

Supernova constraints on lepton flavor violating axions

Yonglin Li and Zuowei Liu^{*}

Department of Physics, Nanjing University, Nanjing 210093, China



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Supernovae offer a unique hot and dense environment to probe new physics beyond the Standard Model. We investigate supernova cooling constraints on lepton-flavor-violating (LFV) axions and axionlike particles (ALPs) that couple to electrons and muons. For LFV-ALP production in supernovae, muon decay and lepton bremsstrahlung have been considered previously. In this work, we identify the electron-muon coalescence channel as an efficient new production mechanism in the high-mass regime. We also include the semi-Compton scattering process, which has recently been shown to provide sizable contributions for electron-coupled ALPs. We find that muon decay dominates in the low-mass regime, electron-muon coalescence becomes the leading channel at high masses, and semi-Compton scattering provides the dominant contribution in the intermediate mass range. We find that the electron-muon coalescence process yields the strongest constraints in the mass range of $\sim(115, 280)$ MeV, probing the ALP-electron-muon coupling down to $\sim 4 \times 10^{-10}$ for an ALP mass of ~ 200 MeV.

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I. INTRODUCTION

Axions are beyond the standard model (BSM) pseudo-scalar particles that were originally proposed to solve the strong CP problem [1–4]. In addition to axions, a number of hypothetical pseudoscalar particles, referred to as axionlike particles (ALPs), can arise in a variety of well-motivated BSM models [5–8]. ALPs can couple to various standard model (SM) particles in ways that lead to violations of SM symmetries, such as lepton flavor conservation [7]. The lepton-flavor-violating (LFV) ALPs can arise either from UV theories of ALPs [6,7,9–13], which are known as flavons or familons, or from radiative corrections [14–16], even if the underlying theory preserves flavor.

Thus, searching for LFV ALPs is an intriguing aspect of new physics studies beyond the SM. In particular, accelerator experiments provide the best constraints on heavy LFV ALPs with mass above GeV [17–25]. In contrast, sub-GeV LFV ALPs are best probed in decay experiments [26–38], and in stellar processes [34,39]. Among the stellar objects, core-collapsed supernovae (CCSNe), the astrophysical events that provide extremely dense and hot environments, probe LFV ALPs with a relatively high mass up to 100 MeV [34]. The most renowned example of CCSNe is the SN 1987A, whose neutrino signals not only

provide crucial insights into the collapse and explosion mechanisms of such supernovae, but also place stringent constraints on potential new energy-loss channels induced by BSM particles, which is known as the supernova (SN) cooling limit [40].

Recently, the SN cooling limits on LFV ALPs that couple both to electrons and muons have been analyzed by Refs. [34,39]. To compute the axion production rate in the SN, Ref. [34] considered the muon decay process, $\mu \rightarrow e + a$, where a is the ALP, and Ref. [39] considered the lepton bremsstrahlung process. In this paper, we further consider the electron-muon coalescence process, $e^\mp + \mu^\pm \rightarrow a$, for the ALP production in the SN. We find that for heavy ALPs with mass of $m_a > m_e + m_\mu$, the electron-muon coalescence process is the dominant ALP production channel in the SN core for the parameter space of interest. Recently, Ref. [41] showed that the semi-Compton process can provide significant contributions for ALPs that couple to electrons. Thus, in this work, we also consider the semi-Compton process, $e/\mu + \gamma \rightarrow \mu/e + a$, for LFV-ALP production in the SN. We find that the semi-Compton process provides the leading contribution in the intermediate mass range, dominating over the lepton bremsstrahlung process in that region.

In computing SN cooling limits, Refs. [34,39] assumed an SN core with constant density and temperature. In this work, we use profiles from recent supernova simulations [42,43], which offer a more accurate description of the SN core. These include profiles of temperature and chemical potentials of various particles, as shown in Fig. 1. Moreover, Refs. [34,39] neglected the effects of ALP absorption. Here, we compute the ALP luminosity by

^{*}Contact author: zuoweiliu@nju.edu.cn

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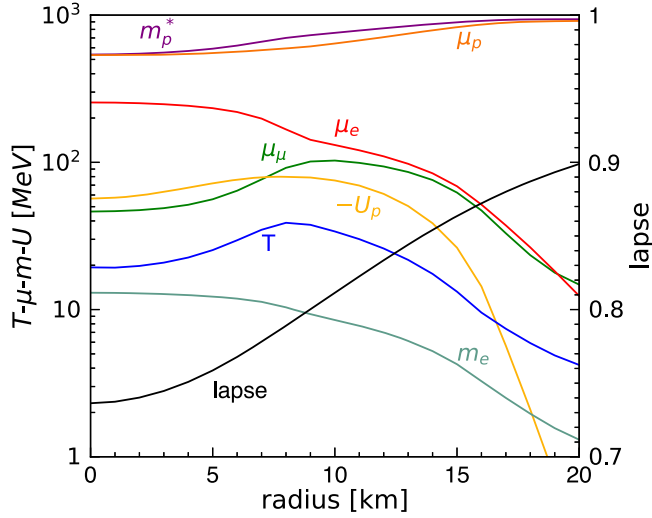


FIG. 1. The profiles of temperature T , electron chemical potential μ_e , muon chemical potential μ_μ , proton chemical potential μ_p , effective proton mass m_p^* , and the negative of the proton interaction potential U_p , as well as the gravitational lapse, for the Garching muonic SN model SFHo-18.8 at 1 second postbounce [43]. The effective electron mass is calculated with T and μ_e using Eq. (3). Note that the proton chemical potential μ_p given in [43] excludes the rest mass; here, we include the rest mass in μ_p .

properly taking into account the ALP absorption effects, following Refs. [44,45]; see also, e.g., Refs. [46–48]. We find that the ALP absorption effects are significant and must be accounted for in order to obtain more accurate constraints.

In this paper, we compute the ALP luminosity by considering the following four production channels in the SN core: (i) muon decay, (ii) lepton bremsstrahlung, (iii) electron-muon coalescence, and (iv) semi-Compton. By using recent SN profiles and taking into account the absorption effects, we find that the SN cooling constraints on LFV ALPs probe currently unexplored parameter space in the mass range of $\sim(105, 280)$ MeV, where the dominant ALP production channels are the semi-Compton process for the mass range of $\sim(105, 115)$ MeV and the electron-muon coalescence process for $\sim(115, 280)$ MeV. Note that in this mass range, LFV ALPs cannot be probed by the rare muon decay experiments due to the kinematical conditions. Collider experiments, on the other hand, only probe the large coupling regime in this mass range, which does not overlap with the parameter space probed by SN processes; see, e.g., Refs. [17,32]. Thus, SN cooling limits arising from the electron-muon coalescence process offer a unique probe of the high ALP mass range.

The rest of the paper is organized as follows. In Sec. II we discuss the LFV-ALP model. In Sec. III we discuss the SN profiles and the ALP luminosity in the SN cooling analysis. In Sec. IV we discuss the ALP production and

absorption rates. Our results are given in Sec. V. We summarize our findings in Sec. VI. We further provide detailed calculations of the ALP production rate in Appendix A, and of the ALP absorption rate in Appendix B.

