

# Electromagnetic couplings of dark domain walls

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We extend Maxwell electrodynamics with a Chern-Simons coupling to a dark-sector top form sourced by domain walls. Cosmic birefringence can arise from a distinct mechanism in which photon polarization is rotated when crossing vacuum interfaces rather than through adiabatic propagation in a background field. Ultrathin walls induce a finite, frequency-independent polarization rotation generated by an electromagnetic Chern-Simons term localized at the interface. The effect persists even in the absence of ultralight axions or other propagating scalar degrees of freedom. For phenomenologically viable parameters, such walls can generate cosmic microwave background polarization rotation at the level  $\Delta\theta \sim 10^{-3}$  rad, providing a signature of the topological structure of the dark-sector vacuum.

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In this work, we show that electromagnetism can couple to domain walls beyond the Standard Model via a Chern-Simons interaction that mixes with a dark top form. As a result, bubbles of evanescent dark energy [1] may induce detectable polarization rotations. This is particularly interesting in light of recent hints of cosmic microwave background (CMB) polarization rotation at the level

$$\Delta\theta \sim 10^{-3} \text{ rad}, \quad (1)$$

which have generated significant interest as possible evidence for new dark sector [2]. The usual interpretation of this effect relies on electromagnetic coupling to a light pseudo-scalar field, whose slow spacetime variation induces an adiabatic rotation of photon polarization which accumulates along the line of sight,  $\Delta\theta \propto \int dx^\mu \partial_\mu \phi$  [3,4]. This is commonly taken to imply that the scalar is extremely light,  $m \lesssim H_{\text{LSS}} \sim 10^{-28}$  eV. Some authors note that an axion does not need to be as light, but *in all cases* they take it to be lighter than the CMB frequency range,  $m < 10^{-4}$  eV, to rely on the adiabatic approximation of [3,4].

Here we demonstrate that cosmic birefringence can arise as a *purely interface-localized effect*, generated when photons cross boundaries between topologically distinct vacua of a dark sector that support a Chern-Simons interaction on the wall. This reveals that electromagnetic birefringence falls into two sharply distinct universality classes:

- (i) *adiabatic birefringence*, in which polarization rotation accumulates continuously along the photon

trajectory due to smooth variation of a light axion, considered in past work; and

- (ii) *interface birefringence*, where polarization rotation changes discretely when a photon crosses a codimension-one vacuum boundary and is independent of propagation history, wall thickness, and photon wavelength below a physical cutoff.

The light axion scenarios considered as sources of cosmic birefringence [3–8] invariably involve axions lighter than CMB frequencies. Even when localized interface effects are mentioned, it is argued that such regimes are difficult to realize or suppressed [8].

In sharp contrast, we focus on *ultrathin walls*, which invalidate adiabatic description. Instead, the polarization rotation is intrinsically a scattering problem across a localized interface (see also [9]). Such walls induce a unitary transformation which rotates photon polarization by an angle set by the Chern-Simons coupling. Solving the wave equation, we find that the crossing implements a finite rotation [9]

$$\Delta\theta \simeq \frac{\zeta Q}{6\mathcal{M}^2}. \quad (2)$$

accompanied by reflection and frequency mixing. In the weak-coupling limit relevant for observations, reflection is suppressed and the effect reduces to a pure polarization rotation.

Crucially, this phenomenon persists even in the absence of any propagating axions. The interaction arises as a topological remnant of heavy microphysics, encoded in a mixed Chern-Simons coupling between electromagnetism and a dark top-form sector. As a result, the effect is sudden rather than adiabatic, localized at vacuum interfaces, and insensitive to the cosmological evolution between crossings. Our mechanism predicts that cosmic birefringence

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probes the topological structure of the dark-sector vacuum rather than the dynamics of light fields. In particular, a single spherical domain wall, or a distribution of such walls, can imprint a uniform polarization rotation across the sky without requiring ultralight particles.

As a concrete realization of interface birefringence, we consider the following gauge invariant and Lorentz invariant low-energy theory, working in the decoupling limit of gravity,  $M_{\text{Pl}} \rightarrow \infty$ , and so ignoring spatial curvature [1,10,11]:

$$S \ni \int d^4x \left\{ -\frac{1}{4g^2} F_{\mu\nu}^2 - A_\mu J^\mu - \frac{\zeta}{4!\mathcal{M}^2 g^2} \mathcal{H} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma} - \frac{1}{2} \mathcal{H}^2 + \frac{1}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu (\mathcal{H}) \mathcal{B}_{\nu\lambda\sigma} \right\} \\ - T \int d^3\xi \sqrt{\left| \det \left( \eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \right) \right|} - \frac{Q}{6} \int d^3\xi \mathcal{B}_{\mu\nu\lambda} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \frac{\partial x^\lambda}{\partial \xi^c} \epsilon^{abc}. \quad (3)$$

