

Next-to-next-to-leading order QCD $\otimes$ QED corrections to unpolarized and polarized semi-inclusive deep-inelastic scattering structure functions

Saurav Goyal^{1,2,*}, Roman N. Lee^{3,†}, Sven-Olaf Moch^{4,‡}, Vaibhav Pathak^{1,2,§} and V. Ravindran^{1,2,||}

¹*The Institute of Mathematical Sciences, Taramani, 600113 Chennai, India*

²*Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400094, India*

³*Budker Institute of Nuclear Physics, 630090, Novosibirsk, Russia*

⁴*II. Institute for Theoretical Physics, Hamburg University, D-22761 Hamburg, Germany*



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We present the first computation of next-to-next-to-leading order (NNLO) pure QED and mixed QCD $\otimes$ QED corrections to unpolarized and polarized semi-inclusive deep-inelastic scattering (SIDIS). Building on our previous NNLO QCD results, these corrections are crucial for improving the theoretical precision. The coefficient functions are derived within the QCD factorization framework using dimensional regularization, with consistent renormalization and mass factorization. A detailed phenomenological analysis shows that the NNLO QED and QCD $\otimes$ QED terms enhance perturbative stability and reduce scale uncertainties. These results are essential for high-precision SIDIS predictions at future facilities such as the Electron-Ion Collider.

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Semi-inclusive deep-inelastic scattering (SIDIS), characterized by the detection of a final-state hadron along with the scattered lepton, serves as a powerful tool for probing the internal structure of both incoming as well as outgoing hadrons. Unlike inclusive DIS, SIDIS provides access to both parton distribution functions (PDFs) and fragmentation functions (FFs), offering a more detailed picture of the nucleon's partonic structure and an opportunity for extracting FFs. QCD factorization separates the perturbatively calculable coefficient functions (CFs) from the nonperturbative PDFs and FFs and allows for the SIDIS cross section to be expressed as a convolution of these ingredients, PDFs, FFs, and CFs. The CFs are computed order-by-order in perturbation theory and are known up to next-to-next-to-next-to-leading order (N³LO) for inclusive DIS [1–4], while for SIDIS, the CFs at next-to-next-to-leading order (NNLO) in QCD have been completed recently [5–11]. More recently, the impact of threshold enhanced contributions to SIDIS using resummed framework has been reported in [12], see also [13,14]. Building

upon these developments, we present the first results on the impact of NNLO QED and mixed QCD $\otimes$ QED contributions on unpolarized and polarized SIDIS. These contributions are essential for enhancing the precision of theoretical predictions, reducing uncertainties resulting from renormalization and factorization scales, and are particularly relevant for upcoming high-precision experiments such as the Electron-Ion Collider (EIC) [15] at Brookhaven National Laboratory.

The reaction $l(k_l) + H(P) \rightarrow l(k'_l) + H'(P_H) + X$ defines the SIDIS process, where $k_l, k'_l (P, P_H)$ are the momenta of the incoming and outgoing leptons (hadrons).

In the single-photon exchange approximation the virtuality of the photon is given by $Q^2 = -q^2$, with $q = k_l - k'_l$. For unpolarized (polarized) SIDIS, the hadronic cross section is described by the structure functions (SFs) $F_{1,2,3} (g_{1,\dots,5})$. The dominant quantum corrections to the CFs of F_1, F_2 and g_1 result from QCD and they have been extensively studied up to NNLO, see [13,14,16–20]. The SFs of SIDIS also receive contributions from electroweak interactions via vector bosons such as $Z, W^\pm$, as well as from additional photons, with the simplest corrections arising from photons—namely, quantum electrodynamic (QED) effects. There have been several studies where QED and mixed QCD $\otimes$ QED corrections are considered for observables in DIS, see [21–29]. QED radiative corrections to the leptonic part largely factorize, a fact exploited in analytic resummations [30] and in formalisms based on lepton distribution functions [31,32]. At the HERA collider, these effects were systematically treated in the experimental analysis through dedicated Monte-Carlo generators for

*Contact author: sauravg@imsc.res.in

†Contact author: r.n.lee@inp.nsk.su

‡Contact author: sven-olaf.moch@desy.de

§Contact author: vaibhavp@imsc.res.in

||Contact author: ravindra@imsc.res.in

QED radiation [33–35]. Nonfactorizable contributions such as two-photon exchange and leptonic-hadronic interference are known to be numerically negligible in DIS (well below 0.1% [36]) and lie beyond the accuracy relevant for a factorized parton-level calculation. In this article, we incorporate QED as well as mixed QCD $\otimes$ QED corrections to F_1 , F_2 and g_1 at NNLO accuracy within the established single-photon exchange framework used in global PDF fits and perturbative QCD analyses.

The perturbative expansion of the SIDIS cross section in both the strong and electromagnetic couplings is given by

$$\sigma = \sigma^{(0,0)} + a_s \sigma^{(1,0)} + a_e \sigma^{(0,1)} + a_s^2 \sigma^{(2,0)} + a_e^2 \sigma^{(0,2)} + a_s a_e \sigma^{(1,1)} + \dots \quad (1)$$

with $a_s = \alpha_s/(4\pi) = g_s^2/(16\pi^2)$ and $a_e = \alpha_e^2/4\pi = e^2/(16\pi^2)$ in terms of strong coupling g_s and electric charge e . The cross sections $\sigma^{(m,n)}$ denote the contributions at order $\mathcal{O}(\alpha_s^m, \alpha_e^n)$. In this work, we focus on the pure QED corrections at $\mathcal{O}(\alpha_e^2)$ as well as the mixed QCD $\otimes$ QED contributions at $\mathcal{O}(\alpha_s \alpha_e)$ to the SFs F_1 , F_2 and g_1 .

The differential SIDIS cross section,

$$\frac{d^3\sigma}{dx dy dz} = \frac{4\pi\alpha_e^2}{Q^2} \left[y F_1(x, z, Q^2) + \frac{(1-y)}{y} F_2(x, z, Q^2) \right], \quad (2)$$

depends on the spacelike momentum transfer Q^2 , the Bjorken variable $x = Q^2/(2P \cdot q)$, the inelasticity $y = P \cdot q/P \cdot k_l$, and the scaling variable $z = P \cdot P_H/P \cdot q$ for the fraction of the initial energy transferred to the final-state hadron. Since we restrict ourselves to photon mediated SIDIS, the SF F_3 does not contribute. Similarly, the spin-dependent cross section is found to be

$$\frac{d^3\Delta\sigma}{dx dy dz} = \frac{4\pi\alpha_e^2}{Q^2} (2-y) g_1(x, z, Q^2). \quad (3)$$

Note that g_2 does not contribute since we restrict ourselves to longitudinally polarized hadron in the initial state.

