

Positivity in the renormalization of effective field theory

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We show that the direction of renormalization in effective field theory is constrained by fundamental principles in the infrared—unitarity, analyticity, and Lorentz invariance. Our theorem, in the spirit of the a theorem in conformal field theory, determines the sign of the one-loop running of couplings in the forward limit, when one inserts two operators whose mass dimensions are identical and even. The theorem holds for a broad class of effective field theories with arbitrary ultraviolet completions. The constraint directly applies to linear positivity bounds derived using tree-level amplitudes in the infrared, providing a criterion for whether renormalization effects can preserve the positivity bounds or lead to their apparent violation. We discuss the phenomenological implications of our theorem in chiral perturbation theory and the Standard Model effective field theory, where our theorem is particularly constraining for the running at dimension eight. We provide several examples and show various extensions and applications even at dimension six.

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Introduction. Renormalization is a central aspect of field theory that governs how physical parameters evolve with energy scale. The structures of the renormalization group (RG) equations—including, notably, the sign of the flow—carry crucial phenomenological implications [1,2]. Although the methods for renormalization are well established, it is often challenging to infer the behavior of the RG without performing explicit calculations. Only a few exceptions are known, mainly in conformal field theory [3,4]. A famous example is the four-dimensional a theorem, which constrains the a anomaly between the infrared (IR) and ultraviolet (UV) fixed points [5–7],

$$a_{\text{IR}} - a_{\text{UV}} \leq 0. \quad (1)$$

This raises a natural question: Do definite signs in RG exist for a broader set of parameters and field theories?

We address this question in the context of effective field theory (EFT). The core concept of EFT is to replace UV physics at scale Λ with a tower of local operators in the IR. A classic example is chiral perturbation theory (χ PT) [8–13] describing the strong interaction among low-energy mesons. In the search for physics beyond the Standard Model (SM),

the Standard Model EFT (SMEFT) has become a prominent platform for systematically parametrizing new physics at the scale Λ by extending the SM with higher-dimensional operators [14]. While much of the focus is on dimension-six operators at tree level, a variety of reasons motivate extensions to loop level [15–19] and to dimension eight [20–29]. The dimension-six RG is fully known [30–33], as well as many sectors at dimension eight [34–42]. Moreover, the community is making rapid progress in renormalization beyond one loop, e.g., Refs. [43–46]. A major challenge in reaching higher orders is the rapid growth in complexity: Even with one generation of fermions, the number of independent couplings increases from 84 at dimension six to 993 at dimension eight [47]. This complexity is compounded at loop level by the operator mixing due to RG. This challenge motivates the search for generic signatures in the SMEFT that are agnostic to UV completions.

A key example of universal structures in EFT is provided by nonrenormalization theorems [48–54], which forbid large classes of operator mixing even in the presence of nontrivial Feynman diagrams. These seemingly miraculous cancellations become transparent in the on-shell approach [55] (see further developments in [34,56–66]). Crucially, these results are only based on fundamental IR principles, and they hold for any UV completion.

Another prominent class of universal structures is the set of positivity bounds on EFT couplings [67,68], which impose powerful constraints on the IR dynamics from basic

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assumptions in the UV. In the SMEFT, these bounds could provide a promising avenue for setting theoretical priors in experimental analyses [22,23,69–78]. The simplest bounds constrain the two-to-two forward amplitudes by truncating the EFT to tree level, writing $A = c_{4n}s^{2n-2}$ at dimension $4n$, where s is the Mandelstam variable. These “tree-level positivity bounds” take the form $c_{4n} \geq 0$. Importantly, they are susceptible to loop corrections from light particles, first discussed in χ PT [79–83] and revived recently by [84–93], which are relevant in the SMEFT as well. For phenomenological applications, it is thus essential to account for loop effects before interpreting tree-level positivity bounds as robust theoretical constraints.

In this paper, we analyze the interplay between RG and tree-level positivity bounds for arbitrary spins. We show that there is an analog of the a theorem in EFT: The sign of the one-loop RG for c_{4n} , arising from the insertion of two dimension- $(2n+2)$ operators with $n \geq 2$, is fixed, a property we dub “RG positivity”:

$$(c_{4n,\text{IR}} - c_{4n,\text{UV}})|_{(\text{dim}-(2n+2))^2} \geq 0. \quad (2)$$

The precise statement will be given later around Eq. (7). Although our theorem applies in the perturbative regime, the proof does not use dispersion relations; it relies only on fundamental principles in the IR: unitarity, analyticity, and Lorentz invariance. It therefore applies to a wide class of theories, including χ PT, the SMEFT, and EFTs for quantum gravity.

