

Testing tree level TeV scale seesaw scenarios in μ TRISTAN

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We investigate TeV scale seesaw scenarios at μ^+e^- and $\mu^+\mu^+$ colliders in the μ TRISTAN (Transposable Ring Intersecting STorage Accelerator in Nippon) experiment. In a minimal type I seesaw scenario, we consider two generations of Standard Model (SM) singlet heavy Majorana-type right-handed neutrinos, which couple with SM-gauge bosons through light-heavy neutrino mixing. We discuss the prospects of probing heavy neutrinos via the processes such as $\mu^+e^- \rightarrow \nu N_i \rightarrow e^+jj\nu$ or $\mu^-jj\nu$ for $\sqrt{s} = 346$ GeV and 1 ab^{-1} luminosity. Studying these process, we estimate limits on the light-heavy neutrino mixing angles as a function of heavy neutrino mass, which could be 2 orders of magnitude stronger than electroweak precision data. Further, we study the effect of a doubly charged scalar boson (H^{++}) from the type II seesaw scenario in $\mu^+\mu^+$ collision at $\sqrt{s} = 2$ TeV. In this case, we consider $\mu^+\mu^+ \rightarrow \ell_i^+\ell_j^+$ and $\mu^+\mu^+ \rightarrow H^{++}Z/\gamma$ processes followed by the same-sign dilepton decay of H^{++} . We find that events involving e^+e^+ among these final states can probe the neutrino mass ordering in the μ TRISTAN experiment at 5σ significance. In addition, we study the production of positively charged triplet fermions in μ TRISTAN following the $\mu^+\mu^+ \rightarrow \mu^+\Sigma^+$ process, where Σ^+ decays into μ^+jj mode through Z boson exchange. Considering a triplet at 1 TeV and studying SM backgrounds, we estimate the discovery potential of the $\mu^+\mu^+jj$ signal at μ TRISTAN with respect to projected luminosity.

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I. INTRODUCTION

The origin of tiny neutrino mass and flavor mixing [1] is an unknown, long-standing puzzle, as Standard Model (SM) fails to explain it. It leads us to extend SM in a variety of ways to explain such an interesting observation. Simply a dimension five operator [2] within SM opened the door to incorporate beyond the Standard Model (BSM) physics through particle extension of the SM. A fascinating approach to study such anomalies is to introduce SM-singlet heavy Majorana-type right-handed neutrinos (RHNs) to generate a tiny neutrino mass through suppression of a lepton number-violating mass scale [3–8]—commonly recognized as a type I seesaw scenario. SM-singlet RHNs mix with the SM light neutrinos to interact with the SM gauge and Higgs bosons.

However, RHNs are hitherto unidentified and could have mass from a light scale up to a very heavy scale, depending on their role in a variety of theoretical aspects [9–21]. Recently, the LHC is also looking for prompt RHNs from same-sign dileptons plus dijet and trilepton plus missing energy modes, respectively, to provide limits on the RHN mass and its mixing with light neutrinos [22–25].

Apart from the singlet fermion extension of the SM, there is another simple but interesting aspect where SM is extended by an $SU(2)$ triplet scalar with hypercharge $Y = +2$ arising as a UV-complete scenario of the dimension five operator. This scenario is called the type II seesaw scenario [26–33]. Through the inclusion of the triplet scalar with small vacuum expectation value (VEV) v_Δ , Majorana-type tiny neutrino mass in $\mathcal{O}(\text{eV})$ scale can be generated at the tree level followed by flavor mixing. Such a scenario could evolve large neutrino Yukawa coupling of $\mathcal{O}(1)$. Due to the gauge structure, the triplet (complex) scalar interacts with the SM gauge bosons, SM lepton, and Higgs doublet, developing a variety of decay modes of the triplet scalar [34,35]. As a result, such triplet scalars can be tested at high-energy colliders following different decay modes of their doubly, singly charged, and neutral scalar multiplets [36–45]. We point out that for $v_\Delta \leq 10^{-4}$, doubly charged Higgs dominantly decays into same-sign dilepton mode,

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whereas for $v_\Delta > 10^{-4}$, it decays into same-sign W bosons. In the first case, doubly charged scalar mass below 1080 GeV [46] was not observed, whereas in the second case, the upper limit is around 350 GeV [47]. The singly charged and neutral multiplets from the type II seesaw scenario have been tested at the LHC from gluon-fusion channels [48–53]. These processes are suppressed by an additional factor of $\mathcal{O}(v_\Delta^2)$, making them irrelevant for the type II seesaw scenario.

Type III seesaw can be considered as another interesting framework, which is realized by extending the SM by an $SU(2)_L$ triplet fermion with zero hypercharge [54]. The triplet fermion consists of both singly charged and charge-neutral multiplets. The mass of the light neutrino is generated by the VEV of the charge-neutral multiplet following the seesaw mechanism. These charge-neutral multiplets can mix with the SM neutrinos, enabling their interaction with the SM bosons. The type III seesaw scenario has been explored at high-energy colliders to investigate various phenomenological aspects, including both prompt and displaced signatures from multilepton and multijet channels [55–60], setting a limit on their masses around 1 TeV.

In this paper, we study the tree level minimal seesaw models at a multi-TeV same-sign positively charged muon collider or positively charged muon-electron collider [61–78] at the μ TRISTAN (Transposable Ring Intersecting Storage Accelerator in Nippon) experiment using ultra-cold antimuon technology from the Japan Proton Accelerator Research Complex (J-PARC) [79]. In this experiment, μ^+e^- collision could occur at $\sqrt{s} = 346$ GeV, where μ^+ energy is $E_\mu = 1$ TeV and e^- energy is $E_e = 30$ GeV ($\sqrt{s} = 2\sqrt{E_\mu E_e}$) at 1 ab^{-1} luminosity. This could be upgraded to a $\mu^+\mu^+$ collider at $\sqrt{s} = 2$ TeV or higher, where muon energy is considered to be 1 TeV with 100 fb^{-1} luminosity or higher. Such a machine can potentially serve as a Higgs factory and participate in various phenomenological aspects [80–86] involving physics beyond the SM.

The unique initial state of the μ TRISTAN experiment allows us to study the tree level seesaw scenarios from interesting perspectives. In the case of μ^+e^- collision, we consider the heavy neutrino production from the type I seesaw where only a t –channel W mediated scenario will appear, provided the RHN mass (M_N) resides in the electroweak or TeV scale. Due to the light-heavy mixing, this process will be suppressed by modulus square of the respective mixing. We consider that there is at least one RHN which resides at the electroweak scale and can be produced at the colliders. In this collider, we can explore this RHN and its corresponding mixing with electrons, muons, and tau. The RHN produced in this scenario can dominantly decay into an electron, muon, or tau and W boson followed by the hadronic decay of its daughter W boson. Finally, from the lepton and jets, it could be possible to reconstruct the RHN. Hence, we can estimate the limits on mass and mixings of RHNs, studying signal and corresponding SM backgrounds.

For the μ^+e^- collision, we consider $45 \text{ GeV} \leq M_N \leq 340 \text{ GeV}$. In case of the $\mu^+\mu^+$ collider, we can study a unique behavior of the RHNs where a t –channel RHN-mediated process could produce a pair of same-sign W bosons, reflecting the Majorana nature of the heavy neutrino. However, this process is suppressed by the fourth power of the modulus of the mixing.

In the case of the type II seesaw scenario, $\mu^+\mu^+$ collision could play a crucial role to probe the effect of the doubly charged scalar multiplet. In this case, we consider two scenarios. In the first case, we consider the doubly charged scalar-mediated same-sign dilepton production scenario in the s –channel process accompanied by the t –channel same-sign dilepton production mode mediated by SM neutral bosons. Here we prefer to consider nonmuonic final states to reduce SM backgrounds. Followed by this scenario, we study the production of the doubly charged scalar in association with a Z boson or photon. If a Z boson is produced, then it is considered to decay into a dominant mode of jets resulting a final state of same-sign dilepton and two jets where same-sign dileptons are produced from the doubly charged scalar. If the doubly charged scalar is produced with a photon, the final state consists of a same-sign dilepton from the doubly charged scalar plus a photon. It is important to note that the dilepton decay of a doubly charged Higgs comes from the Yukawa coupling between the lepton doublet and scalar triplet, which is determined by the oscillation parameters, particularly the neutrino mass ordering. Therefore, studying the above mentioned processes could provide insights into the ordering of light neutrino masses, for example, normal ordering (NO) and inverted ordering (IO). Finally, we compare the left-right asymmetry ($\mathcal{A}_{LR}$) for the SM and doubly charged scalar-induced dilepton production process.

In the type III seesaw, one can have production channels such as $\mu^+e^- \rightarrow \Sigma^+\Sigma^-$ and $\mu^+\mu^+ \rightarrow \Sigma^+\Sigma^+$ at μ TRISTAN. But due to the current bound on the triplet fermion mass, the type III seesaw is not suitable to study at μ TRISTAN. In addition to that, these processes will be suppressed by the fourth power of the light-heavy mixing, making the scenario less interesting. However, there is an interesting scenario that could be studied in the context of the $\mu^+\mu^+$ collider. In this case, the singly charged multiplet of the triplet fermion could be produced in association with a positively charged muon following $\mu^+\mu^+ \rightarrow \Sigma^+\mu^+$, and Σ^+ will decay into μ^+Z followed by hadronic decay of the Z boson. Hence, a same-sign two-muon final state in association with two jets $\mu^+\mu^+jj$ could be studied at $\sqrt{s} = 2$ TeV while $M_\Sigma = 1$ TeV.

The paper is arranged in the following way: in Sec. II, we briefly discuss the type I and type II seesaw models at the tree level, which could be tested at the μ TRISTAN experiment. We give detailed analysis on the BSM searches in Sec. III, followed by discussions on limits obtained on

the light-heavy neutrino mixings. Finally, we conclude our article in Sec. IV.

II. TESTABLE SEESAW MODELS AT THE TREE LEVEL

Among the many proposals of neutrino mass generation mechanism, we discuss neutrino mass generation mechanism at the tree level involving type I, type II, and type III seesaw models, which could reproduce the neutrino oscillation data and flavor mixing.

A. Type I seesaw

In the type I seesaw, SM-singlet RHNs (N_R^β) are introduced to extend the SM, which couples with the SM lepton doublet (ℓ^α) and Higgs doublet (Φ). We can then write down the relevant part of the Lagrangian as

$$\mathcal{L}_{\text{int}} \supset -y_D^{\alpha\beta} \bar{\ell}_L^\alpha \Phi N_R^\beta - \frac{1}{2} M_N^{\alpha\beta} \bar{N}_R^\alpha N_R^\beta + \text{H.c.}, \quad (1)$$

where α and β stand for the generation indices. The second term in Eq. (1) is the lepton number-violating Majorana mass term for the RHNs. After the electroweak symmetry is broken by the generation of VEV of the SM Higgs v_Φ , one obtains the Dirac mass term of the neutrinos as $M_D = \frac{y_D v_\Phi}{\sqrt{2}}$. Hence, we can write the neutrino mass matrix incorporating the Dirac and Majorana masses in the following as

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix}. \quad (2)$$

Now diagonalizing Eq. (2), the seesaw formula for the light Majorana neutrinos mass can be obtained as

$$m_\nu \simeq -M_D M_N^{-1} M_D^T. \quad (3)$$

To obtain a light neutrino mass of $\mathcal{O}(0.1)$ eV from a heavy RHN of mass M_N around $\mathcal{O}(100)$ GeV, the Dirac Yukawa coupling y_D could be $\mathcal{O}(10^{-6})$; however, the Y_D could reach $\mathcal{O}(1)$ if general parametrization for the seesaw scenario is considered [87]. This scenario is considered in our article. We assume $M_D M_N^{-1} \ll 1$, so that the light neutrino flavor eigenstates (ν) can be written in terms of the mass eigenstates of light neutrinos (ν_m) and heavy neutrinos (N_m) as $\nu \simeq \mathcal{N} \nu_m + V N_m$, where $V = M_D M_N^{-1}$ and $\mathcal{N} = (1 - \frac{1}{2}\epsilon) U_{\text{PMNS}}$ with $\epsilon = V^* V^T$ and U_{PMNS} is the usual neutrino mixing matrix applied to diagonalize m_ν in the following way:

$$U_{\text{PMNS}}^T m_\nu U_{\text{PMNS}} = \text{diag}(m_1, m_2, m_3). \quad (4)$$

In presence of the parameter ϵ , the mixing matrix $\mathcal{N}$ is nonunitary [88–90]. Now we replace neutrino flavor eigenstates in charged current (CC) and neutral current (NC) interactions of the SM and obtain

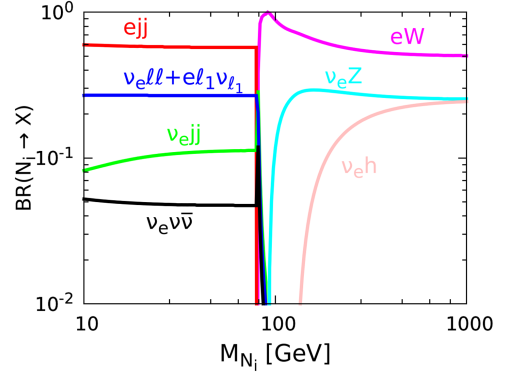


FIG. 1. Branching ratios of N_i to different final states under the assumption $V_{eN_i} \neq 0$, $V_{\mu N_i} = 0$ and $V_{\tau N_i} = 0$. We show the branching ratios to three-body leptonic channels $e\ell_1\nu_{\ell_1} + \nu_e\ell\ell$, $\nu_e\nu\bar{\nu}$, and semileptonic channels such as ejj and $\nu_e jj$, where $\ell = e, \mu, \tau$, $\ell_1 = \mu, \tau$. For relatively large M_{N_i} the two-body decay channels such as eW , $\nu_e Z$, and $\nu_e h$ dominate.

