Decoherence of spatial superpositions along stationary worldlines

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We analyze the decoherence of a particle's spatial superposition moving along a stationary worldline through the Minkowski vacuum. The particle is modeled via an internal degree of freedom that couples to a scalar field, and an external degree of freedom, i.e., its quantized center-of-mass motion around the stationary worldline. Assuming a separation of timescales between the particle's internal and external dynamics, we first obtain an effective redshifted polarizability of the particle, characterizing the trajectory-dependent linear response of the internal oscillator to the field. We then derive a quantum Brownian motion master equation for the particle's center of mass, under the Born-Markov approximation, which describes its decoherence in the position basis, as well as Hamiltonian modifications corresponding to a dispersive potential. The resulting decoherence has two components: (1) arising from a modified field spectrum observed by the particle; and (2) due to a differential time dilation over the particle's extended spatial wave function. For stationary trajectories, both contributions take an effectively thermal form. We evaluate the decoherence rates for two specific cases, hyperbolic and uniform circular motion.

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I. INTRODUCTION

A particle undergoing uniformly accelerated linear motion through the Minkowski vacuum of a quantum field will perceive a thermal field environment. This effective thermality of the environment, also known as the Davies-Unruh effect [1–3], raises the question: what is the backaction of such a thermal environment on the particle? It has been shown, for example, that the effective thermal field can induce a range of remarkable effects such as momentum recoil [4], Casimir-Polder forces between atoms [5] and acceleration-induced transparency and a stimulated Unruh effect [6]. Recent works have also demonstrated that the backaction of such an effective field leads to a momentum diffusion and drag on a polarizable particle, equivalent to that of a particle immersed in a thermal bath at the Davies-Unruh temperature $T_{\mathrm{DU}} = \hbar a/(2\pi k_{\mathrm{B}} c)$ [7]. Considering that the momentum diffusion of a polarizable particle in thermal radiation is linked to decoherence of its quantized center-of-mass [8,9], it is natural to ask: how does a spatial superposition decohere as it undergoes accelerated motion?

A variety of models have been considered for investigating decoherence of particles moving noninertially in flat spacetime, as well as in curved spacetime, as depicted in Fig 1. In general, two different decoherence mechanisms arise: (1) the direct backaction of the modified field spectrum onto the system causes it to decohere, here referred to as Davies-Unruh decoherence; and (2) the differential time dilation over the particle's extended wave function, which engenders which-path information. We will refer to this as time-dilation decoherence.

Davies-Unruh decoherence has been studied in the context of accelerated motion through flat spacetime [10–18] and in connection with black holes, where quantum decoherence near black-hole event horizons offers a compelling window into the interplay between quantum information and general relativity. It is well known that after matter falls into a black hole, it emerges as thermal Hawking radiation, losing any coherence [26]; striking recent results have demonstrated that the mere presence of an event horizon decoheres a spatial superposition nearby [22–25]. Such decoherence of charged (massive) particles is attributed to the measurement of "which-path" information by the black-hole horizon via photons (gravitons). Thus, it has been conjectured that a particle in the vicinity of a horizon will experience

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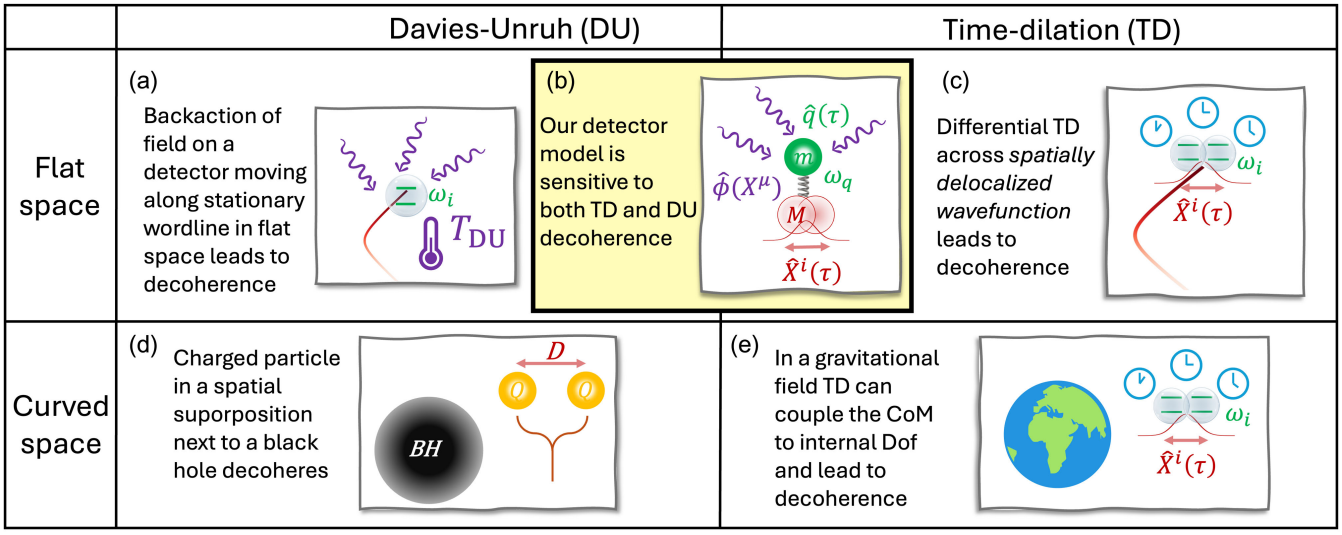


FIG. 1. Summary of commonly studied decoherence mechanisms in flat spacetime as well as curved spacetime. (a): Unruh-DeWitt field detectors with a classical center of mass consisting of either a two-level system (with frequency ω_i) or a harmonic oscillator have been studied in a plethora of situations. Along a hyperbolic worldline such a detector will see an effective temperature T_{DU}. The thermalization and decoherence of such detectors has been studied in [10–18]. (b): Our model (see Fig. 2) where a particle with an internal oscillator degree of freedom in a spatial superposition of $\hat{X}^i(\tau)$ is coupled to a field via its polarizability combines the decoherence mechanisms of (a) and (c). (c): Even in the absence of an external field, a detector's internal degree of freedom can couple to its quantized center of mass via time dilation. This effect has been described in [19–21]. (d): A charged particle without an internal degree of freedom, brought into a spatial superposition of size D next to a black hole, will decohere due to the emission of photons across the black hole horizon [22–25]. (e): Similar to (c), the time dilation in a gravitational field can couple a system's center of mass to its internal degree of freedom without a system-field coupling [19–21].

decoherence both from the presence of the horizon as well as from the Hawking radiation emitted. Time-dilation decoherence, similarly, has been explored in the context of accelerated motion and gravitational fields [19–21]. Here it was shown that a system with internal degrees of freedom as well as a quantized center of mass will experience a system-bath coupling induced by the differential time dilation across the center-of-mass wave function, where the center of mass plays the role of the system and the internal degrees of freedom act as the bath.

In this paper, we analyze the open system dynamics of the quantized center-of-mass motion of a particle moving along a stationary worldline and interacting with a scalar field [27,28]. We show that the effective polarizability of the particle undergoing noninertial motion experiences an effective redshift. We derive a quantum master equation for the center of mass, and show that the interaction of the particle with its effective environment results in: (1) a dispersive potential, and (2) decoherence and dissipation. The decoherence seen by the particle results both from the modified spectrum of the field seen by the particle (Davies-Unruh decoherence), as well as the differential time dilation over the particle's wave function (time-dilation decoherence).

The remainder of this paper is organized as follows: In Sec. II we describe our model and derive the linearized interaction Hamiltonian between the quantized center-of-mass motion and its effective environment, encompassing the particle's internal degrees and the field. In Sec. III we derive the quantum Brownian motion master equation and calculate the decoherence rates arising from two distinct mechanisms: Davies-Unruh decoherence and time dilation. In Secs. IV and V we present calculations of the decoherence under uniform hyperbolic and circular motion, respectively. Finally, we discuss our results and give an outlook in Sec. VI.

II. MODEL

Let us consider a particle moving along a worldline $x_0^\mu(\tau)$ in Minkowski space. The particle is composed of an internal oscillator degree of freedom $\hat{q}$ (conjugate momentum $\hat{p}$) and an external quantized center-of-mass degree of freedom around the worldline $x_0^\mu(\tau)$, which we express in the local rest frame of the particle and denote by $\hat{X}^i$ (conjugate momentum $\hat{P}_i$)[1]. We will assume the particle's worldline $x_0^\mu(\tau)$ to be stationary, such that the state of the

[1]For simplicity, and to avoid the pathologies that arise from writing a relativistic position operator (see, e.g., [29,30]), we treat $\hat{X}^i$ and $\hat{P}_i$ as nonrelativistic observables. This amounts to expressing $\hat{X}^i$ and $\hat{P}_i$ in local normal coordinates around $x_0^\mu(\tau)$ and demanding that the particle's position wave function be confined to within a length scale $\ell \sim c^2/\sqrt{a_\mu a^\mu}$, where a^μ is the proper acceleration along $x_0^\mu(\tau)$ [31].

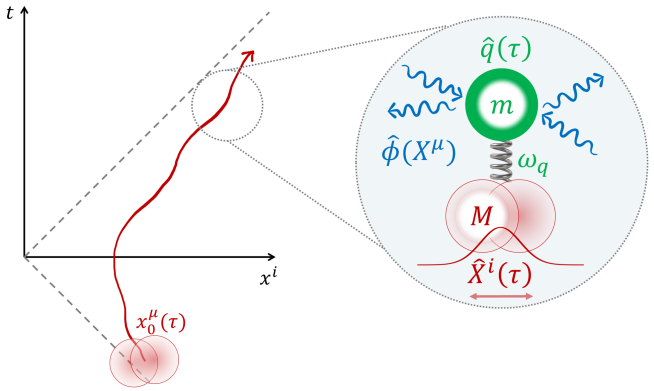


FIG. 2. Schematic representation of a particle with mass M moving along a worldline $X_0^\mu(\tau)$ through Minkowski spacetime. The particle is prepared in a spatial superposition, with $\hat{X}^i(\tau)$ as its quantized center-of-mass position. The particle has an internal oscillator degree of freedom with position $\hat{q}(\tau)$, mass m, and frequency ω_q, and couples to a scalar field $\hat{\phi}(\hat{X}^\mu(\tau))$.

field observed by the detector has a time-independent energy spectrum [27,28]. Here Greek indices $(\mu, \nu, \text{etc.})$ are spacetime indices, whereas Latin indices $(i, j, \text{etc.})$ are spatial indices.

