

Worldvolume fermion as baryon with homogeneous instantons in holographic QCD₃

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We investigate holographically the effective theory of the worldvolume fermion on the flavor branes in the D3/D7 model with homogeneously smeared D(-1)-branes. As a top-down approach in gauge-gravity duality, the D(-1)-branes are instantons and violate the CP symmetry in the dual theory. The background geometry of this model contains black brane (deconfined geometry) and bubble D3-brane solutions (confined geometry), all with a nonzero Ramond-Ramond zero form as axion. The dual theories to the backgrounds are respectively the Super Yang-Mills theory at finite temperature and three-dimensional confining Yang-Mills theory, all with a Chern-Simons term induced by instantons. In the confined geometry, we introduce a baryon vertex as a D5-brane wrapped on S^5 , then identify the fermionic fluxes produced by the N_c open strings on the D7-brane as a baryonic operator. Afterwards, we study the spectrum and the holographic correlation function of the baryonic fermion on the D7-branes. Remarkably, the fermionic spectrum is in agreement with the dispersion curves obtained from the confined correlation function, and the mass ratio of the lowest baryon and meson in our model is close to the associated experimental data. Moreover, the effective interaction terms of the holographic baryon and meson are derived and all the coupling constants take order of $N_c^{1/2}$ agreeing with the evaluation from the large N_c field theory. In the deconfined geometry, the holographic correlation function is also evaluated numerically while the fermion on D7-brane is identified to plasmino instead of baryon. The dispersion curves from the deconfined correlation function basically cover the results from the hard thermal loop approximation and may imply the instanton-induced interaction with spin. Overall, this work constructs a holographic theory about baryonic fermion and mesonic boson with instantons or CP violation.

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I. INTRODUCTION

The investigation of physics with CP violation has a long history. For instance, A. Sakharov in 1967 [1] proposed that C and CP violation is very necessary in the early Universe, because matter and antimatter were created in equal amounts at the Big Bang [2], and they would be annihilated in pairs without matter remaining in today's Universe if C and CP symmetry are strictly preserved. The mechanism of CP violation according to the Standard Model of particle physics arises from the nonzero phase parameter of the Cabibbo-Kobayashi-Maskawa (CKM) matrix which describes successfully the various decays of meson and baryon with CP violation [3,4]. Nevertheless, matter-antimatter asymmetry predicted by the CKM mechanism

is very smaller than some astronomical observations [5], so the discovery of physics beyond the Standard Model's ongoing exploration of CP violation is also significant.

To understand the various phenomena with the hadronic matter-antimatter asymmetry, the CP violation in strong interaction becomes a theory of candidate since, in non-Abelian gauge theory, the CP violation can originate from the topologically nontrivial excitations of the vacuum, i.e., the instanton [6,7]. Instantons are consist of constituents called Bogomol'nyi-Prasad-Sommerfield (BPS) monopoles or dyons and they are related to efforts in linking confinement contributing to the thermodynamics and chiral symmetry breaking [8–17]. On the other hand, mesons and baryons are low-energy confined states in QCD, which is very challenging to describe analytically by using the perturbed method of quantum field theory (QFT), especially in the presence of instantons or CP violation. Therefore, in this work, we focus on the top-down approach of the D3/D7 model [18–22] with instantons through the gauge-gravity duality, in order to construct a holographic theory involving baryons and mesons with instantons or CP violation. Our motivations are as follows: first, gauge-

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gravity duality can be used to analyze a gauge field theory in a strong coupling limit through its dual classical gravity theory [23–25], in particular, mesons and baryons can be included naturally in the top-down holographic approach based on string theory [26–29]. Second, instantons in gauge theory are holographically dual to the D(-1)-branes in the D3/D7 model [18,28], and the corresponding geometric background involving the backreaction of the D(-1)-branes is deformed $\text{AdS}_5 \times S^5$, which can be obtained analytically by turning on homogeneously the Ramond-Ramond (R-R) zero form C_0 in the supergravity action [18,19]. Remarkably, by taking into account a probe D3-brane located at the holographic boundary of the bulk, there would be a Chern-Simons (CS) term in the D3-brane action, as [30]

$$S_{\text{CS}} \sim \int C_0 f \wedge f \sim i \frac{\theta}{64\pi^2} \int f \wedge f \sim \theta \int \omega_3, \quad (1.1)$$

plays the role violating CP symmetry in the dual field theory. Here f refers to the gauge field strength on the probe D3-brane, ω_3 is the CS three-form, and θ is the parameter describing the topological charge carried by the instantonic vacuum. Third, when the baryon vertex as a D5-brane wrapped on S^5 (with N_c open strings end on it) is introduced into the D3/D7 model, the fermionic flux on the D7-brane can be a bulk baryonic operator because it is a gauge-invariant operator combined by N_c open strings (as N_c quarks) taking baryon numbers [29]. Since baryons with instantons lead to several effects which are very difficult to be analyzed analytically in QFT [6,7], gauge-gravity duality may provide an alternative method to this goal. Altogether, the D3/D7 model with D(-1)-branes contains all elements about mesons, baryons, and CP violation in holography, thus it would be a novel approach to construct a holographic theory involving mesons and baryons with instantons or CP violation by using this model.

Let us reveal the outline of this work here: in Sec. II, we collect the essential parts of the D3-brane background with D(-1)-branes as instantons, then discuss the embedding configuration of the D7-branes as flavors and the holographic duality for baryons. The remarkable point here is we can obtain a three-dimensional confining dual theory¹ as QCD 3 by following the classical method in Refs. [27,28,33], and the fermionic flux on the worldvolume of the D7-branes becomes baryonic field when the baryon vertex with N_c open strings is introduced [29,34–36]. In Sec. III, we study the mass spectrum and the holographic correlation function of the fermions on the D7-brane. The onset mass given by the correlation function and by the decomposition of the worldvolume

fermion are consistent in the confined phase. In addition, we find the mass ratio of the lowest baryon and meson in our model is close to the experimental data. In the deconfined phase, the correlation function basically covers the dispersion curves obtained from the hard thermal loop (HTL) approximation in thermal field theory.² In Sec. IV, we demonstrate briefly how to derive the interaction terms involving baryonic fermions and mesonic bosons from the D7-brane action. It indicates that the CP symmetry may not be necessary in the baryon decay. The summary and discussion are given in the last section. In the Appendix, we investigate the spectroscopy of the mesonic boson on the D7-brane, which is useful to evaluate numerically the coupling terms of the baryonic fermions and mesonic bosons in the low-energy effective action.

II. SETUP OF THE D3/D7 MODEL WITH HOMOGENOUS INSTANTON

In this section, let us briefly review the D3/D7 model with homogenous instantons as D(-1)-branes, then focus on the holographic correspondence with respect to baryons.

A. The D3-brane background with D-instanton

The background geometry of the D3/D7 model with homogenous instantons is created by N_c D3-branes with N_D D-instantons, i.e., the N_D D(-1)-branes, in the large- N_c limit [18,19]. Its supergravity solution is in general a ten-dimensional deformed D3-brane solution with a nontrivial R-R zero form C_0 , which is recognized as a marginal “bound state” of D3-branes with homogeneously smeared N_D D(-1)-branes. In our notation, the N_c D3-branes are identified as color branes, hence its number N_c denotes the color number. Then the low-energy dynamics of the background are described by type IIB supergravity action, which is given in string frame as [27]

$$S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[e^{-2\Phi} (\mathcal{R} + 4\partial\Phi \cdot \partial\Phi) - \frac{1}{2} |F_1|^2 - \frac{1}{2} |F_5|^2 \right], \quad (2.1)$$

where $2\kappa_{10}^2 = (2\pi)^7 l_s^8$ is the ten-dimensional gravity coupling constant, and l_s and g_s are, respectively, the string length and the string coupling constant. We use Φ to denote the dilaton field and $F_{1,5}$ the field strength of the R-R zero and four-form $C_{0,4}$ respectively. The solution of N_c D3-branes with N_D D-instantons is the near-horizon solution of nonextremal black D3-branes with a nontrivial C_0 , which in string frame reads [20,21,39]

¹The D3/D7 model is also used to study the third Yang-Mills-Chern-Simons theory [21,22,31,32].

²Searching for the theta vacuum or instantons in hot QCD also attracts great interest [37,38].

$$\begin{aligned}
ds^2 &= e^{\frac{\phi}{2}} \left\{ \frac{r^2}{R^2} [-f(r)dt^2 + d\mathbf{x} \cdot d\mathbf{x}] + \frac{1}{f(r)} \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right\}, \\
e^\phi &= 1 + \frac{Q}{r_H^4} \ln \frac{1}{f(r)}, \quad f(r) = 1 - \frac{r_H^4}{r^4}, \quad F_5 = dC_4 = g_s^{-1} Q_3 \epsilon_5, \\
F_1 &= dC_0, \quad C_0 = -ie^{-\phi} + i\mathcal{C}, \quad \phi = \Phi - \Phi_0, \quad e^{\Phi_0} = g_s,
\end{aligned} \tag{2.2}$$

where $\mathcal{C}$ is a boundary constant for C_0 and ϵ_5 is the volume element of a unit S^5 . The associated parameters are given as

$$R^4 = 4\pi g_s N_c l_s^4, \quad Q_3 = 4R^4 g_s, \quad Q = \frac{N_D (2\pi)^4 \alpha'^2}{N_c V_4} Q_3. \tag{2.3}$$

Here, $x^\mu = \{t, \mathbf{x}\} = \{t, x^i\}$ and $i = 1, 2, 3$ refers to the four-dimensional spacetime along which the D3-branes are extended, and r is the holographic direction perpendicular to the D3-branes. The solution (2.2) is asymptotic $\text{AdS}_5 \times S^5$ at the holographic boundary $r \rightarrow \infty$, which describes geometrically that the D-instanton charge N_D is smeared over the worldvolume V_4 of the N_c coincident black D3-branes homogeneously with a horizon at $r = r_H$. The backreaction of the D-instantons has been taken into account in the background, thus it implies N_D/N_c must be fixed in the large- N_c limit. The dual theory of this background is conjectured as the four-dimensional $\mathcal{N} = 4$ Super Yang-Mills theory (SYM) in a self-dual gauge field (instantonic) background or with a dynamical axion at finite temperature characterized by the parameter Q [20,21,39]. Note that, in the language of hadron physics, C_0 is recognized as axion, and the gluon condensate in this system can be evaluated as

$$\langle \text{Tr} F_{\mu\nu} F^{\mu\nu} \rangle \simeq \frac{N_D}{16\pi^2 V_4} = \frac{Q}{(2\pi\alpha')^2 R^4} \frac{N_c}{(2\pi)^4}, \quad \mu, \nu = 0, 1, \dots, 3. \tag{2.4}$$

In gauge-gravity duality, the black brane background (2.2) is holographically dual to the deconfined SYM theory at high temperature [24,27]. Nevertheless, it is possible to obtain a confined geometry from Eq. (2.2) by following the classical project proposed in Refs. [27,28,33]. Specifically, compactify one of the three spatial dimensions of the D3-branes on a circle S^1 . Hence, below the Kaluza-Klein energy scale $M_{KK} = 2\pi/\delta x^i$, the dual theory becomes effectively three-dimensional, where δx^i refers to the period of S^1 . Then the supersymmetric fermions and scalars in the dual theory will acquire mass of order M_{KK} when the antiperiodic and periodic boundary condition on fermion and bosonic fields are respectively imposed along S^1 . Therefore, they are accordingly decoupled in the low-energy dynamics so that the low-energy effective theory on the compactified D3-branes is purely third Yang-Mills theory. Afterwards, perform a double Wick rotation on the D(-1)-D3 brane background as $t \rightarrow -ix^i, x^i \rightarrow -it$ to identify the bulk gravity solution as its holographic correspondence. We denote the direction along S^1 as x^3 without loss of generality, thus the confining solution obtained from Eq. (2.2) is

$$\begin{aligned}
ds^2 &= e^{\frac{\phi}{2}} \left\{ \frac{r^2}{R^2} [\eta_{\mu\nu} dx^\mu dx^\nu + f(r)(dx^3)^2] + \frac{1}{f(r)} \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right\}, \\
e^\phi &= 1 + \frac{Q}{r_{KK}^4} \ln \frac{1}{f(r)}, \quad f(r) = 1 - \frac{r_{KK}^4}{r^4}, \quad F_5 = dC_4 = g_s^{-1} Q_3 \epsilon_5, \\
F_1 &= dC_0, \quad C_0 = -ie^{-\phi} + i\mathcal{C}, \quad \phi = \Phi - \Phi_0, \quad e^{\Phi_0} = g_s.
\end{aligned} \tag{2.5}$$

The solution (2.5) defined for $r > r_{KK}$ does not have a horizon since the warp factor $e^{\phi/2} \frac{r^2}{R^2}$ never goes to zero. And the asymptotics of the Wilson loop in this geometry lead to an area law in the dual theory. So it means the dual field theory exhibit confinement below the energy scale M_{KK} at very low temperature. In addition, to avoid the conical singularities in the region of $r > r_{KK}$, we have to further require

$$M_{KK} = \frac{2r_{KK}}{R^2}. \tag{2.6}$$

B. Embedding of the D7-brane

To construct a dual theory like QCD, the fundamental quark must be introduced into the dual theory, which can be achieved by embedding a stack of probe N_f D7-branes as flavors. In our setup, we require that the D7-branes are perpendicular to the compactified direction x^3 , spanning the $\mathbb{R}^{1,2}$ denoted by $\{t, x, y\}$, the holographic direction denoted by r , and four of the five directions in Ω_5 as in Refs. [20,22]. Accordingly, the fermion on the D7-branes survives under the dimensional reduction by following

Refs. [27,28,33] and, as we will see, it can be identified to the dual operator of the baryonic field with baryon vertex in the dual theory.