Here  $A_\mu$  and  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  are the  $U(1)$  gauge potential and field strength, respectively,  $g$  is the gauge-matter coupling constant, and  $J^\mu$  is the conserved current of charged matter (normalized to  $g^2 = 1$ ). The field  $\mathcal{H}$  is a pseudoscalar magnetic dual of the electric four-form (also known as top-form)  $U(1)$  field strength  $\mathcal{G}_{\mu\nu\lambda\sigma} = 4\partial_{[\mu} \mathcal{B}_{\nu\lambda\sigma]}$ , with  $\mathcal{B}_{\nu\lambda\sigma}$  the corresponding electric three-form gauge potential. The higher-rank forms are sourced by membranes of tension  $T \geq 0$  and charge  $Q$ . The mass scale  $\mathcal{M}$  originates from the ultraviolet completion of the top-form sector, and the dimensionless parameter  $\zeta$  controls the ratio  $Q/\mathcal{M}^2$ . As discussed in [1,10–14],  $\mathcal{M}$  may be identified with the strong-coupling scale of a non-Abelian gauge theory from which the top form emerges [12]. Below this scale, the naive discrete shift symmetry of the magnetic dual  $\mathcal{H} \rightarrow \mathcal{H} + Q$  is spontaneously broken by  $\mathcal{H}^2/2$  which is absent in the UV, which is restored by monodromy branch structure [15,16].

We imagine that the higher-rank form sector in Eq. (3) originates from a dark sector, while the familiar vector  $U(1)$  corresponds to Maxwell electromagnetism. The mixing between the two sectors may arise, for example, from kinetically mixed very heavy axions—one in each sector—monodromized by top forms [17–21].

A straightforward example of a consistent microscopic completion of the effective theory (3) is provided by a dark sector containing a non-Abelian gauge theory that becomes strongly coupled at a very low scale of the order of meV [1]. As shown by Lüscher [12], such theories naturally generate topological four-forms. If the four-form is taken to be massive, its longitudinal mode corresponds to a propagating pseudoscalar axion [17,18].

Crucially, even though our low-energy theory contains no propagating dark degrees of freedom, electromagnetic couplings can still arise consistently. If the emergent dark-sector axion is sensitive to physics above the dark-sector strong-coupling scale, it can mix either with the Standard Model axion or with a top-form sector that solves the strong  $CP$  problem through nonperturbatively generated operators. This induces a suppressed coupling between the dark sector and electromagnetism without introducing any light

electrically charged particles. Importantly, this interaction survives even if the longitudinal axion mass is raised to the dark-sector cutoff and the axion is integrated out.

Alternatively, one may also introduce heavy particles charged under the dark gauge group and carrying tiny fractional electromagnetic charges, consistent with existing bounds. Integrating out these heavy states can again contribute to the effective interaction Eq. (3), while their radiative corrections in the bulk remain suppressed by large masses or nonperturbative effects. Both approaches lead to the resulting low-energy theory of the form considered here: standard Maxwell electrodynamics in the bulk, with the leading new effect given by domain wall localized Chern-Simons terms.

Adopting this logic, we treat Eq. (3) strictly as a low-energy effective field theory valid below the dark-sector strong-coupling scale, after all propagating dark degrees of freedom have been integrated out. We do not assume the presence of ultralight axions or other light particles coupled to electromagnetism. Instead, the mixed electromagnetic-dark Chern-Simons interaction should be understood as a topological remnant of heavy microphysics, much like the  $\theta$  term in QCD. As a result, conventional cosmological constraints associated with light degrees of freedom (such as contributions to  $N_{\text{eff}}$  and large radiative corrections to low-energy QED) do not apply. The optical activity discussed below persists despite the complete absence of light axion fields which were crucial in the past proposals.

For the most part, the theory (3) behaves as conventional Maxwell electrodynamics. Varying the action with respect to  $A_\mu$  yields

$$\frac{1}{g^2} \partial_\mu F^{\mu\nu} = J^\nu - \frac{\zeta}{6\mathcal{M}^2 g^2} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu (\mathcal{H} F_{\lambda\sigma}). \quad (4)$$

