

$D \rightarrow \pi$ transitions from QCD light-cone sum rules with the chiral currentsDi Gao^{1,2,3,*} Jiangshan Lan^{2,5,6,†} Duojie Jia^{4,7,‡} Xingbo Zhao^{2,5,6,§} and Yanjun Sun^{1,7,||}¹*Institute of Theoretical Physics, College of Physics and Electronic Engineering,
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We present a systematic study of the transition form factors for the semileptonic decays $D \rightarrow Pe^+v_e$ and $D \rightarrow P\mu^+\bar{\nu}_\mu$ using QCD light-cone sum rules with chiral currents, with emphasis on the nonperturbative structure of the pion. The distribution amplitudes of the meson π , including twist-2 and higher-twist components, are analyzed and incorporated in our computation, employing both Gegenbauer polynomial expansions and the basis light-front quantization method, where the latter yields a distribution that rapidly converges to the asymptotic form. Two relations linking transition form factors are derived and found to be consistent with the chiral symmetry of QCD. Our numerical predictions for the D -to-pion form factors agree well with BESIII measurements and lattice QCD calculations. Furthermore, the differential decay widths obtained thereby for the D -to-pion decay also show consistency with the BESIII data in the high-momentum-transfer (q) region with the $q^2 > 1 \text{ GeV}^2$, implying new opportunities for the precise extraction of the Cabibbo-Kabayashi-Maskawa matrix element and the exploration of physics beyond the standard model.

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I. INTRODUCTION

Heavy-to-light decays, encompassing both inclusive [1–4] and exclusive [5–8] processes, provide substantial insights into the dynamics of weak and strong interactions, and play a unique role in extracting the Cabibbo-Kabayashi-Maskawa (CKM) matrix elements (e.g., $|V_{cd}|$) and probing phenomena beyond the standard model. Generally, the inclusive decays of heavy mesons are more challenging experimentally but better understood theoretically than exclusive decays. In contrast, exclusive decays offer higher experimental precision but pose significant theoretical

challenges for nonperturbative methods to compute form factors (FFs) that encode long-distance dynamics.

On the one hand, recent high-statistics measurements of branching fractions and form factors for D/B -meson decays to pseudoscalar mesons, such as those from the Beijing Spectrometer (BESIII) [9–17], have significantly improved precision. On the other hand, the heavy-to-light decays, including leptonic, semileptonic, and nonleptonic modes, have been analyzed by using lattice quantum chromodynamics (LQCD) [18–31], quark model [32–37], QCD sum rule [38–41], heavy quark symmetry, and heavy quark effective field theory (HQEFT) [42–44]. These theoretical efforts have significantly promoted the calculation of the corresponding form factors.

The purpose of this work is to explore the FFs for the semileptonic decays $D \rightarrow P$ using the light-cone QCD sum rule (LCSR), where the relevant distribution amplitudes (DAs) are obtained from the approach of the basis light-front quantization (BLFQ). Our computed DAs are compared to other theoretical predictions. Next, we analyze the semileptonic decay process and reveal simple relations among the FFs for the $D \rightarrow \pi$ transition. Finally, we present

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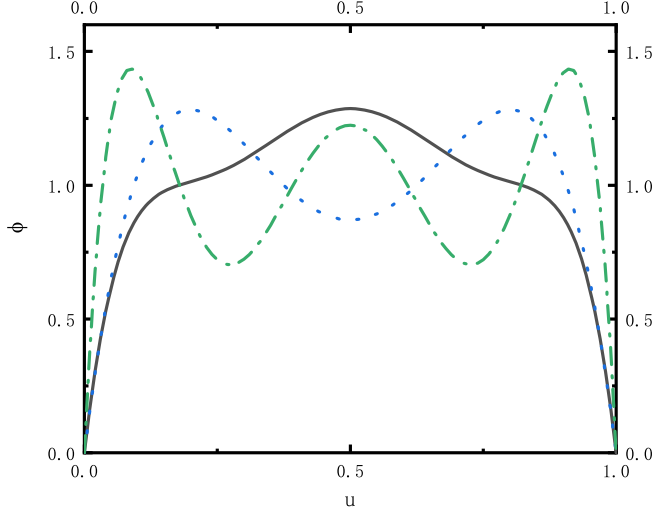


FIG. 1. The distribution amplitudes via Gegenbauer expansions for pions. The solid, dot, and dash dot lines correspond to $\{a_{2,4}\} = \{0.17, 0.06\}$ [45], $a_2 = 0.28$ [46], and $\{a_{2,4}\} = \{0.41, 0.23\}$ [40,41,47,48], respectively.

the differential decay width and make a direct comparison of them with the experimental data reported by BESIII [9].

As illustrated in Fig. 1, many studies adopt a Gegenbauer expansion for the DA truncated at a finite order (typically a_2 or a_4). This approach naturally prompts two critical questions: (a) whether the expansion converges at the chosen truncation order, and (b) whether the extracted coefficients are biased by method-specific parameters (e.g., the Borel parameter and continuum threshold in QCD sum rules). To address these questions, we employ the BLFQ method as a model-independent benchmark. By reconstructing the Gegenbauer coefficients from the BLFQ-derived DA, we observe excellent consistency between the reconstructed and reference DAs, confirming the robustness of the Gegenbauer expansion. This indicates that the variations among previous predictions stem primarily from parameter choices rather than the expansion

framework itself. We discuss this point further in Sec. II.

