

125 GeV Higgs boson rare decays in a flavor-dependent $U(1)_F$ model

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In this work, we analyze the Higgs boson decay channels, specifically, $h \rightarrow \gamma\gamma$, $h \rightarrow VV^*$ (with $V = Z, W$), and $h \rightarrow f\bar{f}$ (for $f = b, c, \tau$) within the flavor-dependent $U(1)_F$ model (FDM). We also investigate processes induced by flavor-changing neutral currents, including the decays $\bar{B} \rightarrow X_s \gamma$ and $B_s^0 \rightarrow \mu^+ \mu^-$, the top quark decays $t \rightarrow ch$ and $t \rightarrow uh$, and the lepton flavor-violating decays $\tau \rightarrow 3e$, $\tau \rightarrow 3\mu$, and $\mu \rightarrow 3e$. Furthermore, we incorporate the electroweak precision observables constraints via the S, T, and U parameters. Compared to the Standard Model, the scalar sector of the FDM is extended by two Higgs doublets and one Higgs singlet, which affects the 125 GeV Higgs properties significantly. Meanwhile, the decays $h \rightarrow Z\gamma$, $h \rightarrow MZ$, and $h \rightarrow M\gamma$ [where M is a vector meson ($\rho, \omega, \phi, J/\Psi, \Upsilon$)] of the Standard-Model-like Higgs are studied, and we illustrate how changes in the scalar sector and the Yukawa coupling influence the signal strengths for the 125 GeV Higgs decay channels and the Higgs mass in the FDM.

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I. INTRODUCTION

The ATLAS and CMS Collaborations have reported significant excess events for a new boson, which is interpreted as the neutral Higgs with mass around 125.20 ± 0.11 GeV [1,2]. The CP properties and couplings of the particle are also being established [3,4]. This implies that the Higgs mechanism breaking electroweak (EW) symmetry has a solid experimental cornerstone [5,6], although there is currently no solid evidence to suggest that experimental results have deviated from the predictions of the Standard Model (SM). The mass of the Higgs boson in the SM is a free parameter; its self-energy correction in the SM is not protected by any symmetry, leading to quadratic divergences, thus raising the issue of naturalness [7]. Although the CP violation of SM explains some experimental results, it fails to account for the Universe's matter-antimatter asymmetry, necessitating new sources of CP violation. The SM also lacks explanations for dark matter and neutrino oscillations [8,9]. These

shortcomings strongly suggest new physics (NP) beyond the SM.

An extension of the SM, known as the flavor-dependent model, introduces an extra local gauge group $U(1)_F$, along with two scalar doublets, one scalar singlet, and two right-handed neutrinos [10,11]. In this model, the additional $U(1)_F$ charges are related to the flavors of fermions. The enhanced fermion sector present in the FDM offers a simultaneous resolution to the puzzles of flavor mixings and fermions mass hierarchy. Moreover, utilizing the type-I seesaw mechanism, the model naturally accounts for the generation of nonzero Majorana neutrino masses [11–14]. In FDM, significant changes have occurred in the extended scalar sector and the Yukawa coupling part, which will directly impact the signal strength [15–17]. Therefore, studying the signal strength of the Higgs is of great significance for testing the coupling strength of the Higgs particle with various fundamental particles and determining the properties of the Higgs particle itself and can be used to verify whether models with different Higgs sectors satisfy experimental data [18–20].

Flavor-changing neutral currents (FCNCs) refer to processes in which particles exchange flavor (quark or lepton) during strong or weak interactions. In recent years, FCNCs have been extensively studied through neutrino scattering experiments and high-energy physics experiments such as those conducted at the Large Hadron Collider (LHC) [21], as exemplified by $\bar{B} \rightarrow X_s \gamma$, $B_s^0 \rightarrow \mu^+ \mu^-$ [22]; $t \rightarrow ch$, $t \rightarrow uh$ [23]; and $\tau \rightarrow 3e$, $\tau \rightarrow 3\mu$, $\mu \rightarrow 3e$. These results may indicate that the SM is incomplete and requires

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additional physics theories to explain such phenomena. New particles or interactions introduced in many extensions of the SM (e.g., supersymmetry, grand unified theories, extra dimensions, etc.) could enhance FCNC processes [24]. Therefore, the study of FCNCs not only deepens our understanding of weak interactions within the SM, but also serves as a powerful tool for validating theories beyond the SM [25].

In this paper, we investigate the 125 GeV Higgs decay channels $h \rightarrow \gamma\gamma$, $h \rightarrow Z\gamma$, $h \rightarrow VV^*$ ($V = Z, W$), and $h \rightarrow f\bar{f}$ ($f = b, c, \tau$), and we study the 125 GeV SM-like Higgs boson rare decays $h \rightarrow MZ$ and $h \rightarrow M\gamma$ in the framework of the FDM, where M denotes vector meson [$\rho, \omega, \phi, J/\psi, \Upsilon(nS)$]. For the processes $h \rightarrow MZ$ [26–30], there are two types of decay topologies: One is the direct contributions induced by the Yukawa couplings of the Higgs boson to the quarks, and the other is the indirect contributions resulting from a $h \rightarrow \gamma^*Z$ transition followed by the conversion of the off-shell boson into a vector meson [26,30,31]. Compared with the SM-like Higgs boson coupling to the quarks in the direct contributions, the effective $h\gamma Z$ vertex in the indirect contributions will give more NP contributions [32,33]. Hence, these rare decays provide complementary information to the decay $h \rightarrow Z\gamma$, both in and beyond the SM [31,34]. In this work, the quantum chromodynamics (QCD) factorization [35–37] is used for the rare SM-like Higgs boson decays $h \rightarrow MZ$. $h \rightarrow \bar{q}q$ is a method for directly studying the coupling of first- and second-generation quarks, although these channels have relatively large branching fractions, while their experimental sensitivity is substantially obscured by large multijet backgrounds [38,39]. Method $h \rightarrow M\gamma$ is considered superior, because, compared to methods $h \rightarrow \bar{c}c$ and $h \rightarrow \bar{b}b$, the process $h \rightarrow J/\psi + \gamma$ or $h \rightarrow \Upsilon(nS) + \gamma$ offers an advantage: Despite their relatively small branching fractions, these radiative decays exhibit a unique experimental signature, effectively suppressing the substantial multijet backgrounds that typically complicate searches for $h \rightarrow \bar{q}q$ [8,40–42]. Additionally, decays of the Higgs boson into charmonium states, specifically, $h \rightarrow J/\psi + \gamma$, provide a unique probe of the charm-quark Yukawa coupling, including both its magnitude and sign [43,44]. Similarly, Higgs decays into bottomonium states, $h \rightarrow \Upsilon(nS) + \gamma$, offer insights into the real and imaginary components of the bottom quark's coupling to the Higgs [45,46]. These processes have also been used by CMS or ATLAS for probing the Yukawa couplings [38,39,47].

Our presentation is organized as follows. In Sec. II, the structure of the FDM including particle content, scalar sector, fermion masses, and gauge sector are collected. We present the decay widths and signal strengths for $h \rightarrow \gamma\gamma$, $h \rightarrow VV^*$ ($V = Z, W$), and $h \rightarrow f\bar{f}$ ($f = b, c, \tau$). We also discuss processes induced by FCNCs, the constraints from electroweak precision observables (the S, T, and U parameters), and the decays $h \rightarrow MZ$ and $h \rightarrow M\gamma$ in Sec. III. The

numerical analysis are given in Secs. IV and V gives a summary.

II. THE FLAVOR-DEPENDENT MODEL

The gauge group of the FDM is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_F$, where the extra $U(1)_F$ local gauge group is related to the particles' flavor [12]. In the FDM, the third generation of fermions obtain masses through the tree-level Yukawa couplings, and the first two generations of fermions achieve masses through the tree-level mixings with the third generation [10]. Hence, two additional scalar doublets are introduced in the FDM to realize the tree-level mixings of the first two generations and the third generation. In addition, to coincide with the observed neutrino oscillations, two right-handed neutrinos and one scalar singlet are introduced. Then, the right-handed neutrinos obtain large Majorana masses after the scalar singlet achieving large vacuum expectation value, and the tiny neutrino masses can be obtained by the type-I seesaw mechanism [12].

All fields in the FDM and the corresponding gauge symmetry charges are presented in Table I; the nonzero constant z denotes the extra $U(1)_F$ charge. It can be noted in Table I that there are only two generations of right-handed neutrinos in the FDM, because both $U(1)_F$ and $U(1)_Y$ charges of the third generation of right-handed neutrinos ν_{R3} are zero, which is trivial. In addition, it is obvious that the chiral anomaly cancellation can be guaranteed for the fermionic charges presented in Table I.

TABLE I. Matter content in the FDM, where the nonzero constant z denotes the extra $U(1)_F$ charge.

Multiplets	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_F$
$l_1 = (\nu_{1L}, e_{1L})^T$	1	2	$-\frac{1}{2}$	z
$l_2 = (\nu_{2L}, e_{2L})^T$	1	2	$-\frac{1}{2}$	$-z$
$l_3 = (\nu_{3L}, e_{3L})^T$	1	2	$-\frac{1}{2}$	0
ν_{1R}	1	1	0	$-z$
ν_{2R}	1	1	0	z
e_{1R}	1	1	-1	$-z$
e_{2R}	1	1	-1	z
e_{3R}	1	1	-1	0
$q_1 = (u_{1L}, d_{1L})^T$	3	2	$\frac{1}{6}$	z
$q_2 = (u_{2L}, d_{2L})^T$	3	2	$\frac{1}{6}$	$-z$
$q_3 = (u_{3L}, d_{3L})^T$	3	2	$\frac{1}{6}$	0
d_{1R}	3	1	$-\frac{2}{3}$	$-z$
d_{2R}	3	1	$-\frac{2}{3}$	z
d_{3R}	3	1	$-\frac{2}{3}$	0
u_{1R}	3	1	$\frac{2}{3}$	$-z$
u_{2R}	3	1	$\frac{2}{3}$	z
u_{3R}	3	1	$\frac{2}{3}$	0
$\Phi_1 = (\phi_1^+, \phi_1^0)^T$	1	2	$-\frac{1}{2}$	z
$\Phi_2 = (\phi_2^+, \phi_2^0)^T$	1	2	$-\frac{1}{2}$	$-z$
$\Phi_3 = (\phi_3^+, \phi_3^0)^T$	1	2	$-\frac{1}{2}$	0
χ	1	1	0	$2z$

A. The scalar sector of the FDM

The scalar potential in the FDM can be written as [10,12]

$$\begin{aligned}
 V = & -M_{\Phi_1}^2 \Phi_1^\dagger \Phi_1 - M_{\Phi_2}^2 \Phi_2^\dagger \Phi_2 - M_{\Phi_3}^2 \Phi_3^\dagger \Phi_3 - M_\chi^2 \chi^* \chi + \lambda_\chi (\chi^* \chi)^2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 \\
 & + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_3^\dagger \Phi_3)^2 + \lambda'_4 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda''_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
 & + \lambda'_5 (\Phi_1^\dagger \Phi_1) (\Phi_3^\dagger \Phi_3) + \lambda''_5 (\Phi_1^\dagger \Phi_3) (\Phi_3^\dagger \Phi_1) + \lambda'_6 (\Phi_2^\dagger \Phi_2) (\Phi_3^\dagger \Phi_3) + \lambda''_6 (\Phi_2^\dagger \Phi_3) (\Phi_3^\dagger \Phi_2) \\
 & + \lambda_7 (\Phi_1^\dagger \Phi_1) (\chi^* \chi) + \lambda_8 (\Phi_2^\dagger \Phi_2) (\chi^* \chi) + \lambda_9 (\Phi_3^\dagger \Phi_3) (\chi^* \chi) + [\lambda_{10} (\Phi_3^\dagger \Phi_1) (\Phi_3^\dagger \Phi_2) + \kappa (\Phi_1^\dagger \Phi_2) \chi + \text{H.c.}], \quad (1)
 \end{aligned}$$

where

$$\begin{aligned}
 \Phi_1 &= \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(iA_1 + S_1 + v_1) \end{pmatrix}, & \Phi_2 &= \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(iA_2 + S_2 + v_2) \end{pmatrix}, \\
 \Phi_3 &= \begin{pmatrix} \phi_3^+ \\ \frac{1}{\sqrt{2}}(iA_3 + S_3 + v_3) \end{pmatrix}, & \chi &= \frac{1}{\sqrt{2}}(iA_\chi + S_\chi + v_\chi), \quad (2)
 \end{aligned}$$

and v_i ($i = 1, 2, 3$) and v_χ are the vacuum expectation values of Φ_i and χ , respectively.

Based on the scalar potential in Eq. (1), the tadpole equations in the FDM can be expressed as

$$\begin{aligned}
 M_{\Phi_1}^2 &= \lambda_1 v_1^2 + \frac{1}{2} \left[(\lambda'_4 + \lambda''_4) v_2^2 + (\lambda'_5 + \lambda''_5) v_3^2 + \frac{v_2}{v_1} v_3^2 \text{Re}(\lambda_{10}) + \sqrt{2} \frac{v_2}{v_1} v_\chi \text{Re}(\kappa) + \lambda_7 v_\chi^2 \right], \\
 M_{\Phi_2}^2 &= \lambda_2 v_2^2 + \frac{1}{2} \left[(\lambda'_4 + \lambda''_4) v_1^2 + (\lambda'_6 + \lambda''_6) v_3^2 + \frac{v_1}{v_2} v_3^2 \text{Re}(\lambda_{10}) + \sqrt{2} \frac{v_1}{v_2} v_\chi \text{Re}(\kappa) + \lambda_8 v_\chi^2 \right], \\
 M_{\Phi_3}^2 &= \lambda_3 v_3^2 + \text{Re}(\lambda_{10}) v_1 v_2 + \frac{1}{2} [(\lambda'_5 + \lambda''_5) v_1^2 + (\lambda'_6 + \lambda''_6) v_2^2 + \lambda_9 v_c^2], \\
 M_\chi^2 &= \lambda_\chi v_\chi^2 + \frac{1}{2} \left[\lambda_7 v_1^2 + \lambda_8 v_2^2 + \lambda_9 v_3^2 + \sqrt{2} \frac{v_1 v_2}{v_\chi} \text{Re}(\kappa) \right]. \quad (3)
 \end{aligned}$$

On the basis (S_1, S_2, S_3, S_χ) , the CP -even Higgs squared mass matrix in the FDM is

$$M_h^2 = \begin{pmatrix} M_{h,11}^2 & M_{h,12}^2 & M_{h,13}^2 & M_{h,14}^2 \\ M_{h,12}^2 & M_{h,22}^2 & M_{h,23}^2 & M_{h,24}^2 \\ M_{h,13}^2 & M_{h,23}^2 & M_{h,33}^2 & M_{h,34}^2 \\ M_{h,14}^2 & M_{h,24}^2 & M_{h,34}^2 & M_{h,44}^2 \end{pmatrix}, \quad (4)$$

where

$$\begin{aligned}
 M_{h,11}^2 &= 2\lambda_1 v_1^2 - \frac{v_2}{2v_1} [v_3^2 \text{Re}(\lambda_{10}) + \sqrt{2} v_\chi \text{Re}(\kappa)], \\
 M_{h,22}^2 &= 2\lambda_2 v_2^2 - \frac{v_1}{2v_2} [v_3^2 \text{Re}(\lambda_{10}) + \sqrt{2} v_\chi \text{Re}(\kappa)], \\
 M_{h,33}^2 &= 2\lambda_3 v_3^2, & M_{h,44}^2 &= 2\lambda_\chi v_\chi^2 - \frac{\sqrt{2} v_1 v_2}{2v_\chi} \text{Re}(\kappa), \\
 M_{h,12}^2 &= (\lambda'_4 + \lambda''_4) v_1 v_2 + \frac{1}{2} \text{Re}(\lambda_{10}) v_3^2 + \frac{\sqrt{2}}{2} \text{Re}(\kappa) v_\chi, \\
 M_{h,13}^2 &= (\lambda'_5 + \lambda''_5) v_1 v_3 + \text{Re}(\lambda_{10}) v_2 v_3, & M_{h,14}^2 &= \lambda_7 v_1 v_\chi + \frac{\sqrt{2}}{2} \text{Re}(\kappa) v_2, \\
 M_{h,23}^2 &= (\lambda'_6 + \lambda''_6) v_2 v_3 + \text{Re}(\lambda_{10}) v_1 v_3, & M_{h,24}^2 &= \lambda_8 v_2 v_\chi + \frac{\sqrt{2}}{2} \text{Re}(\kappa) v_1, \\
 M_{h,34}^2 &= \lambda_9 v_3 v_\chi. \quad (5)
 \end{aligned}$$

The tadpole equations in Eq. (3) are used to obtain the matrix elements above.

On the basis $(\phi_1^+, \phi_2^+, \phi_3^+)$ and $(\phi_1^-, \phi_2^-, \phi_3^-)^T$, the squared mass matrix of singly charged Higgs in the FDM can be written as

$$M_{H^\pm}^2 = \begin{pmatrix} M_{H^\pm,11}^2 & M_{H^\pm,12}^2 & M_{H^\pm,13}^2 \\ (M_{H^\pm,12}^2)^* & M_{H^\pm,22}^2 & M_{H^\pm,23}^2 \\ (M_{H^\pm,13}^2)^* & (M_{H^\pm,23}^2)^* & M_{H^\pm,33}^2 \end{pmatrix}, \quad (6)$$

where

$$\begin{aligned} M_{H^\pm,11}^2 &= -\frac{v_2}{2v_1} [\text{Re}(\lambda_{10})v_3^2 + \sqrt{2}v_\chi \text{Re}(\kappa)] - \frac{1}{2}(\lambda_4''v_2^2 + \lambda_5''v_3^2), \\ M_{H^\pm,22}^2 &= -\frac{v_1}{2v_2} [\text{Re}(\lambda_{10})v_3^2 + \sqrt{2}v_\chi \text{Re}(\kappa)] - \frac{1}{2}(\lambda_4''v_1^2 + \lambda_6''v_3^2), \\ M_{H^\pm,33}^2 &= -\text{Re}(\lambda_{10})v_1v_2 - \frac{1}{2}(\lambda_5''v_1^2 + \lambda_6''v_2^2), \\ M_{H^\pm,12}^2 &= \frac{\sqrt{2}}{2}v_\chi\kappa + \frac{1}{2}\lambda_4''v_1v_2, \quad M_{H^\pm,13}^2 = \frac{1}{2}v_3(\lambda_5''v_1 + \lambda_{10}^*v_2), \\ M_{H^\pm,23}^2 &= \frac{1}{2}v_3(\lambda_6''v_2 + \lambda_{10}^*v_1). \end{aligned} \quad (7)$$

It is easy to verify that there are two neutral Goldstones and one singly charged Goldstone in the FDM.

