

Strange and flavor-singlet axial form factors of the nucleon from lattice QCD

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The singlet axial form factor of the nucleon provides essential input for a complete understanding of the nucleon axial structure. Together with the isovector and isoscalar octet channels, in the forward limit it forms the basis for a full flavor decomposition of the proton spin. In this work we present a lattice QCD determination of the singlet axial form factor $G_A^{u+d+s}(Q^2)$ and related strange contribution $G_A^s(Q^2)$ using a set of $N_f = 2 + 1$ gauge ensembles generated by the Coordinated Lattice Simulations effort with $O(a)$ -improved Wilson fermions, with a full error budget for the extrapolation to the chiral, continuum and infinite-volume limits. Particular focus is placed on the treatment of the disconnected contributions, which constitute the crucial element for the extraction of the strange component. Together with determinations of the isovector and isoscalar octet axial form factors, this work provides a comprehensive lattice QCD determination of the nucleon axial structure across different flavor channels.

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I. INTRODUCTION

The axial form factor of the nucleon plays a central role in our understanding of the electroweak interactions of nucleons as well as the proton's intrinsic structure. The axial form factor enters a wide range of processes such as neutron β decay and neutrino-nucleon scattering, contributing to our theoretical understanding of neutrino cross sections relevant for current and future long-baseline neutrino-oscillation experiments. In addition, charges of the axial current encode information on the fraction of the proton spin carried by quarks [1–3].

In the SU(3) flavor basis, the nucleon axial current can be decomposed into isovector ($u - d$), isoscalar octet ($u + d - 2s$), and singlet ($u + d + s$) components. These channels probe distinct aspects of QCD: the isovector channel is sensitive to W boson exchange, whereas the isoscalar currents are sensitive to Z boson scattering, which also receives contributions from the strange quark. In

particular, the isovector channel is relevant in the analysis of upcoming neutrino experiments [4,5], whereas channels involving Z boson exchange and sea-quark contributions are needed as input for the precise extraction of the weak charge of the proton in the upcoming P2 experiment [6]. Furthermore, the strange component $G_A^s(Q^2)$ is crucial for calculating the cross section within various dark matter models [7]: in this context, it is particularly relevant for the MicroBooNE neutrino detector at Fermilab, which aims to extract $G_A^s(Q^2)$ in the range from $Q^2 = 0.08 \text{ GeV}^2$ to $Q^2 = 1 \text{ GeV}^2$ [8,9].

Lattice QCD provides a nonperturbative framework for computing nucleon axial form factors from first principles. While the isovector combination has seen substantial progress [10–18], the remaining flavor channels have so far been explored less extensively at the level of full form-factor calculations. Existing studies of the singlet and strange channels [19,20] do not include a controlled continuum extrapolation, while several lattice determinations of the corresponding axial charges are available [21–24]. The first complete lattice determination of the isoscalar octet channel has been reported only recently [25,26]. Indeed, while the ($u - d$) combination is free of quark-disconnected contributions, these appear in the isoscalar channels, requiring extra computational cost and a more careful analysis. Moreover, in the singlet case, the

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axial current is further distinguished by the presence of the axial anomaly, which introduces gluonic contributions leading to additional complications in both the lattice calculation and the renormalization of the corresponding matrix elements.

In a series of previous works [14,26], we have investigated the nucleon axial form factor using a set of $N_f = 2 + 1$ gauge ensembles generated by the Coordinated Lattice Simulations (CLS) effort and a common analysis strategy. Our earlier publications presented results for the isovector axial form factor [14] and for the isoscalar octet combination [26]. In the present work, we extend this program to the $SU(3)_f$ singlet axial form factor and, simultaneously, to the strange axial form factor.

Our goal in this work is to determine the momentum-transfer dependence, highlighting the impact of disconnected contributions. In particular, we focus on the treatment of the strange-quark component, which is combined with the previously determined octet contribution to construct the flavor-singlet channel. By completing the analysis of the isovector, octet, and singlet axial form factors within a single computational framework, this work provides a comprehensive lattice QCD determination of the channels relevant for the quark-flavor decomposition of the nucleon axial structure in the $SU(3)$ flavor basis.

This paper is organized as follows. In Sec. II we briefly summarize the lattice setup and correlation-function construction common to the previous works, and we emphasize aspects specific to the singlet axial current. Section III describes the analysis strategy and treatment of disconnected contributions specific to this work, and the extraction of the strange and singlet form factors. We present our results for the form factors in Sec. IV, and conclude in Sec. V.

II. LATTICE SETUP

We consider the axial-vector currents

$$A_\mu^a = \bar{\psi}(x)\gamma_\mu\gamma_5\lambda^a\psi(x), \quad \psi = (u, d, s)^T, \quad (1)$$

where λ^a are the Gell-Mann matrices, $\lambda^0 = \sqrt{2/3}\mathbb{I}$, and explicitly

$$\begin{aligned} A_\mu^0 &\equiv \sqrt{\frac{2}{3}}A_\mu^{u+d+s} = \sqrt{\frac{2}{3}}(\bar{u}\gamma_\mu\gamma_5u + \bar{d}\gamma_\mu\gamma_5d + \bar{s}\gamma_\mu\gamma_5s), \\ A_\mu^8 &\equiv \frac{1}{\sqrt{3}}A_\mu^{u+d-2s} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5u + \bar{d}\gamma_\mu\gamma_5d - 2\bar{s}\gamma_\mu\gamma_5s). \end{aligned} \quad (2)$$

We use the following parametrization of their matrix elements

$$\langle N_{p'} | A_\mu^a | N_p \rangle = \bar{u}(p') \left[G_A^a(Q^2)\gamma_\mu + \frac{q_\mu}{2m} G_P^a(Q^2) \right] \gamma_5 u(p) \quad (3)$$

TABLE I. Summary of ensembles used, where the values for $\beta = 3.40, 3.46, 3.55, 3.70$ correspond to a lattice spacing of roughly $a \sim 0.086, 0.076, 0.064, 0.050$ fm, respectively [32,33].

ID	β	T/a	L/a	M_π (MeV)	M_N (MeV)
H102	3.40	96	32	354	1103
H105	3.40	96	32	280	1045
C101	3.40	96	48	225	980
N101	3.40	128	48	281	1030
S400	3.46	128	32	350	1130
N451	3.46	128	48	286	1011
D450	3.46	128	64	216	978
N203	3.55	128	48	346	1112
N200	3.55	128	48	281	1063
D200	3.55	128	64	203	966
E250	3.55	192	96	129	928
N302	3.70	128	48	348	1146
J303	3.70	192	64	260	1048
E300	3.70	192	96	174	962

where $N_{p'}$ indicates the proton state, p (p') is the momentum of the incoming (outgoing) proton, $q = p' - p$ the momentum transfer, $p^2 = p'^2 = m^2$, and $q^2 = -Q^2$.

The lattice calculation relies on the same data used in our previous works [14,26], i.e., 14 CLS [27] $N_f = 2 + 1$ gauge ensembles with $\mathcal{O}(a)$ improved Wilson fermions [28] and tree-level improved Lüscher-Weisz gauge action [29]. They span a range of lattice spacings from 0.050 fm to 0.086 fm and pion masses from 130 MeV to 350 MeV. The necessary reweighting factors to correct for the treatment of the strange-quark determinant have been computed according to [30,31]. A brief summary of the ensembles employed in this study is provided in Table I, and we refer to [14] for further details.

A. Correlation functions

The setup, conventions, and choice of spin projectors follow those adopted in our previous analyses, which we briefly recall here. For the computation of the relevant correlation functions we employ the nucleon interpolating field

$$\Psi_\alpha(x) = \epsilon_{abc}(\bar{u}_a^T(x)C\gamma_5\bar{d}_b(x))\bar{u}_{c,\alpha}(x), \quad (4)$$

using Gaussian smeared quark fields $\tilde{q}(x)$ [34] and spatially APE-smeared gauge links in the covariant Laplacian [35].