The internal oscillator represents a chargelike degree of freedom and is coupled to a $3+1$ dimensional scalar field at the position of the particle (see Fig. 2), which is given in the local rest frame of the particle as

$$\hat{\phi}(X^\mu) = \sum_{\mathbf{k}} [\hat{g}_{\mathbf{k}} \phi_{\mathbf{k}}(X^\mu) + \hat{g}^\dagger_{\mathbf{k}} \phi^*_{\mathbf{k}}(X^\mu)]. \tag{1}$$

Here $\hat{g}^\dagger_{\mathbf{k}}$ and $\hat{g}_{\mathbf{k}}$ are the creation and annihilation operators associated with the mode functions $\phi_{\mathbf{k}}(X^\mu)$ and $\phi^*_{\mathbf{k}}(X^\mu)$, following the canonical commutation relation $[\hat{g}_{\mathbf{k}}, \hat{g}^\dagger_{\mathbf{k}'}] = \delta_{\mathbf{k},\mathbf{k}'}$.

In the local rest frame of the particle, the total Hamiltonian is $\hat{H} = \hat{H}_S + \hat{H}_F$, with the field Hamiltonian $\hat{H}_F$ given by

$$\hat{H}_F = \sum_{\mathbf{k}} \hbar \Omega_{\mathbf{k}} \hat{g}^\dagger_{\mathbf{k}} \hat{g}_{\mathbf{k}}. \tag{2}$$

The system Hamiltonian in Schrödinger picture $\hat{H}_S$ includes the rest energy of the particle, the kinetic energy of the center of mass as well as the interaction of the particle with the field (see Appendix A for details)

$$\hat{H}_S = \sqrt{-g_{00}(\hat{X}^i)(\hat{E}^2_{\text{rest}} + c^2 \hat{P}_i \hat{P}^i)}, \tag{3}$$

where $g_{00}(\hat{X}^i)$ is the 00-component of the locally static metric $g_{\mu\nu}$ and describes the gravitational time dilation experienced by the internal oscillator degree of freedom, with τ as the proper time of the moving particle. The rest energy of the system is

$$\hat{E}_{\text{rest}} = \mathbb{1} Mc^2 + \frac{1}{2m}[\hat{p} - e\hat{\phi}(\hat{X}^i)]^2 + \frac{1}{2} m\omega_q^2 \hat{q}^2, \tag{4}$$

with M the rest mass and ω_q the internal oscillator frequency seen by a comoving observer. The internal oscillator couples to the field $\hat{\phi}(\hat{X}^i)$ at the position of the particle, with a strength e. Under the assumption that $\Delta P_i/(Mc) \ll 1$ in the particle's rest frame, i.e. the quantized center-of-mass motion of the particle in the proper frame is nonrelativistic, we can simplify the system Hamiltonian to

$$\hat{H}_S \approx \sqrt{-g_{00}(\hat{X}^i)} \left(\hat{E}_{\text{rest}} + \frac{\hat{P}_i \hat{P}^i}{2M} \right). \tag{5}$$

We now make a separation of timescales assumption, considering the internal oscillator dynamics to be much faster than the quantized center-of-mass motion. We first determine the coupled dynamics of the internal oscillator and the field by solving their coupled Heisenberg equations of motion [7] (see Appendix B 1)

$$\ddot{\hat{q}} - g_{00}\omega_q^2 \hat{q} = -\sqrt{-g_{00}} \frac{e}{m} \dot{\hat{\phi}}_{\text{tot}}(\hat{X}^i, \tau). \tag{6}$$

We solve for the dynamics of the internal oscillator, as driven by the field, assuming that the internal degree of freedom has a sufficiently fast equilibration time such that its dynamics is pinned to that of the field. Thus in stationary state we can describe the oscillator amplitude in terms of an effective induced dipole as (see Appendix B 2 for details)

$$\begin{aligned} \hat{d}_{\text{st}}(\hat{X}^i) &\equiv e\hat{q}_{\text{st}} \\ &= -\frac{\mathrm{i}}{2\pi} \sqrt{-g_{00}(\hat{X}^i)} \int_{-\infty}^{\infty} \mathrm{d}\omega\, \omega \alpha(\omega) \underline{\hat{\phi}}(\hat{X}^i, \omega), \end{aligned} \tag{7}$$

with a redshifted linear response

$$\alpha(\omega) \approx \alpha_0(\omega)[1 - \eta(\omega) a_i \hat{X}^i/c^2] \tag{8}$$

and $\underline{\hat{\phi}}(\hat{X}^i, \omega)$ defined via the spectral decomposition:

$$\hat{\phi}(\hat{X}^i) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \underline{\hat{\phi}}(\hat{X}^i, \omega). \tag{9}$$

Such an effective redshifted polarizability, which captures the linear response of a particle undergoing noninertial motion, can be generalized to other particles (e.g., electrons).

Here we have neglected second and higher-order terms in $\hat{X}^i$, assuming that the quantized center-of-mass is sufficiently localized such that $\Delta X^i \ll c^2/a$. The redshifted polarizability stems from the fact that the characteristic

frequencies experience a time dilation due to the spread of the particle's wave function around $x_0^\mu(\tau)$. Here $\alpha_0(\omega)$ is the bare polarizability in the rest frame of the particle and $\eta(\omega)$ is a redshift-induced correction given by

$$\alpha_0(\omega) = \frac{e^2/m}{-\omega^2 + 2i\beta\omega^3 + \omega_q^2}, \tag{10}$$

$$\eta(\omega) = \frac{2i\beta\omega^3 + 2\omega_q^2}{-\omega^2 + 2i\beta\omega^3 + \omega_q^2}, \tag{11}$$

where $\beta = e^2/(8\pi mc^3\epsilon_0)$.[2] We point out that $\alpha_0(\omega)$ is technically not analytic in the upper half of the complex plane, due to the appearance of the ω^3 term in the denominator. However, since β is small compared to $1/\omega_q$, the contributions from this pole can generally be neglected. This allows us to simplify the interaction Hamiltonian into a form similar to the electric-dipole form of the Hamiltonian (see Appendix B 1 for a derivation).

Having solved for the coupled dynamics of the internal oscillator and the field, and assuming a separation of timescales, we now turn to the slow quantized center-of-mass motion. The quantized center-of-mass Hamiltonian consists of a kinetic center-of-mass contribution and an interaction term

$$\hat{H}_{\mathrm{M}} \equiv \sqrt{-g_{00}(\hat{X}^i)}\left(Mc^2 + \frac{\hat{P}_i\hat{P}^i}{2M}\right) - \frac{1}{4}\{\dot{\hat{d}}_{\mathrm{st}}(\hat{X}^i), \hat{\phi}(\hat{X}^i)\}, \tag{12}$$

where we use the steady-state dynamics of the internal oscillator to determine its response to the field [Eq. (7)]. Note that the coupling term between dipole and field is symmetrized to ensure the Hermiticity of $\hat{H}_{\mathrm{M}}$.

We assume that the extent of the spatial wave function of our particle is small enough such that we can expand around the classical center-of-mass worldline of the particle, which now acts as a reference worldline. To make this expansion we write the metric in local normal coordinates [34]

$$g_{00}(\hat{X}^i) \approx -\left(\mathbb{1} + \frac{a_i(\tau)}{c^2}\hat{X}^i\right)^2 - R_{0i0j}(\tau)\hat{X}^i\hat{X}^j, \tag{13}$$

$$g_{0i}(\hat{X}^i) \approx -\frac{2}{3}R_{0jik}\hat{X}^j\hat{X}^k, \tag{14}$$

$$g_{ij}(\hat{X}^i) \approx \delta_{ij} - \frac{1}{3}R_{ikjl}(\tau)\hat{X}^k\hat{X}^l, \tag{15}$$

with $a^\mu(\tau)$ the proper acceleration at $x_0^\mu(\tau)$ and $R_{\mu\nu\sigma\rho}(\tau)$ the curvature tensor evaluated on the particle worldline. We have ignored third and higher order terms in $\hat{X}^i$ in the above expansion. For an observer on a stationary worldline in Minkowski space the curvature tensor $R_{\mu\nu\sigma\rho} = 0$. This allows us to study the quantum dynamics of the center of mass by evaluating field quantities on the prescribed classical worldline $x_0^\mu(\tau)$ only. Expanding the metric as well as the field to first order in $\hat{X}^i$ and neglecting terms proportional to $\hat{P}^i/(Mc)$ leaves us with the system Hamiltonian

$$\hat{H}_{\mathrm{M}} = \mathbb{1}Mc^2 - \frac{1}{4}\{\dot{\hat{d}}_{0,\mathrm{st}}, \hat{\phi}\} + \frac{\hat{P}_i\hat{P}^i}{2M} + Ma_i\hat{X}^i - \frac{1}{2}\hat{X}^i\{\dot{\hat{d}}_{0,\mathrm{st}}, \partial_i\hat{\phi}\} + \frac{a_i\hat{X}^i}{4c^2}\{\dot{\hat{d}}_{1,\mathrm{st}} - \dot{\hat{d}}_{0,\mathrm{st}}, \hat{\phi}\}, \tag{16}$$

where we use the shorthand notation $\hat{\phi} \coloneqq \hat{\phi}(X^i = 0)$ and define induced dipole moments in steady state:

$$\hat{d}_{0,\mathrm{st}} = -\frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega\, \omega\alpha_0(\omega)\underline{\hat{\phi}}(\omega), \tag{17}$$

$$\hat{d}_{1,\mathrm{st}} = -\frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega\, \omega\alpha_0(\omega)\eta(\omega)\underline{\hat{\phi}}(\omega). \tag{18}$$

Having written the Hamiltonian in Eq. (16), we can delineate two different mechanisms that give rise to an interaction of the quantized center of mass with its environment. Moving to the interaction picture (indicated via an explicit τ dependence of the observables), we separate the total interaction Hamiltonian into two components:

(1) The particle's quantized center-of-mass interacts with the field via the internal oscillator, as given by the following interaction Hamiltonian:

$$\hat{H}_{\mathrm{int}}^{\mathrm{DU}}(\tau) \equiv -\frac{1}{2}\hat{X}^i(\tau)\{\dot{\hat{d}}_{0,\mathrm{st}}(\tau), \partial_i\hat{\phi}(\tau)\}. \tag{19}$$

The trajectory-dependent vacuum state observed by the particle exerts a backaction leading to center-of-mass decoherence.

