

Supersymmetric hybrid inflation and current-carrying metastable cosmic strings in $SU(4)_c \times SU(2)_L \times U(1)_R$

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We construct a realistic supersymmetric model for superheavy metastable cosmic strings (CSs) that can be investigated in the current pulsar timing array (PTA) experiments. It is based on shifted μ -hybrid inflation in which the symmetry breaking $SU(4)_c \times SU(2)_L \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_{B-L} \times U(1)_R$ proceeds along an inflationary trajectory such that the topologically unstable primordial monopoles are inflated away. The breaking of $U(1)_{B-L} \times U(1)_R \rightarrow U(1)_Y$ after inflation ends yields the superheavy metastable CSs carrying the right handed neutrino zero modes that generate the stochastic gravitational wave background, which we show is consistent with the current PTA dataset and the LIGO O3 run. The scalar spectral index n_s and the tensor-to-scalar ratio r are compatible with the Planck 2018 and Atacama Cosmology Telescope measurements. Both reheating and leptogenesis are briefly discussed.

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I. INTRODUCTION

The pulsar timing array experiments have recently shared evidence for the presence of a stochastic gravitational wave background [1–5], which may be compatible with the emission of such a background by superheavy metastable [6–11] or quasistable [12] cosmic strings (CSs). For a recent discussion of composite topological structures in $SO(10)$, more precisely Spin (10), see Ref. [13].

In this paper, we describe how the desired metastable strings appear in a realistic supersymmetric inflation model based on the gauge symmetry $G = SU(4)_c \times SU(2)_L \times U(1)_R$, a well-known subgroup of $SO(10)$. Supersymmetric hybrid inflation models have been extensively studied in [14–28], and for tribrid inflation, see [29]. Here we utilize one particular version known as μ -hybrid inflation [30–34]. Following [13,35], the breaking of $SU(4)_c$ to $SU(3)_c \times U(1)_{B-L}$ at a scale close to M_{GUT} produces “red” monopoles, that we show are inflated away

by employing shifted μ -hybrid inflation [36]. The subsequent breaking of $U(1)_{B-L} \times U(1)_R$ to $U(1)_Y$, also close to M_{GUT} , takes place after the inflationary phase is effectively over. This produces the desired metastable strings, that are current-carrying due to the associated right handed neutrino zero modes, and which generate a stochastic gravitational wave background. The strings eventually disappear from the quantum tunneling of the red monopole-antimonopole pairs.

The paper is laid out as follows. In Sec. II, we summarize the salient features of the $SU(4)_c \times SU(2)_L \times U(1)_R$ model, including symmetry breaking and evolution of the gauge couplings. The shifted μ -hybrid inflationary scenario is presented in Sec. III, and in Sec. IV we discuss the supergravity corrections with nonminimal Kähler potential. The global minimum and symmetry breaking after inflation are discussed in Sec. V. Section VI summarizes how reheating and leptogenesis proceed in this model, together with the inflationary predictions. The metastable strings and predictions related to the gravitational waves (GW) are presented in Sec. VII. Our conclusions are given in Sec. VIII.

II. SUPERSYMMETRIC

$SU(4)_c \times SU(2)_L \times U(1)_R$ MODEL

The superfields containing standard model (SM) matter and Higgs content and their $U(1)_R$ charges are given in Table I. The $U(1)_R$ symmetric superpotential for shifted μ -hybrid inflation is written as

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TABLE I. Matter and Higgs superfields together with their decomposition under $SU(4)_c \times SU(2)_L \times U(1)_R = 421$, $SU(3)_c \times SU(2)_L \times U(1)_Y = \text{SM}$, and their R charges.

Superfields	421	SM	$q(R)$
F_i	(4, 2, 0)	$Q_i(3, 2, 1/6)$ $L_i(1, 2, -1/2)$	1/2
F_{ui}^c	($\bar{4}$, 1, -1/2)	$U_i^c(\bar{3}, 1, -2/3)$ $N_i^c(1, 1, 0)$	1/2
F_{di}^c	($\bar{4}$, 1, 1/2)	$D_i^c(\bar{3}, 1, 1/3)$ $E_i^c(1, 1, 1)$	1/2
H	(4, 1, 1/2)	$u_H(3, 1, 2/3)$ $N_H(1, 1, 0)$	0
$\bar{H}$	($\bar{4}$, 1, -1/2)	$\bar{u}_H(\bar{3}, 1, -2/3)$ $\bar{N}_H(1, 1, 0)$	0
S	(1, 1, 1)	(1, 1, 0)	1
Φ	(15, 1, 0)	(1, 1, 0) (8, 1, 0) (3, 1, 1/3) ($\bar{3}$, 1, -1/3)	0
h_u	(1, 2, 1/2)	(1, 2, 1/2)	0
h_d	(1, 2, -1/2)	(1, 2, 1/2)	0

$$\begin{aligned}
W = & S \left(\kappa M^2 - \kappa \text{Tr}(\Phi^2) - \frac{\beta}{M_*} \text{Tr}(\Phi^3) \right) + \lambda_H S H \bar{H} \\
& + \lambda_h S h_u h_d + \lambda_{ij}^{(u,\nu)} F_i F_{uj}^c h_u + \lambda_{ij}^{(d,e)} F_i F_{dj}^c h_d \\
& + \frac{\gamma_{ij}}{M_*} (F_{ui}^c H) (F_{uj}^c H), \quad (1)
\end{aligned}$$

where S is a gauge singlet superfield, M_* is some suitable cutoff scale, and (h_u, h_d) are the Higgs doublet superfields. The third line in W contains Yukawa terms for quarks and leptons. The last term in Eq. (1) corresponds to the Majorana mass term, which plays a crucial role in elucidating the origin of tiny neutrino masses through the utilization of the seesaw mechanism.