Collinear factorization in the QCD improved parton model allows to express the SFs as convolution of PDFs, $(\Delta)f_{a/P}(x_1, \mu_F^2)$ and FFs, $D_{H'/b}(z_1, \mu_F^2)$ with CFs, $(\Delta)\mathcal{C}_{I,ab}(x/x_1, z/z_1, Q^2, \mu_F^2)$

$$F_I = x^{I-1} \sum_{a,b=q,\bar{q},g,\gamma} \int_x^1 \frac{dx_1}{x_1} f_{a/P}(x_1, \mu_F^2) \times \int_z^1 \frac{dz_1}{z_1} D_{H'/b}(z_1, \mu_F^2) \mathcal{C}_{I,ab}\left(\frac{x}{x_1}, \frac{z}{z_1}, Q^2, \mu_F^2\right). \quad (4)$$

The corresponding expression for g_1 will involve polarized PDFs $\Delta f_{a/P}$ and spin-dependent CFs denoted by $\Delta\mathcal{C}_{I,ab}$ (we set $I = 1$). Here x_1 is the incoming parton's momentum

fraction with respect to the hadron $H(P)$, i.e., $p_a = x_1 P$ and z_1 is the momentum fraction of the final state parton b carried away by the outgoing hadron $H'(P_H)$, i.e., $P_H = z_1 P_b$. The above expression is subject to summation over the initial state partons along with the photon, collectively denoted by a , from the incoming hadron and the final state partons that include photons, denoted by b that fragment into the observed hadron. The CFs can be computed in a perturbative expansion in powers of a_s and a_e , cf. Eq. (1),

$$(\Delta)\mathcal{C}_{I,ab} = \sum_{i,j=0}^{\infty} a_s^i(\mu_R^2) a_e^j(\mu_R^2) (\Delta)\mathcal{C}_{I,ab}^{(i,j)}(\mu_R^2), \quad (5)$$

with μ_F , μ_R the factorization and renormalization scales. The computation of the CFs starts from the parton level cross sections denoted by $d(\Delta)\hat{\sigma}_{I,ab}$,

$$d(\Delta)\hat{\sigma}_{I,ab} = \frac{1}{4\pi} \int d\text{PS}_{X+b} \bar{\Sigma} [(\Delta)\overline{M_{ab}}]^2 \delta\left(\frac{z}{z_1} - \frac{P_a \cdot P_b}{P_a \cdot q}\right), \quad (6)$$

where the squared amplitude is defined as $[(\Delta)\overline{M_{ab}}]^2 = (\Delta)\mathcal{P}_I^{\mu\nu} |(\Delta)M_{ab}|_{\mu\nu}^2$ with suitable projectors $(\Delta)\mathcal{P}_I^{\mu\nu}$ for the SF under consideration [37]. The matrix elements M_{ab} denote scattering amplitudes for the parton level subprocess $a(p_a) + \gamma^*(q) \rightarrow b(p_b) + X$ with QCD, QED, and mixed QCD $\otimes$ QED corrections taken into account. For spin-independent squared amplitudes, $\bar{\Sigma}$ represents the average over initial and the sum over final spins/polarizations, and color quantum numbers for partons. For spin-dependent cross sections, instead of averaging over the initial polarizations, we take their difference to account for the parton spin dependence. The integration measure $d\text{PS}_{X+b}$ is the phase space for the fragmenting parton b and the remaining final-state particles X .

The spin-dependent squared amplitudes, $[(\Delta)\overline{M_{ab}}]^2$, in Eq. (6), involve either the Dirac matrix γ_5 or the Levi-Civita tensor $\epsilon_{\mu\nu\rho\sigma}$, arising from helicity-dependent quark (or antiquark) wave functions or gluon(photon) polarization states, respectively; see, e.g., [38]. Since both γ_5 and $\epsilon_{\mu\nu\rho\sigma}$ are intrinsically four-dimensional objects, a consistent prescription is required to define them in dimensional regularization with $d = 4 + \varepsilon$ dimensions.

Several schemes exist for this purpose, though none fully preserves the chiral Ward identities. In this work, we adopt Larin's scheme [39] to define γ_5 in d -dimensions as

$$\not{p}_a \gamma_5 = -\frac{i}{6} \epsilon_{\mu\nu\sigma\lambda} p_a^\mu \gamma^\nu \gamma^\sigma \gamma^\lambda, \quad (7)$$

which provides a consistent framework for handling γ_5 within dimensional regularization, where the product of two Levi-Civita tensors is expressed as a determinant of

Kronecker deltas in d -dimensions. The QCD contributions computed in Larin's scheme can be translated into the $\overline{\text{MS}}$ scheme by applying a finite renormalization. The corresponding renormalization constant [40–43] restores the axial Ward identity and simultaneously transforms the spin-dependent PDFs into the $\overline{\text{MS}}$ scheme, see also [11,37] for a detailed discussion in SIDIS. Currently, no analogous renormalization scheme exists for converting the QED and mixed QCD $\otimes$ QED contributions from Larin's scheme to the $\overline{\text{MS}}$ scheme. However, using Abelianization we determine the finite renormalization constants for QED and mixed QCD $\otimes$ QED from those of QCD. The results for $\Delta\mathcal{C}_{1,ab}$ upto NNLO in the $\overline{\text{MS}}$ -scheme, thus obtained are included in the ancillary file.

Beyond leading order (LO) in perturbation theory, both ultraviolet (UV) and infrared (IR) divergences from soft and collinear partons appear in the computation of $d(\Delta)\hat{\sigma}_{I,ab}$. The UV divergences are removed by renormalization of the bare couplings $\hat{a}_s$ and $\hat{a}_e$ at the scale μ_R . Let Z_{a_c} denote the renormalization factor for the coupling a_c , where $c = s, e$ corresponds to QCD and QED. The bare couplings $\hat{a}_c$ are related to the renormalized ones as

$$\hat{a}_c S_\epsilon = a_c(\mu_R^2) Z_{a_c}(a_s(\mu_R^2), a_e(\mu_R^2), \epsilon) \left(\frac{\mu^2}{\mu_R^2} \right)^{\epsilon/2}. \quad (8)$$

Here μ is an arbitrary mass scale introduced to make $\hat{a}_c$ dimensionless in d -dimension and $S_\epsilon = \exp[\frac{\epsilon}{2}(\gamma_E - \ln 4\pi)]$ with γ_E the Euler-Mascheroni constant. The Z_{a_c} factors satisfy renormalization group equations given by