We build the two-point correlation functions as

$$C_2^\Gamma(t, \mathbf{p}) = \Gamma_{\alpha\beta} \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}} \langle \Psi_\beta(\mathbf{x}, t) \bar{\Psi}_\alpha(0) \rangle \quad (5)$$

as well as the connected three-point functions

$$C_3^{\text{conn},\Gamma'}(t, t_s, \mathbf{q}) = \Gamma'_{\alpha\beta} \sum_{\mathbf{x}, \mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}} \frac{\mathbf{q} \times \mathbf{s}}{|\mathbf{q} \times \mathbf{s}|^2} \cdot \langle \Psi_\beta(\mathbf{x}, t_s) \mathbf{q} \times \mathbf{A}(\mathbf{y}, t) \bar{\Psi}_\alpha(0) \rangle, \quad (6)$$

where the axial current is projected onto the orthogonal component to isolate the corresponding axial form factor, cf. Eq. (3). In practice, we align the nucleon spin along the z axis $\mathbf{s} = \mathbf{e}_3$, and choose the polarization matrix $\Gamma' = \frac{1}{2}(1 + \gamma_0)(1 + i\gamma_5\gamma_3)$. Momentum is injected at the operator insertion, and the sink is projected to zero momentum.

The disconnected contributions to the three-point correlator read

$$C_{3,\mu}^{\text{disc},\Gamma'}(t, t_s, \mathbf{q}) = \langle \mathcal{L}_\mu(t, \mathbf{q}) C_2^{\Gamma'}(t_s, \mathbf{0}) \rangle, \quad (7)$$

where $\mathcal{L}_\mu$ indicates the one-point function

$$\mathcal{L}_\mu(z_0, \mathbf{q}) = - \sum_z e^{i\mathbf{q}\cdot\mathbf{z}} \text{Tr}[S_q^{-1}(z, z) \gamma_\mu \gamma_5], \quad (8)$$

S_q^{-1} being the propagator for the quark q , and C_2 denotes the Wick contraction of the interpolating operators of Eq. (4),

$$C_2^{\Gamma'}(t_s, \mathbf{p}') = \sum_{\mathbf{x}} e^{-i\mathbf{p}'\cdot\mathbf{x}} \text{Tr}[\Gamma' \cdot \overline{\Psi}(\mathbf{x}, t_s) \bar{\Psi}(0)] \quad (9)$$

where the traces are in Dirac space. In this case we average over all three different polarizations, i.e.

$$\Gamma'_i = \frac{1}{2}(1 + \gamma_0)(1 + i\gamma_5\gamma_i), \quad i = 1, 2, 3. \quad (10)$$

For the computation of the disconnected contributions we use the same techniques applied in [36] (see also Appendix C of [37] for full details), namely stochastic estimates of the quark loops with variance reduction.

B. Ratios and matrix elements

Matrix elements are extracted from ratios of three- and two-point correlation functions designed to cancel overlap factors and the leading Euclidean time dependence in the large-time limit. The ratio is given by

$$R(t, t_s, \mathbf{q}) = \frac{C_3^{\Gamma'}(t, t_s, \mathbf{q})}{C_2^{\Gamma'}(t_s, \mathbf{0})} \times \sqrt{\frac{C_2^{\Gamma'}(t_s - t, \mathbf{q}) C_2^{\Gamma'}(t, \mathbf{0}) C_2^{\Gamma'}(t_s, \mathbf{0})}{C_2^{\Gamma'}(t_s - t, \mathbf{0}) C_2^{\Gamma'}(t, \mathbf{q}) C_2^{\Gamma'}(t_s, \mathbf{q})}}. \quad (11)$$

In the connected case, the three-point function in Eq. (6) automatically projects the ratio onto the relevant effective

form factor, i.e.

$$G_A^{\text{conn},\text{eff}}(t, t_s, \mathbf{q}) = \sqrt{\frac{2E_{\mathbf{q}}}{m + E_{\mathbf{q}}}} R^{\text{conn}}(t, t_s, \mathbf{q}). \quad (12)$$

For the disconnected contribution, by contrast, we follow the strategy described in [26] to isolate the signal for G_A , that we extract by solving the corresponding system of equations. In this case, we use two-point functions with higher statistics than in the connected ratio R^{conn} , which relies on the correlations between two-point and connected three-point functions. The final form factor is then given by $G_A^{\text{eff}} = G_A^{\text{conn},\text{eff}} + G_A^{\text{disc},\text{eff}}$, which in the large source-sink separation limit gives

$$G_A^{\text{eff}}(t, t_s, \mathbf{q}) \xrightarrow{t_s - t, t \gg \Delta E^{-1}} G_A(Q^2), \quad (13)$$

where ΔE stands for the energy gap in the one-nucleon sector.

C. Renormalization

The octet current used in [26] makes use of the $O(a)$ -improved and mass-dependent renormalization factor

$$A_{\mu,R}^{8,I} = Z_A \left[\left(1 + 3\bar{b}_A a m_q^{\text{av}} + \frac{b_A}{3} a(m_l + 2m_s) \right) A_\mu^{8,I} + \left(\frac{1}{3} b_A + f_A \right) \sqrt{2} a(m_l - m_s) A_\mu^{0,I} \right], \quad (14)$$

with $m_q^{\text{av}} = (2m_l + m_s)/3$, where we use the improvement coefficients for c_A , b_A from [38,39], and the renormalization constant of the axial-vector current Z_A from [40]. The coefficient $\bar{b}_A$ corresponds to a genuine sea-quark effect and has been found to be small [41]; therefore, we neglect it. We treat f_A analogously, as its contribution is expected to be numerically very small [42]. For the singlet case, on the other hand, we determine the renormalization factors as part of this work. We compute the mass-independent renormalization factor on a set of $N_f = 3$ ensembles, and report the details in the Appendix. We note that this is sufficient for the current level of precision, due to the statistical noise carried by the disconnected contributions.

The renormalized currents read

$$\begin{pmatrix} A_{\mu,R}^3 \\ A_{\mu,R}^8 \\ A_{\mu,R}^0 \end{pmatrix} = \begin{pmatrix} Z_A^{33} & 0 & 0 \\ 0 & Z_A^{88} & Z_A^{80} \\ 0 & Z_A^{08} & Z_A^{00} \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ A_\mu^8 \\ A_\mu^0 \end{pmatrix}, \quad (15)$$

where we defined [cf. Eq. (14)]

$$Z_A^{88} = Z_A \left[1 + 3\bar{b}_A a m_q^{\text{av}} + \frac{b_A}{3} a(m_l + 2m_s) \right], \quad (16)$$

$$Z_A^{80} = Z_A \left[\sqrt{2} \left(\frac{1}{3} b_A + f_A \right) a(m_l - m_s) \right]. \quad (17)$$

Since we determine only the mass-independent singlet renormalization factor, we neglect the off-diagonal coefficient Z_A^{08} . This coefficient is induced only by SU(3)-breaking $O(a)$ effects and hence scales as $Z_A^{08} = O(a(m_l - m_s))$, vanishing in both the continuum and SU(3)-symmetric limits. At the present level of precision this contribution is neglected. The calculation of Z_A^{00} is described in Appendix A, and the corresponding values are listed in Table III. The final results are reported in the $\overline{\text{MS}}$ scheme at 2 GeV.