The $SU(4)_c$ symmetry breaks into $SU(3)_c \times U(1)_{B-L}$ by acquiring a nonzero vacuum expectation value (vev) v in the color singlet direction of the adjoint representation Φ ,

$$SU(4)_c \xrightarrow{\langle \Phi \rangle_{(1,1,0)}} SU(3)_c \times U(1)_{B-L}, \quad (2)$$

with

$$\langle \Phi \rangle_{(1,1,0)} = \frac{v}{2\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}. \quad (3)$$

This breaking creates monopoles that carry $B-L$ and color magnetic charge, which are subsequently diluted during

inflation. A similar mechanism is discussed in the standard version of μ -hybrid inflation; see Ref. [37]. To further break $U(1)_{B-L} \times U(1)_R$ into $U(1)_Y$ at v_H , we consider $H_{(1,1,0)}$ and $\bar{H}_{(1,1,0)}$,

$$U(1)_{B-L} \times U(1)_R \xrightarrow{\langle H \rangle_{(1,1,0)} = \langle \bar{H} \rangle_{(1,1,0)}} U(1)_Y. \quad (4)$$

The SM hypercharge is given by

$$Y = \sqrt{\frac{3}{5}} Y_R + \sqrt{\frac{2}{5}} (B-L), \quad (5)$$

where Y_R is the hypercharge associated with $U(1)_R$ and we have chosen the normalization of Y_R assuming $U(1)_R \subset SU(2)_R$.

The two loop gauge coupling evolution for the gauge symmetries $SU(4)_c \times SU(2)_L \times U(1)_R$, $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$, and $SU(3)_c \times SU(2)_L \times U(1)_Y$ are shown in Fig. 1. We have assumed the degeneracy of supersymmetric particles at $m_S \simeq 10$ TeV. As discussed below, at the MSSM level, a color octet component of the adjoint superfield Φ and color triplets in the $H, \bar{H}$ pair with mass $m_{3/2} \simeq 10$ TeV are present. Note that recent LHC data have established a lower bound on the color-octet mass, $m_{3/2} \gtrsim 2$ TeV [38]. On the other hand, standard cosmology imposes an upper bound of $m_{3/2} \lesssim 2.5$ TeV as discussed in [27,39]. For convenience, we assume that $m_{3/2}$ is degenerate with supersymmetric particles in our analysis. This assumption may be justified in a nonstandard cosmological context, as pointed out in [39], but even if we consider the range $2 \text{ TeV} < m_{3/2} < 2.5 \text{ TeV}$, our results remain unaffected.

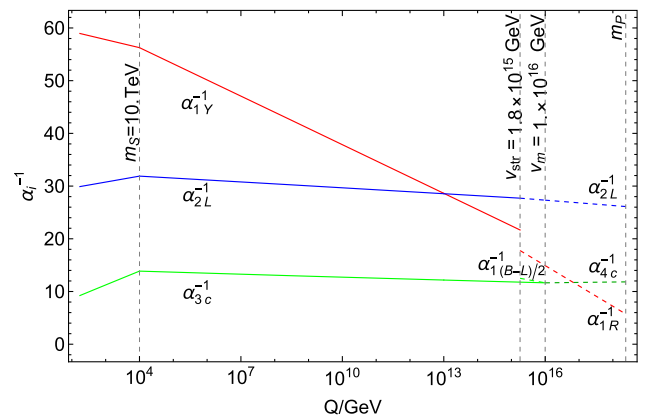


FIG. 1. Two loop RG evolution of the gauge couplings for the symmetries $SU(3)_c \times SU(2)_L \times U(1)_Y$, $U(1)_{B-L}$, and $SU(4)_c \times SU(2)_L \times U(1)_R$. Matching conditions at v_{str} are $\alpha_{1Y}^{-1} = \sqrt{3/5} \alpha_{1R}^{-1} + \sqrt{2/5} \alpha_{1(B-L)}^{-1}$ and $\alpha_{2L}^{-1} = \alpha_{2L}^{-1}$, and at v_m , $\alpha_{3c}^{-1} = \alpha_{4c}^{-1} = \alpha_{1(B-L)}^{-1}$.

III. SHIFTED μ -HYBRID INFLATION IN THE SUPERSYMMETRIC $SU(4)_c \times SU(2)_L \times U(1)_R$ MODEL

In the discussion below, the scalar components of the superfields are denoted by the same symbols as their respective superfields. We express the scalar field Φ in the $SU(4)_c$ adjoint basis $\Phi = \phi_a T^a$, $\text{Tr}[\Phi^2] = \phi_a \phi^a / 2$, $\text{Tr}[\Phi^3] = d^{abc} \phi_a \phi_b \phi_c / 4$. Here $d^{abc} = 2\text{Tr}[T^a \{T^b, T^c\}]$ is totally symmetric and $[T^b, T^c] = i f^{bcd} T^a$ with, f^{bcd} being totally antisymmetric tensors. The indices a, b, c run from 1 to 15, and the repeated indices are summed over.