II. ALP MODEL

We consider an ALP model where the electron number and the muon number are violated by the interaction with ALPs:

$$\mathcal{L}_{\text{int}} = \frac{g_{ae\mu}}{m_e^0 + m_\mu} \bar{e} \gamma^\lambda \gamma_5 \mu \partial_\lambda a + \text{H.c.}, \quad (1)$$

where $g_{ae\mu}$ is the coupling constant, a is the ALP field, e (μ) is the electron (muon) field, and $m_e^0 = 0.511$ MeV ($m_\mu = 105.6$ MeV) is the electron (muon) mass. Note that we use m_e^0 to denote the electron mass in the vacuum, which should be distinguished from the in-medium electron mass in the SN core, m_e , as it receives significant corrections due to plasma effects. At tree level the interaction Lagrangian given in Eq. (1) is equivalent to [49]

$$\mathcal{L}_{\text{int}} = -ig_{ae\mu} a \bar{e} \gamma_5 \mu + \text{H.c.} \quad (2)$$

We note that Eq. (2) is equivalent to Eq. (1) when both fermions are on shell, which can be proven using the equations of motion; see also Refs. [50,51]. However, when there exist off-shell fermions, the equivalence between Eqs. (2) and (1) is lost. In our analysis we use the Lagrangian given in Eq. (2) to compute the SN limits.

III. SN MODEL

The production rate of the LFV ALP depends strongly on the temperature T , and on the number densities of various particles, which in turn depend on their chemical potentials. In our analysis, we adopt the temperature and chemical potential profiles from recent simulations in the Garching muonic SN model SFHo-18.8 [42,43]. Among the models simulated in Refs. [42,43], SFHo-18.8 is the coldest and thus provides a conservative and robust constraint on LFV ALPs. Figure 1 shows various profiles at 1 second post-bounce, including temperature T , electron chemical potential μ_e , muon chemical potential μ_μ , and proton chemical potential μ_p .

We note that muons reach thermal equilibrium with other SM particles via electromagnetic interactions, but their chemical potential is determined by weak processes that change the muon number. In the context of SN cooling, the ALP processes considered here alter the muon number by 1 unit and are required to be comparable in strength to weak interactions. This may lead to non-negligible modifications to the muon distribution in the SN core. These effects, however, are neglected in our analysis.

In a plasma environment like the SN core, the properties of electrons are modified significantly so that the electron

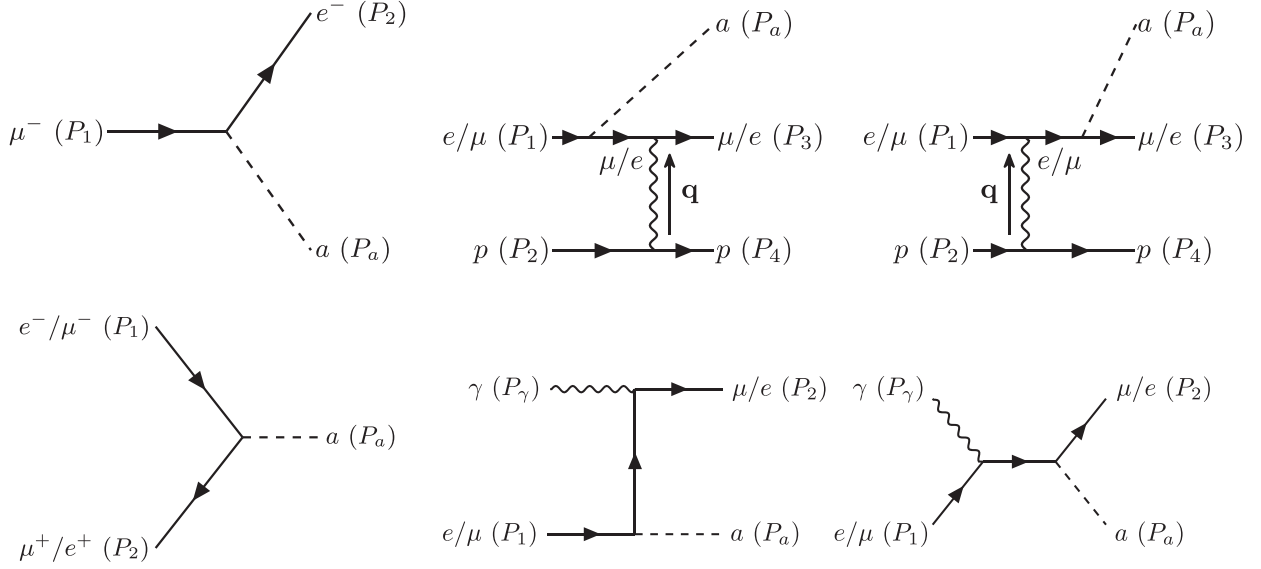


FIG. 2. LFV-ALP production processes in the SN. Upper: muon decay (left) and lepton bremsstrahlung (middle and right). Lower: $e - \mu$ coalescence (left) and semi-Compton (middle and right).

mass in the vacuum ($m_e^0 = 0.511$ MeV) is replaced by the in-medium electron mass [50,52]:

$$m_e = \frac{m_e^0}{\sqrt{2}} + \sqrt{\frac{(m_e^0)^2}{2} + \frac{\alpha}{\pi}(\mu_e^2 + \pi^2 T^2)}, \quad (3)$$

where $\alpha = 1/137$ is the fine-structure constant. We compute m_e as a function of the radius r by using the μ_e and T profiles. As shown in Fig. 1, the in-medium electron mass is in the range of $8.3 \text{ MeV} \lesssim m_e \lesssim 12.9 \text{ MeV}$ for $r \lesssim 10 \text{ km}$.

For the lepton bremsstrahlung process, the ALP production rate depends on the profile of the proton number density, which in turn depends on the profiles of the effective proton mass m_p^* , the proton chemical potential μ_p , and the proton interaction potential U_p , which are shown in Fig. 1; see Appendix A for detailed calculations. The ALP luminosity also depends on the lapse factor, which accounts for both the gravitational redshift as well as contributions from the pressure and energy of both the stellar medium and neutrinos [53–56]. The profile of the lapse factor is shown in Fig. 1.

A. ALP luminosity

ALPs can introduce a new cooling channel of the SN core, which can significantly modify the properties of the SN and thus affects its neutrino luminosity [40]. Based on the observation of SN 1987A [57], Ref. [40] established the criterion that the luminosity of the ALPs from the SN inner core should be smaller than the luminosity of neutrinos at about 1 second postbounce:

$$L_a \leq L_\nu = 3 \times 10^{52} \text{ erg/s}. \quad (4)$$

This is known as the supernova cooling limit. If the couplings between ALPs and SM particles are weak, ALPs can free stream in the SN, and their luminosity can be computed via simple volume integrations, neglecting reabsorption effects. However, if the couplings are strong, ALPs may be rapidly reabsorbed inside the SN core, significantly altering their escape and resulting luminosity.

The SN cooling limits on LFV ALPs have been studied for the muon decay process [34] and for the lepton bremsstrahlung process [39]. Here, we further consider the $e - \mu$ coalescence process and the semi-Compton process for ALP production. Thus, we consider the following four processes: (i) muon decay, (ii) lepton bremsstrahlung, (iii) $e - \mu$ coalescence, and (iv) semi-Compton. The Feynman diagrams of the four processes are shown in Fig. 2. We find that the $e - \mu$ coalescence and the semi-Compton processes, which were neglected in Refs. [34,39], lead to new constraints on LFV ALPs in the mass range of (105,280) MeV.