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} W_\mu \bar{e} \gamma^\mu P_L (\mathcal{N} \nu_m + V N_m) + \text{H.c.}, \quad (5)$$

where e denotes the three generations of the charged leptons in the vector form and

$$\mathcal{L}_{\text{NC}} = -\frac{g}{2c_w} Z_\mu [\bar{\nu}_m \gamma^\mu P_L (\mathcal{N}^\dagger \mathcal{N}) \nu_m + \bar{N}_m \gamma^\mu P_L (V^\dagger V) N_m + \{\bar{\nu}_m \gamma^\mu P_L (\mathcal{N}^\dagger V) N_m + \text{H.c.}\}], \quad (6)$$

where $c_w = \cos \theta_w$ is the weak mixing angle $P_{L(R)} = \frac{1}{2}(1 \mp \gamma_5)$. In our scenario, the RHN mass is a free parameter. If the RHN mass lies between $10 \text{ GeV} \leq M_N < M_W$, the RHN undergoes three-body decay via off shell W , Z , and Higgs bosons. If the RHNs are heavier than the SM gauge and scalar bosons, then dominant two-body decay modes are $N \rightarrow \ell W$, $\nu_\ell Z$, $\nu_\ell h$, respectively. In Fig. 1, we show the branching ratios of heavy neutrino N to various final states for $10 \text{ GeV} \leq M_N \leq 1000 \text{ GeV}$, assuming that $V_{eN} \neq 0$, $V_{\mu N} = 0$ and $V_{\tau N} = 0$. For RHNs lighter than W bosons ($M_N < M_W$) the dominant modes involve three-body leptonic or semileptonic final states, whereas for RHNs heavier than W bosons ($M_N > M_W$), two-body decay modes dominate. Note that for very heavy RHNs, where $M_N \geq \mathcal{O}(1)$ TeV with nonzero light-heavy mixing, the branching ratios can be obtained as

$$\text{BR}(N \rightarrow \ell W) : \text{BR}(N \rightarrow \nu_\ell Z) : \text{BR}(N \rightarrow \nu_\ell h) = 2 : 1 : 1. \quad (7)$$

The above asymptotic behavior of heavy neutrino decay branching ratios is due to the Goldstone equivalence theorem. Note that for sufficiently small mixing and relatively small M_N , heavy RHNs could be long-lived candidates that can travel long distances before their decay

into different daughter particles. This scenario gives rise to displaced vertex scenarios. We consider M_N and $|V_{\ell N}|^2$ as free parameters, and $V_{\ell N}$ is always large enough so that RHNs suffer prompt decay.

B. Type II seesaw scenario

In the type II seesaw scenario, SM is extended by an $SU(2)_L$ triplet scalar $\Delta = (\Delta^{++}, \Delta^+, \Delta^0)^T$. The triplet scalar is described in the matrix form with the help of Pauli spin matrices as

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \quad (8)$$

and due to its $SU(2)_L$ charge, it interacts with the SM gauge bosons. In addition to that, it participates in a Yukawa coupling with SM lepton doublets. The corresponding Lagrangian with the scalar kinetic term can be written as

$$\mathcal{L}_{\text{type II}} = [iY_{\Delta\alpha\beta}L_\alpha^T C^{-1}\tau_2\Delta L_\beta + \text{H.c.}] + (D_\mu\Phi)^\dagger(D^\mu\Phi) + (D_\mu\Delta)^\dagger(D^\mu\Delta) - V(\Phi, \Delta), \quad (9)$$

where $Y_\Delta^{\alpha\beta}$ is the Yukawa coupling and it is a symmetric complex matrix. Here, L_α represents SM lepton doublets, C stands for the charge conjugation operator, D_μ shows covariant derivative of the related scalar field, and $V(\Phi, \Delta)$ is the scalar potential, which could be expressed as

$$\begin{aligned} V(\Phi, \Delta) = & -m_\Phi^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 + \tilde{M}_\Delta^2\text{Tr}[\Delta^\dagger\Delta] \\ & + \lambda_2[\text{Tr}\Delta^\dagger\Delta]^2 + \lambda_3\text{Tr}[\Delta^\dagger\Delta]^2 \\ & + [\mu\Phi^T i\sigma_2\Delta^\dagger\Phi + \text{H.c.}] + \lambda_1(\Phi^\dagger\Phi)\text{Tr}[\Delta^\dagger\Delta] \\ & + \lambda_4\Phi^\dagger\Delta\Delta^\dagger\Phi. \end{aligned} \quad (10)$$

In this case, Higgs triplet UV-completion of the Weinberg operator could be characterized by the induction of a tiny VEV of Δ . The lepton number violation μ -term in Eq. (10) induces a small VEV of triplet scalar as

$$v_\Delta \approx \frac{\mu v_\Phi^2}{\sqrt{2}\tilde{M}_\Delta^2}, \quad (11)$$

where we see that the smallness of v_Δ can be realized by either assuming a small μ or a large value for $\tilde{M}_\Delta$ characterizing the triplet scalar mass [32,33]. The triplet VEV v_Δ could be constrained from the ρ parameter, leading to the upper bound of the triplet VEV as $\mathcal{O}(1 \text{ GeV})$ [1]. After the electroweak symmetry breaking, seven of these scalar fields have definite mass, being evolved as physical fields. These scalar fields can be written as $H^{\pm\pm}$, $H^\pm$ for the charged multiplets; h , H^0 , and A^0 as neutral scalar fields, respectively.

Here, $H^{\pm\pm}$ is simply $\Delta^{\pm\pm}$ as presented in the triplet scalar field Δ . Using the fact $v_\Delta \ll v_\Phi$, the masses of physical Higgs bosons can be approximated as

$$\begin{aligned} m_{H^{\pm\pm}}^2 &\simeq M_\Delta^2 - \frac{\lambda_4}{2}v_\Phi^2, & m_{H^\pm}^2 &\simeq M_\Delta^2 - \frac{\lambda_4}{4}v_\Phi^2, \\ m_h^2 &\simeq 2\lambda v_\Phi^2 & \text{and } m_{H^0}^2 &\simeq m_{A^0}^2 \simeq M_\Delta^2, \end{aligned} \quad (12)$$

so their mass differences can be written as

$$\Delta m \equiv m_{H^0/A^0} - m_{H^\pm} \equiv m_{H^\pm} - m_{H^{\pm\pm}} \approx \frac{\lambda_4}{8} \frac{v_\Phi^2}{M_\Delta}. \quad (13)$$

Hence, we could express all the physical Higgs masses in terms of $m_{H^{\pm\pm}}$ and Δm only. The mass splitting between singly and doubly charged multiplets of the triplet scalar affects S , T , and U parameters, putting a stringent constraint as $|\Delta m| \lesssim 40 \text{ GeV}$ [91]. Depending on the value and sign of λ_4 we obtain three different spectra: (i) $\lambda_4 = 0$: $\Delta m \approx 0$ ($m_{H^{\pm\pm}} \simeq m_{H^\pm} \simeq m_{H^0/A^0}$), (ii) $\lambda_4 < 0$: $\Delta m < 0$ ($m_{H^{\pm\pm}} > m_{H^\pm} > m_{H^0/A^0}$), and (iii) $\lambda_4 > 0$: $\Delta m > 0$ ($m_{H^{\pm\pm}} < m_{H^\pm} < m_{H^0/A^0}$), respectively. The triplet scalar interacts with the SM gauge bosons and fermions.

Through the induced triplet VEV v_Δ , the Yukawa term in Eq. (9) generate the following Majorana mass term for light neutrinos:

$$\mathcal{L}_{\text{Majorana}} = \frac{1}{2} \bar{\nu}_{\alpha L}^c m_{\alpha\beta}^\nu \nu_{\beta L} + \text{H.c.} \quad \text{with } m_{\alpha\beta}^\nu = \sqrt{2}(Y_\Delta)_{\alpha\beta} v_\Delta. \quad (14)$$

From Eq. (14), we find that neutrino mass is directly proportional to the triplet VEV. Therefore, tiny neutrino mass can be ensured by small triplet VEV, which could be satisfied either by a small trilinear term μ or being suppressed by a large $\tilde{M}_\Delta$. Using neutrino oscillation data and $U_{\text{PMNS}}^T m_\nu U_{\text{PMNS}} = \text{diag}(m_1, m_2, m_3)$ to diagonalize the neutrino mass matrix, we express

$$Y_\Delta = \frac{1}{\sqrt{2}v_\Delta} U_{\text{PMNS}} m_\nu^{\text{diag}} U_{\text{PMNS}}^T, \quad (15)$$

where $m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$ are the neutrino mass eigenvalues which carry the information of neutrino oscillation for NO and IO along with U_{PMNS} , which is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix.

In Fig. 2, we show the branching ratios of $H^{\pm\pm}$ for $m_{H^{\pm\pm}} = 1.03 \text{ TeV}$ with $\Delta m \approx 0$ (left panel) and $\Delta m = -20 \text{ GeV}$ (right panel). For small Δm , depending on v_Δ , $H^{\pm\pm}$ dominantly decays either into same-sign dileptons for $v_\Delta \leq 10^{-4} \text{ GeV}$ or gauge bosons for $v_\Delta > 10^{-4} \text{ GeV}$. On the other hand, from the right panel of Fig. 2 with $\Delta m = -20 \text{ GeV}$, the cascade mode $H^{\pm\pm} \rightarrow H^\pm W^{\pm*}$ dominates over the leptonic and diboson decay modes

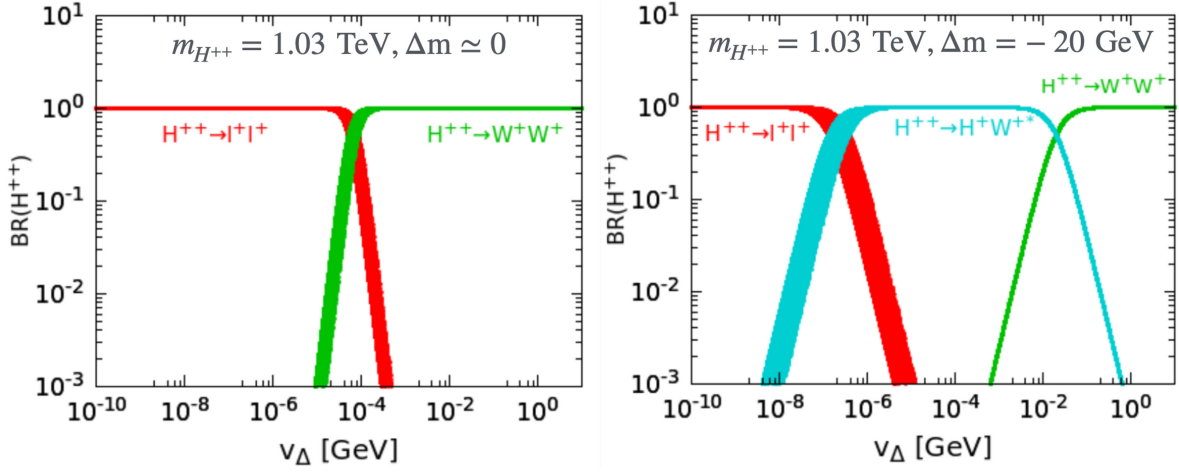


FIG. 2. Branching ratios of $H^{\pm\pm}$ for $m_{H^{\pm\pm}} = 1.03$ TeV with $\Delta m \approx 0$ (left panel) and $\Delta m = -20$ GeV (right panel), respectively. We consider three decay mode of $H^{\pm\pm}$ into $\ell^{\pm}\ell^{\pm}$ (red), $W^{\pm}W^{\pm}$ (green) and $H^{\pm}W^{\pm*}$ (cyan), respectively. The decay mode $H^{\pm\pm} \rightarrow H^{\pm}W^{\pm*}$ is only open for $\Delta m < 0$ and further dominates in the region of intermediate v_{Δ} when Δm is relatively large.

for an intermediate v_{Δ} . The doubly charged multiplet of the triplet scalar provides the tightest constraints from the searches at the LHC. The collider searches strictly depend on v_{Δ} and Δm , which governs the decay modes of the doubly charged multiplets; see Fig. 2. For our study, we consider a degenerate mass spectrum $m_{H^{\pm\pm}} = m_{H^{\pm}} = m_{H^0} = m_{A^0}$ and the triplet VEV in the region $v_{\Delta} < 10^{-4}$ GeV, such that $H^{\pm\pm} \rightarrow \ell_i^{\pm}\ell_j^{\pm}$ is the dominant decay mode. In this case, the LHC has ruled out doubly charged scalar mass below 1080 GeV [46].