The inflation-relevant part of the scalar potential obtained from the superpotential in Eq. (1) is given by

$$V \supset \left| \kappa M^2 - \frac{\kappa}{2} \phi_a \phi_a - \frac{\beta}{M_\star} \frac{d^{abc}}{4} \phi_a \phi_b \phi_c + \lambda_H H \bar{H} + \lambda_h h_u h_d \right|^2 + |S|^2 \left| \kappa \phi_a + \frac{3\beta}{4M_\star} d^{abc} \phi_b \phi_c \right|^2 + \lambda_H^2 |S|^2 (|H|^2 + |\bar{H}|^2) + \lambda_h^2 |S|^2 (|h_u|^2 + |h_d|^2) + V_D, \quad (6)$$

where V_D is the D-term potential given by

$$V_D \supset g_4^2 (f^{abc} \phi_b \phi_c^\star) (f^{ade} \phi_a \phi_e^\star). \quad (7)$$

Vanishing of the D-term is achieved by taking $\phi_a = \phi_{15} \delta_{a,15}$, where ϕ_{15} is chosen to be a real scalar field. In the D-flat direction, we used an appropriate R transformation to rotate the S complex field to the real axis, $S = \sigma / \sqrt{2}$, where σ is a canonically normalized real scalar field playing the role of inflaton. The supersymmetric global minimum of the above potential lies at

$$\frac{\kappa}{2} \phi_a^0 \phi_a^0 + \frac{\beta}{4M_\star} d^{abc} \phi_a^0 \phi_b^0 \phi_c^0 + \lambda_H \langle H \bar{H} \rangle = \kappa M^2, \quad (8)$$

where the superscript “0”; denotes the field value at its global minimum and

$$\langle S \rangle = \langle h_u h_d \rangle = 0, \quad \langle H \bar{H} \rangle = v_H^2. \quad (9)$$

The acquiring of nonzero vev, v_H^2 , by $H \bar{H}$ and the formation of metastable cosmic strings are discussed in later sections. During inflation with $S > M$, the various Higgs fields, namely, $h_u, h_d, H, \bar{H}$, acquire heavy masses, leading them to quickly stabilize at zero. Exploiting the $SU(4)_c$ symmetry, the matrix Φ^0 can be transformed into a diagonal form as shown in Eq. (3). Consequently, Eq. (8) takes the following form:

$$v_\Phi^2 + 4\gamma_H v_H^2 = 4M^2 + \frac{\xi v_\Phi^3}{2M}, \quad (10)$$

where we have used $d_{15,15,15} = -\sqrt{2/3}$, $\gamma_H = \lambda_H / \kappa$, $\xi = \beta M / \sqrt{3} \kappa M_\star$, and only $\phi_{15}^0 = v_\Phi / \sqrt{2} \neq 0$.

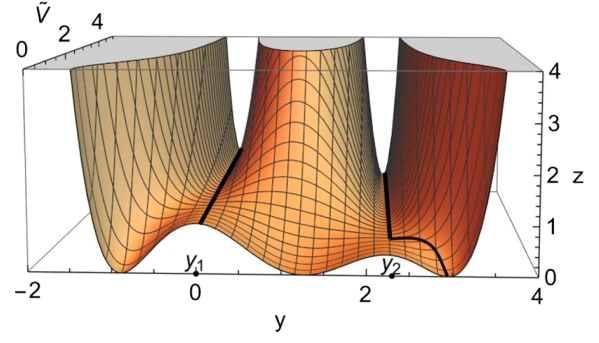


FIG. 2. Schematic display of shifted hybrid track for $\xi = 0.3$. The two minima at $y_1 = 0$ and $y_2 \simeq 2.3$ correspond to standard and shifted trajectory, respectively.

We can rewrite the scalar potential in Eq. (6) in terms of the dimensionless fields

$$y = \frac{\phi_{15}}{\sqrt{2}M} \quad \text{and} \quad z = \frac{\sigma}{\sqrt{2}M}, \quad (11)$$

with

$$\tilde{V} \equiv \frac{V}{\kappa^2 M^4} = (1 - y^2 + \xi y^3)^2 + 2z^2 y^2 (1 - 3\xi y/2)^2. \quad (12)$$

The schematic display of this potential is shown in Fig. 2. Thus, for a constant value of z , the local minima for standard y_1 and shifted y_2 trajectories are

$$y_1 = 0 \quad \text{and} \quad y_2 = \frac{2}{3\xi}. \quad (13)$$

As explained in [40], the interesting region for the parameter ξ in the current analysis is given by $0.27 \lesssim \xi \lesssim 0.38$. In Table II, we summarize the results of the mass

TABLE II. Mass spectrum of the shifted hybrid $SU(4)_c$ model as the system moves along the inflationary trajectory y_2 ($v_2 = 2M y_2 = \frac{4M}{3\xi}$). Here, $M_\xi^2 = M^2(4/27\xi^2 - 1)$, $\gamma_h = \lambda_h / \kappa$, and $\gamma_H = \lambda_H / \kappa$.

