

Frozen states of charged boson stars

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In this paper, we study frozen states of charged boson stars. These solutions are globally regular and exist in a $U(1)$ gauged scalar field model minimally coupled to gravity for suitable choices of the coupling constants. These configurations are field theoretical realizations of the Mazur-Mottola solution with a de Sitter interior, a black hole exterior and a thin shell that interpolates between the two and replaces the event horizon. We demonstrate that standard electrodynamics is sufficient to find these frozen states, but that the self-interaction of the scalar field is crucial. For a sixth-order self-interaction of the scalar field, solutions exist that possess $k = 2n$, $n \in \mathbb{N}$ nodes in the scalar field. The interval in which these nodes exist corresponds to an additional intermediate shell that corresponds to vacuum. Adding Horndeski vector-tensor gravity to the model allows the frozen states to exist without self-interaction though. The frozen states possess one stable and one unstable lightring, the former inside the thin shell, the latter in the black hole exterior.

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I. INTRODUCTION

General Relativity is *per se* a deterministic theory, i.e., given an initial dataset, the entire future and past should be given by the Einstein equation. However, determinism breaks down at space-time singularities. The weak cosmic censorship conjecture [1] suggests that these space-time singularities should always be hidden behind an event horizon, i.e., exist at the center of black holes, and hence be nonobservable from outside the horizon. Nevertheless, they pose a general problem for the deterministic nature of the theory of General Relativity. Hence, *black hole mimickers* are an active area of research. These mimickers are (hypothetical) objects that look very much like black holes from afar, but do not possess an event horizon and—more importantly in the context of the discussion above—no physical singularity at the center (for a recent review see, e.g., [2]). Boson stars are such a viable mimicker [3–8]. Another possibility are regular black holes, i.e., black holes that have a horizon, but no space-time singularity. The first solution of this type was presented by Bardeen [9] and was consequently interpreted to have as its source a magnetic monopole within the framework of nonlinear electrodynamics [10,11]. In contrast to regular black holes, frozen

stars do not possess an event horizon, but a boundary with finite thickness that matches a nonsingular interior to a black hole exterior. The first solution of this type has been given in [12]. This is a compact object made of a perfect fluid with equation of state $\rho = -p$ (i.e., corresponding to de Sitter) in its interior and a Schwarzschild solution in its exterior. The interior and exterior are matched across a shell of finite thickness which replaces the event horizon. A concrete realization motivated from string theory has been discussed in [13,14]. Recent studies [15–17] have extended these ideas by introducing a complex, ungauged scalar field in the original Bardeen model, leading to the construction of boson star solutions with nonlinear electrodynamics. Although horizonless, these configurations can develop a quasihorizon at an intermediate radius, while maintaining regularity at the core.

In this paper, we will demonstrate that nonlinear electrodynamics is not necessary to find these frozen stars. In the first part of this paper, we will use a $U(1)$ gauged scalar field model minimally coupled to gravity. This has been used previously to study spherically symmetric, static and charged black holes that carry a cloud, a so-called Q-cloud, of scalar fields on the horizon [18,19]. In particular, it was shown that both the presence of the electric field as well as the self-interaction are necessary for these solutions to exist. In [20] it was demonstrated that not only one, but up to three branches of charged black holes with scalar hair can exist in this model. Also charged boson star solutions can be constructed in this model and these are what we are concerned with here. These have been investigated before

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in [21]. Here we will point out that in some limits, these solutions have the features of Frozen stars.

In the second part of this paper, we will discuss the same model, but will add a Horndeski vector-tensor term [22,23] which represents a unique nonminimal coupling between the vector field and curvature. The resulting spherically symmetric solutions are deformed Reissner-Nordström black holes, as first studied in [24] and later analyzed in detail in [25]. We will discuss charged boson stars in this model and demonstrate that frozen states exist within this model and have similar features to those observed in models with nonlinear electrodynamics. In particular, we will demonstrate that the scalar self-interaction is not necessary in this case.

II. THE MODEL

The model which we will study in the following is defined by the following four-dimensional action:

$$\mathcal{S} = \int d^4x \sqrt{-g} \mathcal{L}, \quad (2.1)$$

with Lagrangian density given by

$$\mathcal{L} = \frac{1}{16\pi G} R - D^\mu \Psi^\dagger D_\mu \Psi - U(|\Psi|) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \gamma \mathcal{L}_{\text{VT}}, \quad (2.2)$$

R is the Ricci scalar, Ψ is a complex-valued scalar field with potential $U(|\Psi|)$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor of a U(1) gauge field A_μ . The scalar field is minimally coupled to this gauge field through the covariant derivative $D_\mu = \partial_\mu + iqA_\mu$. G is Newton's constant and q the gauge coupling. We also allow for a nonminimal coupling between gravity and the U(1) gauge field by introducing a vector-tensor Horndeski term of the form:

$$\mathcal{L}_{\text{VT}} = -\frac{1}{4} (F_{\mu\nu} F^{\kappa\lambda} R^{\mu\nu}_{\kappa\lambda} - 4F_{\mu\kappa} F^{\nu\kappa} R^\mu_\nu + F_{\mu\nu} F^{\mu\nu} R). \quad (2.3)$$

For $\gamma = 0$ we recover standard Einstein gravity. In the following, we will demonstrate that both the potential as well as the Horndeski term lead to the existence of so-called *frozen stars*. For the potential $U(|\Psi|)$ we will have to choose a potential of at least sixth order for the solutions to exist. We will mainly work with a potential motivated from gauged supersymmetry breaking [26] which reads,

$$U_{(1)}(|\Psi|) = \mu^2 \eta^2 \left[1 - \exp\left(-\frac{|\Psi|^2}{\eta^2}\right) \right], \quad (2.4)$$

where μ is the mass of the scalar field and η is an energy scale. Note that this potential behaves like a mass potential for $|\Psi|/\eta \ll 1$ and is bounded for $|\Psi|/\eta \gg 1$. We will also discuss a second potential that has new features as

compared to $U_{(1)}$, as we will demonstrate below. This potential reads,

$$U_{(2)}(|\Psi|) = \mu^2 (|\Psi|)^2 - \lambda_4 (|\Psi|)^4 + \lambda_6 (|\Psi|)^6, \quad (2.5)$$

where μ is the mass and λ_4 and λ_6 are self-interaction coupling constants.