One can view our theorem (2) as a classification of the RG sectors that preserve the tree-level positivity of c_{4n} . For the SMEFT, this mainly applies to dimension-eight RG from inserting two dimension-six operators but not to the contribution from the mixing of dimension-eight and SM interactions. This is consistent with the violation of tree-level positivity found in [91]; see also the examples in the Supplemental Material [94]. Later, we give a concrete criterion (13) for the preservation or violation of tree-level positivity by RG. See Fig. 1 for a schematic plot.

This study is deeply influenced by the seminal results in [35,91], which first observed nontrivial examples of positivity in the RG at dimension eight in the SMEFT. The authors explained the structure as a consequence of positivity bounds from the UV. See also [37,38,92,93,96,97] for further discussions. Our approach differs from the literature in several ways. The most important difference is that the proof of (2) is solely based on fundamental principles in the IR, without any assumption on UV completions. As a result, our theorem applies very generally, including EFTs with gravitons and operators beyond dimension eight. Our derivation also yields extensions at dimension six, which is typically free from positivity bounds. A more detailed comparison with previous work will be presented later.

Proof of RG positivity. We extract the RG in dimensional regularization from the on-shell formalism [55] to leverage

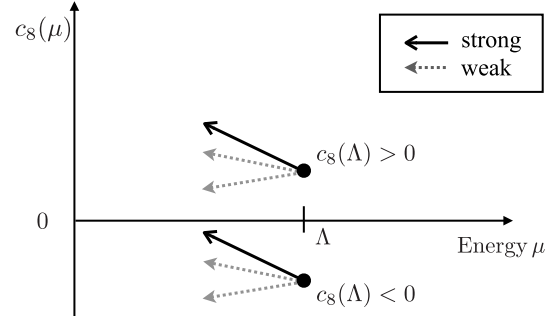


FIG. 1. Schematic plot for the RG of a dimension-eight coupling c_8 subject to our theorem in Eq. (2). The solid arrows indicate the running of $c_8(\mu)$ in a “strongly coupled” EFT, as per the criterion (13), so the RG direction is controlled by (2). The gray arrows show weakly coupled EFTs in which the running can go either way. We consider two values of c_8 at the matching scale Λ ; see examples with $c_8(\Lambda) \leq 0$ in [91].

the connection with fundamental principles. To avoid complications from IR divergences and tadpole diagrams, we restrict to EFTs with massless particles without cubic interactions. This still allows a large range of interactions, such as pions in χ PT in the chiral limit. For the SMEFT, the only higher-dimensional operators we neglect are the F^3 operators at dimension six. We use ϕ, ψ , and F for scalars, fermions, and gauge field strength tensors, and we specify operators only by their field content while omitting details of Lorentz and other indices.

Let us quickly review the on-shell methods. We focus on the RG of a four-point operator renormalized by two four-point operators. The extraction of RG follows from the fact that the full amplitudes are independent of the renormalization scale μ ,

$$\frac{d}{d \ln \mu} A^{\text{full}} = 0. \quad (3)$$

To one-loop level, $A^{\text{full}} = A^{(0)} + A^{(1)}$, where the tree amplitude $A^{(0)}$ is written in terms of the renormalized couplings $c_i(\mu)$. This implies $\frac{dA^{(0)}}{d \ln \mu} = -\frac{dA^{(1)}}{d \ln \mu}$, where the leading μ dependence at one loop comes in the form of $\ln(s_i/\mu^2)$, where $s_i = s, t, u$ are the Mandelstam variables. The scale dependence of the couplings in $A^{(1)}$ is a two-loop effect. Crucially, the coefficients of these logarithms can be computed from the branch cut of each s_i ,

¹Instead of using form factors [55], we use the kinematics $s_i > 0$ to compute the discontinuity of each $\ln s_i$; we then analytically continue to the same kinematics to combine all channels. This analytic continuation is trivial since the RG is given by the coefficient of UV divergence which is local, i.e., a polynomial in s, t, u . We emphasize that, *a priori*, the branch cuts of all channels should be included in the calculation. For instance, this is needed to yield the correct RG in a $\lambda\phi^4$ theory.