The paper is organized as follows: Sec. II presents a comparison of the light-cone distribution amplitudes (LCDAs) for the pseudoscalar meson π based on various theoretical predictions. In Sec. III, we derive the form factors and differential branching fractions for the $D \rightarrow P l \bar{\nu}_l$ semileptonic decays using the LCSR method. Section IV is devoted to the numerical analysis, where we compare our results with other theoretical predictions and experimental data. The paper concludes with a summary in Sec. V.

II. THE DISTRIBUTION AMPLITUDE OF PION

Understanding the intrinsic structure of pions is crucial in exploring the low-energy QCD. Composed of valence up and down quarks and antiquarks, pions are one of Nature's most fundamental particles, known as pseudo-Nambu-Goldstone bosons. They are intrinsically linked to chiral symmetry breaking and QCD bound states in strong interactions. Furthermore, since pions possess leptonlike masses, studying them may shed light on the nonperturbative mechanisms behind the generation of hadron masses. Therefore, elucidating the pion DA, which describes the probability amplitude of finding the pion in a quark-antiquark Fock state on the light cone, is of great significance in subatomic physics. In this section, we introduce the DAs calculated via both conventional and innovative approaches. Thereby, we compare the DA directly obtained from the BLFQ method with the DA expressed in terms of Gegenbauer polynomials (derived from the BLFQ method by assuming it represents the true physical DA and using the definition of Gegenbauer moments) to verify the convergence of the Gegenbauer expansion for the pion.

A. The DAs as a sum of Gegenbauer polynomial

The light-cone distribution amplitudes of the pseudo-scalar meson π made up of one light quark q_1 and light antiquark $\bar{q}_2$, have the form of [49–51]

$$\begin{aligned}
 \langle P(p) | \bar{q}_2(x) \gamma_\mu \gamma_5 q_1(0) | 0 \rangle &= -i p_\mu f_P \int_0^1 du e^{iup \cdot x} \phi_P(u), \\
 \langle P(p) | \bar{q}_2(x) i \gamma_5 q_1(0) | 0 \rangle &= f_P m_P \int_0^1 du e^{iup \cdot x} \phi_P^S(u), \\
 \langle P(p) | \bar{q}_2(x) \sigma_{\mu\nu} \gamma_5 q_1(0) | 0 \rangle &= -i(p_\mu x_\nu - p_\nu x_\mu) f_P m_P \int_0^1 du e^{iup \cdot x} \phi_P^\sigma(u),
 \end{aligned} \tag{1}$$

where ϕ_P is the twist-2 light-cone DA, ϕ_P^S and ϕ_P^σ are the twist-3 light-cone DAs, $u(\bar{u} = 1 - u)$ denotes the momentum fraction carried by the $\bar{q}_2(q_1)$ quark in the pseudoscalar meson, and f_P is the decay constant of the pseudoscalar meson, which is defined by

$$\langle P(p) | \bar{q}_2 \gamma_\mu \gamma_5 q_1 | 0 \rangle = -i f_P p_\mu.$$

We discuss only the twist-2 light-cone DA $\phi_P(u)$, as the twist-3 light-cone DAs $\phi_P^S(u)$ and $\phi_P^\sigma(u)$ vanish when we use chiral currents. In addition, we do not take the higher twist distribution amplitudes into account. The corresponding renormalization group equation at one loop [47,52–55] implies the series expansion of the DA of pions in terms of Gegenbauer polynomials,

$$\phi(u) = 6u(1-u) \sum_n a_n C_n^{\frac{3}{2}}(2u-1), \quad (2)$$

based on the approximate conformal symmetry of QCD with light quarks. Here, the Gegenbauer coefficients a_n determine the deviation of $\phi_P(u)$ from its asymptotic form $6u(1-u)$, and $C_n^{\frac{3}{2}}$, known as Gegenbauer polynomials, can be normalized by

$$\int du C_n^{\frac{3}{2}}(2u-1) C_m^{\frac{3}{2}}(2u-1) = \frac{(n+2)(n+1)}{4(2n+3)} \delta_{mn}. \quad (3)$$

In addition, Gegenbauer coefficients a_n and meson decay constant f_P run as the energy scale μ changes, whose evolution with respect to μ is given by the renormalization group equations [56,57]

$$a_n(\mu) = a_n(\mu_0) \left(\frac{\alpha(\mu_0)}{\alpha(\mu)} \right)^{-(\gamma_{(n)}+4)/b},$$

$$f_P(\mu) = f_P(\mu_0) \left(\frac{\alpha(\mu_0)}{\alpha(\mu)} \right)^{4/b},$$

where the one-loop anomalous dimensions are given by [58,59]

$$\gamma_{(n)} = C_F \left[1 - \frac{2}{(n+1)(n+2)} + 4 \left(\sum_{j=2}^{n+1} \frac{1}{j} \right) \right],$$

with

$$C_F = \frac{N_c^2 - 1}{2N_c}.$$

Given these relations, we are able to explore the Gegenbauer coefficients a_n of the pion mesons, which are closely related to moments of pions at the energy scale $\mu = 1$ GeV. The obtained results are shown in Table I.