Fields	Squared masses
2 real scalars	$\kappa^2 S ^2 \pm \kappa^2 M_\xi^2$
1 Majorana fermion	$\kappa^2 S ^2$
16 real scalars	$4\kappa^2 S ^2 \pm 2\kappa^2 M_\xi^2$
8 Majorana fermions	$4\kappa^2 S ^2$
8 real scalars	$\lambda_h^2 [S^2 \pm M_\xi^2 / \gamma_h]$
4 Majorana fermions	$\lambda_h^2 S^2$
16 real scalars	$\lambda_H^2 [S^2 \pm M_\xi^2 / \gamma_H]$
8 Majorana fermions	$\lambda_H^2 S^2$
6 real scalars	$\frac{2}{3} g^2 v_2^2$
6 Dirac fermions	$\frac{2}{3} g^2 v_2^2$
6 gauge bosons	$\frac{2}{3} g^2 v_2^2$

spectrum of the model along the shifted inflationary trajectory, $y_2 = 2/(3\xi)$. All these supermultiplets satisfy the supertrace rule $\text{Str}[M^2] = 0$.

The inflationary effective potential with 1-loop radiative correction is given by

$$V_{\text{1loop}} = \kappa^2 M_\xi^4 \left(1 + \frac{\kappa^2}{16\pi^2} [F(M_\xi^2, x^2) + 8 \times 4F(2M_\xi^2, 2x^2) + 4\gamma_h^2 F(m_h^2, x_h^2) + 8\gamma_H^2 F(M_H^2, x_H^2)] \right). \quad (14)$$

Here

$$F(M_\xi^2, x^2) = \frac{1}{4} \left((x^4 + 1) \ln \frac{(x^4 - 1)}{x^4} + 2x^2 \ln \frac{x^2 + 1}{x^2 - 1} + 2 \ln \frac{\kappa^2 M_\xi^2 x^2}{Q^2} - 3 \right), \quad (15)$$

where $x = |S|/M_\xi$, $M_\xi^2 = M^2(4/27\xi^2 - 1)$, $m_h^2 = \gamma_h M_\xi^2$, $x_h^2 = \gamma_h x^2$ with $\gamma_h = \lambda_h/\kappa$ and $M_H^2 = \gamma_H M_\xi^2$, $x_H^2 = \gamma_H x^2$ with $\gamma_H = \lambda_H/\kappa$, and Q is the renormalization scale. To ensure the desired symmetry-breaking vacuum, the coupling constants must satisfy the hierarchy $\lambda_h > \lambda_H \geq \kappa$. The $SU(3)_c$ octet multiplet is massless in the supersymmetric limit as a consequence of both the R and gauge symmetries. This will not be a problem, as gauge coupling unification is not a prediction of our model. Moreover, these otherwise massless states are anticipated to acquire masses of order $m_{3/2}$ through soft supersymmetry-breaking effects.

IV. SUGRA CORRECTIONS AND NONMINIMAL KÄHLER POTENTIAL

We take the following general form of the Kähler potential:

$$\begin{aligned} K = & |S|^2 + \text{Tr}|\Phi|^2 + |H|^2 + |\bar{H}|^2 + |h_u|^2 + |h_d|^2 \\ & + \kappa_{S\Phi} \frac{|S|^2 \text{Tr}|\Phi|^2}{m_P^2} + \kappa_{SH} \frac{|S|^2 |H|^2}{m_P^2} + \kappa_{S\bar{H}} \frac{|S|^2 |\bar{H}|^2}{m_P^2} \\ & + \kappa_{H\Phi} \frac{|H|^2 \text{Tr}|\Phi|^2}{m_P^2} + \kappa_{\bar{H}\Phi} \frac{|\bar{H}|^2 \text{Tr}|\Phi|^2}{m_P^2} + \kappa_{H\bar{H}} \frac{|H|^2 |\bar{H}|^2}{m_P^2} \\ & + \kappa_S \frac{|S|^4}{4m_P^2} + \kappa_\Phi \frac{(\text{Tr}|\Phi|^2)^2}{4m_P^2} + \kappa_H \frac{|H|^4}{4m_P^2} + \kappa_{\bar{H}} \frac{|\bar{H}|^4}{4m_P^2} \\ & + \kappa_{SS} \frac{|S|^6}{6m_P^4} + \dots, \end{aligned} \quad (16)$$

where $m_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Additionally, for the sake of simplicity, the contribution of many other terms, e.g., of the form

$$c_2 [\text{Tr}(\Phi^2) + \text{H.c.}] + c_3 [\text{Tr}(\Phi^3)/m_P + \text{H.c.}] + \dots \quad (17)$$

is ignored. Alternatively, we can effectively absorb these extra contributions coming from the Φ superfield into various couplings of the above Kähler potential as only the $|S|$ field plays an active role during inflation. The SUGRA scalar potential is given by

$$V_F = e^{K/m_P^2} (K_{i\bar{j}}^{-1} D_{z_i} W D_{z_j}^* W^* - 3m_P^{-2} |W|^2), \quad (18)$$

with z_i being the bosonic components of the superfields $z_i \in \{S, \Phi, H, \bar{H}, \dots\}$, and we have defined