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z_{a_c} = - \frac{1}{a_c(\mu_R^2)} \beta_{a_c}(a_s(\mu_R^2), a_e(\mu_R^2)), \quad (9)$$

where the β functions admit expansions $\beta_{a_s} = -\sum_{i,j=0}^{\infty} \beta_{ij}^{(s)} a_s^{i+2} a_e^j$ and $\beta_{a_e} = -\sum_{i,j=0}^{\infty} \beta_{ij}^{(e)} a_e^{j+2} a_s^i$, with the coefficients $\beta_{ij}^{(s)}$ and $\beta_{ij}^{(e)}$ accounting for contributions from pure QCD, pure QED, and mixed QCD $\otimes$ QED interactions. To NNLO accuracy the relevant β -function coefficients read $\beta_{00}^{(s)} = \frac{11}{3} C_a - \frac{4}{3} T_f n_f$ and $\beta_{00}^{(e)} = -\frac{4}{3} (N \sum_q^{n_f} e_q^2 + \sum_l^{n_L} e_l^2)$. Here, $C_a = N$ is the quadratic Casimir in the adjoint representation of the color SU(N) and the one for the fundamental representation is denoted by $C_f = \frac{N^2-1}{2N}$. The trace normalization factor for the fundamental representation is $T_f = \frac{1}{2}$ and n_f, n_L are the number of active quark and lepton flavors respectively. e_q and e_l denote the electric charges of the quark q and the lepton l , respectively.

The soft IR divergences cancel among virtual and real emission processes when all the degenerate states are summed thanks to the Kinoshita-Lee-Nauenberg theorem [44,45], except for the collinear ones associated with the initial-state and the final-state fragmenting partons. These residual divergences are factorized into (un)polarized PDFs

and FFs following the standard QCD mass factorization procedure. In the QED and QCD $\otimes$ QED framework, this procedure extends to include the photon and charged leptons in the evolution equations. The partonic cross sections in Eq. (6) are factorized into spacelike Altarelli-Parisi (AP) splitting kernels $(\Delta)\Gamma_{c \leftarrow a}$ for (polarized) unpolarized PDFs and timelike unpolarized AP splitting kernels $\tilde{\Gamma}_{b \leftarrow d}$ for FFs, which contain all $1/\epsilon$ collinear singularities, and the finite CFs $(\Delta)\mathcal{C}_{I,cd}$. This yields the mass factorization formula at scale μ_F ,

$$(\Delta)\hat{\mathcal{C}}_{I,ab}(x, z, \epsilon) = (\Delta)\Gamma_{c \leftarrow a}(x, \epsilon) \otimes (\Delta)\mathcal{C}_{I,cd}(x, z, \epsilon) \tilde{\otimes} \tilde{\Gamma}_{b \leftarrow d}(z, \epsilon), \quad (10)$$

where the convolutions $\otimes, \tilde{\otimes}$ run over the respective momentum fractions. For the QCD $\otimes$ QED interactions, the AP kernels have been extended to include pure QED and mixed QCD $\otimes$ QED corrections up to two-loop order as detailed in [21–23,46].

Beyond LO, the contributions to $d(\Delta)\hat{\sigma}_{I,ab}$ in Eq. (6) can be classified into virtual (V) and real (R) corrections at next-to-leading order (NLO) and virtual-virtual (VV), real-virtual (RV), and real-real (RR) corrections at NNLO. The V (VV) contributions arise from one-(two)-loop virtual corrections to the Born process including combined QCD and QED corrections. The R, RV, and RR contributions incorporate all relevant real emission processes and their interference terms involving QCD, QED, and mixed QCD $\otimes$ QED dynamics. Feynman diagrams are generated using QGRAF [47], see Table I and algebraic manipulations, including the application of Feynman rules, Dirac algebra, Lorentz contractions, and color and charge factor simplifications, are performed with in-house routines written in FORM [48,49].

TABLE I. Partonic subprocesses contributing at NLO, and NNLO, classified by contribution type. All the partons in final state of process can hadronize, here q' is a quark of flavor different from q .

Order	Process	Contribution type
NLO	$q + \gamma^* \rightarrow q$ (1-loop)	QCD, QED
	$q + \gamma^* \rightarrow q + g(\gamma)$	QCD (QED)
	$g(\gamma) + \gamma^* \rightarrow q + \bar{q}$	QCD (QED)
NNLO	$q + \gamma^* \rightarrow q$ (2-loop)	QCD, QED, QCD $\otimes$ QED
	$q + \gamma^* \rightarrow q + g(\gamma)$ (1-loop)	QCD, QED, QCD $\otimes$ QED
	$g(\gamma) + \gamma^* \rightarrow q + \bar{q}$ (1-loop)	QCD, QED, QCD $\otimes$ QED
	$q + \gamma^* \rightarrow q + g(\gamma) + g(\gamma)$	QCD (QED)
	$q + \gamma^* \rightarrow q + g + \gamma$	QCD $\otimes$ QED
	$q + \gamma^* \rightarrow q + q + \bar{q}$	QCD, QED, QCD $\otimes$ QED
	$q + \gamma^* \rightarrow q + q' + \bar{q}'$	QCD, QED
	$g(\gamma) + \gamma^* \rightarrow q + \bar{q} + g(\gamma)$	QCD (QED)
	$g(\gamma) + \gamma^* \rightarrow q + \bar{q} + \gamma(g)$	QCD $\otimes$ QED

The evaluation of phase-space integrals here is more involved than for inclusive DIS cross sections due to the kinematic constraint $\frac{z}{z_1} = \frac{p_a \cdot p_b}{p_a \cdot q}$. This constraint is imposed via a delta function replaced by a cut propagator using the reverse unitarity method [50,51], which enables the use of integration-by-parts (IBP) identities [52] to reduce integrals to a minimal set of master integrals (MIs). The IBP reduction is carried out using the LiteRed package [53]. The master integrals (MIs) needed for the pure QED and mixed QCD $\otimes$ QED contributions coincide with those already known from pure QCD computations. Details of the computation have been described in previous work [37,54,55].