To obtain the induced metric on the D7-branes, we impose the following coordinate transformation:

$$r = \frac{r_H}{\sqrt{2}} \sqrt{\frac{r_H^2}{\rho^2} + \frac{\rho^2}{r_H^2}}, \quad (2.7)$$

so the metric presented in Eq. (2.2) becomes

$$ds^2 = e^{\frac{\phi}{2}} \left\{ \frac{r^2}{R^2} [-f(r)dt^2 + d\mathbf{x} \cdot d\mathbf{x}] + \frac{R^2}{\rho^2} (d\rho^2 + \rho^2 d\Omega_5^2) \right\}. \quad (2.8)$$

We further introduce the coordinate transformation $\zeta = \rho \cos \Theta$, $w = \rho \sin \Theta$, i.e., $\rho^2 = w^2 + \zeta^2$, where Θ is one of the angular coordinates in Ω_5 , then the metric (2.8) takes the final form as

$$ds^2 = e^{\frac{\phi}{2}} \left\{ \frac{r^2}{R^2} [-f(r)dt^2 + d\mathbf{x} \cdot d\mathbf{x}] + \frac{R^2}{\rho^2} (d\zeta^2 + \zeta^2 d\Omega_4^2 + dw^2) \right\}. \quad (2.9)$$

So the induced metric on the D7-brane can be found by choosing the simple embedding solution as $x^3, w = \text{const}$, it leads,

$$ds_{D7}^2 = e^{\frac{\phi}{2}} \left\{ \frac{r^2}{R^2} [-f(r)dt^2 + dx^2 + dy^2] + \frac{R^2}{\rho^2} (d\zeta^2 + \zeta^2 d\Omega_4^2) \right\}. \quad (2.10)$$

Follow the same steps, the confining induced metric can be obtained from Eq. (2.5) as

$$ds_{D7}^2 = e^{\frac{\phi}{2}} \left\{ \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{\rho^2} (d\zeta^2 + \zeta^2 d\Omega_4^2) \right\}, \quad \mu, \nu = 0, 1, 2. \quad (2.11)$$

TABLE I. D-brane configuration in the D3/D7 model with D(-1) branes as homogenous instantons. Dots indicate that the D-brane extends along this direction. x^3 is the compactified direction.

	t	x	y	(x^3)	ζ	Ω_4	w
N_D D(-1)-branes							
N_c D3-branes	...	...	...	...			
N_f D7-branes	...	...	...		...	...	
Baryon vertex D5-brane						...	...

So the dual theory in confined geometry becomes third QCD (QCD 3). The D-brane configuration in our model is given in Table I. We will use the induced metric (2.10) and (2.11) in the following discussion for the dual theory in the deconfined and the confined phases, respectively. To further simplify the calculation, we will set $w = 0$ in this work.³ Since in the D3/D7 model, the boundary value of w is proportional to the quark mass, it means in our setup, we focus on the massless, or namely the light flavored, quarks in the dual theory.

C. The holographic correspondence for baryons

In this subsection, let us outline the holographic correspondence for baryons by taking into account a baryon vertex in our D3/D7 approach. In gauge-gravity duality, a baryon vertex is identified as a probe D-brane wrapped on the dimensions denoted by the spherical coordinates of the background geometry with N_c open strings ending on it [8,29,32]. Accordingly, in the type IIB supergravity on $\text{AdS}_5 \times S^5$, the baryon vertex is a D5-brane wrapped on S^5 , which is illustrated in Table I, with N_c open strings ending on it stretched to the holographic boundary. Consider a probe D3-brane located at the holographic boundary—there are N_c open strings with same orientation stretched to the holographic boundary and ended on the D5-brane as illustrated in Fig. 1. The wrapped D5-brane carries a $U(1)$ charge proportional to N_c , due to the contribution from the R-R flux $N_c \sim \int_{S^5} F_5$. On the other hand, each one of the N_c open strings carries a unit $U(1)$ Chan-Paton charge 1 (or -1 dependent on the orientation), which in total cancels exactly the $U(1)$ charge of the wrapped D5-brane. In this sense, the baryon number, defined as the $U(1)$ charge over N_c , is conserved on the baryon vertex by choosing suitably the orientation of the D5-brane and N_c open strings [8,29].

Then let us specify the holographic duality with respect to baryons. In the viewpoint of the boundary theory, the endpoints of the N_c open strings are inside the D7-branes taking the Chan-Paton charge; thus, they are in the fundamental representation of the color and flavor groups, namely they are flavored fundamental quarks taking baryon number $1/N_c$. Moreover, it must be possible to form an antisymmetric gauge-invariant operator η as the combination of N_c quarks in the boundary theory, and by taking into account the color confinement, this operator η is the color singlet as a baryon, since its baryon number is 1 (or -1 for an antibaryon). Note that this implies that the N_c open strings behave as fermions, since the color singlet η

³One may worry that $w = 0$ causes an issue that the worldvolume of the D7-branes stops at $r = r_{KK}$, which is forbidden. However, this can be figured out by introducing another (anti) D7-brane stretched to $r = r_{KK}$ connecting to D7-brane. The same issue can be found in the D4/D6/ $\overline{\text{D6}}$ approach [40], in which the anti D6-brane is introduced so that the embedding function can be chosen as zero.

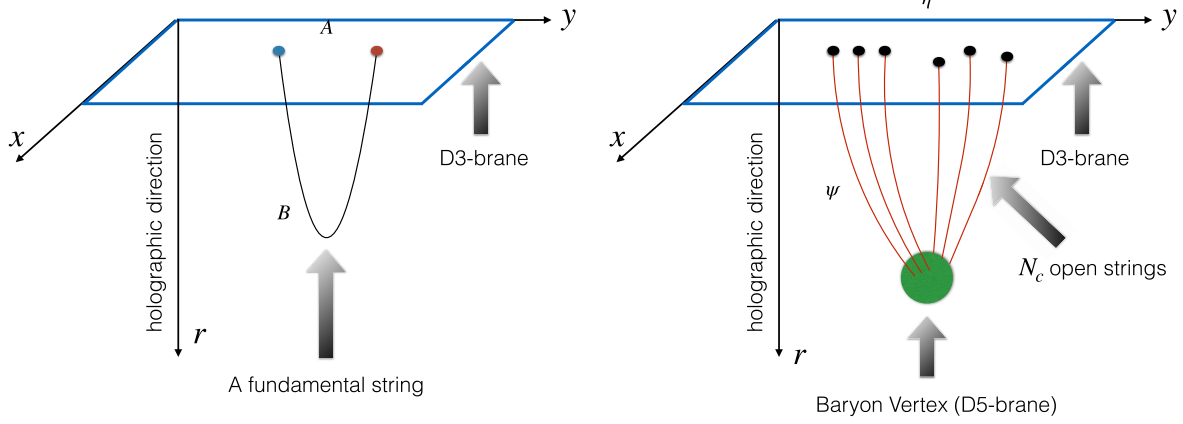


FIG. 1. D-brane configuration of the D3/D7 system for the holographic correspondence with respect to mesons and baryons. The D7-branes extend along x , y , r directions. Mesonic setup (left): the endpoint (blue) and the antiendpoint (red) of a fundamental string refers, respectively, to a quark and an antiquark. A is the bosonic color singlet as a meson consisted of a quark and an antiquark in the boundary theory. B is the bosonic flux produced by the fundamental string as the bulk dual operator to A . Baryonic setup (right): the endpoints (black) of the N_c strings all with same orientation are N_c quarks at boundary. η is the fermionic color singlet as a baryon consisting of N_c quarks at boundary. ψ is the fermionic flux created by the N_c open strings as the bulk dual operator to η .

produced by the endpoints of the N_c open strings is the antisymmetric representation of the color group [29]. In the bulk theory, there must be a dual operator to η produced by the combination of the N_c open strings according to the holographic principle. So we denote the N_c fluxes in bulk produced by the N_c open strings as ψ , and ψ also takes the baryon number as η . Notice that the constructions of the holographic dualities for baryons and mesons are different. In the mesonic case, the baryon vertex is not essential, since the meson is the color singlet consisting of a quark and an antiquark, which in holography corresponds to a fundamental string in the D7-brane ending on the boundary with respect to both the endpoints, as illustrated in Fig. 1. Therefore, the baryon number for a meson is always vanished.

Afterwards, we need to confirm that QCD baryons in the large N_c limit remain to be fermions, in order to find an effective action for the bulk baryonic field ψ . Since baryons consist of N_c quarks, which are the fundamental representation of the spin group $SU(2)$, usually the spin of a baryon can be obtained by decomposing the tensor product of the N_c irreducible representations of the $SU(2)$ group for finite N_c . As a result, we find baryons are fermions when N_c is odd, and are bosons when N_c is even. However, the decomposition of the tensor product of the $SU(2)$ group does not have a definite limit at $N_c \rightarrow \infty$, so it is impossible to obtain exactly the spin of baryons for the case $N_c \rightarrow \infty$ in this way. Fortunately, the spin and flavor groups of baryons combine to form the contracted $SU(2N_f)$ group strictly in the limit of $N_c \rightarrow \infty$, which is the symmetry group of the contracted spin-flavor symmetry [41,43]. Accordingly, baryons in the large N_c limit transform as the irreducible representation of the group of the spin-flavor symmetry, and the simplest irreducible representations of

the spin-flavor group for the case⁴ of $N_f = 2, 3$ are worked out as a tower of states with half-integer spin and half-integer isospin, which illustrates all the baryons are fermions [41–44]. Therefore, in our holographic correspondence in the D3/D7 approach, both the baryonic operator η in the boundary theory and the dual field ψ in bulk should be fermionic to match these analyses of baryons in the large N_c QCD as the boundary theory. Besides, since the N_c open strings are on the D7-branes, their fermionic fluxes ψ as the bulk baryonic field can be described effectively by using the fermionic action for the D7-brane. We note that the analysis in this subsection is valid only in the confined geometry, since there may not be a wrapped configuration of a D5-brane on S^5 in the deconfined geometry [45]. In the rest of this paper, all our discussion in the confined geometry is in the presence of a baryon vertex, so the N_c fermionic fluxes on the D7-brane in the confined geometry are always baryonic.

III. SPECTROSCOPY OF THE WORLDVOLUME FERMION ON THE FLAVOR BRANE

As we have specified that the bulk field for a baryon is the fermionic flux produced by the N_c open strings on the D7-branes, in this section, we study the mass spectrum of the fermion on the D7-brane in the confined geometry and the correlation function of the fermion by using the holographic prescription in AdS/CFT with respect to both the confined and deconfined cases.

⁴We do not need to consider the case of $N_f > 3$ in general, since the flavor number of a baryon usually satisfies $N_f \leq 3$ in the realistic QCD.

