

Transversity generalized parton distributions of the Δ with the diquark spectator model

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We show quark transversity generalized parton distributions (GPDs) of the Δ^+ isobar by using the diquark spectator model for the first time. First, this model is tested by electric charge, magnetic-dipole, and axial charge form factors, and it is used for calculating the transversity GPDs $H_{1,3,5,7}^{qT}$ of Δ^+ . The quark transversity distribution h_1 is then obtained from the transversity GPDs in the forward limit. Then, helicity-flip amplitudes are shown numerically by using relations between the helicity amplitudes and the GPDs. Finally, by taking first moments of the GPDs, tensor form factors are obtained and we predict the tensor charge. Experimentally, N - Δ transition GPDs are investigated in deeply virtual Compton scattering and virtual meson-production processes, and generalized distribution amplitudes, which correspond to the s -channel GPDs, could be investigated by the two-photon processes $\gamma^*\gamma \rightarrow \Delta\bar{\Delta}$ at the electron-positron colliders. Therefore, the spin-3/2 Δ GPDs could become interesting quantities experimentally in the future.

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I. INTRODUCTION

Generalized parton distributions (GPDs) have become important functions to describe the hadron structure since they were proposed [1–4]. The GPDs contain abundant information about quark and gluon distributions inside the hadron, besides the usual parton distributions. It is known that the GPDs have three variables: the parton longitudinal momentum fraction x , the transferred momentum square t , and the skewness ξ , which is defined by the longitudinal transferred momentum. On the other hand, parton-hadron helicity amplitudes, which can be expressed by the GPDs, constrain that there are $2(2J+1)^2$ independent GPDs for a quark or a gluon of a spin- J hadron at the leading twist. Half of the GPDs are helicity-nonflip GPDs, which are unpolarized and longitudinal polarized GPDs, and the other

half are helicity-flip GPDs, i.e., transversity GPDs. For a spin- $\frac{3}{2}$ hadron, there are 16 independent transversity GPDs, and these GPDs are chiral odd, since the nonlocal operator flips the quark chirality in contrast to the chiral-even unpolarized and longitudinal polarized GPDs.

Similar to the deep inelastic scattering process that can be used to measure the parton distributions, off-forward Compton scattering reaction can be utilized to measure the GPDs, like the well-known deeply virtual Compton scattering (DVCS) and deeply virtual meson-production processes. The DVCS reveals the correlations between the momentum and spatial degrees of freedom of the partons, and these correlations contain a wealth of information on the structure of hadrons. In recent years, the proton DVCS has been measured by several accelerator facilities such as Jefferson Lab [5–14] and DESY [15–20]. In addition, there are future GPD projects at EicC [21] and EIC [22], while Japan Proton Accelerator Research Complex (J-PARC) [23,24] and Fermilab [25,26] also have possibilities to measure GPDs. There have been many theoretical studies about the GPDs, especially the spin-0 pion [27–31], the spin- $\frac{1}{2}$ proton [32–37], and the spin- $\frac{1}{2}$ ρ [38,39] and deuteron [40–42].

On the other hand, the s - t crossed process of the DVCS $\gamma^*h \rightarrow \gamma h$ with the photon γ and a hadron h is the two-photon process $\gamma^*\gamma \rightarrow h\bar{h}$. Although the hadron h should be

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generally a stable one in the DVCS, the h could be an unstable hadron in the two-photon process. It enables us to investigate internal structure of unstable hadrons with higher spins such as the spin-1 ρ and spin-3/2 Δ . In the two-photon processes [43], generalized distribution amplitudes (GDAs) are investigated [44], and they could be considered as s -channel GPDs. In fact, the GDAs of the pion were investigated, and its gravitational form factors were extracted from experimental data [44]. The unstable hadron GPDs can be also investigated in the form of transition GPDs [45]. In fact, there are already experimental data on the $N \rightarrow \Delta$ transition at Jefferson Lab and it will be investigated by the future EICs. Under the development of these experimental techniques, it is increasing important to study structure functions of higher-spin hadrons including the spin-3/2 Δ . The GDAs (s -channel GPDs) of Δ could be measured by $\gamma^* \gamma \rightarrow \Delta \bar{\Delta}$, and the transition GPDs were already measured for $N \rightarrow \Delta$. In this work, we study the transversity GPDs of the spin-3/2 particles numerically.

In our previous work [46–48], we have given the decomposition of the spin-3/2 twist-2 GPDs including unpolarized, longitudinal polarized, and transversity polarized GPDs, and the numerical calculations of the quark unpolarized and longitudinal polarized parts have been performed. In the present work, we show the quark transversity GPDs numerically by using the diquark spectator model. In the forward limit, the transversity GPDs give the transversity distribution function $h_1(x)$, which is explained as the number density of a parton with the longitudinal momentum fraction x and polarization parallel to that of the hadron with transverse polarized minus the number density with the antiparallel polarization. The transversity distribution function $h_1(x)$ has been explored by many studies [42,49–58]. Moreover, the first moments

of the transversity GPDs are tensor form factors of Δ , and numerical results of these form factors are also given in this paper.

In a previous paper [47], we showed the general formalism what kind of the transversity GPDs exist for spin-3/2 hadrons. The purpose of this paper is to show the quark transversity GPDs of Δ numerically by using the diquark spectator model. This work is intended to show the magnitude and functional forms of x and t of the transversity GPDs for possible future theoretical and experimental studies. In this paper, we make a qualitative estimate for the transversity GPDs of spin-3/2 particles, taking Δ^+ as an example. In Sec. II, the general formalism is given for the transversity GPDs and corresponding tensor form factors. Then, the diquark spectator model is introduced. In Sec. III, numerical results are shown for the transversity GPDs, the parton distribution functions (PDFs) of Δ^+ , helicity-flip amplitudes, and tensor form factors. Finally, the summary is given in Sec. IV.