In the following, we will discuss stationary, spherically symmetric solutions to the resulting field equations and hence choose the following ansatz for the metric and matter fields:

$$\begin{aligned} ds^2 &= -(\sigma(r))^2 N(r) dt^2 + \frac{1}{N(r)} dr^2 \\ &\quad + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \\ N(r) &= 1 - \frac{2m(r)}{r} \\ A_\mu dx^\mu &= V(r) dt, \quad \Psi = \psi(r) \exp(-i\omega t). \end{aligned} \quad (2.6)$$

We will use dimensionless coordinates and functions in the following. The rescaling is different for the two different potentials. We rescale as follows:

(i) For potential $U_{(1)}$

$$\begin{aligned} r &\rightarrow r/\mu, \quad \omega \rightarrow \mu\omega, \quad \gamma \rightarrow \gamma/\mu^2, \\ \psi &\rightarrow \eta\psi, \quad m(r) \rightarrow m(r)/\mu, \end{aligned}$$

which gives the dimensionless gravitational constant

$$\alpha = 4\pi G \eta^2; \quad (2.7)$$

(ii) For potential $U_{(2)}$

$$r \rightarrow r/\mu, \quad V \rightarrow \frac{\mu}{\sqrt{\lambda_4}} V, \quad \lambda_6 \rightarrow \frac{\lambda_4^2}{\mu^2} \lambda_6, \quad q \rightarrow \sqrt{\lambda_4} q$$

and

$$\omega \rightarrow \mu\omega, \quad \gamma \rightarrow \gamma/\mu^2, \quad \psi \rightarrow \frac{\mu}{\sqrt{\lambda_4}} \psi, \quad m(r) \rightarrow m(r)/\mu,$$

which gives the dimensionless gravitational constant

$$\alpha = \frac{4\pi G \mu^2}{\lambda_4}. \quad (2.8)$$

Using these rescalings and inserting the ansatz (2.6) into the field equations, we find a set of four coupled, nonlinear differential equations. The gravity equations read,

$$m' = \alpha r^2 \left[\frac{V'^2}{2\sigma^2} + N\psi'^2 + U(\psi) + \frac{((\omega - qV)\psi)^2}{N\sigma^2} \right] + \alpha\gamma \frac{(1-N)(V')^2}{\sigma^2}, \quad (2.9)$$

$$\sigma' = 2\alpha r\sigma \left[\psi'^2 + \frac{((\omega - qV)\psi)^2}{N^2\sigma^2} \right] + 2\alpha\gamma \frac{(V')^2}{r\sigma}, \quad (2.10)$$

where $\alpha = 4\pi G\eta^2$. The prime now and in the following denotes the derivative with respect to r . For the matter fields we have

$$\left(1 + \frac{2\gamma(1-N)}{r^2} \right) V'' + \left[\frac{2}{r} - \frac{\sigma'}{\sigma} - \frac{2\gamma}{r^2} \left((1-N)\frac{\sigma'}{\sigma} + N' \right) \right] V' + \frac{2q(\omega - qV)\psi^2}{N} = 0, \quad (2.11)$$

$$\psi'' + \left(\frac{2}{r} + \frac{N'}{N} + \frac{\sigma'}{\sigma} \right) \psi' + \frac{(\omega - qV)^2\psi}{N^2\sigma^2} - \frac{1}{2N} \frac{dU}{d\psi} = 0. \quad (2.12)$$

Due to the U(1) gauge symmetry, the field equations depend only on the combination $\omega - qV(r)$. These equations have to be solved subject to the appropriate boundary conditions. To ensure regularity at the origin, we require

$$N(0) = 0, \quad \psi'(0) = 0, \quad V'(0) = 0. \quad (2.13)$$

Demanding asymptotic flatness and finite energy leads to

$$\sigma(r \rightarrow \infty) \rightarrow 1, \quad \psi(r \rightarrow \infty) \rightarrow 0, \quad V(r \rightarrow \infty) \rightarrow V_\infty, \quad (2.14)$$

with V_∞ a real-valued constant. In the following, we will additionally choose $\psi(0) = C$ with C a real-valued constant. This is to avoid the trivial solution $\psi(r) \equiv 0$. This choice fixes the value of ω which no longer is a free parameter. The scalar field has the following decay:

$$\psi(r \rightarrow \infty) \rightarrow \frac{1}{r} \exp \left(-\sqrt{1 - (\omega - qV_\infty)^2} r \right). \quad (2.15)$$

In order to ensure localization of the scalar field we need to require that $\omega - qV_\infty < 1$. In the following, we will define the parameter $\Omega := 1 - (\omega - qV_\infty)$ and demonstrate that solutions cease to exist when $\Omega \rightarrow 0$.

The physical parameters of the solutions are the (dimensionless) mass M , the (dimensionless) electric charge Q and the (dimensionless) Noether charge Q_N . These can be read off from the metric and matter field functions at infinity,

$$N(r \gg 1) = 1 - \frac{2M}{r} + \frac{2\alpha Q^2}{r^2} + \dots, \quad V(r) = V_\infty - \frac{Q}{r} + \dots, \quad (2.16)$$

and the following integral of the t -component of the locally conserved Noether current:

$$Q_N = \int_0^\infty dr \frac{2r^2 q V \psi^2}{N\sigma}. \quad (2.17)$$

This globally conserved quantity can be interpreted as the number of scalar bosons and hence $qQ_N \equiv Q$.

Note that the coupled differential equations do not possess a globally regular solution for $\psi \equiv 0$. Additionally setting $\gamma = 0$, the system has a closed form solution, the Reissner-Nordström (RN) solution:

$$N = 1 - \frac{2M}{r} + \frac{2\alpha Q^2}{r^2}, \quad \sigma \equiv 1, \quad V(r) = V_\infty - \frac{Q}{r}, \quad (2.18)$$

with event horizon at $r_+ = M(1 + \sqrt{1 - 2\alpha Q^2/M^2})$. The extremal RN solution has $r_+ = M = \sqrt{2\alpha}Q$. For $\gamma \neq 0$ and $\psi \equiv 0$ the solutions were studied in [25].