A. The fermionic action of the D-brane

Since our concern would be the fermionic flux on the flavor brane, we start from the fermionic action on a D_p -brane, which can be obtained by the standard method of T-duality in string theory. In the type IIB string theory, the quadratic action of worldvolume fermionic flux Ψ is given as [46,47]

$$S_{D_p}^f = \frac{iT_p}{2} \int d^{p+1}x e^{-\phi} \sqrt{-(g + \mathcal{F})} \bar{\Psi} (1 - \Gamma_{D_p}) (\Gamma^\alpha \hat{D}_\alpha - \Delta + L_{D_p}) \Psi, \quad (3.1)$$

where

$$\begin{aligned} \hat{D}_\alpha &= \nabla_\alpha + \frac{1}{4 \cdot 2!} H_{\alpha NK} \Gamma^{NK} \bar{\gamma} - \frac{1}{8} e^\phi \left(F_N \Gamma^N \Gamma_\alpha \bar{\gamma} + \frac{1}{3!} F_{KLN} \Gamma^{KLN} \Gamma_\alpha + \frac{1}{2 \cdot 5!} F_{KLMNP} \Gamma^{KLMNP} \Gamma_\alpha \bar{\gamma} \right), \\ \Delta &= \frac{1}{2} \left(\Gamma^M \partial_M \phi + \frac{1}{2 \cdot 3!} H_{MKN} \Gamma^{MKN} \bar{\gamma} \right) + \frac{1}{2} e^\phi \left(F_M \Gamma^M \bar{\gamma} + \frac{1}{2 \cdot 3!} F_{KLN} \Gamma^{KLN} \right), \\ \Gamma_{D_p} &= \frac{1}{\sqrt{-(g + \mathcal{F})}} \sum_q \frac{\epsilon^{\alpha_1 \dots \alpha_{2q} \beta_1 \dots \beta_{p-2q+1}}}{q! 2^q (p-2q+1)!} \mathcal{F}_{\alpha_1 \alpha_2 \dots \alpha_{2q-1} \alpha_{2q}} \Gamma_{\beta_1 \dots \beta_{p-2q+1}} \bar{\gamma}^{\frac{p-2q+1}{2}}, \\ L_{D_p} &= \sum_q \frac{\epsilon^{\alpha_1 \dots \alpha_{2q} \beta_1 \dots \beta_{p-2q+1}}}{q! 2^q (p-2q+1)!} \frac{(-i\sigma_2)(\bar{\gamma})^{\frac{p-2q+1}{2}}}{\sqrt{-(g + \mathcal{F})}} \mathcal{F}_{\alpha_1 \alpha_2 \dots \alpha_{2q-1} \alpha_{2q}} \Gamma_{\beta_1 \dots \beta_{p-2q+1}} \hat{D}_\lambda. \end{aligned} \quad (3.2)$$

Let us clarify the notation presented in Eqs. (3.1) and (3.2). The indices denoted by capital letters $K, L, M, N \dots$ run over the ten-dimensional spacetime and the indices denoted by lowercase letters $a, b, \dots$ run over the tangent space of the ten-dimensional spacetime. T_p refers to the tension of a D_p -brane given as $T_p = g_s^{-1} (2\pi)^{-p} l_s^{-(p+1)}$. The Greek letters α, β, λ refer to the indices running over the worldvolume of the D_p -brane. The metric is written in terms of vielbein as $g_{MN} = e_M^a \eta_{ab} e_N^b$, and the gamma matrices are given by the relations

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \quad \{\Gamma^M, \Gamma^N\} = 2g^{MN}, \quad (3.3)$$

with $e_M^a \Gamma^M = \gamma^a$. Besides, ω_{ab} refers to the spin connection and $\nabla_\alpha = \partial_\alpha + \frac{1}{4} \omega_{ab} \gamma^{ab}$ is the covariant derivative for fermions. The gamma matrix with multiple indices is defined by ranking alternate antisymmetrically or symmetrically the indices, e.g.,

$$\gamma^{ab} = \frac{1}{2} [\gamma^a, \gamma^b], \quad \gamma^{abc} = \frac{1}{2} \{\gamma^a, \gamma^{bc}\}, \quad \gamma^{abcd} = \frac{1}{2} [\gamma^a, \gamma^{bcd}] \dots \quad (3.4)$$

$\Gamma^{MNK \dots}$ shares the same definition as $\gamma^{abc \dots}$. $\bar{\gamma}$ is defined as the chiral gamma matrix as $\bar{\gamma} = \gamma^{01 \dots 9}$ and σ_2 refers to the associated Pauli matrix in ten dimensions. The worldvolume field $\mathcal{F}$ is given as $\mathcal{F} = B + (2\pi\alpha')f$ where B is the NS-NS 2-form in type II string theory with its field strength $H = dB$ and f as the Yang-Mills field strength. $F_M, F_{MN}, F_{KLM} \dots$ refer to the field strength of the massless R-R fields presented in type II string theory. For the D7-brane, let us set $p = 7$ then insert the nonvanished Ramond-Ramond field and dilaton given in the supergravity solution (2.2) or (2.5) into the action (3.1), so the action (3.1) becomes

$$S_{D7}^f = \frac{iT_7}{2} \int d^8x e^{-\phi} \sqrt{-g} \bar{\Psi} \left(\Gamma^\alpha \nabla_\alpha - \frac{1}{8} e^\phi F_N \Gamma^N \Gamma_\alpha \bar{\gamma} - \frac{1}{2 \cdot 8 \cdot 5!} \Gamma^\alpha F_{KLMNP} \Gamma^{KLMNP} \Gamma_\alpha \bar{\gamma} - \frac{1}{2} \Gamma^M \partial_M \phi - \frac{1}{2} e^\phi F_M \Gamma^M \bar{\gamma} \right) \Psi. \quad (3.5)$$

B. The fermionic spectrum

The holographic fermionic spectrum in the dual theory can be obtained by integrating out the extra dimensions in the action (3.5) as most top-down holographic approaches [34–36,48]. Note that the spectrum is only valid in the confined geometry, since its dual theory allows hadronic states exhibiting confinement. To this goal, let us first calculate the Dirac operator in the confined geometry (2.11) as

$$\Gamma^\alpha \nabla_\alpha = e^{-\phi/4} \left(\frac{R}{r} \gamma^\mu \partial_\mu + \frac{\rho}{R} \gamma^4 \partial_4 \right) + \frac{e^{-\phi/4}}{R} \gamma^4 \left(-\frac{2\zeta}{\rho} + \frac{2\rho}{\zeta} + \frac{3\zeta r'}{2r} + \frac{7}{8} \zeta \phi' \right) + e^{-\phi/4} \frac{\rho}{R\zeta} \not{D}_{S^4}, \quad (3.6)$$

where $\mathcal{D}_{S^4}$ is the Dirac operator on a unit S^4 . The coordinate on the D7-brane is denoted as $x^\alpha = \{t, x, y, x^4 = \zeta, \Omega_4\}$ and $x^\mu = \{t, x, y\}$. In addition, we can further calculate

$$\begin{aligned} \frac{1}{8} e^\phi F_N \Gamma^\alpha \Gamma^N \Gamma_\alpha &= -\frac{3}{4} \frac{\rho}{R} e^{3\phi/4} \partial_\zeta C_0 \gamma^4, \\ \frac{1}{2 \cdot 8 \cdot 5!} \Gamma^\alpha F_{KLMNP} \Gamma^{KLMNP} \Gamma_\alpha &= \frac{1}{2} \frac{e^{-5\phi/4}}{R} \gamma^{56789}, \\ \frac{1}{2} \Gamma^M \partial_M \phi + \frac{1}{2} e^\phi F_M \Gamma^M \bar{\gamma} &= \frac{1}{2} \frac{\rho}{R} \gamma^4 e^{-\phi/4} (\partial_\zeta \phi + e^\phi \partial_\zeta C_0 \bar{\gamma}). \end{aligned} \quad (3.7)$$

As the kappa symmetry fixes the chirality condition $\bar{\gamma}\Psi = -i\Psi$ in our notation, the ten-dimensional spinor can be decomposed into a 3 + 1 dimensional part $\psi(x, \zeta)$ with holographic coordinate ζ , an S^4 part φ , and a remaining second part β , as

$$\Psi \sim \psi(x, \zeta) \otimes \varphi(S^4) \otimes \beta, \quad (3.8)$$

satisfying $\gamma^{56789}\Psi = -i\Psi$ and $\beta^\dagger \beta = 1$. The spinor φ is the eigenfunction on S^4 which satisfies the Dirac equation on S^4 as [49]

$$\mathcal{D}_{S^4} \varphi = \Lambda_l^\pm \varphi^{\pm l, s}, \quad \Lambda_l^\pm = \pm(2 + l), \quad l = 0, 1, \dots, \quad (3.9)$$

and normalization

$$\int \sqrt{g_{S^4}} d\Omega_4 \varphi^{\pm l', s'^\dagger} \varphi^{\pm l, s} = \delta^{l, l'} \delta^{s, s'}, \quad (3.10)$$

where s, l represent the angular quantum numbers of the spherical harmonic function, and g_{S^4} refers to the determinant of the metric on a unit S^4 . Afterwards, let us insert Eqs. (3.6)–(3.10) into action (3.5); the fermionic action then takes the form as

$$\begin{aligned} S_{D7} &= i \int d^3 x d\zeta d\Omega_4 (\bar{\Psi} \mathcal{A} \gamma^\mu \partial_\mu \Psi + \bar{\Psi} \mathcal{B} \gamma^4 \partial_4 \Psi + \bar{\Psi} \mathcal{C} \gamma^4 \Psi + \bar{\Psi} \mathcal{D} \Psi), \\ &= i \int d^3 x d\zeta \left[\bar{\psi} \gamma^\mu \partial_\mu \psi + \bar{\psi} \frac{\mathcal{B}}{\mathcal{A}} \gamma^4 \partial_4 \psi + \bar{\psi} \left(\frac{\mathcal{C}}{\mathcal{A}} - \frac{\mathcal{A} \mathcal{B}}{2\mathcal{A}^2} \right) \gamma^4 \psi + \bar{\psi} \frac{\mathcal{D}}{\mathcal{A}} \psi \right], \end{aligned} \quad (3.11)$$

where the gamma matrices have been reduced to third and

$$\begin{aligned} \mathcal{A} &= \frac{T_7}{2} \sqrt{-\frac{g}{g_{S^4}}} e^{-5\phi/4} \frac{R}{r}, \\ \mathcal{B} &= \frac{T_7}{2} \sqrt{-\frac{g}{g_{S^4}}} e^{-5\phi/4} \frac{\rho}{R}, \\ \mathcal{C} &= \frac{T_7}{2} \sqrt{-\frac{g}{g_{S^4}}} \left[\frac{e^{-5\phi/4}}{R} \left(-\frac{2\zeta}{\rho} + \frac{2\rho}{\zeta} + \frac{3\zeta}{2r} \frac{dr}{d\rho} + \frac{7}{8} \zeta \frac{d\phi}{d\rho} \right) - i \frac{3\rho}{4R} e^{-\phi/4} \partial_\zeta C_0 \right], \\ \mathcal{D} &= \frac{T_7}{2} \sqrt{-\frac{g}{g_{S^4}}} \left(\frac{1}{2} \frac{e^{-9\phi/4}}{R} + \frac{\rho}{R\zeta} e^{-5\phi/4} \Lambda_l^\pm \right), \\ \Psi &= \frac{1}{\sqrt{\mathcal{A}}} \psi(x, \zeta) \otimes \varphi^{+l, s}(S^4) \otimes \beta. \end{aligned} \quad (3.12)$$

The 3 + 1 dimensional spinor $\psi(x, \zeta)$ can be further decomposed by a series of completed functions as

$$\psi = \sum_{n,m} \begin{pmatrix} \psi_+^{(n)} f_+^{(n)} \\ \psi_-^{(m)} f_-^{(m)} \end{pmatrix}. \quad (3.13)$$

Defining the gamma matrix specifically as

$$\gamma^\mu = i \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (3.14)$$

where $\sigma^\mu = (1, -\tau^i)$, $\bar{\sigma}^\mu = (1, \tau^i)$, $i = 1, 2, 3$, and τ^i is the Pauli matrix, the action (3.11) becomes

$$\begin{aligned} S_{D7} = \sum_{n,m} \int d^3x d\zeta \Big\{ & -i\psi_+^{(n)\dagger} f_+^{(n)} \bar{\sigma}^\mu \partial_\mu \psi_+^{(m)} f_+^{(m)} - i\psi_-^{(m)\dagger} f_-^{(m)} \sigma^\mu \partial_\mu \psi_-^{(n)} f_-^{(n)} \\ & - \psi_-^{(m)\dagger} \psi_+^{(n)} f_-^{(m)} \left[\frac{\mathcal{B}}{\mathcal{A}} \partial_4 f_+^{(n)} + \left(\frac{\mathcal{C}}{\mathcal{A}} - \frac{\mathcal{B}}{2\mathcal{A}^2} \frac{d\mathcal{A}}{d\zeta} \right) f_+^{(n)} + \frac{\mathcal{D}}{\mathcal{A}} f_+^{(n)} \right] \\ & - \psi_+^{(n)\dagger} \psi_-^{(m)} f_+^{(n)} \left[-\frac{\mathcal{B}}{\mathcal{A}} \partial_4 f_-^{(m)} - \left(\frac{\mathcal{C}}{\mathcal{A}} - \frac{\mathcal{B}}{2\mathcal{A}^2} \frac{d\mathcal{A}}{d\zeta} \right) f_-^{(m)} + \frac{\mathcal{D}}{\mathcal{A}} f_-^{(m)} \right] \Big\}, \end{aligned} \quad (3.15)$$

which implies the normalization condition for the basis functions is

$$\int d\zeta f_\pm^{(n)} f_\pm^{(m)} = \delta^{mn}, \quad (3.16)$$

with eigen equations

$$\begin{aligned} \frac{\mathcal{B}}{\mathcal{A}} \partial_4 f_+^{(n)} + \left(\frac{\mathcal{C}}{\mathcal{A}} - \frac{\mathcal{B}}{2\mathcal{A}^2} \frac{d\mathcal{A}}{d\zeta} \right) f_+^{(n)} + \frac{\mathcal{D}}{\mathcal{A}} f_+^{(n)} &= M_{(n)} f_-^{(n)}, \\ -\frac{\mathcal{B}}{\mathcal{A}} \partial_4 f_-^{(n)} - \left(\frac{\mathcal{C}}{\mathcal{A}} - \frac{\mathcal{B}}{2\mathcal{A}^2} \frac{d\mathcal{A}}{d\zeta} \right) f_-^{(n)} + \frac{\mathcal{D}}{\mathcal{A}} f_-^{(n)} &= M_{(n)} f_+^{(n)}. \end{aligned} \quad (3.17)$$