It is then clear from Eq. (15) that the flavor-decomposed currents mix under renormalization. In particular, we rewrite for convenience the renormalized isoscalar currents in terms of the bare light and strange components as

$$A_{\mu,R}^{u+d-2s} \equiv Z_A^{u+d-2s,u+d} A_{\mu}^{u+d} - 2Z_A^{u+d-2s,s} A_{\mu}^s, \quad (18)$$

$$A_{\mu,R}^{u+d+s} \equiv Z_A^{u+d+s,u+d} A_{\mu}^{u+d} + Z_A^{u+d+s,s} A_{\mu}^s, \quad (19)$$

where we defined

$$Z_A^{u+d-2s,u+d} = Z_A(1 + b_A a m_l), \quad (20)$$

$$Z_A^{u+d-2s,s} = Z_A(1 + b_A a m_s), \quad (21)$$

$$Z_A^{u+d+s,u+d} = Z_A^{u+d+s,s} = Z_A^{00}. \quad (22)$$

For the renormalized strange-quark current we then obtain

$$\begin{aligned} A_{\mu,R}^s &= \frac{1}{3} [A_{\mu,R}^{u+d+s} - A_{\mu,R}^{u+d-2s}] \\ &\equiv Z_A^{s,u+d} A_{\mu}^{u+d} + Z_A^{s,s} A_{\mu}^s, \end{aligned} \quad (23)$$

where we have defined

$$Z_A^{s,u+d} = \frac{1}{3} [Z_A^{u+d+s,u+d} - Z_A^{u+d-2s,u+d}], \quad (24)$$

$$Z_A^{s,s} = \frac{1}{3} [Z_A^{u+d+s,s} + 2Z_A^{u+d-2s,s}]. \quad (25)$$

As a result, a mixing with the $u + d$ channel is present. Note, however, that this contribution is numerically small in our calculation, because $Z_A^{u+d+s,u+d} \simeq Z_A^{u+d-2s,u+d}$.

III. ANALYSIS METHOD

In this Section we describe the analysis strategy used to extract the strange and singlet axial form factor from

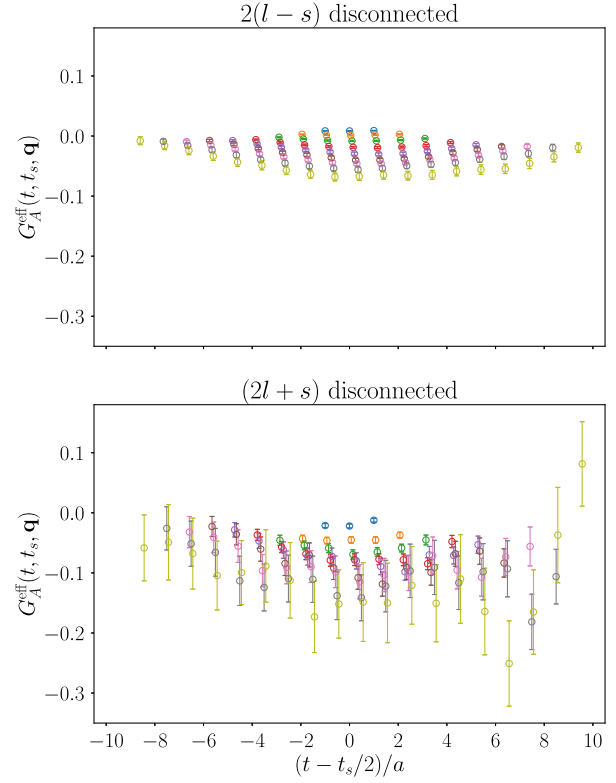


FIG. 1. Comparison between the disconnected contribution to the isoscalar octet channel (top) and singlet (bottom) on D200 at first nonzero momentum $Q^2 \simeq 0.089 \text{ GeV}^2$. The color indicates the different source-sink separations t_s .

the lattice correlation functions. The treatment follows the methodology employed in our previous work on the isoscalar octet, with modifications dictated by the presence of disconnected contributions that exhibit a substantial signal-to-noise problem, since they do not benefit from a cancellation of ultraviolet noise as in the octet case $2(l-s)$. We show an example of this for the ensemble D200 in Fig. 1. For this reason, connected and disconnected contributions are analyzed separately using different strategies, and are subsequently combined in the determination of the physical form factor.

A. Connected contribution

To determine the connected contribution to the form factor we make use of the summation method [43,44] to address the excited-state contamination and perform a linear fit to the summed expression

$$\begin{aligned} S^{\text{conn}}(t_s, \mathbf{q}) &= a \sum_{t=a}^{t_s-a} G_A^{\text{conn,eff}}(t, t_s, \mathbf{q}) \\ &\stackrel{t_s \gg a}{\simeq} b_0(Q^2) + t_s G_A^{\text{conn}}(Q^2) + \mathcal{O}(t_s e^{-\Delta t_s}). \end{aligned} \quad (26)$$

We fit simultaneously for the ranges in Q^2 and $t_s \geq t_{s,\text{min}}$ for all the choices of minimum source-sink separation $t_{s,\text{min}}$

using a z -expansion parametrization

$$G_A^{\text{conn}}(Q^2) = \sum_{k=0}^n a_k z^k(Q^2), \quad (27)$$

$$z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}}, \quad (28)$$

where we set $n = 2$. The fit parameters are therefore $b_0(Q^2)$ for each Q^2 and the three coefficients of the z expansion a_k . We set $t_{\text{cut}} = (4M_\pi)^2$ using the physical pion mass $M_\pi \equiv m_{\pi^0}^{\text{phys}} = 134.977$ MeV across all the ensembles in order to simplify the chiral extrapolation. We note that this choice is common to both A_μ^8 and A_μ^0 , while it differs from the one for A_μ^3 , where $t_{\text{cut}} = (3M_\pi)^2$. This choice also defines the hadronic scheme adopted in this work. Analogously to our previous determination, we perform the extrapolation using data up to $Q^2 = 0.7$ GeV², after which we observe a substantial increase in the statistical noise. The extracted connected contribution is thus obtained in a fully consistent way with respect to our previous analyses. In particular, we apply the same strategy as in Ref. [26] to deal with large and ill-conditioned covariance matrices, namely by applying singular value decomposition cuts to reduce the condition number of the matrices for each ensemble.

As an example, we show in Fig. 2 the results from such a procedure for the connected contributions of two of our more chiral ensembles, comparing it with the more conventional “two-step” approach to confirm their stability and reliability in the presence of a (regularized) large covariance matrix. Note that some of the points correspond to a fit with a p-value smaller than 0.05 or larger than 0.95: this is often due to strong correlations in the data corresponding to different t_s , which lead to very small χ^2 , or small statistical fluctuations in some of these data. We show an example of such a summation-method fit in Appendix.

We continue the practice employed in our previous work [14,26,45] and determine our final results for the z -expansion coefficients through a weighted average obtained by assigning the weights through a window function W defined as

$$W(t_{s,\text{min}}; t_w^{\text{low}}, t_w^{\text{up}}) = \frac{1}{N_w} \left[\tanh\left(\frac{t_{s,\text{min}} - t_w^{\text{low}}}{\Delta t_w}\right) - \tanh\left(\frac{t_{s,\text{min}} - t_w^{\text{up}}}{\Delta t_w}\right) \right], \quad (29)$$

where N_w is a normalization factor and $t_w^{\text{low}} = 0.75$ fm, $t_w^{\text{up}} = 0.95$ fm, $\Delta t_w = 0.1$ fm on each ensemble, in analogy to the window function chosen for the octet case [26].

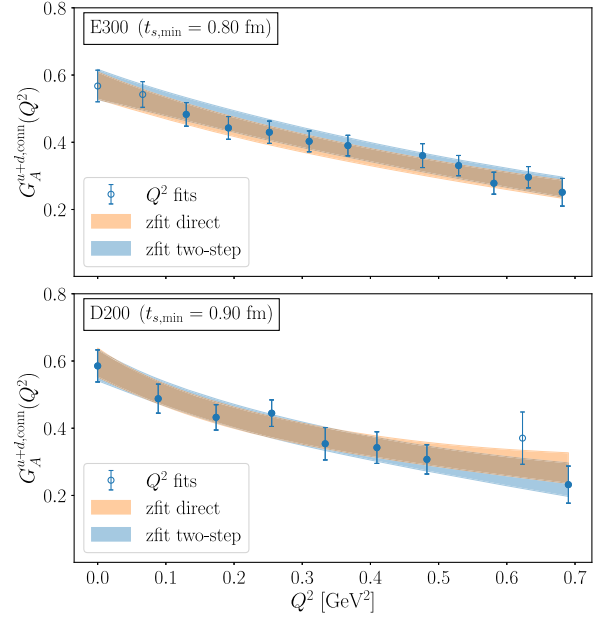


FIG. 2. Comparison of the direct z fit and the two-step procedure for the bare connected contribution to the singlet form factor on the ensembles E300 at $t_{s,\text{min}} = 0.80$ fm (top) and D200 at $t_{s,\text{min}} = 0.90$ fm (bottom). The empty points indicate a p value smaller than 5% or larger than 95% for the first step of the two-step procedure, i.e. the linear fit to the summation expression in Eq. (26), as exemplified in Fig. 8.

