Left-right asymmetry calculation comparisons and projected sensitivity to the weak mixing angle in polarized Bhabha scattering at 10.58 GeV

C. Miller* and J. M. Roney

University of Victoria, 3800 Finnerty Rd, Victoria, BC V8P 5C2, Canada

(Received 8 January 2025; accepted 9 June 2025; published 10 July 2025)

Consideration is being given to upgrading the SuperKEKB electron-positron collider with the introduction of electron-beam polarization. This would enable a unique precision electroweak physics program that opens new ways to search for physics beyond the Standard Model. The upgrade would enable Belle II to make a number of high precision measurements, one of which is the left-right cross-section asymmetry in the $e^+e^- \to e^+e^-$ Bhabha scattering process. The expected level of precision in such a measurement will require the theoretical values of the asymmetry to be calculated at least to the next-to-leading order (NLO) level, and the implementation of simulation event generators with a similar level of precision. In this study, we compare the calculations of the ReneSANCe Monte Carlo generator with those of an independent NLO calculation to determine the level of agreement in this process. An average difference of 0.3% between the calculations is found. Using the published Belle II efficiency for selecting Bhabha events and assuming a 40 ab^{-1} dataset having 70% polarization, the projected uncertainty on the weak mixing angle, $\sin^2\theta_W$, is calculated using ReneSANCe to be ± 0.00032 or better. Combining this with left-right asymmetry measurements from muons and taus under the assumption of lepton universality yields a projected overall uncertainty of ± 0.00019 on $\sin^2\theta_W$ with SuperKEKB upgraded to have polarized electron beams.

DOI: 10.1103/d462-zrcl

I. INTRODUCTION

A proposed upgrade to the SuperKEKB electron-positron collider would introduce polarization to the electron beam [1]. The upgrade, referred to as Chiral Belle, is designed to introduce polarization to the collider without compromising SuperKEKB's ability to reach its design luminosity. The addition of a polarized beam opens a new and unique precision program that enables Belle II to determine the weak mixing angle, $\sin^2\theta_W$, at a 10.58 GeV center-of-mass energy by measuring the left-right cross-section asymmetry, A_{LR}, separately for electrons, muons, taus, b-quarks, and c-quarks. Included in its broader program, Chiral Belle will also measure the τ-lepton anomalous magnetic moment at a level of precision two orders of magnitude higher than has been achieved to date. The Chiral Belle measurement of A_{LR} for the $e^+e^- \to e^+e^-(\gamma)$ process (Bhabha scattering) will complement the proposed measurement of $\sin^2\theta_W$ by the MOLLER experiment [2], which is projected to determine $\sin^2\theta_W$ from electron-electron scattering at the 100 MeV energy scale with an uncertainty of ± 0.00028 [3]. Chiral Belle and MOLLER will probe the running of $\sin^2\theta_W$ via Z^0-electron couplings at two different energies well below the Z^0-pole and will thereby be sensitive to dark sector or other physics beyond the SM that modifies the running of $\sin^2\theta_W$ [4]. The running will also be probed from electron-hadron scattering asymmetries with less precision at the Electron-Ion Collider (EIC) [5].

By measuring ratios of A_{LR} separately for five different species of fermion, the Chiral Belle program will also provide world-leading measurements of the universality of the Z^0-fermion couplings. This paper focuses on predictions of A_{LR} from Bhabha scattering at 10.58 GeV and projections of the uncertainties on $\sin^2\theta_W$ associated with Z^0-electron couplings that can be obtained with Chiral Belle.

In order to prepare for and perform these measurements, the Belle II collaboration will require Monte Carlo (MC) event generators, which, at a minimum, fully account for the next-to-leading order (NLO) effects. The ReneSANCe MC generator [6] is one such NLO generator, which also accounts for the initial state polarization states in the $e^+e^- \to e^+e^-(\gamma)$ process and radiative effects. In presenting a comparison of the ReneSANCe calculations with an

*Contact author: calebmiller@uvic.ca

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

independent NLO calculation [7] and making projections for Chiral Belle's expected sensitivity to $A_{\rm LR}$ and $\sin^2\theta_W$, full NNLO calculations of this process are motivated.