To compute the SN cooling limit, we compute the ALP luminosity at the gain radius R_g [45,56]:

$$L_a = \int_0^{R_g} dr 4\pi r^2 \text{lapse}(r)^2 (1 + 2v_r) \times \int_{m_a'}^\infty dE_a E_a \frac{d^2 n_a}{dt dE_a} \langle e^{-\tau_a(R_g, E_a, r)} \rangle, \quad (5)$$

where E_a is the ALP energy, n_a is the ALP number density, τ_a is the optical depth, $\langle e^{-\tau_a(R_g, E_a, r)} \rangle$ accounts for the absorption of the ALP, $\text{lapse}(r)$ is the lapse factor accounting for the gravitational redshift effects, v_r is the radial

velocity of the emitting medium, and $m'_a = m_a/\text{lapse}(r)$. In our analysis we neglect v_r , since $|v_r| \ll 1$ [56]. We note that the absorption effects of ALPs were neglected in previous studies on SN cooling constraints on LFV ALPs [34,39]. However, we find that the absorption effects, which are properly taken into account in Eq. (5), play a significant role in the ALP luminosity. We also note that L_a computed via Eq. (5) can be directly compared to the neutrino luminosity at infinity, as the gravitational redshift effects have been taken into account. We refer to $d^2 n_a/dtdE_a$, the number of ALPs produced per unit volume per unit time per unit energy, as “the ALP production rate” hereafter. In our analysis we use $R_g = 21.0$ km, which is the gain radius at 1 second postbounce in Ref. [43].

The absorption term is computed via [44,45]

$$\langle e^{-\tau_a(R_g, E_a, r)} \rangle = \int_{-1}^1 \frac{d\mu}{2} e^{-\int_0^{s_{\max}} \frac{ds}{r} \Gamma_{\text{abs}}(E_a, \sqrt{r^2 + s^2 + 2rs\mu})}, \quad (6)$$

where v is the velocity of the ALP, Γ_{abs} is the absorption rate of the ALP, $\mu = \cos\beta$ with β the angle between outward radial direction and the trajectory of the ALP along which ds is integrated, and $s_{\max} = \sqrt{R_g^2 - r^2(1 - \mu^2)} - r\mu$ is the upper limit of the ds integration. Note that the absorption up to the gain radius R_g is taken into account such that $s_{\max}$ satisfies $r^2 + s_{\max}^2 + 2rs_{\max}\mu = R_g^2$.

We note that different treatments on the new particle luminosity have been previously used in the literature. For example, Ref. [58] considered new particle production up to R_ν and computed the absorption up to R_g ; see also Refs. [50,51,59]. In contrast, we consider ALP production within R_g as in Eq. (5), rather than using R_ν as in Refs. [50,51,58,59]. This distinction is motivated by the fact that the energy within R_g contributes to neutrino production [58], and ALP production in the region between R_ν and R_g also depletes the energy that is available for neutrino emission. Therefore, the ALP production between R_ν and R_g should be taken into account when computing the SN cooling constraints. We note that Ref. [45] has computed the ALP luminosity consistently at any radius.

IV. ALP PRODUCTION AND ABSORPTION

To compute the ALP production rate $d^2 n_a/dtdE_a$ in Eq. (5), we consider four ALP production channels in the SN core: (i) muon decay, (ii) lepton bremsstrahlung, (iii) electron-muon coalescence, and (iv) semi-Compton. For light ALPs with $m_a < m_\mu - m_e$, the muon decay process $\mu^- \rightarrow e^- + a$ is kinematically allowed, as shown in the upper left diagram of Fig. 2. For heavy ALPs with $m_a > m_\mu + m_e$, the electron-muon coalescence process $e^\mp + \mu^\pm \rightarrow a$ becomes kinematically accessible, as shown in the lower left diagram of Fig. 2. The lepton bremsstrahlung process, as shown in the upper middle and upper right

diagrams of Fig. 2, contains two subprocesses: $e^- p \rightarrow \mu^- pa$ and $\mu^- p \rightarrow e^- pa$. The semi-Compton process contains two subprocesses: $e^- + \gamma \rightarrow \mu^- + a$ and $\mu^- + \gamma \rightarrow e^- + a$, as shown in the lower middle and lower right diagrams of Fig. 2. Because the lepton bremsstrahlung and semi-Compton processes are kinematically allowed for any ALP mass, they are the only viable ALP production channels in the mass range of $-m_e < m_a - m_\mu < m_e$, as the other two processes are kinematically forbidden. We find that the production rate for the $e^- + \mu^+ \rightarrow a$ channel is higher than that for $e^+ + \mu^- \rightarrow a$ in the SN. We note that the electron-muon coalescence process and the semi-Compton process have not been considered in previous studies on LFV ALPs in SNe [34,39]. We find that, despite the relatively low abundances of positrons and antimuons, the electron-muon coalescence process dominates ALP production in SNe for the mass range of $m_a > m_\mu + m_e$.

To compute the ALP absorption rate Γ_{abs} in Eq. (6), we consider the following four processes: (i) $e^- - a$ coalescence $e^- + a \rightarrow \mu^-$, (ii) inverse bremsstrahlung, (iii) ALP decay $a \rightarrow e^\pm + \mu^\mp$, and (iv) inverse-Compton, which are the inverse of the four ALP production processes shown in Fig. 2. See Appendixes A and B for the detailed calculations on the ALP production rates and absorption rates.

We note that for light ALPs, the muon decay process dominates the ALP production in the SN. In this regime, both the production and absorption rates are largely insensitive to the ALP mass, and the absorption effects are negligible for $g \lesssim 10^{-9}$. In contrast, for heavier ALPs where the electron-muon coalescence process dominates the ALP production, the production rate and absorption rate become sensitive to both the coupling constant g and the ALP mass m_a .

V. RESULTS AND DISCUSSIONS

We compute the SN cooling constraints by computing the ALP luminosity via Eq. (5), where we consider contributions from four ALP production channels: (i) muon decay, (ii) lepton bremsstrahlung, (iii) electron-muon coalescence, and (iv) semi-Compton.

To compare different production channels, we compute the total production rate of ALPs within the gain radius via

$$\frac{dN_a}{dt} = 4\pi \int_0^{R_g} dr r^2 \int_{m'_a}^\infty dE_a \frac{d^2 n_a}{dtdE_a}. \quad (7)$$

Figure 3 compares dN_a/dt from the four different ALP production channels: For $m_a \lesssim 90$ MeV, the muon decay process is the dominant production channel. In the mass range of $m_a \gtrsim 115$ MeV, the electron-muon coalescence process is the dominant production channel. In the mass range of $m_a \sim (90, 115)$ MeV, both the muon decay and electron-muon coalescence processes are kinematically forbidden, and the semi-Compton process provides the leading contributions to the ALP production. We find

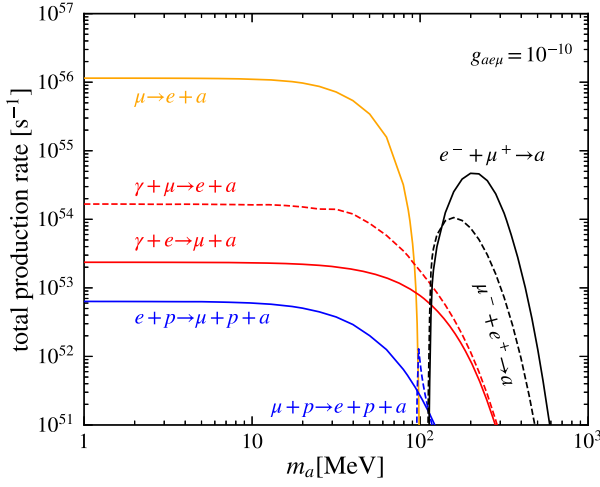


FIG. 3. Total ALP production rate up to the gain radius as a function of the ALP mass, for different production channels: (1) muon decay (orange), (2) lepton bremsstrahlung: $e + p \rightarrow \mu + p + a$ (blue solid) and $\mu + p \rightarrow e + p + a$ (blue dashed), (3) $e - \mu$ coalescence: $e^- + \mu^+ \rightarrow a$ (black solid) and $\mu^- + e^+ \rightarrow a$ (black dashed), (4) semi-Compton: $\gamma + e \rightarrow \mu + a$ (red solid) and $\gamma + \mu \rightarrow e + a$ (red dashed). Here we fix $g_{ae\mu} = 10^{-10}$.

that the lepton bremsstrahlung processes are always subdominant.