Finally, in order to make the connection to the Mazur-Mottola solution [12], we give the components of the energy-momentum tensor for the solutions. The energy density $\rho = -T'_t$, radial pressure $p_r = T'_r$ and transverse pressure $p_t = T'_\theta = T'_\phi$ are given by

$$\rho = \frac{V'^2}{2\sigma^2} + N\psi'^2 + \frac{(\omega - qV)^2\psi^2}{N\sigma^2} + U(\psi), \quad (2.19)$$

$$p_r = -\frac{V'^2}{2\sigma^2} + N\psi'^2 + \frac{(\omega - qV)^2\psi^2}{N\sigma^2} - U(\psi), \quad (2.20)$$

$$p_t = \frac{V'^2}{2\sigma^2} - N\psi'^2 + \frac{(\omega - qV)^2\psi^2}{N\sigma^2} - U(\psi). \quad (2.21)$$

Clearly, $\rho \geq p_r$ and $\rho \geq p_t$, i.e., the model fulfills the causality bound as well as the weak energy condition.

III. NUMERICAL RESULTS

A. Exponential self-interaction $U = U_{(1)}$

In the following, we will discuss our results for the exponential self-interaction potential (2.4).

1. $\gamma = 0$: The role of scalar self-interaction and charge

We first discuss the solutions obtained in the case of a minimal coupling to gravity i.e., for $\gamma = 0$. In order to show that the self-interaction term is crucial for the existence of frozen stars, we first discuss the case of a simple mass potential for the scalar field, i.e., we choose $U(\psi) = \psi^2$. In

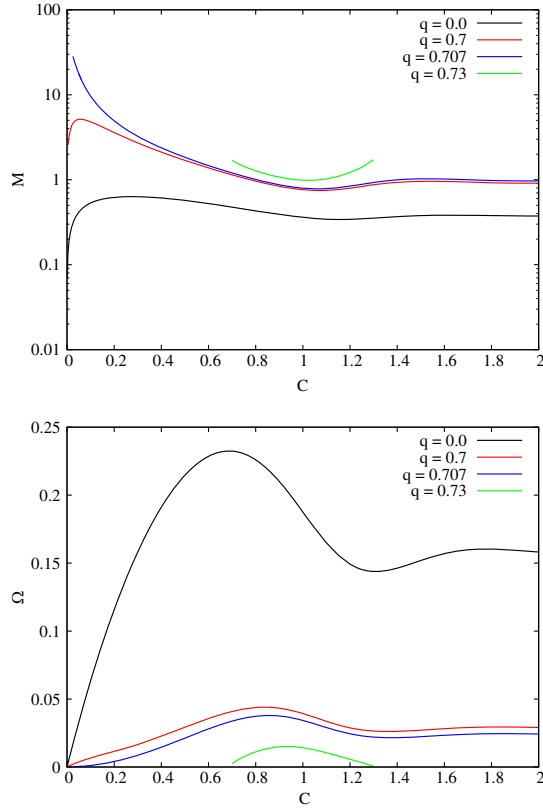


FIG. 1. Left: We show the dependence of the mass M on the value C of the scalar field $\psi(r)$ at the origin for different values of q . Right: The corresponding value of $\Omega = 1 - (\omega - qV_\infty)^2$.

this case, α can be set to unity without loss of generality by applying a rescaling of the matter fields $V \rightarrow V/\alpha$, $\psi \rightarrow \psi/\alpha$. Hence, the only remaining coupling is q that gets rescaled as $q \rightarrow \alpha q$.

These solutions have been studied in [27] and revisited recently in [28]. In order to be able to compare with the self-interacting case, we put the emphasis here on the domain of existence of solutions in the $q - C$ -plane. A family of boson stars corresponding to a specific value of the gauge coupling constant q can be constructed numerically by increasing gradually the parameter C . Increasing C , the value of the function $\sigma(r)$ at the origin, $\sigma(0)$, decreases from unity. Our numerical results strongly suggest that $\sigma(0)$ approaches zero for some maximal value of C . The dependence of the mass M and the parameter Ω on the central value C is shown in Fig. 1. As described in [28], for $q > 1/\sqrt{2}$ we observe that solutions do exist for arbitrary C , but only on a very limited interval. Increasing q beyond this value decreases this interval until solutions cease to exist for $q > 0.75$ [29]. For all solutions constructed, we find that the metric function $N(r)$ presents a minimum that stays finite and hence never approaches zero. This suggests that *charged boson stars in standard Einstein gravity with a simple mass potential cannot reach a frozen state.*

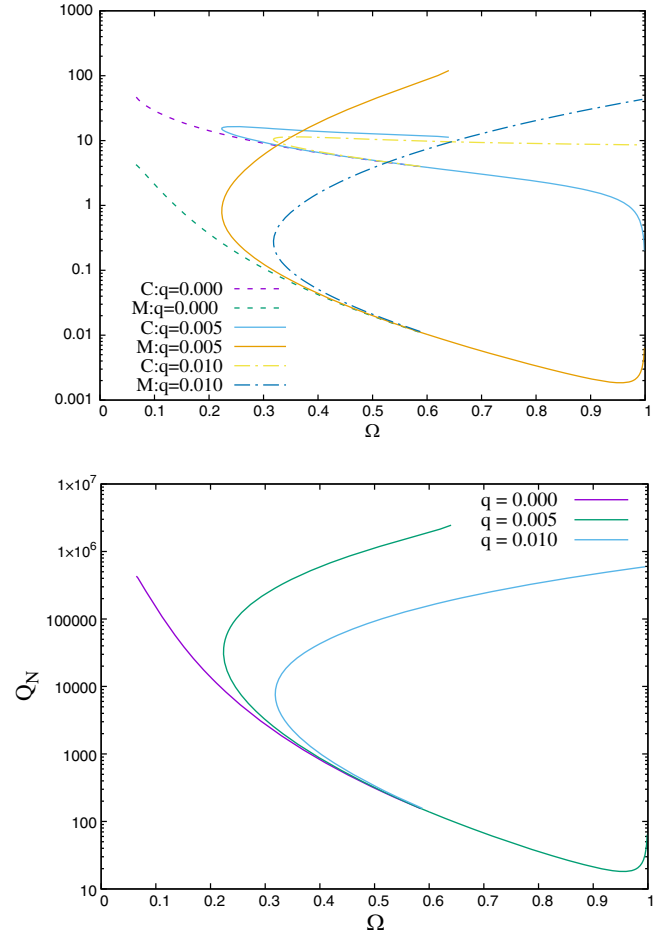


FIG. 2. Left: The show the value C of the scalar field $\psi(r)$ at the origin as well as the mass M of the solutions in dependence on Ω for $\alpha = 0.0001$ and several values of q . Right: We show the dependence of the Noether charge Q_N on Ω for the same solutions.