$$D_{z_i} W \equiv \frac{\partial W}{\partial z_i} + m_P^{-2} \frac{\partial K}{\partial z_i} W, \quad K_{i\bar{j}} \equiv \frac{\partial^2 K}{\partial z_i \partial z_j^*}, \quad (19)$$

and $D_{z_i}^* W^* = (D_{z_i} W)^*$. The soft supersymmetric (SUSY) breaking terms are added in the inflationary potential as

$$V_{\text{soft}} = m_{3/2} \left[z_i \frac{\partial W}{\partial z_i} + (A - 3)W + \text{H.c.} \right], \quad (20)$$

where A is the complex coefficient of the trilinear soft-SUSY-breaking terms. Now along the inflationary trajectory with the D-flat direction ($\phi_i = \delta_{i,15} \phi_{15}$, $|\bar{H}_a| = |H_a|$) and using Eqs. (1), (14), (16), (18), and (20), we obtain the following form of the full potential:

$$\begin{aligned} V = & V_{\text{1loop}} + \kappa^2 M_\xi^4 \left(\left(\frac{4(1 - \kappa_{S\Phi})}{9(4/27 - \xi^2)} - \kappa_S x^2 \right) \left(\frac{M_\xi}{m_P} \right)^2 \right. \\ & + \left(\frac{4((1 - 2\kappa_{S\Phi})^2 + 1 + \kappa_\Phi)}{81(4/27 - \xi^2)^2} \right. \\ & + \left. \frac{4((1 - \kappa_{S\Phi})^2 - \kappa_S(1 - 2\kappa_{S\Phi}))x^2 + \gamma_S x^4}{9(4/27 - \xi^2)} + \frac{\gamma_S x^4}{2} \right) \left(\frac{M_\xi}{m_P} \right)^4 \\ & \left. + a \frac{m_{3/2}}{\sqrt{2}\kappa M_\xi} x + \dots \right) + \dots, \end{aligned} \quad (21)$$

where $x = |S|/M_\xi$ and $\gamma_S = 1 - \frac{7\kappa_S}{2} + 2\kappa_S^2 - 3\kappa_{SS}$. Taking into account the range $0 \lesssim |\kappa_{SS}| \lesssim 1$, the parameter κ_{SS} does not play a significant role in our numerical computations. The parameter a depends on $\arg S$ as follows:

$$a = 2 \left| 2 - A + \frac{A}{2\xi} \right| \cos \left[\arg S + \arg \left(2 - A + \frac{A}{2\xi} \right) \right]. \quad (22)$$

The inflationary slow-roll parameters are given by

$$\epsilon = \frac{m_P^2}{4M_\xi^2} \left(\frac{\partial_x V}{V} \right)^2, \quad \eta = \frac{m_P^2}{2M_\xi^2} \left(\frac{\partial_x^2 V}{V} \right). \quad (23)$$

In the slow-roll approximation, i.e., $(\epsilon, |\eta|) \ll 1$, the scalar spectral index n_s and the tensor to scalar ratio r are given (to leading order) by

$$n_s \simeq 1 + 2\eta - 6\epsilon, \quad r \simeq 16\epsilon. \quad (24)$$

The relevant number of e-folds, N_0 , before the end of inflation, is

$$N_0 = 2 \left(\frac{M_\xi}{m_P} \right)^2 \int_{x_e}^{x_0} \left(\frac{V}{\partial_x V} \right) dx, \quad (25)$$

where $|S_0| = x_0 M_\xi$ is the field value at the pivot scale $k_0 = 0.05 \text{ Mpc}^{-1}$ and x_e denotes the field value at the end of inflation, defined by $|\eta(x_e)| = 1$ (or $x_e = 1$). During inflation, this scale exits the horizon at approximately

$$N_0 = 53 + \frac{1}{3} \ln \left(\frac{T_r}{10^9 \text{ GeV}} \right) + \frac{2}{3} \ln \left(\frac{V^{1/4}(x_0)}{10^{15} \text{ GeV}} \right), \quad (26)$$

where T_r is the reheat temperature, and for the numerical work, we set $T_r = 2 \times 10^9 \text{ GeV}$. This could easily be reduced to lower values if the gravitino problem is considered to be an issue. The amplitude of the curvature perturbation is given by

$$A_s(k_0) = \frac{1}{24\pi^2} \left(\frac{V/m_P^4}{\epsilon} \right) \Big|_{k=k_0}, \quad (27)$$

where $A_s(k_0) = (2.43 \pm 0.11) \times 10^{-9}$ is the Planck 2018 normalization. Note that, for added precision, we include in our calculations the first-order corrections [41] in the slow-roll expansion for the quantities n_s , r , and A_s .

V. GLOBAL MINIMUM AND SYMMETRY BREAKING AFTER INFLATION

We introduce an additional gauge-singlet field X , carrying a global R -charge of 1, to facilitate the breaking of the symmetry $U(1)_R \times U(1)_{B-L}$ to $U(1)_Y$, which is associated with the formation of a CS network. The relevant term in the superpotential is given by

$$W \supset \lambda_X X (H\bar{H} - v_H^2), \quad (28)$$

where, for simplicity, we leave out the other terms. In the early Universe, prior to observable inflation, the field X is assumed to be stabilized at the origin due to its heavy mass,

$$m_X^2 \sim \frac{\kappa_X \lambda_X^2 v_H^4}{m_P^2},$$

which originates from a higher-order term in the Kähler potential,

$$K \supset -\frac{\kappa_X}{4m_P^2} |X|^4,$$

with $\kappa_X \sim 1$ [42].