Taking into account Eqs. (3.16) and (3.17), the action (3.15) can be rewritten in a standard kinetic form, as follows:

$$S_{D7} = -\sum_n \int d^3x \{ i\psi_+^{(n)\dagger} \bar{\sigma}^\mu \partial_\mu \psi_+^{(n)} + i\psi_-^{(n)\dagger} \sigma^\mu \partial_\mu \psi_-^{(n)} + M_n [\psi_-^{(n)\dagger} \psi_+^{(n)} + \psi_+^{(n)\dagger} \psi_-^{(n)}] \}, \quad (3.18)$$

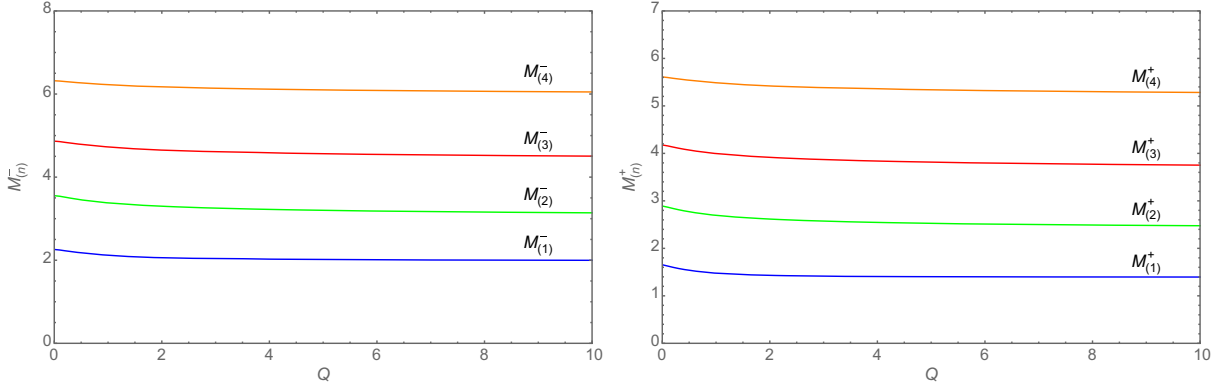
so the mass spectrum of the worldvolume fermion can be evaluated by solving Eq. (3.17) numerically. The resultant mass spectrum is given in Table II, and in particular, its dependence on the density of instanton Q is illustrated in

TABLE II. Spectrum of the worldvolume fermion in the unit of $M_{KK} = 1$ with $\Lambda_l = 2$.

$Q = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
$M_{(n)}^+$	1.65	2.89	4.18	5.61	7.26
$M_{(n)}^-$	2.56	3.55	4.87	6.32	7.94
$Q = 1$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
$M_{(n)}^+$	1.48	2.70	4.00	5.48	7.17
$M_{(n)}^-$	2.12	3.38	4.72	6.22	7.86

Fig. 2. Our numerical calculation indicates that the mass spectrum decreases very slowly at small Q and converges at large Q .

In this work, since the concerned fermionic flux ψ on the D7-brane is the baryonic field produced by the N_c open strings, an overall factor for its action (3.18) must be picked up to include the N_c contributions of the N_c open strings. The simple way to achieve this is to impose the rescaling for quarks in the large N_c QCD as $\psi_\pm^{(n)} \rightarrow \sqrt{N_c} \psi_\pm^{(n)}$ to Eq. (3.18), because the N_c open strings are identified to quarks in the boundary theory. Another reason we can use this rescaling is that a hadron in the large N_c limit behaves like a classical particle [41]. Therefore, we can define the fixed part of the baryonic field as in Ref. [41] in the large N_c limit, which means the exact quadratic form for the baryonic action should be

FIG. 2. Dependence on the instanton density Q of the mass spectrum with $\Lambda_t = 2$.

$$S_{D7} = -N_c \sum_n \int d^3x \left\{ i\psi_+^{(n)\dagger} \bar{\sigma}^\mu \partial_\mu \psi_+^{(n)} + i\psi_-^{(n)\dagger} \sigma^\mu \partial_\mu \psi_-^{(n)} \right. \\ \left. + M_n \left[\psi_-^{(n)\dagger} \psi_+^{(n)} + \psi_+^{(n)\dagger} \psi_-^{(n)} \right] \right\}. \quad (3.19)$$

Thus, the Hamiltonian associated to Eq. (3.19) can be written in the momentum space by using the Fourier transformation $\psi \sim a_{\mathbf{p}}^s u_{\mathbf{p}}^s e^{-iE_{\mathbf{p}}t + \mathbf{p} \cdot \mathbf{x}} + b_{\mathbf{p}}^{s\dagger} v_{\mathbf{p}}^s e^{iE_{\mathbf{p}}t - \mathbf{p} \cdot \mathbf{x}}$ with the creation/annihilation operators as

$$H = N_c \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{s=+,-} E_{\mathbf{p}} (a_{\mathbf{p}}^{s\dagger} a_{\mathbf{p}}^s + b_{\mathbf{p}}^{s\dagger} b_{\mathbf{p}}^s), \quad (3.20)$$

where $E_{\mathbf{p}}$ is the unit energy eigenvalue as $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + M_n^2}$ and M_n is one of the given masses in the fermionic mass spectrum. Therefore, we can see the average value of the Hamiltonian H on a single baryon state $|1\rangle = a_{\mathbf{p}}^{s\dagger}|0\rangle$ is $\langle 1|H|1\rangle = N_c E_{\mathbf{p}}$, and in particular $\langle 1|H|1\rangle = N_c M_n$ for $\mathbf{p} = 0$, which means the baryon mass is proportional to N_c as it is expected for a baryon in the large N_c limit. Moreover, although this model is third, it is interesting to compare our numerical results with the experimental data of $N_c = 3$ baryon. The quantum numbers l, s presented in Eq. (3.9) can be identified to angular momentum and isospin for a baryon, and “ $\pm$ ” refers to the chirality of the fermion under γ^4 , which can be naturally identified to the parity of a baryon. Thus, the lowest fermion in our model takes $l = 0, s = \pm \frac{1}{2}$ and even parity with mass $M_{(1)}^+ \simeq 1.65 M_{KK} N_c \simeq 4.95 M_{KK}$, according to Table II. In the experimental data of baryons, such a baryon state is recognized to be proton or neutron. On the other hand, the lowest vector takes the mass as $m_1 \simeq 3.15 M_{KK}$, according to Table IV and in the experimental data of mesons, such a vector meson is recognized to be ρ meson. So the mass ratio of a proton (or neutron) and ρ meson in our model is $M_{(1)}^+/m_1 \simeq 1.57$, which is close to the experimental data $M_{\text{proton}}/M_{\rho\text{-meson}} \simeq 1.22$.

C. The spectral function

In this section, let us investigate the fermionic spectrum in a parallel way; that is, to evaluate the correlation function as its spectral function with the holographic prescription. Note that the holographic prescription for computing the two-point correlation function is also valid with respect to the deconfined background. In this section, we first review the holographic prescription for computing the two-point correlation function with a spinor, then use it to analyze the spectral function both in confined and deconfined geometry.

1. The holographic prescription

Let us recall the principle of AdS/CFT for computing the two-point correlation function with spinor [50,51]. First, the generating functional of the dual QFT Z_{QFT} is equal to its gravitational partition function Z_{gravity} in the bulk geometry. Therefore, for a spinor field ψ in the bulk, we have

$$Z_{\text{QFT}}[\bar{\psi}_0, \psi_0] = Z_{\text{gravity}}[\bar{\psi}, \psi]|_{\bar{\psi}, \psi \rightarrow \bar{\psi}_0, \psi_0}, \quad (3.21)$$

with

$$Z_{\text{QFT}}[\bar{\psi}_0, \psi_0] = \left\langle \exp \left\{ \int_{\partial\mathcal{M}} (\bar{\eta}\psi_0 + \bar{\psi}_0\eta) d^Dx \right\} \right\rangle,$$

$$Z_{\text{gravity}}[\bar{\psi}, \psi] = e^{-S_{\text{gravity}}^{\text{ren}}},$$

$$S_{\text{gravity}}^{\text{ren}} = \int_{\mathcal{M}} \mathcal{L}_{\text{gravity}}^{\text{ren}}[\bar{\psi}, \psi] \sqrt{-g} d^{D+1}x. \quad (3.22)$$

Here, $\mathcal{M}$ denotes the $D + 1$ dimensional bulk space, and its holographic boundary is given at $\partial\mathcal{M}$. ψ_0 refers to the boundary value of ψ as a source of the boundary fermionic operator η , and $\mathcal{L}_{\text{gravity}}^{\text{ren}}$ represents the renormalized Lagrangian of the bulk field ψ . Following the standard steps in QFT, the one-point function, i.e., average value of $\bar{\eta}$ is given as

$$\langle \bar{\eta} \rangle = \frac{1}{Z_{\text{QFT}}} \frac{\delta Z_{\text{QFT}}}{\delta \psi_0} = \frac{1}{Z_{\text{gravity}}} \frac{\delta Z_{\text{gravity}}}{\delta \psi_0} = -\frac{\delta S_{\text{gravity}}^{\text{ren}}}{\delta \psi_0} \equiv \Pi_0. \quad (3.23)$$

where we have used the relation (3.21). Therefore, the two-point correlation function G_R of η is obtained as

$$\langle \bar{\eta}(\omega, \vec{k}) \rangle = G_R(\omega, \vec{k}) \psi_0, \quad (3.24)$$

i.e.,

$$\Pi_0 = G_R(\omega, \vec{k}) \psi_0, \quad (3.25)$$

where $\omega, \vec{k}$ refers to the frequency and spatial momentum of the associated Fourier modes. Altogether, it is possible to evaluate ψ_0, Π_0 in order to evaluate the two-point correlation function G_R of η by solving the classical gravity action $S_{\text{gravity}}^{\text{ren}}$ in holography. We note that the spinors η and ψ_0 must be gauge-invariant operators under the color group $U(N_c)$, i.e., they should be a baryonic field.⁵

2. Approach to the confined phase

Let us approach the above holographic prescription with the confined metric given in Eq. (2.11). The action for the bulk field ψ on the worldvolume of the D7-brane takes the same form as Eq. (3.5), which can be reduced to an effectively third form as it is given in Eq. (3.11). So its boundary term corresponds to

$$S_{\text{bdry}} = i \int d^3x d\Omega_4 [\bar{\Psi} B \gamma^4 \Psi|_{\partial\mathcal{M}} + \dots]. \quad (3.26)$$

To obtain a standard boundary form, we redefine the spinor ψ by rescaling $\Psi \rightarrow \sqrt{B}\Psi$, then the action (2.11) can be written as

$$S_{\text{D7}} = i \int d^3x d\zeta \left(\bar{\psi} \frac{\mathcal{A}}{B} \gamma^\mu \partial_\mu \psi + \bar{\psi} \gamma^4 \partial_4 \psi - \frac{1}{2} \bar{\psi} \gamma^4 \psi \partial_4 \ln B + \bar{\psi} \frac{C}{B} \gamma^4 \psi + \bar{\psi} \frac{\mathcal{D}}{B} \psi \right), \quad (3.27)$$

and the Dirac equation is derived as

$$\left[\frac{\mathcal{A}}{B} \gamma^\mu \partial_\mu + \gamma^4 \partial_4 + \left(\frac{C}{B} - \frac{1}{2} \partial_4 \ln B \right) \gamma^4 + \frac{\mathcal{D}}{B} \right] \psi = 0. \quad (3.28)$$

