the value of the parameter m (by a factor of 3.5) the mass splittings in this case, readily inferred from Eq. (5), are nearly triple in value compared to that in the preceding case. Therefore, in stark contrast with benchmark I, one expects to discern at least a few individual peaks in the invariant mass distribution of the signal events. We will see that this is indeed the case.

Benchmark II(b): Now, we assume a larger set of CW scalars, namely, $N = 40$, while keeping the rest of the parameters the same as in II(a). Then, from Eq. (15) it is clear that the light axion mass lowers down to a value $m_{a_0} \sim 1.5 \times 10^{-8}$ eV and the effective decay constant increases to $f_0 \approx 4 \times 10^{14}$ GeV. Note that an enhancement in f_0 of this magnitude is incompatible with the DM constraint unless $\theta_0 \ll \mathcal{O}(1)$. Although certain cosmological scenarios could exist where a small initial misalignment angle seems plausible (see Ref. [87] for a discussion), we refrain from delving into those details and, from a strictly phenomenological standpoint of studying different ALP signatures at the colliders, assume a small enough value for θ_0 such that the axion relic density is acceptable. The mass splittings in

¹⁴While we do not consider ALP masses below 10 GeV, collider searches for lighter axions in general have been studied in Refs. [98,99].

¹⁵The notion of the KSVZ axion as dark matter has also been explored in Ref. [100], albeit in the traditional one scalar setup.

¹⁶See Figs. 4 and 5 in Ref. [80] for a summary of the various cosmological constraints on the CW parameter space.

¹⁷This was also hinted at in Ref. [40].

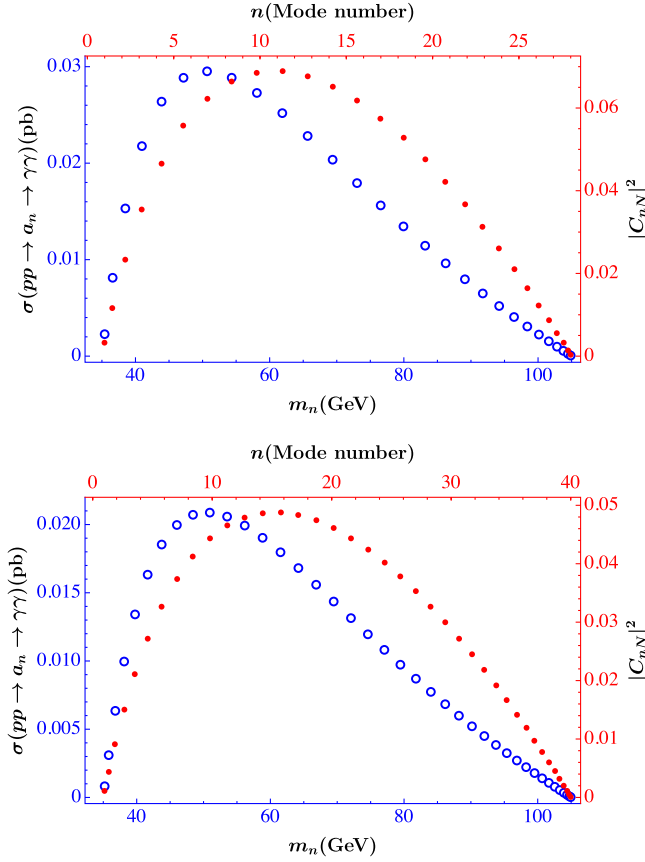


FIG. 3. Masses and couplings for individual ALPs and corresponding diphoton cross sections. Top: benchmark II(a) and (bottom) benchmark II(b).

this case fall in the range ~ 0.5 – 2.7 GeV—which is smaller than what we obtained in benchmark II(a) but not small enough to be completely unresolved at the detector. Consequently, the distribution of signal events, as a function of the diphoton invariant mass, is expected to be an undulating one.

Benchmark III: Exploring ALPs heavier than the SM Higgs, we consider $m = 150$ GeV, $N = 40$ and $f = 1600$ GeV. Because of the increase in the SSB scale f , the QCD axion mass is now even slightly lower than that in benchmark II(b), namely, $m_{a_0} \approx 9.7 \times 10^{-9}$ eV with $f_0 \approx 6 \times 10^{14}$ GeV. Naturally, the arguments pertaining to DM abundance as presented for benchmark II(b) are also applicable here. As before, Fig. 4 shows the diphoton cross sections for the ALP spectrum as well as the characteristic mass splittings.

With the model parameters defined, we now simulate the diphoton signal events and compare with the corresponding SM backgrounds for the 13 TeV LHC. We write the Feynman rules of the models in Feynrules [102] and generate UFO files that are then imported in the MadGraph5 [103] event generator for a Monte Carlo simulation of the signal. Further, PYTHIA8 [104,105] is

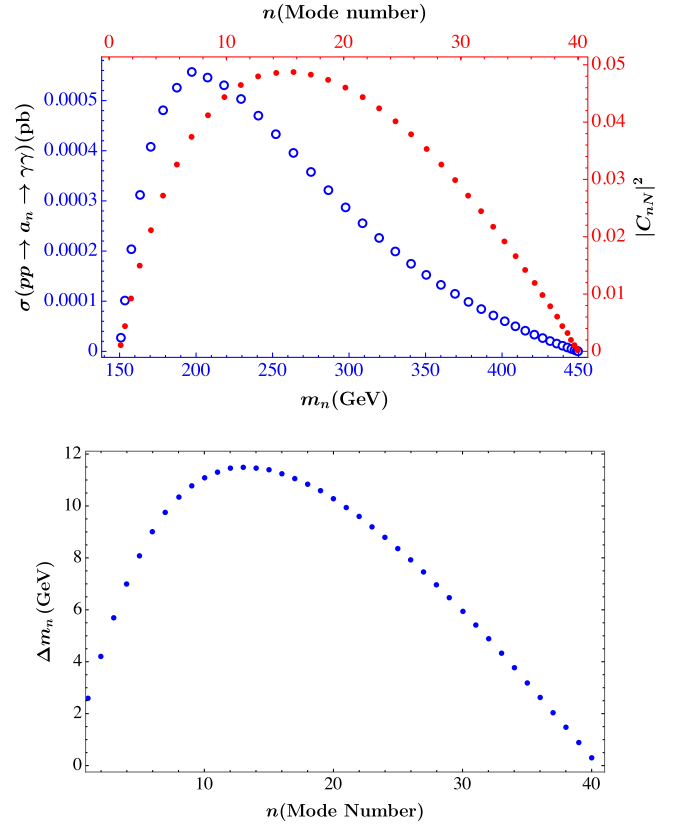


FIG. 4. Diphoton cross sections and mass splittings for benchmark III.

used for parton showering and hadronization. Final state objects at the detector level are reconstructed using the fast simulation tool Delphes [106]. Within Delphes we choose to simulate the detector effects using the ATLAS detector card for the entirety of our study.

The SM diphoton (with and without jets) background at the LHC has been very well studied by both the ATLAS and CMS Collaborations, not only in the context of the SM Higgs, but also for other exotic resonances, light and heavy. Indeed, there have been occasional reports of excesses, only to largely vanish on account of an even more careful recalibration of the backgrounds. We would be largely using the ATLAS analyses to estimate the backgrounds and, by extension, use the same kinematical restrictions to estimate the signal strength as well.

IV. SIGNAL AND BACKGROUND PROFILES

While we would be concentrating on ALPs being produced in gluon fusion and decaying into a diphoton pair, note that the exact final state would be dependent on their average mass. In particular, a very light ALP, produced sans any accompanying high- p_T particle, would result in a pair of relatively soft photons that would, typically, fail trigger requirements. In other words, a minimal number of high- p_T entities must be present,

and thus, both signal and background estimations would need to be done accordingly.

A. Light ALPs

For benchmark point I, the ALP masses are in the 10–30 GeV range, and the final state photons need to be boosted to overcome the detector’s trigger level energy threshold. To achieve this, we follow the strategy employed in the ATLAS search [32] (and also Ref. [89]) for low mass diphoton resonances and consider a diphoton final state with up to two additional jets. To simulate the signal events, we first generate¹⁸ $pp \rightarrow a_n + 0/1/2$ partons at the leading order (LO), using MadGraph5, for $\sqrt{s} = 13$ TeV with the NNPDF2.3LO PDF set, followed by the two-body decay of the on-shell ALPs.¹⁹ This is followed by parton showering and hadronization via PYTHIA8 [104,105]. Matching and merging of the matrix elements (including the avoidance of overcounting) for showering is automated using the MLM matching scheme.

As mentioned previously, for a detector level simulation of the showered events we use the Delphes tool. The following summarizes the criteria used to identify and isolate final state objects.

Jets: Within Delphes, jets are reconstructed using the FastJet package [108]. Jet clustering is performed using the anti- k_T algorithm [109] with the jet cone radius parameter R chosen to be 0.4. To be consonant with the ATLAS analyses, we require the minimum transverse momentum of jets (p_{Tj}) to be 20 GeV and its pseudorapidity to satisfy $|\eta_j| < 2.5$. Finally, any two jets must be separated by $\Delta R(j, j) > 0.7$ where $(\Delta R) \equiv [(\Delta\eta)^2 + (\Delta\phi)^2]^{1/2}$ is the separation in the pseudorapidity-azimuthal angle plane.