(2) The differential time dilation across the center-of-mass wave function, described by the redshift factor g_{00}, gives rise to the interaction Hamiltonian

$$\hat{H}_{\mathrm{int}}^{\mathrm{TD}}(\tau) \equiv \frac{a_i\hat{X}^i(\tau)}{4c^2}\{\dot{\hat{d}}_{1,\mathrm{st}}(\tau) - \dot{\hat{d}}_{0,\mathrm{st}}(\tau), \hat{\phi}(\tau)\}. \tag{20}$$

These interaction terms stem from the redshifted response function $\alpha(\omega)$ of the particle. Although the

[2]While we are not dealing with an electromagnetic (EM) field, the factor of ϵ_0 arises from the choice of normalization of the field mode functions $\{\phi_{\mathbf{k}}, \phi_{\mathbf{k}}^*\}$ such that $|\phi_{\mathbf{k}}|^2 = \frac{\hbar}{2\epsilon_0\Omega_{\mathbf{k}}V}$. With such normalization, one can establish a correspondence with the EM field by identifying the scalar field $\hat{\phi}(X^\mu)$ as a component of the EM vector potential [32,33].

nature of time-dilation induced coupling of the center of mass to a bath is akin to that in [19], we assume in our model that the internal degree of freedom has a sufficiently fast equilibration time such that its dynamics reaches a steady state and is thus pinned to that of the field. It follows that all terms in $\hat{H}_{\rm int}^{\rm TD}$ effectively amount to an interaction between the center of mass and the field.[3]

III. OPEN SYSTEM DYNAMICS

A. Quantum Brownian motion master equation

In order to model the center-of-mass decoherence of our particle, we employ the quantum Brownian motion (QBM) master equation, which describes the reduced dynamics of the quantized center-of-mass density matrix ρ. We expand the interaction Hamiltonian in the interaction picture (indicated by an explicit time dependence) in small $\hat{X}^i$:

$$\begin{aligned}\hat{H}_{\rm int}(\tau) &= \hat{H}^0_{\rm int}(\tau) + \partial_i \hat{H}_{\rm int}(\tau)\hat{X}^i + \mathcal{O}((\hat{X}^i)^2) \\ &= \hat{H}^0_{\rm int}(\tau) + \hat{B}_i(\tau)\hat{X}^i + \mathcal{O}((\hat{X}^i)^2),\end{aligned} \tag{21}$$

with the bath operator defined as

$$\hat{B}_i(\tau) = \partial_i \hat{H}_{\rm int}(\tau). \tag{22}$$

Following our split of the interaction Hamiltonian into time-dilation and Davies-Unruh terms, we can do the same for the bath operator and define

$$\hat{B}_i(\tau) = \hat{B}_i^{\rm DU}(\tau) + \hat{B}_i^{\rm TD}(\tau). \tag{23}$$

Here the time-dilation and Davies-Unruh bath operators are defined, respectively, as

$$\hat{B}_i^{\rm DU}(\tau) = -\frac{1}{2}\{\dot{\hat{d}}_{0,\rm st}(\tau), \partial_i \hat{\phi}(\tau)\}; \tag{24}$$

$$\hat{B}_i^{\rm TD}(\tau) = \frac{a_i}{4c^2}\{\dot{\hat{d}}_{1,\rm st}(\tau) - \dot{\hat{d}}_{0,\rm st}(\tau), \hat{\phi}(\tau)\}. \tag{25}$$

The QBM master equation resulting from the above system-bath interaction is thus [35]

$$\begin{aligned}\dot{\hat{\rho}} = &-\Lambda_{ij}[\hat{X}^i,[\hat{X}^j,\hat{\rho}]] - \frac{\rm i}{\hbar}\Gamma_{ij}[\hat{X}^i,\{\hat{P}^j,\hat{\rho}\}] \\ &+ \frac{\rm i}{\hbar}C_i^{(1)}[\hat{X}^i,\hat{\rho}] - \frac{\rm i}{\hbar}C_{ij}^{(2)}[\hat{X}^i,\{\hat{X}^j,\hat{\rho}\}],\end{aligned} \tag{26}$$

[3]The equilibration time of an Unruh-DeWitt detector has been studied in [18]. Here we assume that $\tau \gg \tau_{\rm thermal}$, such that transient field dynamics can be ignored.

with i, j corresponding to the three spatial dimensions in the rest frame of the particle; repeated indices are summed over.

The derivation of the above QBM master equation relies on the validity of the Born-Markov approximation. The Markov approximation applies for our model of a polarizable particle interacting with a field along a stationary worldline under two specific assumptions. First, it was shown in Ref. [13] that an Unruh-DeWitt detector interacting with a scalar field moving on a stationary worldline reaches a steady state in the limit of $\tau \to \infty$. Second, it was shown in [36,37] that the Markov approximation applies after a time

$$t_{\rm markov} \gg \partial_\omega D^{\pm}(\omega)|_{\omega_q}, \tag{27}$$

which is analogous to saying that once transient effects have decayed away the effective bath spectrum will be sufficiently flat around ω_q, such that this condition can be satisfied [35]. The Born approximation, in addition, requires the interaction between the particle and the field to be weak. Together, these assumptions allow us to make the Born-Markov approximation.

In order, the terms in the above master equation represent:

(1) The diagonal terms Λ_{ii} correspond to the decoherence in the position basis:

$$\Lambda_{ij} = \frac{1}{2\hbar^2}\int_0^\infty {\rm d}\tau' \langle\{\hat{B}_i(\tau), \hat{B}_j(\tau-\tau')\}\rangle. \tag{28}$$

(2) The dissipation of the center-of-mass energy into the environment is

$$\Gamma_{ij} \equiv \frac{\rm i}{2M\hbar}\int_0^\infty {\rm d}\tau' \tau' \langle[\hat{B}_i(\tau), \hat{B}_j(\tau-\tau')]\rangle. \tag{29}$$

The decoherence rate Λ_{ii} is related to Γ_{ii} by the fluctuation-dissipation theorem: $2M\Gamma_{ii}k_{\rm B}T = \hbar^2\Lambda_{ii}$.

(3) The system Hamiltonian is modified by the terms

$$C_i^{(1)} \equiv \langle \hat{B}_i(\tau)\rangle, \quad \text{and} \tag{30}$$

$$C_{ij}^{(2)} \equiv \frac{\rm i}{2\hbar}\int_0^\infty {\rm d}\tau' \langle[\hat{B}_i(\tau), \hat{B}_j(\tau-\tau')]\rangle. \tag{31}$$

Remarkably, these contributions are akin to the first derivative of the first order (in polarizability) Casimir-Polder potential ($C_i^{(1)} \sim \partial_i \mathcal{U}_{\rm CP}^{(1)}$), and second derivative of the second-order Casimir-Polder potential ($C_{ij}^{(2)} \sim \partial_i \mathcal{U}_{\rm CP}^{(2)} \partial_j$) for a polarizable particle near a medium, as defined in [9]. We note that while there is no actual medium or boundary present in the current model, there is a nevertheless a

dispersive potential or force seen by the particle's quantized center of mass.

We will now examine the decoherence (Λ_{ii}) and dispersion potential ($C_i^{(1)}$ and $C_{ij}^{(2)}$) on the particle's center of mass.

B. Decoherence

Using the definition of the bath operators from the previous section and the interaction Hamiltonians Eqs. (20) and (19), we can now derive an expression for the total decoherence coefficient

$$\Lambda = \Lambda^{\mathrm{DU}} + \Lambda^{\mathrm{TD}}. \tag{32}$$

Here Λ^{DU} represents the "Davies-Unruh decoherence", arising from $\hat{H}_{\mathrm{int}}^{\mathrm{DU}}$, i.e., the backaction of the field on the particle, and Λ^{TD} corresponds to the "time dilation decoherence" due to $\hat{H}_{\mathrm{int}}^{\mathrm{TD}}$, i.e., the interaction between the field and the system from time dilation. Since the interaction Hamiltonian is quadratic in the field, the decoherence coefficients thus amounts to calculating fourth-order field correlators along the worldline of the particle. We make the following simplifying assumptions: (i) we assume that the field state observed by a particle moving along a stationary worldline is Gaussian, meaning that we can apply Wick's theorem to write the four-point correlators in terms of two-point correlators along the worldline of the particle. This also implies that the field and its spatial derivative are statistically independent, i.e. that $\langle\hat{\phi}(\tau)\partial_i\hat{\phi}(\tau-\tau')\rangle = 0$ [8]; and, (ii) we assume that different spatial directions of the center-of-mass are uncorrelated. This implies that $\langle\partial_j\hat{\phi}(\tau)\partial_i\hat{\phi}(\tau-\tau')\rangle = 0$ for $i \neq j$, indicating that decoherence along different spatial directions occurs independently of one another. It follows from these assumptions that we can write the four-point correlators in terms of $\langle\hat{\phi}(\tau)\hat{\phi}(\tau-\tau')\rangle$ and $\langle\partial_i\hat{\phi}(\tau)\partial_i\hat{\phi}(\tau-\tau')\rangle$ only.

1. "Davies-Unruh" decoherence

Armed with these assumptions, we can calculate the decoherence coefficient using Eq. (28) and the expression for the Davies-Unruh bath operator from Eq. (19), as follows:

$$\begin{aligned}\Lambda_{ij}^{\mathrm{DU}} &= \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau'\langle\{\hat{B}_i^{\mathrm{DU}}(\tau),\hat{B}_j^{\mathrm{DU}}(\tau-\tau')\}\rangle \\ &= \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau'\langle\dot{\hat{d}}(\tau)\dot{\hat{d}}(\tau-\tau')\rangle\langle\partial_i\hat{\phi}(\tau)\partial_i\hat{\phi}(\tau-\tau')\rangle \\ &\quad + \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau'\langle\dot{\hat{d}}(\tau-\tau')\dot{\hat{d}}(\tau)\rangle\langle\partial_i\hat{\phi}(\tau-\tau')\partial_i\hat{\phi}(\tau)\rangle,\end{aligned} \tag{33}$$

if $i = j$ and zero otherwise and we have used the assumptions stated earlier. To proceed we note that the correlator of the spatial derivatives can be turned into the correlator of the time derivative

$$\langle\partial_i\hat{\phi}(X)\partial_i\hat{\phi}(X')\rangle = \frac{1}{3c^2}\langle\partial_\tau\hat{\phi}(X)\partial_\tau\hat{\phi}(X')\rangle. \tag{34}$$

Since we are working in the local rest frame of the particle, the coordinate time derivative equals the proper time derivative. Spectrally decomposing the field and evaluating the τ integral, we are left with (see Appendix C for details)

$$\Lambda^{\mathrm{DU}} = \frac{1}{6\pi\hbar^2c^2}\int_0^\infty \mathrm{d}\omega\omega^6|\alpha_0(\omega)|^2D^+(\omega)D^-(\omega). \tag{35}$$

Here $D^+(\omega)$ and $D^-(\omega)$ are the Fourier transforms of the forward and backward processes respectively, defined as

$$\begin{aligned}D^+(\omega) &= \int_{-\infty}^\infty \mathrm{d}\tau'\langle\hat{\phi}(\tau+\tau')\hat{\phi}(\tau)\rangle e^{-i\omega\tau'}, \\ D^-(\omega) &= \int_{-\infty}^\infty \mathrm{d}\tau'\langle\hat{\phi}(\tau-\tau')\hat{\phi}(\tau)\rangle e^{-i\omega\tau'}.\end{aligned} \tag{36}$$

They satisfy the relation $D^+(-\omega) = D^-(\omega)$.