B. The fermion masses in the FDM

Based on the matter content listed in Table I, the Yukawa couplings in the FDM can be written as

$$\begin{aligned} \mathcal{L}_Y &= Y_u^{33}\bar{q}_3\tilde{\Phi}_3u_{R_3} + Y_d^{33}\bar{q}_3\Phi_3d_{R_3} + Y_u^{32}\bar{q}_3\tilde{\Phi}_1u_{R_2} + Y_u^{23}\bar{q}_2\tilde{\Phi}_1u_{R_3} + Y_d^{32}\bar{q}_3\Phi_2d_{R_2} \\ &+ Y_d^{23}\bar{q}_2\Phi_2d_{R_3} + Y_u^{21}\bar{q}_2\tilde{\Phi}_3u_{R_1} + Y_u^{12}\bar{q}_1\tilde{\Phi}_3u_{R_2} + Y_d^{21}\bar{q}_2\Phi_3d_{R_1} + Y_d^{12}\bar{q}_1\Phi_3d_{R_2} \\ &+ Y_u^{31}\bar{q}_3\tilde{\Phi}_2u_{R_1} + Y_u^{13}\bar{q}_1\tilde{\Phi}_2u_{R_3} + Y_d^{31}\bar{q}_3\Phi_1d_{R_1} + Y_d^{13}\bar{q}_1\Phi_1d_{R_3} \\ &+ Y_e^{33}\bar{l}_3\Phi_3e_{R_3} + Y_e^{32}\bar{l}_3\Phi_2e_{R_2} + Y_e^{23}\bar{l}_2\Phi_2e_{R_3} + Y_e^{21}\bar{l}_2\Phi_3e_{R_1} + Y_e^{12}\bar{l}_1\Phi_3e_{R_2} \\ &+ Y_e^{31}\bar{l}_3\Phi_1e_{R_1} + Y_e^{13}\bar{l}_1\Phi_1e_{R_3} + Y_R^{11}\bar{\nu}_{R_1}^c\nu_{R_1}\chi + Y_R^{22}\bar{\nu}_{R_2}^c\nu_{R_2}\chi^* + Y_D^{21}\bar{l}_2\tilde{\Phi}_3\nu_{R_1} \\ &+ Y_D^{12}\bar{l}_1\tilde{\Phi}_3\nu_{R_2} + Y_D^{31}\bar{l}_3\tilde{\Phi}_2\nu_{R_1} + Y_D^{32}\bar{l}_3\tilde{\Phi}_1\nu_{R_2} + \text{H.c.} \end{aligned} \quad (8)$$

Then, the mass matrices of quarks and leptons can be written as [10,12]

$$m_q = \begin{pmatrix} 0 & m_{q,12} & m_{q,13} \\ m_{q,12}^* & 0 & m_{q,23} \\ m_{q,13}^* & m_{q,23}^* & m_{q,33} \end{pmatrix}, \quad m_e = \begin{pmatrix} 0 & m_{e,12} & m_{e,13} \\ m_{e,12}^* & 0 & m_{e,23} \\ m_{e,13}^* & m_{e,23}^* & m_{e,33} \end{pmatrix}, \quad m_\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix}, \quad (9)$$

where $q = u, d$, the parameters $m_{q,33}$ and $m_{e,33}$ are real, M_D is 2×3 Dirac mass matrix, and M_R is 2×2 Majorana mass matrix (the nonzero neutrino masses are obtained by the type-I seesaw mechanism). The elements of the matrices in Eq. (9) are

$$\begin{aligned} m_{u,11} &= m_{u,22} = 0, \quad m_{u,33} = \frac{1}{\sqrt{2}}Y_u^{33}v_3, \quad m_{u,12} = \frac{1}{\sqrt{2}}Y_u^{12}v_3, \quad m_{u,13} = \frac{1}{\sqrt{2}}Y_u^{13}v_1, \\ m_{u,23} &= \frac{1}{\sqrt{2}}Y_u^{23}v_2, \end{aligned} \quad (10)$$

$$\begin{aligned}
m_{d,11} = m_{d,22} = 0, \quad m_{d,33} = \frac{1}{\sqrt{2}} Y_d^{33} v_3, \quad m_{d,12} = \frac{1}{\sqrt{2}} Y_d^{12} v_3, \quad m_{d,13} = \frac{1}{\sqrt{2}} Y_d^{13} v_1, \\
m_{d,23} = \frac{1}{\sqrt{2}} Y_d^{23} v_2,
\end{aligned} \tag{11}$$

$$\begin{aligned}
m_{e,11} = m_{e,22} = 0, \quad m_{e,33} = \frac{1}{\sqrt{2}} Y_e^{33} v_3, \quad m_{e,12} = \frac{1}{\sqrt{2}} Y_e^{12} v_3, \quad m_{e,13} = \frac{1}{\sqrt{2}} Y_e^{13} v_1, \\
m_{e,23} = \frac{1}{\sqrt{2}} Y_e^{23} v_2,
\end{aligned} \tag{12}$$

$$\begin{aligned}
M_{D,11} = M_{D,22} = 0, \quad M_{D,12} = \frac{1}{\sqrt{2}} Y_D^{12} v_3, \quad M_{D,31} = \frac{1}{\sqrt{2}} Y_D^{31} v_1, \\
M_{D,32} = \frac{1}{\sqrt{2}} Y_D^{32} v_2, \quad M_{R,12} = M_{R,21} = 0, \quad M_{R,11} = \frac{1}{\sqrt{2}} Y_R^{11} v_\chi, \quad M_{R,22} = \frac{1}{\sqrt{2}} Y_R^{22} v_\chi.
\end{aligned} \tag{13}$$

C. The gauge sector of the FDM

Because of the introduction of an extra $U(1)_F$ local gauge group in the FDM, the covariant derivative corresponding to $SU(2)_L \otimes U(1)_Y \otimes U(1)_F$ is defined as [10,12]

$$D_\mu = \partial_\mu + ig_2 T_j A_{j\mu} + ig_1 Y B_\mu + ig_F F B'_\mu + ig_{YF} Y B'_\mu \quad (j = 1, 2, 3), \tag{14}$$

where (g_2, g_1, g_F) , (T_j, Y, F) , and $(A_{j\mu}, B_\mu, B'_\mu)$ denote the gauge coupling constants, generators, and gauge bosons of groups $(SU(2)_L, U(1)_Y, U(1)_F)$, respectively, and g_{YF} is the gauge coupling constant arises from the gauge kinetic mixing effect which presents in the models with two Abelian groups. Then, the W boson mass can be written as

$$M_W = \frac{1}{2} g_2 (v_1^2 + v_2^2 + v_3^2)^{1/2}, \tag{15}$$

where $(v_1^2 + v_2^2 + v_3^2)^{1/2} = v \approx 246.22$ GeV and we have $v_1 = v_2 < v_3$ in the FDM. The γ , Z , and Z' boson masses in the FDM can be written as

$$M_\gamma = 0, \quad M_Z \approx \frac{1}{2} (g_1^2 + g_2^2)^{1/2} v, \quad M_{Z'} \approx 2|zg_F|v_\chi, \tag{16}$$

and

$$\gamma = c_W B + s_W A_3, \quad Z = -s_W B + c_W A_3 + s'_W B', \\
Z' = s'_W (s_W B - c_W A_3) + c'_W B', \tag{17}$$

respectively, where γ , Z , and Z' are the mass eigenstates, $c_W \equiv \cos \theta_W$, $s_W \equiv \sin \theta_W$ with θ_W denoting the Weinberg angle, and $s'_W \equiv \sin \theta'_W$, $c'_W \equiv \cos \theta'_W$ with θ'_W representing the $Z - Z'$ mixing effect.

III. EXPERIMENTAL CONSTRAINTS AND RARE DECAY OF THE 125 GEV HIGGS BOSON

At the LHC, the gluon fusion process is the main neutral Higgs particle production channel, and the Higgs boson is an unstable particle with a very short lifespan, which quickly decays after being produced. In the FDM, the changes in the scalar sector and the Yukawa coupling have a significant impact on the decays.

A. The decay processes $h \rightarrow gg, \gamma\gamma, h \rightarrow VV^*$ ($V = Z, W$), and $h \rightarrow f\bar{f}$ ($f = c, b, \tau$)

The decay process $h \rightarrow gg$ amplitude can be expressed by incorporating two form factors

$$\begin{aligned}
\mathcal{M}_{ggh}^{ab} = & -\frac{\alpha_s(m_h) m_h^2 \delta^{ab}}{4\pi v} \left\{ S^g(m_h) (\epsilon_{1\perp}^* \cdot \epsilon_{2\perp}^*) \right. \\
& \left. - P^g(m_h) \frac{2}{m_h^2} \langle \epsilon_1^* \epsilon_2^* k_1 k_2 \rangle \right\},
\end{aligned} \tag{18}$$

where a and b (with $a, b = 1-8$) are indices corresponding to the eight generators in the $SU(3)$ adjoint representation, $k_{1,2}$ represent the four-momenta of the two gluons, and $\epsilon_{1,2}$ denote the wave vectors of the respective gluons. m_h is the mass of the lightest CP -even Higgs boson. $\epsilon_{1\perp}^\mu = \epsilon_1^\mu - 2k_1^\mu (k_2 \cdot \epsilon_1) / m_h^2$, $\epsilon_{2\perp}^\mu = \epsilon_2^\mu - 2k_2^\mu (k_1 \cdot \epsilon_2) / m_h^2$, and $\langle \epsilon_1 \epsilon_2 k_1 k_2 \rangle \equiv \epsilon_{\mu\nu\rho\sigma} \epsilon_1^\mu \epsilon_2^\nu k_1^\rho k_2^\sigma$, considering solely the leading contributions originating from third-generation quarks. The scalar and pseudoscalar form factors are then determined based on the methodology outlined in Ref. [48]:

$$\begin{aligned}
S^g(m_h) &= \sum_{f=b,t} \kappa_f F_{sf}(\tau_f) + \Delta S^g; \\
P^g(m_h) &= \sum_{f=b,t} \tilde{\kappa}_f F_{pf}(\tau_f) + \Delta P^g,
\end{aligned} \tag{19}$$

with $\tau_f = m_h^2/(4m_q^2)$ ($q = t, b$) are defined by using the pole masses of the bottom and top quarks. κ_f represent CP -even effective Higgs couplings to the quarks and the leptons, respectively, and $\tilde{\kappa}_f$ represent CP -odd. The loop integral functions $F_{sf}(\tau_f)$ [and $F_{pf}(\tau_f)$, A_1 , and A_0 below] are defined in Appendix A.

In the framework of NP, integration of QCD and EW corrections is performed. These corrections are factorized based on the universal infrared and collinear properties of QCD corrections and the universality of the predominant component of EW corrections. The QCD correction $\delta_{\text{QCD}}^{g:S}$ is computed up to next-to-leading order (NLO), considering the complete quark mass dependence [49], and up to next-to-next-to-next-to-leading order (N³LO) in the heavy top quark limit [48,50–52]. The normalized decay widths of a neutral Higgs boson with mass m_h into gg are calculated by

assuming $\Delta S^g = 0$ and $\tilde{\kappa}_f = \Delta P^g = 0$:

$$\Gamma(h \rightarrow gg) = \frac{m_h^3 \alpha_S^2}{32\pi^3 v^2} [|S^g(m_h)|^2 (1 + \delta_{\text{QCD}}^{g:S}) (1 + \delta_{\text{ew}}^{g:S})]. \quad (20)$$

The concrete expressions of κ_t^S and κ_b^S are formulated, respectively, as

$$\begin{aligned} \kappa_t &= \frac{v_{\text{EW}}}{m_t} C_{hu_3 \bar{u}_3}, \\ \kappa_b &= \frac{v_{\text{EW}}}{m_b} C_{hd_3 \bar{d}_3}. \end{aligned} \quad (21)$$

$\delta_{\text{QCD}}^{g:S}$ and $\delta_{\text{ew}}^{g:S}$ are formulated, respectively, as [49,51,53]

$$\begin{aligned} \delta_{\text{QCD}}^{g:S} &= \left(\frac{95}{4} - \frac{7}{6} N_F^L + \Delta_m^{g:S} \right) \frac{\alpha_s^{(N_F^L)}(m_h)}{\pi} \\ &+ \left[370.20 - 47.19 N_F^L + 0.902 (N_F^L)^2 + (2.375 + 0.667 N_F^L) \log \frac{m_h^2}{M_t^2} \right] \left(\frac{\alpha_s^{(N_F^L)}(m_h)}{\pi} \right)^2 \\ &+ \left[4533.46 - 1062.82 N_F^L + 52.62 (N_F^L)^2 - 0.5378 (N_F^L)^2 \right. \\ &+ (66.66 + 14.60 N_F^L - 0.6887 (N_F^L)^2) \log \frac{m_h^2}{M_t^2} \\ &\left. + (6.53 + 1.44 N_F^L - 0.111 (N_F^L)^2) \log^2 \frac{m_h^2}{M_t^2} \right] \left(\frac{\alpha_s^{(N_F^L)}(m_h)}{\pi} \right)^3, \end{aligned} \quad (22)$$

$$\delta_{\text{ew}}^{g:S} = \frac{G_F M_t^2}{8\sqrt{2}\pi^2}. \quad (23)$$

Considering $N_F^L = 5$ to account for the number of light quark flavors, we use¹ $\Delta_m^{g:S} \approx 0.7$ for the NLO quark-mass effects contributed by the top, bottom, and charm quarks [49]. At $m_h = 125.2$ GeV, the QCD corrections are computed as $\delta_{\text{QCD}}^{g:S} = 0.668 + 0.197 + 0.021$ for the NLO,² next-to-next-to-leading-order (NNLO), and N³LO levels. The EW correction is approximately 0.3%, making it negligibly small.

Under the framework of NP, supplementary enhancements to this leading-order (LO) decay width arise from loops involving third-generation fermions ($f = t, b, \tau$) and W bosons. This phenomenon can be mathematically represented as [5]

$$\begin{aligned} \Gamma_{\text{NP}}(h \rightarrow \gamma\gamma) &= \frac{G_F \alpha^2 m_h^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c Q_f^2 \kappa_f A_{1/2}(x_f) + \kappa_W A_1(x_W) \right. \\ &\left. + \sum_{\alpha=1}^2 \kappa_{H_a^\pm} \frac{m_Z^2}{m_{H_a^\pm}^2} A_0(x_{H_a^\pm}) \right|^2. \end{aligned} \quad (24)$$

Because of charge conjugation invariance and color conservation, gluon radiation from the colored quark loop involved in the $h\gamma\gamma$ vertex is prohibited. Therefore, the complete two-loop QCD corrections can be effectively considered by incorporating scaling factors into the form factors corresponding to the b - and t -quark contributions, as³ [48,49,54–57]

$$A_{1/2}(x_f) \longrightarrow A_{1/2}(x_f) \left[1 + C_{sf}(x_f) \frac{\alpha_s(M_h)}{\pi} \right]; \quad (25)$$

¹This value is obtained from Fig. 7(b) in Ref. [49].

²In this chapter, we denote the pole mass of the fermion f by M_f and its running mass by $m_f(\mu)$.

³For a detailed description of the scaling factors of $C_{sf}(x)$, see Appendix A.

in the limit $x_f \rightarrow 0$, the scaling factors C_{sf} approach -1 . And the expressions of κ_τ , κ_W , κ_Z , and $\kappa_{H_\alpha^\pm}$ are, respectively,

$$\begin{aligned}\kappa_\tau &= \frac{v_{EW}}{m_\tau} C_{e_3 \bar{e}_3 h}, \\ \kappa_W &= -\frac{v_{EW}}{2m_W^2} C_{hW_\sigma^+ W_\mu^-}, \\ \kappa_Z &= -\frac{v_{EW}}{2m_Z^2} C_{hZ_\sigma Z_\mu}, \\ \kappa_{H_\alpha^\pm} &= -\frac{v_{EW}}{2m_Z^2} C_{hH_\alpha^+ H_\alpha^-} \quad (\alpha = 1, 2),\end{aligned}\tag{26}$$

where the couplings $C_{e_3 \bar{e}_3 h}$, $C_{hW_\sigma^+ W_\mu^-}$, $C_{hZ_\sigma Z_\mu}$, and $C_{hH_\alpha^+ H_\alpha^-}$ are defined in Appendix B.

The light doubletlike Higgs with 125 GeV mass can decay through the channels $h \rightarrow WW^*$ and $h \rightarrow ZZ^*$, where W^* and Z^* denote the off-shell EW gauge bosons. Summing over all modes available to the W^* or Z^* , the decay widths are given by [5]

$$\begin{aligned}\Gamma_{NP}(h \rightarrow ZZ^*) &= \frac{e^4 m_h}{2048\pi^3 s_W^4 c_W^4} |\kappa_Z|^2 \left(7 - \frac{40}{3} s_W^2 + \frac{160}{9} s_W^4\right) F\left(\frac{m_Z}{m_h}\right), \\ \Gamma_{NP}(h \rightarrow WW^*) &= \frac{3e^4 m_h}{512\pi^3 s_W^4} |\kappa_W|^2 F\left(\frac{m_W}{m_h}\right),\end{aligned}\tag{27}$$

and the form factor $F(x)$ is formulated in Appendix A. s_W is the Weinberg angle.