Photons: For identification and isolation of photons, a cone size of $\Delta R = 0.2$ around the photon candidate is considered. Denoting the ratio of the sum of transverse momenta of isolated objects (tracks, calorimeter towers, etc.) to the candidate’s transverse momentum as p_T^{ratio} , it is demanded that $p_T^{\text{ratio}} < 0.05$. Also incorporated is a p_T -dependent photon identification efficiency (following the ATLAS analysis of Ref. [32]) that ranges from $\approx 70\%$ at $p_T = 22$ GeV to $\approx 90\%$ for $p_T > 50$ GeV.

In addition, we ensure angular separation between photons or jets by demanding that $\Delta R(\gamma, \gamma) > 0.2$ and $\Delta R(\gamma, j) > 0.4$. Finally, we demand events with at least two photons and at least a jet in the final state by imposing $N_\gamma \geq 2$ and $N_j \geq 1$. The above conditions are summarized in Table I for quick reference.

¹⁸Understandably, this is overwhelmingly dominated by the subprocesses wherein the final state partons are gluons.

¹⁹Given that the ALPs are spinless, there are no nontrivial spin correlations. Furthermore, given that they are very narrow, there is no real loss of information or accuracy in the neglect of possible off-shell effects, as can also be confirmed by the use of tools such as the Madspin [107] module.

TABLE I. Acceptance cuts for the final state objects in benchmarks I and II [32].

Channel: $pp \rightarrow a_n + 0/1/2\text{jets}$	$a_n \rightarrow \gamma\gamma$
Acceptance cuts	
Photon identification	
$\Delta R = 0.2, p_T > 0.5 \text{ GeV}, p_T^{\text{ratio}}(\gamma) < 0.05$	
Jet identification	
$\Delta R = 0.4 \text{ (anti-}k_T\text{)}, p_T^j > 20 \text{ GeV}, \eta_j < 2.5$	
Isolation	
$\Delta R(\gamma, \gamma) > 0.2, \Delta R(\gamma, j) > 0.4, \Delta R(j, j) > 0.7$	
$N_\gamma \geq 2, N_j \geq 1$	

As for the QCD corrections to the process, within the narrow width approximation, we may factorize these separately for the production and the subsequent decay. For such masses of the ALPs, the QCD corrections to the production cross section can largely be summarized in terms of K factors, with $K_\sigma^1 \sim K_\sigma^2 \simeq 2$ [110] where the superscript indicates the number of jets in the final state. As for the decay, while the correction to the photonic branching fraction is small, that to the gluonic one is substantial, viz. $K_{gg} = 2.1$ [94], and serves to scale the diphoton branching fraction (since $\text{BR}_{\gamma\gamma} \approx \Gamma_{\gamma\gamma}/\Gamma_{gg}$). For the effective K factor, then, $K_\sigma^{1,2}/K_{gg} \approx 1$ for the two benchmark points with light ALPs.

Using MadAnalysis5 [111] to analyze the signal events following the acceptance cuts, we order the photons and the jets (wherever applicable) in terms of their p_T and present, in Fig. 5 (top), the corresponding distributions for an integrated luminosity of $\mathcal{L} = 138 \text{ fb}^{-1}$. In this, we use a bin size of 1 GeV, as in the ATLAS analysis [32]. While the sharp edges (at 20 GeV) in the jet²⁰ p_T distributions are but reflections of the acceptance cut imposed, the fast fall off at higher p_T s is characteristic of QCD radiation. The positions of the peaks are just caused by an interplay of the two effects. As for the photons, the decay of an ALP, in its rest frame, is isotropic with each daughter photon having an energy equaling half the ALP mass. This distribution, of course, has to be convoluted with the p_T of the ALP itself (as counterbalancing the jet p_T s). And, finally, there are additional convolutions with both the ALP mass spectrum and the corresponding production cross sections.

Of particular interest is the diphoton invariant mass distribution [Fig. 5 (bottom)] which encompasses the contributions of the entire spectrum of the 28 ALPs. It is instructive to note the difference with the total production cross section [as in Fig. 2 (top)] which shows a maximum for $m_a \approx 15$ GeV, owing, as we have discussed, to a

²⁰Note that the events with 2 jets are only a subset of the accepted set of events.

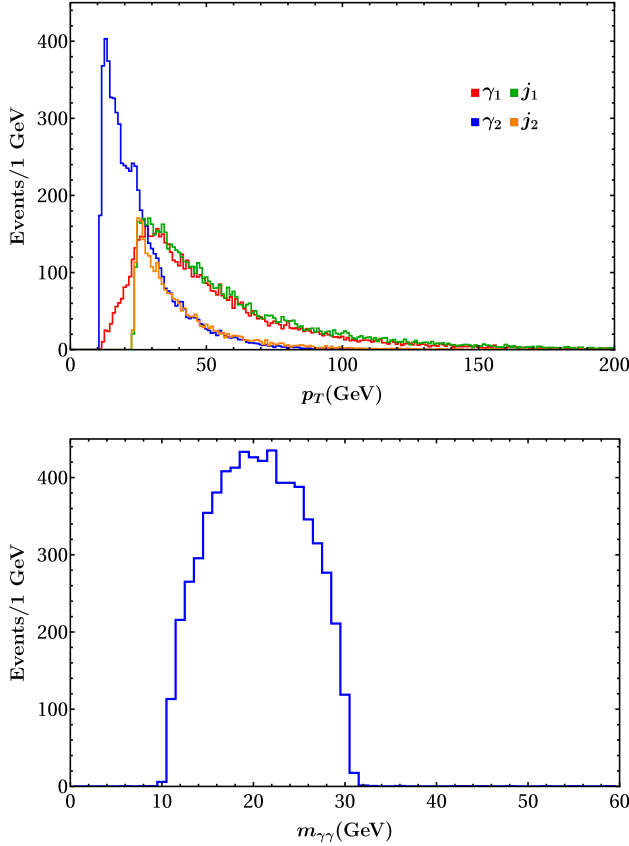


FIG. 5. BP-I: event distributions, after applying acceptance cuts alone, vide. Table I. Top: p_T distributions for the leading photon (γ_1), subleading photon (γ_2), leading jet (j_1) and the sub-leading jet (j_2) and (bottom) the two-photon invariant mass.

combination of the variation in the coupling strength as well as the gluon-gluon flux. However, once the extra jets are required to be radiated, the kinematics (and the mass dependence of the flux) does change considerably. Even more importantly, the requirements²¹ on $p_T(\gamma)$ serve to suppress the contribution from the very low mass ALPs, resulting in the significantly shifted maximum (now around $m_{\gamma\gamma} \approx 20$ GeV) as in Fig. 5 (bottom).

For the background, diphoton (associated with additional jets)—initiated by $q\bar{q}$ as well as gg —within the SM are considered. However, since there is a finite probability for a putative jet masquerading as a photon in the detector (this is especially true of a low-energy jet depositing a large fraction of its energy in the electromagnetic calorimeter), one needs to consider such contributions as well. Thus, the backgrounds emanate primarily from

- (i) $pp \rightarrow \gamma\gamma$,
- (ii) $pp \rightarrow \gamma j$,
- (iii) $pp \rightarrow jj$.

in each case, accompanied by up to two additional jets.

²¹Further corrections arise from the p_T dependence of the detector efficiencies.

TABLE II. Selection cuts applied to form the fiducial signal regions for benchmarks I and II [32].

Channel	Event selection criteria
$pp \rightarrow a_n + 0/1/2$ jets	$N_\gamma = 2, 1 \leq N_j \leq 2,$ $ \eta_\gamma < 2.37$ (excluding barrel-to-endcap region $1.37 < \eta_\gamma < 1.52$),
$a_n \rightarrow \gamma\gamma$	$E_T(\gamma) > 22$ GeV, $p_T^{\gamma\gamma} > 50$ GeV

Other backgrounds emanate from $e^\pm$ not leaving discernible tracks, thereby faking photons. These, though, contribute negligibly and can, therefore, be largely ignored. For an estimate of the relevant background distribution in the diphoton spectra, we again refer to the ATLAS analysis [32] wherein the fiducial region in the phase space is defined using the selection cuts on E_T^γ , $p_T^{\gamma\gamma}$ and η_γ as mentioned in Table II. The diphoton background²² thus determined in the analysis has an uncertainty which is predominantly statistical.

To examine the viability of the diphoton signal in light of the ATLAS analysis, we must subject the former to the same selection criteria as the latter. These are listed in Table II. As an examination of Fig. 5 (top) shows, the $E_T(\gamma)$ cut, especially when applied to the subleading photon, does eliminate a non-negligible fraction of the signal events. Even harder is the cut on $p_T(\gamma\gamma)$, for this requires that the ALP (and, hence, the radiated off jets) must carry a substantial p_T . The consequent reduction of events is reflected in the $m_{\gamma\gamma}$ distribution as shown in Fig. 6.

A striking aspect to note here [this was already evident in Fig. 5 (bottom)] is that although the diphoton distribution receives contributions from all the ALPs in the spectrum, the separation between the individual resonances is smeared by the detector resolution²³ to a degree that the entire distribution appears as a single broad resonance—an *iceberg* of axions, so to say—with a full width at half maximum (FWHM) ~ 16 GeV. This feature, as previously mentioned, is peculiar to the clockwork spectrum for any mass scale m as long as N is adequately large.