2. Davies-Unruh decoherence from detailed balance

Remarkably, there is an analogy between the decoherence rate Λ^{DU} derived above and the decoherence rate of an inertial detector coupled to an effective thermal bath, which suggests an alternative derivation. We start with the decoherence coefficient of an inertial, polarizable particle interacting with a thermal field [8,9]

$$\Lambda_{\mathrm{th}} = \frac{1}{3(2\pi)^3\epsilon_0^2c^8}\int_0^\infty \mathrm{d}\omega\omega^8|\alpha_0(\omega)|^2(n(\omega)+1)n(\omega). \tag{37}$$

Here $n(\omega) = [\exp(\hbar\omega/k_{\mathrm{B}}T)-1]^{-1}$ is the Bose-Einstein distribution. In [13] it was shown that for any observer moving along a stationary worldline, the field state seen by the observer can be described by a temperature T given by the detailed-balance relation

$$\frac{D^-(\omega)}{D^+(\omega)} = e^{\hbar\omega/(k_{\mathrm{B}}T)}. \tag{38}$$

One can thus derive the Davies-Unruh decoherence rate by starting from the expression for the decoherence in thermal radiation of a polarizable particle at rest in Minkowski spacetime given by Eq. (37). Under the assumption that the processes in the environment are in detailed balance [Eq. (38)], we can write the modified spectrum along a stationary worldline in the form of an effective Bose-Einstein distribution with a frequency-dependent temperature as [13,38]

$$n(\omega) = \frac{1}{e^{\hbar\omega/(k_B T(\omega))} - 1} = \frac{D^+(\omega)}{D^-(\omega) - D^+(\omega)}, \quad (39)$$

which immediately allows us to see that

$$n(\omega)(n(\omega)+1) = \frac{D^+(\omega)D^-(\omega)}{(D^+(\omega) - D^-(\omega))^2} = \frac{(2\pi\epsilon_0 c^3)^2}{\hbar^2\omega^2} D^+(\omega)D^-(\omega). \quad (40)$$

In the last step we have used the expression for the density of states in Minkowski spacetime.

Since the Fourier transform of the pullback of the Minkowski spacetime field correlator onto any stationary worldline always satisfies a detailed balance relation, we can associate a temperature with the corresponding field state. For instance, in the case of hyperbolic motion, this temperature would be equivalent to the well known Davies-Unruh temperature. Using Eq. (39) in Eq. (36), we recover Eq. (34). This tells us that the decoherence coefficient of our system can always be expressed in the form of Eq. (36) with an appropriate frequency dependent temperature $T(\omega)$, as long as the processes in the environment are in detailed balance.

3. Decoherence from time dilation

We similarly derive the contribution to the total decoherence arising from the time-dilation interaction Hamiltonian in Eq. (20). As mentioned above, this interaction arises from the differential time dilation over the extent of the spatial wave function of the system. We will thus refer to the associated decoherence effect as time-dilation decoherence, Λ^{TD}. To calculate Λ^{TD}, we first consider the time-dilation bath operator in Eq. (25), wherein both terms depend on the dipole and are thus linear in the polarizability. We can immediately infer the resulting decoherence coefficient $\Lambda^{\mathrm{TD}}_{ij}$, by looking at the similarities between the second and third term in Eq. (25) and Eq. (24). In analogy to Eq. (35), the decoherence coefficient is thus

$$\Lambda^{\mathrm{TD}}_{ij} = \frac{a_i a_j}{6\pi\hbar^2 c^4} \int_0^\infty \mathrm{d}\omega\, \omega^4 |\alpha_0(\omega)|^2 (1 - 2\eta(\omega))^2 D^+(\omega)D^-(\omega). \quad (41)$$

As opposed to the Davies-Unruh decoherence coefficient, the time-dilation decoherence depends on the local acceleration both via the perceived field spectrum of the moving particle ($\sim D^+(\omega)D^-(\omega)$), as well as the direct scaling $\sim a^2$ via the redshift factor.

C. Dispersion potential

The terms $C^{(1)}_i$ and $C^{(2)}_{ij}$ represent derivatives of the dispersion potential arising from the interaction of the particle with the field. Using the split of the bath operator in Eq. (23) into a Davies-Unruh and a time-dilation contribution, we see that both $C^{(1)}_i$ and $C^{(2)}_{ij}$ will have contributions from both bath operators. The only nonvanishing contribution to $C^{(1)}_i$ is due to the bath operator $\hat{B}^{\mathrm{TD}}_i(\tau)$ of the time-dilation Hamiltonian (20)

$$C^{(1),\mathrm{TD}}_i = \frac{a_i}{2\pi c^2} \int_0^\infty \mathrm{d}\omega\, S(\omega)\omega^2 (D^+(\omega) + D^-(\omega)), \quad (42)$$

where $S(\omega)$ is given by

$$S(\omega) = \frac{1}{2}\mathrm{Re}[\alpha_0(\omega)\eta(\omega) - \alpha_0(\omega)]. \quad (43)$$

$C^{(1),\mathrm{TD}}_i$ can be interpreted as the dispersion force arising from the effective temperature gradient across the extent of the particle's wave function. Similar corrections to the flat-space Casimir force between parallel plates that are linear in the local acceleration have been reported in [39–44].

The second-order term $C^{(2)}_i$ receives contributions from both $\hat{B}^{\mathrm{TD}}_i(\tau)$ and the bath operator $\hat{B}^{\mathrm{DU}}_i(\tau)$ defined by the Davies-Unruh Hamiltonian (19). The Davies-Unruh contribution is

$$C^{(2),\mathrm{DU}}_{ij} = -\frac{\delta_{ij}}{24\pi^2\hbar c^2} \mathcal{P} \int_{-\infty}^\infty \mathrm{d}\omega \int_{-\infty}^\infty \mathrm{d}\omega' \frac{\omega^4\omega'^2}{\omega+\omega'} |\alpha_0(\omega)|^2 \times [D^+(\omega)D^+(\omega') - D^-(\omega)D^-(\omega')], \quad (44)$$

and the time-dilation contribution is

$$C^{(2),\mathrm{TD}}_{ij} = -\frac{a_i a_j}{32\pi^2\hbar c^4} \mathcal{P} \int_{-\infty}^\infty \mathrm{d}\omega \int_{-\infty}^\infty \mathrm{d}\omega' \frac{\omega^4}{\omega+\omega'} \times |\alpha_0(\omega)[\eta(\omega) - 1]|^2 \times [D^+(\omega)D^+(\omega') - D^-(\omega)D^-(\omega')]. \quad (45)$$

While $C^{(2),\mathrm{DU}}_{ij}$ represents an effect analogous to the thermal Lamb shift in atoms arising directly from the interaction of the particle with the effective thermal field, $C^{(2),\mathrm{TD}}_{ij}$ once again arises from the effective thermal gradient across the particles. The seeming contradiction of the appearance of a temperature gradient in an equilibrium force, such as $C^{(1),\mathrm{DU}}_i$ and $C^{(2),\mathrm{DU}}_{ij}$, can be related to the Tolman-Ehrenfest effect, which causes the field to equilibrate at a temperature gradient [45–47].

IV. EXAMPLE: HYPERBOLIC MOTION

A. Decoherence

As a first example of decoherence in noninertial reference frames, let us consider linear hyperbolic motion through Minkowski space. Without loss of generality we assume that our particle is accelerated along the x axis with an acceleration a. The classical worldline of the particle can then be written as

$$\begin{aligned} t_0 &= ca^{-1}\sinh(a\tau/c) \\ x_0 &= c^2a^{-1}\cosh(a\tau/c) \\ y_0 &= z_0 = 0. \end{aligned} \tag{46}$$

It was shown in [48,49] that the Fourier transforms of the forward and backward Wightman functions $D^+(\omega)$ and $D^-(\omega)$ pulled back onto a hyperbolic worldline are given by

$$D^{\pm}(\omega) = -\frac{\hbar\omega}{2\pi\epsilon_0 c^3}[\Theta(\mp\omega) + n(\omega)], \tag{47}$$

where $n(\omega)$ can be identified as the Bose-Einstein distribution at the Davies-Unruh temperature $T_{\mathrm{DU}} = \hbar a/(2\pi k_{\mathrm{B}} c)$, and $\Theta(\omega)$ is the Heaviside function. To find all the decoherence coefficients, we substitute these Wightman functions into Eq. (35) and (41) to find

$$\Lambda^{\mathrm{DU}} = \frac{1}{3(2\pi)^3\epsilon_0^2 c^8}\int_0^\infty \mathrm{d}\omega\,\omega^8|\alpha_0(\omega)|^2(n(\omega)+1)n(\omega) \tag{48}$$

and

$$\begin{aligned} \Lambda^{\mathrm{TD}}_{ij} &= \frac{a_i a_j}{3(2\pi)^3\epsilon_0^2 c^4} \\ &\times \int_0^\infty \mathrm{d}\omega\,\omega^6|\alpha_0(\omega)|^2(1-2\eta(\omega))^2(n(\omega)+1)n(\omega). \end{aligned} \tag{49}$$

The Davies-Unruh coefficient Eq. (48) takes on the same form as the decoherence coefficient Eq. (37) for a polarizable particle at rest interacting with a thermal field [8]. The numerical prefactor in the Davies-Unruh decoherence rate differs from that in [9], due to the scalar nature of the field. We further note that the decoherence rate Λ^{DU} is related to the momentum diffusion rate of a particle undergoing hyperbolic motion [7–9]

$$\frac{\langle\Delta P^2\rangle}{\Delta t} = 2\hbar^2\Lambda^{\mathrm{DU}}. \tag{50}$$

This can be understood as follows: the rate at which the effective thermal photons scatter off the particle and impart momentum kicks is the same rate at which they gain "which-path" information and decohere the particle's spatial superposition. Both the decoherence coefficients Λ^{DU} and Λ^{TD} contain the factor $n(\omega)(n(\omega)+1)$, wherein the terms $\sim n(\omega)$ and $\sim n^2(\omega)$ correspond to the particlelike and wavelike contributions of the thermal field, respectively [8]. The appearance of the Bose-Einstein distribution also ensures the convergence of the integral. We also point out that this decoherence mechanism arising from the scattering of thermal photons is unrelated to the transient mechanism described in [11,50].