II. TRANSVERSITY GPDs FOR SPIN-3/2 HADRONS AND THE DIQUARK SPECTATOR MODEL

A. Transversity GPDs for spin-3/2 hadrons

In this work, the leading twist quark transversity GPDs of the spin-3/2 hadron, Δ , are calculated for the first time by using the diquark spectator model. We take the d quark GPDs in Δ^+ as an example, and we can obtain the numerical results of other quarks by counting the corresponding quark number. The basic formalism is shown in this section for the transversity GPDs of the spin-3/2 hadrons [47]. The transversity GPDs of the hadrons are defined through the matrix element of the nonlocal quark operator with spin flip:

$$T_{\lambda\lambda'}^{qi} = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ix(P \cdot z)} \left\langle p', \lambda' \left| \bar{\psi} \left(-\frac{1}{2} z \right) i \sigma^{ni} \psi \left(\frac{1}{2} z \right) \right| p, \lambda \right\rangle \Big|_{z^+ = 0, z_\perp = 0}, \quad (1)$$

The p (p') and λ (λ'), respectively, denote the momentum and helicity of the initial (final) state, and $\lambda, \lambda' = \pm \frac{3}{2}, \pm \frac{1}{2}$ for spin-3/2 hadrons. The light-cone coordinate is employed and any four-vector v can be rewritten as $v = (v^+, v^-, \mathbf{v}_\perp)$, where $v^\pm = v^0 \pm v^3$ and $\mathbf{v}_\perp = (v^1, v^2)$. The scalar product of any two four-vectors then is $u \cdot v = \frac{1}{2} u^+ v^- + \frac{1}{2} u^- v^+ - \mathbf{u}_\perp \cdot \mathbf{v}_\perp$. Moreover, the light-cone vector $n = (0, 2, \mathbf{0}_\perp)$ is needed and $n^2 = 0$. In addition, we use the same kinematical variables with our previous studies [47]:

$$P = \frac{p' + p}{2}, \quad \Delta = p' - p, \quad t = \Delta^2, \\ \xi = -\frac{\Delta^+}{2P^+} (|\xi| \leq 1), \quad x = \frac{k^+}{P^+} (-1 \leq x \leq 1), \quad (2)$$

where $k - \Delta/2$ ($k + \Delta/2$) is the initial (final) parton momentum as displayed in Fig. 1. Here, we use the same symbol Δ to represent the momentum transfer and the hadron name. The following conventions: $a^{[\mu} b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$, $a^{\{\mu} b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$, $\sigma^{ni} = \sigma^{\rho i} n_\rho$, $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$, $\epsilon^{n\rho\delta} = \epsilon^{\mu\rho\delta} n_\mu$, and $\epsilon_{0123} = 1$ are used. Moreover, i in the definition (1) is the transverse index, $i = 1, 2$.

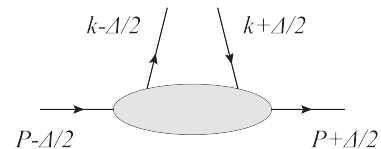


FIG. 1. Diagram describing the quark GPDs.

The quark transversity GPDs are defined by the matrix elements of the transverse nonlocal quark-quark correlator in Eq. (1) as

$$T_{\lambda'\lambda}^{qi} = -\bar{u}_\alpha(p', \lambda') \mathcal{H}^{qT, i, \alpha' \alpha}(x, \xi, t) u_\alpha(p, \lambda), \quad (3)$$

where $u_\alpha(p, \lambda)$ is the spin-3/2 field Rarita-Schwinger spinor, shown in Appendix A in Ref. [47], normalized to $\bar{u}_\alpha(p, \lambda') u^\alpha(p, \lambda) = -2M\delta_{\lambda'\lambda}$. Hermiticity, parity invariance, and time-reversal invariance imply the 16 independent transversity GPDs decomposed from the tensor function $\mathcal{H}^{qT, i, \alpha' \alpha}(x, \xi, t)$ as

$$\begin{aligned} \mathcal{H}^{qT, i, \alpha' \alpha} = & H_1^{qT} \frac{i\sigma^{ni}}{(P \cdot n)} g^{\alpha' \alpha} + H_2^{qT} \frac{n^{[\alpha'} g^{\alpha]i}}{(P \cdot n)} + H_3^{qT} \frac{(\not{n}P^i - P \cdot n\gamma^i)}{M(P \cdot n)} g^{\alpha' \alpha} + H_4^{qT} \frac{(\not{n}P^i - P \cdot n\gamma^i)}{M^3(P \cdot n)} P^{\alpha'} P^\alpha \\ & + H_5^{qT} \frac{(\not{n}\Delta^i - \Delta \cdot n\gamma^i)}{M(P \cdot n)} g^{\alpha' \alpha} + H_6^{qT} \frac{(\not{n}\Delta^i - \Delta \cdot n\gamma^i)}{M^3(P \cdot n)} P^{\alpha'} P^\alpha + H_7^{qT} \frac{(\Delta^i + 2\xi P^i)}{M^2} g^{\alpha' \alpha} + H_8^{qT} \frac{(\Delta^i + 2\xi P^i)}{M^4} P^{\alpha'} P^\alpha \\ & + H_9^{qT} \frac{(\Delta \cdot n n^{\{\alpha'} g^{\alpha\}i} - 2n^{\alpha'} n^\alpha \Delta^i)}{(P \cdot n)^2} + H_{10}^{qT} \frac{(P \cdot n n^{\{\alpha'} g^{\alpha\}i} - 2n^{\alpha'} n^\alpha P^i)}{(P \cdot n)^2} \\ & + H_{11}^{qT} \frac{(\Delta \cdot n P^{[\alpha'} g^{\alpha]i} - P^{[\alpha'} n^{\alpha]} \Delta^i)}{M^2(P \cdot n)} + H_{12}^{qT} \frac{(P \cdot n P^{[\alpha'} g^{\alpha]i} - P^{[\alpha'} n^{\alpha]} P^i)}{M^2(P \cdot n)} \\ & + H_{13}^{qT} \frac{M \not{n} (\Delta \cdot n n^{\{\alpha'} g^{\alpha\}i} - 2n^{\alpha'} n^\alpha \Delta^i)}{(P \cdot n)^3} + H_{14}^{qT} \frac{M \not{n} (P \cdot n n^{\{\alpha'} g^{\alpha\}i} - 2n^{\alpha'} n^\alpha P^i)}{(P \cdot n)^3} \\ & + H_{15}^{qT} \frac{\not{n} (\Delta \cdot n P^{[\alpha'} g^{\alpha]i} - P^{[\alpha'} n^{\alpha]} \Delta^i)}{M(P \cdot n)^2} + H_{16}^{qT} \frac{\not{n} (P \cdot n P^{[\alpha'} g^{\alpha]i} - P^{[\alpha'} n^{\alpha]} P^i)}{M(P \cdot n)^2}, \end{aligned} \quad (4)$$