The left-right electroweak asymmetry: Due to the presence of the weak interaction, the $e^+e^- \to f\bar{f}$ process has an asymmetry in the cross sections for left-handed and right-handed beam polarizations. This left-right asymmetry ($A_{\rm LR}$) is defined as [7,8]

$$A_{\rm LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \tag{1}$$

where $\sigma_L(\sigma_R)$ is the left(right) handed cross section. The Standard Model (SM) provides predictions of $A_{\rm LR}$. For example, at Born level in the s-channel for the $e^+e^- \to f\bar{f}$ process

$$A_{\rm LR}^{\rm meas} = A_{\rm LR}\langle P\rangle = \frac{sG_F}{\sqrt{2}\pi\alpha Q_f} g_A^e g_V^f \langle P\rangle, \tag{2}$$

where s is square of the center-of-mass energy, G_F is the Fermi constant, α is the fine structure constant, Q_f is the electric charge of the final state fermion, g_A^e is the neutral current axial coupling of the initial state electron, g_V^f is the neutral current vector coupling of the final state fermion (f)

$$g_V^f = T_3 - 2Q_f \sin^2\theta_W \tag{3}$$

$$g_A^e = T_3, \tag{4}$$

where T_3 is the weak isospin ($\pm 1/2$) and Q_f is the charge of the fermion. The $\langle P\rangle$ factor in Eq. (2) is the average longitudinal polarization of the mediator in the e^+e^- collision, defined as

$$\langle P\rangle = \frac{R^+L^- - L^+R^-}{R^+L^- + L^+R^-}, \tag{5}$$

where $L^\pm$ ($R^\pm$) is the fraction of positrons (+) or electrons (−) in their respective beams that have left-handed (right-handed) spin, so that $L^\pm + R^\pm \equiv 1$. Chiral Belle plans to operate with $\langle P\rangle = 70\%$ and to switch the beam polarization between left and right polarization states during data taking, in order to collect the left and right polarized datasets from which to extract $A_{\rm LR}$. Compton polarimetry is expected to yield real-time measurements of the polarization at the sub-1% level every five minutes [9]. The polarization will also be precisely measured at the interaction point using the tau decay polarimetry technique developed by *BABAR* [10].

At a 10.58 GeV center-of-mass energy, both the s-channel and t-channel Bhabha processes, depicted in Fig. 1, contribute significantly to the value of $A_{\rm LR}$. The Belle II center-of-mass energy of 10.58 GeV is well below the Z-pole which leads to γ-Z electroweak interference

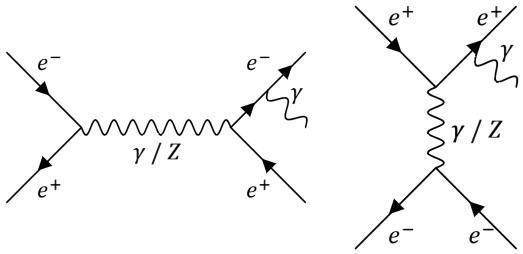


FIG. 1. Sample $e^+e^- \to e^+e^-\gamma$ Feynman diagrams of the s-channel (left) and t-channel (right) processes. Note that the photon can be emitted from any electron or positron.

terms dominating the asymmetry. At Born level [7]

$$A_{\rm LR}^0 = \frac{4v_e^Z a_e^Z s s_\theta^2}{m_Z^2} \times \frac{c_\theta(21 + c_\theta^2) + 12l_\theta}{c_\theta(c_\theta^4 + 26c_\theta^2 - 75) - 24s_\theta^2 l_\theta}, \tag{6}$$

where θ is the angle between the initial state electron momentum vector and the final state electron momentum vector, s_θ is $\sin\theta$, c_θ is $\cos\theta$, and l_θ is $\ln\frac{1-c_\theta}{1+c_\theta}$. The v_e^Z and a_e^Z terms are defined as

$$v_e^Z \equiv \frac{g_V^e}{2\sin\theta_W\cos\theta_W} \tag{7}$$

$$a_e^Z \equiv \frac{g_A^e}{2\sin\theta_W\cos\theta_W}. \tag{8}$$

In Figs. 2 and 3 of Ref. [7] both $A_{\rm LR}$ and $A_{\rm LR\Sigma}$ at 10.58 GeV are presented, where $A_{\rm LR\Sigma}$ is $A_{\rm LR}$ integrated over an angular acceptance

$$A_{\rm LR\Sigma} = \frac{\Sigma_L - \Sigma_R}{\Sigma_L + \Sigma_R} \tag{9}$$

and

$$\Sigma_{L(R)} = \int_{-\cos a}^{\cos a} \frac{d\sigma_{L(R)}}{d\cos\theta} \cdot d\cos\theta. \tag{10}$$

In Ref. [7] $A_{\rm LR}$ and $A_{\rm LR\Sigma}$ are plotted for values of θ, for the electron, ranging from 5° to 85° while constraining the positron to $|\cos\theta_{e^+}| \le \cos 20°$.