We can now estimate the significance of the signal events over the background bin-by-bin using $S = N_{\text{sig}} / \sqrt{N_{\text{sig}} + N_{\text{bkg}}}$, where N_{sig} and N_{bkg} denote the number of signal and background events, respectively, in a particular bin. Since the background uncertainty is statistics dominated, this gives quite a robust estimate of the sensitivity and can be bettered only by a dedicated search. The bin-wise

²²The background distribution as estimated in the ATLAS analysis [32] for a bin size of 1 GeV is available at <https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/HIGG-2019-23/>.

²³We have made use of the energy resolution function for the ATLAS detector's ECAL [112] as implemented within Delphes. For the phase-space region of interest throughout the presented analysis (i.e., for BP-I, II, and III), the energy resolution of the individual photons varies roughly in the range $\sim [0.32, 2.3]$ GeV.

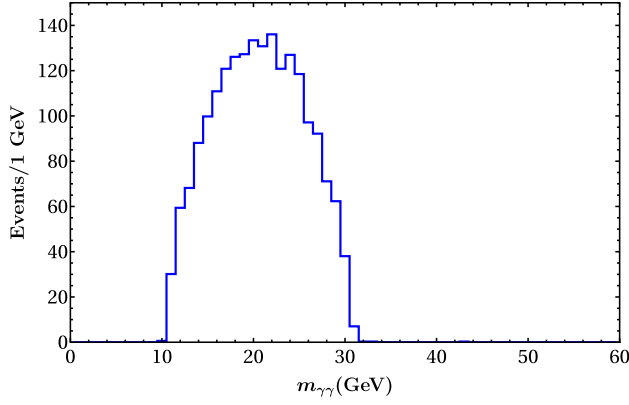


FIG. 6. BP-I: diphoton invariant mass distribution after applying selection cuts.

significances (and the corresponding p -values) for $\mathcal{L} = 138 \text{ fb}^{-1}$ with a bin width of 1 GeV are depicted in Fig. 7, and the maximum individual significance of $S_{138}^{1 \text{ GeV}} \sim 1\sigma$ is obtained in the bin corresponding to $m_{\gamma\gamma} \sim 21\text{--}22 \text{ GeV}$. It is tempting to increase the bin widths as this would be expected to substantially increase the per-bin significance owing to the individual peaks being closely packed within the invariant mass range 10–30 GeV. However, since the existence of a wide (and finite) band of resonances is characteristic to the clockwork scenario, the shape of the invariant mass distribution could itself behave as a discriminator—more so in the case of small mass splittings—and by enlarging the bin width one would, obviously, lose information about the underlying profile of the spectrum. In particular, this would help discriminate between a broad resonance and multiple sharp ones.²⁴ A more useful method of estimating the total signal significance would be to consider the sum $S_c \equiv \sum_i S_i^2$ which, of course, is χ^2 distributed for the appropriate degrees of freedom. Using this, we find for this benchmark point, the expected cumulative significance to be $S_{c,138} = 3.38\sigma$ for $\mathcal{L} = 138 \text{ fb}^{-1}$. On the same note, it is worth alluding to the prospect that the significance can further build up by a sizable amount with an increase in the luminosity, even by as early as the end of the ongoing Run 3 phase of the LHC with a projected luminosity reach of $\sim 300 \text{ fb}^{-1}$. A simple estimate of this can be obtained by scaling the current significance with the luminosity, i.e.

$$S_{c,300} = S_{c,138} \times \sqrt{\frac{300}{138}} \approx 4.98\sigma, \quad (22)$$

²⁴A further discriminant here would be the comparison between the width of the resonance and the total signal size. For a resonance, production cross section is simply related to the total width, whereas the size of the signal is just the product of the cross section, the branching fraction, and the overall detector efficiency (the last being a known quantity).

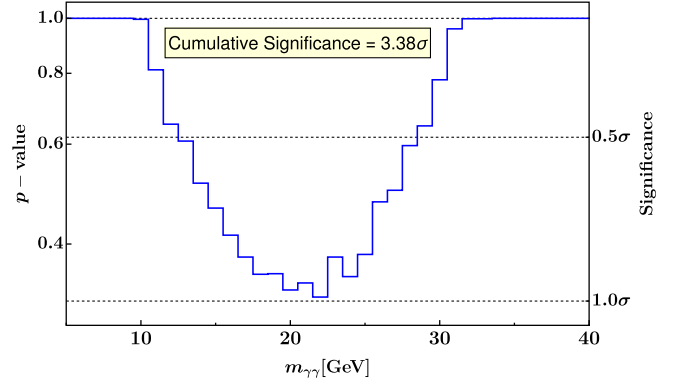


FIG. 7. Bin-wise significance for benchmark I at the LHC for $\mathcal{L} = 138 \text{ fb}^{-1}$.

which makes this benchmark point imminently testable by the end of Run 3.

Apropos of the current benchmark point, Fig. 8 shows the variation of S_c with the SSB scale f for the 138 fb^{-1} luminosity case along with a projection for the 300 fb^{-1} reach of the LHC. The variation follows the approximate relation $S_c \propto f^{-2}$ which can be understood from the fact that, for a particular choice of the CW parameters, the pertinent cross sections scale roughly as f^{-2} with the pole masses being quite insensitive to it.

At this stage, it is worth examining if such a signal could have emanated from any other theory. First and foremost, the decay of the resonance into a diphoton pair rules out a spin-one particle. While spin-two or higher are possible, such fundamental particles are disfavored on theoretical grounds (e.g., a graviton, while well motivated, would not only be much narrower, but would also decay democratically into other channels). Such arguments narrow down the possibilities to only a scalar or a pseudoscalar. This apart, a width comparable to the mass (as is the case here), would require the couplings to be in the nonperturbative regime [113,114]. [For instance, a single ALP resonance

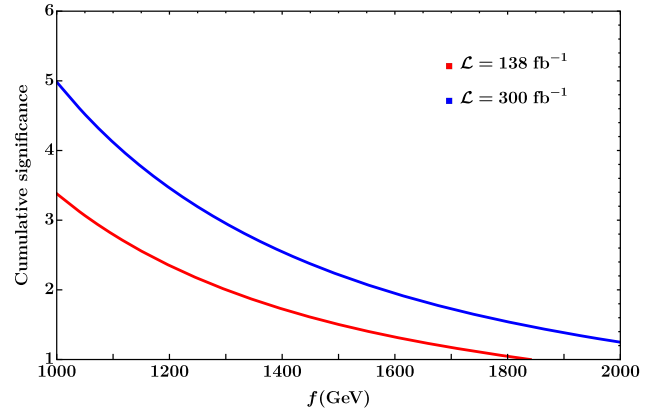


FIG. 8. Cumulative significance as a function of the SSB scale f for benchmark I at the LHC's Run 2 (red curve) and Run 3 (blue curve).

with a total decay width and mass $\Gamma(a) \sim m_a \sim \mathcal{O}(10 \text{ GeV})$ would require its decay constant²⁵ to be $f \lesssim 100 \text{ MeV}$.] Irrespective of the underlying theory, such a large width would have translated, *vide* Eq. (19), to a total production cross section several orders of magnitude larger than what is the case here. This would, immediately, raise question as to the reason for the suppression of the diphoton branching ratio that such a production cross section (along with the observed rates) would imply. Furthermore, an entity that couples so strongly would leave tell-tale signatures in myriad other processes (a trivial example being double diffractive processes leading to a pair of central jets with large rapidity gaps) and, hence, would have been long discovered.

In other words, if such an excess were to be observed, a single resonance hypothesis would be immediately ruled out. If it were to be then interpreted as an overlap of multiple resonances, two issues would need to be considered. The size of the excess imposes restrictions in the plane of the individual total widths and the diphoton branching fraction. On the other hand, a conservative estimate of the number of peaks can be obtained by comparing the width of the excess to the invariant mass resolution operative at such masses. And, finally, the very shape of the excess would dictate that the couplings of the said resonances must conform to a very specific pattern. To reproduce the features in a generic (i.e., nonclockwork) model (or even in multi-axion scenarios involving a new N -flavor confining sector [115,116]) would, thus, require many apparently *ad hoc* assumptions to be made. On similar grounds, it can also be argued that a broad resonance signal—with a finite bandwidth that is comparable with the characteristic mass scale of the resonance $\sim m$ —is incompatible with, say, the Kaluza-Klein spectra of extradimensional scenarios consisting, in principle, of an infinite number of states (up to a natural cutoff of the theory), for then the distribution of resonances would extend over a large mass range and truncate only when the cross sections start to fall near the kinematic limits of the experiment.

It is perhaps worth reiterating at this point that the analysis strategy employed here is complementary to the collider study of the clockwork graviton model carried out in Refs. [49,50]. Of the different strategies investigated therein, a particularly interesting approach involves searching for periodic signals in various kinematic distributions. However, the said technique, while well-suited for the towerlike spectra typically encountered in clockwork models, seems to be more efficient when the resonances are experimentally resolvable. The analysis above, therefore, presents a categorically new perspective to look at clockwork spectra at colliders, especially for the case of light ALPs with tiny mass splittings.

²⁵The inequality takes into account the dependence on the constituents of the anomaly loop.

B. Intermediate mass ALPs

With an intermediate mass range set by the parameters $m = 35 \text{ GeV}$ and $f = 1000 \text{ GeV}$, the analysis particulars in this case are largely the same as described for benchmark I. With the ALP masses in this case spanning the range 35–105 GeV, a sizable fraction of the spectrum still falls in the low mass region for which we require the final state photons to be boosted due to reasons delineated for benchmark I. We, therefore, choose to adopt the same strategy as in benchmark I for the collider simulation and consider a signal consisting of CW axions produced in association with up to two jets (and the axions further decaying to two photons). Photons and jets are reconstructed using the same criteria as in the case of benchmark point I (see Table I).