where the variables x , ξ , and t in the quark transversity GPDs H_i^{qT} are omitted and M is the mass of the spin-3/2 hadron. Here, $H_{3,4,10,11,14,15}^{qT}$ are ξ odd and others are ξ even with respect to the skewness ξ as follows:

$$\begin{aligned} H_i^{qT}(x, \xi, t) &= H_i^{qT}(x, -\xi, t) \quad \text{with } i = 1, 2, 5-9, 12, 13, 16, \\ H_j^{qT}(x, \xi, t) &= -H_j^{qT}(x, -\xi, t) \quad \text{with } j = 3, 4, 10, 11, 14, 15. \end{aligned} \quad (5)$$

Then, all the ξ -odd transversity GPDs vanish in the limit $\xi \rightarrow 0$. In our calculation with the diquark spectator model,

only the negative ξ will be considered. Then, the positive part can be obtained by the parity of GPDs about ξ . In the forward limit, the transversity GPDs change to transversity distribution functions $h_1(x)$:

$$2[H_1^{qT}(x, 0, 0) - H_2^{qT}(x, 0, 0)] = h_1(x), \quad (6)$$

where the number density is from two components, spin-1/2 and spin-1, due to the fact that the Rarita-Schwinger spinor is composed by the fields of spin 1/2 and spin 1.

The nonlocal tensor quark-quark operator gives one local tensor current using the sum rule according to the Mellin moment:

$$\begin{aligned} (P \cdot n)^{a+1} \int dx x^a \int \frac{dz^-}{2\pi} e^{ixP^+ z^-} [\bar{\psi}(-z/2) i\sigma^{\mu\nu} \psi(z/2)] \Big|_{z^+=0, \mathbf{z}=0} &= \left(i \frac{d}{dz^-} \right)^a [\bar{\psi}(-z/2) i\sigma^{\mu\nu} \psi(z/2)] \Big|_{z=0} \\ &= \bar{\psi}(0) i\sigma^{\mu\nu} (i \overleftrightarrow{\partial}^+)^a \psi(0). \end{aligned} \quad (7)$$

Then, the corresponding tensor form factors can be defined using the local tensor current, which is connected to the first Mellin moment ($a = 0$) in x , as

$$\begin{aligned} T^{\mu\nu} &= \langle p', \lambda' | \bar{\psi}(0) i\sigma^{\mu\nu} \psi(0) | p, \lambda \rangle \\ &= -2\bar{u}_\alpha(p', \lambda') \mathcal{F}_q^{\mu\nu, \alpha' \alpha} u_\alpha(p, \lambda), \end{aligned} \quad (8)$$

where the tensor form factors are ξ independent. In the previous work [47], the explicit decomposition of the

matrix element of the tensor current (8) is given as

$$\begin{aligned} \mathcal{F}_q^{\mu\nu, \alpha' \alpha} = & g^{\alpha' \alpha} \left(G_1^{qT}(t) i\sigma^{\mu\nu} + G_5^{qT}(t) \frac{\gamma^{[\mu} \Delta^{\nu]}}{M} + G_7^{qT}(t) \frac{P^{[\mu} \Delta^{\nu]}}{M^2} \right) \\ & + \frac{P^{\alpha'} P^\alpha}{M^2} \left(G_6^{qT}(t) \frac{\gamma^{[\mu} \Delta^{\nu]}}{M} + G_8^{qT}(t) \frac{P^{[\mu} \Delta^{\nu]}}{M^2} \right) \\ & + G_2^{qT}(t) g^{\mu[\alpha'} g^{\alpha]\nu} + G_{12}^{qT}(t) \frac{P^{[\alpha'} g^{\alpha]\nu} P^{\mu]}}{M^2}. \end{aligned} \quad (9)$$

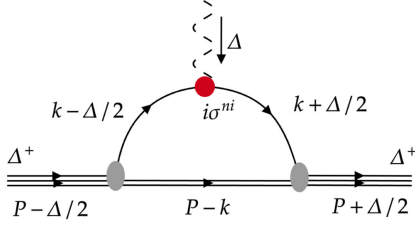


FIG. 2. Feynman diagram for the Δ^+ GPDs using the diquark spectator approach, and the single (double) line stands for the quark (diquark).