However, this changes when we introduce the self-interacting potential (2.4) as we will demonstrate below. Since we cannot perform a rescaling similar to the one used in the mass potential case, α and q are now free coupling constants. In the following, we set $\alpha = 0.0001$. For $q = 0$ the solutions can be characterized by Ω and the parameter C , see Fig. 2. In this figure, we show the mass M and the parameter C in dependence on Ω for several values of q , including $q = 0$. Solutions with large C do exist, and the metric function $N(r)$ possesses a local minimum that, however, remains perfectly finite along the branches. Typically, we find that the minimum of N is ~ 0.5 .

This changes when $q > 0$. As can be clearly seen in Fig. 2, we find two branches of solutions in Ω . These two branches join at a minimal value of $\Omega = \Omega_{\min}$. This minimal value depends on q , e.g., $\Omega_{\min} \approx 0.223$ for $q = 0.005$, and $\Omega_{\min} \approx 0.318$ for $q = 0.01$. In the following, we will refer to the branch of lower M as branch A, while the branch of higher M will be denoted branch B.

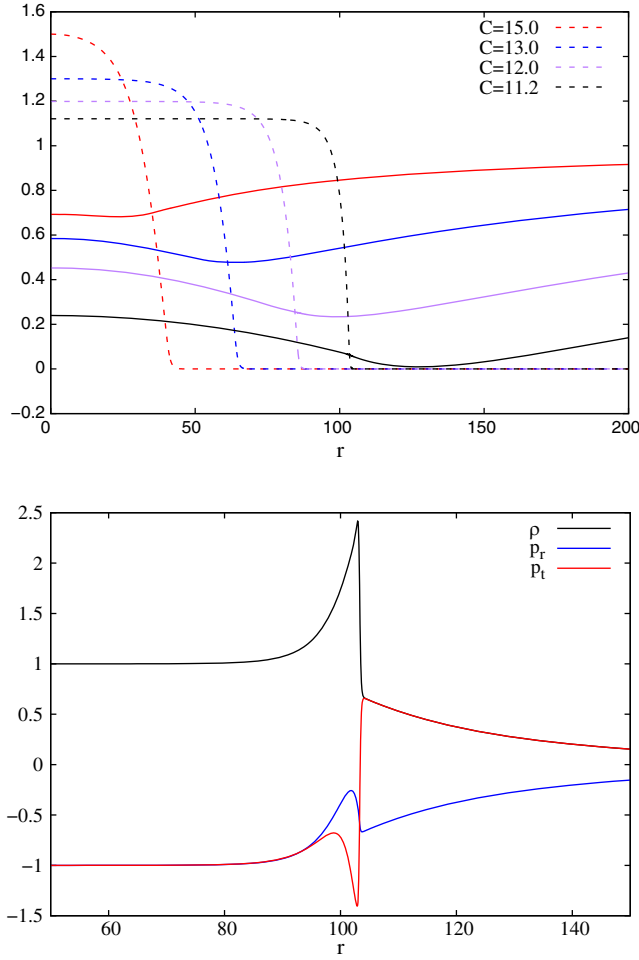


FIG. 3. Left: We show the profiles of the metric function $-g_{tt} = \sigma^2 N$ (solid) and of the scalar field function $\psi(r)$ (dashed) for three charged boson star solutions on the branch B . Here $\alpha = 0.0001$, $q = 0.005$ and the three solutions have different values of C , i.e., different values of the scalar field at the origin. Clearly, we see that the metric function approaches a double zero at some intermediate value of the radial coordinate r when decreasing C . Right: The corresponding energy density ρ , radial pressure p_r and transversal pressure p_t for the solution with $C = 11.2$.

When q is sufficiently large (typically $q \sim 0.01$) the two branches can be extended to $\Omega = 1$ and the local minimum of the metric function $N(r)$, again, is well above zero. Interestingly, for smaller values of q (typically $q \sim 0.005$) the branch B stops at some intermediate value of $\Omega = \Omega_c$ with $\Omega_{\min} < \Omega_c < 1$. We find that the branch terminates here because the minimum of $N(r)$ approaches zero at some value r_c of the radial coordinate. This approach is shown in Fig. 3 (left). Clearly, the solution develops a double zero in this case. These solutions have previously been discussed in the context of forming a thin shell between the boson star interior and its exterior [21]. This behaviour can be understood as follows: for $q = 0$, an arbitrary number Q_N of scalar quanta can coexist bounded by the gravitational force, which is no longer