Including the radiative corrections known up to now, the decay width of Higgs to fermions can be organized as [48,58]

$$\Gamma(h \rightarrow f\bar{f}) = N_C^f \frac{m_f^2 \beta_f m_h}{v^2 8\pi} [\beta_f^2 |\kappa_f|^2 (1 + \delta_{QCD} + \delta_i^{f:S} + \delta_{\text{mixed}}^f) (1 + \delta_{ew}^f) + |\tilde{\kappa}_f|^2 (1 + \delta_{QCD} + \delta_i^{f:P})],\tag{28}$$

where $\beta_f \equiv \sqrt{1 - 4\tau_f}$ with $\tau_f = M_f^2/m_h^2$ and the color factor $N_C^f = 3$ for quarks and 1 for leptons. The lepton mass m_f use their pole mass, while for quarks the $\overline{\text{MS}}$ quark mass $\bar{m}_q(m_h)$ is used [58].

The pure QCD corrections to the Higgs decays into a quark pair $q\bar{q}$ consist of a universal part δ_{QCD} , as well as two types of flavor- and parity-dependent contributions $\delta_i^{q:S}$ and $\delta_i^{q:P}$, which are expressed as [48,59–62]

$$\begin{aligned}\delta_{QCD} &= 5.67 \frac{\alpha_s(m_h)}{\pi} + (35.94 - 1.36N_F) \left(\frac{\alpha_s(m_h)}{\pi}\right)^2 + (164.14 - 25.77N_F + 0.259N_F^2) \left(\frac{\alpha_s(m_h)}{\pi}\right)^3 \\ &\quad + (39.34 - 220.9N_F + 9.685N_F^2 - 0.0205N_F^3) \left(\frac{\alpha_s(m_h)}{\pi}\right)^4, \\ \delta_i^{q:S} &= \frac{\kappa_i}{\kappa_q} \left(\frac{\alpha_s(m_h)}{\pi}\right)^2 \left[1.57 - \frac{2}{3} \log \frac{m_h^2}{M_i^2} + \frac{1}{9} \log^2 \frac{\bar{m}_q^2(m_h)}{m_h^2}\right], \\ \delta_i^{q:P} &= \frac{\tilde{\kappa}_i}{\tilde{\kappa}_q} \left(\frac{\alpha_s(m_h)}{\pi}\right)^2 \left[3.83 - \log \frac{m_h^2}{M_i^2} + \frac{1}{6} \log^2 \frac{\bar{m}_q^2(m_h)}{m_h^2}\right],\end{aligned}\tag{29}$$

where N_F counts the flavor number of quarks lighter than m_h . The QCD coupling constant α_s and the running $\overline{\text{MS}}$ quark mass $\bar{m}_q(m_h)$ are defined at the Higgs mass scale to account for the absorption of significant mass logarithms, and we use program package CRUnDec3.1 to compute these parameters [63,64].

For the EW corrections [65–67], we adopt the approximation [58,68]

$$\delta_{ew}^f = \frac{3\alpha}{2\pi} Q_f^2 \left(\frac{3}{2} - \log \frac{m_h^2}{M_f^2}\right) + \frac{G_F}{8\sqrt{2}\pi^2} \left\{ k_f M_f^2 + m_W^2 \left[-5 + \frac{3}{s_W^2} \log c_W^2\right] - 8m_Z^2 (6v_{Z\bar{f}f}^2 - a_{Z\bar{f}f}^2) \right\}.\tag{30}$$

The vector and axial-vector couplings of the Z boson to fermions are given by $v_{Z\bar{f}f} = \frac{1}{2} I_3^f - Q_f s_W^2$ and $a_{Z\bar{f}f} = \frac{1}{2} I_3^f$, respectively, where I_3^f denotes the third component of the EW isospin and Q_f represents the electric charge of the fermion f .

The large logarithm $\log(m_h^2/M_f^2)$ can be absorbed into the running fermion mass, analogous to the treatment in QCD corrections. For decays involving leptons and light quarks, the coefficient k_f is 7, whereas for b and t quarks, k_f is 1. The EW corrections are below the 1% level for $f = b, c$, while they are of the order of $\mathcal{O}(1\%-5\%)$ for $f = \tau, \mu$ [48].

The mixed corrections, determined using low-energy theorems, can be formulated into the expressions [69,70]

$$\delta_{\text{mixed}}^q = -\frac{G_F M_t^2}{8\sqrt{2}\pi^2} \left(\frac{3}{2} + \zeta_2 \right) \frac{\alpha_s(M_t)}{\pi} \quad \text{for light quarks,}$$

$$\delta_{\text{mixed}}^{b,t} = -\frac{G_F M_t^2}{8\sqrt{2}\pi^2} 4(1 + \zeta_2) \frac{\alpha_s(M_t)}{\pi} \quad \text{for } b \text{ and } t, \quad (31)$$

at NNLO with $\zeta_2 = \pi^2/6$.

Normalized to the SM expectation, the signal strengths for the Higgs decay channels are quantified by the ratios [5,71]

$$\mu_{h\gamma\gamma, hVV^*} = \frac{\sigma_{\text{NP}}(\text{ggF}) \text{BR}_{\text{NP}}(h \rightarrow \gamma\gamma, VV^*)}{\sigma_{\text{SM}}(\text{ggF}) \text{BR}_{\text{SM}}(h \rightarrow \gamma\gamma, VV^*)} \quad (V = Z, W),$$

$$\mu_{hf\bar{f}} = \frac{\sigma_{\text{NP}}(\text{VBF}) \text{BR}_{\text{NP}}(h \rightarrow f\bar{f})}{\sigma_{\text{SM}}(\text{VBF}) \text{BR}_{\text{SM}}(h \rightarrow f\bar{f})} \quad (f = b, c, \tau). \quad (32)$$

One can evaluate the Higgs decay branch ratios by

$$\text{BR}(h_{125}^{\text{NP}} \rightarrow \gamma\gamma) = \frac{\Gamma_{\text{NP}}(h \rightarrow \gamma\gamma)}{\Gamma_{\text{SM}}(h \rightarrow \gamma\gamma)} \text{BR}(h_{125}^{\text{SM}} \rightarrow \gamma\gamma) \frac{\Gamma_{\text{tot},125}^{\text{SM}}}{\Gamma_{\text{tot},125}^{\text{NP}}},$$

$$\text{BR}(h_{125}^{\text{NP}} \rightarrow VV^*) = \frac{\Gamma_{\text{NP}}(h \rightarrow VV^*)}{\Gamma_{\text{SM}}(h \rightarrow VV^*)} \text{BR}(h_{125}^{\text{SM}} \rightarrow VV^*) \frac{\Gamma_{\text{tot},125}^{\text{SM}}}{\Gamma_{\text{tot},125}^{\text{NP}}},$$

$$\text{BR}(h_{125}^{\text{NP}} \rightarrow f\bar{f}) = \frac{\Gamma_{\text{NP}}(h \rightarrow f\bar{f})}{\Gamma_{\text{SM}}(h \rightarrow f\bar{f})} \text{BR}(h_{125}^{\text{SM}} \rightarrow f\bar{f}) \frac{\Gamma_{\text{tot},125}^{\text{SM}}}{\Gamma_{\text{tot},125}^{\text{NP}}}, \quad (33)$$

and, moreover,

$$\frac{\sigma_{\text{NP}}(\text{ggF})}{\sigma_{\text{SM}}(\text{ggF})} \approx \frac{\Gamma_{\text{NP}}(h \rightarrow gg)}{\Gamma_{\text{SM}}(h \rightarrow gg)} = \frac{\Gamma_{\text{NP}}^h}{\Gamma_{\text{SM}}^h} \frac{\Gamma_{\text{NP}}(h \rightarrow gg)/\Gamma_{\text{NP}}^h}{\Gamma_{\text{SM}}(h \rightarrow gg)/\Gamma_{\text{SM}}^h}$$

$$= \frac{\Gamma_{\text{NP}}^h \text{BR}_{\text{NP}}(h \rightarrow gg)}{\Gamma_{\text{SM}}^h \text{BR}_{\text{SM}}(h \rightarrow gg)},$$

$$\frac{\sigma_{\text{NP}}(\text{VBF})}{\sigma_{\text{SM}}(\text{VBF})} \approx \frac{\Gamma_{\text{NP}}(h \rightarrow VV^*)}{\Gamma_{\text{SM}}(h \rightarrow VV^*)} = \frac{\Gamma_{\text{NP}}^h}{\Gamma_{\text{SM}}^h} \frac{\Gamma_{\text{NP}}(h \rightarrow VV^*)/\Gamma_{\text{NP}}^h}{\Gamma_{\text{SM}}(h \rightarrow VV^*)/\Gamma_{\text{SM}}^h}$$

$$= \frac{\Gamma_{\text{NP}}^h \text{BR}_{\text{NP}}(h \rightarrow VV^*)}{\Gamma_{\text{SM}}^h \text{BR}_{\text{SM}}(h \rightarrow VV^*)}, \quad (34)$$

with the 125 GeV Higgs total decay width for the NP

$$\Gamma_{\text{NP}}^h \approx \sum_{f=b,\tau,c} \Gamma_{\text{NP}}(h \rightarrow f\bar{f}) + \sum_{V=Z,W} \Gamma_{\text{NP}}(h \rightarrow VV^*)$$

$$+ \Gamma_{\text{NP}}(h \rightarrow gg) + \Gamma_{\text{NP}}(h \rightarrow \gamma\gamma) + \Gamma_{\text{NP}}(h \rightarrow Z\gamma). \quad (35)$$

Here, we disregard the influences from rare or invisible decays, with Γ_{SM}^h representing the total decay width of the SM Higgs. By using Eqs. (32)–(34), we are able to assess the signal strengths for the Higgs decay modes within the FDM [71] as

$$\mu_{h\gamma\gamma} \approx \frac{\Gamma_{\text{NP}}(h \rightarrow gg)}{\Gamma_{\text{SM}}(h \rightarrow gg)} \cdot \frac{\Gamma_{\text{NP}}(h \rightarrow \gamma\gamma)/\Gamma_{\text{NP}}^h}{\Gamma_{\text{SM}}(h \rightarrow \gamma\gamma)/\Gamma_{\text{SM}}^h}$$

$$= \frac{\Gamma_{\text{SM}}^h}{\Gamma_{\text{NP}}^h} \cdot \frac{\Gamma_{\text{NP}}(h \rightarrow gg)}{\Gamma_{\text{SM}}(h \rightarrow gg)} \cdot \frac{\Gamma_{\text{NP}}(h \rightarrow \gamma\gamma)}{\Gamma_{\text{SM}}(h \rightarrow \gamma\gamma)},$$

$$\mu_{hVV^*} \approx \frac{\Gamma_{\text{NP}}(h \rightarrow gg)}{\Gamma_{\text{SM}}(h \rightarrow gg)} \cdot \frac{\Gamma_{\text{NP}}(h \rightarrow VV^*)/\Gamma_{\text{NP}}^h}{\Gamma_{\text{SM}}(h \rightarrow VV^*)/\Gamma_{\text{SM}}^h}$$

$$= \frac{\Gamma_{\text{SM}}^h}{\Gamma_{\text{NP}}^h} \cdot \frac{\Gamma_{\text{NP}}(h \rightarrow gg)}{\Gamma_{\text{SM}}(h \rightarrow gg)} \cdot |\kappa_V|^2,$$

$$\mu_{hf\bar{f}} \approx \frac{\Gamma_{\text{NP}}(h \rightarrow VV^*)}{\Gamma_{\text{SM}}(h \rightarrow VV^*)} \cdot \frac{\Gamma_{\text{NP}}(h \rightarrow f\bar{f})/\Gamma_{\text{NP}}^h}{\Gamma_{\text{SM}}(h \rightarrow f\bar{f})/\Gamma_{\text{SM}}^h}$$

$$= \frac{\Gamma_{\text{SM}}^h}{\Gamma_{\text{NP}}^h} \cdot |\kappa_V|^2 \cdot |\kappa_f|^2 \quad (V = Z, W; f = b, c, \tau), \quad (36)$$

where κ_V ($V = Z, W$) and κ_f ($f = b, c, \tau$) are defined in Eqs. (21) and (26). The corresponding experimental values are listed in Table II.

B. The flavor-changed neutral currents in the FDM

This work examines effects induced by FCNCs within the FDM, where such currents are mediated by a newly defined Z boson, notably the Z' . The rare decay processes considered include $\bar{B} \rightarrow X_s \gamma$ and $B_s^0 \rightarrow \mu^+ \mu^-$ in the B meson sector and $t \rightarrow ch$ and $t \rightarrow uh$ in the top quark sector, as well as the charged lepton flavor violation decays $\tau \rightarrow 3e$, $\tau \rightarrow 3\mu$, and $\mu \rightarrow 3e$. For simplicity, the nonzero $U(1)_F$ charge is fixed at $z = 1$ throughout the analysis.

TABLE II. Experimental values for the Higgs decay rates.

Signal	Value from PDG [1]
$\mu_{\gamma\gamma}$	1.10 ± 0.06
μ_{ZZ^*}	1.02 ± 0.08
μ_{WW^*}	1.00 ± 0.08
$\mu_{b\bar{b}}$	0.99 ± 0.12
$\mu_{c\bar{c}}$	< 14
$\mu_{\tau\bar{\tau}}$	0.91 ± 0.09
$\mu_{Z\gamma}$	2.2 ± 0.7

The B meson rare decay processes $\bar{B} \rightarrow X_s \gamma$ and $B_s^0 \rightarrow \mu^+ \mu^-$ are related closely to the NP contributions, and the average experimental data on the branching ratios of $\bar{B} \rightarrow X_s \gamma$ and $B_s^0 \rightarrow \mu^+ \mu^-$ are [1]

$$\begin{aligned} \text{Br}(\bar{B} \rightarrow X_s \gamma) &= (3.49 \pm 0.19) \times 10^{-4}, \\ \text{Br}(B_s^0 \rightarrow \mu^+ \mu^-) &= (3.01 \pm 0.35) \times 10^{-9}. \end{aligned} \quad (37)$$

The newly introduced scalars in the FDM including CP -even Higgs, CP -odd Higgs, and charged Higgs can make contributions to these two processes; the analytical calculations of the contributions are collected in the Appendix in Ref. [72].

The branching ratios of the top quark rare decay processes $t \rightarrow ch$ and $t \rightarrow uh$ can be written as [73]

$$\text{Br}(t \rightarrow q_u h) = \frac{|\mathcal{M}_{tq_u h}|^2 \sqrt{((m_t + m_h)^2 - m_{q_u}^2)((m_t - m_h)^2 - m_{q_u}^2)}}{32\pi m_t^3 \Gamma_{\text{total}}^t}, \quad (38)$$

where $q_u = u, c$, the amplitude $\mathcal{M}_{tq_u h}$ can be read directly from the Yukawa couplings in Eq. (8), and $\Gamma_{\text{total}}^t = 1.42 \text{ GeV}$ [1] is the total decay width of top quark. The experimental upper bounds on the branching ratios of $t \rightarrow ch$ and $t \rightarrow uh$ are, respectively,

$$\begin{aligned} \text{Br}(t \rightarrow ch) &< 3.4 \times 10^{-4}, \\ \text{Br}(t \rightarrow uh) &< 1.9 \times 10^{-9}. \end{aligned} \quad (39)$$

Finally, we focus on the lepton flavor violation processes $\tau \rightarrow 3e$, $\tau \rightarrow 3\mu$, and $\mu \rightarrow 3e$ predicted in the FDM. The corresponding amplitude can be written as [74]

$$\begin{aligned} \mathcal{M}(e_j \rightarrow e_i e_i \bar{e}_i) &= C_1^L \bar{u}_{e_i}(p_2) \gamma_\mu P_L u_{e_j}(p_1) u_{e_i}(p_3) \gamma^\mu P_L \nu_{e_i}(p_4) \\ &+ C_1^R \bar{u}_{e_i}(p_2) \gamma_\mu P_R u_{e_j}(p_1) u_{e_i}(p_3) \gamma^\mu P_R \nu_{e_i}(p_4) \\ &+ [C_2^L \bar{u}_{e_i}(p_2) \gamma_\mu P_L u_{e_j}(p_1) u_{e_i}(p_3) \gamma^\mu P_R \nu_{e_i}(p_4) \\ &+ C_2^R \bar{u}_{e_i}(p_2) \gamma_\mu P_R u_{e_j}(p_1) u_{e_i}(p_3) \gamma^\mu P_L \nu_{e_i}(p_4) - (p_2 \leftrightarrow p_3)] \\ &+ [C_3^L \bar{u}_{e_i}(p_2) P_L u_{e_j}(p_1) u_{e_i}(p_3) P_L \nu_{e_i}(p_4) \\ &+ C_3^R \bar{u}_{e_i}(p_2) P_R u_{e_j}(p_1) u_{e_i}(p_3) P_R \nu_{e_i}(p_4) - (p_2 \leftrightarrow p_3)], \end{aligned} \quad (40)$$

where $i = 1, 2$ for $j = 3$, $i = 1$ for $j = 2$, u_{e_i} denotes the spinor of lepton, ν_{e_i} denotes the spinor of antilepton, $P_L = (1 - \gamma_5)/2$, $P_R = (1 + \gamma_5)/2$, and p_k denotes the momentum of charged lepton with $k = 1, 2, 3, 4$. The coefficients $C_{1,2,3}^{L,R}$ from the contributions of Higgs bosons and Z, Z' gauge bosons can be obtained through the Yukawa couplings in Eq. (8) and the definition of covariant derivative in Eq. (14). Then, we can calculate the decay rate [74]

$$\Gamma(e_j \rightarrow e_i e_i \bar{e}_i) = \frac{m_{e_i}^5}{1536\pi^3} \left[\frac{1}{2} (|C_1^L|^2 + |C_1^R|^2) + |C_2^L|^2 + |C_2^R|^2 + \frac{1}{8} (|C_3^L|^2 + |C_3^R|^2) \right]. \quad (41)$$