In the presence of magnetic monopoles, the final term would be the  $CP$ -violating dyonic contribution to the monopole charge [22]. Varying instead with respect to  $\mathcal{B}_{\nu\lambda\sigma}$  gives

$$n^\mu \partial_\mu \mathcal{H} = Q\delta(r - r(t)), \quad (5)$$

where  $r(t)$  describes the wall trajectory,  $n^\mu$  is the outward-pointing normal vector satisfying  $n_\mu dx^\mu = dr(t)$ , and  $r$  is the coordinate along  $n^\mu$ . Away from the walls,  $\mathcal{H}$  is constant and, hence,  $\partial_\mu \mathcal{H} = 0$ . Consequently, in regions without membranes, the final term in Eq. (4) reduces to  $\frac{\zeta \mathcal{H}}{6\mathcal{M}^2 g^2} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu (F_{\lambda\sigma})$ , which vanishes identically in Maxwell theory in the absence of magnetic monopoles, since  $\partial_{[\mu} F_{\lambda\sigma]} = 0$ . One may redefine the field strength as

$$\tilde{F}^{\mu\nu} = F^{\mu\nu} + \frac{\zeta \mathcal{H}}{6\mathcal{M}^2} \epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma}. \quad (6)$$

Note that for the redefined field strength in (6), in general, the field equation for the dual field strength  $\star \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} \tilde{F}_{\lambda\sigma}$  is not a Bianchi identity even away from the wall. Using (4), (6), and  $\partial_{[\mu} F_{\lambda\sigma]} = 0$ , the relevant field equation is  $\partial_\sigma \tilde{F}^{\sigma\lambda} = -\frac{\zeta g^2}{3\mathcal{M}^2} \partial_\sigma (\mathcal{H} F^{\sigma\lambda})$ , and so off the wall,  $\partial_\sigma \tilde{F}^{\sigma\lambda} = -\frac{\zeta \mathcal{H} g^2}{3\mathcal{M}^2} J^\lambda$ . Hence, in terms of  $\tilde{F}^{\mu\nu}$ , all charged particles are dyons [22], carrying magnetic charges  $\mu = -\sqrt{\frac{\zeta \mathcal{H}}{3\mathcal{M}^2}} g$ . Nevertheless, since the ratio of the magnetic to electric charges  $\mu/g$  is the same for all particles, this is merely a matter of perspective and can be removed by returning to the original variables  $F_{\mu\nu}$ , as explained in [23].

The remaining variation of Eq. (3), which will be relevant below, is with respect to  $\mathcal{H}$ :

$$\frac{1}{4!} \epsilon^{\mu\nu\lambda\sigma} \mathcal{H}_{\mu\nu\lambda\sigma} = -\mathcal{H} - \frac{\zeta}{4! \mathcal{M}^2 g^2} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}, \quad (7)$$

where  $\mathcal{H}_{\mu\nu\lambda\sigma} = 4\partial_{[\mu} \mathcal{B}_{\nu\lambda\sigma]}$  is the spectator electric top form, with its flux fixed by the  $CP$ -violating membrane sources. Inverting the Hodge dualization in Eq. (7) gives

$$\mathcal{H}_{\mu\nu\lambda\sigma} = \mathcal{H} \epsilon_{\mu\nu\lambda\sigma} - \frac{\zeta}{\mathcal{M}^2 g^2} F_{[\mu\nu} F_{\lambda\sigma]}, \quad (8)$$

where  $\epsilon_{\mu\nu\lambda\sigma} \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} = -\delta_{\mu\nu\lambda\sigma}^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} = -4! F_{[\mu\nu} F_{\lambda\sigma]}$ . Solving for  $\mathcal{B}$  is straightforward.

The situation becomes more interesting in the presence of walls. In the rest frame of the wall, Eq. (5) implies  $\mathcal{H}_- = \mathcal{H}_+ - \mathcal{Q}$ , where  $\mathcal{H}_-$  and  $\mathcal{H}_+$  are the flux values inside (to the left of) and outside (to the right of) the wall, respectively. In the original coordinates,  $\mathcal{H} = \mathcal{H}_- + \mathcal{Q}\Theta(r - r(t))$ . Then to analyze how a membrane wall affects the electromagnetic fields, we consider walls which are large and treat them as approximately flat. Accordingly, we use

$$\mathcal{H} = \mathcal{H}_- + \mathcal{Q}\Theta(n_\mu (x^\mu - x_0^\mu(t))), \quad (9)$$

where  $x_0^\mu(t)$  is a parametric specification of the wall location and  $n_\mu$  is the outward normal.

We now examine the electromagnetic fields along a line of sight that crosses the wall. This problem shares some

common features with the propagation of light across axion domain walls [3,4], but there are some important differences. As in [3,4], we find that variables  $\hat{F}_{\mu\nu} = F_{\mu\nu} + \frac{\zeta}{12\mathcal{M}^2} \epsilon_{\mu\nu\lambda\sigma} \mathcal{H} F^{\lambda\sigma}$  introduced in Eq. (6) are convenient for understanding how electromagnetic fields cross the wall. There is an important distinction. In [3,4], they consider walls which are thick compared to the wavelength of the incident electromagnetic waves and use adiabatic limit to describe the propagation. In contrast, in our case, the walls are ultrathin compared to light wavelengths, and so we must use the complementary “sudden” approximation. Conceptually, this is like Fresnel’s description of light refraction on interfaces between dielectrics [23]. However, there are important simplifications.