One can then obtain the sum rules connected the tensor form factors $G_i^{qT}(t)$ with the transversity GPDs as

$$\begin{aligned} \int_{-1}^1 dx H_i^{qT}(x, \xi, t) &= G_i^{qT}(t) \quad \text{with } i = 1, 2, 5-8, 12, \\ \int_{-1}^1 dx H_j^{qT}(x, \xi, t) &= 0 \quad \text{with } j = 3, 4, 9, 10, 11, 13-16. \end{aligned} \quad (10)$$

Moreover, the combination $G_1^{qT}(0) - G_2^{qT}(0)$ describes the quark tensor charge carried by the corresponding quark.

B. Diquark spectator model

In the present work, we consider the Δ^+ as an attempt to characterize the multidimensional structure of the spin-3/2 particles. In the picture of the quark model, the Δ^+ isobar is composed of three light quarks, two u quarks and one d quark. The quantum numbers of Δ^+ are $I(J^P) = 3/2(3/2^+)$, and it requires that both the isospin and spin of each pair of quarks be 1. Therefore, it is convenient to regard two quarks in Δ^+ as a whole, i.e., diquark. We treat both quark and diquark as elementary particles, respectively. However, the calculations of the diquark GPDs are difficult because of more complicated integrals, so we calculate the GPDs for each flavor quark by using the diquark spectator model instead of the quark-diquark approach.

The Breit frame $\Delta^+ = -\Delta^-$ is employed for convenience in this work, where the initial and final momenta are

$$\begin{aligned} p &= \left(P^0 - \frac{\Delta_z}{2}, P^0 + \frac{\Delta_z}{2}, -\frac{\Delta_\perp}{2} \right), \\ p' &= \left(P^0 + \frac{\Delta_z}{2}, P^0 - \frac{\Delta_z}{2}, \frac{\Delta_\perp}{2} \right). \end{aligned} \quad (11)$$

Figure 2 displays the Feynman diagram of the diquark spectator approach. The quark transversity GPDs defined in Eq. (3) are calculated according to the Feynman diagram as

$$\begin{aligned} \mathcal{H}^{\alpha\alpha} &= -\frac{i}{2} c_1^2 \int \frac{d^2 k_\perp dk^+ dk^-}{(2\pi)^4} \frac{\delta(k \cdot n - x P \cdot n)}{\mathfrak{D}} \\ &\times \Gamma^{\alpha\beta'} \left(k + \frac{\Delta}{2} + m_q \right) g_{\beta'\beta} \not{n} \left(k - \frac{\Delta}{2} + m_q \right) \Gamma^{\beta\alpha}, \end{aligned} \quad (12)$$

where

$$\begin{aligned} \mathfrak{D} &= \left[\left(k + \frac{\Delta}{2} \right)^2 - m_q^2 + i\epsilon \right] \left[\left(k - \frac{\Delta}{2} \right)^2 - m_q^2 + i\epsilon \right] \\ &\times [(k-P)^2 - m_D^2 + i\epsilon] [(k-P)^2 - m_R^2 + i\epsilon]^2 \\ &\times \left[\left(k + \frac{\Delta}{2} \right)^2 - m_R^2 + i\epsilon \right] \left[\left(k - \frac{\Delta}{2} \right)^2 - m_R^2 + i\epsilon \right], \end{aligned} \quad (13)$$

and the effective form of the vertex function is [59]

$$\Gamma^{\alpha\beta} = g^{\alpha\beta} + c_2 \gamma^\beta \Lambda^\alpha + c_3 \Lambda^\beta \Lambda^\alpha, \quad (14)$$

where Λ is the relative momentum between the spin-1/2 and spin-1 partons. In Eq. (12), we also employ the same scalar function of the loop momentum, $\Xi(p_1, p_2, m_R) = c_1/(p_1^2 - m_R^2 + i\epsilon)(p_2^2 - m_R^2 + i\epsilon)$, where p_1 and p_2 , respectively, represent the momentum of the spin-1/2 and spin-1 partons, with Ref. [48] to perform the regularization. Moreover, we adopt the same simplified diquark propagator $g_{\beta'\beta}$ as used in Ref. [48]. The calculation details are shown in the previous work [48,60].

In Eqs. (13) and (14), there are model parameters m_R and $c_{1,2,3}$. In this work, the same parameter m_R in Ref. [48] is employed. The parameter of c_1 is determined by the electric charge. The parameters $c_{2,3}$ could affect the higher multipole terms, and they are taken zero in this work. Under this approximation, the vertex becomes $g^{\alpha\alpha}$, which implies that the calculation of the spin-3/2 hadron will regress to the spin-1/2 condition, and only four transversity GPDs corresponding to the spin-1/2 will be obtained. However, the spin-1/2 part is the leading term, and the following numerical results verify that the spin-1/2 part can give the reliable leading results. Therefore, we think that the transversity PDF and the tensor form factors from this simplification are also reliable. To extract GPDs from the $\mathcal{H}^{qT,i,\alpha\alpha}$, one needs some identities and on-shell identities like Schouten and Gordon identities which are listed in the Appendixes in Refs. [46–48]. One can find the calculation details in our previous works [48].