Here, the distribution amplitudes in Ref. [45] are derived at next-to-leading order via QCD factorization, whereas the Gegenbauer coefficients in Refs. [40,41,48,50–52,61] are obtained from QCD sum rules. Given these Gegenbauer coefficients and polynomials, one can apply Eq. (2) to compute the distribution amplitudes. The obtained results are plotted in Fig. 1.

TABLE I. Comparison of Gegenbauer coefficients a_2 and a_4 of DA predicted for pion mesons.

	a_2	a_4
Khodjamirian [45]	0.17	0.06
Duplancic [60]	0.16	0.04
Ball [46]	0.28	
Belyaev and Braun <i>et al.</i> [40,41,47,48]	0.41	0.23

B. The DAs with BLFQ method

In the light-front form of QCD field theory, the hadrons can be described by the light-front wave functions (LFWFs), which are given, for the case of mesons, by [62]

$$\Psi_{\{x_i; \vec{p}_\perp; \lambda_i\}}^{N, M_J} = \sum_{n_i; m_i} \psi^N(\{\bar{\alpha}_i\}) \prod_{i=1}^N \phi_{n_i; m_i}(\vec{p}_\perp; b), \quad (4)$$

where $\psi^{N=2}(\{\bar{\alpha}_i\})$ and $\psi^{N=3}(\{\bar{\alpha}_i\})$ are the components of the eigenvectors relevant to the Fock sectors of quark-antiquark configuration $|q\bar{q}\rangle$ and the hybrid meson configuration $|q\bar{q}g\rangle$, with g the valence gluon, $\{\bar{\alpha}\} = \{x, n, m, \lambda\}$, x, n, m, λ are the longitudinal momentum fraction, the radial, angular, and the additional quantum number, respectively, and $\phi_{n_i; m_i}(\vec{p}_\perp; b)$ is the two-dimensional harmonic oscillator basis function, which can be obtained by diagonalizing the full Hamiltonian matrix of QCD, e.g., in the BLFQ method [63–66].

We use the truncation $N_{\max} = 14$, and the six model parameters, all of which are summarized in Table II. With these fixed inputs of the parameters, one can generate the mass spectra of light mesons [62]. Here, the model parameters include the quark mass m_q , the effective mass of gluon m_g , the harmonic oscillator scale parameter b , the strength of the confinement κ , and the two parameters m_f and g (independent of quark mass) in the vertex interaction and coupling constant.

The DAs are defined in terms of the lightlike separated gauge invariant vacuum-to-meson matrix elements. In the light-front formalism, the leading-twist DAs $\phi_P(u, \mu)$ in the light-cone gauge for a pseudoscalar are defined by [52]

$$\langle 0 | \bar{q}_2(z) \gamma^+ \gamma_5 q_1(-z) | P(p) \rangle = i f_P p^+ \int_0^1 dx e^{i p^+ z^- (u - \frac{1}{2})} \phi_P(u). \quad (5)$$

TABLE II. The model parameters for the truncation $N_{\max} = 14$ [62]. All are in units of GeV except g , which is dimensionless.

m_q	m_g	b	κ	m_f	g
0.39	0.60	0.29	0.65	5.69	1.92

The DAs of pseudoscalar states are linked to the LFWF Ψ in the form of [62,67–75]

$$\phi_P(u) = \bar{f}_P \frac{2\sqrt{2N_C}}{N_F} \int [d^2 p_\perp] \Psi^{\uparrow\downarrow+\downarrow\uparrow}(u, p_\perp), \quad (6)$$

with

$$N_F = \sum_{s_1 s_2} \int [d^3 p_1] [d^3 p_2] \Psi^{s_1 s_2*}(p_1, p_2) \Psi^{s_1 s_2}(p_1, p_2),$$

where N_C is the number of colors, $p_{1,2}$ are moments of quarks $q_{1,2}$, with spin $s_{1,2}$, and s_1 and s_2 are spins. In the Nambu-Jona-Lasinio (NJL) model with Hamiltonian of the basis light front quantization form, which we referred to as the BLFQ-NJL model, we use $N_C = 3$ in this work. Recall that in the BLFQ-NJL model the flavor wave function of the neutral pion is the only difference compared to the wave function of the charged pion. Therefore, one can compute the distribution amplitude of the pion, with the obtained results shown in Fig. 2. In the domain $m_\pi^2/Q^2 \simeq 0$ [52–54], the DA of the pion is known to be $\phi(u) = 6u(1-u)$, and this prediction is reproduced by the obtained result of BLFQ basically (Fig. 2).

C. The DAs as a sum of Gegenbauer polynomial with BLFQ method

Given that the DAs of the pion, based on the method of the BLFQ, are precisely determined, the relationship between the moments and the DAs can be established without recourse to any approximations, as follows:

$$\langle \xi^n \rangle = \int_0^1 (2x-1)^n \phi_P(x), \quad (7)$$

where $\phi_P(x)$ is extracted by Eq. (6). Subsequently, one can evaluate the Gegenbauer coefficients at the scale $\mu = 1$ GeV, and one obtains the following sequence of the Gegenbauer coefficients:

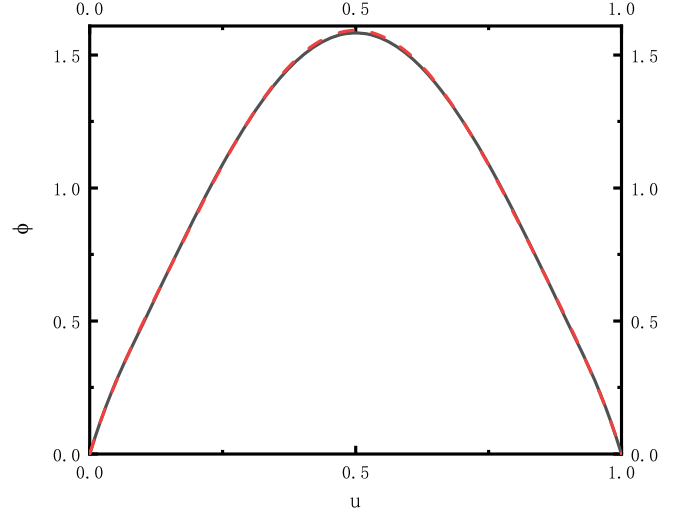


FIG. 2. The two pion DAs based on and with the BLFQ method (solid and dashed lines, respectively). They are almost coincident.

$$\{a_0, a_2, a_4, a_6, a_8, a_{10}, \dots\} \\ = \{1, -0.0283, 0.0130, 0.0024, 0.0012, -0.0010, \dots\}.$$

It is evident that the Gegenbauer coefficients show a clear trend of convergence as the order increases. The distribution amplitude obtained from the BLFQ method is shown in Fig. 2. In addition, we perform a comparative analysis of the DAs based on and with the BLFQ method. One can juxtapose the DAs obtained with the two approaches, and we present the comparison in Fig. 2.

The exact overlap of the two lines representing the DAs based on and from (with) the BLFQ method suggests a high degree of consistency between the distribution amplitudes obtained by the two methods.

III. FORM FACTORS

To calculate the form factor of the decay $D \rightarrow P$, we define the two-point correlation function

$$\begin{aligned} \Pi_{1\mu}(p, q) &= i \int d^4 x e^{iqx} \langle P(p) | T \{ \bar{q}_2(x) \gamma_\mu (1 + \gamma_5) c(x), \bar{c}(0) i(1 + \gamma_5) q_1(0) \} | 0 \rangle, \\ \Pi_{2\mu}(p, q) &= i \int d^4 x e^{iqx} \langle P(p) | T \{ \bar{q}_2(x) \sigma_{\mu\nu} (1 + \gamma_5) q^\nu c(x), \bar{c}(0) i(1 - \gamma_5) q_1(0) \} | 0 \rangle, \end{aligned} \quad (8)$$

where P is light pseudoscalar mesons. The momenta q is defined by $q = p_D - p$, where p_D and p are the four-momentum of the initial and final meson states, respectively. Here, the correlation function $\Pi_{1\mu}$ is associated with the vector current contribution, while $\Pi_{2\mu}$ accounts for the tensor current effects.

To derive the form factors for the $D \rightarrow P$ semileptonic decays, we shall use the LCSR method to deal with the

correlation function on two sides. The standard procedure for this is as follows:

- (1) On the phenomenological side, one inserts a set of hadron complete sets of initial meson states in the middle of two currents in the correlation function.
- (2) On the theoretical side, one uses operator product expansion upon correlation functions.

Then, equating the two representations obtained via the above procedure and applying the Borel transformation to suppress the contributions from higher states and the continuum, one can obtain the sum rule expressions for the form factors. The following subsections are devoted to the details of the relevant analysis.

A. The phenomenological side

For the correlation functions in Eq. (8), one can insert a set of complete sets of mesons, and then isolate the mass term of the pseudoscalar D mesons. The results are obtained, upon applying Fourier transformation to them, as

$$\begin{aligned}\Pi_{1\mu}(p, q) &= \frac{\langle P(p) | \bar{q}_2 \gamma_\mu c | D \rangle \langle D | \bar{c} i \gamma_5 q_1 | 0 \rangle}{m_D^2 - (p + q)^2} + \text{higher states}, \\ \Pi_{2\mu}(p, q) &= \frac{\langle P(p) | \bar{q}_2 \sigma_{\mu\nu} (1 + \gamma_5) q^\nu c | D \rangle \langle D | \bar{c} i \gamma_5 q_1 | 0 \rangle}{m_D^2 - (p + q)^2} + \text{higher states},\end{aligned}\quad (9)$$

where the hadronic matrix elements $\langle P(p) | \bar{q}_2 \gamma_\mu c | D \rangle$ and $\langle P(p) | \bar{q}_2 \sigma_{\mu\nu} (1 + \gamma_5) q^\nu c | D \rangle$ can be parametrized by a set of q -dependent functions in the following forms [60,76–78]:

$$\begin{aligned}\langle P(p) | \bar{q}_2 \gamma_\mu c | D \rangle &= 2f_+^P(q^2)p_\mu + [f_+^P(q^2) + f_-^P(q^2)]q_\mu, \\ \langle P(p) | \bar{q}_2 \sigma_{\mu\nu} (1 + \gamma_5) q^\nu c | D \rangle &= [2p_\mu q^2 - 2q_\mu (p \cdot q)] \frac{if_T^P(q^2)}{m_D + m_P},\end{aligned}\quad (10)$$

where f_+^P and f_-^P are the transition form factors of the semileptonic decays $D \rightarrow P$, $f_T^P(q^2)$ is the penguin form factor, m_D and m_P are the mass of D and pseudoscalar mesons, and m_c and m_{q_1} are the mass of charm quark (c) and light quark, respectively. The D meson decay constant f_D is defined by the other matrix element,

$$\langle D | \bar{c} i \gamma_5 q_1 | 0 \rangle = \frac{m_D^2 f_D}{m_c + m_{q_1}}. \quad (11)$$