B. Disconnected contribution

The disconnected contributions exhibit a much higher level of statistical noise compared to their connected counterparts. The ratio in Eq. (11) leads thus to noisy plateaus with no detectable excited-state contamination within errors, as shown in Fig. 1 (bottom). For this reason, we choose to perform plateau fits to extract the disconnected contribution $G_A^{\text{disc}}(t_s, \mathbf{q})$.

Specifically, for each source-sink separation t_s we fit the effective form factor $G_A^{\text{eff,disc}}(t, t_s, \mathbf{q})$ for both light and strange contributions to a constant in t . To study the dependence on the choice of fit range, we exclude at most two data points from the outer edges of the interval. We retain the fit that provides the best quality as determined by the reduced χ^2 . The final result $G_A^{\text{disc}}(Q^2)$ is then determined in a similar way to the connected part through a weighted average over t_s with the help of the window function $W(t_s; t_w^{\text{low}}, t_w^{\text{up}})$ in Eq. (29), where in this case we pick $t_w^{\text{low}} = 1.1$ fm, $t_w^{\text{up}} = 1.3$ fm and $\Delta t_w = 0.1$ fm on each ensemble.

The procedure is illustrated in Fig. 3 for the case of the bare $(2l + s)$ contribution at vanishing momentum on the ensemble E300, highlighting the noise problem affecting the disconnected contributions. In the lower panel we show the result for the window average over different plateau fits, which is identified with the region where the ratio exhibits

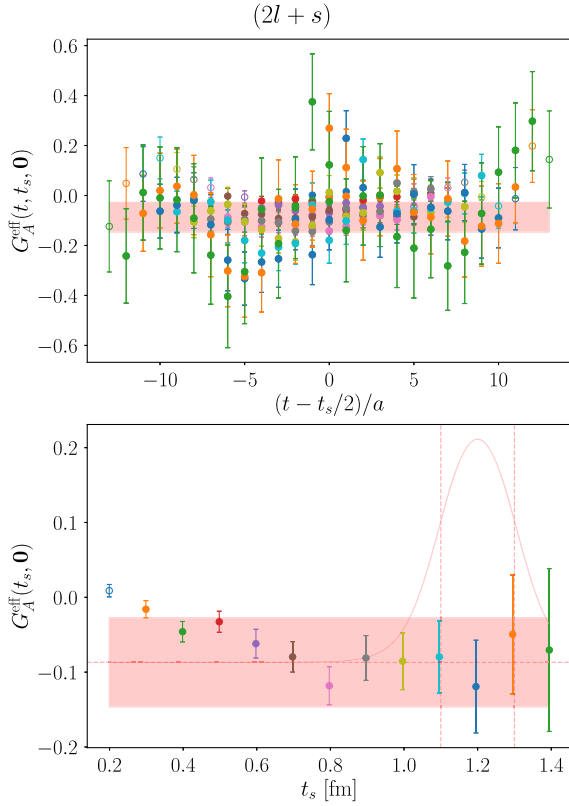


FIG. 3. Example of the fitting procedure on the bare disconnected contribution $(2l + s)$ on E300 at vanishing momentum. The top panel shows the contribution to the effective form factor for the different source-sink separation t_s . The filled symbols indicate the points that were used for the constant fits, whose results are shown in the bottom panel together with the window average indicated by the red band. The red curve indicates the window function used for the weighted average, and the vertical lines correspond to t_w^{low} and t_w^{up} .

no statistically significant dependence on t_s , thereby resulting in a conservative estimate of $G_A^{\text{disc}}(Q^2)$.

C. Combination of contributions

The total axial form factor is obtained by combining the connected and disconnected contributions. In particular, after having obtained the contribution $G_A^{\text{disc}}(Q^2)$ as in Sec. III B, we combine it with the result from the direct fit to the summation expression in Eq. (26) as described in Sec. III A,

$$G_A(Q^2) = G_A^{\text{conn}}(Q^2) + G_A^{\text{disc}}(Q^2), \quad (30)$$

i.e. we combine the z -expansion coefficients obtained from the different contributions. The results are listed in Tabs. IV and V in the Appendix.

In particular, the key analysis of this work focuses on the treatment of the disconnected contribution. First of all, we determine the strange form factor $G_A^s(Q^2)$ in Eq. (23),

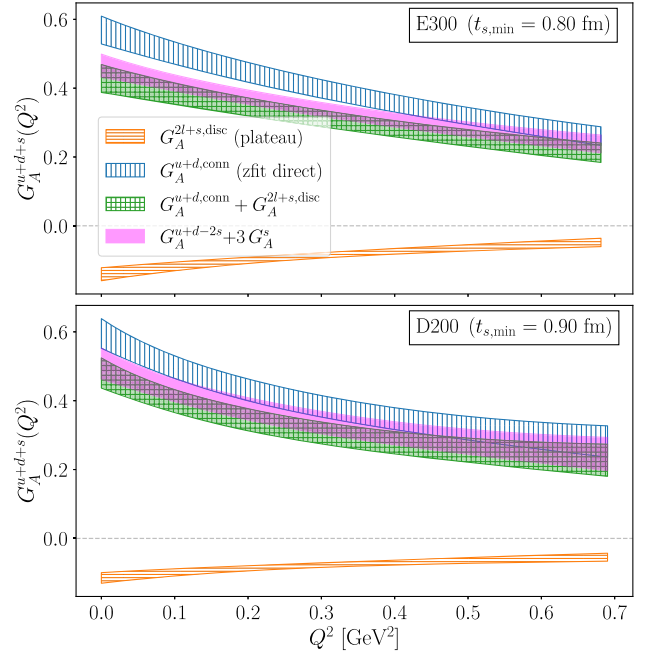


FIG. 4. Singlet axial form factor obtained with z fit for E300 at $t_{s,\text{min}} = 0.80$ fm (top) and D200 at $t_{s,\text{min}} = 0.90$ fm (bottom). We compare our chosen approach of adding the strange form factors G_A^s to the isoscalar octet G_A^{u+d-2s} (magenta) with the full result (green) obtained by treating the connected contribution (blue) and the disconnected one (orange) separately, as described in Secs. III A and III B.

which is dominated by the disconnected contribution, since the renormalization factor in Eq. (24) is numerically small. Then, we determine the singlet by combining (on each ensemble) the results for the octet [26] and the strangeness, i.e. $G_A^{u+d+s} = G_A^{u+d-2s} + 3G_A^s$. An example of this procedure is shown in Fig. 4, where our chosen procedure (magenta) is compared to the sum (green) of individually determined connected and disconnected contributions to the singlet (blue and orange, respectively).