II. ReneSANCe MC GENERATOR

The ReneSANCe generator was developed to provide simulated events for electron-positron collisions extending to the TeV energy regime at full NLO with radiative corrections [6]. By default the generator accounts for the polarization of the initial state particles and allows them to be set symmetrically to a polarization ranging from 0 to 1. As Chiral Belle will feature a single polarized beam, the ReneSANCe authors added asymmetric beam polarization states as a feature to enable this study.

In order to calculate $A_{LR\Sigma}$, ReneSANCe can calculate the difference and sum of the left and right-handed cross sections directly [the numerator and denominator terms seen in Eq. (1)]. Using this method the $A_{LR\Sigma}$ can be calculated for various angular acceptances.

Generator settings: In order to compare the ReneSANCe calculations to those of Ref. [7], the following ReneSANCe default SM parameters were changed to correspond to those in Ref. [7]: the width of the W boson and the top quark are set to 0, $m_H = 125$ GeV, $m_Z = 91.1876$ GeV, $m_W = 80.4628$ GeV, $m_u = 69.83$ MeV, $m_d = 69.84$ MeV, $m_c = 1.2$ GeV, $m_s = 0.15$ GeV, $m_t = 174$ GeV, and $m_b = 4.6$ GeV. The center-of-mass energy is set to 10.577 GeV and the soft photon cutoff, defined as $ome * \sqrt{s}/2$, is set to $ome = 0.002$.

III. A_{LR} COMPARISONS

In order to make the comparisons between the ReneSANCe generator output and the calculations from Ref. [7], the points plotted in Fig. 2 (A_{LR} *vs* θ) and Fig. 3 [$A_{LR\Sigma}$ *vs* a, where a is defined in Eq. (10)] of Ref. [7] were extracted graphically, and an uncertainty of 0.2% assigned from the average value of Table 4 of Ref. [7].

In ReneSANCe, A_{LR} is calculated by finding $A_{LR\Sigma}$ in bins of width 0.1 in $\cos\theta$ between $-1 < \cos\theta < 1$. The central value of the bin reflects the average θ of events in the bin and the calculated $A_{LR\Sigma}$ value is the average value of A_{LR} in the bin. The results are shown in Fig. 2.

For $A_{LR\Sigma}$ the symmetric angular acceptance as defined by equations Eqs. (9) and (10) is varied from $a = 5°$ to $a = 85°$ in steps of 10°. The two calculations are overlaid in Fig. 3. An average difference of 4.4×10^{-7}, equivalent to a relative difference of 0.3%, is evident. The observed difference exceeds any uncertainties associated with extracting the points from the plots in [7] and MC statistics from running ReneSANCe. Both Ref. [7] and ReneSANCe compare their calculations to other sources, with WHIZARD in common between them. These comparisons are carried out for the Born level process and with the addition of radiative effects, and are all in agreement. We therefore assume the difference arises from the calculation of the virtual/internal NLO contributions. However, this difference is also negligible for the purposes on this study.

IV. ESTIMATE OF NNLO SCALE IN $A_{LR\Sigma}$

The scale of the NNLO contributions is conventionally estimated as $\alpha \times$ NLO contribution, i.e., on the order of 1% of the NLO contributions, which translates into an uncertainty on the asymmetry of $\delta A_{LR\Sigma}^{\mathrm{NNLO}} \sim 1.5 \times 10^{-6}$. The NNLO contributions to $e^+e^- \to \tau^+\tau^-$ cross section have been calculated and reported in Table 1 of Ref. [11]. Their results show the NLO effects act as a ~18% correction to the Born level cross sections, while the NNLO effects contribute a ~0.5% correction to the Born level (or a ~3% correction