1. Benchmark point II(a)

The p_T distributions for the leading and the subleading photons thus obtained for the signal events along with the corresponding distributions for the jet p_T 's are shown in Fig. 9 (top), assuming a bin size of 1 GeV. The distributions are similar in nature to what was obtained for benchmark point I, albeit with a shift in the photon distributions

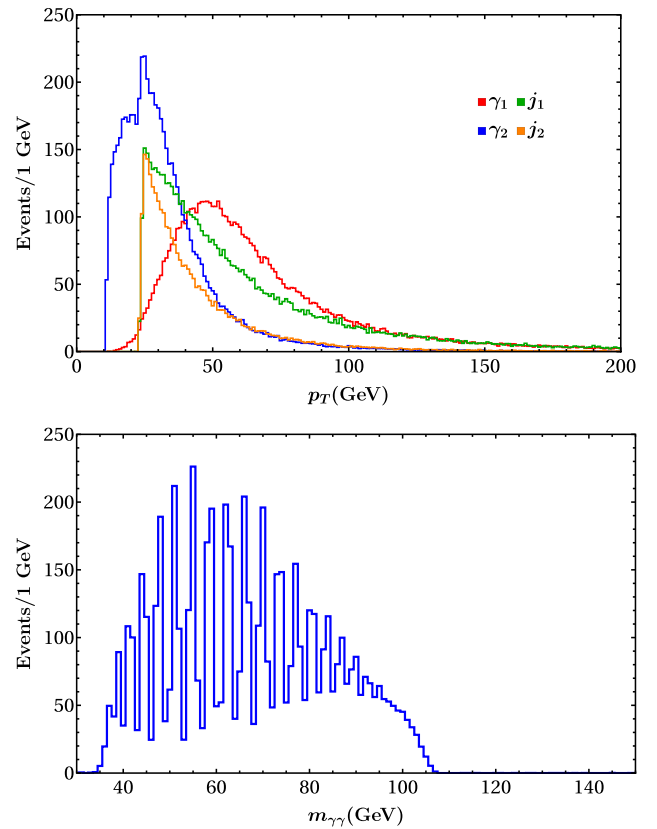


FIG. 9. BP-II(a): event distributions, after applying acceptance cuts alone, *vide* Table I. Top: p_T distributions for the leading photon (γ_1), subleading photon (γ_2), leading jet (j_1), and the subleading jet (j_2) and (bottom) the two-photon invariant mass.

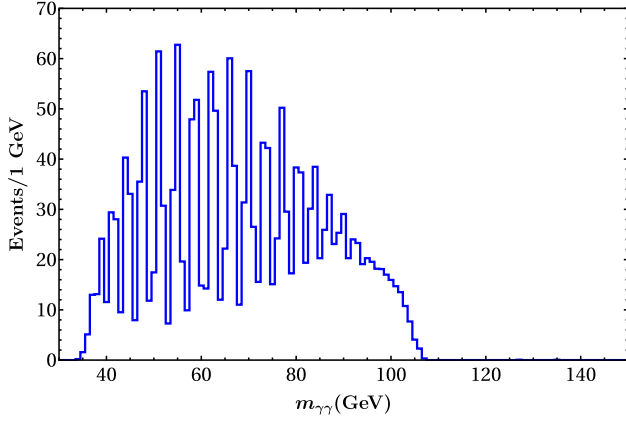


FIG. 10. BP-II(a): diphoton invariant mass distribution after applying selection cuts.

towards higher p_T values due to the ALPs being heavier in this case. The attendant diphoton invariant mass distribution is shown in Fig. 9 (bottom). As for the SM backgrounds, for the mass range $m_{\gamma\gamma} \in [10, 80]$ GeV, once again we adopt the ATLAS analysis [32]. For the $[80, 110]$ GeV window, in the absence of an appropriate analysis,²⁶ we choose to *overestimate* the background, by holding it at the level of $m_{\gamma\gamma} = 80$ GeV. Thus, our significance estimate would be somewhat conservative.

To obtain the signal diphoton distribution in the fiducial region, we employ the selection cuts mentioned in Table II. As for benchmark I, the kinematic cuts substantially reduce the number of events in the signal distribution, shown in Fig. 10. Given the characteristic mass splittings of this benchmark, we see that a large portion of the spectrum (consisting of 28 ALPs) in the diphoton distribution could potentially be resolved at the detector. Now, a bin-wise significance estimation, under the conservative assumption of the background events beyond $m_{\gamma\gamma} \approx 80$ GeV, leads to a distribution as shown in Fig. 11 with a cumulative significance $S_{c,138} = 2.12\sigma$ over the signal region. While spectral techniques [49,50] may enhance sensitivity in scenarios where multiple resonances are moderately separated and experimentally resolvable, the present analysis adopts a more conservative, profile-based strategy. This approach focuses on the overall signal shape, which effectively manifests as a single broad structure due to small mass splittings and finite detector resolution.

²⁶The ATLAS search for diphoton resonances in the mass range $m_{\gamma\gamma} = 66\text{--}110$ GeV [96] does provide a background estimation in the region of interest. However, the event selection criteria employed therein is different than that in the low mass analysis that we refer to, which makes it difficult to match the background events corresponding to the low and intermediate mass ranges in a meaningful way.

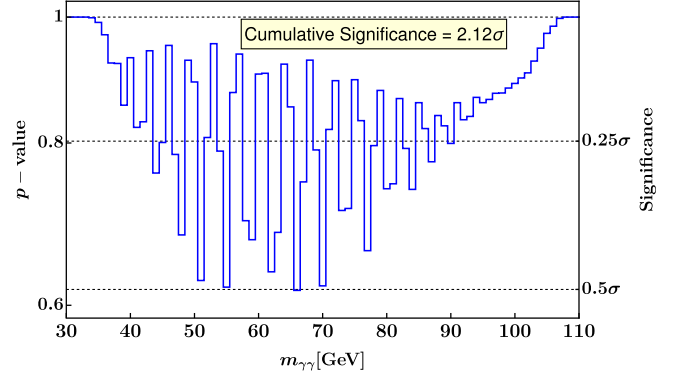


FIG. 11. Bin-wise significance for benchmark II(a) at the LHC for $L = 138 \text{ fb}^{-1}$.

2. Benchmark point II(b)

As the only modification we have in this case is the increase in the number of ALPs to $N = 40$, we follow the same strategy for the analysis as before. The signal's diphoton invariant mass distribution post acceptance cuts is shown in Fig. 12. In contrast with the outcome for benchmark II(a), the sizes of the individual peaks have decreased, with the cross sections scaling as $\sim |C_{nN}|^2/N$, and their smearing is now relatively more pronounced as a result of the smaller mass splittings, both affected by the increase in the number of ALPs. The corresponding distribution in the fiducial region is presented in Fig. 13, exhibiting, expectedly, the same signal shape as in Fig. 12, although with a reduced number of events. The bin-wise significance estimate is shown in Fig. 14 with the corresponding cumulative significance being $S_{c,138} = 1.97\sigma$.

Clearly, the two cases lead to slightly different projections for the 300 fb^{-1} luminosity reach, viz. $S_{c,300}^{(a)} \approx 3.12\sigma$ and $S_{c,300}^{(b)} \approx 2.9\sigma$. Thus, it may not be an overestimate to say that, upon a proper treatment, if the background

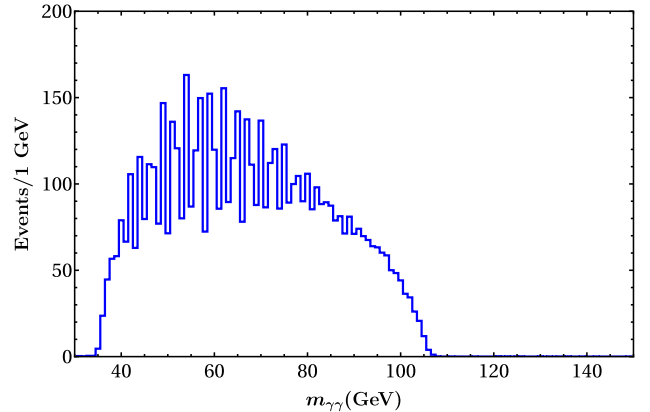


FIG. 12. BP-II(b): diphoton invariant mass distribution after applying acceptance cuts alone.

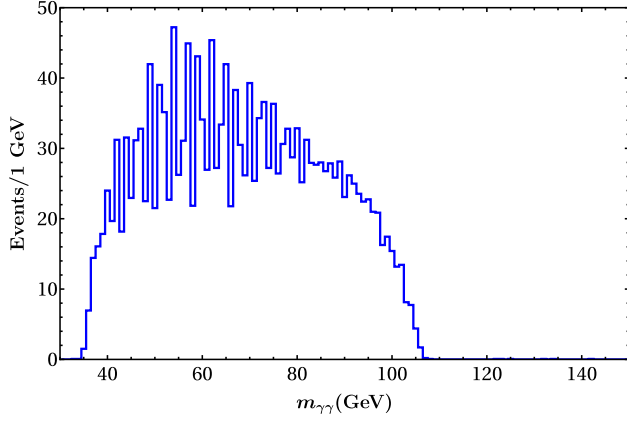


FIG. 13. BP-II(b): diphoton invariant mass distribution after applying selection cuts.

distribution is found to saturate or slowly fall beyond $m_{\gamma\gamma} \sim 80$ GeV, then benchmark II could also be potentially probed by the end of LHC's Run 3 phase.