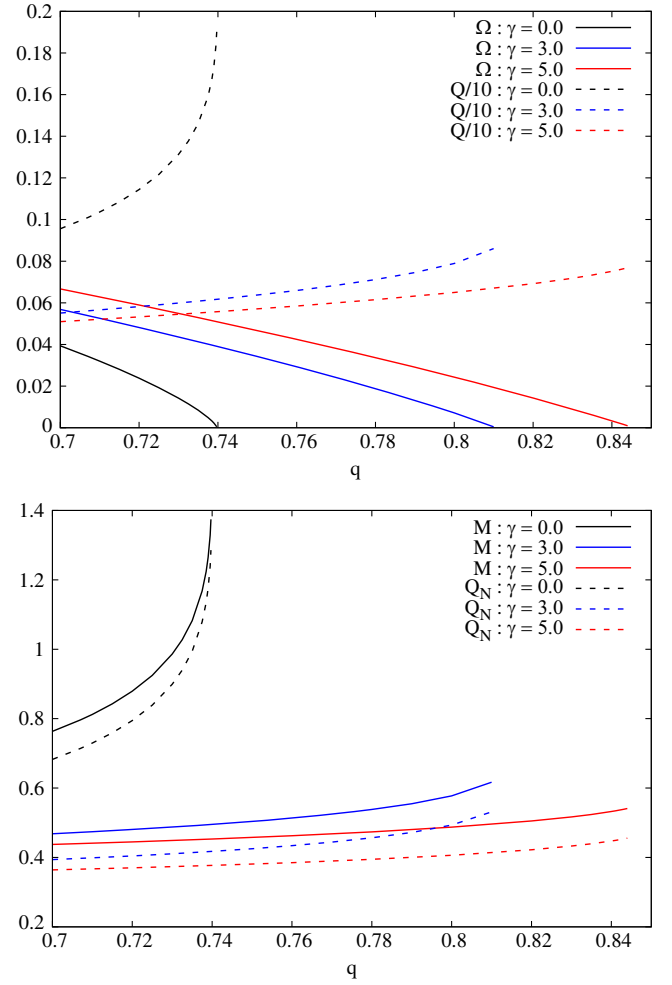


FIG. 4. Left: We show the dependence of Ω and of the electric charge Q on the gauge coupling constant q for constant central value of the scalar field, $\psi(0) = 1$, and three different values of γ . Right: We show the corresponding mass M and Noether charge Q_N of the Horndeski vector-tensor boson star in dependence on q .

true when $q > 0$. In this case, the quanta also carry electric charge and this leads to an electric repulsion. The balance between the gravitational force and the electric repulsion allows for compact boson stars to exist, so dense that a horizon can be formed. For very large values of q , the electric repulsion is too large to allow very compact objects.

In [12] it was argued that in a model with anisotropic fluid that fulfills $\rho = -p_r$ so-called *frozen stars* can be constructed. In Fig. 3 (right) we show the energy-momentum tensor components of our solution with $C = 11.2$. Clearly, the interior of the solution is free from singularities and $\rho = -p_r = -p_t$, i.e., it possesses a de Sitter core. There exists a spherical shell of finite thickness that interpolates with the RN exterior, where $\rho = p_t = -p_r$. Hence, this configuration is globally regular interpolating between a regular interior and a RN exterior with the event horizon replaced by a shell of finite thickness.

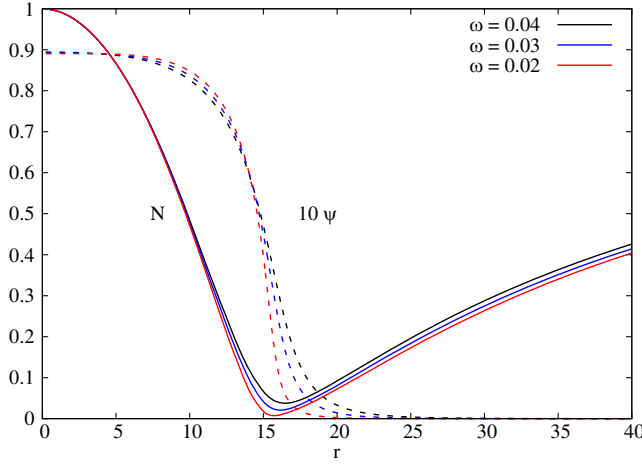


FIG. 5. We show the profiles of the metric function $N(r)$ and of the scalar field function $\psi(r)$ for a vector-tensor Horndeski boson star with $q = 0.7$, $\gamma = -3$ and three values of ω .

2. $\gamma \neq 0$

In the following, we would like to demonstrate that the frozen states of boson stars also appear without self-interaction of the scalar field (which is necessary in standard Einstein gravity, see above), but when considering vector-tensor Horndeski gravity. In the following, we will replace the self-interaction potential by a mass term for the scalar field, i.e., $U(\psi) = \psi^2$.

Our numerical results indicate that for $\gamma > 0$, the properties of the solutions are qualitatively similar to those discussed above. Quantitatively, we observe that boson stars with positive γ exist generically for higher values of the gauge coupling q as compared to the $\gamma = 0$ case. This is shown in Fig. 4, where we give the dependence of Ω and Q on the gauge coupling q for three different values of γ including $\gamma = 0$ (left) for a fixed central value of the scalar field $\psi(0) = 1$. The limit $\Omega \rightarrow 0$ is reached for $q = q_{\max}$. We find that $q_{\max}$ increases with γ . The dependence of the mass M and the Noether charge Q_N on q also changes with γ , see Fig. 4 (right). For $\gamma = 0$ both M and Q_N increase quicker with q than in the $\gamma > 0$ case. Hence, vector-tensor Horndeski boson stars have lower mass and Noether charge for the same value of q as compared to the boson stars in standard Einstein gravity.

Interestingly, choosing $\gamma < 0$ leads to new features. In the following, we will demonstrate this for the case $q = 0.7$, $\gamma = -3$. Let us first note that vector-tensor boson stars do exist in the limit $C \rightarrow 0$ which corresponds to $\Omega \rightarrow 0$. However, decreasing the frequency parameter ω we find that the solution approaches a configuration that looks like a frozen star. This configuration possesses a quasihorizon with the metric function $N(r)$ coming close to zero at some intermediate value of the radial coordinate. This is shown in Fig. 5, where we give the profiles of the metric function $N(r)$ together with the profile of the scalar field function $\psi(r)$ for different values of ω . Decreasing ω