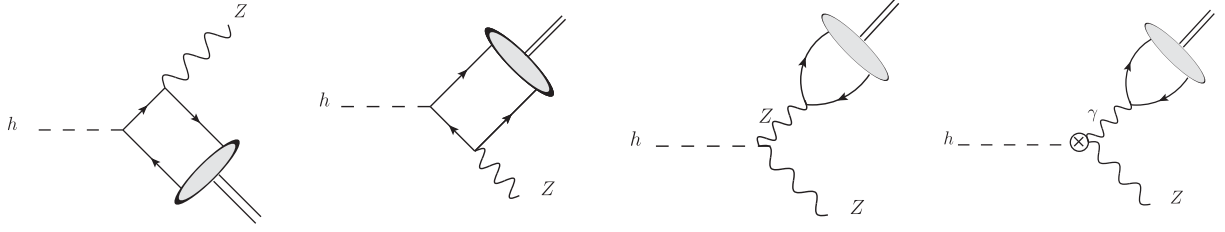
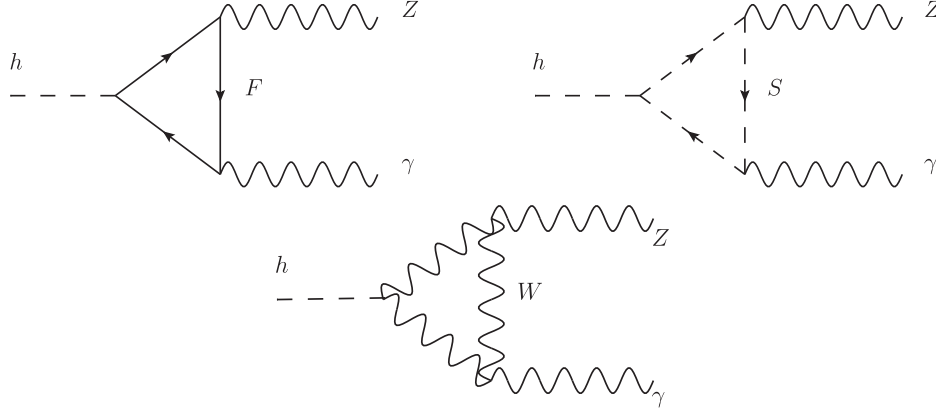
The total decay widths of μ and τ are taken as $\Gamma_{\text{total}}^\mu = 2.996 \times 10^{-19} \text{ GeV}$ and $\Gamma_{\text{total}}^\tau = 2.265 \times 10^{-12} \text{ GeV}$ [1].

The experimental upper bounds on decay widths are [1]

$$\begin{aligned} \text{Br}(\mu^- \rightarrow e^- e^+ e^-) &< 1.0 \times 10^{-12}, \\ \text{Br}(\tau^- \rightarrow e^- e^+ e^-) &< 2.7 \times 10^{-8}, \\ \text{Br}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) &< 1.9 \times 10^{-8}. \end{aligned} \quad (42)$$

C. Constraints from electroweak precision observables

The masses and decay properties of the electroweak bosons, combined with low-energy data, can be used to probe and constrain possible deviations from the SM. At the electroweak one-loop level, such effects on precision observables are predominantly captured by the three oblique parameters S , T , and U , which characterize corrections to the gauge boson self-energies. The parameter T is proportional to the difference between the W and Z

FIG. 1. The dominating Feynman diagrams for $h \rightarrow MZ$, where M is a vector meson ($\rho, \omega, \phi, J/\psi, \Upsilon$) [26].FIG. 2. The one-loop diagrams for $h \rightarrow \gamma Z$ in the FDM, with F denoting charged fermions and $S = H^\pm$ denoting charged Higgs, and W is $W^\pm$ boson.

self-energies at $Q^2 = 0$, while S ($S + U$) is associated with the difference between the Z (W) self-energy at $Q^2 = M_{Z,W}^2$ and $Q^2 = 0$. The current values of these parameters, taken from Ref. [1], are

$$S = -0.04 \pm 0.10, \quad T = 0.01 \pm 0.12, \quad U = -0.01 \pm 0.11. \quad (43)$$

We use SPheno [75] to compute the mass spectrum and the three oblique parameters.

D. The decay processes $h \rightarrow Z\gamma$ and $h \rightarrow MZ$

The weak hadronic decays of the lightest Higgs boson $h \rightarrow MZ$ hold significant interest due to the potential involvement of a longitudinally polarized massive final-state gauge boson [26]. In Fig. 1, we illustrate the predominant Feynman diagrams for $h \rightarrow MZ$. The first two diagrams in Fig. 1 correspond to the direct contributions denoted as F_{direct} , while the last two diagrams depict the indirect

contributions labeled as F_{ind} . In the final diagram, the crossed circle symbolizes the effective vertex $h \rightarrow Z\gamma^*$ originating from the one-loop diagrams. Notably, the hZZ^* vertex is present at the tree level, whereas the $hZ\gamma$ vertex arises through one-loop processes [26] (see Fig. 2). The process $h \rightarrow Z\gamma^*$ serves as a potential probe for NP; thus, we will primarily delve into the discussion of $h \rightarrow Z\gamma^*$.

The experimental investigation conducted in [76] focused on the decay process $h \rightarrow J/\psi Z$, while the exploration of the decays $h \rightarrow \rho Z$ and $h \rightarrow \phi Z$ was elaborated in Ref. [77]. The upper limits on the branch ratio $\text{Br}(h \rightarrow \rho Z)$ is below 1.21%, corresponding to less than 868 times the SM prediction. Correspondingly, the upper limit for $\text{Br}(h \rightarrow \phi Z)$ is below 0.36%, equivalent to 855 times the SM anticipation. These upper bounds are intricately connected to the polarization scenarios as discussed in Ref. [77].

The decay width of $h \rightarrow Z\gamma$ and $h \rightarrow MZ$ can be given as [26]

$$\Gamma(h \rightarrow Z\gamma) = \frac{\alpha^2 m_h^3}{32\pi^3 v^2 \sin^2 \theta_W \cos^2 \theta_W} \left(1 - \frac{m_Z^2}{m_h^2}\right)^3 (|C_{\gamma Z}|^2 + |\tilde{C}_{\gamma Z}|^2). \quad (44)$$

$$\Gamma(h \rightarrow MZ) = \frac{m_h^3}{4\pi v^4} \lambda^{1/2}(1, r_Z, r_M)(1 - r_Z - r_M)^2 \left[|F_{\parallel}^{MZ}|^2 + \frac{8r_M r_Z}{(1 - r_Z - r_M)^2} (|F_{\perp}^{MZ}|^2 + |\tilde{F}_{\perp}^{MZ}|^2) \right]. \quad (45)$$

Here, $\lambda(1, r_Z, r_M) = (1 - r_Z - r_M)^2 - 4r_Z r_M$, $r_Z = \frac{m_Z^2}{m_h^2}$, $r_M = \frac{m_M^2}{m_h^2}$, and m_M is the mass of vector meson. $F_{\parallel}^{MZ}$ represents the CP -even longitudinal form factors, and $F_{\perp}^{MZ}$ and $\tilde{F}_{\perp}^{MZ}$ represent the CP -even and CP -odd transverse form factors, respectively. The mass ratio r_M is very small for all mesons, but it can make the contributions to the transverse polarization states to the $h \rightarrow MZ$ rates significantly, so we still keep the mass ratio r_M in our analysis [26,27,78].

Equation (45) comprises two components of form factors: the direct and the indirect contributions. We initiate the analysis with the indirect contributions, which play a significant role in probing NP. These contributions entail hadronic matrix elements of local currents, enabling calculations to be extended to all orders in QCD [26]. Consequently, the indirect contributions are

$$F_{\parallel \text{ind}}^{MZ} = \frac{g_{hZZ}}{1 - r_M/r_Z} \sum_q f_M^q v_q + C_{\gamma Z} \frac{\alpha(m_M)}{4\pi} \frac{4r_Z}{1 - r_Z - r_M} \sum_q f_M^q Q_q, \quad (46)$$

$$F_{\perp \text{ind}}^{MZ} = \frac{g_{hZZ}}{1 - r_M/r_Z} \sum_q f_M^q v_q + C_{\gamma Z} \frac{\alpha(m_M)}{4\pi} \frac{1 - r_Z - r_M}{r_M} \sum_q f_M^q Q_q, \quad (47)$$

$$\tilde{F}_{\perp \text{ind}}^{MZ} = \tilde{C}_{\gamma Z} \frac{\alpha(m_M)}{4\pi} \frac{\lambda^{1/2}(1, r_Z, r_M)}{r_M} \sum_q f_M^q Q_q, \quad (48)$$

where $v_q = T_3^q/2 - Q_q s_W^2$ are the vector couplings of the Z boson to the quark q , T_3^q and Q_q represent the weak isospin and charge of quark q , respectively, and $s_W = \sin \theta_W$ with θ_W denoting the Weinberg angle. α is the electromagnetic coupling constant can be calculated by AlphaQED [79]. The flavor-specific decay constants f_M^q are defined in terms of the local matrix elements [26,27,30]

$$\langle M(k, \varepsilon) | \bar{q} \gamma^\mu q | 0 \rangle = -i f_M^q m_V \varepsilon^\mu. \quad (49)$$

We use the following relations to simplify our calculation:

$$\sum_q f_M^q Q_q = f_M Q_M, \quad \sum_q f_M^q v_q = f_M v_M. \quad (50)$$

The meson decay constants f_M , Q_M , and v_M for the vector meson $M = (\rho, \omega, \phi, J/\Psi, \Upsilon(1S))$ can be seen in Table III.

The concrete forms of $C_{\gamma Z}$ and $\tilde{C}_{\gamma Z}$ in Eqs. (46)–(48) are given by [26,27]

$$C_{\gamma Z} = C_{\gamma Z}^{\text{SM}} + C_{\gamma Z}^{\text{NP}}, \quad \tilde{C}_{\gamma Z} = \tilde{C}_{\gamma Z}^{\text{SM}} + \tilde{C}_{\gamma Z}^{\text{NP}}, \quad (51)$$

$$C_{\gamma Z}^{\text{SM}} = \sum_q \kappa_q \frac{2N_c Q_q v_q}{3} A_f(\tau_q, r_Z) + \sum_l \kappa_l \frac{2Q_l v_l}{3} A_f(\tau_l, r_Z) - \frac{1}{2} \kappa_W A_W^{\gamma Z}(\tau_W, r_Z), \quad (52)$$

$$\tilde{C}_{\gamma Z}^{\text{SM}} = \sum_q \tilde{\kappa}_q N_c Q_q v_q B_f(\tau_q, r_Z) + \sum_l \tilde{\kappa}_l Q_l v_l B_f(\tau_l, r_Z), \quad (53)$$

where $\tau_f = 4m_f^2/m_h^2$ ($f = q, l$). We use the running quark masses m_q , evaluated at the hadronic scale μ_{hZ} , which is given by $\mu_{hZ} = (m_h^2 - m_Z^2)/m_h \approx 58.78$ GeV [30]. v_l are the vector couplings of the Z boson to the leptons, and Q_l represent the charge of leptons. $\tilde{\kappa}_q$ and $\tilde{\kappa}_l$ are the CP -odd effective Higgs couplings to the quarks and the leptons, respectively. $C_{\gamma Z}^{\text{SM}}$ and $\tilde{C}_{\gamma Z}^{\text{SM}}$ are the corresponding SM CP -even and CP -odd contributions to $h \rightarrow \gamma Z$, respectively. $C_{\gamma Z}^{\text{NP}}$ and $\tilde{C}_{\gamma Z}^{\text{NP}}$ are the NP CP -even and CP -odd contributions to $h \rightarrow \gamma Z$, respectively. The loop functions A_f , $A_W^{\gamma Z}$, and B_f can be found in Refs. [28–30].

In the SM, $\tilde{\kappa}_q = \tilde{\kappa}_l = 0$, so the CP -odd coupling $\tilde{C}_{\gamma Z}^{\text{SM}}$ is 0 [26]. In the NP, $h\gamma Z$ interaction with CP -even and CP -odd parts can be written as $\bar{F}_2 i(A + B\gamma_5)F_1 h$, where A is the CP -even part and B is the CP -odd part [26,27]. Considering the interaction $\bar{F}_2 i(C^L P_L + C^R P_R)F_1 h$ with $P_L = \frac{1-\gamma_5}{2}$ and $P_R = \frac{1+\gamma_5}{2}$, the CP -even part can be $A = \frac{1}{2}(C^L + C^R)$ and the CP -odd part is $B = \frac{1}{2}(C^L - C^R)$. In the FDM, $C_{hH_a^\pm H_a^\mp}^L = C_{hH_a^\pm H_a^\mp}^R$. So the CP -odd coupling $\tilde{C}_{\gamma Z}^{\text{NP}}$ in the FDM can be neglected approximately. The expression of CP -even coupling $C_{\gamma Z}^{\text{NP}}$ in the FDM is

$$C_{\gamma Z}^{\text{NP}} = \frac{c_W}{2} \left[(2c_W^2 - 1) \sum_{a=1}^2 \kappa_{H_a^\pm} \frac{m_Z^2}{m_{H_a^\pm}^2} A_0(x_{H_a^\pm}, \lambda_{H_a^\pm}) \right], \quad (54)$$

where $x_i = 4m_i^2/m_h^2$, $\lambda_i = 4m_i^2/m_Z^2$, and $A_0(x_{H_a^\pm}, \lambda_{H_a^\pm})$ can be found in Appendix A.

Compared to the indirect contributions, the direct contributions to the decay amplitudes can be calculated in a power series in $(\Lambda_{\text{QCD}}/m_h)^2$ or $(m_q/m_h)^2$ [26,27]. Here, the Λ_{QCD} is a hadronic scale where the authors in Ref. [80] provide $\Lambda_{\text{QCD}}/m_Z \sim 0.01$, and m_q are the effective masses

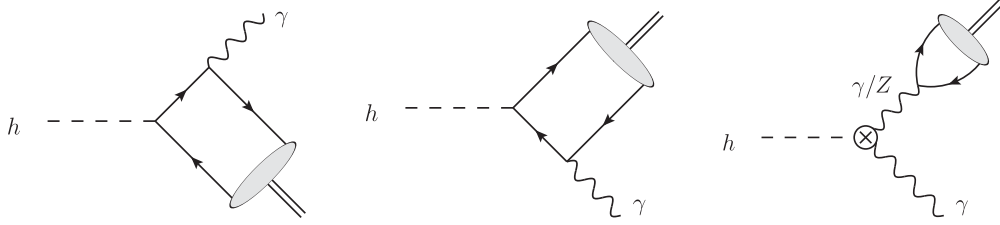


FIG. 3. Direct (left and center) and indirect (right) contributions to the $h \rightarrow M\gamma$ decay amplitude. The crossed circle in the third diagram denotes the off-shell $h \rightarrow \gamma\gamma^*$ and $h \rightarrow \gamma Z^*$ amplitudes, which in the SM arise first at one-loop order.

of the constituent quarks of a given meson, where the asymptotic function $\phi_M^\perp(x) = 6x(1-x)$ [27,81] is needed; then the direct contributions are as follows:

$$F_{\perp\text{direct}}^{MZ} = \sum_q f_M^{q\perp} v_q \kappa_q \frac{3m_q}{2m_M} \frac{1 - r_Z^2 + 2r_Z \ln r_Z}{(1 - r_Z)^2}, \quad (55)$$

$$\tilde{F}_{\perp\text{direct}}^{MZ} = \sum_q f_M^{q\perp} v_q \tilde{\kappa}_q \frac{3m_q}{2m_M} \frac{1 - r_Z^2 + 2r_Z \ln r_Z}{(1 - r_Z)^2}. \quad (56)$$

$f_M^{q\perp}$ represent the flavor-specific transverse decay constants of the meson [30]. Other studies [82] indicate that the direct contribution is significantly smaller than the indirect contribution. Our calculations also show that the direct contribution is relatively small compared to the indirect contributions. Because of the difficulty of observing NP in the direct contribution and the prominence of NP effects in the indirect contribution, we neglect the former in our analysis.