Figure 4 shows the SN cooling constraints on LFV ALPs by demanding $L_a < L_\nu = 3 \times 10^{52}$ erg/s, where the L_a is the ALP luminosity. In the mass range of $m_a \sim (105, 280)$ MeV, the electron-muon coalescence

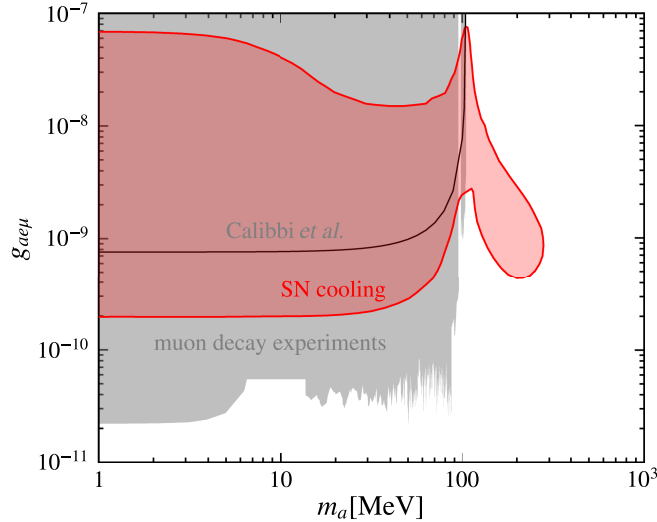


FIG. 4. The SN cooling constraints on LFV ALPs (red), using the Garching muonic SN model SFHo-18.8 [43]. We consider four LFV-ALP production processes: (i) muon decay, (ii) lepton bremsstrahlung, (iii) $e - \mu$ coalescence, and (iv) semi-Compton. Also shown is the SN cooling limit with the muon decay process only (black curve) [34]. The gray shaded region shows the constraints from rare muon decay experiments: Derenzo [26], Bilger *et al.* [29], Jodidio *et al.* [27], TWIST [30], and PIENU [33].

process and semi-Compton process probe currently unexplored parameter regions.

We further compare our results with previous SN limits in Fig. 4. The SN cooling limit from Ref. [34], where only the muon decay process is considered, is approximately four times weaker than ours. This discrepancy is primarily due to the simple SN model used in Ref. [34], where an SN core with constant density and temperature is assumed. In contrast, we adopt the Garching muonic SN model SFHo-18.8 [42,43], which provides a more realistic treatment of the SN core and a more accurate determination of the muon abundance.

We also compare our results with constraints from rare muon decay experiments in Fig. 4. We compute the branching ratio for the rare muon decay process:

$$\text{Br}(\mu \rightarrow ea) = \frac{\Gamma(\mu \rightarrow ea)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})}, \quad (8)$$

where [34]

$$\Gamma(\mu \rightarrow ea) = \frac{g_{ae\mu}^2 m_\mu}{16\pi} \left(1 - \frac{m_a^2}{m_\mu^2}\right)^2 \quad (9)$$

is the decay width of $\mu \rightarrow e + a$ with the electron mass neglected, and

$$\Gamma(\mu \rightarrow e\nu\bar{\nu}) = \frac{G_F^2 m_\mu^5}{192\pi^3} \quad (10)$$

is the SM muon decay width [60], where $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant. We then use the upper limit on $\text{Br}(\mu \rightarrow e + a)$ in Fig. 1 of Ref. [33] to obtain the constraints, including the TWIST constraints for $m_a > 45$ MeV, the constraints from Derenzo [26], from Bilger *et al.* [29], and from PIENU [33]. For the constraints from rare muon experiments by Jodidio *et al.* [27] and TWIST [30] in the mass range of $m_a < 45$ MeV, we adopt the limits from Fig. 4 of Ref. [39].

As shown in Fig. 4, the constraints from rare muon decay experiments are about 1 order of magnitude stronger than the SN cooling constraints, in the mass range of $m_a \lesssim 105$ MeV. However, for $m_a \gtrsim 105$ MeV, the muon decay process is kinematically forbidden, and the only constraints shown in the parameter space of interest are the SN cooling constraints, which probe LFV ALPs up to $m_a \sim 280$ MeV. We note that for $m_a > m_e + m_\mu$, other possible constraints could apply: The decay final states of long-lived ALPs produced in SNe (or binary neutron star mergers) could generate observable x-ray or γ -ray signals [61–63], or form fireballs, leading to sub-MeV photons [64,65].

Recently, Ref. [66] has emphasized the importance of taking into account ALP production in the mantle for the net energy deposition in the mantle, in the strong coupling regime. Reference [66] also suggested that one should incorporate such effects when performing the SN cooling

analysis. Indeed, ALPs produced in the mantle can be reabsorbed in the SN core, thereby reducing the net ALP luminosity that escapes the gain radius. To investigate such effects, we compute ALP luminosity for the following benchmark points: (a) $m_a = 105$ MeV and $g_{ae\mu} = 10^{-7}$, and (b) $m_a = 200$ MeV and $g_{ae\mu} = 4 \times 10^{-9}$, which are slightly above the upper boundary of the exclusion region in Fig. 4. For benchmark point (a), we find that $L_{\text{core}} \simeq 4.7 \times 10^{51}$ erg/s and $L_{\text{mantle}} \simeq 8.5 \times 10^{40}$ erg/s, where L_{core} denotes the luminosity from ALPs produced in the SN core and escaping the gain radius, and L_{mantle} denotes the luminosity from ALPs produced in the mantle and subsequently reabsorbed in the SN core. For benchmark point (b), $L_{\text{core}} \simeq 8.8 \times 10^{51}$ erg/s and $L_{\text{mantle}} \simeq 4.6 \times 10^{32}$ erg/s. Therefore, the effects due to ALP production in the mantle are insignificant for these two benchmark model points. We thus conclude that the upper boundary of the exclusion region in Fig. 4 is essentially not affected by including ALP production in the mantle.

We note that, in the parameter space of interest, the ALP decay length is about 100–1000 km, which is much smaller than the radius of the progenitor star. Consequently, ALPs are likely to decay in the SN mantle, leading to energy deposition that can contribute to the SN explosion energy [56]. Such effects allow one to constrain this parameter space using observations of low-energy SNe (LESNe) [67,68].

VI. SUMMARY

In this paper, we compute the SN cooling constraints on LFV ALPs that mediate $e - \mu$ lepton flavor violations. To properly compute the ALP luminosity, we consider the effects of ALP absorption in the SN. Moreover, we compute both ALP production and absorption terms up to the gain radius R_g . We note that our treatment consistently takes into account the ALP production between R_ν and R_g , which has been neglected in many previous studies.