Inserting the Fourier modes as the ansatz,

$$\psi = \int \frac{d^4p}{(2\pi)^4} e^{ik \cdot x} \chi(\zeta, k) = \int \frac{d^4p}{(2\pi)^4} e^{ik \cdot x} \begin{bmatrix} \chi_+(\zeta, k) \\ \chi_-(\zeta, k) \end{bmatrix}, \quad (3.29)$$

into Eq. (3.28), the Dirac equation (3.28) becomes

⁵The N_c -counting for the two-point correlation function implies that an overall factor N_c^{-1} should be picked up in front of G_R [41] if we use the rescaling $\psi \rightarrow \sqrt{N_c} \psi$.

$$\begin{aligned} \left(\partial_4 + \frac{C}{B} - \frac{1}{2} \partial_4 \ln B + \frac{\mathcal{D}}{B} \right) \chi_+ + \frac{\mathcal{A}}{B} (\tau^3 k + \omega) \chi_- &= 0 \\ \frac{\mathcal{A}}{B} (-\tau^3 k + \omega) \chi_+ + \left(-\partial_4 - \frac{C}{B} + \frac{1}{2} \partial_4 \ln B + \frac{\mathcal{D}}{B} \right) \chi_- &= 0, \end{aligned} \quad (3.30)$$

where the momentum is set to be along x^2 direction as, $k_\mu = (-\omega, 0, k)$ and the gamma matrices are chosen as

$$\gamma^0 = i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^2 = i \begin{pmatrix} 0 & -\tau^3 \\ \tau^3 & 0 \end{pmatrix}, \quad \gamma^4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (3.31)$$

Equation (3.30) reduces to two second-order differential equations for χ_+ and χ_- , respectively, which can be solved analytically near the boundary $\zeta \rightarrow \infty$ as

$$\begin{aligned} \chi_+ &= R_A \zeta^{\Lambda_l^+ - \frac{3}{2}} + R_B \zeta^{-\Lambda_l^+ - \frac{3}{2}}, \\ \chi_- &= L_A \zeta^{\Lambda_l^+ - \frac{1}{2}} + L_B \zeta^{-\Lambda_l^+ - \frac{5}{2}}, \end{aligned} \quad (3.32)$$

where $L_{A,B}, R_{A,B}$ are constant spinors. Therefore, the boundary value of ψ can be defined by taking the dominated term in Eq. (3.32) as

$$\psi|_{\partial\mathcal{M}} = \lim_{\zeta \rightarrow \infty} \zeta^{-\Lambda_l^+ + \frac{1}{2}} \psi = \begin{pmatrix} 0 \\ L_A \end{pmatrix}, \quad (3.33)$$

so the Π_0 given in Eq. (3.23) becomes

$$\Pi_0|_{\partial\mathcal{M}} \sim -\lim_{\zeta \rightarrow \infty} \zeta^{-\Lambda_l^+ + \frac{3}{2}} \chi_+|_{\partial\mathcal{M}} = -R_A. \quad (3.34)$$

Altogether, the renormalized two-point Green function can be obtained as

$$G_R^{(h,h)} = (-1)^h \lim_{\zeta \rightarrow \infty} \zeta \frac{\chi_+^{(h)}}{\chi_-^{(h)}}, \quad h = 1, 2. \quad (3.35)$$

By introducing ratios

$$\xi_{(h)} = (-1)^{h+1} \frac{\chi_+^{(h)}}{\chi_-^{(h)}}, \quad \chi_\pm = \begin{bmatrix} \chi_\pm^{(1)} \\ \chi_\pm^{(2)} \end{bmatrix}, \quad (3.36)$$

the equations of motion for $\xi_{(h)}$ can be obtained from the Dirac equation as

$$\begin{aligned} \frac{B}{\mathcal{A}} \xi'_{(1)} &= -k + \frac{2\mathcal{D}}{\mathcal{A}} \xi_{(1)} + k \xi_{(1)}^2 + \omega + \omega \xi_{(1)}^2, \\ \frac{B}{\mathcal{A}} \xi'_{(2)} &= -k + \frac{2\mathcal{D}}{\mathcal{A}} \xi_{(2)} + k \xi_{(2)}^2 - \omega - \omega \xi_{(2)}^2, \end{aligned} \quad (3.37)$$

where “/” refers to the derivative with respect to ζ . The equations in (3.37) can be solved numerically with

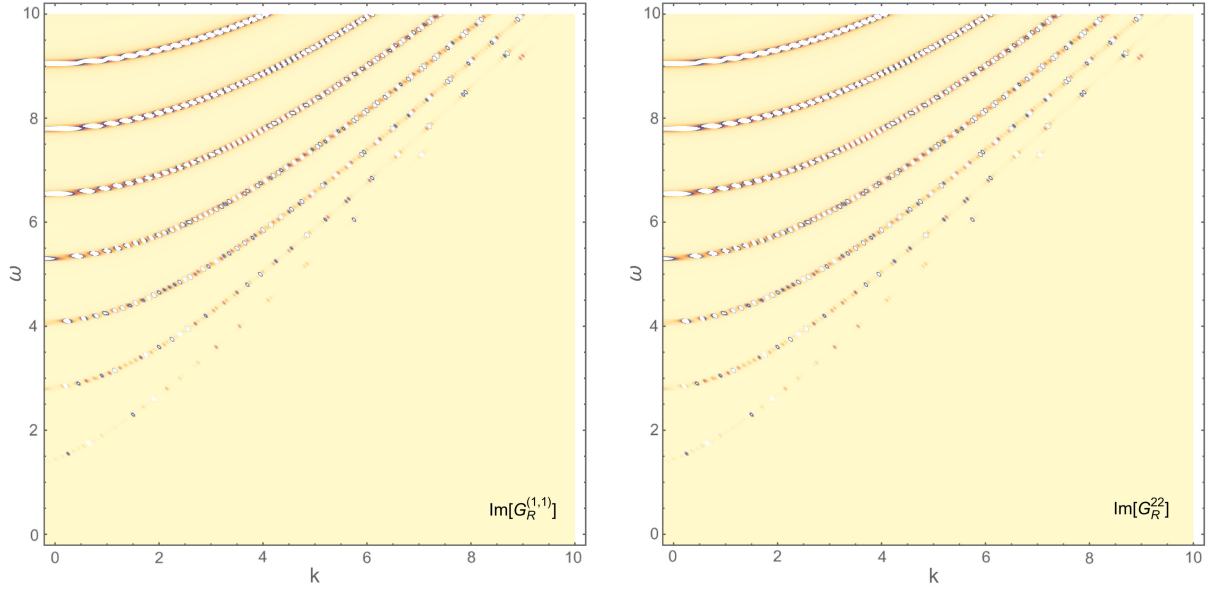


FIG. 3. Confined dispersion curves from the fermionic Green function $G_R^{(1,1)}$ and $G_R^{(2,2)}$ as functions of ω and k in the unit of $M_{KK} = 1$. The red-white points denote the peaks in the correlation function.

the Wick-rotated version of the infalling boundary condition as $\xi_{(h)}|_{r=r_{KK}} = (-1)^h$, since the confined metric (2.11) can be obtained by a double Wick rotation from the deconfined metric (2.10). With all above, the numerical results for the Green functions are given in Figs. 3 and 4.

As the two-point Green function is proportional to $(k - M_n)^{-1}$, the location of the peaks in the Green function refers to the onset mass of the fermion which agrees with the mass spectrum we obtained in Sec. III.2. In addition, Fig. 4 also displays that the onset mass of the fermion decreases very slowly when the instanton density Q

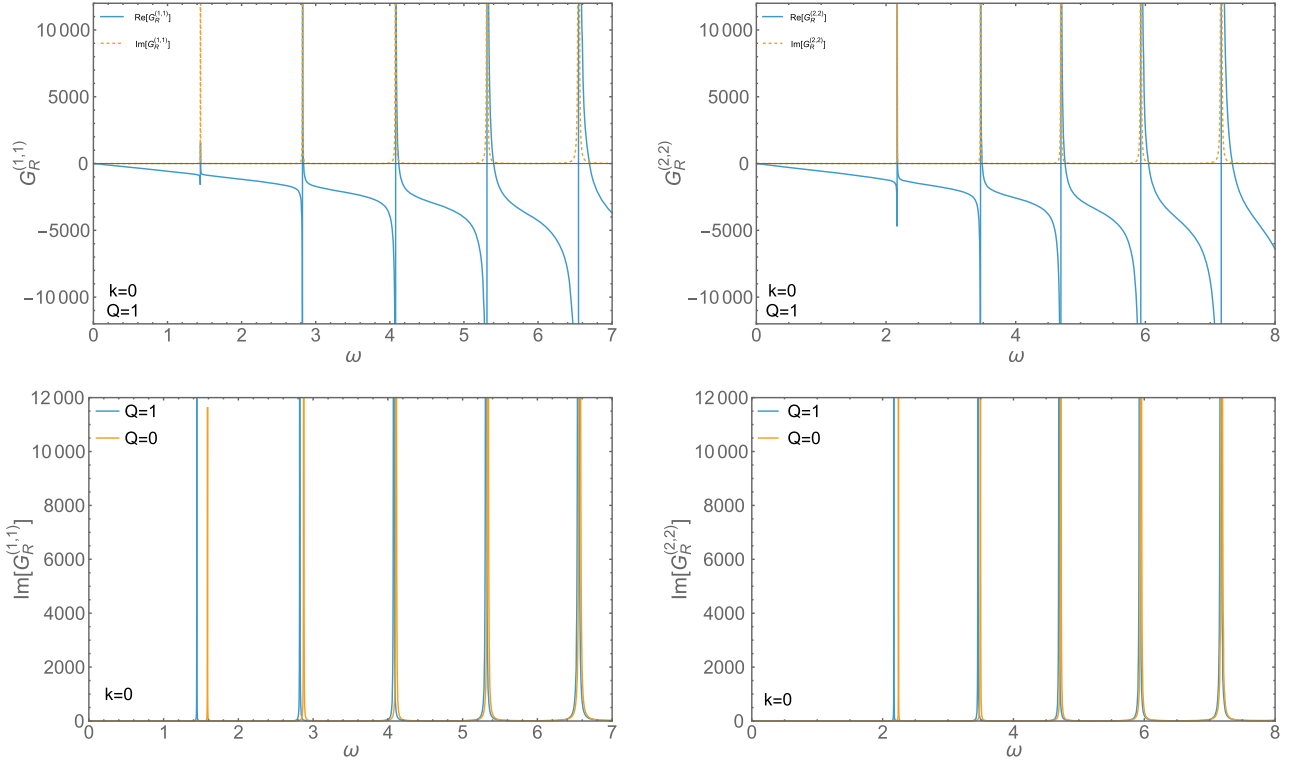


FIG. 4. Confined fermionic Green function $G_R^{(1,1)}$ and $G_R^{(2,2)}$ as functions of ω with $k = 0$ and various Q in the unit of $M_{KK} = 1$.

increases. This conclusion agrees well with the behavior of the mass spectrum given in Fig. 2.