First off, having infinite walls, and going to their rest frame, we pick the coordinates so the wall sits at the  $z = 0$  plane. Then, using the boost invariance of the wall in the  $z$ - $t$  plane [3,4], we can go to the frame where the incident waves come along the normal to the wall. Finally, since  $\mathcal{H}$  off the wall is constant and we are ignoring the curvature of space, the dispersion relation is  $\omega^2 = \vec{k}^2$  on both sides, and the index of refraction is  $n_\pm = 1$ . So, the waves are impacting the wall at right angles, propagating both after and before as waves in vacuum. Any specific case where the wave vector and wall normal are at an angle can be reduced to this by a Lorentz transformation.

Next, since the wall is ultrathin, the derivatives of  $\mathcal{H}$  are large but integrable and ultralocalized. Since the field equations (4) are  $\partial_\mu \tilde{F}^{\mu\nu} = 0$ , to them and the variables  $\tilde{F}^{\mu\nu}$  the large derivatives of  $\mathcal{H}$  are invisible. The problem is with  $F_{\mu\nu}$  and the Bianchi identities  $\partial_{[\mu} F_{\nu\lambda]} = 0$ , which are sensitive to the large derivatives on the wall. The variables  $\tilde{F}^{\mu\nu}$  balance this out, averaging the field variables at the wall and replacing them with principal values, which preserves the  $(+) \leftrightarrow (-)$  symmetry across the wall.

To understand what happens, we switch to  $E^i = F^{0i}$  and  $B^i = \frac{1}{2} \epsilon^{ijk} F_{jk}$ ; the redefined average gauge field strengths are, using  $\sigma = \frac{\zeta \mathcal{H}}{6\mathcal{M}^2}$ ,

$$\vec{\tilde{E}} = \vec{E} - \sigma \vec{B}, \quad \vec{\tilde{B}} = \vec{B} + \sigma \vec{E}. \quad (10)$$

The full set of Maxwell equations with vanishing local charges and currents is

$$\vec{\nabla} \cdot \vec{\tilde{E}} = \vec{\nabla} \cdot (\sigma \vec{B}), \quad \vec{\nabla} \times \vec{\tilde{B}} - \partial_t \vec{\tilde{E}} + \vec{\nabla} \times (\sigma \vec{E}) + \partial_t (\sigma \vec{B}) = 0, \quad (11)$$

$$\vec{\nabla} \cdot \vec{\tilde{B}} = \vec{\nabla} \cdot (\sigma \vec{E}), \quad \vec{\nabla} \times \vec{\tilde{E}} + \partial_t \vec{\tilde{B}} + \vec{\nabla} \times (\sigma \vec{B}) - \partial_t (\sigma \vec{E}) = 0. \quad (12)$$

The first two (11) are  $\partial_\mu \tilde{F}^{\mu\nu} = 0$ , and the second two (12) are the Bianchi identities  $\partial_{[\mu} F_{\nu\lambda]} = 0$  after we added and subtracted terms  $\propto \vec{\nabla} \cdot (\sigma \vec{E})$ ,  $\vec{\nabla} \times (\sigma \vec{B})$ , and  $\partial_t (\sigma \vec{E})$  and

used (10) under derivatives. We now take  $\sigma \ll 1$  and treat the wall interactions as a perturbation of the vacuum equations, motivated by the case of axion walls [3,4].

Since for  $\vec{E}, \vec{B}$  in the absence of charges and currents  $\rho = \vec{j} = 0$  we have  $\vec{\nabla} \cdot \vec{E} = 0$  and  $\vec{\nabla} \times \vec{B} = \partial_t \vec{E}$  to leading order in the  $\sigma$  expansion, and  $\partial_t \sigma = 0$ ,  $\vec{\nabla} \sigma \parallel \vec{n} \parallel \vec{k}$ , where  $\vec{k}$  is the wave vector, and the waves are right-handed triads  $\vec{k}, \vec{E}, \vec{B}$ , Eqs. (11) and (12) reduce to

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= 0, & \vec{\nabla} \times \vec{B} - \partial_t \vec{E} &= -\vec{\nabla} \sigma \times \vec{E}, \\ \vec{\nabla} \cdot \vec{B} &= 0, & \vec{\nabla} \times \vec{E} + \partial_t \vec{B} &= -\vec{\nabla} \sigma \times \vec{B}. \end{aligned} \quad (13)$$