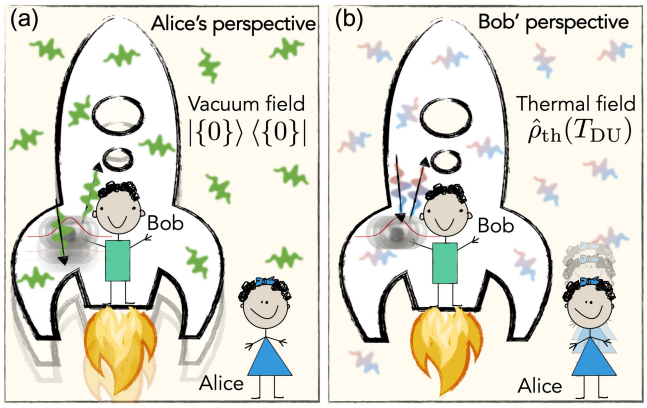


FIG. 3. (a) An inertial observer, Alice, in Minkowski vacuum, observes the decoherence of a particle undergoing uniform acceleration, with Bob as a comoving observer. From Alice's perspective the particle interacts with the zero-temperature vacuum field, scattering quantum fluctuations and experiencing momentum recoil in the process. This recoil is associated with "which-path" measurement, which leads to decoherence. (b) From Bob's perspective, the decoherence results from scattering by a thermal field.

Additionally, the decoherence of the particle undergoing hyperbolic motion can be understood from the perspective of an inertial observer in the Minkowski vacuum as follows. Consider Alice to be an inertial observer, while Bob is located in the comoving frame with the particle (see Fig. 3). Since the field in Bob's frame appears thermal, the decoherence in Bob's frame arises from the scattering of apparent thermal photons by the particle [51,52]. In Alice's frame, as the quantum fluctuations of the vacuum field interact with the accelerating particle and scatter, the particle experiences recoil. This recoil results in both a momentum diffusion, and concomitantly decoherence in the position basis. We note that such an effect is consistent with the fact that Alice does not measure any excitations in the field [53,54].

To illustrate the magnitude of these effects let us consider a levitated silica nanosphere, a platform that is commonly used in tabletop levitation experiments, see for example [55,56]. The response of the nanosphere is described by a polarizability $\alpha_S(\omega)$, which we assume to be described by [57]

$$\alpha_S(\omega) = 3\epsilon_0 V \frac{\omega_p^2}{\omega_i^2 - \omega^2 - i\Gamma\omega}, \tag{51}$$

where V is the particle volume, ω_p is the plasma frequency $\omega_p = 2\pi \times 8.8 \times 10^{12}$ Hz, ω_i is the transition frequency $\omega_i = 2\pi \times 3.4 \times 10^{13}$ Hz and Γ is the internal damping $\Gamma = 2\pi \times 1.5 \times 10^{12}$. Using this model of the particle, we calculate the decoherence coefficients for the following scenarios: (i) Surface of the Earth: assuming that the particle is at rest in the Earth's gravitational field with $a \sim 10\ \mathrm{s}^{-1}\,\mathrm{m}^{-2}$, the decoherence coefficients are $\Lambda^{\mathrm{DU}} \sim 10^{-185}\ \mathrm{s}^{-1}\,\mathrm{m}^{-2}$ and $\Lambda^{\mathrm{TD}} \sim 10^{-132}\ \mathrm{s}^{-1}\,\mathrm{m}^{-2}$. If the particle is in the ground state in an optical tweezer with a wave function spread of $\Delta x \sim 10$ pm, then the corresponding rates are: $\lambda^{\mathrm{DU}} \sim 10^{-207}\ \mathrm{s}^{-1}$ and $\lambda^{\mathrm{TD}} \sim 10^{-154}\ \mathrm{s}^{-1}$. (ii) Surface of a neutron star: the typical surface gravity of a neutron star is $a \sim 10^{12}\ \mathrm{m/s}^2$ [58]. Assuming that the particle is held at rest close to a neutron star we find: $\Lambda^{\mathrm{DU}} \sim 10^{-76}\ \mathrm{s}^{-1}\,\mathrm{m}^{-2}$, $\Lambda^{\mathrm{TD}} \sim 10^{-25}\ \mathrm{s}^{-1}\,\mathrm{m}^{-2}$ and the corresponding rates are: $\lambda^{\mathrm{DU}} \sim 10^{-98}\ \mathrm{s}^{-1}$ and $\lambda^{\mathrm{TD}} \sim 10^{-47}\ \mathrm{s}^{-1}$. We note that the time-dilation induced decoherence can be orders of magnitudes larger than the Davies-Unruh decoherence, however they are both extremely small and unlikely to be observed by experiments in the near future.

B. Dispersion force

Using the same $D^+(\omega)$ and $D^-(\omega)$, we can also calculate the dispersion force experienced by the particle. Substituting the Wightman functions into Eq. (42), we get

$$C_i^{(1),\mathrm{TD}} = \frac{\hbar a_i}{8\pi^2\epsilon_0 c^5} \times \int_0^\infty \mathrm{d}\omega\, \omega^3 \mathrm{Re}[\alpha_0(\omega)(\eta(\omega) - 1)](2n(\omega) + 1)), \tag{52}$$

where $n(\omega)$ is once again the Bose-Einstein distribution at the Davies-Unruh temperature. As explained in Sec. III C, this force is related to the temperature gradient experienced by the particle across its wave function.

V. EXAMPLE: CIRCULAR MOTION

A. Decoherence

The second example we consider is that of relativistic uniform circular motion. Circular motion with relativistic speed is commonly achieved in accelerators and has been studied before in [59–61]. The worldline of a particle undergoing circular motion parametrized by its proper time τ is given by

$$\begin{aligned} t_0 &= \tau\gamma \\ x_0 &= \rho\sin(\xi\tau\gamma) \\ y_0 &= \rho\cos(\xi\tau\gamma) \\ z_0 &= 0. \end{aligned} \tag{53}$$

Here ρ is the radius, $\gamma = 1/(1 - v^2/c^2)^{\frac{1}{2}}$, ξ is the angular frequency and $v = \rho\xi$ is the speed of the particle. The local acceleration experience by the particle in its rest frame is

$$a = v\xi\gamma. \tag{54}$$

To find the decoherence coefficient, we will focus on the ultrarelativistic case $v \approx c$. In this case, the Fourier transform of the Wightman function can be simplified and has been found in [49] to be

$$D^+(\omega) \simeq \frac{\hbar\xi}{8\pi\epsilon_0 c^3 R\gamma}\left[\frac{e^{-2\omega R/\xi\gamma}}{(v/c)\cosh(R) - 1}\right] \tag{55}$$

$$D^-(\omega) \simeq \frac{\hbar\omega}{2\pi\epsilon_0 c^3} + \frac{\hbar\xi}{8\pi\epsilon_0 c^3 R\gamma}\left[\frac{e^{-2\omega R/\xi\gamma}}{(v/c)\cosh(R) - 1}\right], \tag{56}$$

where R is the positive real root of the equation $R = (v/c)\sinh R$. For simplicity we define a function:

$$C(R) = \frac{\xi}{4\pi\gamma}\frac{1}{2R((v/c)\cosh R - 1)}. \tag{57}$$

Substituting these expressions into Eqs. (35) and (41) gives us

$$\Lambda^{\mathrm{DU}} = \frac{1}{6\pi\epsilon_0^2 c^4}\int_0^\infty \mathrm{d}\omega\, \omega^6 |\alpha_0(\omega)|^2 C(R) \times \left[C(R)e^{-2\omega R/\xi\gamma} + \frac{\omega}{2\pi}\right]e^{-2\omega R/\xi\gamma}, \tag{58}$$

$$\Lambda_{ij}^{\mathrm{TD}} = \frac{a_i a_j}{6\pi\epsilon_0^2 c^6}\int_0^\infty \mathrm{d}\omega\, \omega^4 |\alpha_0(\omega)|^2 (1 - 2\eta(\omega))^2 C(R) \times \left[C(R)e^{-2\omega R/\xi\gamma} + \frac{\omega}{2\pi}\right]e^{-2\omega R/\xi\gamma}. \tag{59}$$

The decaying exponential in the integrand once again ensures the convergence of the frequency integral. Compared to hyperbolic motion, the Bose-Einstein distribution is replaced by a distribution determined by the factor $C(R)$.

To estimate the magnitude of the decoherence we consider two different systems. First, recent experiments have been successful in rotating nanosphere dumbbells up to GHz frequencies [62]. Thus assuming that the nanosphere is modeled in the same way as before and that the particle moves along a trajectory with radius $r = 200$ nm

and frequency of about $\Omega \sim 1$ GHz, yields the decoherence coefficients $\Lambda^{\mathrm{DU}} \sim 10^{-3}\ \mathrm{s}^{-1}\,\mathrm{m}^{-2}$ and $\Lambda^{\mathrm{TD}} \sim 10^{4}\ \mathrm{s}^{-1}\,\mathrm{m}^{-2}$. As our second system, we consider a proton in a synchrotron accelerator. We model the proton "polarizability" as [8]

$$\alpha_P(\omega) = -\frac{e^2/m_p}{\omega^2 + \mathrm{i}\tau_p\omega^3}, \tag{60}$$

where m_p is the proton mass and $\tau_p = 2e^2/3m_pc^3$. Assuming that the accelerator has a radius of 4 km and a frequency of 10 kHz, we find that $\Lambda^{\mathrm{DU}} \sim 10^{-20}\ \mathrm{s}^{-1}\,\mathrm{m}^{-2}$ and $\Lambda^{\mathrm{TD}} \sim 10^{-13}\ \mathrm{s}^{-1}\,\mathrm{m}^{-2}$.

B. Dispersion force

We again find the dispersion force by substituting the Wightman functions for circular motion into Eq. (42), which yields

$$C_i^{(1),\mathrm{TD}} = \frac{\hbar a_i}{8\pi^2\epsilon_0 c^5}\int_0^\infty \mathrm{d}\omega\,\omega^2 \mathrm{Re}[\alpha_0(\omega)(1-\eta(\omega))] \times\left(\frac{\omega}{4\pi} + C(R)e^{-2\omega R/\xi\gamma}\right). \tag{61}$$

For the case where the particle moves along a worldline with a constant radius, the local acceleration is the centrifugal force pointing radially outward. The dispersion force thus becomes

$$C_i^{(1),\mathrm{TD}} = \frac{\hbar\xi^2\gamma^2\rho}{8\pi^2\epsilon_0 c^5}\int_0^\infty \mathrm{d}\omega\,\omega^2 \mathrm{Re}[\alpha_0(\omega)(1-\eta(\omega))] \times\left(\frac{\omega}{4\pi} + C(R)e^{-2\omega R/\xi\gamma}\right). \tag{62}$$

Like the decoherence coefficient, the field spectrum is described by a distribution determined by $C(R)$.