III. NUMERICAL RESULTS

In this section, we present the numerical results about the transversity GPDs for d quark of Δ^+ as an example. We use the same masses, Δ resonance mass M , quark mass m_q , diquark mass m_D , and cutoff mass m_R as in our previous papers [48,60], which are listed in Table I. In Ref. [60], we indicated that $c_{2,3}$ in Eq. (14) mainly contribute to the high-order multipole terms, like electric quadrupole and

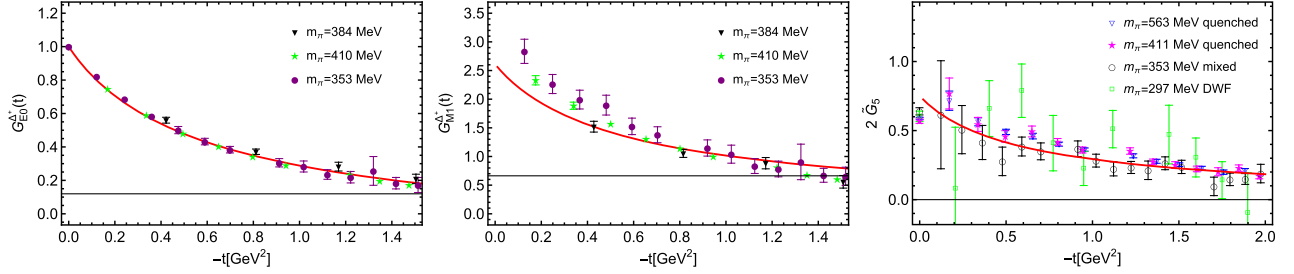
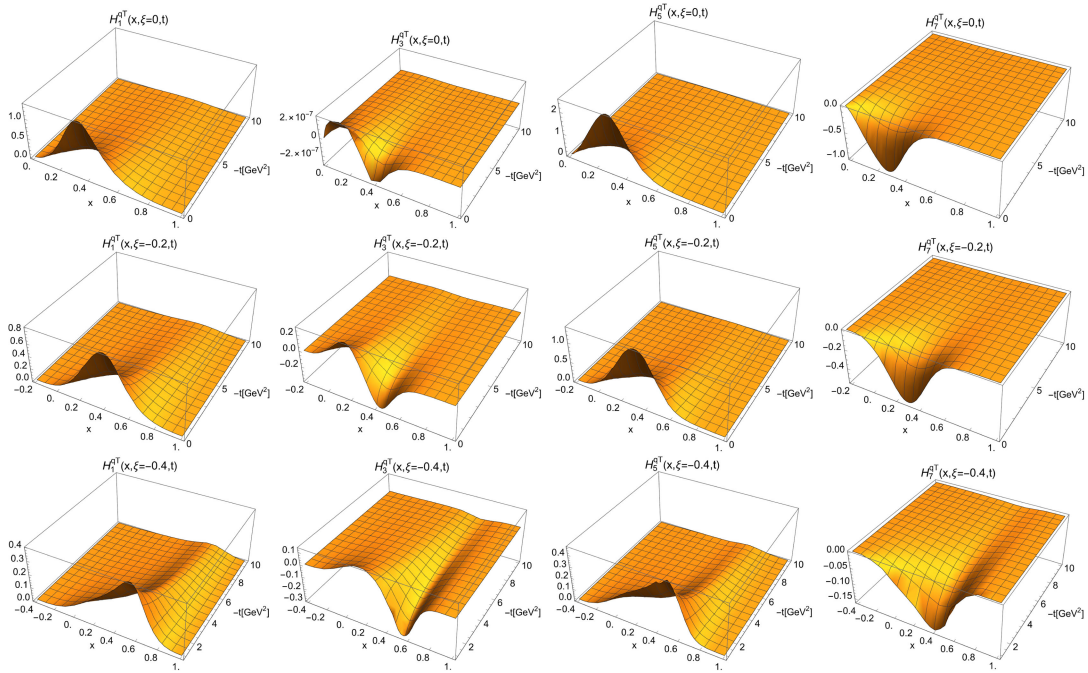
TABLE I. Mass parameters used in this work.

M/GeV	m_q/GeV	m_D/GeV	m_R/GeV	c_2/GeV^{-1}	c_3/GeV^{-2}
1.085	0.4	0.76	1.6	0	0

magnetic octupole form factors, and they are chosen as $c_2 = c_3 = 0$ in this work. To verify that this simplification is reasonable, unpolarized and longitudinal polarized form factors are shown. The electric charge, magnetic moment, and axial charge form factors of Δ^+ are shown in Fig. 3. From Fig. 3, we see that the electric charge and axial charge form factors are consistent with lattice QCD calculations, and the magnetic moment form factor is acceptable. These results illustrate that the spin-1/2 part $g^{\alpha\beta}$ of the vertex $\Gamma^{\alpha\beta}$ in Eq. (14) can provide reliable results in the leading terms. These results indicate that transversity GPDs and tensor form factors corresponding to h_1 are reliable in the following subsections.

A. Numerical transversity GPDs and helicity amplitudes

As an example, but without loss of generality, the d quark transversity GPDs of Δ^+ are given in this section, and the different quark GPDs in all the Δ isobars can be obtained just by counting the corresponding quark number. With the approximation $c_2 = c_3 = 0$, only $H_{1,3,5,7}^{qT}$, all of which respectively corresponds to the Lorentz structure of spin-1/2, survive and other transversity GPDs vanish as explained in Sec. II B. The nonzero transversity GPDs are shown in Fig. 4 as the functions of variables x and t with different skewness ξ , in which $\xi = (0, -0.2, -0.4)$, respectively. The constraint $|\xi| \leq 1/\sqrt{1 - 4M^2/t}$ leads to a bound on the squared momentum transfer $-t \geq \frac{4M^2\xi^2}{1-\xi^2}$, from which one can get the corresponding $-t|_{\min} \sim (0, -0.2, -0.9)$. Because of time reversal constraints in Eq. (5), only the negative ξ region is shown in the results. We have verified the sum rules of the GPDs in Eq. (10), $\int_{-1}^1 dx H_3^{qT}(x, \xi, t) = 0$. Similar with

FIG. 3. The electric charge, magnetic moment, and axial charge form factors of Δ^+ with $c_2 = c_3 = 0$, compared with Refs. [61,62].FIG. 4. The 3D d quark transversity GPDs of Δ^+ $H_{1,3,5,7}^{dT}$ at $\xi = 0, -0.2, -0.4$.