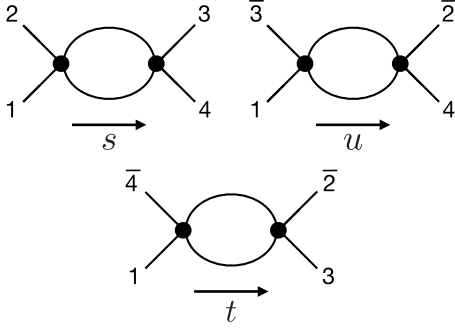


FIG. 2. Unitarity cuts of the s and u channels (top) and the t channel (bottom). All the lines are on shell. We label the incoming states (1 and 2) and outgoing states (3 and 4) by their particle number, and the bar denotes an antiparticle. In the FL, we take the states $|3\rangle = |2\rangle$ and $|4\rangle = |1\rangle$. In this case, the s and u channels have the same initial and final states, but the t channel does not.

$$\begin{aligned} \frac{d}{d \ln \mu} A^{(0)} &= \frac{i}{\pi} \sum_{i=s,t,u} \text{Disc}_i A^{(1)} \\ &= -\frac{1}{\pi} \sum_{i=s,t,u} \int_{\varphi_i''} \langle \varphi_i | \hat{A}^{(0)} | \varphi_i'' \rangle \langle \bar{\varphi}_i'' | \hat{A}^{(0)} | \varphi_i' \rangle, \end{aligned} \quad (4)$$

where we use the optical theorem in the last line to recast the discontinuity into unitarity cuts in terms of tree amplitudes integrated over the phase space $\int_{\varphi_i''}$ of the intermediate state $|\varphi_i''\rangle$. We define $A_{L,i}^{(0)} \equiv \langle \varphi_i | \hat{A}^{(0)} | \varphi_i'' \rangle$ and $A_{R,i}^{(0)\dagger} \equiv \langle \bar{\varphi}_i'' | \hat{A}^{(0)} | \varphi_i' \rangle$. In our case, $A_{L,i}^{(0)}$, $A_{R,i}^{(0)}$, and $A^{(0)}$ are all four-point local amplitudes (see the diagrams in Fig. 2). This formula succinctly derives the one-loop RG from tree amplitudes in the absence of IR divergences. Crucially, the RG derivation does not use dispersion relations and is therefore agnostic about UV completions.

The on-shell RG formula (4) holds for generic kinematics. However, our main result is that additional structure can be exposed by considering special limits. In particular, we take the forward limit (FL) where the incoming particles 1 and 2 have the same momenta and quantum numbers as the outgoing particles 4 and 3; i.e., their states satisfy $|3\rangle = |2\rangle$ and $|4\rangle = |1\rangle$. This implies $t = (p_1 - p_4)^2 \rightarrow 0$; see Fig. 2. In the FL, the tree amplitude behaves as

$$A^{(0)} = \sum_{m=0} c_{2m+4} s^m, \quad (5)$$

where the coefficient c_{2m+4} depends on the EFT parameters. By defining c_{2m+4} through the on-shell amplitudes, we have (partially) fixed the choice of operator basis. The formula (4) then computes the RG of c_{2m+4} , thereby providing direct insight into the RG corrections to tree-level positivity bounds, $c_{2m+4} \geq 0$. In the following, it is a key property of the s and u channels that they have the same initial and final states in the FL, $|\varphi_i\rangle = |\varphi_i'\rangle$, while this is

not true in the t channel. Let us consider these cases separately.

For the s and u channels, the total integrand in (4) is positive definite due to unitarity in their respective physical regimes (when s or u is positive). However, the RG for c_{2m+4} requires us to split the right-hand side of (4) into a power series in s which may break positivity of individual couplings. The direction of the RG running for c_{2m+4} then hinges on the truncation of (4) to order s^m , as well as the combination between the s and u channels. First, we impose that both $\hat{A}^{(0)}$ come from operators of the same mass dimension $(m+4)$, such that each integrand is always non-negative: $\int_{\varphi_i''} A_{L,i}^{(0)} A_{R,i}^{(0)\dagger} = d_{i,m} s_i^m$, for a non-negative coefficient $d_{i,m}$ and $s_i = s$ or u . Second, since $s = -u$, the sum over the two channels is manifestly sign definite only when m is even.² To satisfy both criteria, we find that the contribution from these channels has a definite sign when we insert a pair of operators whose mass dimensions are even and equal. For instance, we have control on the RG of dimension-eight operators from a pair of dimension-six operators but not from the interference between dimension-four and dimension-eight operators, nor the RG of dimension-six operators from a pair of dimension-five operators.