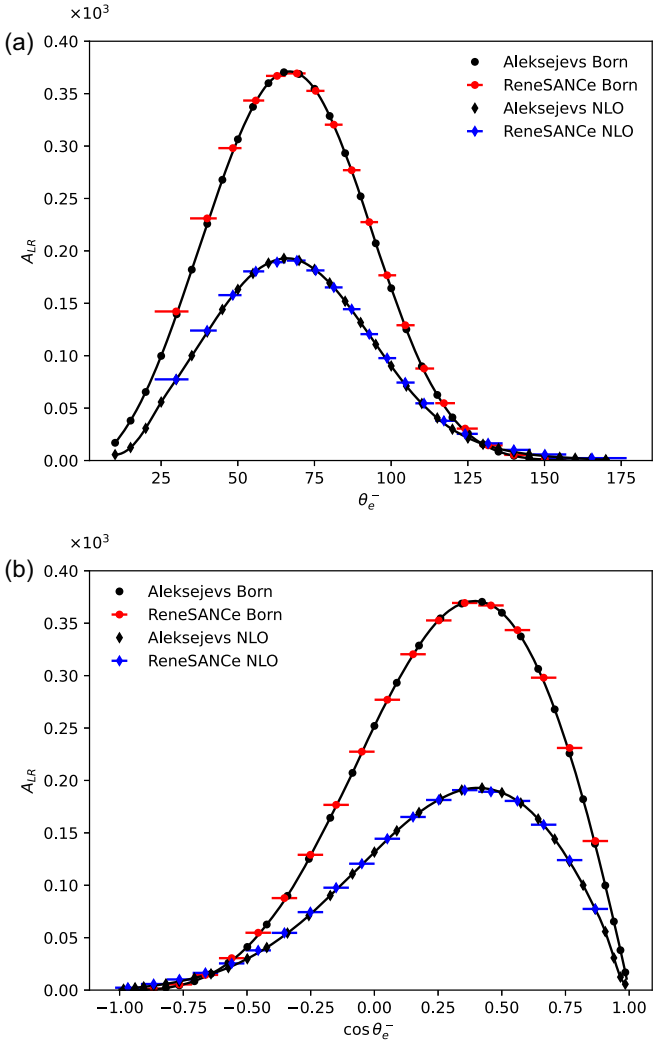


FIG. 2. Comparison of the calculations of A_{LR} in Bhabhas from Ref. [7] and ReneSANCe. A cubic spline is employed to illustrate the trend. The horizontal error bars represent the bin width of $\cos\theta = 0.10$. The exponential nature of the Bhabha cross section results in the central value being biased toward the edge with higher statistics. Subfigure (a) shows the distribution as a function of the final-state electron angle as presented in Fig. 2 of Ref. [7], while subfigure (b) shows the same data distributed in $\cos\theta$.

to the NLO values). However the lack of the t-channel in $e^+e^- \to \tau^+\tau^-$ limits the value of using these calculations in providing an estimate for NNLO contributions to Bhabha scattering. NNLO calculations have also been carried out for the MOLLER experiment for polarized e^-e^- scattering, which has t and u-channel processes but no s-channel, and the NNLO quadratic and reducible contributions were found to contribute a ~5% correction to the asymmetry (equivalently a ~12% correction to the NLO contribution) [12–17]. The irreducible NNLO contributions are expected to reduce the size of the correction slightly, but it is expected to remain of order 1% [12]. Based on these calculations, the scale of the NNLO effects on the Bhabha scattering $A_{LR\Sigma}$ are expected to be significant compared to the projected