For either the intermediate or the heavy-mass cases, the individual peaks are, thus, expected to be visible. The presence of such a bunch of closely packed resonances, with the specific envelope, would immediately point towards a clockworklike scenario.²⁷

C. Heavy ALPs

For ALPs that are significantly heavier than considered hitherto, the decay photons would be expected to carry sufficient energy (and p_T) for the event to be triggered even without any additional jet. An example is afforded by the aforementioned benchmark III, wherein the ALP masses span the range ~ 150 – 450 GeV. However, although extra jets are not needed for triggering, events with such jets do contribute significantly to the signal cross sections. Hence, we define the signal as a semi-inclusive one composed of a pair of energetic photons with up to two additional jets.²⁸ The corresponding K factors, for this mass range, are taken from Refs. [93,117].

The background, thus, would receive contributions from essentially the same channels as assumed in the low mass case. The signal profile being different, we, though, would need to adequately tune the assessment, and, in this, we are guided by the ATLAS diphoton analysis [95] germane to this mass range. Post showering and hadronization, using PYTHIA8, we take for the identification and isolation of photons in the Delphes

²⁷Naively, an alternative could be a composite sector with multiple excitations. However, not only would such a compositeness scale be very surprising, the abrupt ending of the excitations as well as the shape of the envelope would be hard to explain.

²⁸The inclusion of events with three or more jets does not improve the signal to noise ratio.

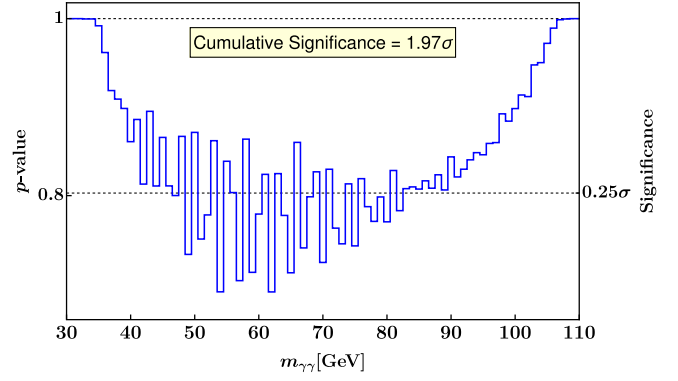


FIG. 14. Bin-wise significance for benchmark II(b) at the LHC for $L = 138 \text{ fb}^{-1}$.

detector simulation a cone size of $\Delta R = 0.4$ around the photon candidate. The requirement on the parameter p_T^{ratio} , as defined earlier, is now reset to $p_T^{\text{ratio}} < 0.12$. The complete photon and jet identification criteria, as adapted from the ATLAS analysis in Ref. [95], are displayed in Table III. The p_T distributions of the photons

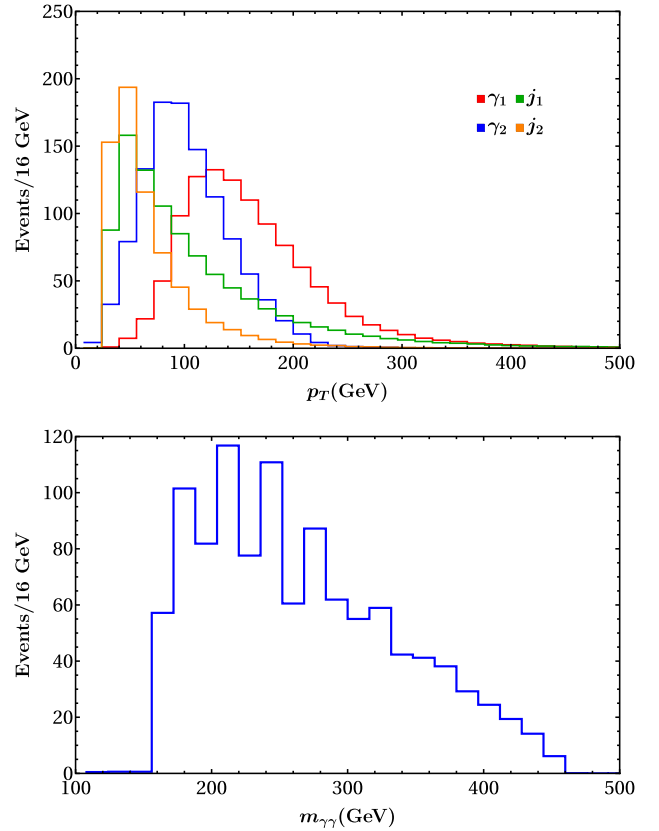


FIG. 15. BP-III: event distributions, after applying acceptance cuts alone, *vide*. Table III. Top: p_T distributions for the leading photon (γ_1), subleading photon (γ_2), leading jet (j_1), and the subleading jet (j_2) and (bottom) the two-photon invariant mass.

TABLE III. Acceptance cuts for the final state objects in benchmark III [95].

Channel	Acceptance cuts
Photon identification	
$pp \rightarrow a_n$	$\Delta R = 0.4, p_T > 0.4 \text{ GeV}, p_T^{\text{ratio}}(\gamma) < 0.12$
Jet identification	
$a_n \rightarrow \gamma\gamma$	$\Delta R = 0.4 \text{ (anti-}k_T\text{)}, p_T^j > 20 \text{ GeV}, \eta_j < 2.5$
Isolation	
	$\Delta R(\gamma, \gamma) > 0.4, \Delta R(\gamma, j) > 0.4, \Delta R(j, j) > 0.7 N_\gamma \geq 2$

TABLE IV. Event selection cuts applied to form the fiducial signal regions for benchmark III [95].

Channel	Event selection criteria
$N_\gamma = 2, N_j \leq 2$	
$pp \rightarrow a_n$	$ \eta_\gamma < 2.37 \text{ (excluding barrel-to-endcap region)}$
	$1.37 < \eta_\gamma < 1.52,$
$a_n \rightarrow \gamma\gamma$	$E_T(\gamma_1) > 0.3m_{\gamma\gamma}, E_T(\gamma_2) > 0.25m_{\gamma\gamma}$
	$p_T^j > 20 \text{ GeV}, \eta_j < 2.5$

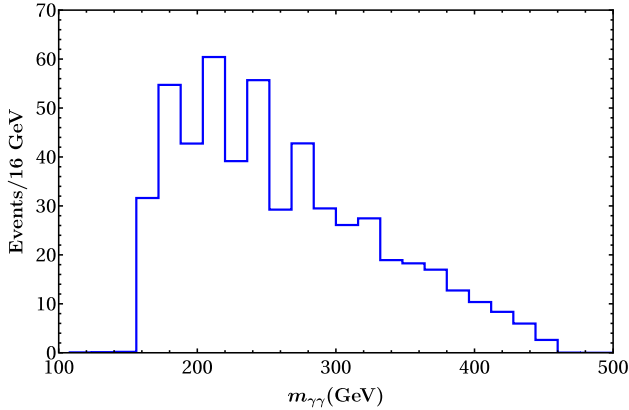
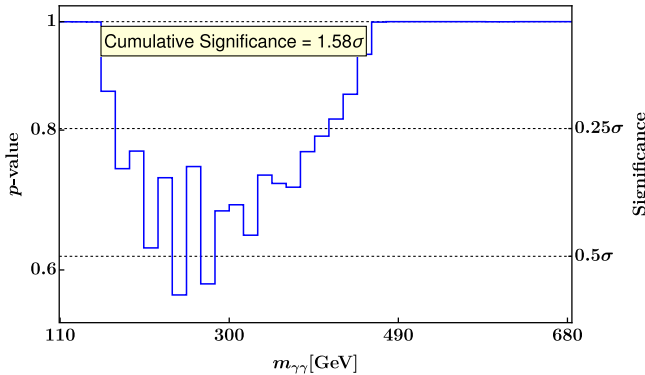


FIG. 16. BP-III: diphoton invariant mass distribution after applying selection cuts.

FIG. 17. Bin-wise significance for benchmark III at the LHC for $\mathcal{L} = 138 \text{ fb}^{-1}$.

and the jets for the generated signal²⁹ events are shown in Fig. 15 (top), adopting a bin size of 16 GeV as in the ATLAS search [95].

Figure 15 (bottom) shows the corresponding diphoton invariant mass distribution. Taking cues from Ref. [95], we then apply the selection cuts mentioned in Table IV in order to define the fiducial volume in the phase space.

The resulting diphoton invariant mass distribution is shown in Fig. 16, whereas Fig. 17 shows the corresponding bin-wise significance (with the background profile adopted from Ref. [95]) over the diphoton signal region with a cumulative significance $S_{c,138} \approx 1.58\sigma$. For this case, the projected enhancement for a 300 fb⁻¹ luminosity reach is $S_{c,300} \approx 2.33\sigma$. Thus, in contrast to benchmarks I and II, the high mass scenario will not be readily accessible in the LHC's Run 3. It is perhaps worth speculating, though, that benchmark III, with heavy ALPs, could potentially reach the discovery threshold during the forthcoming high luminosity phase of the LHC with a projected luminosity enhancement by a factor of nearly 20 times the current value.