D. Physical-point extrapolation and model average

In order to obtain the final result, we extrapolate the final coefficients of the z expansions for both singlet and strange form factors to the chiral and continuum limit using an ansatz linear in M_π^2 and a^2 for all the coefficients a_i , i.e.

$$a_i = d_{i,0} + d_{i,\pi} M_\pi^2 + d_{i,a}^{(1)} \frac{a}{\sqrt{t_0}} + d_{i,a}^{(2)} \frac{a^2}{t_0}, \quad (31)$$

including a linear term in a to account for effects due to the mass-independent determination of the renormalization constant Z_A^{00} . In practice, since we expect our data to be dominated by $\mathcal{O}(a^2)$ effects, we impose a prior $d_{i,a}^{(1)} \sim \mathcal{N}(0, 0.1 \cdot |d_{i,a}^{(2)}|)$, where $|d_{i,a}^{(2)}|$ refers to the absolute value of

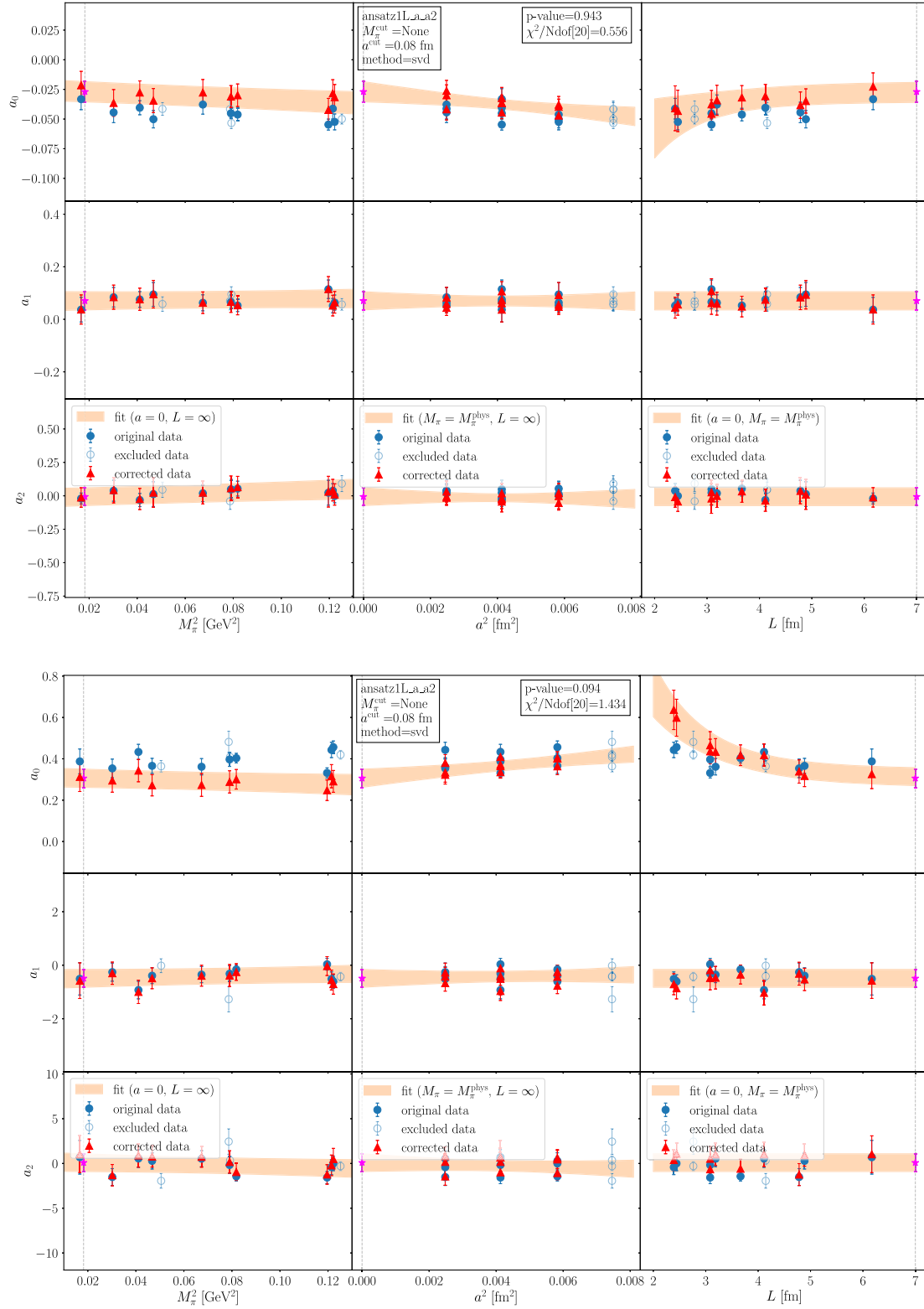


FIG. 5. Chiral continuum extrapolation for the strange (top) and singlet (bottom) with finite-volume term and with no cut on M_π and without the coarsest lattice spacing. The results are plotted as a function of M_π^2 (left), a^2 (center) and L (right). The blue circles indicate the original data, and the filled ones are the ones used after applying the cuts; the red triangles represent the corrected data with respect to the fit ansatz, as indicated in the legend for the orange bands.

the coefficient $d_{i,a}^{(2)}$ determined by a fit with $d_{i,a}^{(1)} = 0$. We include the term [46]

$$\frac{M_\pi^2}{\sqrt{M_\pi L}} e^{-M_\pi L} \quad (32)$$

in the axial charge a_0 to account for possible finite-volume effects.¹ We perform multiple fits with cuts in the pion mass, $M_\pi^{\text{cut}}[\text{MeV}] = \{300, 285, 265\}$, and by removing the coarsest lattice spacing, preserving the correlations among the three coefficients on each ensemble.

This ansatz appears to be sufficient to describe the data well thanks to their flat behavior in the variables M_π^2 , a^2 and the lattice spatial extent L . We show an example of the chiral-continuum extrapolation in Fig. 5.

To finalize the results, we employ a model average [48] procedure based on the Akaike Information Criterion (AIC) [49]. Each fit model k is assigned a weight

$$w_k^{\text{AIC}} \propto e^{-\frac{1}{2}(\chi_k^2 + 2n_{\text{par},k} - n_{\text{data},k})}, \quad (33)$$

with $n_{\text{par},k}$ being the number of parameters and $n_{\text{data},k}$ the number of data points entering the fit. As discussed in [26], given the Gaussianity of the weighted joint distribution built out of the above values w_k^{AIC} , we perform a weighted average of the different models, thus accounting for both statistical and systematic uncertainties.

IV. RESULTS

Our results for the coefficients of the z expansion of the strange nucleon axial form factor in the continuum and at the physical pion mass are

$$\begin{aligned} a_0^s &= -0.0325 \pm 0.0071(\text{stat}) \pm 0.0033(\text{sys}), \\ a_1^s &= +0.0651 \pm 0.0310(\text{stat}) \pm 0.0077(\text{sys}), \\ a_2^s &= -0.0116 \pm 0.0592(\text{stat}) \pm 0.0048(\text{sys}), \end{aligned} \quad (34)$$

with correlation matrix

$$C^s = \begin{pmatrix} 1.00000 & -0.57655 & 0.11734 \\ -0.57655 & 1.00000 & -0.56446 \\ 0.11734 & -0.56446 & 1.00000 \end{pmatrix}. \quad (35)$$

The result is plotted in Fig. 6 (top) and extrapolated up to $Q^2 = 1 \text{ GeV}^2$ (as indicated by the shaded areas). The charge is then given by

$$g_A^s = -0.0325(78), \quad (36)$$

and it is reported in comparison to other recent results in Fig. 7. We also note that the FLAG review 2024 [50] quotes

¹We do not include any higher order terms [47] as we are not sensitive to them with our current statistics.