C. Type III seesaw scenario

In the type III seesaw model, SM is extended by three generations of an $SU(2)_L$ triplet fermion (Ψ) with zero hypercharge [54]. Inclusion of such triplets helps the generation of nonzero but tiny neutrino mass through the seesaw mechanism. The Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \text{Tr}(\bar{\Psi}i\gamma^{\mu}D_{\mu}\Psi) - \frac{1}{2}M\text{Tr}(\bar{\Psi}\Psi^c + \bar{\Psi}^c\Psi) - \sqrt{2}(\bar{\ell}_L Y_D^{\dagger}\Psi H + H^{\dagger}\bar{\Psi}Y_D\ell_L), \quad (16)$$

where D_{μ} represents the covariant derivative and M is the Majorana mass term. We consider three degenerate generation of the triplets. Y_D is the Dirac Yukawa coupling between the SM lepton doublet (ℓ_L), SM Higgs doublet (H), and the triplet fermion (Ψ). For brevity, we have suppressed the generation indices. In this analysis, we represent the relevant SM candidates, the triplet fermion, and its charged conjugate ($\Psi^c = C\bar{\Psi}^T$) in the following way:

$$\begin{aligned} \ell_L &= \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad H = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}, \\ \Psi &= \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix} \\ \text{and } \Psi^c &= \begin{pmatrix} \Sigma^{0c}/\sqrt{2} & \Sigma^{-c} \\ \Sigma^{+c} & -\Sigma^{0c}/\sqrt{2} \end{pmatrix}. \end{aligned} \quad (17)$$

To study the mixing between the SM charged leptons and $\Sigma^{\pm}$, we write the 4 degrees of freedom of each $\Sigma^{\pm}$ in terms of a Dirac spinor such as $\Sigma = \Sigma_R + \Sigma_R^{+c}$, where Σ^0 are two component fermions with 2 degrees of freedom. The corresponding Lagrangian after the electroweak symmetry breaking can be written as

$$\begin{aligned} -\mathcal{L}_{\text{mass}} &= \begin{pmatrix} \bar{e}_L & \bar{\Sigma}_L \end{pmatrix} \begin{pmatrix} m_{\ell} & Y_D^{\dagger}v \\ 0 & M \end{pmatrix} \begin{pmatrix} e_R \\ \Sigma_R \end{pmatrix} \\ &+ \frac{1}{2} \begin{pmatrix} \bar{\nu}_L^c & \bar{\Sigma}_R^0 \end{pmatrix} \begin{pmatrix} 0 & Y_D^T \frac{v}{\sqrt{2}} \\ Y_D \frac{v}{\sqrt{2}} & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \Sigma_R^{0c} \end{pmatrix} + \text{H.c.}, \end{aligned} \quad (18)$$

where m_{ℓ} is the Dirac type SM charged lepton mass. The 3×3 Dirac mass of the triplets can be written as $M_D = Y_D^T v/\sqrt{2}$. Diagonalizing the neutrino mass matrix in Eq. (18), we can write the light neutrino mass eigenvalue as

$$m_{\nu} \simeq -\frac{v^2}{2} Y_D^T M^{-1} Y_D = M_D M^{-1} M_D^T. \quad (19)$$

The light neutrino flavor eigenstate can be expressed in terms of the light and heavy mass eigenstates in the

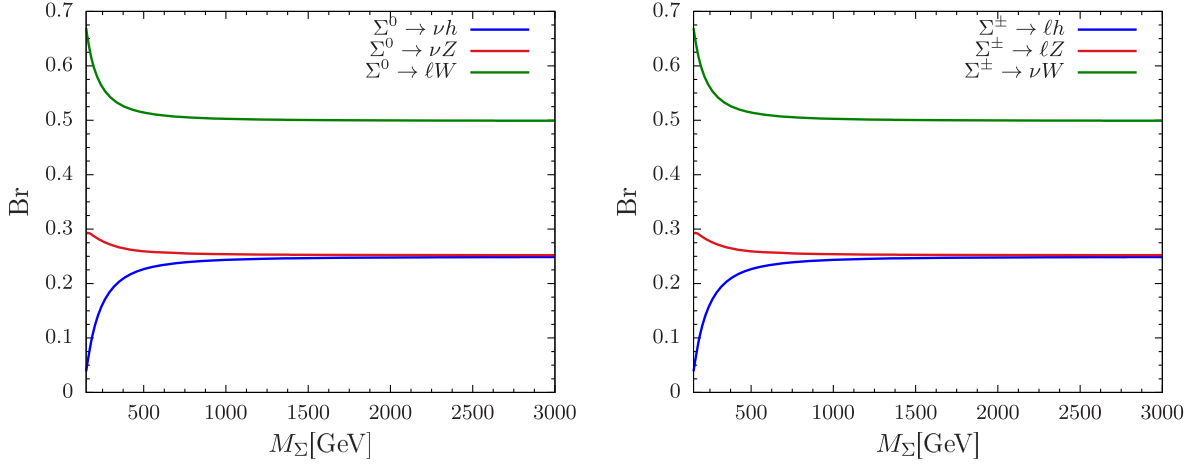


FIG. 3. Branching ratio of Σ^0 (left) and $\Sigma^\pm$ (right) into the SM particles as a function of M_Σ for $|V_{\mu\Sigma}|^2 = 2.89 \times 10^{-4}$ considering $V_{e\Sigma} = V_{\tau\Sigma} = 0$ from [94,95].

following way:

$$\nu \approx \nu_m + V\Sigma_m, \quad (20)$$

where ν_m and Σ_m represent the light and heavy mass eigenstates, respectively, where $V = M_D M^{-1}$. The relevant charged current, neutral current, and Higgs interaction involving SM leptons and triplet fermions can be found in Refs. [57,89,92,93]. Estimating partial decay widths [57], we show the branching ratios $\Sigma^{0,+}$ in Fig. 3. Note that when the triplet is very heavy, the branching ratios of $\Sigma^{0,+}$ to different modes follow the same relation as of Eq. (7) and this is again due to the Goldstone equivalence theorem. In this analysis, we consider the scenario where limit on the light-heavy mixing $|V_{\mu\Sigma}|^2 = 2.89 \times 10^{-4}$ considering $V_{e\Sigma} = V_{\tau\Sigma} = 0$ from electroweak precision data (EWPD), following Refs. [94,95]. We use this mixing as the benchmark limit in further analysis.

III. RESULTS AND DISCUSSIONS

In type I seesaw scenarios, we investigate a single electron (muon) final state in association with jets and neutrinos. In the type II seesaw scenario, we study the same-sign dilepton final state and doubly charged scalar multiplet production in association with the Z boson or photon. We also study left-right asymmetry from the same-sign dilepton mode. We study the production of positively charged multiplets of the triplet fermions at the $\mu^+\mu^+$ collider in association with μ^+ . Following the decay of the positively charged multiplets of the triplets, we study same-sign dileptons and jets in the final state from the type III seesaw scenario. We describe complete search strategies and results in the following way:

A. Heavy neutrino search

At the μ^+e^- collider of the μ TRISTAN experiment, the heavy neutrino can be produced as $\mu^+e^- \rightarrow \nu N$ through a

W mediated t -channel process as shown in Fig. 4. The differential scattering cross section for this t -channel process can be given by

$$\begin{aligned} \frac{d\sigma(\mu^+e^- \rightarrow N\nu)}{d\cos\theta} &= (3.89 \times 10^8 \text{ pb}) \times \frac{(s - M_N^2)}{32\pi s^2} \\ &\times \left[\frac{g^4 \left\{ (s - M_N^2)(1 \pm \cos\theta) \times \left(\frac{1}{2}s - \frac{1}{4}(s - M_N^2)(1 \mp \cos\theta) \right) \right\}}{\left(\frac{1}{2}(s - M_N^2)(1 \mp \cos\theta) + M_W^2 \right)^2 + \Gamma_W^2 M_W^2} \right], \end{aligned} \quad (21)$$

where g, M_W , and Γ_W are the $SU(2)$ gauge coupling, W boson mass, and the total decay width, respectively. To obtain the signal cross section, we integrated over the scattering angle θ in the range of $-1 \leq \cos\theta \leq 1$. Note that the production cross section is normalized by the relevant light-heavy neutrino mixing square. The normalized production cross section in the μ TRISTAN experiment at $\sqrt{s} = 346$ GeV center of mass energy is shown in Fig. 5. We find that the production process $\mu^+e^- \rightarrow N\nu$ followed by the decay chains such as $N \rightarrow \mu^- W^+ / e^+ W^-$ and $W^\pm \rightarrow jj$ lead to collider signatures such as $\mu^- jj\nu$ or $e^+ jj\nu$. For the signal, we only consider the final state with a hadronically decaying W boson, because this channel suffers from lower background than the leptonic decaying W channel at the lepton collider. We find that final state $\mu^+e^- \rightarrow N\nu \rightarrow \mu^- jj\nu$ can constrain either $|V_{\mu N}|^2$ (when N is from a μ^+ beam) or $|V_{eN}^* V_{\mu N}|$ (when N is from an e^- beam), whereas the final state $\mu^+e^- \rightarrow N\nu \rightarrow e^+ jj\nu$ can constrain either $|V_{eN}|^2$ (when N is from an e^- beam) or $|V_{eN}^* V_{\mu N}|$ (when N is from a μ^+ beam).

To study the signal and SM backgrounds, we simulate corresponding events using the event generation package MadGraph5_aMC@NLO [96] using the Universal FeynRules

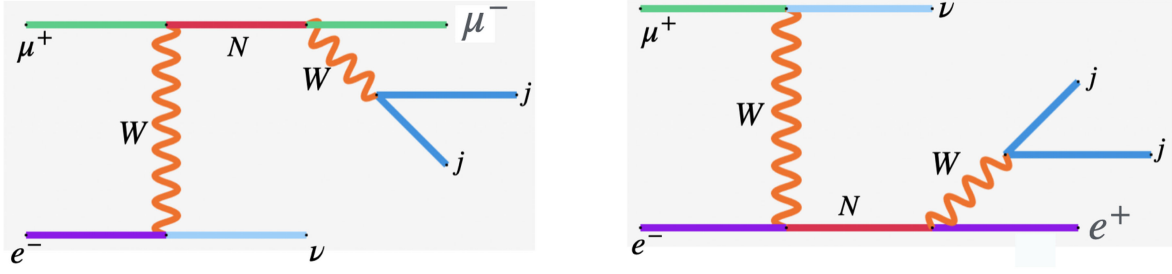


FIG. 4. Heavy neutrino production processes in t -channel and its dominant decay in the μ TRISTAN experiment. Following the heavy neutrino production, we consider the heavy neutrino decays to either positron or muon in association with two jets.