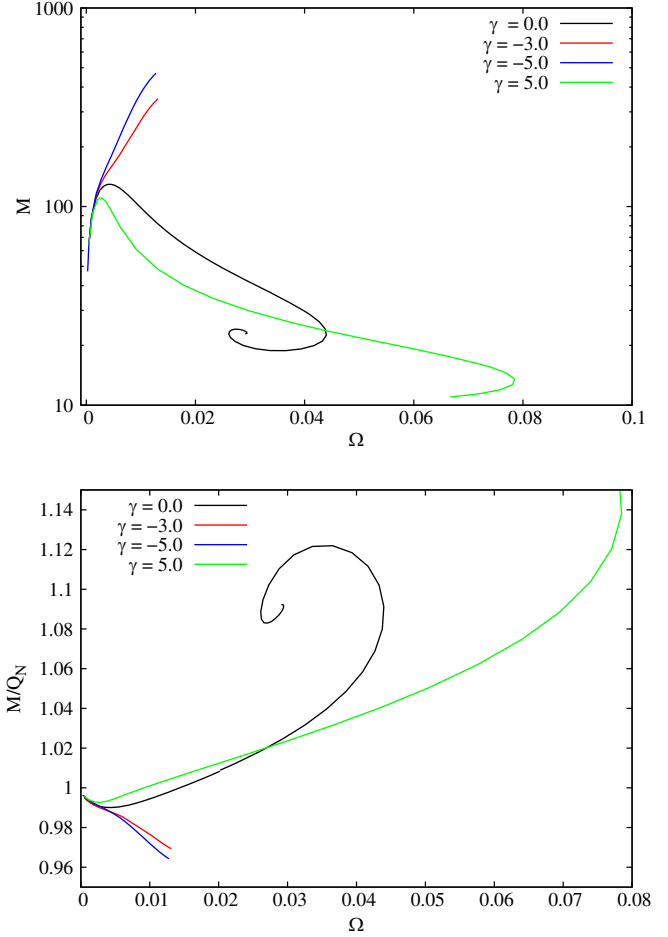


FIG. 6. Left: We show the dependence of the mass M on the parameter Ω for $q = 0.7$ and different values of γ . Right: We show the ratio M/Q_N in dependence on Ω for the same solutions.

(which corresponds to increasing Ω in this case), the value of the minimum of $N(r)$ approaches zero, while the scalar field is well-behaved.

In general, we observe that this limiting behaviour leads to the fact that the branches of solutions with $\gamma < 0$ are very limited in both C and Ω . This can be seen in Fig. 6, where we compare the dependence of the mass M (left) and the ratio between the mass M and the Noether charge Q_N (right) on Ω . For $\gamma \geq 0$, we find that more than one branch of solutions exists, while for $\gamma < 0$ only one branch appears. In this latter case, the branches end exactly due to the limiting behaviour described above, i.e., the branches end at a frozen state. To be able to compare this, we note that the values of $\omega = 0.04$, 0.03 and 0.02 correspond to $\Omega = 0.01314$, 0.01336 , and 0.01360 , respectively. It is clear from our numerical data that the frozen states of Horndeski vector-tensor boson stars are larger in mass M than their nonfrozen counterparts. However, we find that $M/Q_N < 1$ for all solutions with $\gamma < 0$. This means that these configurations are stable with respect to the decay into Q_N individual bosons of mass $\mu = 1$.

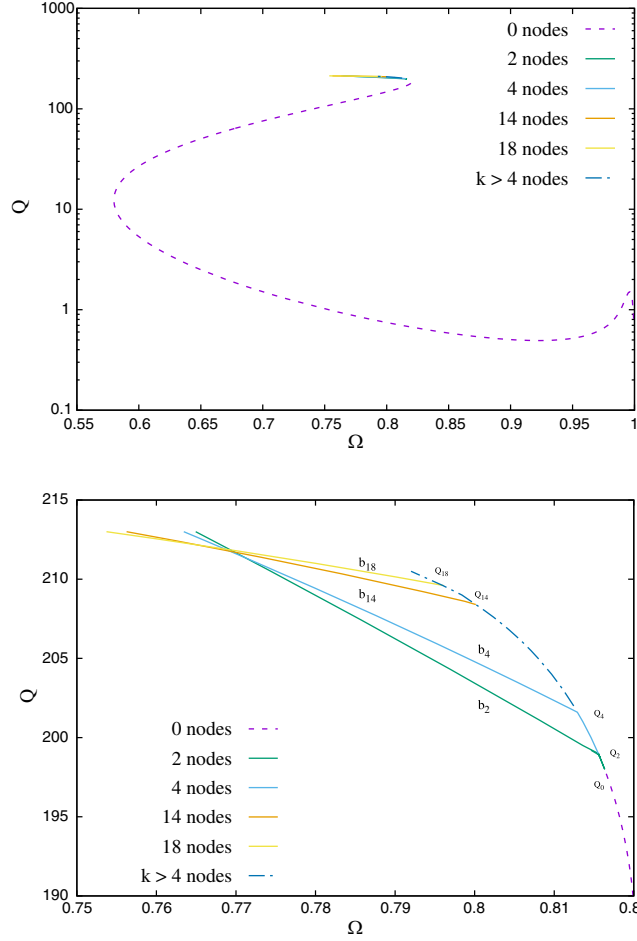


FIG. 7. Left: We show the dependence of the electric charge Q on Ω for solutions $\alpha = 0.012$, $q = 0.08$, $\lambda_6 = 9/32$ and different node numbers k of the scalar field function. Right: We show the details of the plot on the left in the region of bifurcations.

B. Polynomial potential $U_{(2)}$

We will now consider the polynomial potential $U_{(2)}$ and demonstrate that new features appear. Note that the mass parameter μ and the coupling constant λ_4 can be set to unity without loss of generality by suitable rescaling of r and ψ . Very similar to the case of the exponential potential, we can find frozen states of charged boson stars. What is new now and was already discussed in [21] is that the scalar field can now possess a number $k = 2n$, $n \in \mathbb{N}$ of nodes in the interval of the radial coordinate r in which the quantity

$$\frac{1}{N} - \frac{q^2 V^2}{\sigma^2 N^2} < 0 \quad (3.1)$$

becomes negative. The above expression constitutes the effective (and position-dependent) mass of the scalar field when linearizing the Klein-Gordon equation. This is possible whenever $N(r)$ is sufficiently close to zero. In [21] a number of solutions of this type were presented, but no systematic study was performed and, in particular, no