We consider four LFV-ALP production channels in the SN core: (i) muon decay, (ii) lepton bremsstrahlung, (iii) $e - \mu$ coalescence, and (iv) semi-Compton. The muon decay process is the dominant production channel in the low-mass regime, while the $e - \mu$ coalescence process is the dominant production channel at high masses. In the intermediate mass range, the semi-Compton process provides the leading contribution, dominating over the lepton bremsstrahlung process. We note that the $e - \mu$ coalescence and semi-Compton processes have not been considered previously in the literature for the SN cooling constraints on LFV ALPs. We find that the two new channels lead to the most stringent constraints on LFV ALPs in the mass range of $\sim(105, 280)$ MeV.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: ALP PRODUCTION RATE

In this section we compute the ALP production rate. For this purpose, we consider a generic LFV-ALP production process $i \rightarrow j$, where i and j denote collectively the initial and final state particles, respectively, and one of the final state particles is an ALP. The other initial and final state particles are SM particles that are in the equilibrium state. In this case, the ALP production rate is given by (see, e.g., Refs. [51,70,71])

$$\frac{d^2 n_a}{dt dE_a} = \frac{|\mathbf{p}_a|}{4\pi^2} \int \prod_i d\Phi_i f_i \prod_{j \neq a} d\Phi_j (1 \pm f_j) (2\pi)^4 \delta^{(4)}(P_i - P_j) |\mathcal{M}|^2, \quad (\text{A1})$$

where $d\Phi_i = d^3 p_i (2\pi)^{-3} (2E_i)^{-1}$ is the Lorentz invariant phase space for particle i with momentum $P_i^\mu = (E_i, \mathbf{p}_i)$, f_i is the equilibrium distribution function of particle i , P_i (P_j) is the total four-momentum of the initial (final) particles, and $|\mathcal{M}|^2$ is the squared matrix element with spin sum for both initial and final states. We next compute the ALP production rates in the SN core for the following four processes: (i) muon decay, (ii) lepton bremsstrahlung, (iii) $e - \mu$ coalescence, and (iv) semi-Compton, as shown in Fig. 2.

1. Muon decay

We first consider the muon decay process, $\mu^- \rightarrow e^- + a$, as shown in the upper left diagram of Fig. 2. This process occurs when $m_\mu > m_a + m_e$. Because the effective electron mass is ~ 10 MeV in the SN core, the muon decay process occurs for $m_a \lesssim 100$ MeV. The matrix element for the muon decay process is given by

$$\mathcal{M}_d = -ig_{ae\mu} \bar{u}_e(P_2) \gamma_5 u_\mu(P_1), \quad (\text{A2})$$

where P_1 and P_2 are the four-momenta of the μ^- and e^- particles, respectively. The ALP production rate for the muon decay process is

$$\frac{d^2 n_d}{dt dE_a} = \frac{|\mathbf{p}_a|}{4\pi^2} \int d\Phi_1 d\Phi_2 f_\mu (1 - f_e) (2\pi)^4 \delta^{(4)}(P_1 - P_2 - P_a) |\mathcal{M}_d|^2, \quad (\text{A3})$$

where f_e (f_μ) is the Fermi-Dirac distribution for e^- (μ^-), and $|\mathcal{M}_d|^2 = 2g_{ae\mu}^2 [(m_\mu - m_e)^2 - m_a^2]$ is the spin-summed matrix element. We first integrate out the phase space of the muon by using three of the four delta functions. Because $|\mathcal{M}_d|^2$ is constant, we can write $d^3 p_2 = 2\pi |\mathbf{p}_2| E_2 dE_2 d\cos\theta_{2a}$ so that

$$\frac{d^2 n_d}{dt dE_a} = \frac{|\mathcal{M}_d|^2}{32\pi^3} \int \frac{|\mathbf{p}_2|}{E_1} dE_2 d\cos\theta_{2a} f_\mu (1 - f_e) \delta(E_1 - E_2 - E_a). \quad (\text{A4})$$

Noting that $E_1 = \sqrt{m_1^2 + \mathbf{p}_2^2 + \mathbf{p}_a^2 + 2|\mathbf{p}_2||\mathbf{p}_a|\cos\theta_{2a}}$, we integrate out $\cos\theta_{2a}$ to obtain

$$\frac{d^2 n_d}{dt dE_a} = \frac{|\mathcal{M}_d|^2}{32\pi^3} \int_{E_2^-}^{E_2^+} dE_2 f_\mu (1 - f_e), \quad (\text{A5})$$

where

$$E_2^\pm = \frac{E_a(m_1^2 - m_2^2 - m_a^2)}{2m_a^2} \pm \frac{\sqrt{E_a^2 - m_a^2} I}{2m_a^2}, \quad (\text{A6})$$

where $I = \sqrt{(m_1^2 - m_2^2 - m_a^2)^2 - 4m_2^2 m_a^2}$.

2. Lepton bremsstrahlung

We next consider the lepton bremsstrahlung process, as shown in the upper middle and upper right diagrams of Fig. 2. There are two different lepton bremsstrahlung processes: $e^- p \rightarrow \mu^- p a$ and $\mu^- p \rightarrow e^- p a$. Note that for the case where $m_\mu > m_a + m_e$, we consider the muon decay process of $\mu^- \rightarrow e^- + a$, but not the $\mu^- p \rightarrow e^- p a$ process; we consider the latter process only in the case where $m_\mu < m_a + m_e$.

For both $e^- p \rightarrow \mu^- p a$ and $\mu^- p \rightarrow e^- p a$, there are two diagrams (ALPs emitted either from the initial or from the final states), and the matrix element for each process is given by

$$i\mathcal{M}_b = -4\pi\alpha g_{ae\mu} [\bar{u}_4 i\gamma^\nu u_2] \frac{ig_{\mu\nu}}{(P_2 - P_4)^2} \left[\bar{u}_3 i\gamma^\mu \frac{\not{P} + m_3}{P^2 - m_3^2} i\gamma_5 u_1 + \bar{u}_3 i\gamma_5 \frac{\not{Q} + m_1}{Q^2 - m_1^2} i\gamma^\mu u_1 \right], \quad (\text{A7})$$

where $u_j \equiv u(P_j)$ is the Dirac spinor with $j = (1, 2, 3, 4)$, $P = P_1 - P_a$, and $Q = P_3 + P_a$. In this case, the ALP production rate is given by

$$\frac{d^2 n_b}{dt dE_a} = \frac{|\mathbf{p}_a|}{4\pi^2} \int \prod_{i=1}^4 d\Phi_i f_1 f_2 (1 - f_3) (1 - f_4) (2\pi)^4 \delta^{(4)}(P_i - P_j) |\mathcal{M}_b|^2, \quad (\text{A8})$$

where $P_i = P_1 + P_2$ and $P_j = P_3 + P_4 + P_a$.

To simplify the computation, we note that the proton mass is significantly larger than the temperature, $T \sim 30$ MeV. We thus follow Ref. [72] to use the approximation where the proton is static such that $E_2 \simeq E_4 \simeq m_p$, which leads to $\bar{u}_4 \gamma^\nu u_2^s \simeq 2m_p \delta^{\nu 0} \delta^{rs}$ and $(P_2 - P_4)^2 \simeq -|\mathbf{q}|^2$, where $\mathbf{q} = \mathbf{p}_2 - \mathbf{p}_4 = \mathbf{p}_3 + \mathbf{p}_a - \mathbf{p}_1$. We further include plasma effects on the photon propagator, such that the matrix element is

$$i\mathcal{M}_b = 2e^2 g m_p \frac{1}{|\mathbf{q}| \sqrt{\mathbf{q}^2 + k_s^2}} \left[\bar{u}_3 \gamma^0 \frac{\not{P} + m_3}{P^2 - m_3^2} \gamma_5 u_1 + \bar{u}_3 \gamma_5 \frac{\not{Q} + m_1}{Q^2 - m_1^2} \gamma^0 u_1 \right], \quad (\text{A9})$$

where $k_s^2 = (4\pi\alpha/T) \sum_j Z_j^2 n_j$ is the Debye screening scale with n_j being the number density of ion j with charge $Z_j e$ [73]; in our analysis, we only consider the contribution of protons.