Substituting the matrix elements Eqs. (11) and (10) into Eq. (9) and taking account of Lorentz structures of the correlation functions, one gets the phenomenological part of two correlation functions. We can obtain the phenomenological part of two correlation functions in terms of the form factors as

$$\begin{aligned}\Pi_{1\mu}(p, q) &= -\frac{2f_+(q^2)p_\mu + [f_+(q^2) + f_-(q^2)]q_\mu}{m_D^2 - (p + q)^2} \frac{m_D^2 f_D}{m_c + m_{q_1}} + \int_{s_0^h}^{\infty} \frac{\rho_\mu(s) ds}{s - (p + q)^2}, \\ \Pi_{2\mu}(p, q) &= \frac{[-q^2 2p_\mu - 2q_\mu (p \cdot q)]}{m_D^2 - (p + q)^2} \frac{if_T}{m_D + m_P} \frac{m_D^2 f_D}{m_c + m_{q_1}} + \int_{s_0^h}^{\infty} \frac{\rho_\mu(s) ds}{s - (p + q)^2}.\end{aligned}\quad (12)$$

Since calculation of the contributions from higher excited and continuum states on the phenomenological side is highly nontrivial, one can invoke quark-hadron duality to parametrize the corresponding spectral integral [39,59],

$$\int_{s_0^h}^{\infty} ds \frac{\rho_\mu(s)}{s - (p + q)^2} \simeq \int_{s_0}^{\infty} ds \frac{1}{\pi} \frac{\text{Im} \Pi_\mu^{\text{pert}}(s)}{s - (p + q)^2}. \quad (13)$$

Here, s_0^h denotes the hadronic continuum threshold (a physical parameter representing the onset of higher

resonances), while s_0 denotes the duality threshold (an effective parameter introduced by the quark-hadron duality assumption), and $\rho_\mu(s)$ is the spectral density of the higher excited and continuum states.

B. The theoretical side

In the region of large spacelike momentum, the computation is based on the expansion of the T-product in the correlation functions near the light cone. By contracting c and $\bar{c}$ quark fields, we obtain

$$\begin{aligned}\Pi_{1\mu}(p, q) &= i \int d^4x e^{iqx} \langle P(p) | \bar{q}_2(x) \gamma_\mu (1 + \gamma_5) S^c(x, 0) i(1 + \gamma_5) q_1(0) | 0 \rangle, \\ \Pi_{2\mu}(p, q) &= i \int d^4x e^{iqx} \langle P(p) | T \bar{q}_2(x) \sigma_{\mu\nu} (1 + \gamma_5) q^\nu S^c(x, 0) i(1 - \gamma_5) q_1(0) | 0 \rangle,\end{aligned}\quad (14)$$

where $S^c(x, 0)$ is the propagator of the charm (c) quark,

$$S^c(x, 0) = -i \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot x} \frac{k + m_c}{m_c^2 - k^2}. \quad (15)$$

In the method of the LCSR, the nonvanishing matrix elements, which are exactly the light-cone DAs, are defined in terms of twist functions. As a result, the DA matrix element obtained takes the form of

$$\langle P(p) | \bar{q}_2(x) \gamma_\mu \gamma_5 q_1(0) | 0 \rangle = -i p_\mu f_P \int_0^1 du e^{iup \cdot x} \phi(u). \quad (16)$$

By employing the LCDAs and performing some straightforward calculations, two correlation functions on the theoretical side are obtained as

$$\begin{aligned} -\frac{2f_+(q^2)p_\mu + [f_+(q^2) + f_-(q^2)]q_\mu}{m_D^2 - (p+q)^2} \frac{m_D^2 f_D}{m_c + m_{q_1}} &= 2f_P m_c \int_\Delta^1 du \phi(u) \frac{p_\mu}{m_c^2 - (q+up)^2}, \\ \frac{-i[q^2 2p_\mu - 2q_\mu(p \cdot q)]}{m_D^2 - (p+q)^2} \frac{f_T}{m_D + m_P} \frac{m_D^2 f_D}{m_c + m_{q_1}} &= -i \int_\Delta^1 du f_P [2p_\mu q^2 - 2q_\mu(p \cdot q)] \frac{\phi(u)}{m_c^2 - (q+up)^2}, \end{aligned} \quad (18)$$

where Δ defines the integration lower limit, depending on the duality threshold s_0 of the D meson, through

$$\Delta = \frac{\sqrt{(s_0 - m_P^2 - q^2)^2 + 4(m_c^2 - q^2)m_P^2} - (s_0 - m_P^2 - q^2)}{2m_P^2}.$$