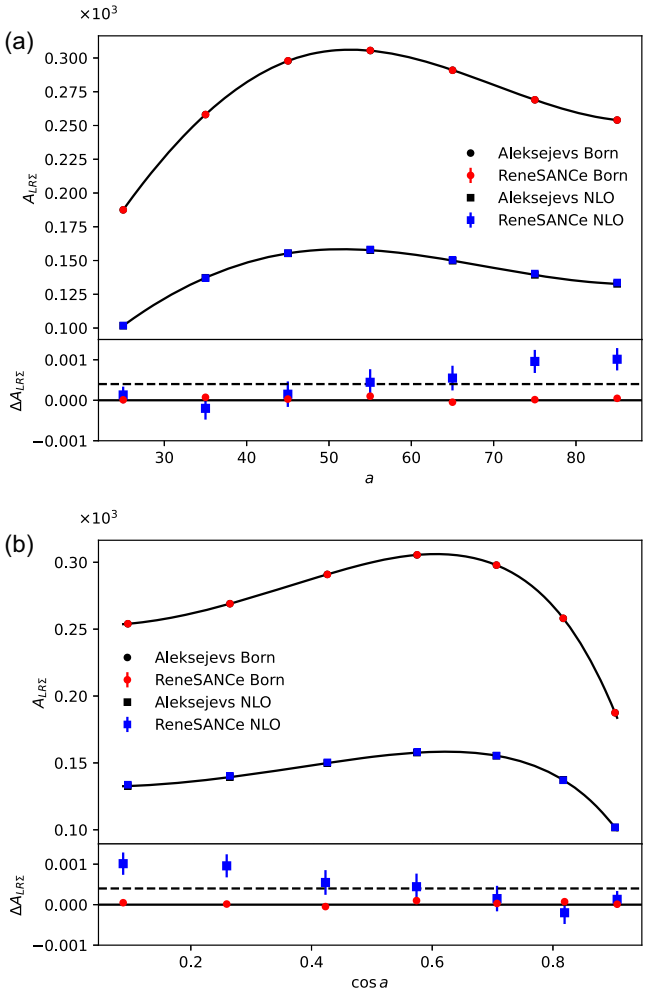


FIG. 3. Comparison of the calculations of $A_{\mathrm{LR}\Sigma}$ in Bhabhas from Ref. [7] and ReneSANCe for an angular acceptance integrated between a and $180° - a$. A cubic spline is employed to illustrate the trend. The lower half of each plot shows the absolute difference between the Ref. [7] and ReneSANCe calculations, with a $\Delta A_{\mathrm{LR}\Sigma} = 4.4 \times 10^{-7}$ horizontal line shown as the mean difference. Subfigure (a) shows the distribution as a function of the final-state electron angle as presented in Fig. 2 of Ref. [7], while subfigure (b) shows the same data distributed in $\cos\theta$.

experimental uncertainties from Chiral Belle, discussed in the following section. Therefore a dedicated NNLO calculation of this process is required.

V. EXPERIMENTAL EXPECTATIONS

To estimate the sensitivity Chiral Belle will have to $A_{\mathrm{LR}\Sigma}$ measurements the following assumptions are made. Belle II reports a Bhabha cross section of 17.4 nb with a detection efficiency for Bhabha events of 0.3593 for an angular acceptance of $|\cos\theta| < 0.819$ [18]. Assuming Chiral Belle achieves its goal of a 70% polarized e^- beam, and taking $A_{\mathrm{LR}\Sigma} = 0.00012$ from ReneSANCe for the angular acceptance of $|\cos\theta| < 0.819$, a measured $A_{\mathrm{LR}\Sigma}$ of $A_{\mathrm{LR}\Sigma}^{\mathrm{meas}} = \langle P_e \rangle A_{\mathrm{LR}\Sigma} = 0.000098$ is predicted, with a statistical uncertainty of 2.3% for 40 ab^{-1} of data. Dominant systematic uncertainties are expected to arise from knowledge of the beam polarization, background modeling, angular acceptance, and knowledge of the center-of-mass energy of the collisions. The A_{LR} scales linearly with the beam polarization and uncertainty in its measurement translates directly to measuring $A_{\mathrm{LR}\Sigma}$. The *BABAR* collaboration has demonstrated a technique to extract the beam polarization at the interaction point from tau decay kinematics with a systemic uncertainty of ± 0.0029 [10]. This represents a relative uncertainty of 0.4% for $\langle P \rangle = 0.70$. The dominant backgrounds for this selection are expected to be a mix of $u\bar{u}, \tau^+\tau^-$, and $d\bar{d}$ events which contribute 0.07% of the selection [18]. The $d\bar{d}$ events contribute the largest effect as they carry an asymmetry roughly two orders of magnitude times largely than the Bhabhas, $A_{\mathrm{LR}\Sigma}^{d\bar{d}} \approx -0.020$. If we conservatively assume the entire background is composed of $d\bar{d}$, and the backgrounds are controlled to the 10% level, a systematic uncertainty of 1% is projected if the backgrounds are not further controlled. The Belle II calorimeter location is known to within 0.11 mm along the beam axis [19]. This corresponds to an acceptance uncertainty of 36 micro-radians or 2×10^{-3} degrees. Propagating this uncertainty through ReneSANCe results in a 0.06% uncertainty on $A_{\mathrm{LR}\Sigma}$. The center-of-mass energy is expected to be known to within ~5 MeV [19] and a similar evaluation in ReneSANCe results in a 0.07% uncertainty in $A_{\mathrm{LR}\Sigma}$. Adding all the sources of systematic uncertainty in quadrature gives a total systematic uncertainty estimate of 1%, dominated by the background projections.

A. Weak mixing angle

In order to determine Chiral Belle's sensitivity to $\sin^2\theta_W$, the value of the mass of the W-boson (m_W) is varied in the generator, as recommended by the ReneSANCe authors [20]. This correspondingly changes the value of $\sin^2\theta_W$, which can be expressed at the Born level as $\sin^2\theta_W = 1 - m_W^2/m_Z^2$. As a baseline, the generator setup is changed to use the PDG value of m_W, 80.377 GeV [21], and from the world average uncertainty of 12 MeV, $\pm 1, 3, 5\sigma$ changes in the mass are applied to study the effect on A_{LR} and $A_{\mathrm{LR}\Sigma}$. The effects are shown for A_{LR} in Fig. 4 and for $A_{\mathrm{LR}\Sigma}$ in Fig. 5.