Normalized to the SM expectation, the signal strengths for the Higgs decay channels are quantified by the ratios [78,83]

$$\mu_{hMZ} = \frac{\sigma_{\text{NP}}(\text{ggF}) \text{Br}_{\text{NP}}(h \rightarrow MZ)}{\sigma_{\text{SM}}(\text{ggF}) \text{Br}_{\text{SM}}(h \rightarrow MZ)}, \quad (57)$$

$$\mu_{hZ\gamma} = \frac{\sigma_{\text{NP}}(\text{ggF}) \text{Br}_{\text{NP}}(h \rightarrow Z\gamma)}{\sigma_{\text{SM}}(\text{ggF}) \text{Br}_{\text{SM}}(h \rightarrow Z\gamma)}. \quad (58)$$

Through Eqs. (57), (58), and (34), we quantify the signal strengths for $h \rightarrow MZ$ and $h \rightarrow Z\gamma$:

$$\begin{aligned} \mu_{hMZ} &\approx \frac{\Gamma_{\text{NP}}(h \rightarrow \text{gg})}{\Gamma_{\text{SM}}(h \rightarrow \text{gg})} \frac{\Gamma_{\text{NP}}(h \rightarrow MZ)/\Gamma_{\text{NP}}^h}{\Gamma_{\text{SM}}(h \rightarrow MZ)/\Gamma_{\text{SM}}^h} \\ &= \frac{\Gamma_{\text{SM}}^h}{\Gamma_{\text{NP}}^h} \frac{\Gamma_{\text{NP}}(h \rightarrow \text{gg})}{\Gamma_{\text{SM}}(h \rightarrow \text{gg})} \frac{\Gamma_{\text{NP}}(h \rightarrow MZ)}{\Gamma_{\text{SM}}(h \rightarrow MZ)}, \end{aligned} \quad (59)$$

$$\begin{aligned} \mu_{hZ\gamma} &\approx \frac{\Gamma_{\text{NP}}(h \rightarrow \text{gg})}{\Gamma_{\text{SM}}(h \rightarrow \text{gg})} \frac{\Gamma_{\text{NP}}(h \rightarrow Z\gamma)/\Gamma_{\text{NP}}^h}{\Gamma_{\text{SM}}(h \rightarrow Z\gamma)/\Gamma_{\text{SM}}^h} \\ &= \frac{\Gamma_{\text{SM}}^h}{\Gamma_{\text{NP}}^h} \frac{\Gamma_{\text{NP}}(h \rightarrow \text{gg})}{\Gamma_{\text{SM}}(h \rightarrow \text{gg})} \frac{\Gamma_{\text{NP}}(h \rightarrow Z\gamma)}{\Gamma_{\text{SM}}(h \rightarrow Z\gamma)}. \end{aligned} \quad (60)$$

E. Higgs radiative decay to vector quarkonium

Analogous to the $h \rightarrow MZ$ process, the $h \rightarrow M\gamma$ decay rate is governed by the destructive interference of two amplitudes, one of which involves the Higgs coupling to the quark-antiquark pair inside the vector meson, and the other amplitude arises from the loop-induced effective $h\gamma\gamma^*$ and $h\gamma Z^*$ couplings, where the off-shell gauge boson converts into the vector meson [30,84,85]. The LO Feynman diagrams are shown in Fig. 3, the first two graphs are the so-called direct contributions, and the third one is the indirect contributions. In this paper, we will use the results in Ref. [86] to calculate the indirect and direct amplitudes by computing the order- v^4 correction to the decay rate in the NRQCD factorization formalism.

For the process $h \rightarrow M\gamma$ ($M = \rho, \omega, \phi$), the most general parametrization of the decay amplitude is [30,85]

$$\begin{aligned} iA(h \rightarrow M\gamma) &= -\frac{ef_V}{2} \left[\left(\varepsilon_V^\mu \cdot \varepsilon_\gamma^* - \frac{q \cdot \varepsilon_V^* k \cdot \varepsilon_\gamma^*}{k \cdot q} \right) F_1^V \right. \\ &\quad \left. - i\epsilon_{\mu\nu\alpha\beta} \frac{k^\mu q^\nu \varepsilon_V^{*\alpha} \varepsilon_\gamma^{*\beta}}{k \cdot q} F_2^V \right], \end{aligned} \quad (61)$$

with k representing the momentum of the M boson and q representing the momentum of γ , so that $\varepsilon_V^\mu \cdot k = \varepsilon_\gamma^\mu \cdot q = 0$. ε_V^* and ε_γ^* are, respectively, the polarization vectors of vector mesons. F_1^V and F_2^V will be defined below. f_V is a decay constant and can be seen in Table III. All of them are influenced by the direct and indirect contributions. The decay width is

TABLE III. The mesons decay constants f_M , Q_M , and v_M will be used in the numerical analysis. $f_M^\perp$ and $f_M^{q\perp}$ represent the transverse decay constants and the flavor-specific transverse decay constants, respectively [30].

Mesons M	m_M/GeV	f_M/GeV	Q_M	v_M	$f_M^\perp/f_M = f_M^{q\perp}/f_M^q$
ρ	0.77	0.216	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}(\frac{1}{2} - s_W^2)$	0.72
ω	0.782	0.194	$\frac{1}{3\sqrt{2}}$	$-\frac{s_W^2}{3\sqrt{2}}$	0.71
ϕ	1.02	0.223	$-\frac{1}{3}$	$-\frac{1}{4} + \frac{s_W^2}{3}$	0.76
J/ψ	3.097	0.403	$\frac{2}{3}$	$\frac{1}{4} - \frac{2s_W^2}{3}$	0.91
$\Upsilon(1S)$	9.46	0.648	$-\frac{1}{3}$	$-\frac{1}{4} + \frac{s_W^2}{3}$	1.09

$$\Gamma(h \rightarrow M\gamma) = \frac{\alpha f_V^2}{8m_h} (|F_1^V|^2 + |F_2^V|^2). \quad (62)$$

Here, $\alpha = 1/137.036$ is the fine-structure constant evaluated at $q^2 = 0$.

For direct contributions, we have

$$F_{1,\text{direct}}^V = \bar{\kappa}_V Q_V F_V, \quad F_{2,\text{direct}}^V = i\bar{\kappa}_V Q_V F_V, \quad (63) \quad \text{where}$$

$$F_V = \frac{6m_b(\mu)}{v} \frac{f_V^\perp(\mu)}{f_V} \left[1 - \frac{C_F \alpha_s(\mu)}{\pi} \ln \frac{m_h^2}{\mu^2} \right] I_V(m_h), \quad (64)$$

$$I_V(m_h) = \sum_{n=0}^{\infty} C_{2n}(m_h, \mu) a_{2n}^{V\perp}(\mu), \quad (65)$$

$$C_n(m_h, \mu) = 1 + \frac{C_F \alpha_s(\mu)}{4\pi} \left[-4(H_{n+1} - 1) \left(\ln \frac{m_h^2}{\mu^2} - i\pi \right) + 4H_{n+1}^2 - 3 + 4i\pi \right] + \mathcal{O}(\alpha_s^2) \quad (66)$$

and $H_{n+1} = \sum_{k=1}^{n+1} \frac{1}{k}$ are the harmonic numbers, $a_{2n}^{V\perp}(\mu)$ are the Gegenbauer moments, $\bar{\kappa}_V$ characterizes the CP -even gauge boson couplings in scenarios where the Higgs field interactions deviate from the SM expectations, and $\bar{\kappa}_V$ is CP -odd. For indirect contributions, we follow the treatment in Ref. [30]:

$$\begin{aligned} F_{1,\text{indirect}}^V &= \frac{\alpha(m_M)}{\pi} \frac{m_h^2 - m_M^2}{m_M v} \left[Q_V C_{\gamma\gamma}(r_M) - \frac{v_M}{(s_W c_W)^2} \frac{m_M^2}{m_Z^2 - m_M^2} C_{\gamma Z}(r_M) \right], \\ F_{2,\text{indirect}}^V &= i \frac{\alpha(m_M)}{\pi} \frac{m_h^2 - m_M^2}{m_M v} \left[Q_V \tilde{C}_{\gamma\gamma}(r_M) - \frac{v_M}{(s_W c_W)^2} \frac{m_M^2}{m_Z^2 - m_M^2} \tilde{C}_{\gamma Z}(r_M) \right], \end{aligned} \quad (67)$$

$$\begin{aligned} C_{\gamma\gamma}(r_M) &= \sum_q \kappa_q \frac{2N_c Q_q^2}{3} A_f(\tau_q, r_M) + \sum_l \kappa_l \frac{2Q_l^2}{3} A_f(\tau_l, r_M) - \frac{\kappa_W}{2} A_W^{\gamma\gamma}(\tau_W, r_M), \\ C_{\gamma Z}(r_M) &= \sum_q \kappa_q \frac{2N_c Q_q v_q}{3} A_f(\tau_q, r_M) + \sum_l \kappa_l \frac{2Q_l v_l}{3} A_f(\tau_l, r_M) - \frac{\kappa_W}{2} A_W^{\gamma Z}(\tau_W, r_M), \end{aligned} \quad (68)$$

the first two terms in each coefficient are the contributions from the quarks and leptons, and the third term in $C_{\gamma\gamma}$ and $C_{\gamma Z}$ arises from gauge-boson loops,

$$\begin{aligned} \tilde{C}_{\gamma\gamma}(r_M) &= \sum_q \tilde{\kappa}_q N_c Q_q^2 B_f(\tau_q, r_M) + \sum_l \tilde{\kappa}_l Q_l^2 B_f(\tau_l, r_M), \\ \tilde{C}_{\gamma Z}(r_M) &= \sum_q \tilde{\kappa}_q N_c Q_q v_q B_f(\tau_q, r_M) + \sum_l \tilde{\kappa}_l Q_l v_l B_f(\tau_l, r_M), \end{aligned} \quad (69)$$

where $\tau_W = 4m_W^2/m_h^2$; in the τ_q or τ_l we use the running masses $m_q(m_h)$ or $m_l(m_h)$. All loop functions A_f , B_f , $A_W^{\gamma\gamma}$, and $A_W^{\gamma Z}$ can be found in Appendix A.

As discussed in $h \rightarrow MZ$, the CP -odd couplings $\tilde{C}_{\gamma Z}^{\text{SM}}$ and $\tilde{C}_{\gamma\gamma}^{\text{SM}}$ are 0 in the SM. Through the same discussion, the CP -odd couplings $\tilde{C}_{\gamma Z}^{\text{NP}}$ and $\tilde{C}_{\gamma\gamma}^{\text{NP}}$ in the FDM can be neglected approximately. In the SM, we have $\kappa_q = \kappa_l = \kappa_W = 1$. The effective Higgs couplings $\tilde{\kappa}_i$ entering in Eq. (69) all vanish in the SM. The expression of CP -even couplings $C_{\gamma Z}^{\text{NP}}$ and $C_{\gamma\gamma}^{\text{NP}}$ in the FDM are

$$C_{\gamma Z}^{\text{NP}} = \frac{c_W}{2} \left[(2c_W^2 - 1) \sum_{\alpha=1}^2 \kappa_{H_a^\pm} \frac{m_Z^2}{m_{H_a^\pm}^2} A_0(x_{H_a^\pm}, \lambda_{H_a^\pm}) \right], \quad (70)$$

$$C_{\gamma\gamma}^{\text{NP}} = \sum_{\alpha=1}^2 \kappa_{H_a^\pm} \frac{m_Z^2}{m_{H_a^\pm}^2} A_0(x_{H_a^\pm}), \quad (71)$$

where $x_{H_a^\pm} = 4m_{H_a^\pm}^2/m_h^2$, $\lambda_{H_a^\pm} = 4m_{H_a^\pm}^2/m_Z^2$, and $A_0(x_{H_a^\pm}, \lambda_{H_a^\pm})$ can be found in Appendix A. And, for reasons similar to those of $h \rightarrow MZ$, we ignore the direct contributions.

But in the process of $h \rightarrow M\gamma$ [$M = J/\psi, \Upsilon(nS)$], the direct contributions cannot simply be ignored, because its value is significantly larger than that from indirect contributions. The decay rate $\Gamma(h \rightarrow M\gamma)$ is given by

$$\Gamma(h \rightarrow M\gamma) = |A_{\text{dir}} + A_{\text{ind}}|^2, \quad (72)$$

where

$$A_{\text{dir}} = \frac{1}{2} \sqrt{\Phi} e e_Q \kappa_q \sum_{n=0,2,4,\alpha_s} U_{f_V^\perp}(\mu, \mu_0) f_V^\perp(\mu_0) \times \int_0^1 dx T_H(x, \mu) \phi_V^{\perp(n)}(x, \mu), \quad (73)$$

$$A_{\text{ind}} = -\sqrt{\Phi} \frac{e_Q}{|e_Q|} \frac{\sqrt{24\pi m_h}}{m_M} \left[\frac{\Gamma(M \rightarrow \ell^+ \ell^-) \Gamma(h \rightarrow \gamma\gamma)}{\alpha(0)} \right]^{\frac{1}{2}} \quad (74)$$

and

$$\int_0^1 dx T_H(x, \mu) \phi_V^{\perp(n)}(x, \mu) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \hat{T}_H(n_1, \mu) U_{n_1 n_2}(\mu, \mu_0) \hat{\phi}_V^{\perp(n)}(n_2, \mu_0), \quad (75)$$

$$\Phi = \frac{1}{2m_h} \frac{m_M(m_h^2 - m_M^2)}{2\pi m_h^2}. \quad (76)$$

The definitions of the parameters evolution matrix $U_{f_V^\perp}(\mu, \mu_0)$, the decay constant $f_V^\perp(\mu_0)$, the perturbative hard part $T_H(x, \mu)$, light-cone distribution amplitude $\phi_V^{\perp(n)}(x, \mu)$, and evolution matrix $U_{n_1 n_2}(\mu, \mu_0)$ can be found in Refs. [8,85,86].

We take the values of m_M and $\Gamma(M \rightarrow \ell^+ \ell^-)$ listed in Table IV as inputs. The average experimental data on the branch ratios are [1]

$$\begin{aligned} \text{Br}(h \rightarrow \rho\gamma) &< (1.04) \times 10^{-3}, \\ \text{Br}(h \rightarrow \omega\gamma) &< (5.5) \times 10^{-4}, \\ \text{Br}(h \rightarrow \phi\gamma) &< (5) \times 10^{-4}, \\ \text{Br}(h \rightarrow J/\psi\gamma) &< (2.0) \times 10^{-4}, \\ \text{Br}(h \rightarrow \Upsilon(1S)\gamma) &< (2.5) \times 10^{-4}, \\ \text{Br}(h \rightarrow \Upsilon(2S)\gamma) &< (4.2) \times 10^{-4}, \\ \text{Br}(h \rightarrow \Upsilon(3S)\gamma) &< (3.4) \times 10^{-4}. \end{aligned} \quad (77)$$

TABLE IV. The meson masses m_M and the leptonic decay rates $\Gamma(M \rightarrow \ell^+ \ell^-)$ are all from [1].

M	m_M/GeV	$\Gamma(M \rightarrow \ell^+ \ell^-)/\text{keV}$
J/ψ	3.0969	5.529 ± 0.03
$\Upsilon(1S)$	9.4604	1.291 ± 0.043
$\Upsilon(2S)$	10.0234	0.6108 ± 0.05
$\Upsilon(3S)$	10.3551	0.443 ± 0.4

Similarly, we define $\mu_{hM\gamma}$ [$M = \rho, \omega, \phi, J/\psi, \Upsilon(nS)$] as follows:

$$\begin{aligned} \mu_{hM\gamma} &\approx \frac{\Gamma_{\text{NP}}(h \rightarrow gg)}{\Gamma_{\text{SM}}(h \rightarrow gg)} \cdot \frac{\Gamma_{\text{NP}}(h \rightarrow M\gamma)/\Gamma_{\text{NP}}^h}{\Gamma_{\text{SM}}(h \rightarrow M\gamma)/\Gamma_{\text{SM}}^h} \\ &= \frac{\Gamma_{\text{SM}}^h}{\Gamma_{\text{NP}}^h} \cdot \frac{\Gamma_{\text{NP}}(h \rightarrow gg)}{\Gamma_{\text{SM}}(h \rightarrow gg)} \cdot \frac{\Gamma_{\text{NP}}(h \rightarrow M\gamma)}{\Gamma_{\text{SM}}(h \rightarrow M\gamma)}. \end{aligned} \quad (78)$$

IV. NUMERICAL ANALYSIS

There are many free parameters in the FDM. In order to obtain a transparent numerical results, we make some assumptions on parameter space without losing the generality. For the free parameters in the scalar sector of the FDM, we take $v_1 = v_2 = v_{1,2}$, $\lambda'_4 = \lambda''_4 = \lambda_4/2$, $\lambda'_5 = \lambda''_5 = \lambda_5/2$, $\lambda'_6 = \lambda''_6 = \lambda_6/2$, and all parameters to be real for simplicity. The Higgs properties are affected negligibly by the gauge coupling constants g_F and g_{YF} and also considering the constraints from collider direct searches for new gauge bosons or scalars [87]; we take $g_F = 0.20$, $g_{YF} = 0.10$, and $v_\chi \geq 4$ TeV in the following analysis. Considering the requirements of tree-level perturbative unitary bounds condition, we adopt the following parameter ranges [12]:

$$\begin{aligned} |\lambda_1|, |\lambda_2|, |\lambda_3|, |\lambda_\chi| &\leq \frac{4\pi}{3}, \quad |\text{Re}(\lambda_{10})| \leq 4\pi, \\ |\lambda'_4 + \lambda''_4|, |\lambda'_5 + \lambda''_5|, |\lambda'_6 + \lambda''_6|, |\lambda_7|, |\lambda_8|, |\lambda_9| &\leq 8\pi. \end{aligned} \quad (79)$$

And the vacuum stability requirement the scalar potential should be bounded by the conditions [87]

$$\begin{aligned} \lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > 0, \quad \lambda_\chi > 0, \quad |\lambda_{10}|^2 &\leq (L_{13} + 2\sqrt{\lambda_1 \lambda_3})(L_{23} + 2\sqrt{\lambda_2 \lambda_3}), \\ L_{12} + 2\sqrt{\lambda_1 \lambda_2} \geq 0, \quad L_{13} + 2\sqrt{\lambda_1 \lambda_3} \geq 0, \quad L_{23} + 2\sqrt{\lambda_2 \lambda_3} &\geq 0, \\ \lambda_7 + 2\sqrt{\lambda_1 \lambda_\chi} \geq 0, \quad \lambda_8 + 2\sqrt{\lambda_2 \lambda_\chi} \geq 0, \quad \lambda_9 + 2\sqrt{\lambda_3 \lambda_\chi} &\geq 0, \\ \sqrt{\lambda_1 \lambda_2 \lambda_\chi} + \frac{L_{12}\sqrt{\lambda_\chi} + \lambda_7\sqrt{\lambda_2} + \lambda_8\sqrt{\lambda_1}}{2} + \frac{1}{2} \sqrt{(L_{12} + 2\sqrt{\lambda_1 \lambda_2})(\lambda_7 + 2\sqrt{\lambda_1 \lambda_\chi})(\lambda_8 + 2\sqrt{\lambda_2 \lambda_\chi})} &\geq 0, \end{aligned}$$