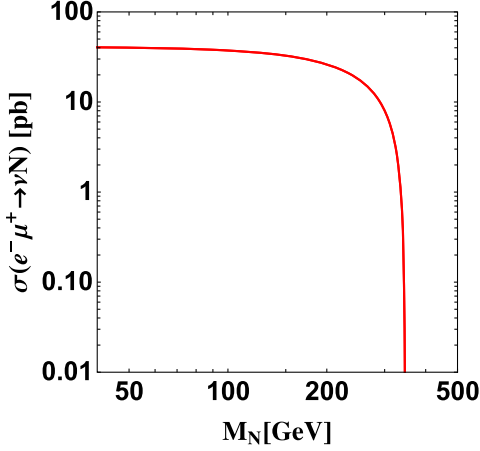


FIG. 5. Heavy neutrino production cross section as a function of heavy neutrino mass in the μ TRISTAN experiment at $\sqrt{s} = 346$ GeV center of mass energy normalized by the light-heavy mixing squared.

Output model files generated by FeynRules [97]. The Pythia8 [98] is used to implement parton shower, hadronization, and decay of the SM hadrons. The detector effects are simulated by Delphes [99] with the default international linear collider (ILC) configuration card. The leading-order cross sections of the generic SM backgrounds $\mu^+ e^- \rightarrow e^+ jj + \cancel{E}_T$ and $\mu^- jj + \cancel{E}_T$ are listed in Table I. While generating the background, the transverse momenta of the final state charged lepton and jets are required to be greater than 10 and 20 GeV, respectively. The dominant SM background

TABLE I. Leading-order production cross section of generic SM backgrounds at $\sqrt{s} = 346$ in the μ TRISTAN experiment. The jet in the final state should satisfy $p_T > 20$ GeV and $|\eta| < 4.5$, and the lepton is required have $p_T > 10$ GeV and $|\eta| < 2.5$.

Process	σ [pb]
$\mu^+ e^- \rightarrow e^+ jj + \nu\bar{\nu}$	7.97×10^{-5}
$\mu^+ e^- \rightarrow \mu^- jj + \bar{\nu}\nu$	8.42×10^{-5}
$\mu^+ e^- \rightarrow e^- jj + \bar{\nu}$	1.33×10^{-1}
$\mu^+ e^- \rightarrow \mu^+ jj + \nu$	1.46×10^{-2}

processes are $\mu^+ e^- \rightarrow e^+ jj + \nu\bar{\nu}$, $\mu^- jj + \bar{\nu}\nu$, $e^- jj + \bar{\nu}$, and $\mu^+ jj + \nu$, respectively. Their leading-order cross sections are listed in Table I. The latter two processes becomes the background when the charge of the lepton is misidentified. In our analysis, the charge misidentification rate is taken to be 0.001 [100].

To increase the signal significance, we will adopt the gradient boosted decision tree (GBDT) method [101] that takes into account several feature variables for signal and background discrimination. Only events that fulfill the following preselection cuts will be used in the training and testing of the GBDT: (1) exactly one charged lepton with $p_T(\ell^\pm) > 20$ GeV and $|\eta_\ell| < 2.5$; (2) either one or two jets with $p_T(j) > 20$ GeV, $|\eta_j| < 4.5$, and $m_{jj} \in [70, 90]$ GeV for two jets events¹; (3) missing transverse momentum $\cancel{E}_T < 60$ GeV. From the preselected event sample, we reconstruct the following set of observables:

$$p^\mu(\ell^\pm), \cos(\theta_{\ell^\pm}), p^\mu(\ell jj), m_{\ell jj}, \quad (22)$$

where $p^\mu(\ell^\pm)$ denotes the four-momentum of the isolated charged lepton, $\cos(\theta_{\ell^\pm})$ corresponds to its polar angle in the laboratory frame, while $p^\mu(\ell jj)$ and $m_{\ell jj}$ represent, respectively, the four-momentum and invariant mass of the ℓjj system formed by the vector sum of the lepton and dijet momenta. The distributions for some of the feature variables are shown in Fig. 6. The GBDT method uses a 100-tree ensemble that requires a minimum training events in each leaf node of 1 and a maximum tree depth of three, with a learning rate of 0.1. It is training on benchmark signals with heavy neutrino masses in the range of [39, 300] GeV with a step size 14.5 GeV. For each mass, half of the preselected signal ($\sim 10^4$) and background ($\sim 10^4$) events are used in the training and testing of the GBDT, respectively. To avoid overtraining, the subsample is required to be less than 0.8. After the training, the GBDT is able to assign a BDT score for each event. The score combines all discriminating abilities of the input feature variables, which can be used for signal and background discrimination. The BDT scores

¹The one-jet final state corresponds to a boosted hadronically decaying W boson.

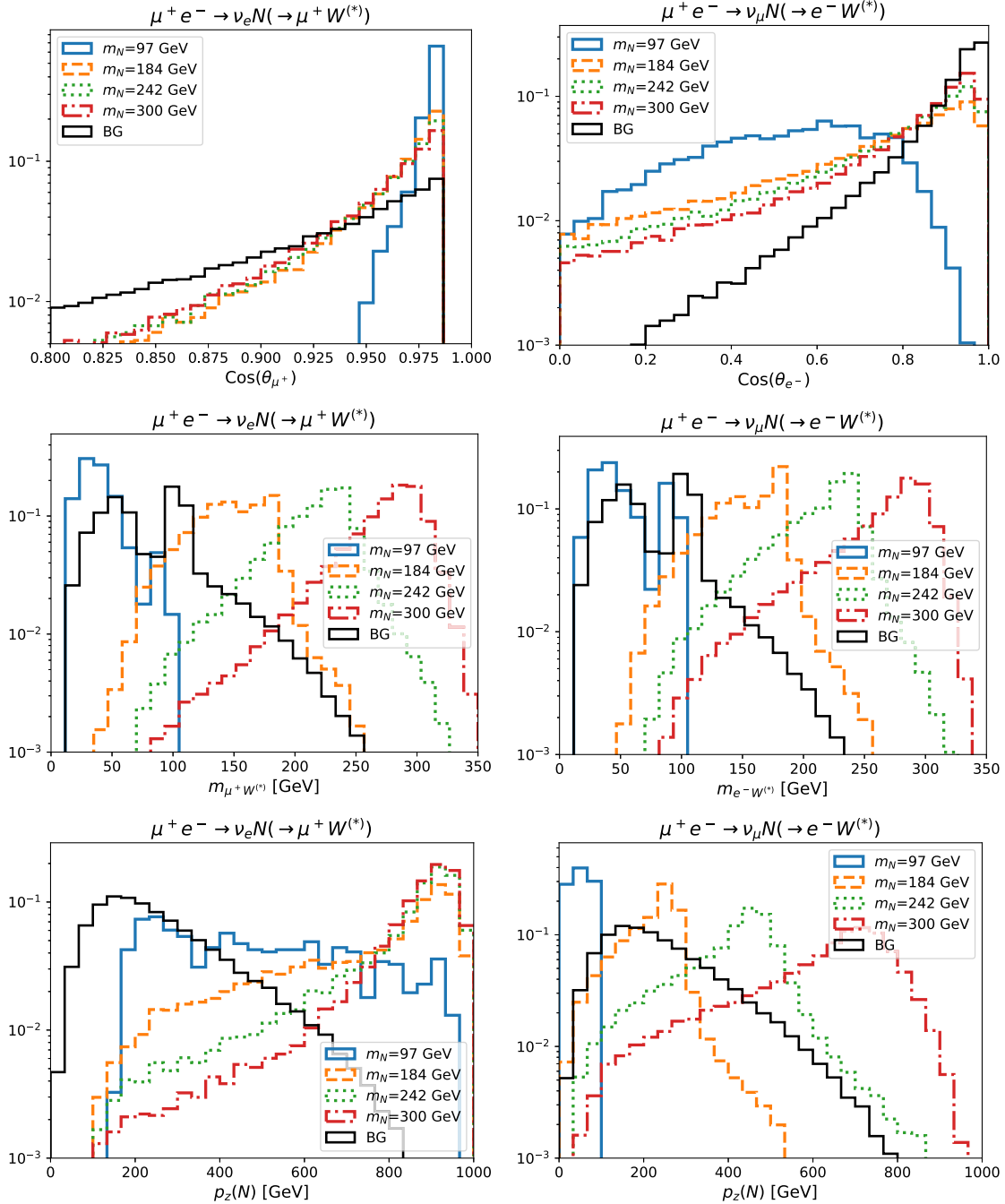


FIG. 6. Feature variables that input to the BDT from top to bottom: $\cos \theta$ of μ/e , invariant mass, and longitudinal momentum of the reconstructed heavy neutrino. Left panels: muonic flavor heavy neutrino; Right panels: electronic flavor heavy neutrino.

for signal and background are around unit and zero, respectively. In Fig. 7, the BDT score for a few benchmark signals and the corresponding backgrounds are shown. The signal and background are well separated for each case, indicating the strong discriminating power of the GBDT. After the application of the kinematic cuts, we write down the signal and background events in Table II for some benchmark heavy neutrino masses (M_N) considering the decay of the heavy neutrinos in the ℓjj final state. After the application of all the kinematic cuts, we estimate a 2σ

contour on the $M_N - |V_{e(\mu)N}|^2$ and $M_N - |V_{eN}^* V_{\mu N}|$ planes solving the following equation:

$$2 = \frac{S|V_{e(\mu)N}|^2}{\sqrt{S|V_{e(\mu)N}|^2 + B}}, \quad (23)$$

where S and B stands for signal (when mixing squared is taken to be unity) and corresponding background events, respectively, at 1 ab^{-1} luminosity. Corresponding estimated

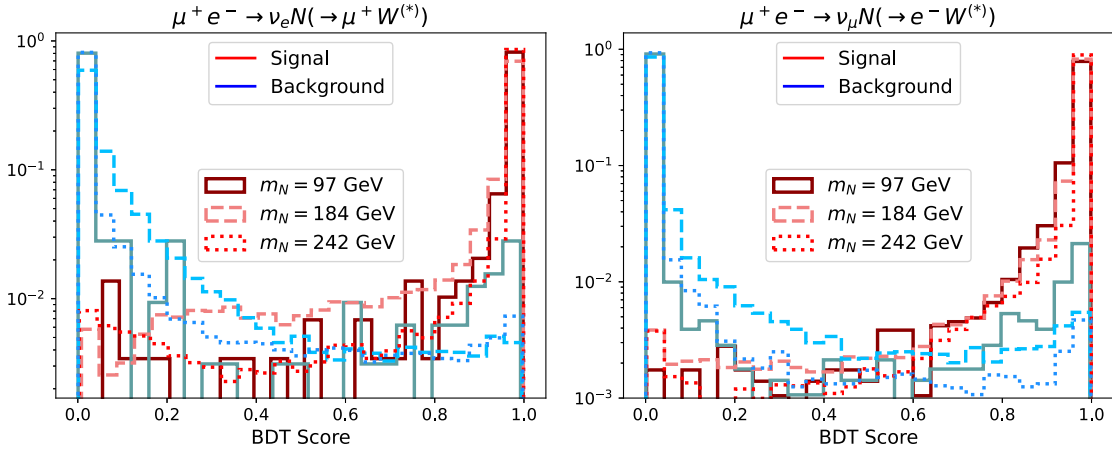


FIG. 7. BDT score for a few benchmark signals and the corresponding backgrounds.

bounds on the light-heavy mixings $|V_{eN}|^2$, $|V_{\mu N}|^2$, and $|V_{eN}^* V_{\mu N}|$ are shown in the upper left, upper right, and bottom panels of Fig. 8, respectively. The shaded regions in each panel are already ruled out by different existing searches. From our analysis, we find that in case of the light-heavy mixing angle $|V_{eN}|^2$, prospective limits could be stronger than existing bounds from prompt heavy neutrino searches from compact muon solenoid (CMS) [102,103] and EWPd [94,104,105] for $77 \text{ GeV} \leq M_N \leq 315 \text{ GeV}$. We find that for $93 \text{ GeV} \leq M_N \leq 295 \text{ GeV}$, prospective limit on $|V_{eN}|^2$ could vary in the range $4.2 \times 10^{-6} \leq |V_{eN}|^2 \leq 10^{-5}$. In addition, we estimate prospective limits on the mixing $|V_{\mu N}|^2$ which could provide stronger limits

TABLE II. Signal normalized by light-heavy mixing squared and SM background cross sections before and after cuts for several benchmark points of the heavy neutrino masses. The cuts here include the preselection cuts as well as a BDT cut that maximizes the signal significance for each heavy neutrino mass.