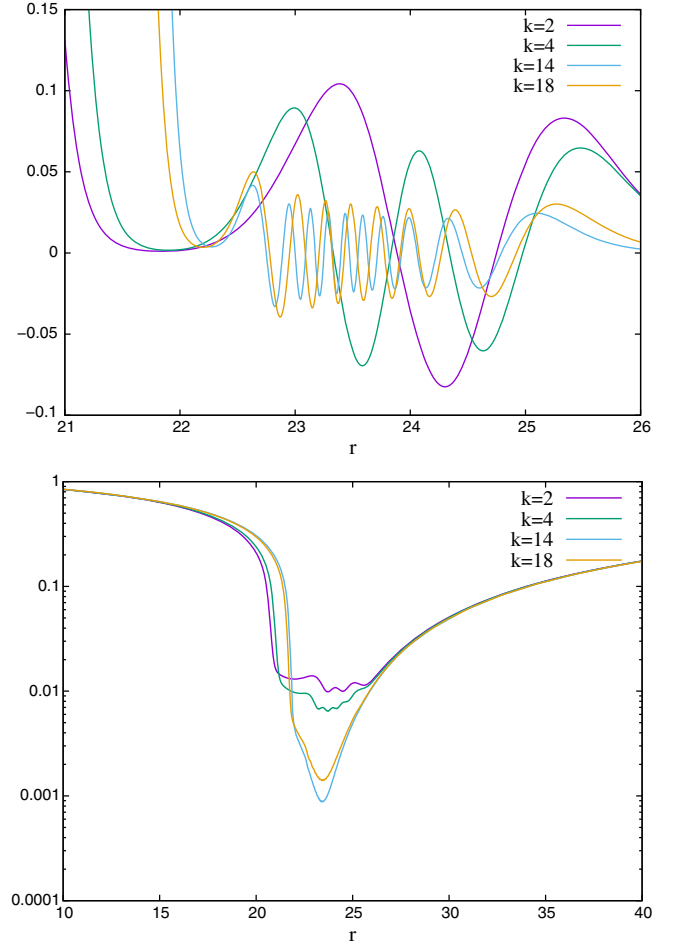


FIG. 8. Left: We show the profile of the scalar field $\psi(r)$ for solutions with $k = 2$, $k = 4$, $k = 14$, and $k = 18$ nodes for $\alpha = 0.012$, $q = 0.08$, $\lambda_6 = 9/32$ and $Q = 213$. Right: The corresponding profiles for the metric function $N(r)$.

connection to the Mazur-Motolla solution was made. In the following, we will choose $\alpha = 0.012$, $q = 0.08$, and $\lambda_6 = 9/32$ to attempt a systematic approach. For this choice of coupling constants we were able to construct solutions with zero nodes in the scalar field ($k = 0$) and with electric charge Q up to $Q \approx 197$. This is seen in Fig. 7, where the relation between Ω and Q is presented. For larger values of Q the pattern becomes more involved and in the following, we will discuss it with the help of the labels Q_k and b_k (see Fig. 7). One solution with no node of the scalar field exists for each value of Q for $Q \leq Q_0$ with $Q_0 \sim 197.0$. This corresponds to $\Omega \approx 0.816$. Let us call this family of solutions the “main branch.” This is the purple dashed curve in Fig. 7 (right). For $Q > Q_0$, the scalar field starts to develop nodes. In the interval $Q \in [Q_0, Q_2] \approx [197.0, 199.0]$ a single solution with two nodes ($k = 2$) exists. For $Q > Q_2$ solutions with $k = 4$ exist and the branch of these solutions continues the main branch. Moreover, our results indicate the emergence of a new branch of solutions with $k = 2$, which is the green line with label b_2 in Fig. 7. This branch joins the main

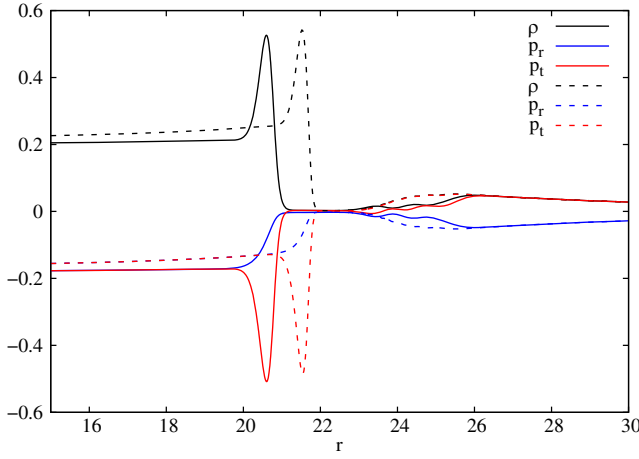


FIG. 9. We show the nonvanishing components of the energy-momentum tensor, i.e., the energy density ρ , radial pressure p_r and transversal pressure p_t for solutions with $k = 2$ (solid) and $k = 18$ (dashed) and for $\alpha = 0.012$, $q = 0.08$, $\lambda_6 = 9/32$, and $Q = 213$.

branch in the limit $Q \rightarrow Q_2$. In other words the branch b_2 bifurcates from the main branch at $Q = Q_2$. In the interval $Q \in [Q_2, Q_4] \sim [199.0, 201.6]$ there is a single solution with $k = 4$ and solutions with $k = 6$ exist for $Q > Q_4$. Similar to the case with two lesser nodes described above, for $Q > Q_4$, a new branch of solutions with $k = 4$ bifurcates from the main branch at $Q = Q_4$. This is the blue curve, labeled b_4 in Fig. 7 (right). This pattern of bifurcations repeats itself for larger k ; new branches of solutions bifurcate from the main branch with $k = 2n$, $n \in \mathbb{N}$. We were able to construct solutions up to $k = 16$ [labeled b_{16} in Fig. 7 (right)] and $k = 18$ [labeled b_{18} in Fig. 7 (right)].

In Fig. 8 we show the scalar field function $\psi(r)$ (left) and the metric function $N(r)$ (right) for $Q = 213.0$ and $k = 2, 4, 16, 18$. We observe that the amplitude of the spatial oscillation in $\psi(r)$ decreases with increasing node number. Moreover, the metric function $N(r)$ shows spatial oscillations around its minimum, that, however, seem only to be of appreciable amplitude for low node numbers.