V. THE VECTORLIKE QUARKS AND THE HEAVY SCALARS

Beyond the low-energy spectrum of the pseudoscalars, the model, as described in Sec. II, also contains a heavy quark with the $SU(3)_c \times SU(2)_L \times U(1)_Y$ charge assignment $(3, 1, 2/3)$ as well as $N + 1$ heavy radial scalar singlets (the partners of the pseudoscalars). Although not germane to the main objectives of our analysis, it is worth outlining here the dynamics of both the vectorlike quark (VLQ) and the heavy scalars so as to establish the consistency of the model, especially in view of the fact that the assumed mass scales (characterized by the SSB scale f) of these heavy particles are, in principle, accessible at the LHC.

Considering that $m^2 \ll f^2$ in all the benchmarks, the off-diagonal quadratic terms for the radial scalars in the Lagrangian (eq. (4)) are hierarchically smaller than the diagonal mass terms governed by the coupling λ , which is presumed to assume $\mathcal{O}(1)$ values. Consequently, all the radial scalars are nearly degenerate with masses $\sim \sqrt{2\lambda}f$. To simplify matters, we assume the other new quartic coupling, viz. $\lambda_{\Phi H}$, is also small, thereby automatically relaxing constraints from the stability of the SM Higgs potential, triviality, etc. Similar to the pseudoscalars, these heavy scalars have effective interactions with the gluons, photons, and the EW vector bosons with the respective couplings given by $g_{\pi VV}/\xi$, where $g_{\pi VV}$ represent the pseudoscalar couplings as shown in Eq. (17).

As stated in the discussion following Eq. (8), we assume, for the sake of simplicity, that the singlet VLQ mixes

²⁹In contrast with benchmarks I and II, events without jets in the final state also contribute significantly in this case and, therefore, $N(2j) < N(1j) < N_{\text{Tot}}$.

predominantly with the SM top quark, thereby automatically suppressing flavour-changing neutral currents (FCNCs) involving the first two generations of quarks. Post-electroweak symmetry breaking (EWSB), the two quarks mix and the corresponding transformations can be expressed (in the limit of neglecting the very small mixings with the first two generations) as

$$\begin{aligned} (\Psi_L \ u_L^{(3)})^T &= U(T_L \ t_L)^T, \quad \text{and} \\ (\Psi_R \ u_R^{(3)})^T &= V(T_R \ t_R)^T, \end{aligned} \quad (23)$$

where T, t represent the mass eigenstates and U, V are the (special unitary) mixing matrices. Since Ψ_R and $u_R^{(3)}$ have identical quantum numbers, there is no gauge-mediated FCNC involving T_R and t_R and the weak gauge couplings of the VLQ-like state T can be expressed as

$$\begin{aligned} \mathcal{L}_{T-V} &= \left(\frac{g}{\sqrt{2}} U_{iT} \right) W_\mu \bar{b}_L \gamma^\mu T_L \\ &+ \left(\frac{g}{2c_w} U_{it}^* U_{iT} \right) Z_\mu \bar{t}_L \gamma^\mu T_L \\ &+ \frac{g}{c_w} Z_\mu \bar{T} \gamma^\mu \left[-\frac{2}{3} s_w^2 + \frac{g}{2c_w} U_{Ti}^* U_{iT} P_L \right] T + \text{H.c.} \end{aligned} \quad (24)$$

where $P_{L,R}$ are the usual chiral projection operators. On the other hand, the FCNCs involving the scalars, as derived from Eq. (10), are given by

$$\begin{aligned} \mathcal{L}_{T-\phi} &= -\frac{1}{\sqrt{2}} \bar{t}_L [(\lambda_\Psi U_{iT}^* V_{TT} + y'_\Psi U_{iT}^* V_{iT}) \phi_N \\ &+ (\lambda_h U_{it}^* V_{iT} + y_\Psi U_{it}^* V_{TT}) h] T_R \\ &- \frac{1}{\sqrt{2}} \bar{T}_L [(\lambda_\Psi U_{TT}^* V_{Ti} + y'_\Psi U_{TT}^* V_{ii}) \phi_N \\ &+ (\lambda_h U_{Ti}^* V_{ii} + y_\Psi U_{Ti}^* V_{Ti}) h] t_R + \text{H.c.}, \end{aligned} \quad (25)$$

where λ_h denotes the top-Higgs Yukawa coupling. To obtain the pseudoscalar FCNCs, it is perhaps the most convenient to make use of the shift symmetry and rescale the field $\Psi_L \rightarrow e^{i\xi\pi_N/f} \Psi_L$ in Eq. (10). This, in turn, results in a pseudovector FCNC term in the full Lagrangian through the kinetic term of the field Ψ_L , namely,

$$\mathcal{L}_{T-\pi} = \frac{\partial_\mu \pi_N}{f} \bar{T}_L \gamma^\mu (U_{TT}^* U_{Ti}) t_L + \text{H.c.}, \quad (26)$$

with π_N being expressed in terms of the mass eigenstates a_n through $\pi_N = \sum_k C_{Nk}^{-1} a_k$ with the matrix C as defined in Eq. (6). For $f \gtrsim 1000$ GeV and $m_T \sim \lambda_\Psi f / \sqrt{2} > (m_{W,Z,h,\phi} + m_t)$, the primary decay channels for the VLQ are clearly $T \rightarrow bW$, $T \rightarrow tZ$, $T \rightarrow th$, $T \rightarrow t\phi_N$ (if kinematically allowed) and $T \rightarrow ta_n$ (see Appendix B for the decay width expressions [118]). Note that the decays to the new scalars and pseudoscalars appear in addition to the

conventional VLQ decay modes to the SM particles and, therefore, it is worth comparing the branching fractions pertaining to the different channels. As this needs specifying a few further parameters, we begin by doing so for each of the benchmark points:

BP-I and II: We assume $\lambda_\Psi = 2.2$, $y_\Psi = \epsilon y'_\Psi$, $\lambda = 1.8$ and $y'_\Psi \lesssim 0.1$ with $\epsilon = 0.1$. This choice of λ_Ψ is motivated by the experimental lower bound on the VLQ mass ($m_T \gtrsim 1500$ GeV) to be discussed below. However, as can be expected from such a large value of the coupling, the evolution is fast and an examination of the three-loop renormalization group equations [119] leads to the conclusion that the theory becomes a very strongly coupled one at a scale $\mu \sim 10$ TeV, signaling that new physics must take over well before this scale.³⁰ Given that the clockwork model is *not* a UV-complete one, this, per se, may still be overlooked. However, we return to this point in the next section.

Note that the aforementioned collider limits assume that the VLQ decays within the detector. If, on the other hand, its decay length was large enough that it does not decay within the LHC detector,³¹ it would hadronize and leave very distinctive signals. While such “R hadrons” have been considered in the literature [121–130], the corresponding experimental studies are not as exhaustive [131]. The limits (typically ranging from ~ 900 GeV to ~ 1250 GeV) depend on the details of the hadronization model assumed. We refrain from considering this possibility in our analysis.

BP-III: The considerably larger value of the SSB scale f allows us the luxury of choosing a relatively smaller value for the Yukawa coupling λ_Ψ , and we consider, instead, $\lambda_\Psi = 1.5$, $\lambda = 0.7$ and $y_\Psi = \epsilon y'_\Psi$, $y'_\Psi \lesssim 0.1$ with $\epsilon = 0.1$. For such a choice, the running of λ_Ψ is significantly slower and the strong coupling phase of the theory lies near 10^8 GeV.

While in Eqs. (24) and (25) we have not listed any alterations in the SM couplings of the top-quark, it is obvious that certain changes would be wrought. However, these changes are only higher order in the T – u_3 mixing U_{iT} , and with the latter not being large, are well below the current sensitivity limits (the strongest being that for the SM CKM element V_{tb} [19]), whether from flavor physics or from top decay.

The constraints on the VLQ sector would, thus, come from direct observations at colliders.³² At the large hadron collider, the overwhelmingly leading production

³⁰A cutoff near 10 TeV also serves to mitigate probable issues related to naturalness in the extended scalar sector, exhibiting only a *little* hierarchy problem.

³¹This could happen if both y'_Ψ and y_Ψ are tiny. The VLQ, nonetheless, has to decay on cosmological timescales so as not to violate the strong constraints on colored and electrically charged dark matter candidates. The possibility of a bound state, made of stable neutral VLQs, being a DM candidate is discussed in [120].

³²Collider phenomenology of such KSVZ-VLQs have also been studied in Refs. [132–134].

TABLE V. VLQ branching fractions for BP-I,II and III (The SM column refers to the assumed branching fractions for VLQ searches [52,135]). The bottom row displays the resulting lower bounds on m_T , as derived from the limits on the VLQ pair-production cross section presented in Ref. [135].

Channel	Branching ratios		
	SM	BP-I & II	BP-III
$T \rightarrow bW$	0.5	0.44	0.47
$T \rightarrow tZ$	0.25	0.21	0.23
$T \rightarrow th$	0.25	0.23	0.25
$T \rightarrow ta_{(\text{all})}$	...	0.12	0.05
m_T lower limit	1540 GeV [135,136]	1500 GeV	≈ 1540 GeV

mechanism is the QCD-driven one.³³ Once pair produced, the T 's would decay promptly. Table V lists the leading VLQ branching ratios for the three benchmark points.³⁴ It can be readily ascertained that the branching fractions have a very small dependence on the free parameter y'_Ψ .