$\sqrt{s}$ (GeV)	M_{N_1} (GeV)	Signal (S)		Background (B)	
		Before cuts (pb)	After cuts (fb)	Before cuts (fb)	After cuts (fb)
346	45	24.62	3.7	0.0988	5.525×10^{-3}
	68	23.6	71.1	0.0988	9.375×10^{-4}
	97	23.5	137.3	0.0988	9.023×10^{-4}
	184	11.5	430.2	0.0988	2.780×10^{-4}
	300	2.9	363.24	0.0988	3.776×10^{-4}
$\sqrt{s}$ (GeV)	M_{N_2} (GeV)	Signal		Background	
		Before cuts (pb)	After cuts (fb)	Before cuts (fb)	After cuts (fb)
346	45	24.62	767.4	0.213	3.681×10^{-4}
	68	23.6	1242.6	0.213	6.519×10^{-4}
	97	23.5	697.6	0.213	1.089×10^{-3}
	184	11.5	1675.0	0.213	8.560×10^{-4}
	300	2.9	582.1	0.213	3.473×10^{-4}

compared to prompt heavy neutrino searches from CMS [102,103] (gray solid) and EWPd (cyan solid [dashed] for electron [muon]) [94,104,105] for $61.5 \text{ GeV} \leq M_N \leq 321 \text{ GeV}$. We find that for $88 \text{ GeV} \leq M_N \leq 304 \text{ GeV}$, prospective limit could vary in the range $2.0 \times 10^{-6} < |V_{\mu N}|^2 \leq 10^{-5}$.

We compare the prospective bounds with existing results from a variety of experiments represented by the gray shaded region in Fig. 8. Bounds from the same-sign dilepton search by the ATLAS and CMS experiments at $\sqrt{s} = 8 \text{ TeV}$ LHC are shown by the red dashed and dotted lines from Refs. [106,107]. Experimental bounds from the L3 detector of the large electron-positron collider experiment are shown by the light green dot-dashed line from Ref. [108] in Fig. 8 (upper panel). Bounds from $\sqrt{s} = 13 \text{ TeV}$ LHC using the same-sign dilepton searches from CMS [23] using cyan dot-dashed and trilepton searches from ATLAS [109] using cyan dashed and CMS [22] brown dot-dashed lines, respectively. Bounds obtained from the prompt heavy neutrinos searches from the DETector with Lepton, Photon and Hadron Identification (DELPHI) [110] experiment are represented by black dot-dashed and dashed lines for electron and muon flavors, respectively. Bounds on the mixing using long-lived particle searches for Majorana-type heavy neutrinos from the CMS [25] are shown by a purple thin dot-dashed line in the upper panel of Fig. 8. Displaced vertex searches in ATLAS from Majorana-type heavy neutrinos are taken from [111,112] where bounds are shown for single flavor case with normal hierarchy using red dot-dashed line, lines for muon flavor in the lower panel of Fig. 8. We show corresponding CMS bounds for displaced vertex from Majorana-type heavy neutrinos by a pink dot-dashed line [103,103] in the upper and lower panels of Fig. 8. We also estimate bounds on the light-heavy mixing from the top quark decay into heavy Majorana neutrinos shown by light green solid line in the shaded region [113]. Limits on the light-heavy mixing due to the second-generation heavy neutrino studying the vector

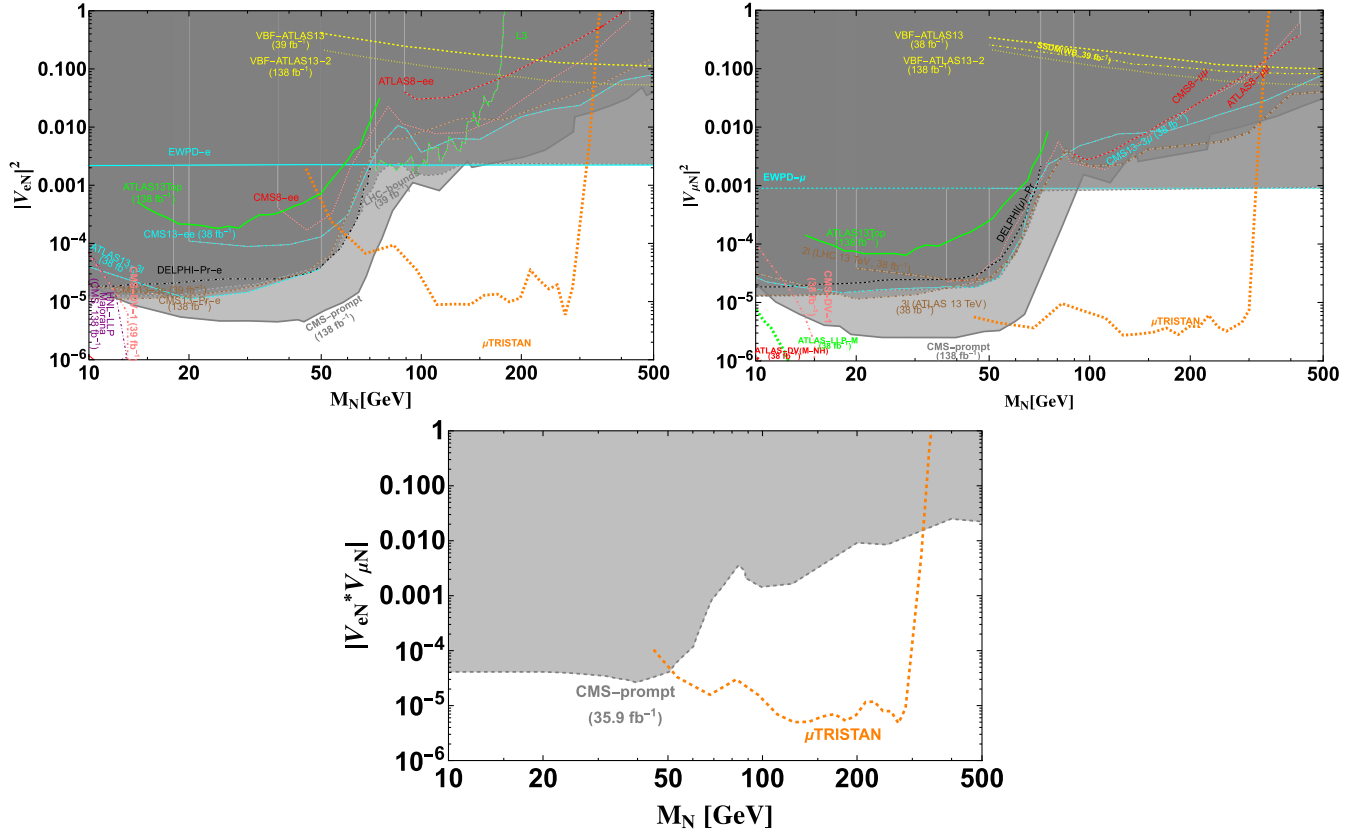


FIG. 8. Limits on light-heavy mixing with respect to first (left) and second (right) generation heavy neutrino masses from the μ TRISTAN experiment (orange dashed line) at $\sqrt{s} = 346$ GeV and 1 ab^{-1} luminosity. Shaded regions are already ruled out by different existing experimental searches (see text for other prospective limits on mixing).

boson fusion process from ATLAS (VBF-ATLAS13, VBF-ATLAS13-2) [112,114,115] and CMS (SSDM[W]) [24] at different luminosities are shown by different yellow lines in the gray shaded region, which are weaker than the bounds obtained from the EWPd. These limits could be probed in the future. We show the limits on $|V_{eN} V_{\mu N}|$ in the lower panel of Fig. 8. We compare our results with the bounds obtained from the 13 TeV LHC [103,107] at 35.9 fb^{-1} luminosity. We show the excluded region by LHC in gray.

Finally, we mention that limits on light-heavy mixing of neutrinos were studied from a variety of production modes and decays of the RHNs at the hadron colliders involving a variety of initial states [116,117], considering next-to-leading order QCD corrections at the initial states [118], involving boosted objects like fat jets [119,120] where we found that $\mathcal{O}(100)$ GeV RHNs could be probed within $10^{-4} \leq |V_{eN}|^2 \leq 10^{-3}$ at 3 ab^{-1} luminosity. For RHNs lighter than $\mathcal{O}(100)$ GeV [121,122], which could be probed from Higgs decay at the LHC, the prospective limit on light-heavy mixing square could reach around $\mathcal{O}(10^{-5})$ at 3 ab^{-1} luminosity. If the heavy neutrinos mass is around and below $\mathcal{O}(10)$ GeV, the bounds comes from the meson decay and could be stronger than $\mathcal{O}(10^{-5})$ [21,123,124]. RHNs having mass $\mathcal{O}(1)$ TeV can be

produced at the electron-positron colliders, and considering their decay into boosted objects [125–129] prospective limits on light-heavy neutrino mixing square could be projected around $\mathcal{O}(10^{-5})$. On the other hand, the prospective bound for the RHN mass in the range of $\mathcal{O}(10)$ GeV to $\mathcal{O}(100)$ GeV was studied in electron-positron and electron-photon colliders, where the predicted bound is of the order of $\mathcal{O}(10^{-5})$ and slightly stronger than that [130], depending on the RHN mass. We find that our obtained bound on the light-heavy mixings are comparable with most of these above-mentioned prospective bounds.

B. Triplet scalar search

We consider the type II seesaw scenario where doubly charged component of the $SU(2)$ scalar triplet could play a crucial role in $\mu^+ \mu^+$ collision of the proposed μ TRISTAN collider. We focus mainly on the small value of triplet VEV v_Δ such that doubly charged Higgs exclusively decay only to the leptonic mode. The corresponding branching ratios are given by

$$\text{BR}(H^{\pm\pm} \rightarrow l_i^\pm l_j^\pm) = \frac{2}{(1 + \delta_{ij})} \frac{|Y_\Delta^{ij}|^2}{\sum_{ab} |Y_\Delta^{ab}|^2}, \quad (24)$$

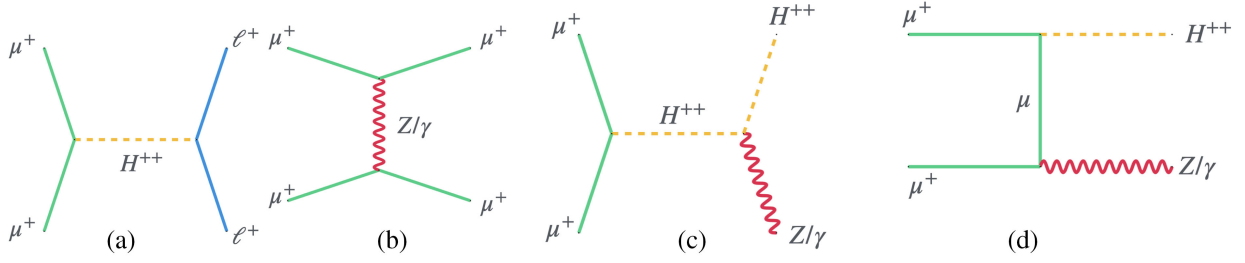


FIG. 9. Doubly charged scalar multiplet (H^{++}) mediated $\mu^+\mu^+ \rightarrow \ell^+\ell^+$ processes in (a) and the SM backgrounds (γ, Z) in (b) in the μ TRISTAN experiment within the type II seesaw framework. Feynman diagrams for the $\mu^+\mu^+ \rightarrow H^{++}Z/\gamma$ process are shown in (c) and (d), followed by the decay $H^{++} \rightarrow \ell^+\ell^+$, which could be tested in the μ TRISTAN experiment at $\sqrt{s} = 2$ TeV.

with Y_Δ as given in Eq. (15). The flavor structure of the Yukawa coupling Y_Δ plays a crucial role here, as it governs both the properties of neutrinos and the branching ratio of $H^{\pm\pm}$ decaying into charged leptons of various flavors. Hence, studying the branching ratios of doubly charged Higgs, one can shed some light on neutrino mass ordering. In view of this we first consider the doubly charged scalar mediated same-sign dilepton channel $\mu^+\mu^+ \rightarrow \ell_i^+\ell_j^+$. Note that for the final state $\ell_i^+\ell_j^+ = \mu^+\mu^+$, there would be large SM background coming from t -channel Z/γ mediation. Corresponding Feynman diagrams are given in Fig. 9. Total production cross section of $\mu^+\mu^+ \rightarrow \ell_i^+\ell_j^+ (\neq \mu^+\mu^+)$ signal process can be given by

$$\sigma(\mu^+\mu^+ \rightarrow \ell_i^+\ell_j^+) = \frac{|Y_\Delta^{\mu\mu} Y_\Delta^{ij}|^2}{4\pi(1 + \delta_{ij})} \frac{s}{(s - m_{H^{\pm\pm}}^2)^2 + m_{H^{\pm\pm}}^2 \Gamma_{H^{\pm\pm}}^2}, \quad (25)$$

neglecting initial and final state lepton masses. Here $\sqrt{s} = 2$ TeV, δ_{ij} is Kronecker delta vanishing for leptons of a different kind, giving unity for same-type leptons, and $\Gamma_{H^{\pm\pm}}$ is the total decay width of $H^{\pm\pm}$.