VI. SUMMARY AND OUTLOOK

In this paper we analyze the open quantum system dynamics of the center of mass of a pointlike particle coupled to a scalar field, and moving along a stationary worldline through $3+1$ dimensional Minkowski space. In our model the particle has two distinct degrees of freedom: (1) an internal oscillator that couples to the field, and (2) its quantized center-of-mass motion, which is considered to be spatially delocalized around its worldline. We first derive an effective redshifted polarizability of the particle undergoing noninertial motion by solving the coupled dynamics of the internal oscillator and the field [Eq. (8)].

We then describe the open system dynamics of the particle's quantized center-of-mass motion via a QBM master equation, derived under the Born-Markov approximation [Eq. (26)]. The resulting QBM master equation consists of (1) a dispersive contribution corresponding to a center-of-mass Hamiltonian modification [Eqs. (30) and (31)], akin to Casimir-Polder forces on particles near media [9]; and (2) dissipative terms that include center-of-mass decoherence in the position basis. For stationary worldlines, we find that the dispersive potential seen by the particle scales linearly with the local acceleration a_i, resembling the gravitational corrections to Casimir forces found in [39–44].

We highlight two distinct sources of decoherence arising from two distinct system-bath coupling mechanism:

(1) Davies-Unruh decoherence. A particle moving along a stationary worldline registers a modified field spectrum, which differs from that of the Minkowski vacuum seen by an inertial observer. When this field interacts with the particle and imparts recoil, which-path information about the particle's center-of-mass position is acquired, and this manifests itself as decoherence. Such decoherence has a form that is effectively thermal, Eq. (35). This is further confirmed by the fact that in the special case of hyperbolic motion, the decoherence rate of a uniformly accelerating particle [Eq. (48)] is equivalent to that of a particle at rest immersed in a thermal field at temperature $T_{\mathrm{DU}} = \hbar a/(2\pi k_{\mathrm{B}}c)$ [8,9]. Additionally, we find that the same decoherence coefficient can be arrived at by considering the thermal decoherence rate of a polarizable point particle in flat spacetime [Eq. (37)], and imposing the condition that the effective field observed by the particle on a stationary worldline satisfies the detailed balance condition [Eq. (38)].

(2) Time-dilation decoherence. Since the particle contains an internal oscillator with a characteristic frequency, it is sensitive to time dilation across its delocalized center of mass. A spatial superposition parallel to the acceleration experienced by the particle leads to a differential time dilation across the particle's wave function. This induces an effective coupling between the center of mass and the field, resulting in decoherence. Along a stationary worldline, such time-dilation decoherence [Eq. (41)] is also thermal in nature, represented by the appearance of the factor $D^+(\omega)D^-(\omega)$, but with an explicit dependence on the local acceleration as well as on the particle's redshifted polarizability [Eq. (8)].

We evaluate both the dispersion force and decoherence rates for two specific examples of stationary worldlines: hyperbolic motion (Sec. IV) and circular motion (Sec. V). One of the distinguishing features between these two types of motion is that hyperbolic motion gives rise to a Rindler horizon, whereas circular motion does not. Considering the recent interest in decoherence near black holes [22–25,63] and the role played by the presence of horizons in engendering it [64], we remark that in our model we find a finite decoherence rate for both hyperbolic and circular

motion. Furthermore, we note that there are striking similarities between the Davies-Unruh decoherence rate of a particle undergoing noninertial motion [Eq. (35)], and the decoherence of a polarizable particle at rest near a medium, immersed in a thermal field [9,65,66]. This correspondence motivates further investigations into the open system dynamics of test quantum systems undergoing noninertial motion, or equivalently in a curved spacetime, suggesting a characterization of curved spacetime as an effective reservoir.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: SYSTEM LAGRANGIAN AND HAMILTONIAN

We start with a particle whose center-of-mass moves along a prescribed classical worldline $x_0^\mu(\tau)$ parametrized by the proper time τ. We describe the spatial quantized center-of-mass fluctuations of the particle around $x_0^\mu(\tau)$ in the local rest frame of the particle and denote them by $\hat{X}^i$. The conjugate momentum in the local rest frame is $\hat{P}_i$ and we impose the canonical commutation relation $[\hat{X}^i, \hat{P}_j] = i\hbar\delta^i_j$. Following the description in [19,67], we can write the Lagrangian in the local rest frame as

$$L = L_{\text{rest}}[g_{00}(\hat{X}^i) - g_{mn}(\hat{X}^i)\dot{\hat{X}}^m\dot{\hat{X}}^n/c^2]^{\frac{1}{2}}, \tag{A1}$$

where for simplicity we have assumed that $g_{0m} = g_{m0} = 0$. The rest Lagrangian L_{rest} is given by

$$L_{\text{rest}} = -\mathbb{1}Mc^2 + \frac{m\dot{\hat{q}}(\tau)^2}{2} - \frac{m\omega_q^2\hat{q}(\tau)^2}{2}. \tag{A2}$$

Here M is the mass of the particle, $\hat{q}(\tau)$ is the internal degree of freedom position, which is assumed to be pointlike and thus does not transform under general coordinate transformations, ω_q is the frequency in the rest frame of the particle, and m is the mass associated with the internal oscillator. Using the Lagrangian in the local rest frame, we can calculate the Hamiltonian in the local rest frame by taking the Legendre transform with respect to both the internal and center-of-mass degrees of freedom, which yields

$$\hat{H} = \sqrt{g_{00}(\hat{X}^i)(\hat{E}^2_{\text{rest}} + \hat{P}^i\hat{P}_i c^2)}. \tag{A3}$$

We note that the metric $g_{\mu\nu}$ which appears in the form of the redshift factor g_{00} and in the inner product of $\hat{P}^i$ as g_{ij} is evaluated around the particle worldline $x_0^\mu(\tau)$ and can thus be expanded in the local rest frame in terms of $\hat{X}^i$. Finally, we assume that the particle moves nonrelativistically in the local rest frame such that the kinetic contribution to total system energy is much smaller than the rest energy contribution. In other words, we assume that there are no special relativistic time-dilation effects in the local rest frame of the particle. We then end up with Eq. (5).

APPENDIX B: INTERNAL OSCILLATOR DYNAMICS AND POLARIZABILITY

1. Coupled dynamics of the internal oscillator and the field

In order to describe the response of the particle to the external field, we will now derive an effective polarizability of the particle, defined through its linear response to the field. Using Eq. (5), the Heisenberg equations of motion for the internal oscillator are given by

$$\dot{\hat{q}} = -\frac{\mathrm{i}}{\hbar}[\hat{q}, \hat{H}_{\text{S}}] = \frac{\sqrt{-g_{00}}}{m}(\hat{p} - e\hat{\phi}(\hat{X}^i, \tau)) \tag{B1}$$

$$\dot{\hat{p}} = -\frac{\mathrm{i}}{\hbar}[\hat{p}, \hat{H}_{\text{S}}] = -\sqrt{-g_{00}}m\,\omega_q^2\hat{q}. \tag{B2}$$

Combining these two equations leaves us with

$$m\ddot{\hat{q}} - g_{00}m\omega_q^2\hat{q}^2 = -\sqrt{-g_{00}}\,e\dot{\hat{\phi}}(\hat{X}^i, \tau), \tag{B3}$$

from which we can see that the time derivative of the field acts as a driving force on the internal oscillator.

Similarly, the Heisenberg equation of motion for the field mode operators are

$$\begin{aligned}\dot{\hat{g}}_{\mathbf{k}} &= -\frac{\mathrm{i}}{\hbar}[\hat{g}_{\mathbf{k}}, \hat{H}_{\text{F}} + \hat{H}_{\text{S}}] \\ &= -\mathrm{i}\Omega_k\hat{g}_{\mathbf{k}} + \frac{\mathrm{i}e\sqrt{-g_{00}}}{\hbar m}(\hat{p} - e\hat{\phi}(\hat{X}^i, \tau))\phi^*_{\mathbf{k}}(\hat{X}^i, \tau),\end{aligned} \tag{B4}$$

which we solve to get

$$\hat{g}_{\mathbf{k}}(\tau) = \hat{g}_{\mathbf{k}}(0)e^{-i\Omega_{\mathbf{k}}\tau} + \frac{ie}{\hbar}\int_0^\tau d\tau' \frac{d\hat{q}(\tau')}{d\tau'}\phi^*_{\mathbf{k}}(\hat{X}^i,\tau')e^{-i\Omega_{\mathbf{k}}(\tau-\tau')}. \tag{B5}$$

Therefore the field is given by

$$\hat{\phi}_{\mathrm{tot}}(\hat{X}^i,\tau) = \hat{\phi}(\hat{X}^i,\tau) + \frac{ie}{\hbar}\int_0^\tau d\tau' \frac{d\hat{q}(\tau')}{d\tau'}\sum_{\mathbf{k}}\left[\phi_{\mathbf{k}}(\hat{X}^i,\tau)\phi^*_{\mathbf{k}}(\hat{X}^i,\tau')e^{-i\Omega_{\mathbf{k}}(\tau-\tau')} - \phi^*_{\mathbf{k}}(\hat{X}^i,\tau)\phi_{\mathbf{k}}(\hat{X}^i,\tau')e^{i\Omega_{\mathbf{k}}(\tau-\tau')}\right] \tag{B6}$$

$$= \hat{\phi}(\hat{X}^i,\tau)\underbrace{+\frac{ie}{\hbar}\int_0^\tau d\tau' \frac{d\hat{q}(\tau')}{d\tau'}[\langle\hat{\phi}(\hat{X}^i,\tau)\hat{\phi}(\hat{X}^i,\tau')\rangle - \langle\hat{\phi}(\hat{X}^i,\tau')\hat{\phi}(\hat{X}^i,\tau)\rangle]}_{\hat{\phi}_{\mathrm{rr}}(\hat{X}^i,\tau)}, \tag{B7}$$

where $\hat{\phi}(\hat{X}^i,\tau)$ is the free-evolving field (i.e., the part evolved by $\hat{H}_{\mathrm{F}}$). The radiation-reaction part $\hat{\phi}_{\mathrm{rr}}(\hat{X}^i,\tau)$ can be simplified by taking the Fourier transform with respect to τ', using the definitions of $D^{\pm}(\Omega)$ in Eq. (36) and neglecting $\mathcal{O}((\hat{X}^i)^2)$ terms

$$\hat{\phi}_{\mathrm{rr}}(\tau) \simeq \frac{ie}{2\pi\hbar}\int_0^\tau d\tau' \int_{-\infty}^{\infty} d\Omega \frac{d\hat{q}(\tau')}{d\tau'}[D^+(\Omega) - D^-(\Omega)]e^{i\Omega(\tau-\tau')} \tag{B8}$$

$$= \frac{e}{2\pi^2\epsilon_0 c^3}\int_0^\tau d\tau' \int_0^{\infty} d\Omega \frac{d\hat{q}(\tau')}{d\tau'}\Omega\sin(\Omega(\tau-\tau')). \tag{B9}$$