We integrate Eq.(13) over Gaussian pillboxes encroaching the wall, with bases parallel to it. The first and the third give the answers for  $\vec{E}, \vec{B}$  which look just like the standard results for  $\vec{E}, \vec{B}$  across the charge- and current-free interfaces between dielectrics, since those equations look the same [23]. The second and the fourth equations involve an extra term  $\propto \vec{\nabla} \sigma$ , and we need its circulation along a closed infinitesimal rectangle straddling the interface. Using  $\vec{\nabla} \sigma \parallel \vec{n}$ , in both cases the contributions from the paths parallel with the interface vanish, since  $\vec{\nabla} \sigma = 0$  on either side, and those on the paths crossing the interface are orthogonal to the paths. So the boundary conditions controlling the fields across the wall are

$$\begin{aligned} \vec{n} \cdot (\vec{E}_+ - \vec{E}_-) &= 0, & \vec{n} \times (\vec{B}_+ - \vec{B}_-) &= 0, \\ \vec{n} \cdot (\vec{B}_+ - \vec{B}_-) &= 0, & \vec{n} \times (\vec{E}_+ - \vec{E}_-) &= 0. \end{aligned} \quad (14)$$

These look precisely the same as the standard boundary conditions on dielectric interfaces—but for  $\vec{E}, \vec{B}$ . Ergo, it is these variables that cross the wall “smoothly.” In turn, the canonically normalized fields undergo a propagation-induced *electromagnetic duality transformation* of the original monopole-free  $U(1)$  field strengths [23,24] on the wall. Defining the rotation angle  $\vartheta$  via  $\cos \vartheta = (1 + \sigma^2)^{-1/2}$  and  $\sin \vartheta = \sigma(1 + \sigma^2)^{-1/2}$ , so that  $\sigma = \tan \vartheta$ , and factorizing the transformation encoded by (14) into a rotation matrix and a rescaling, we find

$$\begin{aligned} & \frac{1}{g^2 \cos \vartheta_-} \begin{pmatrix} \cos \vartheta_- & -\sin \vartheta_- \\ \sin \vartheta_- & \cos \vartheta_- \end{pmatrix} \begin{pmatrix} \vec{E}_- \\ \vec{B}_- \end{pmatrix} \\ &= \frac{1}{g^2 \cos \vartheta_+} \begin{pmatrix} \cos \vartheta_+ & -\sin \vartheta_+ \\ \sin \vartheta_+ & \cos \vartheta_+ \end{pmatrix} \begin{pmatrix} \vec{E}_+ \\ \vec{B}_+ \end{pmatrix}. \end{aligned} \quad (15)$$

The resulting optical activity is the field-space rotation [23,24]

$$\begin{aligned} \mathcal{R}(+ \rightarrow -) &= \begin{pmatrix} \cos \vartheta_- & \sin \vartheta_- \\ -\sin \vartheta_- & \cos \vartheta_- \end{pmatrix} \begin{pmatrix} \cos \vartheta_+ & -\sin \vartheta_+ \\ \sin \vartheta_+ & \cos \vartheta_+ \end{pmatrix} \\ &= \begin{pmatrix} \cos \Delta \vartheta & -\sin \Delta \vartheta \\ \sin \Delta \vartheta & \cos \Delta \vartheta \end{pmatrix}, \end{aligned} \quad (16)$$

which strictly speaking in our case holds only for small  $\Delta \vartheta$ , as we neglected  $\mathcal{O}(\sigma^2)$  contributions in deducing (14). Specifically, if  $\vec{E}$  field initially points along the  $x$  direction and  $\vec{B}$  field along  $y$  outside the wall, then after crossing both are rotated by the same frequency-independent angle:

$$\Delta \vartheta \simeq \frac{\zeta Q}{6\mathcal{M}^2}. \quad (17)$$

Curiously, the transformation in Eq. (15) also suggests a rescaling of electric and magnetic fields across the wall. However, for small  $\sigma$  this effect enters only at subleading order  $\mathcal{O}(\sigma^2)$ . In a complete description [9], this rescaling is countermanded by unitarity, since the subleading reflection from the wall, neglected here since it appears at higher orders in perturbation theory, compensates these terms. By contrast, the polarization rotation arises already at linear order and is robust. We, therefore, focus on it as the leading physical signature of the interface interaction.