As the table shows, the branching fractions relevant to the standard search algorithms are not overly affected. Consequently, the derived limits are only slightly relaxed at best. On the other hand, it might be interesting to consider exotic channels such as $T \rightarrow t + a_n \rightarrow t + \gamma\gamma$. Hitherto (largely) unexplored, these might be of interest at future runs of the LHC.

VI. SUMMARY AND CONCLUSION

We have examined the minimal QCD axion model within the *clockwork* paradigm and investigated the prospects of observing the massive ALPs that the model engenders at current and future hadron colliders. To this end, we employ a Kim–Shifman–Vainshtein–Zakharov (KSVZ)-like setup with a heavy colored $SU(2)_L$ singlet quark (as a top-partner) which couples to one end of a CW chain of $(N + 1)$ complex scalars. Being chirally charged under the site-specific $U(1)$ —which acts as an analog of the PQ symmetry—this quark generates the requisite chiral anomaly for addressing the strong CP problem in a natural manner. The lightest pseudo-Nambu-Goldstone boson (pNGB) in the CW sector identifies as the QCD axion while the accompanying heavier pNGBs in the spectrum behave as ALPs with a characteristic mass scale $\sim mq$. Through the chiral anomaly, these axions then couple to the gluons, the photon and the Z boson, albeit with hierarchically³⁵ different decay constants for the QCD axion and the

ALPs, namely $f_0 \sim q^N f$ and $f_{n>0} \sim f$, thanks to the CW mechanism. Being naturally consistent with the current experimental and observational constraints, the large suppression in the effective couplings of the light axion renders it practically invisible at high-energy colliders (as is the case with all traditional QCD axions).

For a reasonable parameter configuration such as $q > 1$, $N \sim \mathcal{O}(10)$, $m \gtrsim 10$ GeV and $f \sim 1$ TeV, what is *more attractive* instead is the production and detection of the ALPs, especially at the currently operating LHC and its future derivatives. To perform a quantitative analysis, we classify three benchmark scenarios according to the ALP masses in the range 10–30 GeV (BP-I), 35–105 GeV (BP-II) and 150–450 GeV (BP-III). Such varied mass scales for the ALPs are enabled by the fact that the CW mechanism is based on the premise of localization in the theory space and, thus, is practically independent of the mass scales assumed in the theory. We performed an analysis of the expected signal profiles pertaining to the three benchmark points for the production of ALPs (with up to two additional jets) via gluon fusion and their subsequent decay to two photons at the LHC for an integrated luminosity of 138 fb^{-1} . As anchor points for the analysis and for estimates of the pertinent background distributions, we referred to the ATLAS searches for diphoton resonances in the relevant invariant mass regions [32,95].

What stands out as particularly interesting is the case of the light ALPs where the number of particles N that is consistent with the allowed window for the QCD axion's decay constant also corresponds to very small mass splittings among the ALPs. The resulting diphoton invariant mass distribution for the signal events turns out to be such that the individual resonances are smeared by the detector's resolution (implemented in the form of a fast simulation of the ATLAS detector) and, thereby, overlap with each other with the full CW spectrum of light ALPs manifesting itself, instead, as a single but very broad resonance. It becomes immediately apparent, though, that a single such resonance is entirely untenable, and that such an excess must have resulted from the merging of several individual narrow resonances. On the other hand, treating the spectrum as a whole also offers the possibility of a better, more inclusive, estimation of the signal's statistical significance over the background. For instance, the simplistic *cumulative* estimation defined in this work gives a sizable significance value of $S_c \sim 3.4\sigma$ (for an integrated luminosity of only 138 fb^{-1}) in BP-I, even though the significance estimates of the individual bins in the signal distribution are comparatively small. Moreover, when scaled by the projected luminosity reach of the ongoing Run-3 phase of the LHC, viz. $\sim 300 \text{ fb}^{-1}$, the cumulative significance gets enhanced to $S_c \sim 5\sigma$. Of course, a dedicated search for such signals would call for a more detailed statistical analysis that is tailored to the fact that the

³³With U_{iT} not being large, single production is suppressed and the consequent bounds [52] are relaxed.

³⁴For the chosen set of couplings, a decay of T to radial scalars is kinematically forbidden. For any phenomenologically viable set of couplings, the branching fractions remain much smaller than those listed in the table.

³⁵The hierarchy is generated dynamically.

individual resonances of the CW spectrum are characteristically correlated.

As for the heavy ALPs, e.g., those encountered in BP-III, the cumulative significance hovers just below 3σ even with 300 fb^{-1} of data. It should be borne in mind, though, that our analysis and projections are strictly based on the published ATLAS data. It is conceivable that a more refined analysis would serve to improve the sensitivity. What is exciting, though, is the prospect of probing a wide window of the CW ALPs at the HL-LHC with a planned luminosity reach of $\sim 3000 \text{ fb}^{-1}$, where larger portions of the diphoton invariant mass profile could start to become apparent—the structure of the axion *iceberg* revealing itself—and even scenarios like BP-III could possibly surpass the discovery threshold. We note that the kind of multi-ALP signals discussed here possess a very broad and distinct quasiperiodic profile which cannot be reproduced in models with a single heavy ALP or in generic multiscalar scenarios.

A key issue in our analysis has been the opposing pulls on the SSB scale f from the need to increase the effective ALP-gluon-gluon coupling (scaling as f^{-1}) on the one hand and the lower bound on the VLQ mass (scaling as f) on the other. While modifying the number of sites N or the exponent parameter q does make a difference, the effect is a muted one (especially given that renormalizability of the low-energy theory, as well as vacuum stability, requires that $q \leq 3$). However, given that the light ALP solutions presented here (benchmark points I and II) indicate that further new physics must exist by $\sim 10 \text{ TeV}$ scale, such a restriction might seem unwarranted and $q > 3$ may be considered. However, since the ALP couplings to the gluons and photon have a dependence on q which appears only through the elements of the CW transformation matrix, namely, $C_{(n>0)N}$, the corresponding gain is marginal. It can be similarly argued that a change in the position of the VLQ coupling over the CW lattice (i.e., to assume $j < N$) also does not lead to a substantial gain.

Naively, a second possibility would be to postulate additional VLQs such that the effective $a_n gg$ coupling may be raised by way of all the VLQs contributing in the loop diagram. However, the corresponding change in the axion potential would putatively lead to the formation of stable domain walls/cosmic strings in the early Universe and is phenomenologically disfavored. While one might attempt to mitigate the issue by invoking some of the solutions proposed in the literature, e.g., by assuming a soft breaking of the discrete symmetry or an inflation-induced statistical bias in the axion potential, the model would, nevertheless, be severely constrained due to the inherent drawbacks of such solutions [137]. Consider, instead, the introduction of one (or more) vectorlike leptons (VLLs). This does not contribute to the $a_n gg$ couplings (thereby evading the domain wall constraints), but does contribute to the $a_n \gamma \gamma$ couplings, which now get enhanced by a factor of

$$1 + \frac{1}{3Y_\Psi^2} \sum N_L Q_L^2$$

where N_L is the number of VLLs of charge Q_L . Even the simplest choice of a single VLL of charge $Q = -1$ enhances the coupling by a factor of 1.75, thereby allowing for a similar enhancement in f without changing the rates. This, in turn, allows for a smaller Yukawa λ_Ψ , and postponement of the strong coupling phase until a cutoff scale $\mu \gtrsim 10^7 \text{ GeV}$. As for the VLL phenomenology, the LHC limits are understandably much weaker. For it to decay, it must have at least a small Yukawa coupling with a SM lepton, and postulating this to be the τ would not only escape low-energy constraints, but also allow for intriguing signals at the LHC.

An entirely different mechanism of raising f without suppressing the signal is afforded by the choice of the $U(1)_N$ charge ξ for the Ψ_L and Φ_N . Increasing ξ and f proportionately keeps f_{eff} (and, therefore, the $a_n gg$ coupling) unchanged.

Whatever such mechanisms may be, it is quite apparent that the clockwork-axion-ALP scenario would be manifested at the LHC in terms of interesting signals, perhaps the most interesting being the case of the possibility of a strong coupling regime.

In addition to diphoton channels, several other prospective final states could enhance our search efforts at the collider, offering complementary avenues to test the model. Of particular interest are processes involving the heavy VLQ or the radial scalars which, while lying below the current LHC sensitivities, can be probed at the upcoming and proposed high-intensity and high-energy upgrades. For instance, as previously mentioned, the exotic decay of the VLQ to ALPs in association with a SM quark, e.g. $T \rightarrow t + a_n (\rightarrow \gamma\gamma)$, presents an interesting channel to study. Processes mediated by the heavy radial scalars ϕ_n such as $pp \rightarrow \phi_n \rightarrow a_n (\rightarrow 2\gamma/2j) a_n (\rightarrow 2\gamma/2j)$ are also worth exploring, more so because of the recent interest in the search for boosted diphotons [138]. Another noteworthy process is the axion mediated production of a Z boson in association with a photon, with the Z subsequently decaying into dijets ($pp \rightarrow a_n \rightarrow Z (\rightarrow jj) \gamma$). While the cross section for this process is low for the chosen benchmark points in the parameter space and the background is substantial, it remains a promising channel for probing the model at future colliders, particularly for benchmark point III.