The time derivative of the radiation-reaction term is therefore

$$\dot{\hat{\phi}}_{\mathrm{rr}}(\tau) = \frac{e}{2\pi^2\epsilon_0 c^3}\int_0^\tau d\tau' \int_0^{\infty} d\Omega \frac{d\hat{q}(\tau')}{d\tau'}\Omega^2\cos(\Omega(\tau-\tau')) \tag{B10}$$

$$= -\frac{e}{2\pi^2\epsilon_0 c^3}\int_0^\tau d\tau' \int_0^{\infty} d\Omega \frac{d\hat{q}(\tau')}{d\tau'}\partial^2_\tau\cos(\Omega(\tau-\tau')) \tag{B11}$$

$$= -\frac{e}{2\pi\epsilon_0 c^3}\int_0^\tau d\tau' \frac{d\hat{q}(\tau')}{d\tau'}\partial^2_\tau\delta(\tau-\tau') \tag{B12}$$

$$= -\frac{e}{4\pi\epsilon_0 c^3}\left(\frac{d^3\hat{q}(\tau)}{d\tau^3} - \frac{d^2\hat{q}(\tau)}{d\tau^2}\delta(0)\right). \tag{B13}$$

Then from Eqs. (B3), (B7), and (B13) we get

$$(m+\delta m)\ddot{\hat{q}} - g_{00}m\omega_q^2\hat{q} - \sqrt{-g_{00}}\frac{e^2}{4\pi\epsilon_0 c^3}\dddot{\hat{q}} = -\sqrt{-g_{00}}\,e\dot{\hat{\phi}}(\hat{X}^i,\tau). \tag{B14}$$

δm is a contribution to the particle mass resulting from the coupling of the particle to its own radiation reaction field, and is given by the divergent expression $(e^2/4\pi\epsilon_0 c^3)\delta(0)$ in our calculation assuming a pointlike particle. To be consistent with our nonrelativistic approximation for the internal dynamics of the particle we can replace $\delta(0)$ by Ω/π following the description in [68], where Ω is a high-frequency cutoff; this results in the replacement of $\delta m/m$ by

$$\frac{\delta m}{m} = \frac{e^2\Omega}{4\pi^2\epsilon_0 mc^3} = \frac{r_0}{\pi c}\Omega, \tag{B15}$$

where $r_0 = e^2/(4\pi\epsilon_0 mc^2) \cong 2.8\times10^{-15}$ m is the classical electron radius. Thus $\delta m/m$ may be assumed to be negligible for any cutoff frequency Ω much smaller than 10^{23} Hz, and then we can ignore the mass renormalization in Eq. (B14).

2. Effective polarizability

Taking the Fourier transform of Eq. (B14) allows us to write the steady state induced dipole $\hat{d}_{\mathrm{st}}$ as

$$\hat{d}_{\mathrm{st}}(\hat{X}^i,\tau) = e\hat{q}(\hat{X}^i,\tau) = -\frac{i}{2\pi}\sqrt{-g_{00}}\int d\omega\,\omega\alpha(\omega)\underline{\hat{\phi}}(\hat{X}^i,\omega)e^{i\omega\tau}, \tag{B16}$$

where $\alpha(\omega)$ is the polarizability of the particle, which describes its response to an external field

$$\alpha(\omega) \coloneqq \frac{e^2/m}{2\mathrm{i}\sqrt{-g_{00}}\beta\omega^3 - \omega^2 - g_{00}\omega_q^2}; \qquad \beta \coloneqq \frac{e^2/m}{8\pi\epsilon_0 c^3}. \tag{B17}$$

From this expression we can see that the characteristic frequencies that determine the response of the particle are redshifted across the spread of the wave function, which leads to a coupling between the quantized center of mass and the field. This can be seen by expanding the polarizability to first order in $\hat{X}^i$ using Eq. (15) to get

$$\alpha(\omega) \approx \alpha_0(\omega)[1 - \eta(\omega)a_i\hat{X}^i/c^2]. \tag{B18}$$

Here $\alpha_0(\omega)$ is the bare polarizability in the rest frame of the particle and $\eta(\omega)$ is a redshift-induced correction (see Eqs. (10) and (11) respectively). Combining Eq. (4), Eq. (B1) and Eq. (8) we can rewrite Eq. (5) in the Schrödinger picture as given by Eq. (12).

APPENDIX C: DECOHERENCE RATE

1. Davies-Unruh decoherence from first principles

The derivation of $\Lambda_{ij}^{\mathrm{DU}}$, with $i = j$ corresponding to the Davies-Unruh decoherence coefficient, goes as follows. Using the bath operator $B_i^{\mathrm{DU}}(\tau)$ in the interaction picture [Eq. (24)]

$$\Lambda_{ij}^{\mathrm{DU}} = \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau' \langle\{\hat{B}_i^{\mathrm{DU}}(\tau), \hat{B}_j^{\mathrm{DU}}(\tau-\tau')\}\rangle \tag{C1}$$

$$= \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau' \langle\hat{B}_i^{\mathrm{DU}}(\tau)\hat{B}_j^{\mathrm{DU}}(\tau-\tau')\rangle + \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau' \langle\hat{B}_j^{\mathrm{DU}}(\tau-\tau')\hat{B}_j^{\mathrm{DU}}(\tau)\rangle. \tag{C2}$$

Substituting the expressions for the Davies-Unruh bath operator gives

$$\Lambda_{ij}^{\mathrm{DU}} = \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau' \langle\dot{\hat{d}}_{0,\mathrm{st}}(\tau)\partial_i\hat{\phi}(\tau)\dot{\hat{d}}_{0,\mathrm{st}}(\tau-\tau')\partial_j\hat{\phi}(\tau-\tau')\rangle + \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau' \langle\dot{\hat{d}}_{0,\mathrm{st}}(\tau-\tau')\partial_i\hat{\phi}(\tau-\tau')\dot{\hat{d}}_{0,\mathrm{st}}(\tau)\partial_j\hat{\phi}(\tau)\rangle, \tag{C3}$$

where we use the shorthand notation $\hat{\phi}(\tau) \equiv \hat{\phi}(x_0^\mu(\tau))$. In the next step we apply Wick's theorem to break up the 4-point correlators into products of 2-point correlators. Additionally, we make use of our assumption that the field and its first derivative are statistically independent. Together these assumptions yield

$$\Lambda_{ij}^{\mathrm{DU}} = \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau' \langle\dot{\hat{d}}_{0,\mathrm{st}}(\tau)\dot{\hat{d}}_{0,\mathrm{st}}(\tau-\tau')\rangle\langle\partial_i\hat{\phi}(\tau)\partial_j\hat{\phi}(\tau-\tau')\rangle + \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau' \langle\dot{\hat{d}}_{0,\mathrm{st}}(\tau-\tau')\dot{\hat{d}}_{0,\mathrm{st}}(\tau)\rangle\langle\partial_i\hat{\phi}(\tau-\tau')\partial_j\hat{\phi}(\tau)\rangle. \tag{C4}$$

Now we use the assumption that different spatial directions of the center-of-mass are uncorrelated. This implies that $\langle\partial_j\hat{\phi}(\tau)\partial_i\hat{\phi}(\tau-\tau')\rangle = 0$ and leaves us with

$$\Lambda_{ii}^{\mathrm{DU}} = \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau' \langle\dot{\hat{d}}_{0,\mathrm{st}}(\tau)\dot{\hat{d}}_{0,\mathrm{st}}(\tau-\tau')\rangle\langle\partial_i\hat{\phi}(\tau)\partial_i\hat{\phi}(\tau-\tau')\rangle + \frac{1}{2\hbar^2}\int_0^\infty \mathrm{d}\tau' \langle\dot{\hat{d}}_{0,\mathrm{st}}(\tau-\tau')\dot{\hat{d}}_{0,\mathrm{st}}(\tau)\rangle\langle\partial_i\hat{\phi}(\tau-\tau')\partial_i\hat{\phi}(\tau)\rangle. \tag{C5}$$

Since T_{eff} is a function of $\sqrt{a_i a^i}$ and thus isotropic, we assume that $\Lambda_{ij}^{\mathrm{DU}}$ is independent of which axis the superposition is prepared in. In combination with $\langle\partial_j\hat{\phi}(\tau)\partial_i\hat{\phi}(\tau-\tau')\rangle = 0$ this allows us to simplify the correlator of the spatial derivatives

$$\langle\partial_i\hat{\phi}(X)\partial_i\hat{\phi}(X')\rangle = \frac{1}{3c^2}\langle\partial_\tau\hat{\phi}(X)\partial_\tau\hat{\phi}(X')\rangle. \tag{C6}$$

After this, Λ^{DU} is

$$\Lambda^{\mathrm{DU}} = \frac{1}{6\hbar^2c^2}\int_0^\infty \mathrm{d}\tau' \langle\dot{\hat{d}}_{0,\mathrm{st}}(\tau)\dot{\hat{d}}_{0,\mathrm{st}}(\tau-\tau')\rangle\langle\partial_\tau\hat{\phi}(\tau)\partial_\tau\hat{\phi}(\tau-\tau')\rangle + \frac{1}{6\hbar^2c^2}\int_0^\infty \mathrm{d}\tau' \langle\dot{\hat{d}}_{0,\mathrm{st}}(\tau-\tau')\dot{\hat{d}}_{0,\mathrm{st}}(\tau)\rangle\langle\partial_\tau\hat{\phi}(\tau-\tau')\partial_\tau\hat{\phi}(\tau)\rangle. \tag{C7}$$