3. Approach to the deconfined phase

In the deconfined geometry, there is no warp configuration for a D5-brane on S^5 in the background (2.2) [45];

therefore, as in most works about gauge-gravity duality, the bulk fermion ψ is identified as baryonic plasmino instead of baryons [50–53]. With this in hand, let us follow the above steps with respect to the deconfined geometry given in Eq. (2.10) in order to compute the correlation function. To begin with, the Dirac operator is computed as

$$\Gamma^\alpha \nabla_\alpha = \frac{e^{-\frac{\phi}{4}} R}{\sqrt{f}} \gamma^0 \partial_0 + e^{-\frac{\phi}{4}} \frac{R}{r} \gamma^i \partial_i + e^{-\frac{\phi}{4}} \frac{\rho}{R} \gamma^4 \partial_4 + \frac{e^{-\frac{\phi}{4}}}{R} \gamma^4 \left(-\frac{2\zeta}{\rho} + \frac{2\rho}{\zeta} + \frac{\zeta}{4f} \frac{df}{d\rho} + \frac{3\zeta}{2r} \frac{dr}{d\rho} + \frac{7\zeta}{8} \frac{d\phi}{d\rho} \right) + e^{-\phi/4} \frac{\rho}{R\zeta} \mathcal{D}_{S^4}. \quad (3.38)$$

Putting Eq. (3.38) with Eq. (3.7) into the fermionic action (3.5), we can obtain

$$S_{D7}^f = i \int d^3x d\zeta \bar{\psi} \left[\frac{\mathcal{A}}{\mathcal{C}} \gamma^0 \partial_0 + \frac{\mathcal{B}}{\mathcal{C}} \gamma^i \partial_i + \gamma^4 \left(\partial_4 - \frac{1}{2} \partial_4 \ln \mathcal{C} \right) + \frac{\mathcal{D}}{\mathcal{C}} \gamma^4 + \frac{\mathcal{E}}{\mathcal{C}} \right] \psi, \quad (3.39)$$

by using the dimensional reduction given in Sec. III 2. And the corresponding coefficient terms are collected as

$$\begin{aligned} \mathcal{A} &= \frac{T_7}{2} e^{-5\phi/4} \sqrt{-\frac{g}{g_{S^4} r \sqrt{f}}}, \\ \mathcal{B} &= \frac{T_7}{2} e^{-5\phi/4} \sqrt{-\frac{g}{g_{S^4} r}}, \\ \mathcal{C} &= \frac{T_7}{2} e^{-5\phi/4} \sqrt{-\frac{g}{g_{S^4} R}}, \\ \mathcal{D} &= \frac{T_7}{2} \sqrt{-\frac{g}{g_{S^4}}} \left[\frac{e^{-5\phi/4}}{R} \left(-\frac{2\zeta}{\rho} + \frac{2\rho}{\zeta} + \frac{\zeta}{4f} \frac{df}{d\rho} + \frac{3\zeta}{2r} \frac{dr}{d\rho} + \frac{7\zeta}{8} \frac{d\phi}{d\rho} \right) - i \frac{3\rho}{4R} e^{\phi/4} \partial_\zeta C_0 \right], \\ \mathcal{E} &= \frac{T_7}{2} \sqrt{-\frac{g}{g_{S^4}}} \left(\frac{1}{2} \frac{e^{-9\phi/4}}{R} + e^{-5\phi/4} \frac{\rho}{R\zeta} \Lambda_l^+ \right), \\ \Psi &= \frac{1}{\sqrt{\mathcal{C}}} \psi(x, \zeta) \otimes \varphi^{+l,s}(S^4) \otimes \beta. \end{aligned} \quad (3.40)$$

Then, the Dirac equation derived from Eq. (3.39) is

$$\left[\frac{\mathcal{A}}{\mathcal{C}} \gamma^0 \partial_0 + \frac{\mathcal{B}}{\mathcal{C}} \gamma^i \partial_i + \gamma^4 \left(\partial_4 - \frac{1}{2} \partial_4 \ln \mathcal{C} \right) + \frac{\mathcal{D}}{\mathcal{C}} \gamma^4 + \frac{\mathcal{E}}{\mathcal{C}} \right] \psi = 0. \quad (3.41)$$

Recalling the ansatz (3.29) and gamma matrices given in Eq. (3.31), the Dirac equation becomes

$$\begin{aligned} \left(\partial_4 - \frac{1}{2} \partial_4 \ln \mathcal{C} + \frac{\mathcal{D}}{\mathcal{C}} + \frac{\mathcal{E}}{\mathcal{C}} \right) \chi_+ + \left(\omega \frac{\mathcal{A}}{\mathcal{C}} + \tau^3 k \frac{\mathcal{B}}{\mathcal{C}} \right) \chi_- &= 0, \\ \left(\omega \frac{\mathcal{A}}{\mathcal{C}} - \tau^3 k \frac{\mathcal{B}}{\mathcal{C}} \right) \chi_+ + \left(-\partial_4 + \frac{1}{2} \partial_4 \ln \mathcal{C} - \frac{\mathcal{D}}{\mathcal{C}} + \frac{\mathcal{E}}{\mathcal{C}} \right) \chi_- &= 0, \end{aligned} \quad (3.42)$$

which can be solved analytically near the holographic boundary $\zeta \rightarrow \infty$, as

$$\begin{aligned} \chi_+ &= R_A \zeta^{\Lambda_l^+ + 1} + R_B \zeta^{-\Lambda_l^+ - 1}, \\ \chi_- &= L_A \zeta^{\Lambda_l^+} + L_B \zeta^{-\Lambda_l^+}. \end{aligned} \quad (3.43)$$

Following the same steps given in Eqs. (3.33) and (3.34), the Green function remains to be Eq. (3.35). So by introducing ratios given in Eq. (3.36), the equations of motion for $\xi_{(s)}$ can be obtained from the Dirac equation (3.42) as

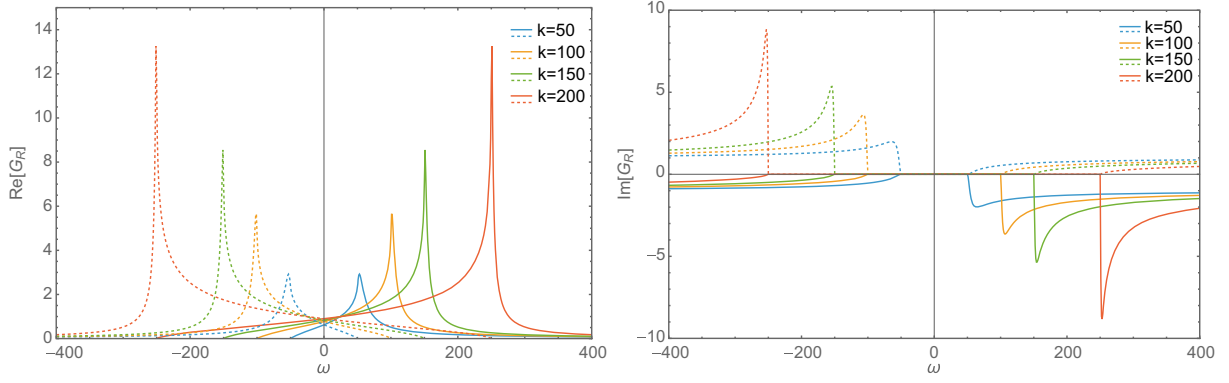


FIG. 5. Deconfined fermionic Green function from D3/D7 model with instanton density $Q = 1$ and temperature $T = \frac{1}{\pi}$. The solid and dashed lines refer respectively to $G_R^{(1,1)}$ and $G_R^{(2,2)}$ with various momentums k .

$$\begin{aligned} \frac{C}{B} \xi'_{(1)} &= -k - \frac{A}{B} \omega - \frac{2\mathcal{E}}{B} \xi_{(1)} + \left(k - \frac{A}{B} \omega\right) \xi_{(1)}^2, \\ \frac{C}{B} \xi'_{(2)} &= -k + \frac{A}{B} \omega - 2 \frac{\mathcal{E}}{B} \xi_{(2)} + \left(k + \frac{A}{B} \omega\right) \xi_{(2)}^2, \end{aligned} \quad (3.44)$$

which can be solved numerically with the infalling boundary condition $\xi_{(h)}|_{r=r_H} = (-1)^h i$. The numerical results are collected in Figs. 5 and 6.

It indicates that dispersion curves satisfy $\omega \simeq k$ for large k while $\omega \simeq k^2$ for small k . And this behavior is very close to the fermionic dispersion curves obtained from the method of HTL in thermal field theory, that is [54],

$$\begin{aligned} \omega(k) &\simeq m_f \pm \frac{1}{3}k + \frac{1}{3m_f}k^2, \quad k \ll 1; \\ \omega(k) &\simeq k, \quad k \gg 1, \end{aligned} \quad (3.45)$$

where m_f is the effective mass generated by the medium effect of fermion as

$$m_f = \sqrt{\frac{C_F}{8}} g_{YM} T. \quad (3.46)$$

Here, g_{YM} refers to the Yang-Mills coupling constant and C_F is a constant suggested to be $C_F = 4/3$ for fundamental quarks or $C_F = 1$ for electron. While the hard thermal loop approximation [54] displays two branches of the dispersion curves for $\omega > 0$ and $\omega < 0$, it is out of reach for the present holographic works with spinor, e.g., Refs. [50–53].

Finally, let us investigate the instanton dependence in the spectral function. The above numerical calculation illustrates that the spectral function basically holds when the instanton charge increases, since the dependence of the instanton density Q in Eq. (3.44) is not dominant at both boundary and horizon. Nevertheless, to see explicitly the

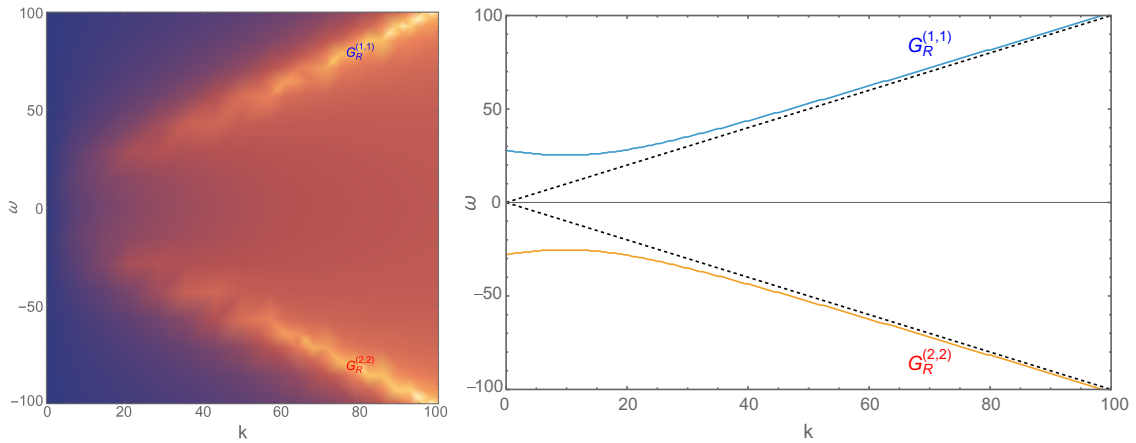


FIG. 6. Dispersion curves from the fermionic Green function with $Q = 1$ and $T = \frac{1}{\pi}$. Left: density plot of the fermionic Green functions, where the white region refers to the peaks in the Green function. Right: numerical evaluation of the dispersion curves from fermionic Green functions, where the dashed lines refer to the light cone.

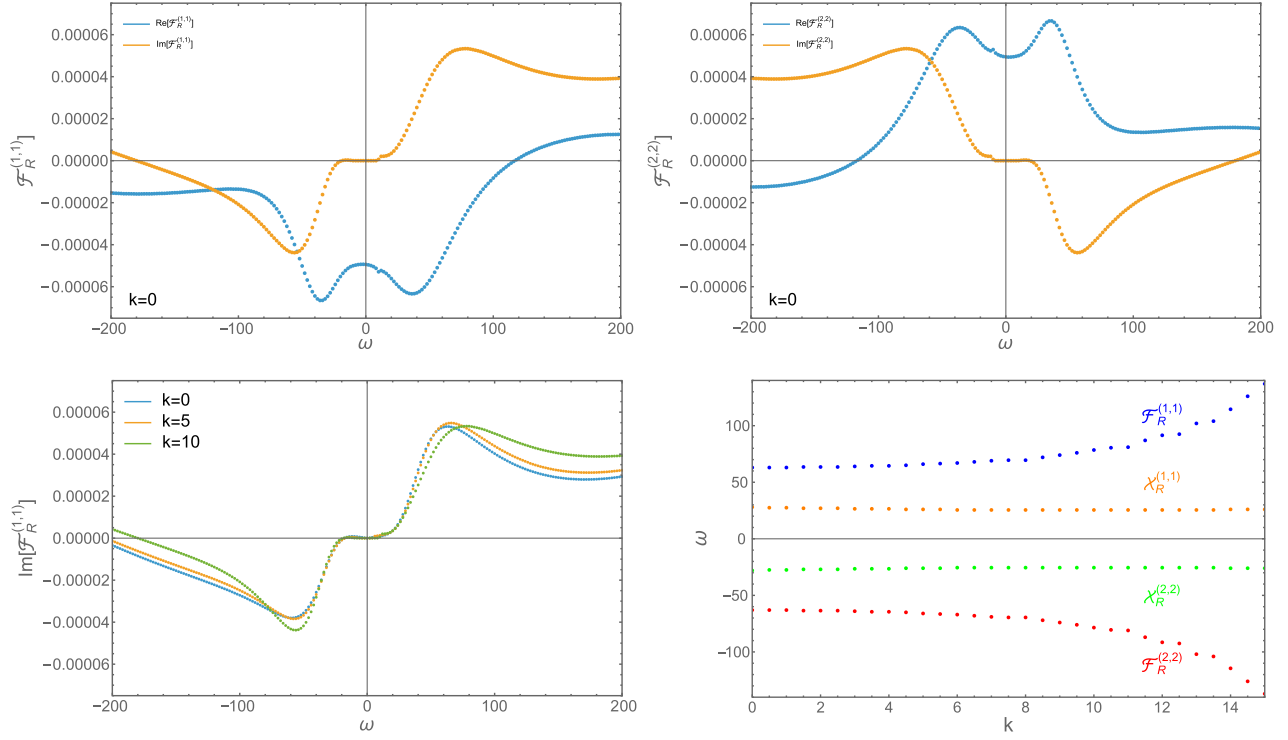


FIG. 7. First-order Green function and the associated dispersion curves in the deconfined phase.