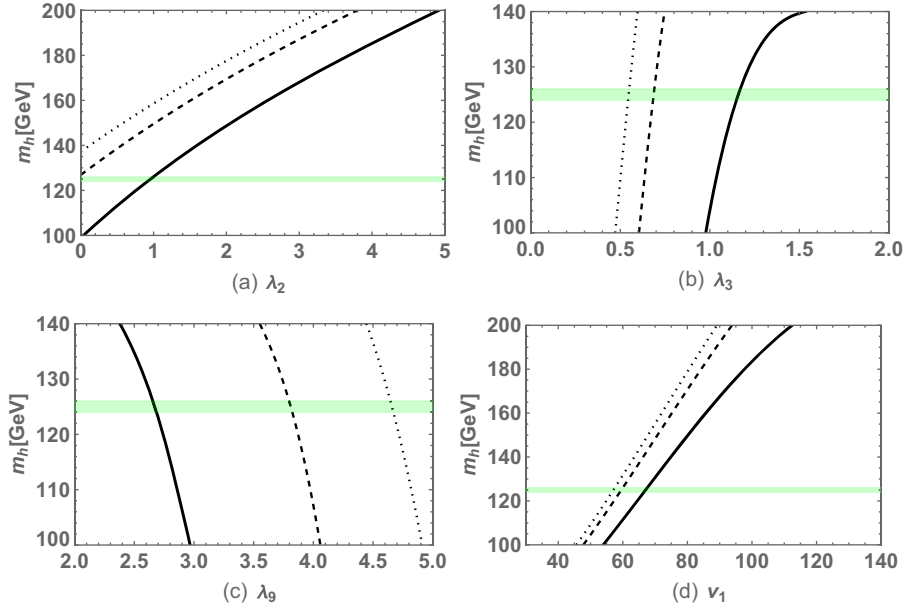


FIG. 4. The plots show m_h versus λ_2 (a), m_h versus λ_9 (b), m_h versus λ_2 (c), and m_h versus v_1 (d), where the solid, dashed, and dotted curves represent the results for $\lambda_\chi = 2, 3, 4$, respectively. The light green areas indicate the range of $124 \text{ GeV} < m_h < 126 \text{ GeV}$.

$$\begin{aligned}
& \sqrt{\lambda_1 \lambda_3 \lambda_\chi} + \frac{L_{13} \sqrt{\lambda_\chi} + \lambda_7 \sqrt{\lambda_3} + \lambda_9 \sqrt{\lambda_1}}{2} + \frac{1}{2} \sqrt{(L_{13} + 2\sqrt{\lambda_1 \lambda_3})(\lambda_7 + 2\sqrt{\lambda_1 \lambda_\chi})(\lambda_9 + 2\sqrt{\lambda_3 \lambda_\chi})} \geq 0, \\
& \sqrt{\lambda_2 \lambda_3 \lambda_\chi} + \frac{L_{23} \sqrt{\lambda_\chi} + \lambda_8 \sqrt{\lambda_3} + \lambda_9 \sqrt{\lambda_2}}{2} + \frac{1}{2} \sqrt{(L_{23} + 2\sqrt{\lambda_2 \lambda_3})(\lambda_8 + 2\sqrt{\lambda_2 \lambda_\chi})(\lambda_9 + 2\sqrt{\lambda_3 \lambda_\chi})} \geq 0, \\
& \sqrt{\lambda_1 \lambda_2 \lambda_3} + \frac{L_{12} \sqrt{\lambda_3} + \tilde{L}_{13} \sqrt{\lambda_2} + \tilde{L}_{23} \sqrt{\lambda_1}}{2} + \frac{1}{2} \sqrt{(L_{12} + 2\sqrt{\lambda_1 \lambda_2})(\tilde{L}_{13} + 2\sqrt{\lambda_1 \lambda_3})(\tilde{L}_{23} + 2\sqrt{\lambda_2 \lambda_3})} \geq 0, \quad (80)
\end{aligned}$$

where

$$\begin{aligned}
L_{12} &= \lambda'_4 + \min(0, \lambda''_4), & L_{13} &= \lambda'_5 + \min(0, \lambda''_5), & L_{23} &= \lambda'_6 + \min(0, \lambda''_6), \\
\tilde{L}_{13} &= L_{13} - |\lambda_{10}| \sqrt{\frac{L_{13} + 2\sqrt{\lambda_1 \lambda_3}}{L_{23} + 2\sqrt{\lambda_2 \lambda_3}}}, \\
\tilde{L}_{23} &= L_{23} - |\lambda_{10}| \sqrt{\frac{L_{23} + 2\sqrt{\lambda_2 \lambda_3}}{L_{13} + 2\sqrt{\lambda_1 \lambda_3}}}. \quad (81)
\end{aligned}$$

For relevant parameters in the SM, we choose [1]

$$\begin{aligned}
\alpha_s(m_Z) &= 0.118, & m_t &= 172.57 \text{ GeV}, & m_c &= 1.273 \text{ GeV}, & m_Z &= 91.1880 \text{ GeV}, \\
\alpha(m_Z) &= 1/127.93, & m_b &= 4.183 \text{ GeV}, & m_\tau &= 1.777 \text{ GeV}, & m_W &= 80.3692 \text{ GeV}. \quad (82)
\end{aligned}$$

In the FDM, the mass of the lightest Higgs boson can be obtained by diagonalizing the squared mass matrix in Eq. (4). Among the three Higgs doublets, Φ_3 is identified as the SM-like Higgs boson. As indicated by Eq. (5), λ_3 exerts a significant influence on m_h . Furthermore, the mixing between S_3 and S_χ implies that both λ_9 and λ_χ

play a crucial role in determining m_h . In our chosen parameter space, v_1 not only affect the Yukawa couplings directly, but also affect the mixing effect of three Higgs doublets. It indicates that v_1 plays a crucial role in determining the numerical results. To investigate the effects of λ_2 , λ_3 , λ_9 , S_χ , and v_1 on m_h , we take

TABLE V. Scanning parameters for scalar potential coefficient on the Higgs decays.

Parameters	Min	Max
v_1/GeV	30	142
λ_i ($i = 2, 3, 9, \chi$)	0	5
λ_{10}	-4	0
v_χ/TeV	4	10

$\lambda_1 = \lambda_2 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = 2$, $\lambda_{10} = -2$, and $\kappa = -1$ TeV and plot m_h versus λ_2 , λ_3 , λ_9 , and S_χ in Figs. 4(a)–4(d), respectively, where the light green zone represents the range of $124 \text{ GeV} < m_h < 126 \text{ GeV}$ and the solid, dashed, and dotted curves denotes the results for $\lambda_\chi = 2, 3, 4$, respectively. These plots show that m_h increases with higher values of λ_3 , λ_χ , λ_2 , and v_1 and decreases with higher values of λ_9 . Furthermore, the parameters λ_2 , λ_3 , λ_9 , v_1 , and λ_χ have a significant impact on the theoretical predictions of m_h .

To explore the combined effects of λ_2 , λ_3 , λ_9 , λ_{10} , λ_χ , and v_1 on the Higgs signal strengths, we fix $\lambda_1 = \lambda_2 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = 2$ and $\kappa = -1$ TeV and scan the parameter space listed in Table V. In addition, in this work, we perform a χ^2 test to explore the best fit describing 125 GeV Higgs mass and the signal strength in the model. Generally, the χ^2 function can be constructed as

$$\chi^2 = \sum_i \left(\frac{O_i^{\text{th}} - O_i^{\text{exp}}}{\sigma_i^{\text{exp}}} \right)^2, \quad (83)$$

where O_i^{th} denotes the i th observable computed theoretically, O_i^{exp} is the corresponding experimental value, and σ_i^{exp} is the uncertainty in O_i^{exp} . O_i^{th} , σ_i^{exp} , and O_i^{exp} are listed in Table VI and Eq. (43). The parameters obtaining the best fit are

$$\begin{aligned} \lambda_2 &= 0.907, & \lambda_3 &= 2.201, & \lambda_9 &= 3.885, \\ \lambda_{10} &= -3.135, & \lambda_\chi &= 4.807, & v_1 &= 128.267, \\ v_\chi &= 7689.94. \end{aligned} \quad (84)$$

TABLE VI. The results obtained for the best-fit point corresponding to $\chi^2 = 3.94$.

Observables	O_i^{th}	O_i^{exp}	σ_i^{exp}
μ_{hZZ^*}	0.988	1.02	0.08
μ_{hWW^*}	0.988	1.00	0.08
$\mu_{h\tau\tau}$	0.984	0.91	0.09
$\mu_{hb\bar{b}}$	0.998	0.99	0.12
$\mu_{h\gamma\gamma}$	1.094	1.10	0.06
$\mu_{hZ\gamma}$	1.0253	2.2	0.7
m_h	125.20	125.20	0.11

Additionally, to explore the mixing effects among the three Higgs doublets, we scan the parameter space in Table V, keeping $\lambda_1 = \lambda_2 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = 2$ and $\kappa = -1$ TeV to examine the variation of $|Z_{h,11}|$, $|Z_{h,12}|$, and $|Z_{h,13}|$ with respect to v_1 in Figs. 5(a)–5(c), respectively, where a green triangle denotes the best-fit point. We take $v_3 > v_{1,2}$ in the chosen parameter space, which indicates that Φ_3 dominates the SM-like Higgs, i.e., $|Z_{h,13}| > |Z_{h,11}| \approx |Z_{h,12}|$. With the increasing of $v_{1,2}$, the contributions from Φ_1 and Φ_2 to the lightest SM-like Higgs play more important roles, and the ones from Φ_1 , Φ_2 , and Φ_3 are comparable when $v_3 \approx v_{1,2} \approx 142.2 \text{ GeV}$. All these facts can be seen obviously in Fig. 5.

Then, based on the points obtained in Fig. 5, we will explore the impact of λ_2 , λ_3 , λ_9 , λ_{10} , λ_χ , v_χ , and v_1 on the signal strength in the following subsections.

A. $h \rightarrow \gamma\gamma, gg, h \rightarrow VV^* (V=Z, W), h \rightarrow f\bar{f} (f=c, b, \tau)$, and $h \rightarrow Z\gamma$

We focus on the precisely measured signal strengths μ_{hVV^*} , $\mu_{h\tau\tau}$, and $\mu_{hZ\gamma}$ first, and the results of μ_{hZZ^*} (a), μ_{hWW^*} (b), $\mu_{h\tau\tau}$ (c), $\mu_{hb\bar{b}}$ (d), $\mu_{hc\bar{c}}$ (e), $\mu_{h\gamma\gamma}$ (f), and $\mu_{hZ\gamma}$ (g) versus v_1 are plotted in Fig. 6, where these plots are obtained by scanning the parameter space listed in Table V and keeping $\lambda_1 = \lambda_2 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = 2$, $\kappa = -1$ TeV, and $124 \text{ GeV} < m_h < 126 \text{ GeV}$. In addition, we employ the conditions of several FCNC processes in Sec. III B and electroweak precision observables in Sec. III C described as additional constraints. All signal

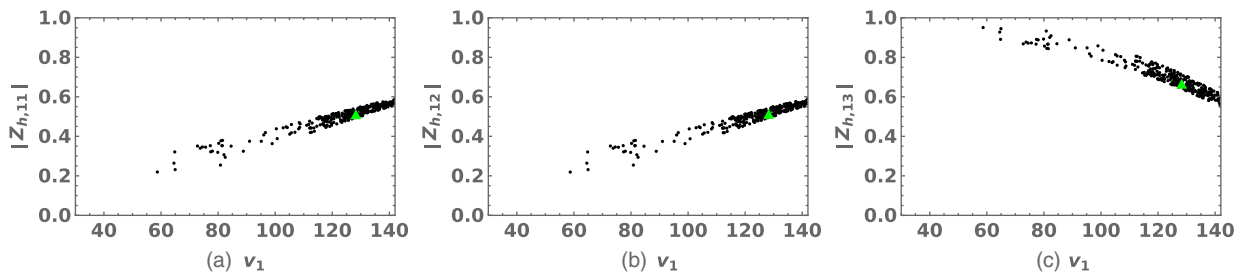


FIG. 5. Scanning the parameter space in Table V keeping $\lambda_1 = \lambda_2 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = 2$ and $\kappa = -1$ TeV, and the results of $v_1 - |Z_{h,11}|$ (a), $v_1 - |Z_{h,12}|$ (b), and $v_1 - |Z_{h,13}|$ (c) are plotted, where the green triangle denotes the best-fit point $\chi^2 = 3.94$.

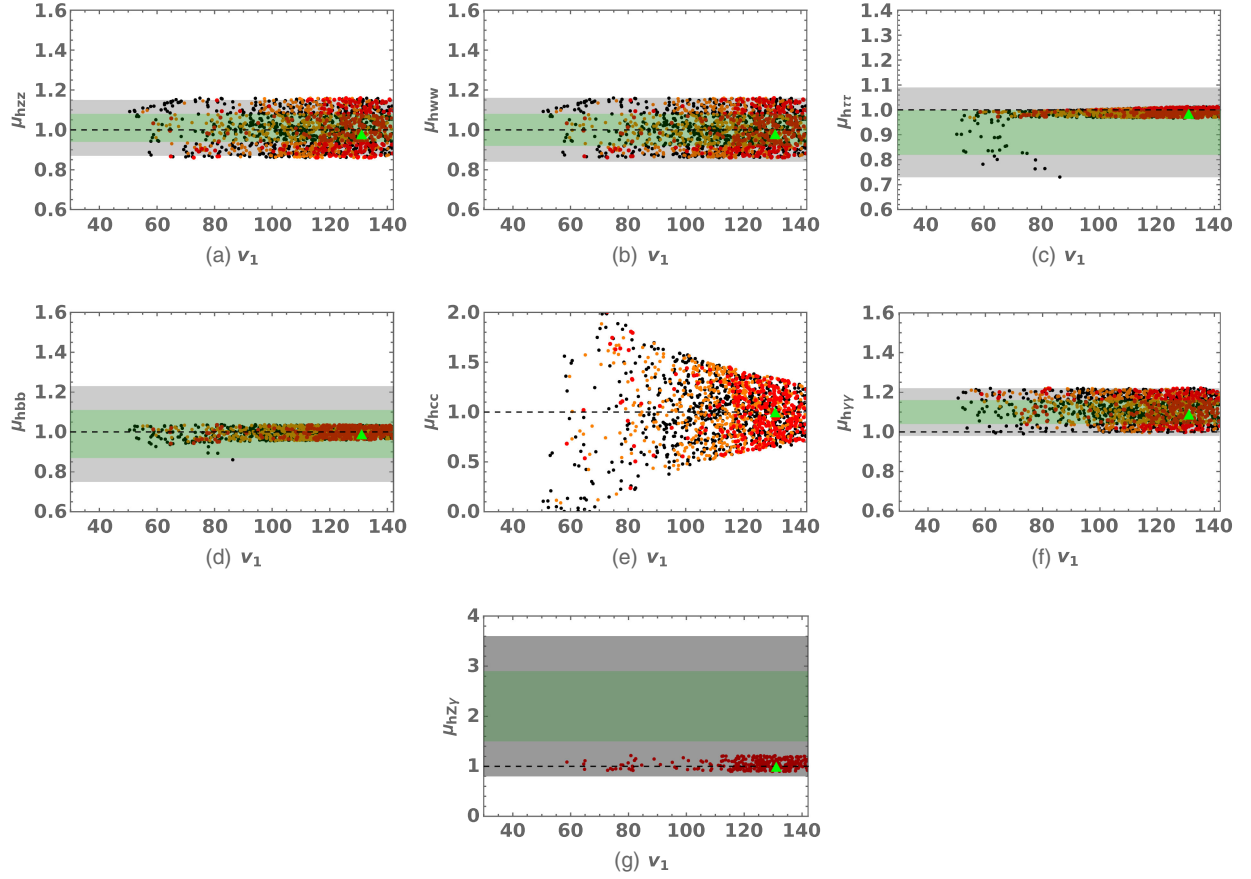


FIG. 6. Keeping $\lambda_1 = \lambda_2 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = 2$ and $\kappa = -1$ TeV and scanning the parameter space in Table V. μ_{hZZ}^* (a), μ_{hWW}^* (b), $\mu_{h\tau\tau}$ (c), μ_{hbb} (d), μ_{hcc} (e), $\mu_{h\gamma\gamma}$ (f), and $\mu_{hZ\gamma}$ (g) versus v_1 are plotted, respectively. The gray areas represent the 2σ confidence interval, the green areas represent the 1σ confidence interval, and the horizontal dashed line represents a signal strength of 1, where the green triangle denotes the best-fit point $\chi^2 = 3.94$.

strengths lie within the 2σ confidence intervals. Figure 6 depicts the 1σ (light green region) and 2σ (gray region) confidence intervals. The dashed line indicates a signal strength of 1. The best-fit point is denoted by the green triangle. The black dots denote points satisfying only the FCNC constraints, the orange dots correspond to points that fulfill both the FCNCs and signal strength constraints, and red dots correspond to the cases that are consistent with all three constraints. Notably, a significant region of parameter space with $v_1 > 60$ GeV accommodates signal strengths consistent with experimental constraints even as $v_3 \approx v_{1,2} \approx 142.2$ GeV. This demonstrates the viability of the chosen parameter space in reproducing the observed data. In the process in Fig. 6(e), due to the Yukawa coupling between Higgs and charm quark with respect to v_1 sensitively, the overall trend shows a noticeable change.