In order to suppress the contribution of the higher states and continuum, we shall perform Borel transformation B_{M^2} [79,80] on both sides of Eq. (18). In the present case, this amounts to employing the following relation:

$$\begin{aligned} B_{M^2} \frac{1}{m_D^2 - (q+p)^2} &= \frac{1}{M^2} e^{-\frac{m_D^2}{M^2}}, \\ B_{M^2} \frac{1}{m_c^2 - (q+up)^2} &= \frac{1}{uM^2} e^{-\frac{m_c^2 + u(1-u)p^2 - (1-u)q^2}{uM^2}}, \end{aligned}$$

where M^2 is the Borel parameter. Upon Borel transformation on Eq. (18), one can get three relations, from which one can solve the following three FFs:

$$\begin{aligned} f_+(q^2) &= \frac{m_c + m_{q_1}}{m_D^2 f_D} f_P m_c \int_\Delta^1 \frac{\phi(u)}{u} du e^{FF}, \\ f_-(q^2) &= -\frac{m_c + m_{q_1}}{m_D^2 f_D} f_P m_c \int_\Delta^1 \frac{\phi(u)}{u} du e^{FF}, \\ f_T(q^2) &= (m_D + m_P) \frac{m_c + m_{q_1}}{m_D^2 f_D} f_P \int_\Delta^1 \frac{\phi(u)}{u} du e^{FF}, \end{aligned} \quad (19)$$

with

$$\begin{aligned} \Pi_{1\mu} &= 2f_P m_c p_\mu \int_0^1 du \frac{\phi(u)}{m_c^2 - (q+up)^2}, \\ \Pi_{2\mu} &= -if_P [2p_\mu q^2 - 2q_\mu(p \cdot q)] \int_0^1 du \frac{\phi(u)}{m_c^2 - (q+up)^2}. \end{aligned} \quad (17)$$

C. The form factors

Now, we are in the position to use the LCSR method for the correlation function on two sides. Let the coefficients of Lorentz structures in the correlation functions (9) on the phenomenological side in Sec. III A be equal to that of corresponding Lorentz structures in the correlation functions (14) the theoretical side in Sec. III B, and we get

$$FF = -\frac{1}{uM^2} (m_c^2 + u\bar{u}p^2 - \bar{u}q^2) + \frac{m_D^2}{M^2}. \quad (20)$$

Simple comparison of the three relations (19) enables us to obtain two relations associated with three form factors, f_+^P , f_-^P , and $f_T^P(q^2)$,

$$\begin{aligned} f_-(q^2) &= -f_+(q^2), \\ f_T(q^2) &= \frac{m_D + m_P}{m_c} f_+(q^2). \end{aligned}$$

D. Branching fractions

Equipped with the FFs derived above, we can now compute the differential decay widths for the processes under consideration. Specifically, for the semileptonic decay $D \rightarrow Pl\bar{\nu}_l$, which is the focus of this work, the LCSR framework allows us to express the differential decay width in terms of the FFs as follows:

$$\begin{aligned} \frac{d\Gamma(D \rightarrow Pl\bar{\nu}_l)}{dq^2} &= \frac{G_F^2 |V_{cd}|^2}{24\pi^2} \left[\left(\frac{m_D^2 + m_P^2 - q^2}{2m_D} \right)^2 - m_P^2 \right]^{\frac{3}{2}} \\ &\quad \times |f_+(q^2)|^2, \end{aligned} \quad (21)$$

TABLE III. Parameter inputs for form factors and branching fractions [81].

m_{D^0} [GeV]	m_{D^+} [GeV]	$ V_{cd} $	τ_{D^+} [10^{-12} s]	τ_{D^0} [10^{-13} s]
1.865	1.870	0.220	1.040 ± 0.007	4.101 ± 0.015
m_u [MeV]	m_c [GeV]	f_π [MeV]	f_D [MeV]	f_B [MeV]
2.16	1.27	130.3	205.4	190
G_F [GeV $^{-2}$]	m_e [GeV]	m_μ [GeV]	m_d [MeV]	$m_{\pi^{\pm(0)}}$ [MeV]
1.1664×10^{-5}	0.511×10^{-3}	0.106	4.67	139.57(134.98)

where G_F is the Fermi coupling constant and $|V_{cd}|$ is the CKM matrix element. One sees that the differential decay width is proportional to the FFs squared.

IV. NUMERICAL ANALYSES AND DISCUSSION

A. Input parameters

Before going to numerical analyses of the form factors and differential branching fractions, let us briefly discuss the parameters involved in these semileptonic decay processes. For this purpose, we shall use the following inputs [81] for the parameters of the form factors and branching fractions, which are shown collectively in Table III. The duality threshold s_0 has been explored and estimated in several effective scenarios [82–86]. Since s_0 is near to the squared mass of the lowest pseudoscalar D meson, we adopt $s_0 = 5.37 \text{ GeV}^2$, corresponding to the D channel. Despite that the form factors should be independent of the Borel parameter ideally, the Borel mass must, in practice, be chosen within a window that satisfies the following criteria:

- (1) The contributions from the higher excited and continuum states are sufficiently suppressed.
- (2) The form factors exhibit weak dependence on the Borel parameter.