We consider a luminosity of 1 ab^{-1} to estimate number of events for the signal and background. We generate the signal and background events with transverse momentum of the leptons as $p_T^\ell > 400 \text{ GeV}$ and missing energy as $\cancel{E}_T < 60 \text{ GeV}$. As we are considering the doubly charged Higgs mass in the range $m_{H^{\pm\pm}} > 1 \text{ TeV}$, the transverse momentum of charged leptons coming from $H^{\pm\pm}$ decay will be very high. We find that usage of these cuts does not reduce the signal that much, while background will be highly suppressed. We find that SM background is very large for $\mu^+\mu^+$ final state; as a result, we did not show the results for this final state. However, for the other combinations of the signal events, such as $e^+\mu^+$, $\mu^+\tau^+$, $e^+\tau^+$, $\tau^+\tau^+$, and e^+e^+ , the number of signal events are large compared to the SM backgrounds. The dominant background comes from the WW scattering and becomes $\mathcal{O}(10^{-7} \text{ pb})$ after we apply the p_T^ℓ and $\cancel{E}_T$ cuts. We find that although the expected number of events can be large for all the final states, only the e^+e^+ final state potentially

can differentiate between the NO and IO case for neutrino mass ordering. In Fig. 10, we show the significance for the $\mu^+\mu^+ \rightarrow e^+e^+$ channel for NO and IO scenarios as a function of $m_{H^{++}}$ (left panel) and m_{lightest} (right panel), respectively. We find that the significance for NO case are different from the IO case where the significance for IO case could be at least 2 orders of magnitude larger than the NO case. We find that for $m_{\text{lightest}} < 0.01 \text{ eV}$, the significance for the NO and IO scenarios become nearly 2 orders of magnitude different. Therefore $\mu^+\mu^+ \rightarrow e^+e^+$ process mediated by H^{++} can be tested in the μ TRISTAN experiment to probe IO of the neutrino mass generation mechanism. In this analysis, v_Δ and $m_{H^{++}}$ are taken in such a way that the branching ratios of the lepton flavor-violating (LFV) decay mode $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ are satisfied by the experimental limits $\text{BR}(\mu \rightarrow e\gamma) = 4.2 \times 10^{-13}$ and $\text{BR}(\mu \rightarrow 3e) = 10^{-12}$ [131,132], respectively. The neutrino oscillation parameters are varied within their allowed 3σ range [34]; whereas the Majorana phases (ϕ_1, ϕ_2) are set to be zero. The gray shaded region in Fig. 10 is excluded from the combined analysis of Cosmic Microwave Background (CMB)+Baryon Acoustic Oscillations (BAO) [133].

We study another production mode of H^{++} following $\mu^+\mu^+ \rightarrow H^{++}\gamma/Z$, and the corresponding Feynman diagrams are given in Fig. 9. We first consider the scenario $\mu^+\mu^+ \rightarrow H^{++}\gamma \rightarrow e^+e^+\gamma$, where doubly charged Higgs decays to e^+e^+ final state. The dominant SM background for this channel comes from the W -boson fusion channel such as $\mu^+\mu^+ \rightarrow W^+W^+ + \bar{\nu}\bar{\nu}$, following the leptonic decay of W bosons and photons radiated from any of the charged leptons. In this analysis, we apply the following cuts: $p_T^\gamma > 50 \text{ GeV}$, $p_T^\ell > 400 \text{ GeV}$, and $\cancel{E}_T < 60 \text{ GeV}$, respectively. After cuts, signal events become almost half of the events before cut, whereas the SM background reduces to $\sim 10^{-9} \text{ pb}$ after the application of kinematic cuts from the cross section $\sim 10^{-3} \text{ pb}$ obtained before application of kinematic cuts. These SM backgrounds could be large for $\mu^+\mu^+$ mode due to the following W -boson fusion channel, such as $\mu^+\mu^+ \rightarrow \mu^+\mu^+W^+W^-$, as well as due to the t -channel γ/Z mediated SM process $\mu^+\mu^+ \rightarrow \mu^+\mu^+$. Therefore, we do not consider $H^{++} \rightarrow \mu^+\mu^+$ mode.

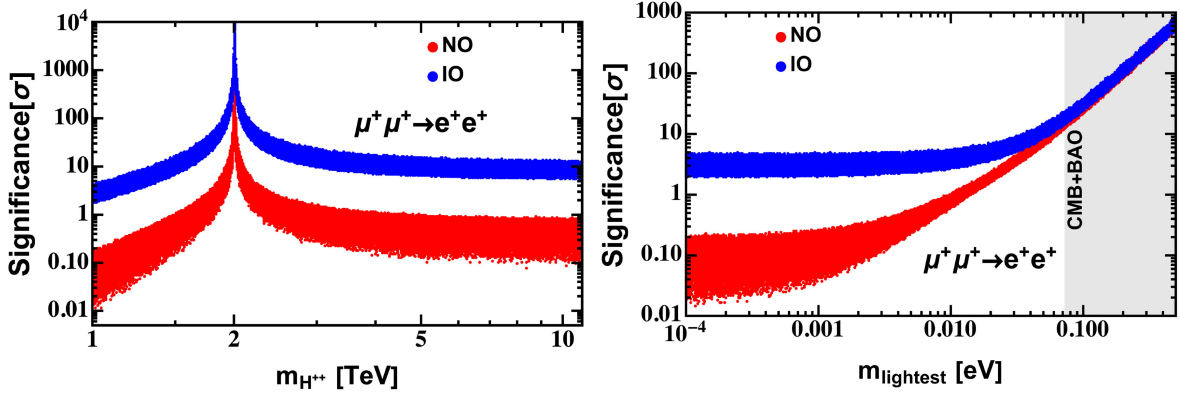


FIG. 10. We show the significance for $\mu^+\mu^- \rightarrow e^+e^-$ process as function of $m_{H^{++}}$ (left panel) and lightest neutrino mass for $m_{H^{++}} = 1.03$ TeV (right panel) in μ TRISTAN collider considering $\sqrt{s} = 2$ TeV and luminosity as $\mathcal{L} = 1\text{ab}^{-1}$ assuming v_Δ and $m_{H^{++}}$ to be taken in such a way that the branching ratios of the LFV processes $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ are satisfied by the experimental limits $\text{BR}(\mu \rightarrow e\gamma) = 4.2 \times 10^{-13}$ and $\text{BR}(\mu \rightarrow 3e) = 10^{-12}$ [131,132], respectively. The neutrino oscillation parameters are varied within their allowed 3σ range [34] whereas the Majorana phases (ϕ_1, ϕ_2) are set to be zero. The gray shaded region in the bottom right panel is excluded from the combined analysis of CMB + BAO [133].

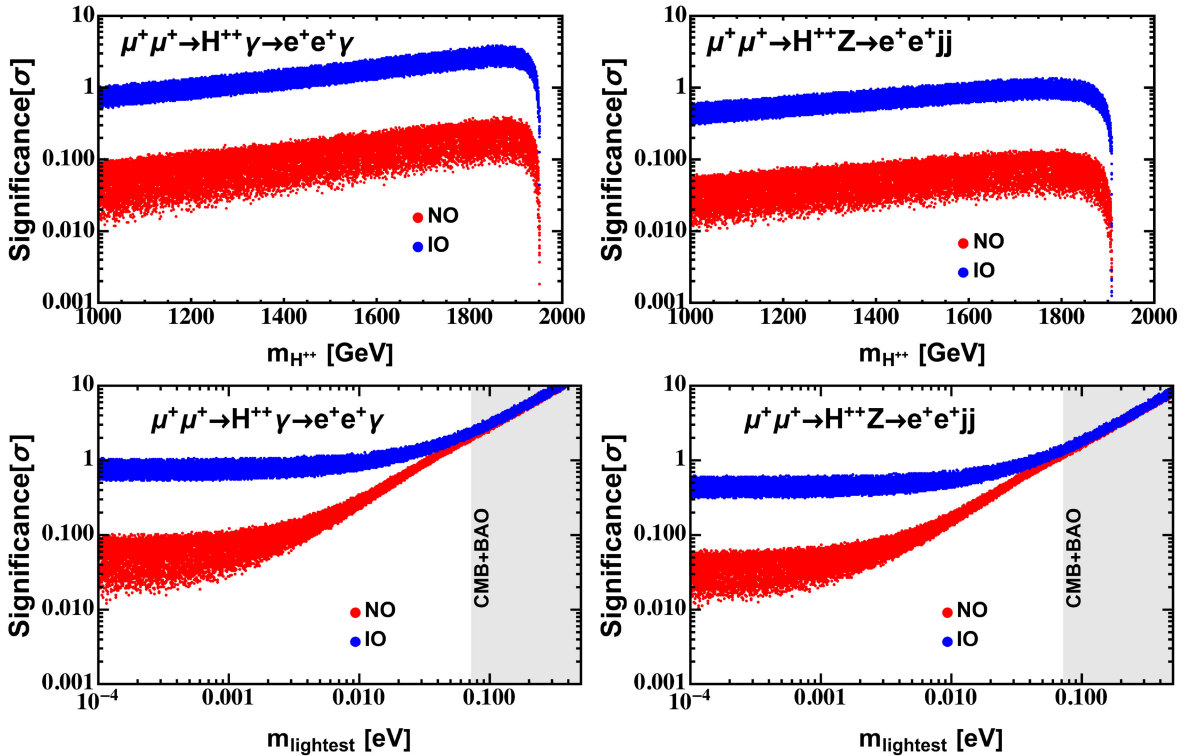


FIG. 11. Significance of the $\mu^+\mu^- \rightarrow e^+e^+\gamma$ (upper left panel) and e^+e^+jj (upper right panel) processes after cut with $\sqrt{s} = 2$ TeV and $m_{\text{lightest}} = 0$ in the μ TRISTAN experiment. Fixing $m_{H^{++}} = 1.03$ TeV distinction between NO and IO cases is shown (bottom panel) as a function of m_{lightest} . We assume v_Δ and $m_{H^{++}}$ to be taken in such a way that the branching ratios of the LFV processes $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ are satisfied by the experimental limits $\text{BR}(\mu \rightarrow e\gamma) = 4.2 \times 10^{-13}$ and $\text{BR}(\mu \rightarrow 3e) = 10^{-12}$ [131,132], respectively. The neutrino oscillation parameters are varied within their allowed 3σ range [34]; whereas the Majorana phases (ϕ_1, ϕ_2) are set to be zero. The gray shaded region in the bottom right panel is excluded from the combined analysis of CMB + BAO [133].

Next, we consider the process $\mu^+\mu^- \rightarrow H^{++}Z \rightarrow e^+e^+jj$; see Fig. 9. We are considering hadronic decays of Z bosons due to larger branching ratios over leptonic mode and smaller SM background. To analyze the signal

and SM background events, we apply kinematic cuts as $p_T^e > 400$ GeV, $p_T^j > 20$ GeV, and $\cancel{E}_T < 60$ GeV. The dominant SM background for this channel comes from the following SM processes: $W^+W^+ + \bar{\nu}\bar{\nu}$ and

$W^+W^+(Z/h) + \bar{\nu}\bar{\nu}$. Signal events after the cuts are almost 84% of the events before cuts, and SM background reduces to 10^{-10} pb. The upper and bottom panels of Fig. 11 shows the significance as a function of $m_{H^{\pm\pm}}$ and m_{lightest} , respectively. We see that as long as lightest neutrino mass is in the range $m_{\text{lightest}} \leq 0.01$ eV, depending on the doubly charged Higgs mass, the significance for both the processes $e^+e^+\gamma$ and e^+e^+jj clearly differs for the NO case compared to the IO case.