The expected sensitivity to $\sin^2\theta_W$ obtained by measuring A_{LR} in each $\cos\theta$ bin is calculated from the change in the average A_{LR} in each bin, ΔA_{LR}, associated with a change in $\sin^2\theta_W$, $\Delta\sin^2\theta_W$, in that bin as determined by varying the value of m_W. The overall selection efficiency of 0.3593 is assumed to apply to each bin. For each $\cos\theta$ bin the projected uncertainty is $\sigma_{\sin^2\theta_W} = \sigma_{A_{\mathrm{LR}}} \times (\Delta\sin^2\theta_W/\Delta A_{\mathrm{LR}})$. The value of $\sigma_{A_{\mathrm{LR}}}$

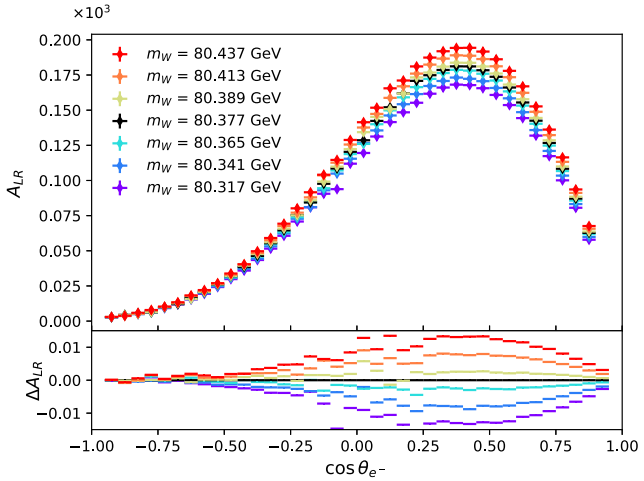


FIG. 4. A_{LR} in Bhabhas for bins of $\cos\theta = 0.05$ across the angular acceptance in NLO. m_W is input as 1σ, 3σ, and 5σ deviations from the world average of 80.377 ± 0.012 GeV.

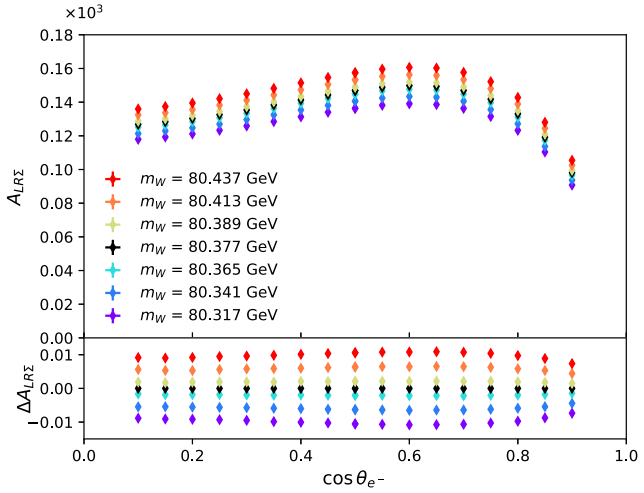


FIG. 5. $A_{\mathrm{LR}\Sigma}$ in Bhabhas for an angular acceptance of $\theta < a < 180° - \theta$. m_W is input as 1σ, 3σ, and 5σ deviations from the world average of 80.377 ± 0.012 GeV.

in each bin is determined from the cross section in that $\cos\theta$ bin as calculated by ReneSANCe. The overall $\sin^2\theta_W$ uncertainty is determined from the weighted mean of the values in all bins under the assumption of independent measurements. From these studies the weighted average value of $(\Delta \sin^2\theta_W/\Delta A_{\mathrm{LR}})$ is found to be 230. For 40 ab^{-1}, a 70% polarization, and detector acceptance of $|\cos\theta| < 0.90$, the projected precision from the Bhabha A_{LR} measurement is $\sigma_{\sin^2\theta_W} = \pm 0.00028$. For $|\cos\theta| < 0.819$, the Belle II fiducial acceptance used for the luminosities studies of [18], the uncertainty increases to $\sigma_{\sin^2\theta_W} = \pm 0.00032$.