For the t channel, the integrand in (4) is not necessarily positive in the FL since $|\varphi_i\rangle \neq |\varphi_i'\rangle$. We instead prove that $\text{Disc}_t A^{(1)} = 0$ in the FL when inserting any higher-dimensional operator.³ Consider the two internal particles to have momenta ℓ_j and helicity h_j . Recall that a local amplitude containing a particle with helicity $-|h|$ or $|h|$ must be proportional to $|p\rangle^{2h}$ or $|p\rangle^{2h}$ in the spinor-helicity variables. Any extra derivative on the particle adds another factor of $|p\rangle[p]$. Applying this to the local amplitudes $A_{L,i}^{(0)}$ and $A_{R,i}^{(0)\dagger}$ in (4) yields

$$\begin{aligned} \text{Disc}_t A^{(1)} &\propto \int_{\varphi_i''} (|\ell_1\rangle[\ell_1])^{2|h_1|+n_1} (|\ell_2\rangle[\ell_2])^{2|h_2|+n_2} \\ &\propto (p_1 - p_4)^{2|h_1|+2|h_2|+n_1+n_2}, \end{aligned} \quad (6)$$

where n_i are the number of derivatives on ℓ_i and we only keep track of the ℓ_i dependence but do not assume any details on index contractions. In the second line, we crucially use Lorentz invariance to replace the internal $\ell_{1,2}$ vectors by the external momentum $p_1 - p_4$ after integration. It is important that we only have a bubble diagram so that the phase-space integration involves a single scale. This shows that $\text{Disc}_t A^{(1)}$ can be nonzero only when both internal lines are scalar particles without any extra derivative. However,

²The same constructive interference also appears in the derivation of tree-level positivity bounds.

³Setting $t = 0$ directly leads to vanishing scaleless bubbles. However, for the extraction of the RG, we need to show that the coefficient of the scaleless bubble, $\text{Disc}_t A^{(1)}$, vanishes instead.

in such a case, all external momenta or spinors must contract with the ones from particles on the same side of the unitarity cut which vanish in the FL, due to $p_i^2 = 0$ and $\langle ii \rangle = [ii] = 0$. This singles out ϕ^4 theory as the only case where $\text{Disc}_t A^{(1)} \neq 0$ in the forward limit.⁴ In other words, the t channel does not contribute to the RG of any higher-dimensional operator which survives the FL, given that $A_{L,i}^{(0)}$ and $A_{R,i}^{(0)}$ are local. Notably, this includes dimension-six operators.

Let us combine the analysis for all channels in (4). The s and u channels for the RG of c_{4n} induced by two dimension- $(2n+2)$ operators have definite sign, and the t -channel contribution vanishes for $n \geq 2$. We have thus proven the RG positivity:

$$\left. \frac{dc_{4n}}{d \ln \mu} \right|_{(\dim - (2n+2))^2} \leq 0. \quad (7)$$

While examples of this theorem are known in the scalar case [84–86] by explicit calculations and at dimension eight in the SMEFT [35,91], our proof holds for arbitrary spins and mass dimension. This theorem leads to the solid lines in Fig. 1 for the running of c_{4n} . The vanishing of the t -channel discontinuity for higher-dimensional operators also leads to a corollary for a new type of nonrenormalization theorem which we will discuss later.

As we have emphasized, our proof relies only on unitarity, analyticity, and Lorentz invariance in the IR EFT regime while being agnostic on the UV completion. This is in contrast to the argument for the signs and zeros in the RG at the dimension-eight level in [91], which uses positivity assumptions in the UV, together with a classification of operators in weakly coupled UV completions. The RG positivity dictated by (7) thus explains and generalizes the results of [91] from a pure IR perspective.

Phenomenological implications. The generality of the above derivation implies that our result applies to a wide range of EFTs. Here, we discuss the phenomenological implications for χ PT and the SMEFT.