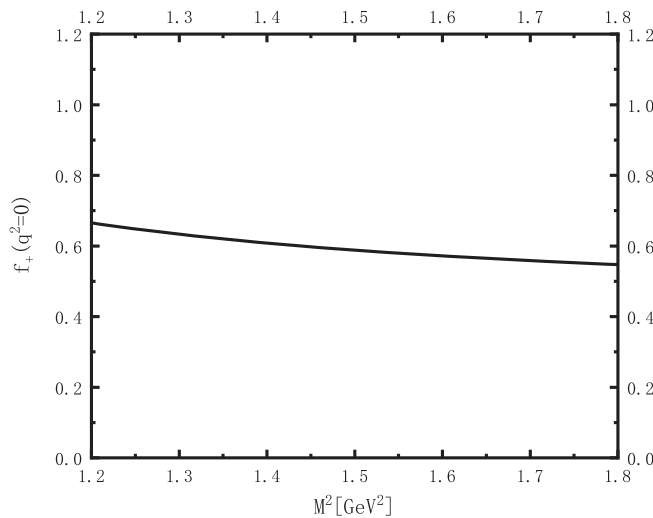


FIG. 3. $D^{+(0)} \rightarrow \pi^{0(-)}$ transition form factor function on Borel parameter M^2 at threshold $s_0 = 5.37 \text{ GeV}^2$ and $q^2 = 0$.

Given all these parameters in Table III, one can use Eq. (19) to perform calculation of the LCSR form factor $f_+(q^2 = 0)$ and obtain the Borel parameter (M^2) dependence of the LCSR form factor $f_+(q^2 = 0)$.

With these numerical analyses, one can numerically determine the thresholds and Borel windows for various semileptonic decay processes in this work. In Figs. 3 and 4, we plot the numerical predictions of the form factors for $D^0 \rightarrow \pi^-$. As seen in Fig. 3, f_+ demonstrates a high stability against changes of M^2 with the range of $1.2 \text{ GeV}^2 < M^2 < 1.8 \text{ GeV}^2$. This range is to be chosen as the optimized working interval. As shown in Fig. 4, the dependence of the form factors for $D^{+(0)} \rightarrow \pi^{0(-)}$ upon s_0 is quite weak, and the corresponding range of the s_0 is to be chosen as the optimized working interval. Furthermore, we list, in Table IV, our computed value of form factors and compare it with that obtained via other approaches for the decay $D \rightarrow \pi$ at $q^2 = 0$. For comparison, we have also computed the $B \rightarrow \pi$ form factors using the same BLFQ-based DA and LCSR framework. The results are listed in Table IV along with other theoretical predictions.

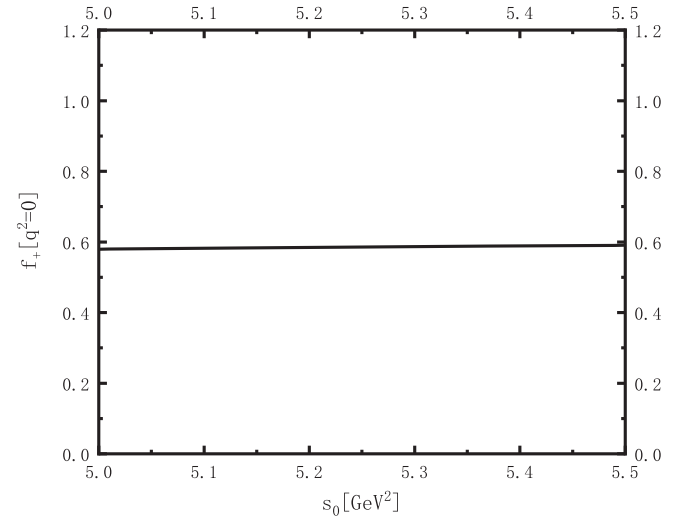


FIG. 4. $D^{+(0)} \rightarrow \pi^{0(-)}$ transition form factor function on threshold s_0 at Borel parameter $M^2 = 1.5 \text{ GeV}^2$ and $q^2 = 0$.

TABLE IV. Comparison of form factors in this work with that by other theoretical methods for $D/B \rightarrow P$. The first error is due to the Borel parameter M^2 , and the second is due to the duality threshold s_0 .

Processes	$B \rightarrow \pi$		$D \rightarrow \pi$	
	f_+	f_-	f_+	f_-
This work	$0.27^{+0.005+0.004}_{-0.005-0.004}$	$-0.27^{+0.005+0.004}_{-0.005-0.004}$	$0.59^{+0.07+0.0057}_{-0.05-0.0106}$	$-0.59^{+0.07+0.0057}_{-0.05-0.0106}$
LCSR [41]	0.28		0.65	
LCSR [45]	0.281			
LCSR [46]			0.63	
LCSR [87]	0.28		0.62	
LCSR [77]	0.258			
LCSR [88]	0.26			
LCSR [78]			0.67	
LQCD [25]			0.57	
LQCD [26,27]			0.64	
LQCD [28]			0.74	
LQCD [29]			0.65	
LQCD [30]			0.666	
LQCD [31]			0.625	
HQEFT ^a [89]	0.35		0.67	
QCDSR ^b [90]	0.26		0.5	
RQM ^c [91]			0.640	
CCQM ^d [6]			0.63	
CLFQM ^e [92]			0.66	
BESIII [9]			0.622	
BESIII [14,15]			0.635	
HFLAV [93]			0.635	
ETM [94]			0.612	

^aThe effective field theory of heavy quark.

^bThe QCD sum rule.

^cThe relativistic quark model.

^dThe covariant confining quark model.

^eThe covariant light-front quark model.