During inflation, as long as the inflaton field S satisfies $S > M_\xi$, the mass-squared of some components of the waterfall fields is positive:

$$\lambda_H^2 [S^2 - M_\xi^2/\gamma_H] > 0, \quad (29)$$

provided $\gamma_H \equiv \lambda_H/\kappa \geq 1$. This ensures that $|H| = |\bar{H}|$ remains stabilized at zero as the field Φ rolls toward its minimum after becoming destabilized at $S = M_\xi$. After the inflationary phase ends, when S drops below M_ξ/γ_H , the SM-singlet components $|N_H| = |\bar{N}_H|$ of the fields $H, \bar{H}$ become tachyonic and acquire nonzero vacuum expectation values,

$$\langle |N_H| \rangle = \langle |\bar{N}_H| \rangle = v_H. \quad (30)$$

This spontaneous symmetry breaking of $U(1)_R \times U(1)_{B-L}$ triggers the formation of the cosmic string network. The mass of the $\theta_H = (\delta N_H + \delta \bar{N}_H)/\sqrt{2}$ component of the waterfall fields, where $\delta N_H = N_H - v_H$, $\delta \bar{N}_H = \bar{N}_H - v_H$, is given by

$$M_H \simeq \sqrt{2} \lambda_X v_H, \quad (31)$$

while the color-triplet components $(u_H, \bar{u}_H) \subset (H, \bar{H})$ acquire masses of order of the gravitino mass, $m_{3/2}$, via soft SUSY-breaking effects [43].

We assume that the formation of the cosmic string network takes place immediately after the end of inflation, near the critical value $S_c \simeq M_\xi$ with $\gamma_H \simeq 1$. To avoid a secondary phase of inflation triggered by the rolling of the fields $|N_H| = |\bar{N}_H|$, we require that the scalar mass M_H exceeds the Hubble scale during inflation,

$$M_H > \mathcal{H} \simeq \frac{\kappa M_\xi^2}{\sqrt{3} m_P}.$$

This condition ensures that the waterfall fields rapidly settle into their global minima following the end of inflation. Assuming $\lambda_X \sim \kappa$, this leads to a lower bound on the symmetry-breaking scale:

$$v_H > \frac{M_\xi^2}{\sqrt{6} m_P}.$$

Furthermore, under the assumption $\kappa M_\xi^2 \gg \lambda_X v_H^2$, the vacuum energy during inflation is dominated by the term $\kappa^2 M_\xi^4$, confirming that inflation is primarily driven by the inflaton sector. This hierarchy also implies that $v_H \ll M_\xi$, consistent with the earlier assumption $\lambda_X \sim \kappa$.

VI. INFLATON DECAY AND REHEATING

After inflation ends, the system falls towards the SUSY vacuum and performs damped oscillations about it.

The oscillating system mainly consists of two complex scalar fields S and $\theta_\phi = \delta\phi$ with the same mass,

$$m_{\text{inf}} \simeq \frac{\kappa v_\phi}{\sqrt{2}} \left(-1 + \frac{3\xi v_\phi}{4M} \right). \quad (32)$$

The dominant decay channels of the inflaton (oscillating system) are into a pair of higgsinos ($\tilde{h}_u, \tilde{h}_d$) and higgses (h_u, h_d), each with a decay width, Γ_h , given by [30,34,44]

$$\Gamma_h = \Gamma(S \rightarrow \tilde{h}_u \tilde{h}_d) = \Gamma(\theta_\phi \rightarrow h_u h_d) = \frac{\lambda_h^2}{8\pi} m_{\text{inf}}. \quad (33)$$

A comparable decay rate exists involving the fields ($H, \tilde{H}$) and their superpartners ($\tilde{H}, \tilde{\tilde{H}}$), each with a decay width, Γ_H , given by

$$\Gamma_H = \Gamma(S \rightarrow \tilde{H} \tilde{\tilde{H}}) = \Gamma(\theta_\phi \rightarrow H \tilde{H}) = \frac{\lambda_H^2}{8\pi} m_{\text{inf}}, \quad (34)$$

where we take $\lambda_h = 2\lambda_H$.

Another decay mode for S and θ_H , via the higher-dimensional superpotential term $(\gamma_{ij}/M_*) N_H^2 N_i^c N_j^c$, leads to a pair of right-handed neutrinos (N) and sneutrinos ($\tilde{N}$), respectively, with the equal decay width given by

$$\Gamma_N = \Gamma(\theta_H \rightarrow NN) = \Gamma(S \rightarrow \tilde{N} \tilde{N}) \quad (35)$$

$$= \frac{M_H}{8\pi} \left(\frac{M_N}{v_H} \right)^2 \left(1 - \frac{4M_N^2}{M_H^2} \right)^{1/2}, \quad (36)$$

provided the lightest right-handed neutrino mass M_N satisfies the kinematic condition $M_H \geq 2M_N$. This channel is crucial for generating the baryon asymmetry via leptogenesis, which requires us to consider $M_H \simeq 2M_N$. The required kinematic suppression, in turn, makes the decay width of this channel Γ_N suppressed compared to $\Gamma_{h,H}$. We further require $M_N \approx 10T_r$ to suppress the washout effects. With $H = 3\Gamma_{\text{inf}}$, we define the reheat temperature T_r in terms of the inflaton decay width Γ_{inf} ,

$$T_r = \left(\frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma_{\text{inf}} m_P}, \quad (37)$$

where $g_* = 228.75$ for MSSM and $\Gamma_{\text{inf}} \simeq \Gamma_h + \Gamma_H$.