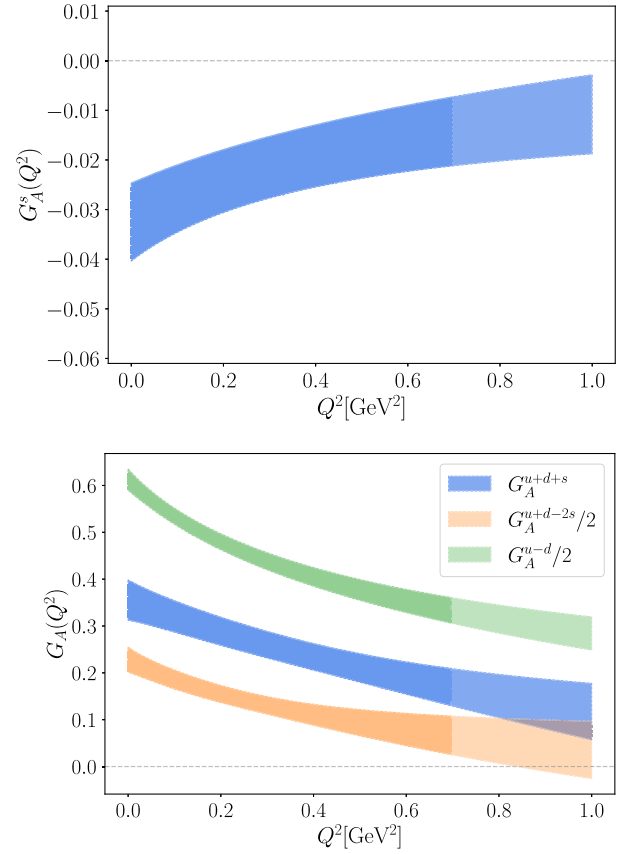


FIG. 6. Final result for the strange (top) and singlet (bottom) form factors. The data are used up to $Q^2 = 0.7 \text{ GeV}^2$, and the lighter-shaded bands represent extrapolations based on the respective parametrizations. In the lower panel, we also plot the rescaled isovector and isoscalar octet form factors $G_A^{u-d}/2$ [14] and $G_A^{u+d-2s}/2$ [26] for comparison.

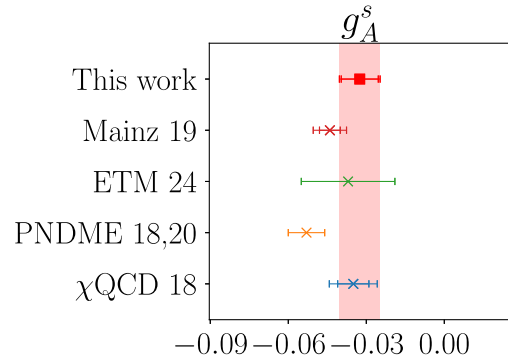


FIG. 7. Our result for the axial charge g_A^s (red square) compared to other collaborations. The red cross shows our previous result [51]. The green point shows the result from ETM [24], the orange from PNDME [22,23], and the blue from χQCD [21].

averages for $N_f = 2 + 1$ and $N_f = 2 + 1 + 1$ values for the strange axial charge, which are identified with the single χQCD 18 [21] and PNDME 18 [22] calculations, respectively.

The singlet case has been obtained through a combination of the octet channel and the strange component, taking advantage of the better signal quality of the octet, which allows us to use the summation method on the whole dataset. The final result is given by

$$\begin{aligned} a_0^{u+d+s} &= +0.355 \pm 0.041(\text{stat}) \pm 0.013(\text{sys}), \\ a_1^{u+d+s} &= -0.423 \pm 0.322(\text{stat}) \pm 0.064(\text{sys}), \\ a_2^{u+d+s} &= -0.691 \pm 0.947(\text{stat}) \pm 0.448(\text{sys}), \end{aligned} \quad (37)$$

with correlation matrix

$$C^{u+d+s} = \begin{pmatrix} 1.00000 & -0.57561 & 0.22890 \\ -0.57561 & 1.00000 & -0.85035 \\ 0.22890 & -0.85035 & 1.00000 \end{pmatrix}. \quad (38)$$

Our result for the singlet axial charge is (in the $\overline{\text{MS}}$ scheme at a renormalization scale of 2 GeV)

$$g_A^{u+d+s} = 0.355(43), \quad (39)$$

and the full form factor is plotted in Fig. 6 (bottom). Therefore, our result suggests that the intrinsic quark spin contributes roughly 35% to the proton spin, while the remaining fraction must be supplied by gluon and orbital angular momentum contributions. Other lattice determinations of the singlet axial charge can be found in [21,22,24]. Historically, the polarized deep-inelastic scattering experiment EMC [52,53] already found a value on the order of 30%. Our result is fully consistent with the determination based on the more recent deep-inelastic scattering data of the COMPASS experiment [2] supplemented by an estimate of the octet axial charge. Subtracting from g_A^{u+d+s} the $\text{SU}(3)_f$ -symmetry estimate of g_A^{u+d-2s} based on hyperon beta decays is, however, too crude to reliably determine the (small) strangeness contribution to the nucleon spin [3].

V. CONCLUSIONS

This work provides the first lattice QCD determination with a full error budget of the singlet axial form factor $G_A^{u+d+s}(Q^2)$ of the nucleon in a large Q^2 range, together with the strange $G_A^s(Q^2)$ contribution. The calculation completes our program of studies of the nucleon axial form factor across flavor channels, following our previous determinations of the isovector [14] and isoscalar octet [26] combinations performed within the same lattice setup.

The singlet and the strange channels introduce qualitatively new challenges compared to the nonsinglet cases, most notably the presence of noisy disconnected quark-loop contributions. In this work, the connected contribution is analyzed using the summation method to suppress excited-state contamination, while the disconnected contribution is determined using plateau fits to the effective form factor.

The momentum dependence of the form factor is parametrized using the z expansion, and the results for its coefficients are extrapolated to the physical point through a linear ansatz in M_π^2 and a^2 [plus a small parametrization for possible $O(a)$ effects]. Our final result is obtained through a model average inspired by the AIC and includes a complete error budget that incorporates both statistical and systematic uncertainties.

Together with our previous results, this work provides a comprehensive lattice QCD determination of the nucleon axial structure across flavor channels within a unified computational framework. Future extensions of this program will include a full flavor decomposition through a unified treatment of the three channels with the use of $\mathbf{p}' \neq \mathbf{0}$ frames. In this way, the charges of the single-flavor components can be extracted with the best possible statistics, providing insight into the composition of the proton spin.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: RENORMALIZATION

Our calculation of the renormalization factors is described in detail in Ref. [59]; here we summarize the pertinent formulae and concentrate on the results for the

TABLE II. The $N_f = 3$ flavor ensembles with periodic boundary conditions used to determine the renormalization constants.

ID	β	a/fm	T/a	L/a	κ	M_π/MeV
rqcd.019	3.40	0.086	32	32	0.1366	600
rqcd.016	3.40	0.086	32	32	0.13675962	420
rqcd.021	3.40	0.086	32	32	0.136813	340
rqcd.017	3.40	0.086	32	32	0.136865	230
rqcd.029	3.46	0.076	64	32	0.1366	700
rqcd.030	3.46	0.076	64	32	0.1369587	320
X450	3.46	0.076	64	48	0.136994	250
B250	3.55	0.064	64	32	0.1367	710
B251	3.55	0.064	64	32	0.137	420
X250	3.55	0.064	64	48	0.13705	350
X251	3.55	0.064	64	48	0.13710	270

singlet case, for which we follow the preliminary work presented in [60]. We list the set of $N_f = 3$ ensembles employed in Table II.