After including the MIs results in the partonic cross sections, soft divergences cancel between real and virtual contributions and collinear divergences are removed by mass factorization using space- and timelike AP kernels. We apply Eq. (10) to systematically extract the collinear finite CFs, $(\Delta)\mathcal{C}_{I,ab}$, order by order from the partonic cross sections. The AP kernels can be expanded as a power series in the strong coupling a_s and the electromagnetic coupling a_e as follows:

$$\Gamma_{c \leftarrow d} = \delta_{cd} \delta(1 - \xi) + \sum_{i+j=1}^{\infty} a_s^i(\mu_F^2) a_e^j(\mu_F^2) \Gamma_{c \leftarrow d}^{(i,j)}(\xi, \epsilon), \quad (11)$$

where we abbreviate the AP kernels collectively by the set $\Gamma \in \{\Gamma, \Delta\Gamma, \tilde{\Gamma}\}$. They satisfy renormalization group evolution equations, $\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{c \leftarrow d} = \frac{1}{2} \mathbb{P}_{ck}(a_s(\mu_F^2), a_e(\mu_F^2)) \otimes \Gamma_{k \leftarrow d}$ in terms of splitting functions, denoted by the set $\mathbb{P} \in \{\mathbb{P}, \Delta\mathbb{P}, \tilde{\mathbb{P}}\}$ consisting of unpolarized spacelike ($\mathbb{P}$), polarized spacelike ($\Delta\mathbb{P}$) and unpolarized timelike ($\tilde{\mathbb{P}}$) splitting functions respectively. Their perturbative expansion reads $\mathbb{P}_{ab} = \sum_{i+j=1}^{\infty} a_s^i a_e^j \mathbb{P}_{ab}^{(i,j)}$ where i and j are the QCD and QED indices, respectively. For removal of initial state collinear singularities in the present study we use the QCD spacelike (polarized) unpolarized splitting functions $(\Delta)\mathbb{P}_{ab}^{(i,0)}$ with $i = 1, 2$ [4,43,56–64], the spacelike splitting functions $(\Delta)\mathbb{P}_{ab}^{(0,j)}$ with $j = 1, 2$ for QED and $(\Delta)\mathbb{P}_{ab}^{(1,1)}$ for the mixed QCD $\otimes$ QED, all available in the literature [21–23]. The timelike splitting functions needed to remove final state collinear singularities are known for QCD interactions [63,64]. For QED and the mixed QCD $\otimes$ QED interactions, the corresponding expressions are not

available in the literature. We derive them here by assuming that mass factorization holds with QED interactions and by requiring finite CFs. In detail, we determine the timelike splitting functions $\tilde{\mathbb{P}}_{ab}^{(0,j)}$, $j = 1, 2$ for QED and $\tilde{\mathbb{P}}_{ab}^{(1,1)}$ for mixed QCD $\otimes$ QED:

$$\begin{aligned} \text{LO: } & \tilde{\mathbb{P}}_{qq}^{(0,1)}, \tilde{\mathbb{P}}_{q\gamma}^{(0,1)}, \tilde{\mathbb{P}}_{\gamma q}^{(0,1)}, \tilde{\mathbb{P}}_{\gamma\gamma}^{(0,1)} \\ \text{NLO: } & \tilde{\mathbb{P}}_{qq}^{(0,2)}, \tilde{\mathbb{P}}_{gq}^{(1,1)}, \tilde{\mathbb{P}}_{\gamma q}^{(1,1)}, \tilde{\mathbb{P}}_{qq}^{(0,2),\text{NS}}, \tilde{\mathbb{P}}_{qq}^{(0,2),\text{S}}, \tilde{\mathbb{P}}_{q\bar{q}}^{(0,2)} \end{aligned} \quad (12)$$

The QCD CFs for the polarized SF g_1 are already known [7,8,11]. As mentioned before, we use Larin's prescription for γ_5 , with polarized spacelike splitting functions in Larin's scheme and unpolarized timelike splitting functions in $\overline{\text{MS}}$ to obtain the QED and mixed QCD $\otimes$ QED CFs. The CFs in the $\overline{\text{MS}}$ -scheme are then derived with the help of a finite scheme transformation $Z = 1 + \sum_{i+j=1} a_s^i a_e^j Z^{(i,j)}$. The ones for QCD, $Z^{(i,0)}$ are already known for $i = 1, 2$. The remaining $Z^{(i,j)}$ i.e., $(i, j) = (0, 1), (0, 2), (1, 1)$ relevant for QED and the mixed QCD $\otimes$ QED CFs are obtained from the former through Abelianization, see, e.g., [21]. The set of polarized $\overline{\text{MS}}$ splitting functions given below for QED and mixed QCD $\otimes$ QED evolution, that has been obtained in this way, i.e., using Z for g_1 , is in complete agreement with [23,46].

$$\begin{aligned} \text{LO: } & \Delta\mathbb{P}_{qq}^{(0,1)}, \Delta\mathbb{P}_{q\gamma}^{(0,1)}, \Delta\mathbb{P}_{\gamma q}^{(0,1)}, \Delta\mathbb{P}_{\gamma\gamma}^{(0,1)} \\ \text{NLO: } & \Delta\mathbb{P}_{q\gamma}^{(0,2)}, \Delta\mathbb{P}_{qg}^{(1,1)}, \Delta\mathbb{P}_{\gamma\gamma}^{(1,1)}, \Delta\mathbb{P}_{qq}^{(0,2),\text{NS}}, \\ & \Delta\mathbb{P}_{qq}^{(0,2),\text{S}}, \Delta\mathbb{P}_{q\bar{q}}^{(0,2)} \end{aligned}$$

where the superscripts NS (S) refer to the nonsinglet (singlet) parts of the qq channel, respectively. This completes the NNLO derivation of the CFs via Eq. (10); as in the known QCD case, the finite CFs can be expressed in terms of double and single distributions, as well as regular terms, cf. [37] for computational details. The convolution of the CFs with PDFs and FFs provides $(g_1)F_1 = \sum_{i+j=0} a_s^i a_e^j (g_1)F_1^{(i,j)}$, such that at LO $F_1^{(0,0)} = \sum_q e_q^2 (\Delta)H_{qq}$ (e_q being the electric charge of quark q). Since we have already presented the QCD results for F_i and g_1 , that is, $(g_1)F_i^{(i,0)}$ for $i = 1, 2$ in [37], in the following we give results for pure QED and mixed QCD $\otimes$ QED:

$$(g_1)F_1^{(0,1)} = \sum_q e_q^4 \left((\Delta)H_{qq} \hat{\otimes} (\Delta)\mathcal{C}_{1,q}^{(0,1)} + (\Delta)H_{q\gamma} \hat{\otimes} (\Delta)\mathcal{C}_{1,q\gamma}^{(0,1)} + (\Delta)H_{\gamma q} \hat{\otimes} (\Delta)\mathcal{C}_{1,\gamma q}^{(0,1)} \right), \quad (13)$$