Table III summarizes two representative benchmark points that accommodate metastable cosmic strings, are consistent with current cosmic microwave background (CMB) data, and yield successful nonthermal leptogenesis. The corresponding predictions for the scalar spectral index are aligned with the central values from Planck 2018 ($n_s = 0.965$) [45,46] and the recent ACT DR6 datasets ($n_s = 0.9743$) [47]. The baryon asymmetry of the Universe is explained through a nonthermal leptogenesis scenario,

TABLE III. Benchmark points for inflationary and reheating parameters consistent with metastable cosmic strings, CMB observations, and successful leptogenesis ($n_L/s \simeq 2.5 \times 10^{-10}$).

Parameter	Benchmark 1	Benchmark 2
$G\mu_{\text{cs}}$	1.3×10^{-7}	1.0×10^{-6}
M_ξ [GeV]	1.8×10^{15}	4.9×10^{15}
$m_{3/2}$ [TeV]	10	10
m_{inf} [GeV]	5×10^9	5×10^9
v_ϕ [GeV]	10^{16}	2×10^{16}
M_H [GeV]	4×10^{10}	4×10^{10}
T_r [GeV]	2×10^9	2×10^9
M_N [GeV]	2×10^{10}	2×10^{10}
x_0	1.008(1.007)	1.023(1.02)
x_e	1	1
n_s	0.965(0.9743)	0.965(0.9743)
r	9×10^{-14}	5×10^{-12}
N_0	50.6	51.2
κ	10^{-4}	10^{-4}
κ_{SS}	1	1
ξ	0.2–0.3	0.2–0.3
a	0.3(0.2)	9.8(7.0)
γ_h	2	2
γ_H	1	1
κ_S	0.014(0.009)	0.017(0.012)

generating the required lepton asymmetry $n_L/s \simeq 2.5 \times 10^{-10}$ [45].

In supersymmetric hybrid inflation, preheating is typically suppressed unless both the inflaton and the waterfall fields couple to an additional scalar sector. In such cases, efficient preheating may occur if the inflaton S possesses relatively large couplings to those scalars [48]. In our framework, the electroweak Higgs doublets h_u, h_d , and the Higgs pair $H, \tilde{H}$ could in principle play this role. However, efficient preheating would require either $\lambda_h \gg \kappa$ or $\lambda_H \gg \kappa$. Since in our benchmark scenario we consider $\lambda_h \sim \lambda_H \sim \kappa \sim 10^{-4}$, non-perturbative effects from preheating are expected to be strongly suppressed. Moreover, preheating into fermionic channels is generically inefficient due to Pauli blocking and thus does not alter this conclusion.

VII. CONSTRAINTS ON CURRENT-CARRYING METASTABLE COSMIC STRINGS

The strength of the string's gravitational interaction is expressed in terms of the dimensionless string tension, $G\mu_{\text{cs}}$, where $G = 1/8\pi m_P^2$ and μ_{cs} is mass per unit length of the string. The CMB bound on the standard CS tension is [45,46]

$$G\mu_{\text{cs}} \lesssim 1.3 \times 10^{-7}. \quad (38)$$

The quantity μ_{cs} can be written in terms of the $U(1)_R \times U(1)_{B-L}$ gauge symmetry breaking scale M_ξ as $\mu_{\text{cs}} \simeq 2\pi M_\xi^2$. In the standard scenario involving metastable cosmic strings, one typically obtains $M_\xi \lesssim 1.8 \times 10^{15}$ GeV corresponding

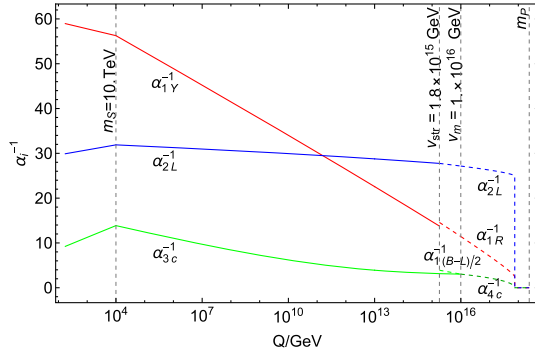


FIG. 3. Two loop RG evolution of the gauge couplings for the symmetries $SU(3)_c \times SU(2)_L \times U(1)_Y$, $U(1)_{B-L}$, and $SU(4)_c \times SU(2)_L \times U(1)_R$ with an additional 15-plet and a pair of $H, \bar{H}$. With this additional content, we obtain $g_m \sim 2$.

to $G\mu_{cs} \lesssim 1.3 \times 10^{-7}$, which is compatible with a metastable cosmic string network, as discussed in [49]. This possibility not only circumvents the CMB bound on $G\mu_{cs}$, but also evades other bounds coming from LIGO O3 [50]. In the case of current-carrying cosmic strings considered in Ref. [51], the LIGO O3 bounds can be evaded even with $G\mu_{cs} \sim 10^{-6}$ corresponding to $M_\xi \sim 4.9 \times 10^{15}$ GeV.