We next integrate out $d\Phi_4$ by using of three of the four delta functions to obtain [50,51]

$$\frac{d^2 n_b}{dtdE_a} = \frac{n_{\text{eff}} |\mathbf{p}_a|}{8m_p^2 4\pi^2} \int d\Phi_1 d\Phi_3 f_1(1-f_3)(2\pi)\delta(E_1 - E_a - E_3) |\mathcal{M}_b|^2, \quad (\text{A10})$$

where we have used the fact that the matrix element given in Eq. (A9) is independent of p_2 , and

$$n_{\text{eff}} = 2 \int \frac{d^3 p_2}{(2\pi)^3} f_p(E_2)(1 - f_p(E_4)) \quad (\text{A11})$$

is the effective proton number density, where f_p is the proton distribution function. Note that the Pauli blocking effects can be significant: Including $f_p(E_4)$ as in Eq. (A11) can decrease n_{eff} up to 60% in the SN core [51]. For protons in the SN core, we use

$$f_p(E_p) = \frac{1}{e^{(E_p - \mu_p)/T} + 1}, \quad (\text{A12})$$

where μ_p is the proton chemical potential, and

$$E_p = \sqrt{m_p^{*2} + \mathbf{p}_2^2} + U_p, \quad (\text{A13})$$

where m_p^* is the effective proton mass, and U_p is the proton interaction potential. In our analysis, we use $p_2 \simeq p_4$ and adopt the profiles of m_p^* , μ_p , and U_p from the Garching profiles [43], which are shown in Fig. 1.

The ALP production rate can be further simplified as follows [50,51]:

$$\frac{d^2 n_b}{dtdE_a} = \frac{n_{\text{eff}} |\mathbf{p}_a|}{(2\pi)^6 32m_p^2} \int_{m_3}^{\infty} dE_3 \int_{-1}^1 dz_a dz_3 \int_0^{2\pi} d\phi |\mathbf{p}_1| |\mathbf{p}_3| f_1(1-f_3) |\mathcal{M}_b|^2, \quad (\text{A14})$$

where $|\mathbf{p}_1| = \sqrt{(E_3 + E_a)^2 - m_1^2}$, ϕ is the angle between the $\mathbf{p}_1 - \mathbf{p}_a$ plane and the $\mathbf{p}_1 - \mathbf{p}_3$ plane, $z_a = \cos \theta_{1a}$ with θ_{1a} being the angle between $\mathbf{p}_1$ and $\mathbf{p}_a$, and $z_3 = \cos \theta_{13}$ with θ_{13} being the angle between $\mathbf{p}_1$ and $\mathbf{p}_3$.

3. Electron-muon coalescence

The electron-muon coalescence process of $e^\mp + \mu^\pm \rightarrow a$, as shown in the lower left diagram of Fig. 2, occurs when $m_a > m_\mu + m_e$. We note that the $e^- + \mu^+ \rightarrow a$ process is more important than the $e^+ + \mu^- \rightarrow a$ process, primarily due to the higher distribution functions of the initial-state particles in the former case. The matrix element for these two processes is $\mathcal{M}_c = ig_{ae\mu} \bar{v}(P_2) \gamma_5 u(P_1)$, where P_1 and P_2 are the four-momenta of the e^-/μ^- and μ^+/e^+ particles, respectively. The ALP production rate for this process is

$$\frac{d^2 n_c}{dtdE_a} = \frac{|\mathbf{p}_a|}{4\pi^2} \int d\Phi_1 d\Phi_2 (f_\mu f_e^+ + f_e f_\mu^+) \times (2\pi)^4 \delta^{(4)}(P_1 + P_2 - P_a) |\mathcal{M}_c|^2, \quad (\text{A15})$$

where $f_i^+ = [e^{(E_i + \mu_i)/T} + 1]^{-1}$ is the distribution of antifermions with $i = e, \mu$, and $|\mathcal{M}_c|^2 = 2g_{ae\mu}^2 [m_a^2 - (m_\mu - m_e)^2]$ is the spin-summed matrix element. Thus, we have

$$\begin{aligned} \frac{d^2 n_c}{dtdE_a} &= \frac{|\mathbf{p}_a|}{4\pi} \int \frac{d\Phi_2}{E_1} (f_\mu f_e^+ + f_e f_\mu^+) \delta(E_1 + E_2 - E_a) |\mathcal{M}_c|^2 \\ &= \frac{|\mathcal{M}_c|^2}{32\pi^3} \int_{E_2^-}^{E_2^+} dE_2 (f_\mu f_e^+ + f_e f_\mu^+), \end{aligned} \quad (\text{A16})$$

where

$$E_2^\pm = \frac{E_a(m_a^2 - m_1^2 + m_2^2)}{2m_a^2} \pm \frac{\sqrt{E_a^2 - m_a^2}}{2m_a^2} I. \quad (\text{A17})$$

4. Semi-Compton

Finally we consider the semi-Compton processes: $e + \gamma \rightarrow \mu + a$ and $\mu + \gamma \rightarrow e + a$. Each process proceeds via two channels, the u and s channel, shown in the lower middle and lower right diagrams of Fig. 2, respectively. Recently, Ref. [41] found that the semi-Compton process dominates over the lepton bremsstrahlung process, for the $a - e - e$ coupling. While we confirm the qualitative conclusion of Ref. [41], we also identify an error in their matrix element, which led to an overestimation of the axion production rate.

For both $\gamma(P_\gamma) + e(P_1) \rightarrow \mu(P_2) + a(P_a)$ and $\gamma(P_\gamma) + \mu(P_1) \rightarrow e(P_2) + a(P_a)$, the matrix element is

$$i\mathcal{M}_{\text{Com}} = eg_{ae\mu} \left[\bar{u}_2(-i\gamma^\mu)\epsilon_\mu \frac{i(\not{P}_2 - \not{P}_\gamma + m_2)}{u - m_2^2} \gamma_5 u_1 + \bar{u}_2 \gamma_5 \frac{i(\not{P}_1 + \not{P}_\gamma + m_1)}{s - m_1^2} (-i\gamma^\mu)\epsilon_\mu u_1 \right], \quad (\text{A18})$$

where P_γ is the four-momentum of the initial photon, P_1 (P_2) is the four-momentum of the initial (final) lepton, $m_1^2 = P_1^2$, $m_2^2 = P_2^2$, $s = (P_1 + P_\gamma)^2$, and $u = (P_2 - P_\gamma)^2$. The squared matrix element is thus

$$|\mathcal{M}_{\text{Com}}|^2 = 4\pi\alpha g_{ae\mu}^2 (\mathcal{F}_u + \mathcal{F}_s + \mathcal{F}_{su}), \quad (\text{A19})$$

where

$$\mathcal{F}_u = 4 \frac{-(s - m_1^2)(u - m_2^2) + 2(m_1 m_2 - m_2^2)(u - m_1 m_2) + 2m_2^2 m_a^2}{(u - m_2^2)^2}, \quad (\text{A20})$$

$$\mathcal{F}_s = 4 \frac{-(s - m_1^2)(u - m_2^2) + 2(m_1 m_2 - m_1^2)(s - m_1 m_2) + 2m_1^2 m_a^2}{(s - m_1^2)^2}, \quad (\text{A21})$$

$$\mathcal{F}_{su} = 8 \frac{-(s - m_1 m_2 - m_a^2 + m_2^2)(u - m_1 m_2 - m_a^2 + m_1^2) + m_2^2 s + m_1^2 u - m_1^2 m_2^2}{(u - m_2^2)(s - m_1^2)}. \quad (\text{A22})$$

Note that here we have neglected the plasma corrections on photons. Also note that in the limit of $m_1 \rightarrow 0$ and $m_2 \rightarrow 0$, Eqs. (A20) and (A21) agree with $\mathcal{F}_u$ and $\mathcal{F}_s$ in Ref. [41], respectively; however, Eq. (A22) differs from $\mathcal{F}_{su}$ in Ref. [41] by an overall sign. We find that the error in $\mathcal{F}_{su}$ in Ref. [41] led to an overestimate of the axion production rate.