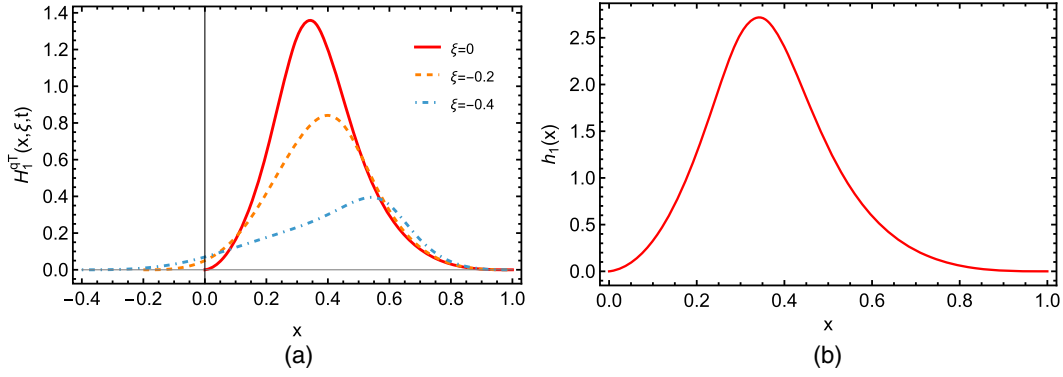


FIG. 5. (a) The GPDs $H_1^{qT}(x, \xi, -|t|_{\min})$ with different skewness ξ and the corresponding minimal $|t|$. The red solid line, orange dashed line, and blue dot-dashed line, respectively, represent the GPDs with $(\xi, -t) = (0, 0 \text{ GeV}^2)$, $(-0.2, 0.2 \text{ GeV}^2)$, and $(-0.4, 0.9 \text{ GeV}^2)$. (b) The transversity PDF $h_1(x)$, where $h_1(x) = 2H_1^{qT}(x, 0, 0)$.

unpolarized and longitudinal polarized GPDs [48], there is also a tendency that the maximums or minimums of ξ -even transversity GPDs $H_{1,5,7}^{qT}$ shift to large x as $|\xi|$ increases. In order to illustrate this character more intuitively and to observe the change with different ξ , the 2D cutting planes of H_1^{qT} with different ξ and the corresponding minimal $-t$ are listed in Fig. 5(a). One can assume that the ratio of the parton momentum fraction equals the mass ratio, i.e., $\frac{x+|\xi|}{1-x} = \frac{m_q}{m_D}$, and this assumption determines a position $x_{\max} = \frac{m_q + m_D |\xi|}{m_q + m_D}$. The positions $x_{\max} = 0.345, 0.476, 0.607$, respectively, correspond to $\xi = 0, -0.2, -0.4$. Furthermore, one can find that the ξ -even GPDs are concentrated near $x_{\max}$. This character implies that the parton momenta are distributed by their

masses and it agrees with our intuition. Moreover, the GPDs corresponding to the unpolarized and longitudinally polarized PDFs also have this feature [48].

In the forward limit, the transversity GPDs degenerate to transversity PDFs according to Eq. (6). Because of our choice $c_2 = c_3 = 0$, only the h_1 exists. Moreover, the choice makes that only the spin-1/2 structure $i\sigma^{ni}g^{\alpha\alpha}$ can be embodied in the transversity PDF and there is no contribution from H_2^{qT} corresponding to the spin-1 structure $n^{[\alpha}g^{\alpha]i}$. Figure 5(b) shows the transversity PDF of the d quark in Δ^+ .

As the combination of the GPDs, the quark helicity amplitude is another important quantity in the particle inner structure description. The quark amplitudes with helicity flip can be defined as

$$\mathcal{A}_{\lambda'-\lambda+}^q = \int \frac{dz^-}{2\pi} e^{ix(P \cdot z)} \left\langle p', \lambda' \left| \bar{\psi} \left(-\frac{1}{2} z \right) \frac{1}{4} (-i\sigma^{+1} + ii\sigma^{+2}) \psi \left(\frac{1}{2} z \right) \right| p, \lambda \right\rangle_{z^+=0, z_{\perp}=0}, \quad (15)$$

where the labels $-$ and $+$ are the emitted and reabsorbed quark helicities, respectively, and the explicit forms of the helicity-flip amplitudes are given in Ref. [47]. According to the explicit forms in Ref. [47], the helicity amplitudes can be expressed as $\mathcal{A}_{\lambda'-\lambda+}^q = F(\zeta) \mathcal{A}_{\lambda'-\lambda+}^{iq}$, where $F(\zeta)$ is the complex part and $\mathcal{A}_{\lambda'-\lambda+}^{iq}$ is the real part. The specific form of the complex part is $F(\zeta) = C^\zeta \theta(\zeta) + C^{*- \zeta} \theta(-\zeta)$ with $\zeta = \lambda' - \lambda + 1$, where C^ζ represents the ζ powers of C and $\theta(\zeta)$ is the Heaviside step function with $\theta(0) = \frac{1}{2}$ and $F(0) = 1$. The factor $F(\zeta)$ will be real when the helicity conserves, $\lambda' - \lambda = -1$ (i.e., helicity conservation implies that the helicity transfer of the hadron corresponds to that of the parton). Moreover, the factor C carries all the complex phase information:

$$C = \sqrt{\frac{1-\xi}{1+\xi}} \frac{|\mathbf{p}_{\perp}|}{M} e^{-i\phi} - \sqrt{\frac{1+\xi}{1-\xi}} \frac{|\mathbf{p}'_{\perp}|}{M} e^{-i\phi'} = -\frac{(\Delta + 2\xi P)^x - i(\Delta + 2\xi P)^y}{M\sqrt{1-\xi^2}}, \quad (16)$$

where $|\mathbf{p}_{\perp}|e^{\pm i\phi} \equiv p^x \pm ip^y$ and $|\mathbf{p}'_{\perp}|e^{\pm i\phi'} \equiv p'^x \pm ip'^y$. In the above equation, C represents the transverse momentum transfer and also orbital angular momentum from the initial to the final hadronic states. One can obtain $|C| = \frac{|\Delta_{\perp}|}{M\sqrt{1-\xi^2}}$ after removing the complex phase in the Breit frame. Furthermore, the numerical results of the helicity-flip amplitudes without the complex phase are shown in Fig. 6.