All these results suggest that, for sufficiently large Q , a discrete family of frozen states of boson stars exists that are labeled by the number of nodes k of the scalar field function. To understand the connection to the Mazur-Motolla (MM) solution, we give the components of the energy-momentum tensor for solutions with two and eighteen nodes for $Q = 213$ in Fig. 9. This demonstrates the existence of four shells, not three, as in the original MM solution (and the solutions presented for the exponential self-interaction potential above):

$$\begin{aligned}
 r \in [0; r_1]: \quad & \rho = -p_r = -p_t, \\
 r \in [r_1; r_2]: \quad & \rho = -p_t \neq -p_r, \\
 r \in [r_2; r_3]: \quad & \rho = p_r = p_t \approx 0, \\
 r \in [r_3; \infty]: \quad & \rho = -p_r \approx p_t.
 \end{aligned} \tag{3.2}$$

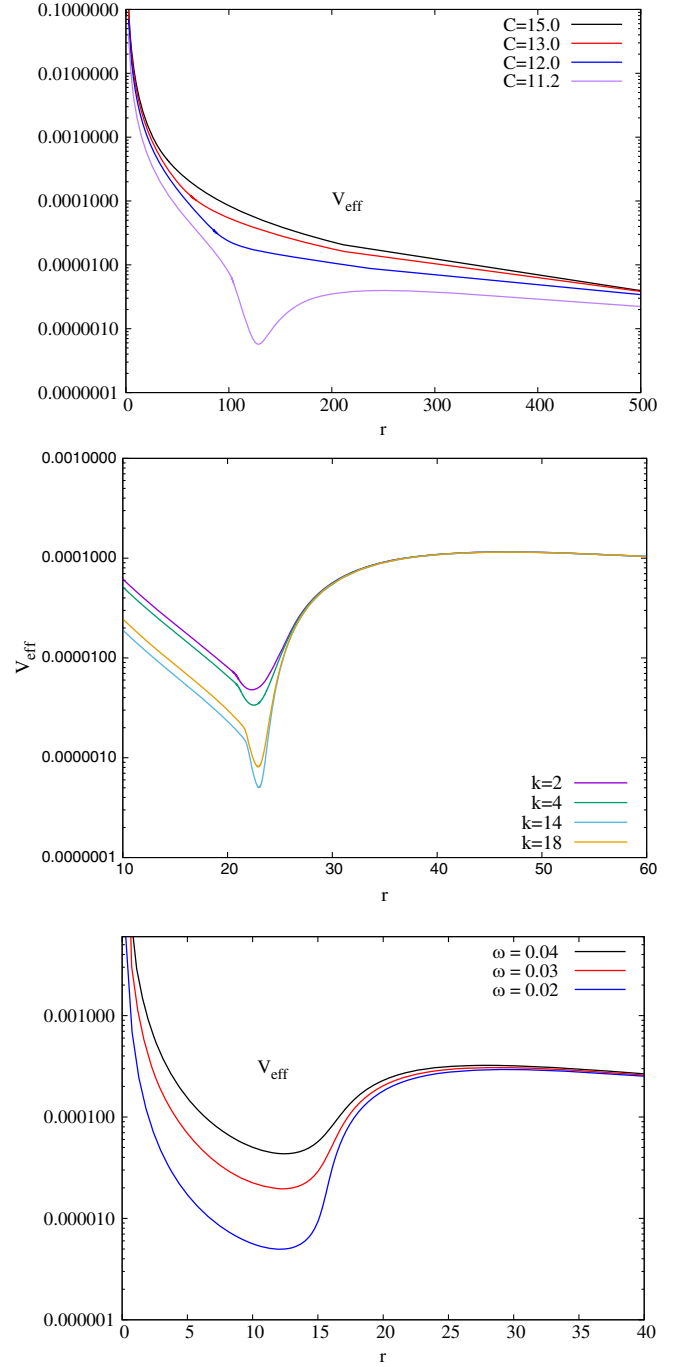


FIG. 10. We show the effective potential V_{eff} for lightlike particle motion in the space-time of (a) charged frozen boson stars with exponential self-interaction of the scalar field (top left), (b) charged frozen boson stars with polynomial self-interaction, and different node numbers k of the scalar field (top right) (c) Horndeski charged frozen boson stars (bottom).

The solution possesses a de Sitter core, an intermediate shell which consists of two shells, the outer of which is vacuum, and an exterior shell corresponding to the RN solution.

IV. DISCUSSION

In this paper, we have studied the existence of frozen states of charged boson stars. We find that the standard ungauged scalar field model does not allow for these states. However, when gauging the U(1) symmetry and introducing a self-interaction for the scalar field, we find that these states appear for intermediate values of the gauge coupling. Moreover, adding a vector-tensor Horndeski term, we find that these frozen states also appear without the scalar self-interaction. In order to understand observational signatures of these configurations, let us consider the (equatorial) geodesic motion of lightlike test particles which can be described by the following equation:

$$\sigma^2 \dot{r}^2 + \tilde{V}_{\text{eff}}(r) = E^2, \quad \tilde{V}_{\text{eff}} = L_z^2 \frac{N\sigma^2}{r^2}, \quad (4.1)$$

where E is the energy of the test particle, while L_z is its angular momentum with respect to the z -axis. The RN solution possesses a stable and one unstable lighttring, but only the latter can be observed as it lies outside the event horizon. In Fig. 10, we show the effective potential $V_{\text{eff}} = \tilde{V}_{\text{eff}}/L_z^2$ at the approach to the frozen state. This demonstrates that these configurations possess an inner stable and an outer unstable lighttring. The stable lighttring lies within the thin, intermediate shell, while the unstable lighttring can be found in the black hole exterior, which for $\gamma = 0$ corresponds to the RN solution. When increasing the node number k in the case of the polynomial self-interaction, the location of the lighttrings is essentially unchanged, however, the value of the minimum of V_{eff} changes.

Possessing a lighttring, these objects qualify as ultra-compact objects (UCO) and it has been shown that for

UCOs lighttrings always come in pairs [30]. The existence of a stable lighttring has been suggested to lead to instabilities of UCOs [31], which has been confirmed by a full numerical simulation in pure scalar field models [32]. The frozen states in our model possess additional gauge fields and, in particular, a de Sitter core. So, it would be interesting to understand whether the results of [32] also apply to the frozen states discussed here.

Finally, let us comment on the (in)stability of these objects. The stability of gauged boson stars was first addressed in [33] and it was found that below a critical density the solutions (even those with nodes) are dynamically stable. The difference to our model is that only self-interaction of fourth-order was taken into account. In [28], charged boson stars without self-interaction have been discussed and it was found that solutions for gauge coupling lower than a critical value are stable, while solutions for gauge coupling above a critical value (so-called “super-critical” boson stars) have been found to be unstable. A three-dimensional numerical evolution of the full system confirmed that the stability depends crucially on the gauge coupling [34]. While gauged boson stars without self-interaction were shown to eventually form black holes, it was also shown that the self-interaction renders these configurations stable against gravitational collapse. Hence, we believe that the solutions discussed in this paper will eventually be stable, although a full numerical solution as that performed in [34] would be able to confirm this.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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