$$\begin{aligned}
(g_1)F_1^{(0,2)} = & \sum_q e_q^6 \left((\Delta)H_{qq} \hat{\otimes} (\Delta)C_{1,qq,[1]}^{(0,2)} + (\Delta)H_{q\bar{q}} \hat{\otimes} (\Delta)C_{1,q\bar{q}}^{(0,2)} + (\Delta)H_{q\gamma} \hat{\otimes} (\Delta)C_{1,q\gamma}^{(0,2)} + (\Delta)H_{\gamma q} \hat{\otimes} (\Delta)C_{1,\gamma q}^{(0,2)} \right) \\
& + \sum_q e_q^4 \left(N \sum_{q_i} e_{q_i}^2 + \sum_{l_i} e_{l_i}^2 \right) (\Delta)H_{qq} \hat{\otimes} (\Delta)C_{1,qq,[2]}^{(0,2)} + \sum_q e_q^2 \left(N \sum_{q_i} e_{q_i}^4 + \sum_{l_i} e_{l_i}^4 \right) (\Delta)H_{qq} \hat{\otimes} (\Delta)C_{1,qq,[3]}^{(0,2)} \\
& + \sum_q \sum_{q' \neq q} \left(e_q^4 e_{q'}^2 (\Delta)H_{qq'}^+ \hat{\otimes} (\Delta)C_{1,qq',[1]}^{(0,2)} + e_q^2 e_{q'}^4 (\Delta)H_{qq'}^+ \hat{\otimes} (\Delta)C_{1,qq',[2]}^{(0,2)} + e_q^3 e_{q'}^3 (\Delta)H_{qq'}^- \hat{\otimes} (\Delta)C_{1,qq',[3]}^{(0,2)} \right) \\
& + \left(N \sum_{q_i} e_{q_i}^6 + \sum_{l_i} e_{l_i}^6 \right) \left((\Delta)H_{\gamma\gamma} \hat{\otimes} (\Delta)C_{1,\gamma\gamma}^{(0,2)} \right), \tag{14}
\end{aligned}$$

$$\begin{aligned}
(g_1)F_1^{(1,1)} = & \sum_q e_q^4 \left((\Delta)H_{qq} \hat{\otimes} (\Delta)C_{1,qq}^{(1,1)} + (\Delta)H_{q\bar{q}} \hat{\otimes} (\Delta)C_{1,q\bar{q}}^{(1,1)} + (\Delta)H_{qg} \hat{\otimes} (\Delta)C_{1,qg}^{(1,1)} + (\Delta)H_{gq} \hat{\otimes} (\Delta)C_{1,gq}^{(1,1)} \right. \\
& \left. + (\Delta)H_{\gamma\gamma} \hat{\otimes} (\Delta)C_{1,\gamma\gamma}^{(1,1)} + (\Delta)H_{\gamma q} \hat{\otimes} (\Delta)C_{1,\gamma q}^{(1,1)} \right) + \left(\sum_{q_i} e_{q_i}^4 \right) \left((\Delta)H_{g\gamma} \hat{\otimes} (\Delta)C_{1,g\gamma}^{(1,1)} + (\Delta)H_{\gamma g} \hat{\otimes} (\Delta)C_{1,\gamma g}^{(1,1)} \right), \tag{15}
\end{aligned}$$

where $e_{q'}$ is the electric charge of quark (q') of different flavor from quark q and $\hat{\otimes}$ denotes the convolution with $(\Delta)H_{ab}$ in both variables x and z ,

$$\begin{aligned}
(\Delta)H_{qq} &= (\Delta)f_q(x)D_q(z) + (\Delta)f_{\bar{q}}(x)D_{\bar{q}}(z), & (\Delta)H_{q\bar{q}} &= (\Delta)f_q(x)D_{\bar{q}}(z) + (\Delta)f_{\bar{q}}(x)D_q(z), \\
(\Delta)H_{qg} &= (\Delta)f_q(x)D_g(z) + (\Delta)f_{\bar{q}}(x)D_g(z), & (\Delta)H_{q\gamma} &= (\Delta)f_q(x)D_\gamma(z) + (\Delta)f_{\bar{q}}(x)D_\gamma(z), \\
(\Delta)H_{\gamma q} &= (\Delta)f_\gamma(x)D_q(z) + (\Delta)f_\gamma(x)D_{\bar{q}}(z), & (\Delta)H_{gq} &= (\Delta)f_g(x)D_q(z) + (\Delta)f_g(x)D_{\bar{q}}(z), \\
(\Delta)H_{gg} &= (\Delta)f_g(x)D_g(z), & (\Delta)H_{\gamma\gamma} &= (\Delta)f_\gamma(x)D_\gamma(z), & (\Delta)H_{g\gamma} &= (\Delta)f_g(x)D_\gamma(z), \\
(\Delta)H_{\gamma g} &= (\Delta)f_\gamma(x)D_g(z), & (\Delta)H_{q\gamma}^\pm &= (\Delta)f_q(x)D_{q'}(z) \pm (\Delta)f_q(x)D_{\bar{q}'}(z) \pm (\Delta)f_{\bar{q}}(x)D_{q'}(z) + (\Delta)f_{\bar{q}}(x)D_{\bar{q}'}(z). \tag{16}
\end{aligned}$$

We now illustrate the numerical impact of our $(\Delta)C_{1,ab}$ results for various center-of-mass energies $\sqrt{s}$ for a range of x and z values. Specifically, we examine the effects of pure QED and mixed QCD $\otimes$ QED corrections on the SFs $F_1(g_1)$, included alongside the pure QCD terms, providing a more complete description up to $\mathcal{O}(\alpha_e \alpha_s)$. In Fig. 1, the left (right) panel shows results for F_1 (g_1) as a function of x ,

after integration of y between 0.5 and 0.9 and of z between 0.2 and 0.85. We have set both renormalization and factorization scale equal to central scale, $\mu_F = \mu_R = Q_{\text{avg}}$, where $Q_{\text{avg}}^2 = xy_{\text{avg}}s$. Similarly, in Fig. 2, the left (right) panel presents F_1 (g_1) as a function of z , after integration of x between 0.1 and 0.8 and of y between 0.5 and 0.9, we set $\mu_F = \mu_R = Q_{\text{avg}}$ here, $Q_{\text{avg}}^2 = x_{\text{avg}}y_{\text{avg}}s$. We