Any bounds on the electromagnetic couplings to the walls depend on encountering walls, which is controlled by the nucleation rates of membranes charged under  $\mathcal{H}$ . In the toy model of thin walls coupled to electromagnetism in Eq. (3), the nucleation rates depend on the background field values and theory parameters. A crucial subtlety arises from the UV behavior of the theory. If the top-form description remains valid up to a high cutoff, so that the membrane radius at nucleation exceeds the inverse cutoff, the semi-classical description of nucleation is reliable. In this case, the rate can be computed using standard techniques [25–27], treating the process as a Schwinger-like discharge of extended objects [28–30].

By contrast, if the top form emerges only below a symmetry-breaking scale—as in the case of QCD [12–14] or in discretely evanescent dark-energy proposal [1]—symmetry restoration above that scale renders vacua degenerate and removes the top form from the spectrum. So if the membrane size at nucleation were shorter than this scale, a full UV completion is required.<sup>1</sup> In our examples, the UV completion is provided by asymptotically free Yang-Mills theories with chiral symmetry and degenerate vacua in the ultraviolet. In this regime, there is neither a top form to discharge nor an energy difference between vacua. They emerge only in the IR. Therefore, the nucleation rate must be extremely suppressed.

<sup>1</sup>A simple way to understand this is the Heisenberg uncertainty principle  $\Delta x \Delta p \gtrsim 1$ , which says that if we cut off the momenta at scales  $p_* \sim \Lambda$ , we cannot describe phenomena of spatial extent less than  $\Delta x \sim 1/\Lambda$ .

To proceed, we rewrite Eq. (3) in Euclidean space [31,32]. Defining the Euclidean action via  $iS = -S_E$  and integrating the bilinear term by parts to account for membrane boundary conditions, we obtain

$$S_E = \int d^4x \left( \frac{\zeta}{6\mathcal{M}^2 g^2} \mathcal{H} \vec{E} \cdot \vec{B} + \frac{1}{2} \mathcal{H}^2 + \frac{1}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu (\mathcal{H}) \mathcal{B}_{\nu\lambda\sigma} \right) + \mathcal{T} \int d^3\xi \sqrt{\gamma_B} - \frac{\mathcal{Q}}{6} \int d^3\xi \mathcal{B}_{\mu\nu\lambda} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} \frac{\partial x^\lambda}{\partial \xi^\gamma} \epsilon^{\alpha\beta\gamma}, \quad (18)$$

with the standard Lorentzian-to-Euclidean mapping [31,32]. We retain the background contribution  $\vec{E} \cdot \vec{B}/g^2$ . When the fields are approximately constant, the dominant contribution arises from  $O(4)$ -symmetric tunneling; subleading channels yield at most  $\mathcal{O}(1)$  corrections and will be neglected. Treating  $\vec{E} \cdot \vec{B}/g^2$  as stationary and constant over the region relevant for the tunneling is a good approximation for small nucleated bubbles.

Using (18), the tunneling configurations correspond to slices of a four-dimensional sphere  $S^4$  glued along a fixed  $S^3$  latitude. These configurations describe Euclidean world volumes of spherical membranes with tension  $\mathcal{T}$  and charge  $\mathcal{Q}$ . Detailed analyses of such processes can be found in [29–32]. In the limit  $M_{\text{Pl}} \rightarrow \infty$ , the only relevant configuration is the Euclidean bounce connecting backgrounds with vacuum energy

$$V = \frac{1}{2} \mathcal{H}^2 + \frac{\zeta}{6\mathcal{M}^2 g^2} \mathcal{H} \vec{E} \cdot \vec{B}, \quad (19)$$

where the interior and exterior values of  $\mathcal{H}$  differ by a single unit of charge,  $\Delta\mathcal{H} = \mathcal{Q}$ , as implied by the Euclideanized equation (5). At least the initial vacuum energy must be non-negative. The corresponding energy difference  $\Delta V$  inside the membrane, arising from the flux discharge, must balance the cost of creating a membrane with tension  $\mathcal{T}$  [25].

The “statics” of the configuration are governed by d’Alembert’s principle of virtual work, which compares the action (18) evaluated on a configuration containing a single bubble to that of the smooth background. For constant  $\Delta V$  and  $\mathcal{T}$ , the relevant volume factors are  $V_{S^4} = \pi^2 r_0^4/2$  and  $V_{S^3} = 2\pi^2 r_0^3$ , yielding

$$S_{\text{membrane}} = 2\pi^2 r_0^3 \mathcal{T} - \frac{1}{2} \pi^2 r_0^4 \Delta V, \quad (20)$$

where, from (19),

$$\Delta V \simeq \left( \mathcal{H} + \frac{\zeta}{6\mathcal{M}^2 g^2} \vec{E} \cdot \vec{B} \right) \mathcal{Q}. \quad (21)$$

Minimizing (20) with respect to  $r_0$  gives

$$r_0 = \frac{3\mathcal{T}}{\Delta V}, \quad B = \frac{\pi^2}{2} \mathcal{T} r_0^3. \quad (22)$$