B. The differential decay width

Further, we employ the obtained FFs in Sec. III C to compute the differential decay width by Eq. (21). The numerical results for the differential decay width as a function of the transfer momentum q^2 are plotted in Fig. 5. In Fig. 5, we present a comparison of the differential decay width $d\Gamma/dq^2$ for $D^+ \rightarrow \pi^0 e^+ \nu_e$ between our LCSR predictions (solid line) and the BESIII data [9] (dots). Excellent agreement is observed in the high- q^2 region ($q^2 > 1 \text{ GeV}^2$), where the LCSR framework is valid. However, a notable discrepancy emerges at $q^2 < 1 \text{ GeV}^2$, attributed to the breakdown of the light-cone expansion in the low momentum-transfer regime. We note that this discrepancy is consistent with theoretical expectations, as the LCSR formalism is built upon the assumption of large momentum transfer ($q^2 \gg \Lambda_{\text{QCD}}^2$, where $\Lambda_{\text{QCD}}^2 \sim 0.04\text{--}0.09 \text{ GeV}^2$). Such limitations are inherent to this theoretical approach and underscore the necessity of including higher-order corrections.

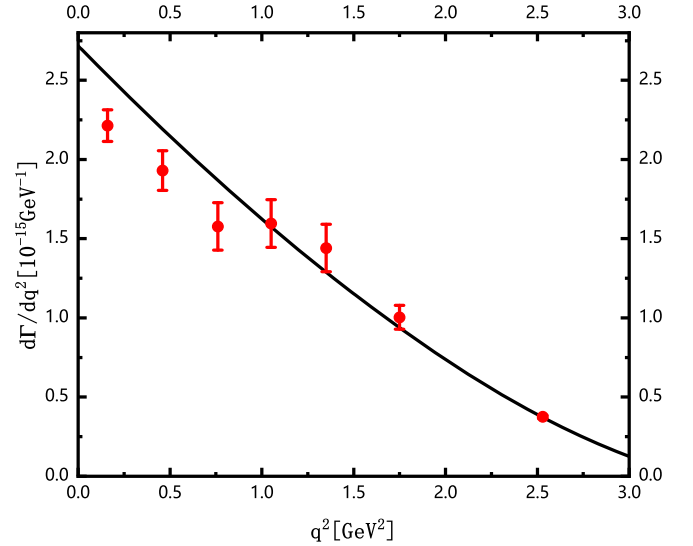


FIG. 5. The differential decay width for $D^+ \rightarrow \pi^0 e^+ \nu_e$ as a function of q^2 . The data (the dots) reported by BESIII [9] are compared to the prediction (solid line) by the obtained DA based on BLFQ.

The quoted uncertainties stem from variations in the Borel parameter M^2 and the duality threshold s_0 , while the leading-twist distribution amplitude ϕ_P incorporates contributions from the $|q\bar{q}\rangle$ and $|q\bar{q}g\rangle$ Fock states. To improve our prediction in this work, future refinements should systematically include additional effects, such as higher Fock states (e.g., $|q\bar{q}gg\rangle$), higher-twist light-cone distribution amplitudes, and next-to-leading-order corrections to the light-cone operator product expansion.

V. SUMMARY

In this work, we have computed the transition FFs for the semileptonic decays $D \rightarrow \pi$ using QCD LCSRs with chiral currents, with a focus on the role of pion LCDAs. The twist-2 pion LCDA is systematically analyzed using both Gegenbauer polynomial expansions and the BLFQ method. The BLFQ-derived LCDA, which incorporates contributions from the $|q\bar{q}\rangle$ and $|q\bar{q}g\rangle$ Fock states, converges rapidly to the asymptotic form $\phi(u) = 6u(1-u)$, yielding Gegenbauer coefficients of $a_2 = -0.0283$ and $a_4 = 0.0130$ at $\mu = 1$ GeV. Comparisons with previous theoretical predictions (e.g., QCD sum rules and lattice QCD) are given, underscoring the significance of nonperturbative dynamics of the strong interaction.

The $D \rightarrow \pi$ transition form factors f_+ , f_- , and f_T , derived from the LCSR framework, reveal two simplified relationships: $f_- = -f_+$ and $f_T = \frac{m_D + m_\pi}{m_c} f_+$, which are consistent with chiral symmetry. At $q^2 = 0$, our numerical result $f_+(0) = 0.59$ agrees with experimental measurements from BESIII (0.622–0.635) [9,14,15] and lattice QCD (0.57–0.74) [25–31]. The stability of the obtained FFs against the Borel parameter ($1.2 \text{ GeV}^2 < M^2 < 1.8 \text{ GeV}^2$) and duality threshold ($s_0 = 5.37 \text{ GeV}^2$) is verified via systematic uncertainty analyses.

Our predictions for the differential decay width $d\Gamma/dq^2$ show good agreement with BESIII data in the high- q^2 region ($>1 \text{ GeV}^2$), while deviations at low q^2 are attributed to the validity domain of LCSR ($q^2 \gg \Lambda_{\text{QCD}}^2$). The theoretical uncertainties may stem from neglected higher-twist corrections, omitted Fock states, and the truncation of the Gegenbauer expansion. These results offer promising

prospects for extracting the CKM matrix element (e.g., $|V_{cd}|$) via Eq. (21) with high precision and exploring new physics beyond the standard model through precision tests of heavy-to-light decays.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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