We use the χ PT Lagrangian at $\mathcal{O}(p^2)$ given by $\mathcal{L}_2 = f^2 \text{tr}(\partial_\mu U^\dagger \partial^\mu U)/4$, where $U = \exp(2iT^a \pi^a/f)$, with $\text{tr}(T^a T^b) = \delta^{ab}/2$. The $\mathcal{O}(p^4)$ Lagrangian with $SU(2)_V$ symmetry is then $\mathcal{L}_4 = c_1 \text{tr}(\partial_\mu U^\dagger \partial^\mu U)^2 + c_2 \text{tr}(\partial_\mu U^\dagger \partial_\nu U) \times \text{tr}(\partial^\nu U^\dagger \partial^\mu U)$ [12], which yields the tree-level forward amplitude

⁴One can also see this from the partial wave perspective [62], noting that in the t -channel partial-wave decomposition, only the $J = (n-4)/2$ coefficient survives the FL of a dimension- n operator.

$$A_{ij \rightarrow ji} = \begin{cases} 16s^2(c_1 + c_2) & \text{if } i = j \\ 8s^2 c_2 & \text{if } i \neq j, \end{cases} \quad (8)$$

where i, j label the flavor of pions in the s channel. The RG positivity (7) requires $\frac{dc_2}{d \ln \mu} \leq 0$ and $\frac{d(c_1+c_2)}{d \ln \mu} \leq 0$ when considering all pion species. Indeed, this agrees with the explicit RG calculation:

$$\frac{d(c_1 + c_2)}{d \ln \mu} = -\frac{1}{16\pi^2} \frac{1}{4}, \quad \frac{dc_2}{d \ln \mu} = -\frac{1}{16\pi^2} \frac{1}{6}. \quad (9)$$

Moreover, the loop corrections induced by the $\mathcal{O}(p^2)$ Lagrangian are naturally comparable to the tree-level $\mathcal{O}(p^4)$ terms [98], leading to $\mathcal{O}(c_i) \sim 0.01$. Crucially, the scale dependence would exhibit the same qualitative behavior in the presence of electromagnetic interactions between charged pions, whose effects on (9) are negligible because $\mathcal{O}(\frac{\alpha_{\text{EM}}}{4\pi}) \times \mathcal{O}(c_i) \lesssim 10^{-5}$. Therefore, our theorem (7) phenomenologically captures the direction of the RG flow even though it does not determine the sign of all terms.

Let us turn to the SMEFT. The dimension-eight RG in the SMEFT in the unbroken phase takes the form

$$\frac{dc_{8,i}}{d \ln \mu} = \gamma_{ij} c_{8,j} + \gamma'_{ijk} c_{6,j} c_{6,k}, \quad (10)$$

where $c_{n,i}$ denotes a coupling associated with a dimension- n operator $\mathcal{O}_{n,i}$. The anomalous dimensions γ and γ' may depend on the SM couplings. The sign of the γ' term is determined by RG positivity if $c_{8,i}$ can be probed in the FL.⁵ For tree-level scattering in the SMEFT, the appearance of $c_{8,i}$ in the FL has been systematically studied in [22], where the combination $c_{H^4 D^4}^{(1)} + c_{H^4 D^4}^{(2)}$ appears as one of them.⁶ Thus, the γ' for this combination is indeed negative:

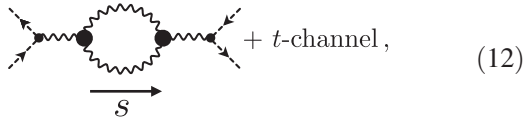
$$\begin{aligned} 16\pi^2 \frac{d(c_{H^4 D^4}^{(1)} + c_{H^4 D^4}^{(2)})}{d \ln \mu} \\ = -16(c_{H^2 W B}^2 + c_{H^2 \tilde{W} B}^2) - 12g_2^2(c_{W^3}^2 + c_{\tilde{W} W^2}^2) + \dots, \end{aligned} \quad (11)$$

where we only show the gauge boson loops.⁷ Remarkably, double insertions of the F^3 operators also mix into $H^4 D^4$ with a negative sign, even though we assumed the absence of cubic vertices in our proof. In the present case, we only have bubble diagrams,

⁵Note that further definite signs and zeros were identified in some entries of γ in [96,97].