In the best-fit point in Fig. 6(g), the branch ratio is $\text{Br}[h \rightarrow Z\gamma] = (1.56 \pm 0.06) \times 10^{-3}$, and the error in the branch ratio mainly stems from the uncertainty associated with the full width of the Higgs boson. The prediction of the FDM for the branch ratio is notably below the measured

value that our results do not fall within the 1σ confidence interval range. While the FDM prediction differs significantly from experimental results, it is too early to dismiss the SM or FDM. The discrepancy may stem from experimental uncertainties rather than a fundamental flaw in either model. Future, higher-precision measurements from the HL-LHC and potential Higgs factories (CEPC, FCC-ee) are crucial to resolve this issue [88].

B. $h \rightarrow MZ$

To help to analyze the results of the processes $h \rightarrow MZ$, we define the ratio

$$R_M = \frac{\Gamma_{U(1)_F}(h \rightarrow MZ)}{\Gamma_{\text{SM}}(h \rightarrow MZ)}, \quad (85)$$

and the results of the corresponding calculations are listed in Table VII.

Taking the points obtained in Fig. 6, we plot μ_{hMZ} versus R_M in Fig. 7, where $M = \rho, \omega, \phi, J/\psi, \Upsilon(nS)$ for Figs. 7(a)–7(e), respectively. The orange dots in the figure

TABLE VII. The indirect contributions for the decay width of $h \rightarrow MZ$, where $C_{\gamma Z} = C_{\gamma Z}^{\text{SM}} + C_{\gamma Z}^{\text{NP}}$, with $C_{\gamma Z}^{\text{SM}} \simeq -2.43$.

	ρ	ω	ϕ	J/ψ	$\Upsilon(1S)$
$F_{\text{ind},\text{NP}}^{MZ}$	$0.042 + 4.05 \times 10^{-4} C_{\gamma Z}$	$-0.01 + 1.2 \times 10^{-4} C_{\gamma Z}$	$-0.039 - 2 \times 10^{-4} C_{\gamma Z}$	$0.040 + 7.3 \times 10^{-4} C_{\gamma Z}$	$-0.114 - 6 \times 10^{-4} C_{\gamma Z}$
$F_{\text{ind},\text{SM}}^{MZ}$	0.041	-0.01	-0.038	0.039	-0.11
$F_{\text{ind},\text{NP}}^{MZ}$	$0.042 + 1.10 C_{\gamma Z}$	$-0.01 + 0.32 C_{\gamma Z}$	$-0.039 - 0.31 C_{\gamma Z}$	$0.040 + 0.12 C_{\gamma Z}$	$-0.114 - 0.011 C_{\gamma Z}$
$F_{\text{ind},\text{SM}}^{MZ}$	-2.63	-0.79	0.71	-0.26	-0.088
$\frac{8f_M \Gamma_Z}{(1-\Gamma_Z - \Gamma_M)^2}$	7.35×10^{-4}	7.59×10^{-4}	1.29×10^{-3}	1.19×10^{-2}	0.114
$R_M - 1$	$-0.74 + 0.132 \times -2.37 + C_{\gamma Z}^{\text{NP}} ^2$	$-0.013 + 0.135 \times -2.473 + C_{\gamma Z}^{\text{NP}} ^2$	$-0.29 + 0.058 \times -2.24 + C_{\gamma Z}^{\text{NP}} ^2$	$-0.295 + 0.079 \times -1.93 + C_{\gamma Z}^{\text{NP}} ^2$	$-0.172 + 0.001 \times 13.32 + C_{\gamma Z}^{\text{NP}} ^2$

represent points in the range of $(-4, -3)$ for λ_{10} , the blue dots represent points in the range of $(-3, -2)$ for λ_{10} , and the red dots represent points in the range of $(-2, 0)$ for λ_{10} . The green triangle represents the best-fit point. In Fig. 7(a) or 7(c), the graph indicates that, as λ_{10} increases, R_ρ or R_ϕ increases, and the value of $\mu_{h\rho Z}$ or $\mu_{h\phi Z}$ gradually decreases. When λ_{10} is within the range of $(-4, -3)$, $\mu_{h\rho Z}$ and $\mu_{h\phi Z}$ can deviate significantly from the SM, exceeding 20%. In Fig. 7(b), the distribution of $\mu_{h\omega Z}$ and R_ω is very similar to those in Fig. 7(a). The calculation in Table VII shows that the only variable in the change process of $R_M - 1$ for these two processes is $C_{\gamma Z}^{\text{NP}}$, with coefficients of 0.132 and 0.135 which differ very slightly, indicating that the small difference between R_ρ and R_ω . For a similar reason, the results in Fig. 7(d) are similar to those in Fig. 7(c). Figure 7(e) shows $R_{\Upsilon(1S)}$ versus $\mu_{h\Upsilon(1S)Z}$. In Table VII, $\frac{F_{\text{ind},\text{NP}}^{MZ}}{F_{\text{ind},\text{SM}}^{MZ}} \approx 1$, so the ratio of $\frac{F_{\text{ind},\text{NP}}^{MZ}}{F_{\text{ind},\text{SM}}^{MZ}}$ is the main influencing factor of $R_{\Upsilon(1S)}$. The value of $R_{\Upsilon(1S)}$ approaching 1 indicates that the values of FDM and SM are almost

the same. This is because the quality of $m_{\Upsilon(1S)}$ and the value of f_Υ are relatively high, leading to a significant contribution from SM.

Figures 7(a)–7(e) show obviously that λ_{10} affects all results of μ_{hMZ} significantly, because the dominant contributions to these processes in the R_M come from the charged Higgs, and λ_{10} affects the lightest charged Higgs mass $M_{H_1^\pm}$ acutely as shown in Fig. 7(f).

C. $h \rightarrow M\gamma$

The parameter space, scanned using data points from Fig. 6, is marked by a green triangle indicating the best-fit point from previous analysis and a dashed line representing the contour where the signal strength equals 1. Figures 8(a)–8(c) exhibit signal strengths that deviate by 40% from SM predictions. This deviation arises from the indirect contributions of the new scalar sector and strongly suggests the potential presence of NP. Figure 8(d) shows a relatively small deviation (up to 20%) in $\mu_{J/\psi\gamma}$ due to less significant cancellation between direct and indirect

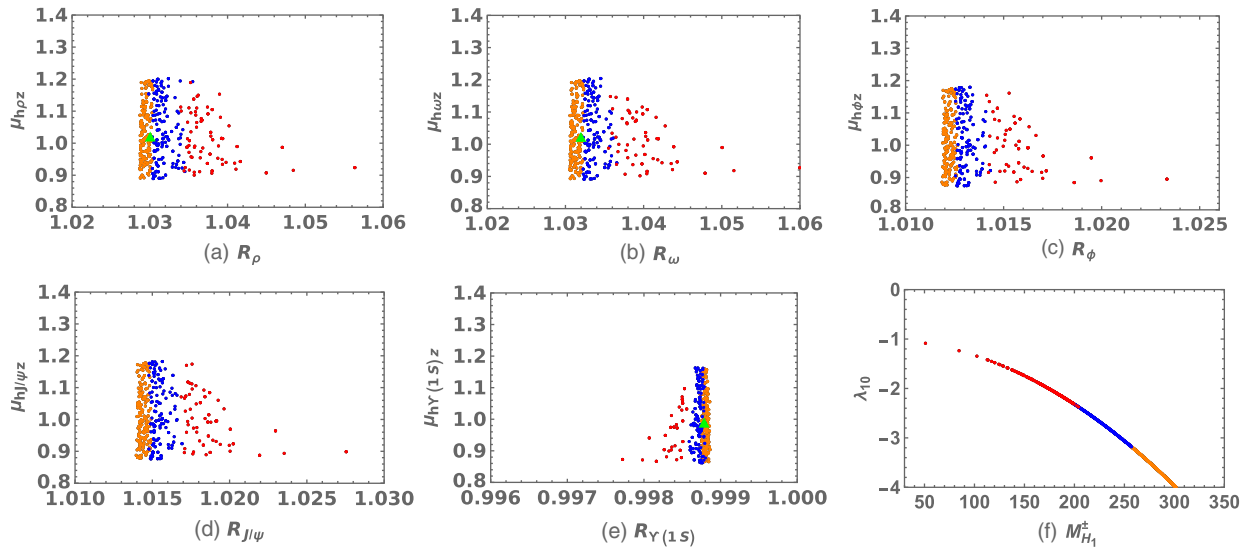


FIG. 7. Using the points obtained in Fig. 6, μ_{hMZ} versus R_M are plotted, where $M = \rho, \omega, \phi, J/\psi, \Upsilon(nS)$ for (a)–(e), respectively. And the relationship between λ_{10} and $M_{H_1^\pm}^2$ is plotted (f), where the orange dots in the figure represent points in the range of ≤ 1.03 for R_ρ , the blue dots in the figure represent points in the range of $(1.03, 1.33)$ for R_ρ , and the red dots in the figure represent points in the range of ≥ 1.033 for R_ρ . The green triangles represent the best-fit point $\chi^2 = 3.94$.

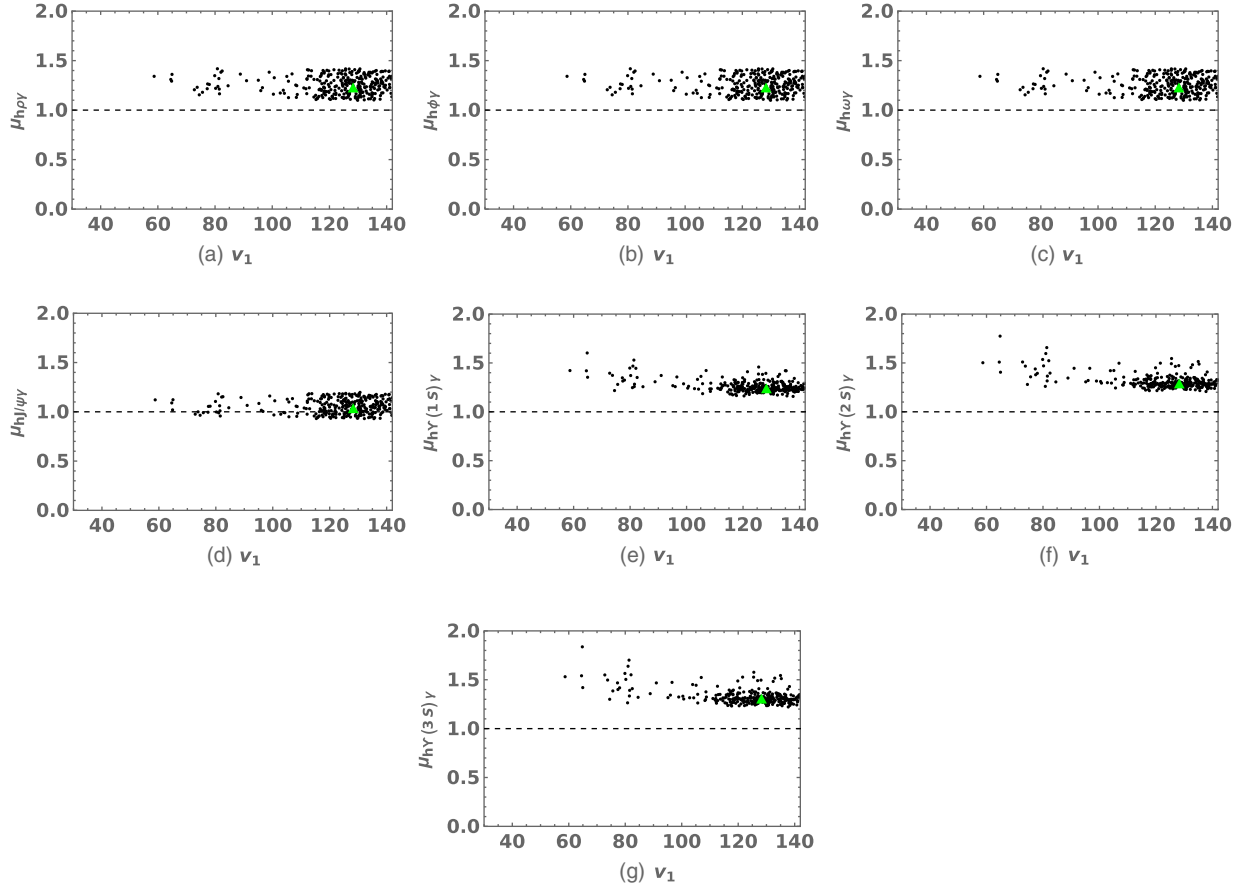


FIG. 8. Scanning the parameter space in Table V, and the results of $\mu_{\rho\gamma} - v_1$ (a), $\mu_{\phi\gamma} - v_1$ (b), $\mu_{\omega\gamma} - v_1$ (c), $\mu_{J/\psi\gamma} - v_1$ (d), and $\mu_{\Upsilon(nS)\gamma} - v_1$ (e)–(g) are plotted, and the green triangles represent the best-fit point $\chi^2 = 3.94$.

contributions. Conversely, Figs. 8(e)–8(g) show substantial enhancements (up to 50%) in $\mu_{h\Upsilon(nS)\gamma}$ signal strengths, despite the large masses and small branching ratios. This sensitivity of SM predictions to the interplay of direct and indirect contributions provides a valuable opportunity to probe for NP.

The numerical results for m_M and $\Gamma(M \rightarrow \ell^+ \ell^-)$ are taken from Table IV. The amplitudes of the direct and indirect contributions are presented in Table VIII, and the decay widths and branching ratios are further computed and summarized in Table IX. These results are consistent with the experimental values given in Eq. (77), with all parameters determined based on the best-fit point obtained

TABLE VIII. Numerical results for the amplitudes A_{ind} and A_{dir} .

M	$A_{\text{ind}} \times 10^5 (\text{GeV}^{1/2})$	$A_{\text{dir}} \times 10^5 (\text{GeV}^{1/2})$
J/ψ	-11.81	$0.6295 + 0.06488i$
$\Upsilon(1S)$	3.26	$-2.6688 - 0.2857i$
$\Upsilon(2S)$	2.178	$-1.8941 - 0.1968i$
$\Upsilon(3S)$	1.825	$-1.614 - 0.165i$

from prior analysis. For $h \rightarrow J/\psi\gamma$, A_{ind} is larger by an order of magnitude compared to A_{dir} . For $\Gamma_{\Upsilon(nS)+\gamma}$ ($n = 1, 2, 3$), as n increases, the absolute value of A_{dir} gets closer and closer to A_{ind} ; this leads to the corresponding decay width rapidly decreasing.

We calculate $\Gamma(h \rightarrow M + \gamma)$ through $|\kappa_q A_{\text{dir}} + A_{\text{ind}}|^2$ to study the deviation of the Yukawa coupling κ_q from the SM; the $c\bar{c}\gamma$ and $b\bar{b}\gamma$ vertexes are highly constrained by QED, so typically modifications to the direct contribution amplitude can be made only by modifying $hc\bar{c}$ and $hb\bar{b}$ vertexes. As shown in Fig. 9(a), as κ_c increases from -4 to 4, the decay width of $\Gamma_{J/\psi+\gamma}$ decreases gradually, which helps us narrow down the range of κ_c . The FDM decay width is larger ($\kappa_c < 1$) or smaller ($\kappa_c > 1$) than the SM's, primarily due to the dominance of $|A_{\text{ind}}|$ over $|A_{\text{dir}}|$ and their opposite signs. However, direct contributions are negligible ($\approx 5\%$) in the $h \rightarrow J/\Psi + \gamma$ process, resulting in a near-linear relationship. Figure 9(b) shows the $h \rightarrow \Upsilon(nS)\gamma$ decay width as a concave function of κ_b (for all nS), with a minimum near $\kappa_b = 1$. This concavity facilitates experimental constraints on κ_b . Significant cancellation between direct and indirect contributions occurs in the $\Gamma_{\Upsilon(3S)+\gamma}$ process, particularly in the SM ($\kappa_b = 1$). This

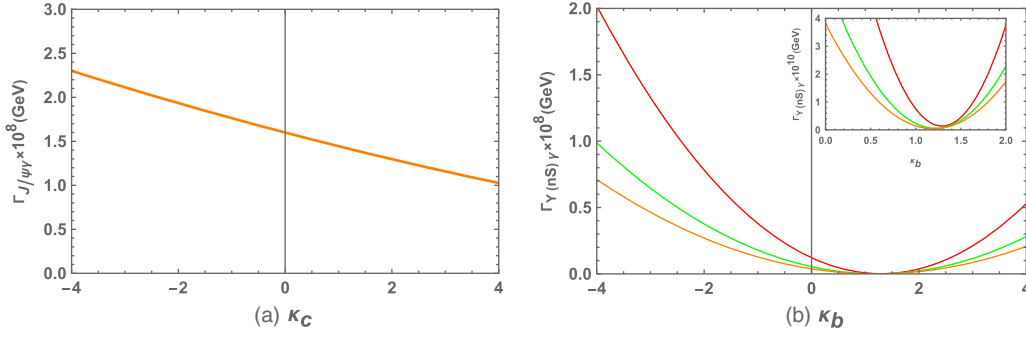


FIG. 9. The decay rate $\Gamma_{J/\Psi+\gamma} = \Gamma(h \rightarrow J/\Psi + \gamma)$ is determined by scaling the Higgs charm coupling with a factor κ_c and is shown in (a), while the decay rates $\Gamma_{\Upsilon(nS)+\gamma} = \Gamma(h \rightarrow \Upsilon(nS) + \gamma)$ for $n = 1, 2, 3$ are computed by scaling the Higgs bottom coupling with a factor κ_b , represented by the red, green, and orange solid lines respectively, and are shown in (b). In the SM prediction, we have $\kappa_c = \kappa_b = 1$.

TABLE IX. Decay rates and branching ratios for $h \rightarrow M + \gamma$.