The nucleation rate per unit spacetime volume is  $\Gamma = Ae^{-B}$  [25,26]. In the limit  $M_{\text{Pl}} \rightarrow \infty$ , the prefactor

$A$  was computed in [27], yielding

$$\Gamma \simeq \mathcal{T}^2 r_0^2 \exp\left(-\frac{\pi^2}{2} \mathcal{T} r_0^3\right). \quad (23)$$

Requiring  $r_0 > 0$  implies  $\Delta V > 0$ . From (21), two regimes arise: (i)  $\vec{E} \cdot \vec{B}/g^2 \ll 3\mathcal{M}^2 \mathcal{H}/\zeta$  and (ii)  $\vec{E} \cdot \vec{B}/g^2 \gg 3\mathcal{M}^2 \mathcal{H}/\zeta$ .

In the first regime, the bounds reduce to the familiar cosmological constraints from the CMB, dating back to [33] for large, horizon-crossing domain walls. Recently, it has been suggested that additional bounds may arise if the bubble-sector scales are very low and nucleations become prolific when late-time vacuum energy dominates [34]. Both situations have been explored in the context of relaxing strong  $CP$  violation [10,11] and discretely evanescent dark energy [1]. Such phenomena, including possible gravitational-wave signatures, could provide additional observational probes of these dynamics.

We now estimate how the nucleation rate is affected by the electromagnetic  $\vec{E} \cdot \vec{B}/g^2$  term. The most controlled laboratory scenario for enhancing nucleation could involve stationary, uniform, and parallel electric and magnetic fields. In this case, the magnitude of  $\vec{E} \cdot \vec{B}/g^2$  is limited by the maximum steady fields achievable in practice. Laboratory magnetic fields do not exceed  $\sim \mathcal{O}(10)$  T, while pulsed lasers can produce electric fields as large as  $\sim 10^{13} - 10^{14}$  V/m, implying  $\vec{E} \cdot \vec{B}/g^2 \simeq 0.1$  (keV)<sup>4</sup>.

When  $\mathcal{M}^2 \mathcal{Q}/\zeta \gg 0.1$  (keV)<sup>4</sup>, the electromagnetic correction to the bubble production rate is negligible. The nucleation rate is then suppressed by the vacuum tunneling barrier, similar to the previous case, and the parameter ranges relevant for restoring strong  $CP$  violation [10,11] remain valid. In that example,  $\sqrt{\mathcal{Q}} \sim \mathcal{M} \sim 3$  keV and  $\zeta \lesssim 1$ , dominating over  $\vec{E} \cdot \vec{B}/g^2$ , which implies that the electromagnetic coupling of the walls can be substantial, with the coefficient of  $\vec{E} \cdot \vec{B}/g^2$  in (3) being close to unity.

A key difference relative to cosmological bubble production is that the relevant four-volume is not the Hubble volume but the “world volume of the experiment,”  $\Gamma \Omega_{\text{lab}}$ , adapting the approach of [35,36]. Here,  $\Omega_{\text{lab}}$  is the spacetime volume occupied by the region with nonzero  $\vec{E} \cdot \vec{B}/g^2$  during the experiment. For steady fields probed with laser pulses, a generous estimate gives  $\Omega_{\text{lab}} \sim (\text{m})^3 \times 10^{-18} \text{ s} \sim 10^{12} \text{ eV}^{-4}$ .

Putting it together, if the membranes responsible for  $CP$  restoration [10,11] couple to electromagnetism as proposed here, and if their cosmological production rate satisfies  $\Gamma \sim H_{\text{QCD}}^4 \sim 10^{-36} \text{ eV}^4$  so that all discharges complete before big bang nucleosynthesis [10,11], then the expected laboratory event rate is  $\Gamma \Omega_{\text{lab}} \sim 10^{-24}$  for  $\zeta \sim \mathcal{O}(1)$ . Further reducing the electromagnetic coupling suppresses this rate even more. The bubble radius at nucleation in this regime is  $r_0 \sim \mathcal{T}/\Delta V \sim \text{few keV}^{-1}$ , consistent with the experimental

energy scales and the cutoff of the theory, which validates our estimate. In sum, the nucleation rate is extremely suppressed.

If, instead, the membrane tension and charge are much smaller, as in the dark-energy model of [1], the semiclassical barrier suppression is absent unless the electromagnetic coupling is extremely small,  $\zeta \lesssim 10^{-23}$ . In this case, the semiclassical nucleation radius would be  $r_0 \sim \mathcal{T}/\Delta V \sim 10^{-21} \text{ eV}^{-1}$ , requiring the effective description to remain valid up to energies  $\mathcal{E} \gtrsim 10^{12} \text{ GeV}$ . However, the cutoff of this sector is only of the order of meV [1]. Above the cutoff, the dark sector becomes an asymptotically free Yang-Mills theory with degenerate vacua and no top form. Thus, steady lab fields will not yield prolific nucleation rates.