⁶We use $\mathcal{L}_8 = \sum_i c_{H^4 D^4}^{(i)} \mathcal{O}_{H^4 D^4}^{(i)}/\Lambda^4$, where $\mathcal{O}_{H^4 D^4}^{(i)} \equiv \mathcal{O}_{H^4}^{(i)}$ in the basis of [99].

⁷We use $\mathcal{L}_6 = \sum_i c_i \mathcal{O}_i/\Lambda^2$, where $\mathcal{O}_i$ is defined in the basis of [100] as $\mathcal{O}_{H^2 X B} \equiv \mathcal{O}_{\phi X B}$, $\mathcal{O}_{W^3} \equiv \mathcal{O}_W$, and $\mathcal{O}_{\tilde{W} W^2} \equiv \mathcal{O}_{\tilde{W}}$.



(12)

so the t -channel cut still depends on a single scale and must vanish by the same counting as in (6). We thus conclude that the RG positivity constrains the sign of the full RG of $H^4 D^4$ operators induced by two dimension-six insertions. We provide this RG in the Supplemental Material [94].⁸ Together with the results of [35,36], this completes the RG of the $H^4 D^4$ operators from double insertions of dimension-six operators.

The relative importance of the two terms in (10) depends on the UV completion. The contribution from a pair of dimension-six operators is dominant whenever the EFT is more strongly coupled than the SM,

$$\mathcal{O}(\hat{c}_6)^2 \gg \mathcal{O}(\hat{c}_8)\mathcal{O}(\hat{c}_4), \quad (13)$$

where $\mathcal{O}(\hat{c}_n)$ is the order of a dimension- n coefficient normalized according to naive dimensional analysis [98,101]. Here, $\mathcal{O}(\hat{c}_4)$ is known to be small since the SM is weakly coupled above the electroweak scale. If (13) holds, which is natural in a range of UV completions [102–106], the dimension-eight couplings increase by running to the IR.

Conversely, if the heavy particles are (very) weakly coupled to the SM, the Wilson coefficients are naturally also small, in which case the RG positivity theorem no longer fixes the overall sign of the running at the dimension-eight level. This may even lead to violations of the tree-level positivity bounds in the IR [91]. See Fig. 1 for a schematic plot of the RG flow in various UV scenarios.

We emphasize that violations of tree-level positivity bounds on dimension-eight couplings are consistent with the constraints from dispersion relations. As explained in [84], even though c_{4n} in (5) is positive in a mass-deformed theory below the mass gap, such a deformation is not small at low energies and may lead to different behavior in the deep IR compared to massless theories. Alternatively, an arc with a smaller radius in the picture of [85] is more positive since the discontinuity is positive definite. However, the arc may be dominated by the loops of dimension-four couplings [91–93,96], such that the positivity of the arc does not constrain the sign of c_{4n} . We exemplify the violation of tree-level positivity bounds in the Supplemental Material [94], including a comparison to the dispersion relations.

⁸In the Supplemental Material [94], we also discuss an example with three-point interactions, to which our derivation may not apply. One can see that the optical theorem does not imply a definite sign of the UV divergence. Thus, the systematic extension of our theorem to include three-point interactions requires further investigation that is beyond the scope of this paper.

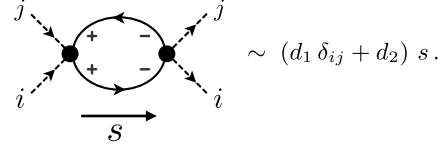


FIG. 3. The only nonzero channel in the RG formula (4) for the mixing of a pair of dimension-five operators into the $H^4 D^2$ operators in the SMEFT. The dashed and solid lines correspond to the Higgs boson and lepton doublets, respectively. The i, j denote $SU(2)$ indices of the external Higgs doublets. Helicities are indicated in the all-outgoing convention. It follows from the RG positivity that the constants satisfy $d_2 \geq 0$ and $d_1 + d_2 \geq 0$, as verified by Eq. (14).