This apart, there also exist other intriguing channels, the nonstandard decay $Z \rightarrow a_n \gamma$ being one interesting example. Although the process is hardly relevant for an investigation at the LHC, given the strong limits placed on the Z exotic decay fractions at LEP, it could be well within the projected sensitivities of future lepton colliders, e.g. the proposed ILC running in the Giga- Z mode [139]. Furthermore, while the minimal setup considered in this work does not engender coupling of the ALPs to the light quarks or leptons even at the one-loop level, one could envision extending or modifying

the model to include mixing of the VLQ with the light quarks and introduce additional VLLs that mix with the SM leptons in order to induce ALP couplings with the light SM fermions. In that case, several interesting channels open up, most notable of them being $h \rightarrow a_n a_n \rightarrow b \bar{b} \mu \bar{\mu}$ (with both the ATLAS and CMS experiments having observed intriguing signals [34,35] in this channel) and $h \rightarrow Z a_n, a_n \rightarrow \text{hadrons}$ [37]. This would also have interesting ramifications for flavor physics, including tests for lepton flavor universality in B decays.

A complementary test of the model can also be found in the context of the direct detection of the light axion. A quick inspection shows that the masses and the effective photon couplings of the QCD axion in BP I and II(a) are close to the current sensitivities of several cavity haloscope experiments such as ADMX, CAPP, HAYSTAC, QUAX, etc. Thus, a collider probe of the characteristic multi-ALP signatures, which would largely constrain the parameters m and f_{eff} , along with the complementary probe of the light axion at the current or next generation of direct detection experiments, which is sensitive to the parameters f_{eff} , q and N , present a robust experimental platform to test the model in the near future.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: THE AXIAL ANOMALY

In the KSVZ model described in this work, the left- and the right-handed projections of the heavy fermion Ψ are chirally charged under the global symmetry $U(1)_N$. As is usually the case in KSVZ models, the chiral (or axial) $U(1)_N$ [analogous to $U(1)_{\text{PQ}}$ in the original setup] symmetry is anomalous which, in turn, results in the couplings of the pseudoscalar π_N to gluons and the hypercharge boson given in Eq. (11). This is most succinctly realized in the path integral formulation of the KSVZ theory, with the partition function given by

$$Z[0]_A = \int \mathcal{D}\bar{\Psi} \mathcal{D}\Psi \exp \left[i \int d^4x \mathcal{L}_\Psi \right]. \quad (\text{A1})$$

Here, the subscript A denotes a fixed background with respect to the gauge fields and $\mathcal{L}_\Psi[\Psi, \bar{\Psi}, D_\mu \Psi, D_\mu \bar{\Psi}]$ marks the full Lagrangian for Ψ including the kinetic terms as well. Post-SSB (at the scale f), the Yukawa term for Ψ given in Eq. (10) (ignoring the mixing terms with the SM fermions for brevity) can be rewritten in the convenient form

$$-\frac{\lambda_\Psi}{\sqrt{2}}(\phi + f)e^{i\xi\pi_N\gamma_5/f}\bar{\Psi}\Psi. \quad (\text{A2})$$

The anomalous nature of $U(1)_N$ is apparent when we see that while the tree level action is invariant under the chiral transformation

$$\Psi' = U\Psi = e^{-i\xi\beta\gamma_5/(2f)}\Psi, \quad \pi_N \rightarrow \pi_N + \beta, \quad (\text{A3})$$

where β is a constant transformation parameter, the measure of the path integral is not. The measure would transform as

$$\mathcal{D}\bar{\Psi} \mathcal{D}\Psi \rightarrow \mathcal{J}^{-2} \mathcal{D}\bar{\Psi} \mathcal{D}\Psi, \quad (\text{A4})$$

where $\mathcal{J}$ is the Jacobian corresponding to the chiral transformation. The Jacobian, after appropriate regularization, is given by [140,141]

$$\mathcal{J} = \exp \left[i \int d^4x \frac{1}{32\pi^2} \frac{\xi\beta}{f} \text{Tr}\{F^{\mu\nu} \tilde{F}_{\mu\nu}\} \right], \quad (\text{A5})$$

where $F^{\mu\nu} \tilde{F}_{\mu\nu} \equiv F^{A\mu\nu} \tilde{F}_{\mu\nu}^B T^A T^B$ which stands for the gauge fields for all the local symmetries under which Ψ is charged with T^A being the respective generators. For brevity, we define T^A such that they include the corresponding gauge couplings. Tr denotes trace over the gauge representation of the field Ψ . Therefore, the anomaly term which appears in the full Lagrangian is given by

$$\mathcal{A}(x) = -\frac{\xi\beta}{16\pi^2 f} \text{Tr}\{F^{\mu\nu} \tilde{F}_{\mu\nu}\}. \quad (\text{A6})$$

Similarly, if we use the freedom to redefine the field Ψ so as to absorb the pseudoscalar field through the chiral transformation

$$\Psi \rightarrow e^{-i\xi\pi_N\gamma_5/(2f)}\Psi, \quad (\text{A7})$$

we obtain the corresponding anomaly term in the Lagrangian given by

$$\mathcal{A}(x) = -\frac{\xi\pi_N}{16\pi^2 f} \text{Tr}\{F^{\mu\nu} \tilde{F}_{\mu\nu}\}. \quad (\text{A8})$$

A straightforward evaluation of the trace in the preceding expression leads to the terms in Eq. (11).

APPENDIX B: VLQ DECAY WIDTHS

In the following we list the decay width expressions for the heavy fermion Ψ with the labels q and V denoting the SM quarks (b, t) and the EW gauge bosons (W, Z) respectively.

$$\begin{aligned}\Gamma(T \rightarrow qV) &= |g_L^V|^2 \frac{\sqrt{m_q^4 - 2m_q^2(m_T^2 + m_V^2) + (m_T^2 - m_V^2)^2}}{32\pi m_T^3 m_V^2} [(m_T^2 - m_q^2)^2 + m_V^2(m_T^2 + m_q^2) - 2m_V^4] \\ &\approx \frac{|g_L^V|^2 m_T^3}{32\pi m_V^2} \quad (\text{for } m_T \gg m_{q,V}).\end{aligned}\quad (\text{B1})$$

Here, the couplings g_L^V are given by

$$g_L^W = \frac{g}{\sqrt{2}} U_{iT}, \quad g_L^Z = \frac{1}{2} \frac{g}{c_w} U_{iT}^* U_{iT}, \quad (\text{B2})$$

whereas the corresponding (off-diagonal) right-handed couplings vanish identically thanks to an analog of the GIM mechanism.

For decays to scalars we ignore the small mixing between the SM Higgs and the heavy scalars ϕ_n . The corresponding expressions are

$$\begin{aligned}\Gamma(T \rightarrow th) &= \frac{\sqrt{m_h^4 - 2m_h^2(m_T^2 + m_t^2) + (m_T^2 - m_t^2)^2}}{64\pi m_T^3} \{(|y_{Tt}^h|^2 + |y_{iT}^h|^2)[m_T^2 + m_t^2 - m_h^2] + 4\text{Re}(y_{Tt}^{h*} y_{iT}^h) m_T m_t\} \\ &\approx \frac{|y_{Tt}^h|^2 + |y_{iT}^h|^2}{64\pi} m_T \quad (\text{for } m_T \gg m_{t,h}),\end{aligned}\quad (\text{B3a})$$

$$\begin{aligned}\Gamma(T \rightarrow t\phi_N) &= \frac{\sqrt{m_{\phi_N}^4 - 2m_{\phi_N}^2(m_T^2 + m_t^2) + (m_T^2 - m_t^2)^2}}{64\pi m_T^3} \{(|y_{Tt}^\Phi|^2 + |y_{iT}^\Phi|^2)[m_T^2 + m_t^2 - m_{\phi_N}^2] + 4\text{Re}(y_{Tt}^{\Phi*} y_{iT}^\Phi) m_T m_t\} \\ &\approx \frac{|y_{Tt}^\Phi|^2 + |y_{iT}^\Phi|^2}{64\pi} m_T \left(1 - \frac{m_{\phi_N}^2}{m_T^2}\right)^2 \quad (\text{for } m_{T,\phi_N} \gg m_t)\end{aligned}\quad (\text{B3b})$$

and

$$\begin{aligned}\Gamma(T \rightarrow ta_n) &= \frac{\xi^2 C_{nN}^2 \sqrt{m_n^4 - 2m_n^2(m_T^2 + m_t^2) + (m_T^2 - m_t^2)^2}}{32\pi f^2 m_T^3} (|U_{TT}|^2 |U_{Tt}|^2 [(m_T^2 - m_t^2)^2 - (m_T^2 + m_t^2) m_n^2]) \\ &\approx \frac{\lambda_\Psi^2 \xi^2 C_{nN}^2 |U_{TT}|^2 |U_{Tt}|^2}{32\pi} m_T \quad (\text{for } m_T \gg m_{t,a_n}),\end{aligned}\quad (\text{B3c})$$

where the effective Yukawa couplings are given by

$$\begin{aligned}y_{Tt}^h &\approx \lambda_h U_{Tt}^* V_{tt}, \\ y_{iT}^h &= \lambda_h U_{it}^* V_{iT} + y_\Psi U_{it}^* V_{TT}, \\ y_{iT}^\Phi &\approx \lambda_\Psi U_{iT}^* V_{TT}, \\ y_{Tt}^\Phi &= \lambda_\Psi U_{TT}^* V_{Tt} + y'_\Psi U_{TT}^* V_{tt}.\end{aligned}\quad (\text{B4})$$

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