The metastable string network decays via the Schwinger production of monopole-antimonopole pairs with a rate per string unit length of [49,52–54]

$$\Gamma_d = \frac{\mu_{cs}}{2\pi} \exp(-\pi\kappa_{cs}), \quad (39)$$

where κ_{cs} quantifies the metastability of CSs network and is given by

$$\kappa_{cs}^{1/2} \approx \frac{M_m}{\sqrt{\mu_{cs}}} \sim \frac{4\pi M_X}{g_m^2} \frac{1}{\sqrt{2\pi} v_{str}}. \quad (40)$$

Here M_m is the monopole mass and $M_X = \sqrt{2/3}(g_m v_m)$ is the gauge boson mass associated with the grand unified theory (GUT) symmetry breaking responsible for monopole production. The parameters v_m and v_{str} represent the scales for the production of monopoles and CSs, respectively. The metastable CSs can explain the NANOGrav observations for $\sqrt{\kappa_{cs}} \sim 8$ (see Fig. 4), which implies

$$\frac{v_m}{v_{str}} \sim 2g_m, \quad (41)$$

where $g_m = g_4(M_X) = g_3(M_X)$ with $v_m = 2M/(3\xi)$ and $v_{str} = M_\xi$. Therefore, we obtain

$$\frac{v_m}{v_{str}} \sim \frac{2}{\sqrt{\frac{4}{3} - 9\xi^2}}, \quad (42)$$

implying $g_m \sim 1.4(1.0)$ for $\xi \sim 0.3(0.2)$. Thus, the existence of metastable CSs is compared to account for the

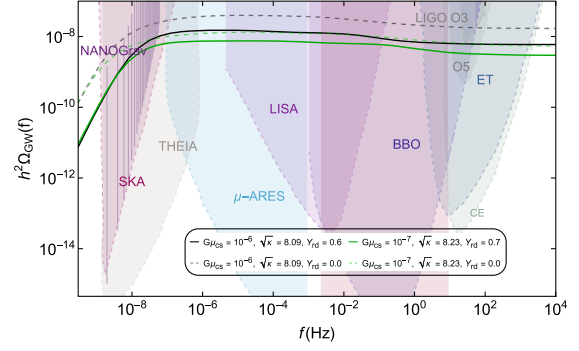


FIG. 4. GW spectra of the current carrying CSs for the benchmark points in Table III. This not only explains the NANOGrav 15-year dataset for $G\mu_{cs} \simeq 10^{-6}(10^{-7})$ with $\sqrt{\kappa_{cs}} \simeq 8.09(8.23)$ but also is consistent with the LIGO O3 run. The standard scenario has also been presented with the dashed lines. The colored shaded regions indicate the sensitivity curves of present (solid boundaries) LIGO O3 [50], NANOGrav [1], and future (dashed boundaries) SKA [57], THEIA [58], LISA [59], ET [60], BBO [61], μ -ARES [62], and CE [63] experiments.

NANOGrav results [2]. With the minimal matter content described in Table I, we obtain $g_m \sim 1$ as shown in Fig. 1. The evolution of gauge couplings under this scenario is illustrated in Fig. 3. For the benchmark points in Table III, with $G\mu_{cs} \simeq 10^{-6}(10^{-7})$ and $\sqrt{\kappa_{cs}} \simeq 8.09(8.23)$, the GW spectrum for the current-carrying CSs is shown in Fig. 4. We follow the dynamics given in [55,56] for the calculation of current-carrying CSs. The right handed neutrino zero modes residing on the CSs constitute a chiral current, and the major contribution arises from the cusps. In Fig. 4, we have also included the standard case of metastable CSs, and one can see that the GUT scale CSs ($G\mu \simeq 10^{-6}$) are in tension with the LIGO O3 run. But if they are current-carrying, as in our model, they not only evade the LIGO O3 bound but also are consistent with the NANOGrav 15-year dataset as well. Similar results for metastable CSs have previously been presented in [51].

VIII. CONCLUSIONS

We have shown that superheavy metastable strings can evade inflation in a supersymmetric model based on the gauge symmetry $SU(4)_c \times SU(2)_L \times U(1)_R$. The strings described here are current-carrying due to the presence of right handed neutrino zero modes, and they produce a GW spectrum that appears to be compatible with the recent pulsar timing array experiments and the LIGO O3 run. Measurements in other frequency ranges should provide additional tests of the model. Moreover, our considerations here can be extended to the breaking chain $SO(10) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_\chi \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$, previously discussed in [34,64]. The first breaking produces the $SO(10)$ GUT monopole as well as the “ χ ” monopole, with the latter being the analog of the “red” monopole in our $SU(4)_c \times SU(2)_L \times U(1)_R$ model.

Additional care is required to make sure that the symmetry-breaking scales are compatible with the NANOGrav and LIGO-VIRGO measurements.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [65].

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