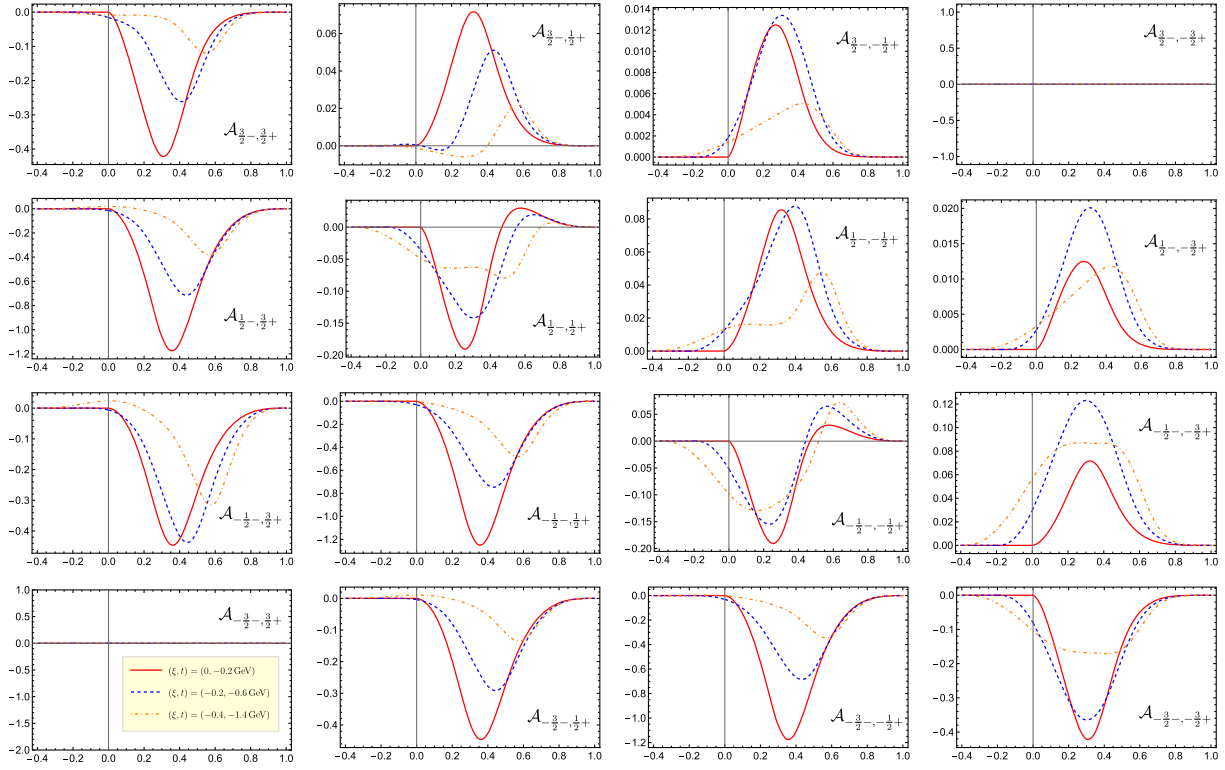


FIG. 6. The helicity-flip amplitudes of the d quark in Δ^+ , where the abscissa represents momentum fraction x .

The bound $-t \geq \frac{4M^2\xi^2}{1-\xi^2}$ implies that $-t$ reaches its minimum when $\Delta_\perp = \mathbf{0}$. Consequently, under the condition that $-t = |t|_{\min}$, the factor C vanishes and the helicity amplitudes violating the helicity conservation will be zero. Therefore, only the helicity amplitudes with helicity conservation exist. Moreover, to compare the helicity-flip amplitudes with different ξ , the specific variable values $(\xi, t) = (0, -0.2 \text{ GeV}), (-0.2, -0.6 \text{ GeV}), (-0.4, -1.4 \text{ GeV})$ are selected in Fig. 6. Moreover, other helicity-flip amplitudes can be obtained by the constraints [47]:

$$\begin{aligned} \mathcal{A}_{-\lambda' - \mu', -\lambda - \mu}^q(P, \Delta, n) &= (-1)^{(\lambda' - \lambda) - (\mu' - \mu)} \mathcal{A}_{\lambda' \mu', \lambda \mu}^{q*}(P, \Delta, n), \\ \mathcal{A}_{-\lambda' - \mu', -\lambda - \mu}^q(P, \Delta, n) &= (-1)^{(\lambda' - \lambda) - (\mu' - \mu)} \mathcal{A}_{\lambda \mu, \lambda' \mu'}^q(P, -\Delta, n), \end{aligned} \quad (17)$$

from Hermiticity, parity, and time reversal transformations.