dependence of Q in the spectral function, let us consider the situation that the instanton density is sufficiently small, i.e., $Q \ll 1$. And we assume the Green function can be decomposed as a series of Q ,

$$\begin{aligned} G_R &= \mathcal{G}_R + Q\mathcal{F}_R + \mathcal{O}(Q^2), \\ \xi_{(h)} &= X_{(h)} + QY_{(h)} + \mathcal{O}(Q^2), \end{aligned} \quad (3.47)$$

where $\mathcal{G}_R$ is zeroth-order Green function given by solving Eq. (3.44) with $Q = 0$ and $\mathcal{F}_R$ is first-order Green function, which is given through Eq. (3.35) as

$$\mathcal{F}_R^{(h,h)} = (-1)^h \lim_{\zeta \rightarrow \infty} \zeta Y_{(s)}. \quad (3.48)$$

The equation of motion for $Y_{(h)}$ can be obtained by plugging Eq. (3.47) into Eq. (3.44), which is

$$\begin{aligned} -\left(\frac{1}{\rho} + \frac{2\Lambda_l}{\zeta}\right)Y_{(h)} + X_{(h)} \left[\frac{2\ln(1 - \frac{\rho^2}{r^2})}{\rho^3(\rho^2 - 2r^2)} \right. \\ \left. + \frac{2R^2}{z\rho} \left(k + \frac{\omega}{1 - \frac{\rho^2}{r^2}} Y_{(h)} \right) \right] - Y'_{(h)} = 0. \end{aligned} \quad (3.49)$$

And this equation can be solved numerically with the boundary condition for $Y_{(h)}$ as $Y_{(h)}|_{r=r_H} = 0$ which is reduced from the infalling boundary condition $\xi_{(h)}|_{r=r_H} = (-1)^h i$. Afterwards, the numerical evaluation for the first-order Green function $\mathcal{F}_R$ is plotted in Fig. 7. It shows

that the onset mass increases by the momentum, which leads to a parallel dispersion curve additional to those from the first-order Green function. While there are two branches to each dispersion curve with this approximation and it is close to the calculation from the HTL, this approximation is very slight due to our numerical calculation. Obviously, for $Q = 0$, the approximated dispersion curve disappears.

IV. THE INTERACTION OF THE LOWEST BARYON AND MESON

In this section, let us investigate the interaction of the lowest fermion and gauge boson as the lowest baryon and meson, respectively, on the worldvolume of the D7-branes. The spectrum of the meson on the flavor can be reviewed in the Appendix, which is also useful in this section. As our concern is the interaction of hadrons, the following analysis is valid in the confined phase, only since quarks in deconfined geometry cannot form a hadron [28,45] in this model. Keeping this in mind, let us focus on the trilinear Yukawa-like couplings of the fermion and boson given in the action (3.1), since both L_{D7} and Γ_{D7} include a piece linear in the gauge field strength f as

$$\begin{aligned} \Gamma_{D7} &= \gamma^4 \left[1 - \frac{1}{2} \bar{\gamma} \Gamma^{\alpha\beta} (2\pi\alpha') f_{\alpha\beta} + \mathcal{O}(f^2) \right], \\ L_{D7} &= -\gamma^4 \bar{\gamma} \Gamma^\alpha (2\pi\alpha') f_{\alpha\beta} g^{\beta\delta} D_\delta + \mathcal{O}(f^2), \end{aligned} \quad (4.1)$$

where α, β, δ runs over the D7-brane. Inserting Eq. (4.1) into action (3.1), we can obtain the trilinear coupling terms as the lowest interaction terms involving mesons in the action as

$$\begin{aligned} S_{\text{int}} &= S_{\text{int}}^{(1)} + S_{\text{int}}^{(2)}, \\ S_{\text{int}}^{(1)} &= -i \frac{T_7}{2} (2\pi\alpha') \int d^8x \sqrt{-g} e^{-\phi} \bar{\Psi} \frac{1}{2} \gamma^4 \Gamma^{\alpha\beta} f_{\alpha\beta} \Gamma^\gamma D_\gamma \Psi, \\ S_{\text{int}}^{(2)} &= -\frac{T_7}{2} (2\pi\alpha') \int d^8x \sqrt{-g} e^{-\phi} \bar{\Psi} (1 - \gamma^4) \tilde{\gamma} \Gamma^\alpha f_{\alpha\beta} g^{\beta\gamma} D_\gamma \Psi. \end{aligned} \quad (4.2)$$

Following the dimensional reduction in Sec. III 2, then picking up the confined metric (2.10), the actions presented in Eq. (4.2) can be written as

$$\begin{aligned} S_{\text{int}}^{(1)} &= i \int d^3x d\zeta d\Omega_4 \bar{\psi} \gamma^\mu \partial_\mu \pi (W_1 \gamma^\nu \partial_\nu + W_2 \gamma^4 \partial_4 + W_3 \gamma^4 + W_4) \psi, \\ S_{\text{int}}^{(2)} &= -\frac{1}{2} \int d^3x d\zeta \bar{\psi} (\gamma^4 - 1) [P_1 \gamma^\mu \partial_\mu \pi \partial_4 - P_1 \gamma^4 \partial_\mu \pi + P_2 \gamma^\mu \partial_\mu \pi] \psi, \end{aligned} \quad (4.3)$$

where the potential functions dependent on $x^4 \equiv \zeta$ are collected as

$$\begin{aligned} W_1 &= \frac{T_7}{2} (2\pi\alpha') d^{(0)} \sqrt{-\frac{g}{g_{44}g_{xx}g_{S^5}}} e^{-5\phi/4} \frac{R}{r}, \\ W_2 &= \frac{T_7}{2} (2\pi\alpha') d^{(0)} \sqrt{-\frac{g}{g_{44}g_{xx}g_{S^5}}} e^{-5\phi/4} \frac{\rho}{R}, \\ W_3 &= \frac{T_7}{2} (2\pi\alpha') d^{(0)} \sqrt{-\frac{g}{g_{44}g_{xx}g_{S^5}}} \left[\frac{e^{-5\phi/4}}{R} \left(-\frac{2\zeta}{\rho} + \frac{2\rho}{\zeta} + \frac{3\zeta}{2r} \frac{dr}{d\rho} + \frac{7}{8} \zeta \frac{d\phi}{d\rho} \right) - i \frac{3}{4} \frac{\rho}{R} e^{-\phi/4} \partial_\zeta C_0 \right], \\ W_4 &= \frac{T_7}{2} (2\pi\alpha') d^{(0)} \sqrt{-\frac{g}{g_{44}g_{xx}g_{S^5}}} \left(\frac{1}{2} \frac{e^{-9\phi/4}}{R} + \frac{\rho}{R\zeta} e^{-5\phi/4} \Lambda_l^\pm \right), \\ P_1 &= T_7 (2\pi\alpha') d^{(0)} \sqrt{-\frac{g}{g_{44}g_{xx}g_{S^5}}} e^{-\phi}, \\ P_2 &= T_7 (2\pi\alpha') d^{(0)} \sqrt{-\frac{g}{g_{44}g_{xx}g_{S^5}}} \frac{\zeta e^{-\phi}}{R^2} \left(\frac{1}{2} \frac{dr}{d\rho} + \frac{r}{8} \frac{d\phi}{d\rho} \right), \end{aligned} \quad (4.4)$$

and we have imposed the decomposition of $f_{\alpha\beta}$ given (A15) by picking up the lowest mesonic scalar identified as $\varphi^{(0)} \equiv \pi$. Afterwards, let us further recall the decomposition of ψ given in Eq. (3.13) and pick its lowest baryonic fermion as $\psi_\pm \equiv \psi_\pm^{(0)}, f_\pm \equiv f_\pm^{(0)}$; the actions in Eq. (4.3) can then be rewritten as the effective coupling terms as

$$S_{\text{int}} = \int d^3x d\mu \pi \left[\frac{ig_1}{M_{KK}} (\psi_+^\dagger \bar{\sigma}^\mu \sigma^\nu \partial_\nu \psi_- - \psi_-^\dagger \bar{\sigma}^\mu \sigma^\nu \partial_\nu \psi_+) + g_2 \psi_+^\dagger \bar{\sigma}^\mu \psi_+ - g_3 \psi_-^\dagger \bar{\sigma}^\mu \psi_- + g_4 (\psi_+^\dagger i \partial^\mu \psi_- - \psi_-^\dagger i \partial^\mu \psi_+) \right], \quad (4.5)$$

where the coupling constants $g_{1,2,3,4}$ are functions of the instanton density Q , which can be evaluated by the potentials given in Eq. (4.4) as

$$\begin{aligned} g_1(Q) &= \int d\zeta f_+ W_1 f_-, \\ g_2(Q) &= \int d\zeta f_+ (W_2 \partial_4 + W_3 + W_4) f_+ - \int d\zeta (f_+ P_1 \partial_4 f_+ + f_+ P_2 f_+), \\ g_3(Q) &= \int d\zeta f_- (W_2 \partial_4 + W_3 - W_4) f_- - \int d\zeta (f_- P_1 \partial_4 f_- + f_- P_2 f_-), \\ g_4(Q) &= \int d\zeta f_+ P_1 f_-. \end{aligned} \quad (4.6)$$

TABLE III. Coupling constants of trilinear Yukawa-like terms at $Q = 0$ in the unit of $\lambda^{-\frac{1}{2}} N_c^{-\frac{1}{2}} M_{KK}^{-1}$. The $\lambda = g_s N_c$ is the dimensionless 't Hooft coupling constant.

$Q = 0$	g_1	g_2	g_3	g_4
$\lambda^{-\frac{1}{2}} N_c^{-\frac{1}{2}} M_{KK}^{-1}$	1.65	12.08	-8.92	1.46

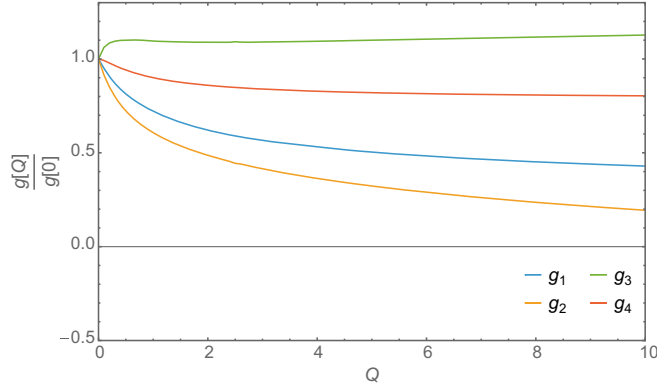


FIG. 8. Ratios of $g(Q)/g(0)$ as functions of Q . $g(Q)$ refers to the coupling constants $g_{1,2,3,4}$.

Therefore, all the coupling constants can be evaluated numerically where the associated results are given in Table III, and in particular, their dependences on the instanton density Q are illustrated numerically in Fig. 8. We can see that all the coupling constants converge to a finite value at large Q , which implies the amplitude of the baryonic fermion and mesonic scalar basically hold when the instanton density Q increases. In addition, as the fermionic fields are expected to be baryonic, there are N_c quarks in N_c possible insertions of the fermion bilinear. In large N_c , as we have mentioned, we can rescale the fermionic field by $\psi_{+,-}^{(n)} \rightarrow \sqrt{N_c} \psi_{+,-}^{(n)}$ to include the contributions of the N_c quarks in a baryonic field. In this sense, all the coupling constants presented in Eq. (4.6) are proportional to $N_c^{1/2}$, which agrees with that the coupling constant of baryon-meson interaction takes order of $N_c^{1/2}$ in large N_c QCD [41]. We finally note that action (4.5) could be P violated if we identify “ $\pm$ ” to be the parity of the fermion.