The ALP production rate in the semi-Compton process is given by [41]

$$\begin{aligned} \frac{d^2 n_{\text{Com}}}{dt dE_a} &= \int d\Phi_1 d\Phi_\gamma d\Phi_2 \frac{|\mathbf{p}_a| d\Omega_a}{2(2\pi)^3} f_1 f_\gamma (1 - f_2) (2\pi)^4 \delta^{(4)}(P_1 + P_\gamma - P_2 - P_a) |\mathcal{M}_{\text{Com}}|^2 \\ &= \int d\Phi_1 d\Phi_\gamma \frac{1}{2E_2} \frac{|\mathbf{p}_a| d\Omega_a}{2(2\pi)^3} f_1 f_\gamma (1 - f_2) (2\pi) \delta(E_1 + E_\gamma - E_2 - E_a) |\mathcal{M}_{\text{Com}}|^2, \end{aligned} \quad (\text{A23})$$

where $f_\gamma(E_\gamma) = (e^{E_\gamma/T} - 1)^{-1}$, $|\mathbf{p}_a| = \sqrt{E_a^2 - m_a^2}$, and we have used the three delta functions for momentum conservation so that $E_2 = \sqrt{(\mathbf{p}_1 + \mathbf{p}_\gamma - \mathbf{p}_a)^2 + m_2^2}$ in the second line of Eq. (A23). The $\gamma + \mu \rightarrow e + a$ process is divergent when the energy of the initial photon goes to zero. To remove such an IR divergence, we use the plasma frequency, ω_p , as the lower cutoff on the photon energy, such that the dispersion relation of photons is given by $E_\gamma^2 = |\mathbf{p}_\gamma|^2 + \omega_p^2$. In the relativistic limit, the plasma frequency is given by [74]

$$\omega_p^2 = \frac{4\alpha}{3\pi} \left(\mu_e^2 + \frac{1}{3} \pi^2 T^2 \right). \quad (\text{A24})$$

Although Eq. (A23) requires integration over eight variables, several of these variables do not enter the integrand (the matrix element and the delta function that ensures energy conservation) and are thus associated with integration symmetries, allowing them to be integrated out trivially. In our case, the matrix element is a function of s and u , which can be expressed as

$$s = 2E_\gamma E_1 - 2|\mathbf{p}_\gamma||\mathbf{p}_1|X + m_1^2 + \omega_p^2, \quad (\text{A25})$$

$$u = m_1^2 + m_a^2 - 2E_1E_a + \frac{2|\mathbf{p}_1||\mathbf{p}_a|}{|\mathbf{p}_{\text{tot}}|} \left((|\mathbf{p}_1| + |\mathbf{p}_\gamma|X)z_a + |\mathbf{p}_\gamma|\sqrt{1-X^2}\sqrt{1-z_a^2}\cos\phi_{1a} \right), \quad (\text{A26})$$

where $X \equiv \cos\theta_{1\gamma}$ with $\theta_{1\gamma}$ denoting the angle between $\mathbf{p}_1$ and $\mathbf{p}_\gamma$, $\mathbf{p}_{\text{tot}} = \mathbf{p}_1 + \mathbf{p}_\gamma$ such that $|\mathbf{p}_{\text{tot}}| = \sqrt{\mathbf{p}_1^2 + \mathbf{p}_\gamma^2 + 2|\mathbf{p}_1||\mathbf{p}_\gamma|X}$, $z_a \equiv \cos\theta_a$ with θ_a denoting the angle between $\mathbf{p}_a$ and $\mathbf{p}_{\text{tot}}$, and ϕ_{1a} is the angle between the $\mathbf{p}_{\text{tot}} - \mathbf{p}_1$ plane and the $\mathbf{p}_{\text{tot}} - \mathbf{p}_a$ plane. The energy-conservation delta function can be expressed as

$$\delta(E_1 + E_\gamma - E_2 - E_a) = \delta\left(E_1 + E_\gamma - E_a - \sqrt{m_2^2 + \mathbf{p}_a^2 + \mathbf{p}_{\text{tot}}^2 + 2|\mathbf{p}_a||\mathbf{p}_{\text{tot}}|z_a}\right). \quad (\text{A27})$$

Thus, we find that the integrand only depends on the following five variables: $|\mathbf{p}_1|$, $|\mathbf{p}_\gamma|$, X , z_a , and ϕ_{1a} .

We next integrate out the integration variables that correspond to integration symmetries. Note that neither the orientation of the photon momentum ($\mathbf{p}_\gamma$) nor that of the initial lepton ($\mathbf{p}_1$) appears in the integrand; only their relative angle (X) does. Thus, we can choose the z axis along $\mathbf{p}_\gamma$ so that

$$d^3p_1 d^3p_\gamma = 8\pi^2 \mathbf{p}_1^2 d|\mathbf{p}_1| dX \mathbf{p}_\gamma^2 d|\mathbf{p}_\gamma|, \quad (\text{A28})$$

where we have integrated out the solid angle of the photon and the azimuthal angle of $\mathbf{p}_1$, which are integration symmetries. Because the azimuthal angle of $\mathbf{p}_1$ is an integration symmetry, we can set it to be zero so that $\phi_{1a} = \phi_a$ and $d\Omega_a = dz_a d\phi_{1a}$. We then use the last delta function to remove the integration on z_a in Eq. (A23):

$$\frac{d^2 n_{\text{Com}}}{dt dE_a} = \int_{m_1}^{\infty} |\mathbf{p}_1| dE_1 \int_{\omega_p}^{\infty} |\mathbf{p}_\gamma| dE_\gamma \int_{-1}^1 dX \int_0^{2\pi} d\phi_{1a} \frac{f_1 f_\gamma (1 - f_2)}{512\pi^6 |\mathbf{p}_1 + \mathbf{p}_\gamma|} |\mathcal{M}_{\text{Com}}|^2 \Theta(1 - |z_a^0|), \quad (\text{A29})$$

where

$$z_a^0 = \frac{2(E_1 + E_\gamma)E_a - 2E_1E_\gamma + 2|\mathbf{p}_1||\mathbf{p}_\gamma|X - m_a^2 - \omega_p^2 - m_1^2 + m_2^2}{2|\mathbf{p}_a||\mathbf{p}_{\text{tot}}|}, \quad (\text{A30})$$