M	$\Gamma_{\text{SM}}(h \rightarrow M + \gamma)$ (GeV)	$\text{Br}_{\text{SM}}(h \rightarrow M + \gamma)$	$\Gamma_{\text{NP}}(h \rightarrow M + \gamma)$ (GeV)	$\text{Br}_{\text{NP}}(h \rightarrow M + \gamma)$
J/ψ	1.253×10^{-8}	3.047×10^{-6}	1.445×10^{-8}	3.45×10^{-6}
$\Upsilon(1S)$	4.316×10^{-11}	1.049×10^{-8}	7.54×10^{-11}	1.80×10^{-8}
$\Upsilon(2S)$	1.193×10^{-11}	2.90×10^{-9}	2.292×10^{-11}	5.47×10^{-9}
$\Upsilon(3S)$	7.123×10^{-12}	1.73×10^{-9}	1.413×10^{-11}	3.37×10^{-9}

cancellation reduces the decay width, increasing experimental challenges. However, the FDM predicts a larger decay width than the SM, improving its experimental prospects.

Table IX (calculated at $\kappa_q = 1$) shows that the FDM increases both decay widths and branching ratios for all four processes, with some branching ratios nearly doubled. The consistency of these branching ratios with experimental data strongly suggests the presence of NP.

V. CONCLUSION

In the framework of the FDM, the redefined scalar sector can have a significant impact on the properties of the 125 GeV Higgs boson, and, in order to explain the fermion mass hierarchy problem, the FDM also redefines the Yukawa couplings, which will impact the couplings between the Higgs and fermions. Under stringent experimental constraints on the signal strengths $\mu_{\gamma\gamma, Z\gamma}$, μ_{VV^*} ($V = Z, W$), and $\mu_{f\bar{f}}$ ($f = b, c, \tau$) in the scalar sector, $\bar{B} \rightarrow X_s \gamma$ and $B_s^0 \rightarrow \mu^+ \mu^-$ in the B meson sector, $t \rightarrow ch$ and $t \rightarrow uh$ in the top quark sector, the charged lepton flavor violation decays $\tau \rightarrow 3e$, $\tau \rightarrow 3\mu$, and $\mu \rightarrow 3e$, as well as the S, T, U parameters, the properties of the Higgs boson are determined via fits to these observables. Our results confirm that the FDM can explain the experimentally observed signal strengths, FCNCs, and the electroweak precision observables and identify the appropriate parameter space.

We present an in-depth analysis of $h \rightarrow MZ$ and $h \rightarrow M\gamma$, where M is vector mesons [$\rho, \omega, \phi, J/\psi, \Upsilon(nS)$]. Our findings indicate an inverse relationship between λ_{10} and R_M , mediated by the lightest charged Higgs boson mass: Decreasing λ_{10} and increasing the lightest charged Higgs boson mass result in a smaller R_M . At low values of R_M , μ_{hMZ} ($M = \rho, \omega, \phi, J/\psi$) exhibits a deviation of up to 20% above the SM; however, $\mu_{h\Upsilon(nS)Z}$ and $R_{\Upsilon(nS)}$ have an inverse relationship. These provide reference value for probing at the possible future high-energy colliders. The FDM partially reduces the cancellation between direct and indirect contributions of $h \rightarrow M\gamma$ processes in the SM, the signal strength can exceed the SM by up to 50%, and the strength of the Higgs effective couplings also show a significant impact on the related decay widths. In addition, the parameter $v_{1,2}$ significantly impacts theoretical predictions of signal strengths, primarily by altering the mixing strength of the three Higgs doublet fields and other Yukawa couplings.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: FORM FACTORS

The form factors are

$$\begin{aligned} A_0(x) &= -(x - g(x))/x^2, \\ A_{1/2}(x) &= 2[x + (x - 1)g(x)]/x^2, \\ A_1(x) &= -[2x^2 + 3x + 3(2x - 1)g(x)]/x^2, \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} F_{sf}(\tau) &= \tau^{-1}[1 + (1 - \tau^{-1})g(\tau)], \\ F_{pf}(\tau) &= \tau^{-1}g(\tau), \end{aligned} \quad (\text{A2})$$

with

$$\begin{aligned} g(x) &= \begin{cases} \arcsin^2 \sqrt{x}, & \text{if } x \leq 1, \\ -\frac{1}{4} \left[\ln \left(\frac{1 + \sqrt{1-1/x}}{1 - \sqrt{1-1/x}} \right) - i\pi \right]^2, & \text{if } x > 1, \end{cases} \\ f(x) &= x(1 - x)g'(x), \end{aligned} \quad (\text{A3})$$

and

$$\begin{aligned} F(x) &= -(1 - x^2) \left(\frac{47}{2}x^2 - \frac{13}{2} + \frac{1}{x^2} \right) - 3(1 - 6x^2 + 4x^4) \ln x \\ &\quad + \frac{3(1 - 8x^2 + 20x^4)}{\sqrt{4x^2 - 1}} \cos^{-1} \left(\frac{3x^2 - 1}{2x^3} \right), \end{aligned} \quad (\text{A4})$$

$$m_Q(\mu_Q^2) \rightarrow 0: \begin{cases} \text{Re } C_{sf} \rightarrow -\frac{1}{18}[\log^2(4x) - \pi^2] - \frac{2}{3}\log(4x) + 2\log \frac{\mu_Q^2}{m_Q^2}, \mu_Q = M_h/2 \\ \text{Im } C_{sf} \rightarrow \frac{\pi}{3} \left[\frac{1}{3}\log(4x) + 2 \right], \end{cases} \quad (\text{A5})$$

$$\begin{aligned} A_f(\tau, x) &= \frac{3\tau}{2(1-x)} \left\{ 1 - \frac{2x}{1-x} [f(\tau) - f(\tau/x)] + \left(1 - \frac{\tau}{1-x} \right) [g(\tau) - g(\tau/x)] \right\}, \\ B_f(\tau, x) &= \frac{\tau}{1-x} [g(\tau) - g(\tau/x)], \\ A_W^{\gamma\gamma}(\tau, x) &= \frac{2+3\tau}{1-x} \left\{ 1 - \frac{2x}{1-x} [f(\tau) - f(\tau/x)] \right\} + \frac{3\tau}{(1-x)^2} \left(2 - \tau - \frac{8x}{3} \right) [g(\tau) - g(\tau/x)], \\ A_W^{\gamma Z}(\tau, x) &= \frac{1}{1-x} \left[1 - 2s_W^2 + \left(\frac{5}{2} - 3s_W^2 \right) \tau \right] \left\{ 1 - \frac{2x}{1-x} [f(\tau) - f(\tau/x)] \right\} \\ &\quad + \frac{\tau}{(1-x)^2} \left[\left(\frac{5}{2} - 3s_W^2 \right) (2 - \tau) - 2x(3 - 4s_W^2) \right] [g(\tau) - g(\tau/x)], \end{aligned} \quad (\text{A6})$$

$$\begin{aligned}
A_{1/2}(x, \lambda) &= I_1(x, \lambda) - I_2(x, \lambda), \\
A_1(x, \lambda) &= c_W \left\{ 4 \left(3 - \frac{s_W^2}{c_W^2} \right) I_2(x, \lambda) + \left[\left(1 + \frac{2}{x} \right) \frac{s_W^2}{c_W^2} - \left(5 + \frac{2}{x} \right) \right] I_1(x, \lambda) \right\}, \\
A_0(x, \lambda) &= I_1(x, \lambda), \\
I_1(x, \lambda) &= \frac{x\lambda}{2(x-\lambda)} + \frac{x^2\lambda^2}{2(x-\lambda)^2} [f(x^{-1}) - f(\lambda^{-1})] + \frac{x^2\lambda}{(x-\lambda)^2} [g(x^{-1}) - g(\lambda^{-1})], \\
I_2(x, \lambda) &= \frac{x\lambda}{2(x-\lambda)} [f(x^{-1}) - f(\lambda^{-1})].
\end{aligned} \tag{A7}$$

APPENDIX B: THE RELATED COUPLING VERTEXES

The related coupling vertexes are

$$C_{hW_\sigma^+ W_\mu^-} = \frac{i}{2} g_2^2 (v_{\phi_1} Z_{i1}^H + v_{\phi_2} Z_{i2}^H + v_{\phi_3} Z_{i3}^H) (g_{\sigma\mu}) \quad (i = 1), \tag{B1}$$

$$\begin{aligned}
C_{hZ_\sigma Z_\mu} &= \frac{i}{2} (v_{\phi_1} (-(2g_F + g_{YF}) \sin \theta'_W + (2g_{FY} + g_1) \cos \theta'_W \sin \theta_W + g_2 \cos \theta_W \cos \theta'_W)^2 Z_{i1}^H \\
&\quad + v_{\phi_2} (-(2g_F + g_{YF}) \sin \theta'_W + (-2g_{FY} + g_1) \cos \theta'_W \sin \theta_W + g_2 \cos \theta_W \cos \theta'_W)^2 Z_{i2}^H \\
&\quad + g_2^2 v_{\phi_3} \cos^2 \theta_W \cos^2 \theta'_W Z_{i3}^H + g_1^2 v_{\phi_3} \cos^2 \theta_W \sin^2 \theta_W Z_{i3}^H \\
&\quad + g_1 g_2 v_{\phi_3} \cos^2 \theta'_W \sin 2\theta_W Z_{i3}^H - 2g_{YF} g_2 v_{\phi_3} \cos \theta_W \cos \theta'_W \sin \theta'_W Z_{i3}^H \\
&\quad - 2g_1 g_{YF} v_{\phi_3} \cos \theta'_W \sin \theta_W \sin \theta'_W Z_{i3}^H + g_{YF}^2 v_{\phi_3} \sin^2 \theta'_W Z_{i3}^H \\
&\quad + 16g_F^2 v_\chi v_\lambda \cos^2 \theta'_W \sin^2 \theta_W Z_{i4}^H - 32g_{FY} v_\chi \cos \theta'_W \sin \theta_W \sin \theta'_W Z_{i4}^H \\
&\quad + 16g_F^2 v_\chi \sin^2 \theta'_W Z_{i4}^H) (g_{\sigma\mu}) \quad (i = 1, \sin \theta'_W = 0, \cos \theta'_W = 1),
\end{aligned} \tag{B2}$$

$$\begin{aligned}
C_{hd_j \bar{d}_k} &= -i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} (U_{R,i2}^{d,*} (Y_{d12} U_{L,j1}^{d,*} Z_{k3}^H + Y_{d32} U_{L,j3}^{d,*} Z_{k2}^H) + U_{R,i1}^{d,*} (Y_{d21} U_{L,j2}^{d,*} Z_{k3}^H + Y_{d31} U_{L,j3}^{d,*} Z_{k1}^H) \\
&\quad + U_{R,i3}^{d,*} (Y_{d13} U_{L,j1}^{d,*} Z_{k1}^H + Y_{d23} U_{L,j2}^{d,*} Z_{k2}^H + Y_{d33} U_{L,j3}^{d,*} Z_{k3}^H)) \left(\frac{1 - \gamma_5}{2} \right) \\
&\quad + i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} (Y_{d13}^* U_{R,j3}^d U_{L,i1}^d Z_{k1}^H + Y_{d31}^* U_{R,j1}^d U_{L,i3}^d Z_{k1}^H + Y_{d23}^* U_{R,j3}^d U_{L,i2}^d Z_{k2}^H + Y_{d32}^* U_{R,j2}^d U_{L,i3}^d Z_{k2}^H \\
&\quad + Y_{d12}^* U_{R,j2}^d U_{L,i1}^d Z_{k3}^H + Y_{d21}^* U_{R,j1}^d U_{L,i2}^d Z_{k3}^H + Y_{d33}^* U_{R,j3}^d U_{L,i3}^d Z_{k3}^H) \left(\frac{1 + \gamma_5}{2} \right) \quad (i = 1),
\end{aligned} \tag{B3}$$

$$\begin{aligned}
C_{hu_j \bar{u}_k} &= -i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} (U_{R,i2}^{u,*} (Y_{u12} U_{L,j1}^{u,*} Z_{k3}^H + Y_{u32} U_{L,j3}^{u,*} Z_{k1}^H) + U_{R,i1}^{u,*} (Y_{u21} U_{L,j2}^{u,*} Z_{k3}^H + Y_{u31} U_{L,j3}^{u,*} Z_{k2}^H) \\
&\quad + U_{R,i3}^{u,*} (Y_{u13} U_{L,j1}^{u,*} Z_{k2}^H + Y_{u23} U_{L,j2}^{u,*} Z_{k1}^H + Y_{u33} U_{L,j3}^{u,*} Z_{k3}^H)) \left(\frac{1 - \gamma_5}{2} \right) \\
&\quad + i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} (Y_{u23}^* U_{R,j3}^u U_{L,i2}^u Z_{k1}^H + Y_{u32}^* U_{R,j2}^u U_{L,i3}^u Z_{k1}^H + Y_{u13}^* U_{R,j3}^u U_{L,i1}^u Z_{k2}^H + Y_{u31}^* U_{R,j1}^u U_{L,i3}^u Z_{k2}^H \\
&\quad + Y_{u12}^* U_{R,j2}^u U_{L,i1}^u Z_{k3}^H + Y_{u21}^* U_{R,j1}^u U_{L,i2}^u Z_{k3}^H + Y_{u33}^* U_{R,j3}^u U_{L,i3}^u Z_{k3}^H) \left(\frac{1 + \gamma_5}{2} \right) \quad (i = 1),
\end{aligned} \tag{B4}$$

$$\begin{aligned}
C_{e_i \bar{e}_j h} = & -i \frac{1}{\sqrt{2}} (U_{R,12}^{e,*} (Y_{e12} U_{L,j1}^{e,*} Z_{k3}^H + Y_{e32} U_{L,j3}^{e,*} Z_{k2}^H) + U_{R,11}^{e,*} (Y_{e21} U_{L,j2}^{e,*} Z_{k3}^H + Y_{e31} U_{L,j3}^{e,*} Z_{k1}^H) \\
& + U_{R,i3}^{e,*} (Y_{e13} U_{L,j1}^{e,*} Z_{k1}^H + Y_{e23} U_{L,j2}^{e,*} Z_{k2}^H + Y_{e33} U_{L,j3}^{e,*} Z_{k3}^H)) \left(\frac{1-\gamma_5}{2} \right) \\
& + i \frac{1}{\sqrt{2}} (Y_{e13}^* U_{R,j3}^e U_{L,i1}^e Z_{k1}^H + Y_{e31}^* U_{R,j1}^e U_{L,i3}^e Z_{k1}^H + Y_{e23}^* U_{R,j3}^e U_{L,i2}^e Z_{k2}^H + Y_{e32}^* U_{R,j2}^e U_{L,i3}^e Z_{k2}^H \\
& + Y_{e12}^* U_{R,j2}^e U_{L,i1}^e Z_{k3}^H + Y_{e21}^* U_{R,j1}^e U_{L,i2}^e Z_{k3}^H + Y_{e33}^* U_{R,j3}^e U_{L,i3}^e Z_{k3}^H) \left(\frac{1+\gamma_5}{2} \right) \quad (i = 1), \quad (B5)
\end{aligned}$$

$$\begin{aligned}
C_{hH_j^+ H_k^-} = & \frac{i}{2} (-2\lambda'_5 v_{\phi_3} Z_{i3}^H Z_{j1}^+ Z_{k1}^+ - 2\lambda_7 v_\chi Z_{i4}^H Z_{j1}^+ Z_{k1}^+ - \sqrt{2}\kappa Z_{i4}^H Z_{j2}^+ Z_{k1}^+ \\
& - \lambda''_5 v_{\phi_1} Z_{i3}^H Z_{j3}^+ Z_{k1}^+ - v_{\phi_2} \lambda_{10}^* Z_{i3}^H Z_{j3}^+ Z_{k1}^+ - \sqrt{2}\kappa^* Z_{i4}^H Z_{j1}^+ Z_{k2}^+ \\
& - 2\lambda'_6 v_{\phi_3} Z_{i3}^H Z_{j2}^+ Z_{k2}^+ - 2\lambda_8 v_\chi Z_{i4}^H Z_{j2}^+ Z_{k2}^+ - \lambda''_6 v_{\phi_2} Z_{i3}^H Z_{j3}^+ Z_{k2}^+ \\
& - v_{\phi_1} \lambda_{10}^* Z_{i3}^H Z_{j3}^+ Z_{k2}^+ - \lambda''_5 v_{\phi_1} Z_{i3}^H Z_{j1}^+ Z_{k3}^+ - \lambda_{10} v_{\phi_2} Z_{i3}^H Z_{j1}^+ Z_{k3}^+ \\
& - \lambda_{10} v_{\phi_1} Z_{i3}^H Z_{j2}^+ Z_{k3}^+ - \lambda''_6 v_{\phi_2} Z_{i3}^H Z_{j2}^+ Z_{k3}^+ - 4\lambda_3 v_{\phi_3} Z_{i3}^H Z_{j3}^+ Z_{k3}^+ - 2\lambda_9 v_\chi Z_{i4}^H Z_{j3}^+ Z_{k3}^+ \\
& - Z_{i1}^H (Z_{j3}^+ (2\lambda'_5 v_{\phi_1} Z_{k3}^+ + \lambda''_5 v_{\phi_3} Z_{k1}^+ + v_{\phi_3} \lambda_{10}^* Z_{k2}^+) \\
& + Z_{j2}^+ (2\lambda'_4 v_{\phi_1} Z_{k2}^+ + \lambda_{10} v_{\phi_3} Z_{k3}^+ + \lambda''_4 v_{\phi_2} Z_{k1}^+) \\
& + Z_{j1}^+ (4\lambda_1 v_{\phi_1} Z_{k1}^+ + \lambda''_4 v_{\phi_2} Z_{k2}^+ + \lambda''_5 v_{\phi_3} Z_{k3}^+)) \\
& - Z_{i2}^H (Z_{j3}^+ (2\lambda'_6 v_{\phi_2} Z_{k3}^+ + \lambda''_6 v_{\phi_3} Z_{k2}^+ + v_{\phi_3} \lambda_{10}^* Z_{k1}^+)) \quad (i = 1). \quad (B6)
\end{aligned}$$

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