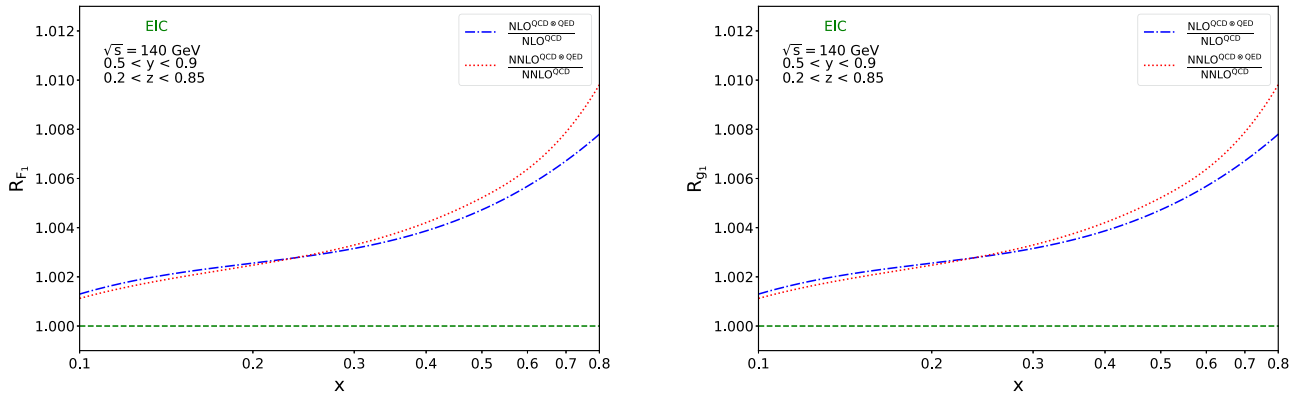


FIG. 1. Ratio of the SFs F_1 (left panel) and g_1 (right panel) with QCD $\otimes$ QED contributions at NLO and NNLO to those with only QCD corrections applied at the respective order as a function of x at the central scale $\mu_R^2 = \mu_F^2 = Q_{\text{avg}}^2$ for the EIC at $\sqrt{s} = 140$ GeV. Integration ranges for y and z are indicated in the plots.

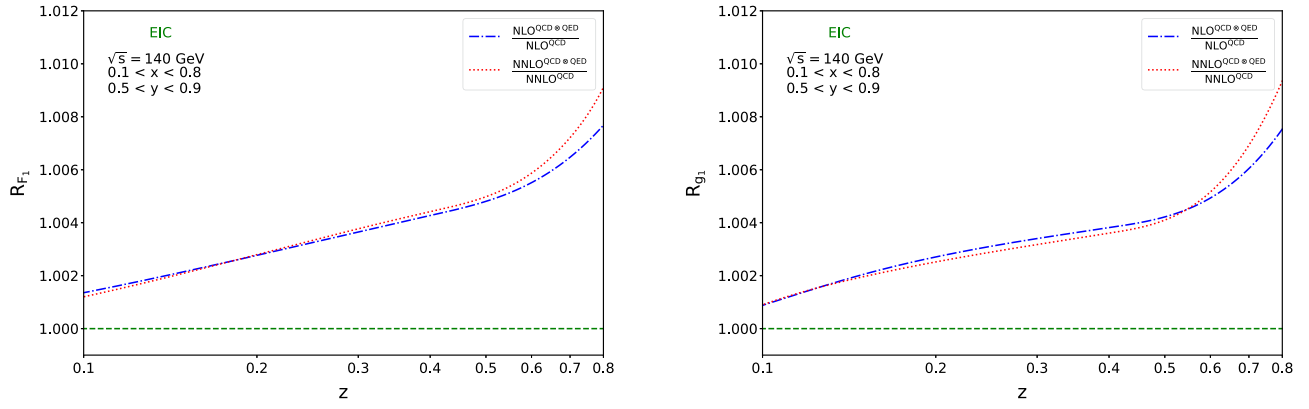


FIG. 2. Same as Fig. 1 for the SFs F_1 (left panel) and g_1 (right panel) as a functions of z and integration ranges for x and y indicated in the plots.

have used $n_f = 3$ as active number of quark and lepton flavors for all numerical calculations. In Table II (III), we present the results of F_1 (g_1) from pure QCD and mixed QCD $\otimes$ QED for $z = 0.15, 0.25, 0.45, 0.6, 0.75$ along with percentage error coming from varying μ_R and μ_F . Both in the plots and in the tables, $\text{NLO}^{\text{QCD} \otimes \text{QED}}$ indicates that contributions at order a_s from QCD and at a_e from QED are added to LO; in $\text{NNLO}^{\text{QCD} \otimes \text{QED}}$ we have added to LO the pure QCD contributions up to order a_s^2 , pure QED to order a_e^2 and mixed QCD $\otimes$ QED upto $a_s a_e$. For F_1 , we have used the LUXqed PDF sets [65] at LO, NLO and NNLO, extended to include QED effects. In contrast, the predictions for g_1 have been obtained using the BDSSV24NLO

PDF set at LO and NLO, and the BDSSV24NNLO PDF set [66] at the NNLO level, also incorporating QED effects. In both cases, for F_1 and g_1 , the NNFF10PIp FF sets [67] have been utilized at the respective orders (LO, NLO, NNLO). The contribution coming from QED and mixed QCD $\otimes$ QED is comparable in size to the N^3LO QCD corrections. We put the polarized PDFs and unpolarized FFs for photons to be zero throughout. The polarized PDFs for photons can be found in [46,68]. We observe that the dominant contribution come from the QCD sector while those from QED and mixed QCD $\otimes$ QED are generally small but not negligible. However, the later is expected to be comparable in size to the N^3LO QCD corrections.

TABLE II. Values of the SF F_1 at various orders at the central scale $\mu_R^2 = \mu_F^2 = Q_{\text{avg}}^2$ and variation of scales $\{\mu_R^2, \mu_F^2\} \in [Q_{\text{avg}}^2/2, 2Q_{\text{avg}}^2]$ as function of z . (See text for integration ranges of x, y).

z	F_1				
	LO	NLO^{QCD}	$\text{NLO}^{\text{QCD} \otimes \text{QED}}$	NNLO^{QCD}	$\text{NNLO}^{\text{QCD} \otimes \text{QED}}$
0.15	1.23119 ^{+4.644%} _{-4.288%}	1.04009 ^{+0.952%} _{-1.202%}	1.04266 ^{+0.865%} _{-1.121%}	0.99712 ^{+0.092%} _{-0.176%}	0.99894 ^{+0.175%} _{-0.077%}
0.25	0.44911 ^{+5.814%} _{-5.283%}	0.38139 ^{+1.341%} _{-1.620%}	0.38257 ^{+1.245%} _{-1.532%}	0.33861 ^{+0.156%} _{-0.291%}	0.33962 ^{+0.156%} _{-0.211%}
0.45	0.08062 ^{+7.564%} _{-6.741%}	0.07737 ^{+2.583%} _{-2.825%}	0.07771 ^{+2.474%} _{-2.727%}	0.07296 ^{+0.433%} _{-0.720%}	0.07329 ^{+0.393%} _{-0.597%}
0.6	0.02339 ^{+8.625%} _{-7.611%}	0.02469 ^{+3.588%} _{-3.767%}	0.02483 ^{+3.471%} _{-3.666%}	0.02463 ^{+1.031%} _{-1.372%}	0.02479 ^{+0.885%} _{-1.234%}
0.75	0.00651 ^{+10.229%} _{-8.902%}	0.00682 ^{+5.019%} _{-5.069%}	0.00687 ^{+4.894%} _{-4.964%}	0.00699 ^{+1.897%} _{-2.294%}	0.00705 ^{+1.731%} _{-2.141%}