This Chiral Belle uncertainty using only Bhabha events, and consequently only sensitive to the Z^0-electron coupling, is similar to the ± 0.00026 precision of $\sin^2\theta_W$ determined at the Z^0 pole by the SLD collaboration from A_{LR} measurements [22] for the combined lepton states, which are dominated by the measurements sensitive to the Z^0-electron couplings. Combining this SLD result with the forward-backward Z-pole asymmetry measurements from LEP involving only electrons, the Z^0-electron effective vector coupling constant was determined to be $g_{Ve} = -0.03816 \pm 0.00047$ (see Table 7.7 in [22]), corresponding to a combined SLD-LEP uncertainty of ± 0.00024 on $\sin^2\theta_W$ at the Z^0 pole involving only the Z^0-electron coupling. We note that CMS has published results on the forward-backward asymmetry in e^+e^- and $\mu^+\mu^-$ events near the Z^0 pole that yield a precision of ± 0.00032 on $\sin^2\theta_W$ [23]. The projected Chiral Belle sensitivity with 40 ab^{-1} is also comparable to the $\sin^2\theta_W$ projected uncertainty of the MOLLER experiment (± 0.00028) at the lower 100 MeV energy scale.

Note that Chiral Belle will also measure $\sin^2\theta_W$ using muons [24] and tau leptons as well as b-quarks and c-quarks. Combined with the electron measurements discussed here, these different measurements will yield precision neutral-current universality tests, as well as an uncertainty on $\sin^2\theta_W$ assuming universality of ± 0.00019 taking into account the common uncertainty on $\langle P \rangle$.

VI. CONCLUSIONS

The $A_{\mathrm{LR}\Sigma}$ studies on the Bhabha process show a negligible 4.4×10^{-7} difference, corresponding to a relative difference of 0.3%, between the theory work from Ref. [7] and the ReneSANCe generator [6]. The data generated by ReneSANCe for this study are publicly available [25]. Projections for the Chiral Belle experiment suggest an $A_{\mathrm{LR}\Sigma}$ measurement with a $\pm 2.3\%_{\mathrm{stat}} \pm 1\%_{\mathrm{sys}}$ relative uncertainty is achievable with 40 ab^{-1} of data. While the NLO effects are understood at a high level of precision, the effects of the NNLO contributions have yet to be calculated, and are expected to be significant in comparison to the experimental uncertainties.

A study of the sensitivity to $\sin^2\theta_W$ indicates that with 40 ab^{-1} of polarized data Chiral Belle would make a measurement only involving the Z^0-electron couplings with a statistics-dominated precision of ± 0.00032 or better. Combining this with Chiral Belle $A_{\mathrm{LR}\Sigma}$ measurements from muons and taus under the assumption of lepton universality would yield an overall uncertainty of ± 0.00019 on $\sin^2\theta_W$. These precisions are similar to the sensitivities of the most precise previous experiments. As these past high precision measurements were performed at the Z^0 pole, the measurements enabled by Chiral Belle provide a unique probe for new physics through the running of neutral current couplings.

At this level of precision, combined with its measurements of the other fermion species, Chiral Belle will

also provide world-leading measurements of the universality of the Z^0-fermion couplings. The projected level of precision motivates a full calculation of the NNLO contributions at 10.58 GeV in order to fully interpret future Chiral Belle measurements in the context of the SM.

ACKNOWLEDGMENTS

The authors would like to thank Renat Sadykov and Vitaly Yermolchyk for their efforts to extend the capabilities of the ReneSANCe generator to enable this study, as well as their insights into our analysis methods. The authors would further like to thank Aleksandrs Aleksevejs and Svetlana Barkanova for their insights and helpful comments. This work is supported by the Natural Sciences and Engineering Research Council.

DATA AVAILABILITY

The data that support the findings of this article are openly available [25].

[1] A. Accardi *et al.* (US Belle II Group, Belle II/SuperKEKB e- Polarization Upgrade Working Group), Snowmass 2021 white paper on upgrading SuperKEKB with a polarized electron beam: Discovery potential and proposed implementation, in *2022 Snowmass Summer Study* (2022), arXiv:2205.12847.

[2] J. Benesch *et al.* (MOLLER Collaboration), The MOLLER experiment: An ultra-precise measurement of the weak mixing angle using Møller scattering, arXiv:1411.4088.

[3] Z. S. Demiroglu (MOLLER Collaboration), The MOLLER experiment: An ultra-precise measurement of the weak mixing angle using Moller scattering, Proc. Sci. SPIN2023 (**2024**) 141.