Much stronger but highly localized electromagnetic fields occur in particle colliders and have been studied in connection with  $CP$  violation and the chiral magnetic effect [37]. At the RHIC and LHC, heavy-ion collisions can generate magnetic fields  $\sim 10^{14}\text{--}10^{15} \text{ T}$  and electric fields  $\sim 10^{18}\text{--}10^{19} \text{ V/m}$ . Because the ions are ultrarelativistic, the fields satisfy  $|\vec{E}| \sim |\vec{B}|$ . While individual ions primarily generate radiative fields with  $\vec{E} \cdot \vec{B} \ll \vec{B}^2$ , stochastic fluctuations in the swarm of  $\mathcal{O}(100)$  ions produce peak regions with  $\vec{E} \cdot \vec{B} \sim \vec{B}^2$ . This yields  $\vec{E} \cdot \vec{B}/g^2 \lesssim 10^{30} \text{ eV}^4$  [38,39], but these fields persist only over spatiotemporal scales  $\sim 10^{-16} \text{ m} - 10^{-10} \text{ eV}^{-1}$ , so that  $\Omega_{\text{lab}} \sim 10^{-40} \text{ eV}^{-4}$ .

When  $\Delta V$  is dominated by  $\vec{E} \cdot \vec{B}/g^2$  in (21), the semiclassical bubble radius at nucleation  $r_0 \sim \mathcal{T}/\Delta V$  will be small, and the semiclassical barrier is absent. For the discretely evanescent dark-energy model of [1], with  $\mathcal{M} \sim \mathcal{T}^{1/3} \sim \mathcal{Q}^{1/2} \sim 10^{-3} \text{ eV}$ , one finds  $\Delta V \sim \zeta \times 10^{30} \text{ eV}^4$  and  $r_0 \sim 10^{-39} \text{ eV}^{-1}/\zeta$ . This is far above any reasonable UV cutoff of the effective theory. The IR description fails, but since the UV completion has degenerate vacua and no top forms, the nucleations are extremely suppressed.

For the QCD-motivated scenario of [10,11], the bubble radius is larger by the cube of the ratio of cutoffs,  $r_0 \sim 10^{-21} \text{ eV}^{-1}$ , corresponding to energies  $\mathcal{E} \sim 10^{12} \text{ GeV}$ . If the discharging sector in this case is UV completed by an asymptotically free gauge theory which enters strong coupling and chiral symmetry breaking below  $10^{12} \text{ GeV}$ , below which the top form is induced, the membrane

production in these circumstances would also evade copious production in heavy-ion collisions, despite large electromagnetic fields and significant wall couplings to electromagnetism at low energies. A similar conclusion applies to magnetars, whose enormous electromagnetic fields also involve energies where top forms and membranes have already decoupled.

Additional constraints from cosmology concern a straggler bubble nucleated far from the peak nucleation epoch could survive percolation and later reenter our Hubble volume. If its tension were too large, the resulting wall would distort the CMB. Requiring that its total energy be smaller than the energy in a Hubble volume,  $E_{\text{hubble}} \simeq \rho_0/H_0^3$ , by a factor of  $10^{-6} - 10^{-7}$  [33] implies  $\mathcal{T} < 10^{-6} M_{\text{Pl}}^2 H_0 \sim \text{few} \times (100 \text{ keV})^3$ .

The membranes considered in [1,10,11] can easily satisfy both this bound and the limits derived from the absence of nucleations in strong electromagnetic fields while still coupling nontrivially to electromagnetism via the Maxwell-Chern-Simons term in (3). This is particularly intriguing in light of claims that weak optical activity might have been observed [2], at the level of, roughly,

$$\Delta\theta \sim 10^{-3} \text{ rad}, \quad (24)$$

based on CMB polarization surveys. Such effects may easily arise from electromagnetic interactions with an evanescent dark-energy domain wall, with an electromagnetic Chern-Simons coupling  $\zeta \mathcal{Q}/(6\mathcal{M}^2) \sim \text{few} \times 10^{-3}$ . Note that in our cases the optical activity arises purely due to the topological features of the dark-sector gauge theories, without requiring ultralight axions, distinguishing it from standard realizations in the literature [3–8]. In the discretely evanescent dark-energy scenario, this would also be accompanied by an  $\mathcal{O}(1)$  jump in the dark-energy density at low redshift. These coincidences merit further scrutiny.

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