Further applications. The RG positivity generally does not apply to the RG of dimension-six operators, due to potential cancellations between the s and u channels. However, whenever one (or both) of these channels is absent, definite signs or zeros are still implied. For instance, in the SMEFT, the RG of the $H^4 D^2$ operators (using the basis of [100]) induced by double insertions of the Weinberg operators $\psi^2 H^2$ [107] only have the s channel shown in Fig. 3. Thus, the $\psi^2 H^2$ contributions to the RG of $H^4 D^2$ operators are sign definite,⁹ as confirmed by the explicit results in [108,109],

$$\begin{aligned} \frac{dc_{HD}}{d \ln \mu} &= -\frac{4}{16\pi^2} \text{tr}(c_5 c_5^*), \\ \frac{d(c_{HD} + 2c_{H\Box})}{d \ln \mu} &= -\frac{8}{16\pi^2} \text{tr}(c_5 c_5^*), \end{aligned} \quad (14)$$

where the trace sums over the flavor space. We also analyze the full mixing of the Weinberg operators into other dimension-six operators in the Supplemental Material [94].

A corollary of (6) is the nonrenormalization of a higher-dimensional operator that can be probed in the FL if the t -channel cut is the only available one. This predicts, for example, the nonrenormalization of dimension-eight operators of the form $H^2 F^2 D^2$ from inserting $H^4 D^2$ and $H^2 F^2$ types of operators. Even at dimension six, this explains the zero contribution from the λH^4 potential to the diagonal anomalous dimension of the $\bar{\psi}\psi H^2 D$ operators [30]. In contrast, the $H^2 F^2$ operators vanish in the FL, so the corollary does not apply.

An alternative perspective on the nonrenormalization of $\bar{\psi}\psi H^2 D \rightarrow \bar{\psi}\psi H^2 D$ follows from the angular-momentum selection rules identified in [52,62]. Namely, in the $\bar{\psi}\psi \rightarrow HH$ channel, these operators generate amplitudes with angular momentum (spin) $J = 1$, while λH^4 has spin

⁹In the FL, the dimension-six four-Higgs amplitude is $A_{ij \rightarrow ij} = (c_{HD} + 2\delta_{ij}c_{H\Box})s$, where i, j are the $SU(2)$ indices of the Higgs doublets. This implies $\dot{c}_{HD} < 0$ and $\dot{c}_{HD} + 2\dot{c}_{H\Box} < 0$ from Weinberg operators.

zero and is thus orthogonal. The same argument applies to the nonrenormalization of dimension-eight operators that survive in the FL by those that vanish in this limit (again, if only the t channel exists) [96,97]. This hinges on the fact that only the former class of operators has a nonzero $J = (n - 4)/2$ t -channel partial wave.¹⁰

Conclusion. We systematically analyzed the direction of the one-loop RG flow of EFT couplings that are subject to tree-level positivity bounds. The RG positivity, derived solely from basic principles in the IR, singles out the contributions that preserve the positivity at the matching scale under the RG flow but does not exclude potential violation from other sectors [91]. If the violation of tree-level positivity bounds in the SMEFT are found experimentally, our results select weakly-coupled UV models as one of the possible scenarios. We also discussed applications to the running of dimension-six couplings, and we derived new nonrenormalization theorems.

Our work opens several promising avenues for future studies. A natural extension is to include cubic interactions in both gauge and gravitational theories. These contributions introduce additional complexity due to infrared divergences, though the positivity structure may still persist [62], potentially offering sharper insights into the weak gravity conjecture in the deep IR [110].

It is also of interest to generalize these constraints to higher-point scattering, higher-loop corrections and massive particles, as well as to the two-point spectral density. For constraints on the RG of higher-point operators, it is crucial to judiciously generalize the forward limit; see, e.g., Refs. [111,112]. Furthermore, as we have focused on linear positivity bounds, another natural extension would be to

apply similar methods to nonlinear bounds [84–86]. Furthermore, while we have illuminated important constraints on the RG flow of couplings, it would be valuable to study the behavior of UV matching using on-shell methods [113]. Our theorem may also lead to interesting interplay when combined with the geometric renormalization [36,39,114–118].

Given the deep connection between RG flow and the counting of degrees of freedom, it would be intriguing to consider whether the RG positivity theorem can be reformulated from the quantum information perspective [119–122]. Even decades after its development, we still have much to learn from the structures and implications of renormalization.

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¹⁰More intricate are the definite signs in the RG mixing among a certain class of dimension-eight operators with nonzero FL [96,97]. We postpone a detailed study of these signs to future work.

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