Finally, we study the left-right asymmetry for the process $\mu^+\mu^+ \rightarrow \ell^+\ell^+$ using the polarized cross section at the μ TRISTAN experiment, considering $\sqrt{s} = 2$ TeV and 1 ab^{-1} luminosity. Corresponding Feynman diagrams are given in Fig. 9. We will mostly follow Ref. [134] for the methodology. The relevant scattering process is given by

$$\mu^+(k_1, \sigma_1) + \mu^+(k_2, \sigma_2) \rightarrow \ell^+(k_3, \sigma_3) + \ell^+(k_4, \sigma_4), \quad (26)$$

where $\sigma_i(k_i)$ indicate helicity (momenta) and $\ell = \{e, \mu, \tau\}$. If the doubly charged Higgs mass is very high, the effective Lagrangian responsible for the doubly charged Higgs mediated process $\mu^+\mu^+ \rightarrow \ell^+\ell^+$ can be described as

$$\mathcal{L}_{\text{eff}} = \frac{(Y_\Delta^\dagger Y_\Delta)_{\mu\ell}}{2m_{H^{\pm\pm}}^2} (\bar{\mu}\gamma^\mu P_L \mu) (\bar{\ell}\gamma_\mu P_L \ell). \quad (27)$$

When the final state is $\ell^+\ell^+ = \mu^+\mu^+$, there will be additional contribution from SM interaction; see Fig. 9. The squared helicity amplitudes $|\mathcal{M}_{\{\sigma_i\}}|^2 = |\mathcal{M}(\sigma_1\sigma_2\sigma_3\sigma_4)|^2$ for the $\mu^+\mu^+ \rightarrow \mu^+\mu^+$ process are calculated as

$$|\mathcal{M}(+-+)|^2 = |\mathcal{M}(-+-)|^2 = s^2(1 + \cos\theta)^2 \left[\frac{e^2}{t} + \frac{g_L g_R}{t_Z} \right]^2, \quad (28)$$

$$|\mathcal{M}(+-+)|^2 = |\mathcal{M}(-+-)|^2 = s^2(1 - \cos\theta)^2 \left[\frac{e^2}{u} + \frac{g_L g_R}{u_Z} \right]^2, \quad (29)$$

$$|\mathcal{M}(++++)|^2 = 4s^2 \left[e^2 \left(\frac{1}{t} + \frac{1}{u} \right) + g_R^2 \left(\frac{1}{t_Z} + \frac{1}{u_Z} \right) \right]^2, \quad (30)$$

$$|\mathcal{M}(----)|^2 = 4s^2 \left[e^2 \left(\frac{1}{t} + \frac{1}{u} \right) + g_L^2 \left(\frac{1}{t_Z} + \frac{1}{u_Z} \right) - \frac{(Y_\Delta^\dagger Y_\Delta)_{\mu\mu}}{m_{H^{\pm\pm}}^2} \right]^2, \quad (31)$$

where $g_L = e(-1/2 + s_W^2)/(s_W c_W)$, $g_R = es_W/c_W$, $t = -s(1 - \cos\theta)/2$, $u = -s(1 + \cos\theta)/2$, $u_Z = u - m_Z^2 + im_Z\Gamma_Z$, $t_Z = t - m_Z^2 + im_Z\Gamma_Z$, and $\cos\theta$ is the scattering polar angle. For the $\mu^+\mu^+ \rightarrow e^+e^+(\tau^+\tau^+)$ process, we do

not have the SM contribution, and the amplitude is only

$$|\mathcal{M}'(----)|^2 = 4s^2 \left[\frac{(Y_\Delta^\dagger Y_\Delta)_{\mu e(\tau)}}{m_{H^{\pm\pm}}^2} \right]^2. \quad (32)$$

Here, we write the differential cross section for the purely polarized initial state as

$$\frac{d\sigma_{\sigma_1\sigma_2}}{d\cos\theta} = \frac{1}{32\pi s} \sum_{\sigma_3, \sigma_4} |\mathcal{M}_{\{\sigma_i\}}|^2. \quad (33)$$

For a realistic case, we consider the partially polarized initial state with the degree of polarization P_{μ^+} for the μ^+ beams, and the polarized cross section is given as

$$\begin{aligned} \frac{d\sigma(P_{\mu^+}, P_{\mu^+})}{d\cos\theta} &= \frac{1 + P_{\mu^+}}{2} \frac{1 + P_{\mu^+}}{2} \frac{d\sigma_{++}}{d\cos\theta} \\ &+ \frac{1 + P_{\mu^+}}{2} \frac{1 - P_{\mu^+}}{2} \frac{d\sigma_{+-}}{d\cos\theta} \\ &+ \frac{1 - P_{\mu^+}}{2} \frac{1 + P_{\mu^+}}{2} \frac{d\sigma_{-+}}{d\cos\theta} \\ &+ \frac{1 - P_{\mu^+}}{2} \frac{1 - P_{\mu^+}}{2} \frac{d\sigma_{--}}{d\cos\theta}. \end{aligned} \quad (34)$$

For μ TRISTAN, we consider the following right-handed and left-handed cases:

$$\frac{d\sigma_R}{d\cos\theta} = \frac{d\sigma(P_{\mu^+} = -0.8, P_{\mu^+} = -0.8)}{d\cos\theta}, \quad (35)$$

$$\frac{d\sigma_L}{d\cos\theta} = \frac{d\sigma(P_{\mu^+} = 0.8, P_{\mu^+} = 0.8)}{d\cos\theta}. \quad (36)$$

To study the sensitivity to the doubly charged scalar effects on the $\mu^+\mu^+ \rightarrow \mu^+\mu^+$ scattering process, we consider the left-right asymmetry defined by

$$A_{LR} = \frac{N_R - N_L}{N_R + N_L}, \quad (37)$$

where

$$N_{L(R)} = \mathcal{L} \cdot \int_{-0.95}^{0.95} d\cos\theta \frac{d\sigma_{L(R)}}{d\cos\theta}, \quad (38)$$

where $\mathcal{L} = 1 \text{ ab}^{-1}$ is the luminosity we consider in this analysis. To estimate total cross sections, we integrate over the range $-0.95 \leq \cos\theta \leq 0.95$ of the scattering angle. Note that the sensitivity to new physics decreases when we integrate over the range $|\cos\theta| \leq 1$, since the photon exchanging contribution is large around $|\cos\theta| \simeq 1$. Here, we adopt the range taken from the previous study in Ref. [134]. We also consider a statistical error of the asymmetry given by

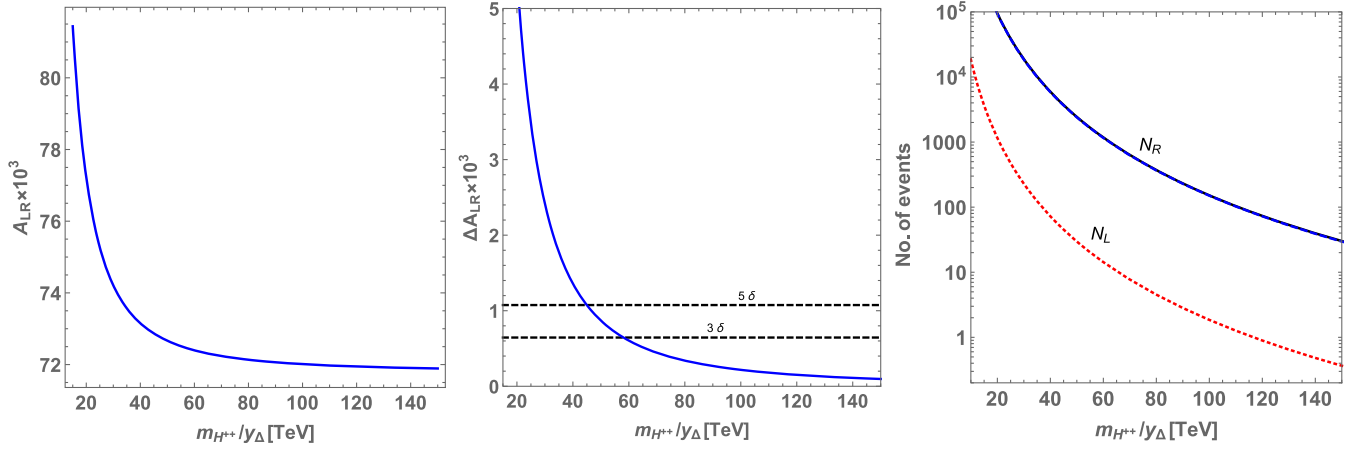


FIG. 12. Left-right asymmetry (leftmost panel) and the deviation (middle panel) from the SM at 3σ (lower black dashed line) and 5σ (upper black dashed line) significance as a function of $m_{H^{++}}/Y_\Delta$ for $\mu^+\mu^+ \rightarrow \mu^+\mu^+$ process in the μ TRISTAN experiment at $\sqrt{s} = 2$ TeV at 1 ab^{-1} luminosity. In the rightmost panel, we show total number of events for e^+e^+ and $\tau^+\tau^+$ final states for left (N_L) and right (N_R) polarizations, represented by red dotted and blue-black solid lines.

$$\delta A_{LR} = \sqrt{\frac{1 - A_{LR}^2}{N_R + N_L}}. \quad (39)$$

The sensitivity to the new physics scenario is estimated at some confidence level ($x\sigma$) by requiring

$$|A_{LR}(\text{SM} + \delta^{++}) - A_{LR}(\text{SM})| \geq x \times \delta A_{LR}(\text{SM}), \quad (40)$$

where $A_{LR}(\text{SM})[\delta A_{LR}(\text{SM})]$ can be obtained by considering the limit $m_{H^{++}} \rightarrow \infty$. For the $\mu^+\mu^+ \rightarrow e^+e^+(\tau^+\tau^+)$ process, we simply consider the number of events $N_{L,R}$, since there is no SM contribution. The corresponding results are shown in the left and middle panels of Fig. 12, where we estimate left-right asymmetry (A_{LR}) for the $\mu^+\mu^+ \rightarrow \mu^+\mu^+$ process and its corresponding deviation (ΔA_{LR}) from the SM as a function of $m_{H^{++}}/Y_\Delta$. We find that the deviation could be probed by the μ TRISTAN collider at 3σ (upper black dashed line) and 5σ (lower black dashed line) levels with $\sqrt{s} = 2$ TeV and $\mathcal{L} = 1 \text{ ab}^{-1}$ for $m_{H^{++}}/Y_\Delta > 45$ TeV. Note that we here assume no systematic uncertainty. In fact, we find $\Delta A_{LR}/A_{LR} \simeq 0.003$ at $m_{H^{++}}/Y_\Delta = 50$ TeV, and roughly speaking, systematic uncertainty would need to be less than the 0.3% level to obtain sensitivity to new physics at the point. It can be feasible as the systematic uncertainty is

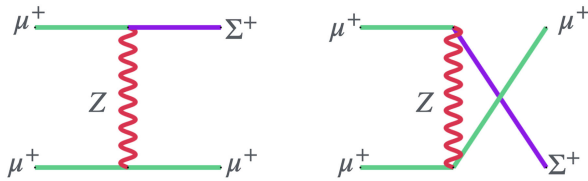


FIG. 13. Production of Σ^+ in t -channel (left) and u -channel (right) process in association with μ^+ in μ TRISTAN followed by the decay $\Sigma^+ \rightarrow \mu^+Z$ and $Z \rightarrow jj$ in a $\mu^+\mu^+jj$ final state.

considered to be at the 0.2% level for leptonic signals at the ILC [135]. In the rightmost panel of Fig. 12, we show the number of events for left (N_L) and right (N_R) polarizations represented by red dotted and blue-black solid lines obtained from e^+e^+ and $\tau^+\tau^+$ final states as a function of $m_{H^{++}}/Y_\Delta$, where there is no SM background. We find that $N_R > N_L$ by 2 orders of magnitude throughout the range of $m_{H^{++}}/Y_\Delta$ under consideration. In the μ TRISTAN experiment, we could probe a high scale of triplet mass though sizable left-right asymmetry for different same-sign dilepton modes and deviation of left-right asymmetry from the SM scenario for the e^+e^+ final state.