[4] H. Davoudiasl, H.-S. Lee, and W. J. Marciano, Low Q^2 weak mixing angle measurements and rare higgs decays, Phys. Rev. D **92**, 055005 (2015).

[5] A. Accardi *et al.*, Electron-ion collider: The next QCD frontier, Eur. Phys. J. A **52**, 268 (2016).

[6] R. Sadykov and V. Yermolchyk, Polarized NLO EW e^+e^- cross section calculations with ReneSANCe-v1.0.0, Comput. Phys. Commun. **256**, 107445 (2020).

[7] A. G. Aleksejevs, S. G. Barkanova, Y. M. Bystritskiy, and V. A. Zykunov, Electroweak corrections with allowance for hard bremsstrahlung in polarized Bhabha scattering, Phys. At. Nucl. **83**, 463 (2020).

[8] SuperB Collaboration, SuperB Technical Design Report, arXiv:1306.5655.

[9] D. Charlet, T. Ishibashi, A. Martens, M. Masuzawa, F. Mawas, Y. Peinaud, D. Zhou, and F. Zomer, Conceptual study of a Compton polarimeter for the upgrade of the SuperKEKB collider with a polarized electron beam, J. Instrum. **18**, P10014 (2023).

[10] J. P. Lees *et al.* (*BABAR* Collaboration), Precision e^- beam polarimetry at an e^+e^- B factory using tau-pair events, Phys. Rev. D **108**, 092001 (2023).

[11] S. Kollatzsch and Y. Ulrich, Lepton pair production at NNLO in QED with EW effects, SciPost Phys. **15**, 104 (2023).

[12] A. Aleksejevs (private communication).

[13] A. Aleksejevs, S. Barkanova, Y. Kolomensky, E. Kuraev, and V. Zykunov, Quadratic electroweak corrections for polarized Moller scattering, Phys. Rev. D **85**, 013007 (2012).

[14] A. Aleksejevs, S. Barkanova, Y. Kolomensky, E. Kuraev, and V. Zykunov, High precision calculations of electroweak radiative corrections for polarized Moller scattering at one loop and beyond, Nuovo Cimento C **035**, 192 (2012).

[15] A. G. Aleksejevs, S. G. Barkanova, and V. A. Zykunov, Precise calculations of observables of polarized Moller scattering: From JLAB to ILC energies, Phys. At. Nucl. **75**, 209 (2012).

[16] A. G. Aleksejevs, S. G. Barkanova, Y. M. Bystritskiy, E. A. Kuraev, and V. A. Zykunov, NNLO electroweak corrections for polarized Møller scattering: One-loop insertions to boxes, Phys. Part. Nucl. Lett. **12**, 645 (2015).

[17] A. G. Aleksejevs, S. G. Barkanova, Y. M. Bystritskiy, E. A. Kuraev, and V. A. Zykunov, Two-loop electroweak vertex corrections for polarized Møller scattering, Phys. Part. Nucl. Lett. **13**, 310 (2016).

[18] F. Abudinén *et al.* (Belle II Collaboration), Measurement of the integrated luminosity of the Phase 2 data of the Belle II experiment, Chin. Phys. C **44**, 021001 (2020).

[19] I. Adachi *et al.* (Belle II Collaboration), Measurement of the integrated luminosity of data samples collected during 2019-2022 by the Belle II experiment, Chin. Phys. C **49**, 013001 (2025).

[20] R. Sadykov and V. Yermolchyk (private communication).

[21] M. Tanabashi, K. Hagiwara, K. Hikasa, K. Nakamura, Y. Sumino, F. Takahashi, J. Tanaka *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **98**, 030001 (2018).

[22] S. Schael *et al.* (ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group Collaborations), Precision electroweak measurements on the z resonance, Phys. Rep. **427**, 257 (2006).

[23] A. Khukhunaishvili (CMS Collaboration), Measurement of the effective leptonic electroweak mixing angle and Drell-Yan forward-backward asymmetry using pp collisions at $\sqrt{s} = 13$ TeV, Phys. Lett. B **866**, 139526 (2025).

[24] A. Aleksejevs, S. Barkanova, C. Miller, J. M. Roney, and V. Zykunov, NLO radiative corrections for forward-backward and left-right asymmetries at a B-factory, Phys. Rev. D **101**, 053003 (2020).

[25] C. Miller, ReneSANCe 10.58 GeV study, 10.5281/zenodo.15616559 (2025).