TABLE III. Same as Table II for the SF g_1 .

z	g_1				
	LO	NLO^{QCD}	$\text{NLO}^{\text{QCD} \otimes \text{QED}}$	NNLO^{QCD}	$\text{NNLO}^{\text{QCD} \otimes \text{QED}}$
0.15	0.59417 ^{+4.356%} _{-4.034%}	0.45428 ^{+0.471%} _{-0.741%}	0.45525 ^{+0.467%} _{-0.653%}	0.42511 ^{+0.142%} _{-0.183%}	0.42599 ^{+0.237%} _{-0.278%}
0.25	0.22688 ^{+5.579%} _{-5.080%}	0.17305 ^{+0.804%} _{-1.121%}	0.17353 ^{+0.702%} _{-1.026%}	0.14799 ^{+0.154%} _{-0.158%}	0.14837 ^{+0.250%} _{-0.268%}
0.45	0.04229 ^{+7.416%} _{-6.614%}	0.03790 ^{+2.001%} _{-2.313%}	0.03806 ^{+1.888%} _{-2.212%}	0.03430 ^{+0.266%} _{-0.373%}	0.03443 ^{+0.305%} _{-0.286%}
0.6	0.01232 ^{+8.500%} _{-7.505%}	0.01236 ^{+3.017%} _{-3.284%}	0.01242 ^{+2.896%} _{-3.178%}	0.01192 ^{+0.628%} _{-0.990%}	0.01199 ^{+0.558%} _{-0.845%}
0.75	0.00341 ^{+10.116%} _{-8.809%}	0.00339 ^{+4.422%} _{-4.583%}	0.00342 ^{+4.293%} _{-4.473%}	0.00337 ^{+1.414%} _{-1.856%}	0.00340 ^{+1.245%} _{-1.698%}

In this letter, we present the complete NNLO coefficient functions for both unpolarized and polarized SIDIS in the $\overline{\text{MS}}$ -scheme, including pure QED and mixed QCD $\otimes$ QED corrections. We also provide the relevant timelike unpolarized splitting functions, the spacelike polarized splitting functions in $\overline{\text{MS}}$ -schemes, and the renormalization constant Z required for the scheme transformation at NNLO. These results address a key gap in the literature and facilitate high-precision predictions for SIDIS, supporting future global fits of (un)polarized PDFs and unpolarized FFs. In conclusion, our results underscore the importance of including QED and mixed QCD $\otimes$ QED corrections in precision studies, as they enhance the robustness and accuracy of theoretical predictions at the EIC.

Supplemental Materials [69] are provided with this work, including *Mathematica* notebooks containing the unpolarized CFs, $C_{1,ab}^{(i,j)}$ and polarized CF, $\Delta C_{1,ab}^{(i,j)}$ in the $\overline{\text{MS}}$ scheme, the required splitting functions and renormalization constant for scheme transformation Z . All CFs are

simplified to eliminate the θ functions making all the terms real-valued and continuous throughout the entire range $0 \leq x', z' \leq 1$.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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- [1] S. Moch, J. A. M. Vermaseren, and A. Vogt, *Phys. Lett. B* **606**, 123 (2005).
 - [2] J. Vermaseren, A. Vogt, and S. Moch, *Nucl. Phys. B* **724**, 3 (2005).
 - [3] S. Moch, J. A. M. Vermaseren, and A. Vogt, *Nucl. Phys. B* **813**, 220 (2009).
 - [4] J. Blümlein, P. Marquard, C. Schneider, and K. Schönwald, *J. High Energy Phys.* **11** (2022) 156.
 - [5] S. Goyal, S.-O. Moch, V. Pathak, N. Rana, and V. Ravindran, *Phys. Rev. Lett.* **132**, 251902 (2024).
 - [6] L. Bonino, T. Gehrmann, and G. Stagnitto, *Phys. Rev. Lett.* **132**, 251901 (2024).
 - [7] L. Bonino, T. Gehrmann, M. Löchner, K. Schönwald, and G. Stagnitto, *Phys. Rev. Lett.* **133**, 211904 (2024).
 - [8] S. Goyal, R. N. Lee, S.-O. Moch, V. Pathak, N. Rana, and V. Ravindran, *Phys. Rev. Lett.* **133**, 211905 (2024).
 - [9] L. Bonino, T. Gehrmann, M. Löchner, K. Schönwald, and G. Stagnitto, *Phys. Rev. Lett.* **135**, 211902 (2025).
 - [10] L. Bonino, T. Gehrmann, M. Löchner, K. Schönwald, and G. Stagnitto, *J. High Energy Phys.* **10** (2025) 016.
 - [11] L. Bonino, T. Gehrmann, M. Löchner, K. Schönwald, and G. Stagnitto, *J. High Energy Phys.* **03** (2026) 109.
 - [12] S. Goyal, S.-O. Moch, V. Pathak, N. Rana, and V. Ravindran, *Phys. Rev. D* **113**, 034004 (2026).
 - [13] M. Abele, D. de Florian, and W. Vogelsang, *Phys. Rev. D* **106**, 014015 (2022).
 - [14] M. Abele, D. de Florian, and W. Vogelsang, *Phys. Rev. D* **104**, 094046 (2021).
 - [15] R. Abdul Khalek *et al.*, *Nucl. Phys. A* **1026**, 122447 (2022).
 - [16] G. Altarelli, R. K. Ellis, G. Martinelli, and S.-Y. Pi, *Nucl. Phys. B* **160**, 301 (1979).
 - [17] W. Furmanski and R. Petronzio, *Z. Phys. C* **11**, 293 (1982).
 - [18] M. Cacciari and S. Catani, *Nucl. Phys. B* **617**, 253 (2001).
 - [19] D. P. Anderle, F. Ringer, and W. Vogelsang, *Phys. Rev. D* **87**, 034014 (2013).
 - [20] D. P. Anderle, F. Ringer, and W. Vogelsang, *Phys. Rev. D* **87**, 094021 (2013).
 - [21] D. de Florian, G. F. R. Sborlini, and G. Rodrigo, *Eur. Phys. J. C* **76**, 282 (2016).
 - [22] D. de Florian, G. F. R. Sborlini, and G. Rodrigo, *J. High Energy Phys.* **10** (2016) 056.
 - [23] D. de Florian and L. P. Conte, *Eur. Phys. J. C* **86**, 118 (2026).
 - [24] A. H. Ajjath, P. Banerjee, A. Chakraborty, P. K. Dhani, P. Mukherjee, N. Rana, and V. Ravindran, *Phys. Rev. D* **100**, 114016 (2019).
 - [25] A. H. Ajjath, P. Mukherjee, and V. Ravindran, *J. High Energy Phys.* **08** (2020) 156.
 - [26] L. Cieri, G. Ferrera, and G. F. R. Sborlini, *J. High Energy Phys.* **08** (2018) 165.
 - [27] D. de Florian, M. Der, and I. Fabre, *Phys. Rev. D* **98**, 094008 (2018).
 - [28] L. Cieri, D. de Florian, M. Der, and J. Mazzitelli, *J. High Energy Phys.* **09** (2020) 155.
 - [29] W. B. Kilgore and C. Sturm, *Phys. Rev. D* **85**, 033005 (2012).
 - [30] J. Blümlein and H. Kawamura, *Eur. Phys. J. C* **51**, 317 (2007).
 - [31] T. Liu, W. Melnitchouk, J.-W. Qiu, and N. Sato, *Phys. Rev. D* **104**, 094033 (2021).
 - [32] J. Cammarota, J.-W. Qiu, K. Watanabe, and J.-Y. Zhang, *Phys. Rev. D* **112**, 056007 (2025).
 - [33] A. Kwiatkowski, H. Spiesberger, and H. J. Möhring, *Comput. Phys. Commun.* **69**, 155 (1992).
 - [34] K. Charchula, G. A. Schuler, and H. Spiesberger, *Comput. Phys. Commun.* **81**, 381 (1994).