V. SUMMARY AND DISCUSSION

In this work, we investigate the flavored worldvolume theory in the top-down approach of D3/D7 model with homogenous D(-1)-branes. In the gauge-gravity duality, D(-1)-branes are holographic instantons which can violate the CP symmetry in the dual theory. Taking into account the double Wick rotation and dimensional reduction, we obtain the confined and deconfined geometry in this model, which refers respectively to the bubble and black brane

solution. In the confined geometry, we construct a third holographic theory of QCD involving baryons, mesons with instantons, or CP violation. And the spectrum of the fermion is obtained by decomposing the spinor on the worldvolume of the D7-brane, which is consistent quantitatively with the confined correlation function. Moreover, our numerical evaluation illustrates that the mass spectrum decreases slightly by the instanton density Q and almost holds at large Q . So the CP violation may not affect evidently the mass of the hadrons predicted by this model. Interestingly, when the baryon vertex as a D5-brane with N_c open strings is introduced, the worldvolume fermion as a gauge-invariant operator of $U(N_c)$ further takes the baryon number; therefore, it can be identified as a baryon in the confined geometry. In this sense, we compare the mass of the lowest baryon and meson in our model with the associated experimental data, then find that they are basically close. Afterwards, we demonstrate how to obtain the effective interaction terms of the baryonic fermion and mesonic boson, and evaluate the coupling constants by using the data from the spectroscopy on the D7-brane. In the large N_c limit, we find all the coupling constants are proportional to $N_c^{1/2}$, which agrees with the evaluation from the large N field theory [41], and it may not be necessary to preserve the CP symmetry in interaction terms. So employing it to analyze the hadron decay with CP violation additional to the CKM mechanism may be possible in future works. Besides, in the deconfined geometry, although the wrapped configuration of a baryon vertex (D5-brane) may not exist, we also work out the fermionic correlation function by using the holographic prescription and interpret the fermion as plasmino [52,53]. The dispersion curve obtained from the deconfined correlation function basically covers the results from HTL approximation [54]. In summary, this work is a systematic study about baryons and mesons involving instantons or CP violation through gauge-gravity in the top-down approach.

Let us give some more comments here to finalize this work. First, the approach of the D3/D7 model with instantons is in fact based on the type IIB string theory, thus only two parameters in our theory, λ and M_{KK} , which correspond, respectively, to the string coupling constant and the confining scale, are free to fit the experimental data. Therefore “fewer parameters” is a characteristic feature of the top-down approach different from the bottom-up approaches and phenomenological models. Second, in the deconfined geometry, as we have seen, most holographic approaches cannot work out the two branches in the fermionic dispersion curve, e.g., Refs. [52,53]; the reason might be that the background geometry we used corresponds to the ideal fluid, where the dissipative terms are absent. In this sense, reevaluating the holographic fermionic correlation function with dissipative terms in the background geometry (e.g., Ref. [55]) may be interesting in future works. Third, although we obtain the second

branch in the fermionic dispersion curve (displayed in Fig. 7) by solving the first-order equations for the correlation function in Eq. (3.49), it is due to the presence of the instantons. And this separation in the dispersion curve may be caused by the instanton-induced interaction with spin, as discussed in Refs. [6,7], so the gauge-gravity duality may provide a powerful way to investigate it, since confirming the correlation function of baryons with instantons is quite difficult in the QFT approach. Last but not least, in the confined geometry, while the dual field theory in our model is definitely QCD 3 in the large N_c limit, we believe that most of the evaluation in this work is less dependent on the dimension. The reason is that the spectroscopy and the coupling constants of both fermions and bosons are determined by the basis functions given in Eqs. (3.13) and (A6), whose eigen equations are basically independent of the dimension given in Eqs. (3.17) and (A10). Therefore, as a holographic approach, our current evaluation may also be suitable for four-dimensional QCD.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: THE MESONIC SPECTRUM FROM THE DBI ACTION OF THE FLAVOR BRANE

In this appendix, let us outline the spectrum of the mesonic gauge boson on the worldvolume of the (flavor) D7-brane by using the Dirac-Born-Infeld (DBI) action. Since mesons are confined states, the following analysis is valid only in the confined geometry given in Eq. (2.5). Recall that the DBI action for a D7-brane is given as

$$S_{D7} = -T_{D7} \int d^8x e^{-\phi} \sqrt{-\det(g_{\alpha\beta} + 2\pi\alpha' f_{\alpha\beta})},$$

$$\simeq -T_{D7} \int_{D7} d^3x d\zeta d\Omega_4 e^{-\phi} \sqrt{-g} \left[1 + \frac{1}{4} (2\pi\alpha')^2 g^{\alpha\gamma} g^{\beta\delta} f_{\alpha\beta} f_{\gamma\delta} + \mathcal{O}(f^4) \right], \quad (A1)$$

where $f_{\alpha\beta}$ and $g_{\alpha\beta}$ refer, respectively, to the gauge field strength and induced metric on the flavor brane. To obtain the standard kinetic form for vector and scalar meson, the quadratic term in the action is necessary, so we need

$$S_{D7} = -\frac{1}{4} (2\pi\alpha')^2 T_{D7} \int d^3x d\zeta d\Omega_4 e^{-\phi} \sqrt{-g} g^{\alpha\gamma} g^{\beta\delta} f_{\alpha\beta} f_{\gamma\delta}, \quad (A2)$$

where the indices α, β, δ run over the D7-brane. Plugging the induced metric on the D7-brane given in Eq. (2.11) into Eq. (A2) and assuming the nonvanished components of the gauge field are ($x^4 \equiv \zeta$),

$$A_\alpha(x, \zeta) = \{A_\mu(x, \zeta), A_4(x, \zeta)\}, \quad (A3)$$

the action (A2) can be written as

$$S_{D7} = -\frac{1}{4} \int d^3x d\zeta U_1 (\eta^{\mu\rho} \eta^{\nu\sigma} f_{\mu\nu} f_{\rho\sigma} + 2U_2 f_{\mu 4} f_{\nu 4}), \quad (A4)$$

where $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the kinetic term of the gauge field and

$$U_1(\xi) = \frac{2}{3\pi^2} \frac{N_c}{M_{KK}} e^{2\phi} \frac{\xi^4 r_{KK}}{\rho^5 r}, \quad U_2(\xi) = \frac{r^2}{4r_{KK}^2} M_{KK}^2 \rho^2. \quad (A5)$$

Then we can decompose the gauge field by a series of complete functions as

$$A_\mu(x, \zeta) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x) b^{(n)}(\zeta), \quad A_4(x, \zeta) = \sum_{n=1}^{\infty} \varphi^{(n)}(x) d^{(n)}(\zeta). \quad (A6)$$

Inserting Eq. (A6) into Eq. (A4) leads to

$$S_{D7} = -\frac{1}{4} \sum_{n,m=1}^{\infty} \int d^3x d\zeta U_1 \{ [\partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)}] [\partial^\mu B^{\nu(m)} - \partial^\nu B^{\mu(m)}] b^{(n)} b^{(m)} \\ + 2U_2 [\partial_\mu \varphi^{(n)} d^{(n)}(\zeta) - B_\mu^{(n)} \partial_\zeta b^{(n)}] [\partial^\mu \varphi^{(m)} d^{(m)} - B^{\mu(n)} \partial_\zeta b^{(n)}] \}. \quad (A7)$$

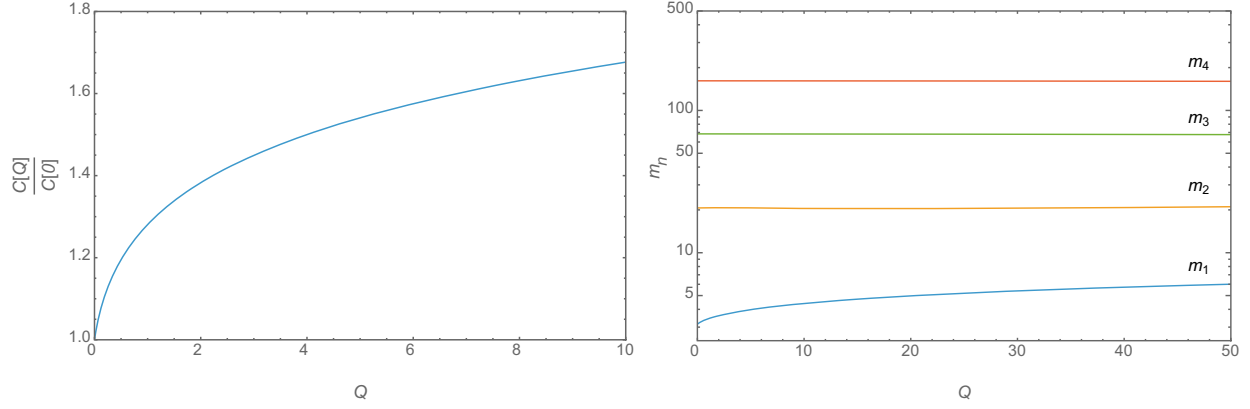


FIG. 9. Left: ratio of $C(Q)/C(0)$ as a function of Q . Right: mesonic spectrum as a function of Q .

In order to obtain the standard form of the kinetic term, the following normalization conditions are required for the basis functions as

$$\begin{aligned} \int d\zeta U_1 b^{(n)} b^{(m)} &= \delta^{mn}, \\ \int d\zeta U_1 U_2 \partial_\zeta b^{(n)} \partial_\zeta b^{(m)} &= m_n \delta^{mn}, \end{aligned} \quad (\text{A8})$$

with $m_n = \lambda_n M_{KK}$ and

$$\int d\zeta U_1 U_2 d^{(n)} d^{(m)} = \delta^{mn}. \quad (\text{A9})$$

So these conditions reduce to the solution for the $d^{(n)}$, $\partial_\zeta b^{(n)}$ and the eigen equations for the basis function $b^{(n)}$ ($n \geq 1$) as

$$\begin{aligned} \partial_\zeta [U_1 U_2 \partial_\zeta b^{(n)}] &= \lambda_n M_{KK} U_1 b^{(n)}, \\ d^{(n)} &= m_n^{-1} \partial_\zeta b^{(n)}. \end{aligned} \quad (\text{A10})$$

Notice that the basis functions $b^{(n)}$'s have to satisfy the Dirichlet boundary condition, and there is an additional function $d^{(0)}$ orthogonal to all the $d^{(n)}$'s for $n \geq 1$ if we choose

$$d^{(0)} = \frac{C}{U_1 U_2}. \quad (\text{A11})$$

So, one can verify that

$$\int d\zeta U_1 U_2 d^{(0)} d^{(n)} = C m_n^{-1} \int d\zeta \partial_\zeta b^{(n)} = 0, \quad n \geq 1. \quad (\text{A12})$$

Further, using the normalization condition,

$$r_{KK}^{-1} \int d\zeta U_1 U_2 d^{(0)} d^{(0)} = 1, \quad (\text{A13})$$

we obtain

$$C = \left(r_{KK}^{-1} \int d\zeta U_1 U_2 \right)^{-1/2}, \quad (\text{A14})$$

where the constant C must depend on the density of instanton Q . Dependence of Q is numerically evaluated in Fig. 9. Keeping all the above in mind, it means we need to shift the start point in the summary given in A_4 in Eq. (A6) as

$$A_4(x, \zeta) = \varphi^{(0)}(x) d^{(0)}(\zeta) + \sum_{n=1}^{\infty} \varphi^{(n)}(x) d^{(n)}(\zeta). \quad (\text{A15})$$

Afterwards, performing the gauge transformation as

$$V_\mu^{(n)}(x) = B_\mu^{(n)} - m_n^{-1} \partial_\mu \varphi^{(n)}, \quad (\text{A16})$$

then substituting $\partial_\zeta b^{(n)}$ for $d^{(n)}$ and inserting Eqs. (A8) and (A15) into Eq. (A7), the action in Eq. (A7) becomes the standard form of the vector and scalar field as

$$\begin{aligned} S_{D7} &= - \sum_{n=1}^{\infty} \int d^3x \left[\frac{1}{4} W_{\mu\nu}^{(n)} W^{\mu\nu(n)} + \frac{1}{2} m_n V_\mu^{(n)} V^{\mu(n)} \right] \\ &\quad - \frac{1}{2} \int d^3x \partial_\mu \varphi^{(0)} \partial^\mu \varphi^{(0)}, \end{aligned} \quad (\text{A17})$$

where

$$W_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu. \quad (\text{A18})$$

The mass spectrum for the vector meson V_μ can be obtained by solving the eigen equations for $b^{(n)}$, which is given in Eq. (A10); the numerical results are given in Table IV and Fig. 9. Our numerical results indicate that the mass spectrum is less dependent on the instanton density Q . Beside, since the confined phase described by Eq. (2.11) is valid below the scale M_{KK} , we note that only the massless scalar $\varphi^{(0)}$ arises strictly in the low-energy theory.

Finally, the action of the D7-brane also contains CS terms, as follows:

$$S_{\text{CS}} = \frac{\mu_7}{8!} \int C_8 + \frac{\mu_7}{2!} (2\pi\alpha')^2 \int f \wedge f \wedge C_4 + \text{fermionic terms}, \quad (\text{A19})$$

which are of order $\mathcal{O}(f^2)$ and are CP -violated terms.

TABLE IV. Spectrum of the mesonic boson in the unit of $M_{KK} = 1$ with various Q .

m_n	$n = 1$	$n = 2$	$n = 3$	$n = 4$
$Q = 0$	3.15	20.64	68.43	161.5
$Q = 1$	3.43	20.71	68.47	161.51

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Correction: The omission of an affiliation for the second author has been corrected.