B. Tensor form factors

The sum rules in Eq. (10) derived from the Mellin moments of GPDs give the connections between the GPDs and the form factors. Here, the matrix element of the local tensor current $\bar{\psi}(0) i\sigma^{\mu\nu} \psi(0)$ corresponding to the transversity GPDs can be decomposed in terms of the tensor form factors in Eq. (9). According to the sum rules and the numerical GPDs, three nonzero tensor form factors are obtained and shown in Fig. 7. Note that we just give the single-quark contribution because

of the unknown physical meaning of the local tensor current. In particular, one can calculate the tensor charge g_T from the integral of the transversity PDF $h_1(x)$ [63] over the parton momentum fraction x :

$$g_T = \delta u - \delta d, \quad (18)$$

with

$$\begin{aligned} \delta u &= \int_0^1 dx (h_1^u(x) - h_1^{\bar{u}}(x)), \\ \delta d &= \int_0^1 dx (h_1^d(x) - h_1^{\bar{d}}(x)), \end{aligned} \quad (19)$$

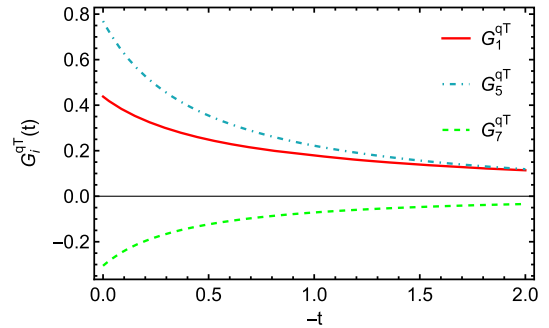


FIG. 7. The tensor form factors of Δ^+ contributed by the d quark.

where u and d , respectively, represent up and down quarks. There is no antiquark in our model $h_1^{\bar{u}}(x) = h_1^{\bar{d}}(x) = 0$, and the spin and flavor wave functions are symmetric for the Δ resonance. According to Eqs. (6) and (10), one therefore can obtain $\delta d = 2[G_1^{qT}(0) - G_2^{qT}(0)] = 0.876$ and $\delta u = 2\delta d$ due to the number ratio between u and d quarks in hadron Δ^+ . Therefore, we can obtain the tensor charge of the Δ resonance:

$$\begin{aligned} g_T^{++} &= 2.628, & g_T^{+-} &= 0.876, \\ g_T^{0+} &= -0.876, & g_T^{--} &= -2.628, \end{aligned} \quad (20)$$

from the quark number, and the results can be predicted by other models and the lattice QCD. Experimentally, one can extract the tensor charge using the transverse-momentum-dependent observables, like the Collins effect in semi-inclusive deep-inelastic scattering (SIDIS) [64,65] and semi-inclusive $e^+e^- \rightarrow h\bar{h}$ (SIA, h represents hadron here) [66,67], GPDs [68], and the hadron observables in SIDIS [69–71] and SIA [72,73].

IV. SUMMARY AND CONCLUSION

Based on the diquark spectator model, we calculated the transversity GPDs, the transversity PDF, the helicity-flip amplitudes, and the tensor form factors of Δ^+ for the first time. According to our previous studies and our analyses here, the $c_{2,3}$ terms of the hadron-quark-diquark vertex mainly contribute to the high-order physical quantities, like the magnetic-dipole and electric-quadrupole form factors. In this work, the hadron-quark-diquark vertex is simplified to be the $g^{\alpha\beta}$ form by neglecting the $c_{2,3}$ terms. Then, only the GPDs corresponding to the Lorentz structures that occurred in spin-1/2 transversity GPD definition are nonzero. Meanwhile, our numerical results verify that this simplification is sufficient to describe the electric charge and axial charge form factors, while the magnetic-dipole form factor is acceptable. Therefore, we believe that one can obtain the reasonable tensor charge form factor and the transversity PDF h_1 .

The nonzero transversity GPDs of the d quark in Δ^+ are calculated and satisfy the corresponding sum rules. The even terms $H_{1,5,7}^{qT}$ with respect to the skewness ξ have the same feature; they are mainly distributed around $x_{\max} = \frac{m_q + m_D |\xi|}{m_q + m_D}$, with the unpolarized and longitudinal

polarized ones. In the forward limit, $\xi = 0$ and $t = 0$, the transversity PDF can be obtained from the transversity GPDs, and only the leading order h_1 exists. Moreover, the helicity-flip amplitude as the combination of the GPDs is another significant physical quantity. In the helicity amplitudes, the imaginary part is from the transverse momentum transfer, i.e., the orbital angular momentum from the initial to the final hadronic states. The numerical results are calculated taking the absolute value of the transverse momentum transfer in the Breit frame.

Moreover, the Mellin moment connects the nonlocal transversity quark-quark operator $\bar{\psi}(-z/2)i\sigma^{ni}\psi(z/2)$ with the local tensor current $\bar{\psi}(0)i\sigma^{\mu\nu}\psi(0)$. Meanwhile, the tensor form factors and the sum rules connecting the GPDs and the form factors have been derived in our previous work. The numerical tensor form factors of Δ^+ contributed by the d quark are immediately obtained after integrating the parton momentum fraction x and only $G_{1,5,7}^{qT}$ are nonzero. Integral of x in the transversity PDF h_1 and the tensor form factor in the forward limit give the tensor charge of the d quark, $\delta_d = \int_0^1 dx (h_1^d(x) - h_1^{\bar{d}}(x)) = 2[G_1^{qT}(0) - G_2^{qT}(0)] = 0.876$ because there is no antiquark in our model. Because of the symmetry of the spin and flavor wave functions for the Δ resonance, one can get the relation $\delta_u = 2\delta_d$ in Δ^+ , and we can give the prediction of the tensor charge of Δ^+ , $g_T = 0.876$. Moreover, the tensor charge of other Δ resonances can be derived by the quark number. And the tensor charge can be detected by various processes, such as SIDIS, SIA, and the GPD processes.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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