- [35] A. Arbuzov, D. Y. Bardin, J. Blümlein, L. Kalinovskaya, and T. Riemann, *Comput. Phys. Commun.* **94**, 128 (1996).
- [36] H. Spiesberger, *Radiative corrections to deep inelastic ep scattering at HERA*, (1989), DESY-89-175, <https://lib-ext-01.kek.jp/preprints/PDF/1990/9003/9003109.pdf>.
- [37] S. Goyal, R. N. Lee, S.-O. Moch, V. Pathak, N. Rana, and V. Ravindran, *Phys. Rev. D* **111**, 094007 (2025).
- [38] E. B. Zijlstra and W. L. van Neerven, *Nucl. Phys.* **B417**, 61 (1994); **B426**, 245(E) (1994); **B773**, 105(E) (2007); **B501**, 599(E) (1997).
- [39] S. Larin and J. Vermaseren, *Phys. Lett. B* **303**, 334 (1993).
- [40] Y. Matiounine, J. Smith, and W. L. van Neerven, *Phys. Rev. D* **58**, 076002 (1998).
- [41] V. Ravindran, J. Smith, and W. L. van Neerven, *Nucl. Phys.* **B647**, 275 (2002).
- [42] V. Ravindran, J. Smith, and W. L. van Neerven, *Nucl. Phys.* **B682**, 421 (2004).
- [43] S. Moch, J. A. M. Vermaseren, and A. Vogt, *Nucl. Phys.* **B889**, 351 (2014).
- [44] T. Kinoshita, *J. Math. Phys. (N.Y.)* **3**, 650 (1962).
- [45] T. Lee and M. Nauenberg, *Phys. Rev.* **133**, B1549 (1964).
- [46] D. de Florian and L. P. Conte, *Eur. Phys. J. C* **83**, 695 (2023).
- [47] P. Nogueira, *J. Comput. Phys.* **105**, 279 (1993).
- [48] J. Kuipers, T. Ueda, J. A. M. Vermaseren, and J. Vollinga, *Comput. Phys. Commun.* **184**, 1453 (2013).
- [49] B. Ruijl, T. Ueda, and J. Vermaseren, [arXiv:1707.06453](https://arxiv.org/abs/1707.06453).
- [50] C. Anastasiou, K. Melnikov, and F. Petriello, *Phys. Rev. D* **69**, 076010 (2004).
- [51] C. Anastasiou, S. Buehler, C. Duhr, and F. Herzog, *J. High Energy Phys.* **11** (2012) 062.
- [52] K. G. Chetyrkin and F. V. Tkachov, *Nucl. Phys.* **B192**, 159 (1981).
- [53] R. N. Lee, *J. Phys. Conf. Ser.* **523**, 012059 (2014).
- [54] T. Ahmed, S. Goyal, S. M. Hasan, R. N. Lee, S.-O. Moch, V. Pathak, N. Rana, A. Rapakoulas, and V. Ravindran, *Phys. Rev. D* **112**, 014020 (2025).
- [55] L. Bonino, T. Gehrmann, M. Marcoli, R. Schürmann, and G. Stagnitto, *J. High Energy Phys.* **08** (2024) 073.
- [56] S. Moch, J. A. M. Vermaseren, and A. Vogt, *Nucl. Phys.* **B688**, 101 (2004).
- [57] A. Vogt, S. Moch, and J. A. M. Vermaseren, *Nucl. Phys.* **B691**, 129 (2004).
- [58] R. Mertig and W. L. van Neerven, *Z. Phys. C* **70**, 637 (1996).
- [59] W. Vogelsang, *Phys. Rev. D* **54**, 2023 (1996).
- [60] W. Vogelsang, *Nucl. Phys.* **B475**, 47 (1996).
- [61] J. Blümlein, P. Marquard, C. Schneider, and K. Schönwald, *Nucl. Phys.* **B971**, 115542 (2021).
- [62] J. Blümlein, P. Marquard, C. Schneider, and K. Schönwald, *J. High Energy Phys.* **01** (2022) 193.
- [63] A. A. Almasy, S. Moch, and A. Vogt, *Nucl. Phys.* **B854**, 133 (2012).
- [64] H. Chen, T.-Z. Yang, H. X. Zhu, and Y. J. Zhu, *Chin. Phys. C* **45**, 043101 (2021).
- [65] R. D. Ball *et al.* (NNPDF Collaboration), *Eur. Phys. J. C* **77**, 663 (2017).
- [66] I. Borsa, M. Stratmann, W. Vogelsang, D. de Florian, and R. Sassot, *Phys. Rev. Lett.* **133**, 151901 (2024).
- [67] V. Bertone, S. Carrazza, N. P. Hartland, E. R. Nocera, and J. Rojo (NNPDF Collaboration), *Eur. Phys. J. C* **77**, 516 (2017).
- [68] D. de Florian, L. P. Conte, and G. F. Volonnino, *Eur. Phys. J. C* **84**, 905 (2024).
- [69] See Supplemental Materials at <http://link.aps.org/supplemental/10.1103/d9b5-cgwg> for the quantities $(\Delta)\mathcal{C}_{l,ab}^{(i,j)}$ calculated within the $\overline{\text{MS}}$ scheme, as presented in Eqs. (13)–(15) and the required splitting functions and renormalization constant for scheme transformation.