We work in the RI'-MOM scheme [61], where the renormalization condition is specified by matching the renormalized matrix element of an operator O_Γ between two quark states of momentum p at vanishing momentum transfer to its tree-level value [62]

$$Z_O \langle p | O_\Gamma | p \rangle|_{p^2=\mu^2} = \langle p | O_\Gamma | p \rangle_{\text{tree}}|_{p^2=\mu^2}. \quad (\text{A1})$$

We consider the flavour diagonal currents for a generic Γ , i.e.

$$O_\Gamma^a(x) = \bar{\psi}(x) \Gamma \lambda^a \psi(x). \quad (\text{A2})$$

The matrix element is related to the amputated Green functions, which read

$$\begin{aligned} \Lambda_{O_\Gamma}^f(p) &= S_f^{-1}(p) G_{O_\Gamma}^f(p) S_f^{-1}(p), \\ G_{O_\Gamma}^f(p) &= \frac{1}{V} \sum_{x,y,z} e^{-ip(x-y)} \langle f(x) O_\Gamma^a(z) \bar{f}(y) \rangle, \end{aligned} \quad (\text{A3})$$

where f denotes the quark field with a given flavour, and $S_f(p)$ is the quark propagator

$$S(p) = \frac{1}{V} \sum_{x,y} e^{-ip(x-y)} \langle f(x) \bar{f}(y) \rangle. \quad (\text{A4})$$

For the renormalization condition we use

$$Z_A = \frac{12Z_q}{\frac{1}{3} \sum_{\mu\nu} (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) \text{tr}[\Lambda_{A_\mu}(p) \Lambda_{A_\nu}^{-1, \text{tree}}(p)]|_{p^2=\mu^2}} \quad (\text{A5})$$

which is compatible with the axial Ward identity [19]. The renormalization constants for the singlet case are then obtained from the generalized amputated Green functions in Eq. (A3), which renormalize as [19,61]

$$\Lambda_{O_\Gamma^a}^f(p)_R = \frac{Z_\Gamma^{ab}}{Z_q} \Lambda_{O_\Gamma^b}^f(p). \quad (\text{A6})$$

Z_q denotes the quark normalization constant, which is determined by matching the renormalized quark propagator to the free massless lattice propagator, i.e.

$$Z_q = \frac{1}{12} \text{tr}[S^{-1}(p) S_{\text{free}}(p)]|_{p^2=\mu^2}, \quad (\text{A7})$$

with

$$S_{\text{free}}(p) = \frac{-ia \sum_\mu \gamma_\mu \sin(ap_\mu)}{\sum_\mu \sin^2(ap_\mu)}. \quad (\text{A8})$$

For flavor diagonal currents and degenerate quark masses the renormalization matrix is diagonal, i.e.

$$Z_\Gamma^{ab} = \begin{pmatrix} Z_\Gamma^{33} & 0 & 0 \\ 0 & Z_\Gamma^{88} & 0 \\ 0 & 0 & Z_\Gamma^{00} \end{pmatrix}. \quad (\text{A9})$$

We note that $Z_\Gamma^{33} = Z_\Gamma^{88}$, and Z_Γ^{88} is identical to the isovector result. Finally, the renormalization conditions read

$$(Z_\Gamma^{-1})^{da} = \sum_{f=u,d,s} \text{diag}(\lambda^a) \frac{1}{12Z_q} \text{tr}[\Lambda_{O_\Gamma^a}^f(p) \Gamma^{-1}]|_{p^2=\mu^2}. \quad (\text{A10})$$

For the flavor-diagonal currents we obtain

$$(Z_\Gamma^{-1})^{00}(p) = \frac{1}{12Z_q} (\Sigma_\Gamma^{\text{con}}(p) + 3\Sigma_\Gamma^{\text{dis}}(p)), \quad (\text{A11})$$

$$(Z_\Gamma^{-1})^{88}(p) = \frac{1}{12Z_q} \Sigma_\Gamma^{\text{con}}(p), \quad (\text{A12})$$

where the connected and disconnected contributions of a single quark flavor read

$$\begin{aligned} \Sigma_\Gamma^{\text{con}}(p) &= \text{tr}[S^{-1}(p) \mathcal{O}_\Gamma^{\text{conn}}(p) S^{-1}(p) \Gamma^{-1}], \\ \mathcal{O}_\Gamma^{\text{conn}}(p) &= \left\langle \frac{1}{V} \sum_z \gamma_5 \mathcal{S}(z, p)^\dagger \gamma_5 \Gamma \mathcal{S}(z, p) \right\rangle, \end{aligned} \quad (\text{A13})$$

for the connected and

$$\begin{aligned} \Sigma_\Gamma^{\text{dis}}(p) &= -\text{tr}[S^{-1}(p) \mathcal{O}_\Gamma^{\text{dis}}(p) S^{-1}(p) \Gamma^{-1}], \\ \mathcal{O}_\Gamma^{\text{dis}}(p) &= \left\langle \mathcal{S}(p) \frac{1}{V} \sum_z \text{tr}[\mathcal{S}(z, z) \Gamma] \right\rangle, \end{aligned} \quad (\text{A14})$$

for the disconnected. $\mathcal{S}$ refers to the estimate of the propagator on each configuration, i.e. before taking the

TABLE III. Renormalization constants for the axial singlet current on CLS $N_f = 3$ ensembles in the RGI scheme. Note that the value for $\beta = 3.7$ is the result of a linear extrapolation based on the three coarser lattice spacings.

β	3.4	3.46	3.55	3.7
Z_A^{00}	0.721(6)(18)	0.729(5)(21)	0.743(5)(16)	0.77(4)

gauge average. The traces are taken over spin and color indices, and the brackets denote the gauge average. We obtain the renormalization constant for each value of the lattice spacing at different quark masses. We extrapolate the renormalization constants to the massless limit, accounting for systematics by performing different chiral extrapolations, i.e. with or without a M_π^4 -term and a term proportional to $e^{-M_\pi L}$, as well as different fit intervals in the scale μ . We then subtract lattice artifacts, c.f. [59], which to first order in the coupling coincide with the isovector case. As in [59] we explore several alternatives for defining the coupling used for the subtraction, where the quoted values in Table III are based on the BLM coupling of [63,64]. Finally we transform the renormalization constants to the RGI scheme. For the singlet case we implement the one-loop anomalous dimensions from Ref. [65], in the conversion factor $\Delta Z^{\overline{\text{MS}}}$ between the $\overline{\text{MS}}$ to the RGI scheme. The renormalization constants are determined from a simultaneous fit to all lattice data using

$$Z_{\text{sub}}^{\text{RGI}}(a, \mu) = Z^{\text{RGI}}(\beta) \{ 1 + c_1 g^{\overline{\text{MS}}}(\mu)^4 \} + c_2(\beta)(a\mu)^2 \Delta Z^{\overline{\text{MS}}}(\mu) Z_{\text{RI}'\text{-MOM}}^{\overline{\text{MS}}}(\mu). \quad (\text{A15})$$

We note that the fit constant c_1 is independent of β and parametrizes the neglected higher-order contributions in the conversion factors. The central values of these fits are summarized in Tab. III, where the value for $\beta = 3.7$ is the result based on an extrapolation of the other lattice spacings. Indeed, employing periodic boundary conditions is essential for the RI-MOM scheme, which is impossible at our finest lattice spacing, where open boundary conditions are used to mitigate topological freezing. Thus, in the absence of a viable ensemble at our finest lattice spacing, we extrapolate the renormalization constant using the result from the three other lattice spacings.

The singlet current renormalization is scheme dependent. In Table III we quote the RGI value obtained from Eq. (A15). For the final results we transcribe our results to the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV. For the conversion from the RGI values we use the anomalous dimensions of the singlet current from Ref. [65]

$$\begin{aligned} \gamma(a_s)^{\text{singlet}} &= a_s^2(-6C_F n_f) \\ &+ a_s^3 \left(-\frac{142}{3} C_F C_A n_f + \frac{4}{3} C_F n_f^2 + 18 C_F^2 n_f \right), \\ &= a_s^2 \gamma_1 + a_s^3 \gamma_2, \end{aligned} \quad (\text{A16})$$

where $a_s = \alpha_s/(4\pi)$, $n_f = 3$, $C_A = 3$ and $C_F = 4/3$. For the running of the strong coupling we use [66]

$$\begin{aligned} \beta_0 &= 11 - \frac{2}{3} n_f, \\ \beta_1 &= 102 - \frac{38}{3} n_f, \\ \beta_2 &= \frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2, \\ \beta_3 &= \frac{149753}{6} + 3564 \zeta_3 - \left(\frac{1078361}{162} + \frac{6508}{27} \zeta_3 \right) n_f \\ &+ \left(\frac{50065}{162} + \frac{6472}{81} \zeta_3 \right) n_f^2 + \frac{1093}{729} n_f^3 \end{aligned} \quad (\text{A17})$$