We now write the field and induced polarization in the interaction picture in terms of their spectral decomposition

$$\hat{\phi}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega\, \underline{\hat{\phi}}(\omega,\tau),$$
$$\hat{d}_{0,\mathrm{st}}(\tau) = \frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega\, \omega\alpha_0(\omega)\underline{\hat{\phi}}(\omega,\tau), \tag{C8}$$

which yields

$$\Lambda^{\mathrm{DU}} = \frac{1}{(2\pi)^2 6\hbar^2 c^2}\int_0^{\infty}\mathrm{d}\tau'\int_{-\infty}^{\infty}\mathrm{d}\omega_1\,\omega_1^4|\alpha_0(\omega_1)|^2\langle\underline{\hat{\phi}}(\omega_1,\tau)\underline{\hat{\phi}}(\omega_1,\tau-\tau')\rangle\int_{-\infty}^{\infty}\mathrm{d}\omega_2\,\omega_2^2\langle\underline{\hat{\phi}}(\omega_2,\tau)\underline{\hat{\phi}}(\omega_2,\tau-\tau')\rangle$$
$$+\frac{1}{(2\pi)^2 6\hbar^2 c^2}\int_0^{\infty}\mathrm{d}\tau'\int_{-\infty}^{\infty}\mathrm{d}\omega_1\,\omega_1^4|\alpha_0(\omega_1)|^2\langle\underline{\hat{\phi}}(\omega_1,\tau-\tau')\underline{\hat{\phi}}(\omega_1,\tau)\rangle\int_{-\infty}^{\infty}\mathrm{d}\omega_2\,\omega_2^2\langle\underline{\hat{\phi}}(\omega_2,\tau-\tau')\underline{\hat{\phi}}(\omega_2,\tau)\rangle. \tag{C9}$$

In the second term we now make the transformation $\tau' \to -\tau'$, which will cause the correlators to change

$$\langle\underline{\hat{\phi}}(\omega,\tau-\tau')\underline{\hat{\phi}}(\omega,\tau)\rangle \to \langle\underline{\hat{\phi}}(\omega,\tau+\tau')\underline{\hat{\phi}}(\omega,\tau)\rangle = \langle\underline{\hat{\phi}}(\omega,\tau)\underline{\hat{\phi}}(\omega,\tau-\tau')\rangle, \tag{C10}$$

where in the last equality we have used the fact that the pullback of the correlator onto the worldline of the particle only depends on τ'; this follows from the stationarity of the particle's worldline. Thus the correlation function is invariant under proper time translations along the worldline. With this we can simplify the decoherence expression to

$$\Lambda^{\mathrm{DU}} = \frac{1}{(2\pi)^2 6\hbar^2 c^2}\int_{-\infty}^{\infty}\mathrm{d}\tau'\int_{-\infty}^{\infty}\mathrm{d}\omega_1\,\omega_1^4|\alpha_0(\omega_1)|^2\langle\underline{\hat{\phi}}(\omega_1,\tau)\underline{\hat{\phi}}(\omega_1,\tau-\tau')\rangle\int_{-\infty}^{\infty}\mathrm{d}\omega_2\,\omega_2^2\langle\underline{\hat{\phi}}(\omega_2,\tau)\underline{\hat{\phi}}(\omega_2,\tau-\tau')\rangle. \tag{C11}$$

Remembering that the field operators are in the interaction picture, we can write

$$\Lambda^{\mathrm{DU}} = \frac{1}{(2\pi)^2 6\hbar^2 c^2}\int_{-\infty}^{\infty}\mathrm{d}\tau'\int_{-\infty}^{\infty}\mathrm{d}\omega_1\int_{-\infty}^{\infty}\mathrm{d}\omega_2\omega_1^4\omega_2^2|\alpha_0(\omega_1)|^2 D^+(\omega_1)D^+(\omega_2)e^{-i(\omega_1+\omega_2)\tau'}. \tag{C12}$$

Simplifying further

$$\Lambda^{\mathrm{DU}} = \frac{1}{12\pi\hbar^2 c^2}\int_{-\infty}^{\infty}\mathrm{d}\omega\,\omega^6|\alpha_0(\omega)|^2 D^+(\omega)D^-(\omega) \tag{C13}$$

$$= \frac{1}{6\pi\hbar^2 c^2}\int_0^{\infty}\mathrm{d}\omega\,\omega^6|\alpha_0(\omega)|^2 D^+(\omega)D^-(\omega), \tag{C14}$$

which is our final form of the Davies-Unruh decoherence coefficient. Here $D^+(\omega)$ and $D^-(\omega)$ are the Fourier transforms of the pullback of the forward and backward processes respectively.

2. Time-dilation decoherence

As before, we start by writing the time-dilation decoherence coefficient as

$$\Lambda_{ij}^{\mathrm{TD}} = \frac{1}{2\hbar^2}\int_0^{\infty}\mathrm{d}\tau'\langle\{\hat{B}_i^{\mathrm{TD}}(\tau),\hat{B}_j^{\mathrm{TD}}(\tau-\tau')\}\rangle$$
$$= \frac{1}{2\hbar^2}\int_0^{\infty}\mathrm{d}\tau'\langle\hat{B}_i^{\mathrm{TD}}(\tau)\hat{B}_j^{\mathrm{TD}}(\tau-\tau')\rangle + \frac{1}{2\hbar^2}\int_0^{\infty}\mathrm{d}\tau'\langle\hat{B}_j^{\mathrm{TD}}(\tau-\tau')\hat{B}_j^{\mathrm{TD}}(\tau)\rangle. \tag{C15}$$

Comparing the bath operators $\hat{B}_i^{\mathrm{DU}}(\tau)$ and $\hat{B}_i^{\mathrm{TD}}(\tau)$ [Eqs. (24) and (25)], we can obtain immediately the time-dilation decoherence coefficient by realizing that it is of the same structure as Eq. (C7), except with redshifted polarizabilities:

$$\Lambda_{ij}^{\mathrm{TD}} = \frac{a_i a_j}{6\pi\hbar^2 c^2}\int_0^\infty \mathrm{d}\omega\, \omega^4 |\alpha_0(\omega)|^2 (1-2\eta(\omega))^2 D^+(\omega)D^-(\omega). \tag{C16}$$

APPENDIX D: SECOND-ORDER DISPERSION POTENTIAL

Here we compute the second-order contributions to the dispersion potential by substituting the bath operators $\hat{B}_i^{\mathrm{DU}}(\tau)$ and $\hat{B}_i^{\mathrm{TD}}(\tau)$ [Eqs. (25) and (24)] into Eq. (31).

1. Davies-Unruh dispersion potential

The Davies-Unruh term of the dispersion potential is

$$C_{ij}^{(2),\mathrm{DU}} = \frac{\mathrm{i}}{2\hbar}\int_0^\infty \mathrm{d}\tau' \langle [\hat{B}_i^{\mathrm{DU}}(\tau), \hat{B}_j^{\mathrm{DU}}(\tau-\tau')]\rangle. \tag{D1}$$

As done in the decoherence calculations, we substitute $\hat{B}_i^{\mathrm{DU}}(\tau)$ and split the 4-point correlators into 2-point correlators,

$$C_{ij}^{(2),\mathrm{DU}} = \frac{\mathrm{i}}{2\hbar}\int_0^\infty \mathrm{d}\tau' \langle \dot{\hat{d}}_{0,\mathrm{st}}(\tau)\dot{\hat{d}}_{0,\mathrm{st}}(\tau-\tau')\rangle \langle \partial_i\hat{\phi}(\tau)\partial_j\hat{\phi}(\tau-\tau')\rangle - (\tau \leftrightarrow \tau-\tau') \tag{D2}$$

$$= \frac{\mathrm{i}\delta_{ij}}{(2\pi)^2 6\hbar c^2}\int_0^\infty \mathrm{d}\tau' \int_{-\infty}^\infty \mathrm{d}\omega \int_{-\infty}^\infty \mathrm{d}\omega' \omega^4 \omega'^2 |\alpha_0(\omega)|^2 e^{\mathrm{i}(\omega+\omega')\tau'} [D^+(\omega)D^+(\omega') - D^-(\omega)D^-(\omega')] \tag{D3}$$

$$= -\frac{\delta_{ij}}{(2\pi)^2 6\hbar c^2}\mathcal{P}\int_{-\infty}^\infty \mathrm{d}\omega \int_{-\infty}^\infty \mathrm{d}\omega' \frac{\omega^4\omega'^2}{\omega+\omega'} |\alpha_0(\omega)|^2 [D^+(\omega)D^+(\omega') - D^-(\omega)D^-(\omega')], \tag{D4}$$

in the second line using Eq. (C6) to simplify the correlator of spatial derivatives. Here $\mathcal{P}$ denotes the Cauchy principal value.

2. Time-dilation dispersion potential

The time-dilation term of the dispersion potential is computed in a similar manner

$$C_{ij}^{(2),\mathrm{TD}} = \frac{\mathrm{i}}{2\hbar}\int_0^\infty \mathrm{d}\tau [\langle \hat{B}_i^{\mathrm{TD}}(\tau), \hat{B}_j^{\mathrm{TD}}(0)\rangle] \tag{D5}$$

$$= \frac{\mathrm{i}a_i a_j}{8\hbar c^4}\int_0^\infty \mathrm{d}\tau' \langle (\dot{\hat{d}}_{\mathrm{st},1}(\tau) - \dot{\hat{d}}_{\mathrm{st},0}(\tau))(\dot{\hat{d}}_{\mathrm{st},1}(\tau-\tau') - \dot{\hat{d}}_{\mathrm{st},0}(\tau-\tau'))\rangle \langle \hat{\phi}(\tau)\hat{\phi}(\tau-\tau')\rangle - (\tau \leftrightarrow \tau-\tau') \tag{D6}$$

$$= \frac{\mathrm{i}a_i a_j}{(2\pi)^2 8\hbar c^4}\int_0^\infty \mathrm{d}\tau' \int_{-\infty}^\infty \mathrm{d}\omega \int_{-\infty}^\infty \mathrm{d}\omega' \omega^4 |\alpha_0(\omega)\eta(\omega) - \alpha_0(\omega)|^2 e^{\mathrm{i}(\omega+\omega')\tau'} [D^+(\omega)D^+(\omega') - D^-(\omega)D^-(\omega')] \tag{D7}$$

$$= -\frac{a_i a_j}{(2\pi)^2 8\hbar c^4}\mathcal{P}\int_{-\infty}^\infty \mathrm{d}\omega \int_{-\infty}^\infty \mathrm{d}\omega' \frac{\omega^4}{\omega+\omega'} |\alpha_0(\omega)\eta(\omega) - \alpha_0(\omega)|^2 [D^+(\omega)D^+(\omega') - D^-(\omega)D^-(\omega')]. \tag{D8}$$

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