C. Triplet fermion search

At the $\mu^+\mu^+$ collider of the μ TRISTAN experiment, Σ^+ production can be produced from the t - and u - channel

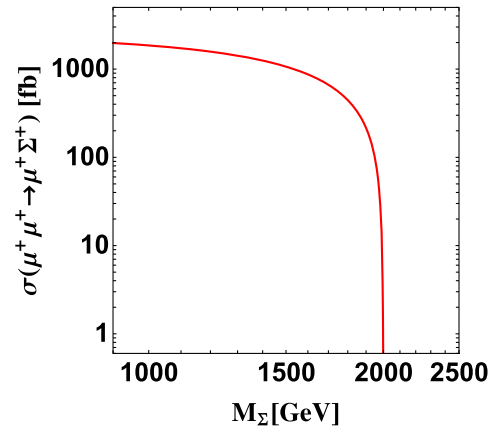


FIG. 14. Σ^+ production cross section as a function of triplet fermion mass in the μ TRISTAN experiment at $\sqrt{s} = 2$ TeV center of mass energy normalized by the light-heavy mixing squared.

processes following $\mu^+\mu^+ \rightarrow \Sigma^+\mu^+$ mediated by Z bosons, as shown in Fig. 13. The differential scattering cross section of $\mu^+\mu^+ \rightarrow \Sigma^+\mu^+$ process, including the effect of interference, can be given by

$$\frac{d\sigma(\mu^+\mu^+ \rightarrow \Sigma^+\mu^+)}{d\cos\theta} = (3.89 \times 10^8 \times 10^3 \text{ fb}) \times \frac{s - M_\Sigma^2}{32\pi s^2} (|M_1|^2 + 2\text{Re}(M_1 M_2^*) + |M_2|^2), \quad (41)$$

$$|M_{1,2}|^2 = \frac{g^4}{2c_W^4} \frac{(-\frac{1}{2} + s_W^2)^2}{(\frac{1}{2}(s - M_\Sigma^2)(1 \mp \cos\theta + m_Z^2) + \Gamma_Z^2 m_Z^2)} \quad (42)$$

$$2\text{Re}(M_1 M_2^*) = -\frac{g^4}{16c_W^4} \frac{s}{2} \left(-\frac{1}{2} + s_W^2\right)^2 (4s_W^4 + 4s_W^2 + 1)(s - M_\Sigma^2) \times \frac{(\frac{1}{2}(s - M_\Sigma^2)(1 - \cos\theta) + m_Z^2)(\frac{1}{2}(s - M_\Sigma^2)(1 + \cos\theta) + m_Z^2) + \Gamma_Z^2 m_Z^2}{((\frac{1}{2}(s - M_\Sigma^2)(1 - \cos\theta) + m_Z^2)^2 + \Gamma_Z^2 m_Z^2)((\frac{1}{2}(s - M_\Sigma^2)(1 + \cos\theta) + m_Z^2)^2 + \Gamma_Z^2 m_Z^2)} \quad (43)$$

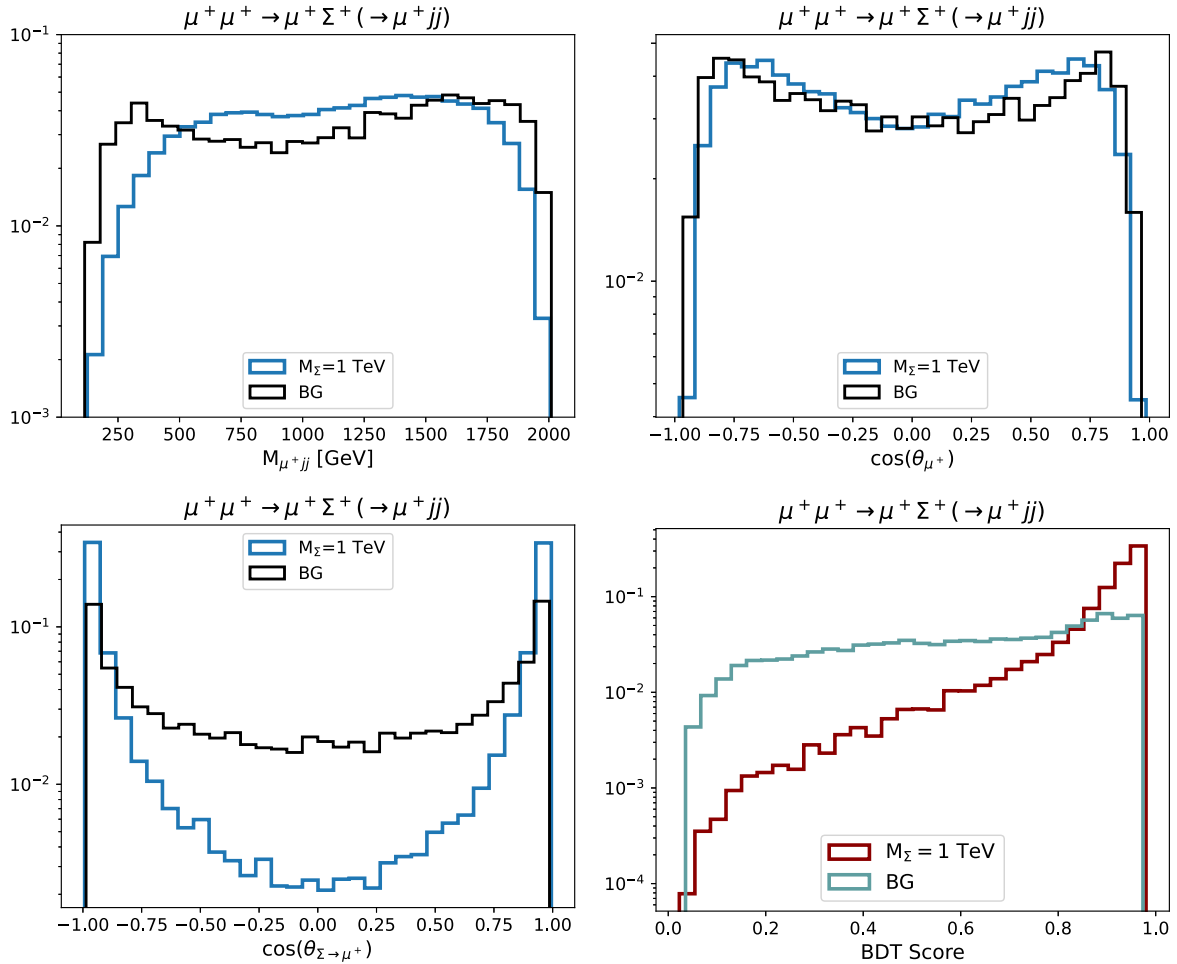


FIG. 15. Distributions of some kinematic variables as well as the BDT score for the signal ($\mu^+\mu^+ \rightarrow \mu^+\Sigma^+ (\rightarrow Z\mu^+)$) and background.

The total production cross section as a function of the triplet mass M_Σ is shown in Fig. 14. After considering $\text{BR}(\Sigma^+ \rightarrow \mu^+ Z) = 25\%$ and $\text{BR}(Z \rightarrow jj) = 69.9\%$, the cross section becomes 324 fb being normalized by the square of the light-heavy mixing before cuts for $\mu^+ \mu^+ jj$ final state when $M_\Sigma = 1$ TeV. However, as $m_\Sigma \gg m_Z + m_\mu$, the Z boson is highly boosted. As a result, only the Z jet can be reconstructed at the detector. The cross section of the background process $\mu^+ \mu^+ \rightarrow \mu^+ \mu^+ jj$ is 0.0341 pb. To enhance the signal-background separation, we employ a boosted decision tree algorithm trained on the following kinematic observables:

$$\begin{aligned} & p^\mu(\mu^+), p^\mu(\mu_{\Sigma \rightarrow \mu}), p^\mu(j_Z), m_\Sigma, \\ & \cos(\theta(\mu^+)), \cos(\theta(\mu_{\Sigma \rightarrow \mu})), \end{aligned} \quad (44)$$

where p^μ denotes the four-momentum and $\cos(\theta)$ specifies the polar angle (measured in the lab frame) of the corresponding particle. The reconstructed mass m_Σ is derived from the vector sum of momenta of: (i) the boosted jet j_Z (identified as originating from a hadronically decaying Z boson in signal processes), and (ii) the muon $\mu_{\Sigma \rightarrow \mu}$, exhibiting minimal angular separation $\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$ relative to j_Z . In Fig. 15, we present the distributions of some kinematic variables as well as the BDT score for our signal and background. Due to the relatively large width of the Σ and ambiguity in combining the μ and the jet, the discrimination of signal and background is mild. Optimizing the BDT cut to maximize the signal significance, we obtain the signal and background after the cut to be 29.9 and 0.01633 fb, respectively. Considering the light-heavy mixing squared from EWPD [94,95] as $|V_{\mu\Sigma}|^2 = 0.000289$, we estimate significance (σ) as a function of luminosity using Eq. (23) and show the significance as a function of luminosity in Fig. 16. We find that to study the $\mu^+ \mu^+ jj$ final state for the Σ^+ induced final state, a 2σ significance

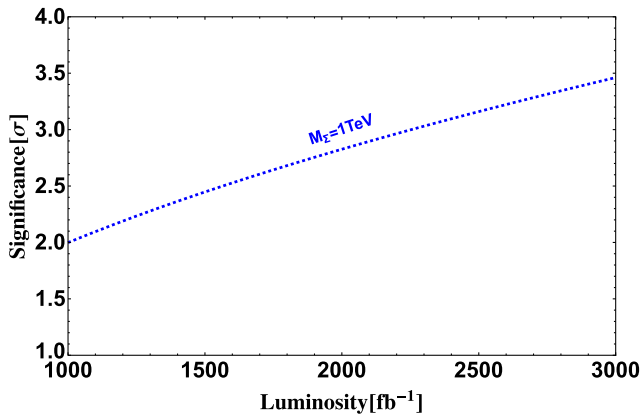


FIG. 16. Significance of Σ^+ production with respect to luminosity in the $\mu^+ \mu^+ jj$ final state in the $\mu^+ \mu^+$ collider of the μ TRISTAN experiment at $\sqrt{s} = 2$ TeV considering $M_\Sigma = 1$ TeV.

could be obtained around 1000 fb^{-1} luminosity, and could reach around 3σ with a luminosity of 2500 fb^{-1} when $M_\Sigma = 1$ TeV.

IV. CONCLUSIONS

We study different tree level seesaw scenarios, namely, TeV scale type I, type II, and type III seesaw models at $\mu^+ e^-$ ($\sqrt{s} = 346$ GeV) and $\mu^+ \mu^+$ ($\sqrt{s} = 2$ TeV) collider in the proposed μ TRISTAN experiment. In the case of the $\mu^+ e^-$ collider, we study the production of heavy RHNs in association with light neutrinos. These heavy neutrinos then decay through the mode of a charged lepton and jets, providing a single lepton, missing momentum, and jets. Generating events for the signal and SM backgrounds, we estimate prospective limits on the light-heavy mixing angles for 1 ab^{-1} luminosity and 2σ significance. We find that mixing $|V_{eN}|^2$ could be probed between $4.2 \times 10^{-6} \leq |V_{eN}|^2 \leq 10^{-5}$ for $93 \text{ GeV} \leq M_N \leq 295 \text{ GeV}$ which is 2 orders of magnitude stronger than EWPD limits. Similarly, the mixing $|V_{\mu N}|^2$ could be probed between $2.0 \times 10^{-6} < |V_{\mu N}|^2 \leq 10^{-5}$ for $88 \text{ GeV} \leq M_N \leq 304 \text{ GeV}$, which is also nearly 2 orders of magnitude stronger than the limits obtained from EWPD. We could also provide a prospective limit on $|V_{eN}^* V_{\mu N}|$ around $\mathcal{O}(10^{-5})$ being stronger than existing bounds from the LHC for $50 \text{ GeV} \leq M_N \leq 300 \text{ GeV}$. In addition, we have studied same-sign dilepton production from the doubly charged scalar-induced interactions from the type II seesaw scenario. We found that such a scenario depends on neutrino oscillation data and hence on neutrino mass hierarchy. More specifically, we find that a two-positron final state could provide nearly 2 orders of magnitude more events if inverted hierarchy of the neutrino mass is considered, compared to the normal hierarchy scenario, whereas the dilepton final state with different flavor cannot distinguish between the neutrino mass hierarchies. We also study production modes such as of $\mu^+ \mu^+ \rightarrow H^{++} \gamma / Z \rightarrow e^+ e^+ \gamma$ or $e^+ e^+ jj$. The cross section for this process after all kinematic cuts could make a nearly 2 orders of magnitude difference between normal and inverted ordering of the neutrino mass hierarchy. We find that generic SM background events could be negligibly small after the application of kinematic cuts, allowing for 5σ significance of these signals for different neutrino mass hierarchies in future. We study the left-right asymmetry from the same-sign dilepton mode of mediating the doubly charged scalar boson. We find that the positively charged dimuon final state could significantly deviate from the SM results, and the corresponding deviations could be probed in the μ TRISTAN experiment with 3σ to 5σ significances in future. Finally, we study the $\mu^+ \mu^+ jj$ final state from the production of singly charged multiplet of the triplet fermion in association with a muon for $M_\Sigma = 1$ TeV. This final state could be observed at 2σ significance with a luminosity of 1000 fb^{-1} and the

significance could reach up to a significance of 3σ with a luminosity of 2500 fb^{-1} , which could be tested in the $\mu\text{TRISTAN}$ experiment in future.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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