$$a_s(\mu) = \frac{1}{\beta_0 L} \left(1 - \frac{\beta_1 \ln L}{\beta_0^2 L} + \frac{\beta_1^2 \ln^2 L - \beta_1^2 \ln L - \beta_1^2 + \beta_0 \beta_2}{\beta_0^4 L^2} \right), \quad (\text{A18})$$

with

$$L = \ln \frac{\mu^2}{\Lambda^2}. \quad (\text{A19})$$

The conversion of the RGI values to the $\overline{\text{MS}}$ values proceeds via

$$\begin{aligned} Z_A^{\overline{\text{MS}}} &= \frac{Z_A^{\text{RGI}}}{\Delta Z^{\overline{\text{MS}}}(\mu)}, \\ \Delta Z^{\overline{\text{MS}}}(\mu) &= (2\beta_0 a_s)^{-\frac{\gamma_0}{2\beta_0}} (1 + c_1 a_s + c_2 a_s^2 + \mathcal{O}(a_s^3)) \end{aligned} \quad (\text{A20})$$

$$c_1 = \frac{\beta_1 \gamma_0 - \beta_0 \gamma_1}{2\beta_0^2} \quad (\text{A21})$$

$$\begin{aligned} c_2 &= \frac{1}{8\beta_0^4} [\beta_1^2 \gamma_0^2 - 2\beta_0 \beta_1 \gamma_0 (\beta_1 + \gamma_1) \\ &+ \beta_0^2 [2\beta_2 \gamma_0 + \gamma_1 (2\beta_1 + \gamma_1)] - 2\beta_0^3 \gamma_2]. \end{aligned} \quad (\text{A22})$$

We note that $\gamma_0 = 0$ for the anomalous dimensions of the singlet current of Eq. (A16). To evaluate the strong coupling a_s we use $\Lambda_{\text{QCD}} = 344.4(8.7)$ MeV [67] as input.

1. Form factor data

We report the numerical results of the form factor extraction on each ensemble as described in Sec. III. In Tabs. IV and V we collect the results of the simultaneous fits in Q^2 and t_s for the z -expansion for the strange and

singlet form factors, respectively. We also report an example of the summation method applied to the connected contribution in Fig. 8 for a single value of Q^2 , which corresponds to the first step of the “two-step” procedure discussed in Sec. III A.

TABLE IV. Results for the coefficients of the z -expansion for the strange form factor $G_A^s(Q^2)$ for each ensemble, as well as their correlations ρ .

ID	a_0	a_1	a_2	ρ_{a_0,a_1}	ρ_{a_0,a_2}	ρ_{a_1,a_2}
H102	−0.0500(43)	0.057(22)	0.091(59)	−0.57966	0.20849	−0.84444
H105	−0.0417(69)	0.070(34)	−0.040(61)	−0.79042	0.35483	−0.65399
C101	−0.0415(54)	0.058(27)	0.046(56)	−0.66147	0.00103	−0.56508
N101	−0.0534(45)	0.095(29)	0.045(81)	−0.56226	0.31775	−0.91011
S400	−0.0524(68)	0.064(28)	−0.0009(500)	−0.70915	0.03087	−0.47276
N451	−0.0462(54)	0.053(23)	0.054(55)	−0.59169	−0.00101	−0.66223
D450	−0.0501(73)	0.095(45)	0.013(95)	−0.77778	0.33501	−0.69722
N203	−0.0547(47)	0.114(35)	0.023(92)	−0.66042	0.33043	−0.87438
N200	−0.0451(42)	0.066(26)	0.049(68)	−0.62386	0.27643	−0.84611
D200	−0.0405(58)	0.076(29)	−0.031(50)	−0.83238	0.20169	−0.50324
E250	−0.0332(88)	0.037(47)	−0.020(41)	−0.87477	0.16563	−0.33639
N302	−0.0413(88)	0.052(27)	0.038(53)	−0.65772	−0.08478	−0.52741
J303	−0.0378(81)	0.063(31)	0.019(64)	−0.62573	0.18523	−0.65222
E300	−0.0444(87)	0.084(38)	0.038(81)	−0.63405	−0.13794	−0.54834

TABLE V. Results for the coefficients of the z -expansion for the singlet form factor $G_A^{u+d+s}(Q^2)$ for each ensemble, as well as their correlations ρ .

ID	a_0	a_1	a_2	ρ_{a_0,a_1}	ρ_{a_0,a_2}	ρ_{a_1,a_2}
H102	0.420(21)	−0.43(16)	−0.30(49)	−0.55475	0.24112	−0.87515
H105	0.482(51)	−1.27(47)	2.4(1.4)	−0.67007	0.40437	−0.90007
C101	0.364(28)	−0.02(26)	−1.94(81)	−0.58428	0.28658	−0.86096
N101	0.409(22)	−0.41(15)	0.35(48)	−0.58757	0.31939	−0.86613
S400	0.456(31)	−0.61(25)	0.007(819)	−0.55464	0.21355	−0.87501
N451	0.403(23)	−0.16(17)	−1.44(51)	−0.57789	0.22473	−0.83573
D450	0.366(36)	−0.39(29)	0.29(91)	−0.68929	0.36487	−0.83337
N203	0.332(25)	0.04(22)	−1.57(66)	−0.59985	0.28622	−0.87190
N200	0.397(34)	−0.32(30)	−0.18(95)	−0.58087	0.29409	−0.88870
D200	0.433(36)	−0.93(33)	0.55(98)	−0.64949	0.35969	−0.87590
E250	0.388(60)	−0.51(62)	0.7(1.9)	−0.73652	0.46906	−0.89671
N302	0.443(37)	−0.51(26)	−0.42(82)	−0.50539	0.15406	−0.87375
J303	0.362(40)	−0.35(31)	0.53(93)	−0.62964	0.34276	−0.86904
E300	0.354(45)	−0.26(35)	−1.52(97)	−0.67212	0.35520	−0.86714

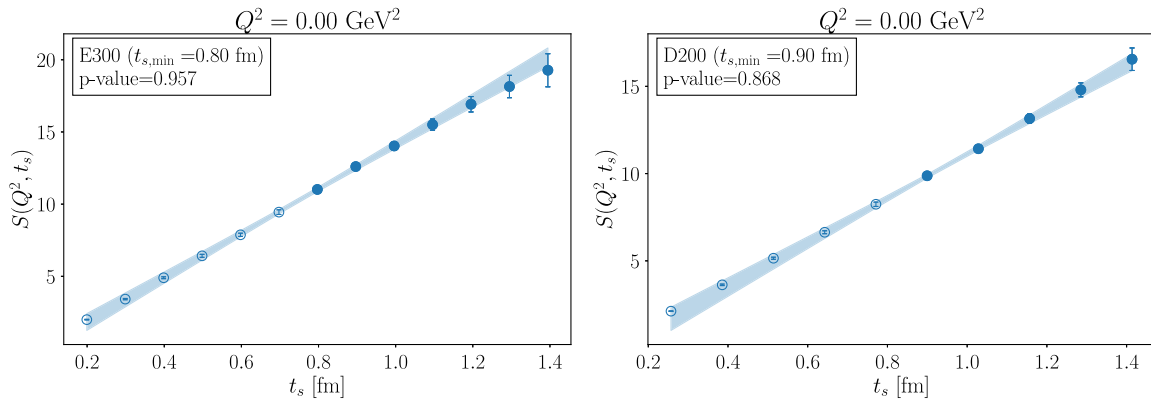


FIG. 8. Linear fit to the summation expression Eq. (26) for the bare connected contribution to the singlet form factor at vanishing momentum for E300 at $t_{s,min} = 0.80$ fm and D200 at $t_{s,min} = 0.90$ fm. The filled points indicate the values that enter the fit; the quality of fit is indicated in the box on the figure.

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