and we have changed the integration variables from $d|\mathbf{p}_1|d|\mathbf{p}_\gamma|$ to dE_1dE_γ . We note that in the limit of $m_1 = m_2 = 0$ and $\omega_p = 0$, Eq. (A29) agrees Ref. [41], which neglects both the electron mass and the plasma corrections to photons. The Heaviside function Θ in Eq. (A29) selects the physical region of the parameter space: $|z_a^0| \leq 1$, which is equivalent to

$$X_{\min} \leq X \leq X_{\max}, \quad (\text{A31})$$

where $X_{\max/\min} = \frac{1}{2|\mathbf{p}_a||\mathbf{p}_{\text{tot}}|} [(|\mathbf{p}_a| \pm \sqrt{A})^2 - |\mathbf{p}_1|^2 - |\mathbf{p}_2|^2]$ with $A = (E_1 + E_\gamma - E_a)^2 - m_2^2$. Note that $A > 0$ is always valid since $E_1 + E_\gamma - E_a \geq m_2$ is guaranteed by the energy conservation. Thus, we have

$$\frac{d^2 n_{\text{Com}}}{dt dE_a} = \int_{m_1}^{\infty} |\mathbf{p}_1| dE_1 \int_{E_{\gamma\min}}^{\infty} |\mathbf{p}_\gamma| dE_\gamma \int_{X_{\min}}^{X_{\max}} dX \int_0^{2\pi} d\phi_{1a} \frac{f_1 f_\gamma (1 - f_2)}{512\pi^6 |\mathbf{p}_1 + \mathbf{p}_\gamma|} |\mathcal{M}_{\text{Com}}|^2 \Theta(1 - |X|), \quad (\text{A32})$$

where $E_{\gamma\min} = \max\{\omega_p, E_a + m_2 - E_1\}$. We use Eq. (A32) in the numerical analysis for the semi-Compton process.

APPENDIX B: THE ABSORPTION OF ALPS

In this section we calculate the absorption rate of LFV ALPs in the SN. Consider a general ALP absorption process of $j \rightarrow i$, where j and i denote the initial and final state particles, respectively, and one of the initial state particles is an ALP a ; all other particles in the $j \rightarrow i$ process are SM particles that are in equilibrium. The ALP absorption rate is given by [58,75]

$$\Gamma_{\text{abs}}(E_a) = \frac{1}{2E_a} \int \prod_{j \neq a} d\Phi_j f_j \prod_i d\Phi_i (1 \pm f_i) (2\pi)^4 \delta^{(4)}(P_j - P_i) |\mathcal{M}|^2, \quad (\text{B1})$$

where the quantities have the same definitions as in Eq. (A1). Note that in the 2-to-N process of $a + 1 \rightarrow i$, Eq. (B1) can be understood via $\Gamma_{\text{abs}}^a = n_1 v \sigma(a + 1 \rightarrow i)$, where σ is the cross section, v is the relative velocity between a and 1, and n_1 is the number density of particle 1.

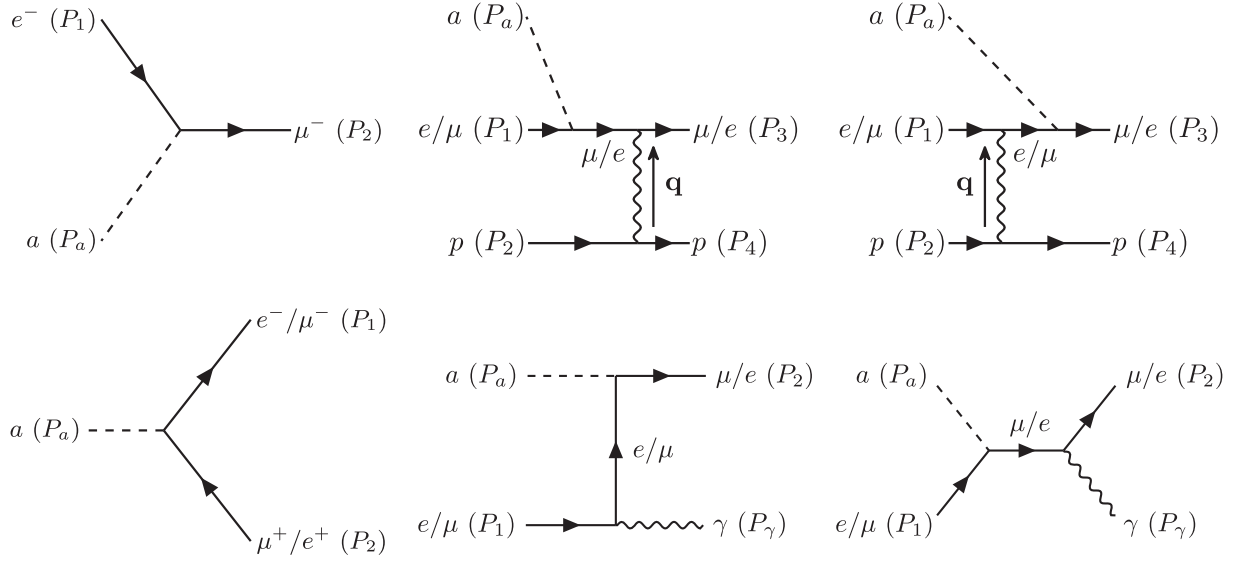


FIG. 5. Absorption processes of LFV ALP in the SN. Upper: $e - a$ coalescence (left) and inverse bremsstrahlung (middle and right). Lower: ALP decay (left) and inverse Compton (middle and right).

In our analysis, we consider ALP absorption through the following four processes: (1) $e - a$ coalescence, (2) inverse bremsstrahlung, (3) ALP decay, and (4) inverse-Compton, which are shown in Fig. 5; these correspond to the inverse processes of those shown in Fig. 2.

The absorption rate given in Eq. (B1) is related to the production rate given in Eq. (A1). First note that for both Fermi-Dirac and Bose-Einstein distributions, the distribution function f obeys $1 \pm f = e^{(E-\mu)/T} f$, where the plus and minus signs on the left-hand side correspond to the Bose-Einstein and Fermi-Dirac distributions, respectively, and E (μ) is the energy (chemical potential) [75]. This then leads to

$$\begin{aligned} \Gamma_{\text{abs}}(E_a) &= \frac{1}{2E_a} \int \prod_{j \neq a} d\Phi_j e^{-(E_j - \mu_j)/T} (1 \pm f_j) \prod_i d\Phi_i e^{(E_i - \mu_i)/T} f_i (2\pi)^4 \delta^{(4)}(P_j - P_i) |\mathcal{M}|^2, \\ &= e^{(E_a - \mu_a^0)/T} \frac{1}{2E_a} \int \prod_i d\Phi_i f_i \prod_{j \neq a} d\Phi_j (1 \pm f_j) (2\pi)^4 \delta^{(4)}(P_j - P_i) |\mathcal{M}|^2, \end{aligned} \quad (\text{B2})$$

where we have used the energy conservation $E_a = \sum_i E_i - \sum_{j \neq a} E_j$ and defined $\mu_a^0 \equiv \sum_i \mu_i - \sum_{j \neq a} \mu_j$. By comparing Eq. (B2) with Eq. (A1), one obtains the following relation between the absorption rate and the production rate:

$$\Gamma_{\text{abs}}(E_a) = e^{(E_a - \mu_a^0)/T} \frac{2\pi^2}{|\mathbf{p}_a| E_a} \frac{d^2 n_a}{dt dE_a}. \quad (\text{B3})$$

Thus the absorption rates of the four processes shown in Fig. 5 can be obtained through the production rates of their inverse processes, which are given in Eqs. (A5), (A14), (A16), and (A32).

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