

Probing fundamental physics with extreme mass ratio inspirals: Full Bayesian inference for scalar charge

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Extreme mass ratio inspirals (EMRIs) are key sources for the future space-based gravitational wave detector Laser Interferometer Space Antenna (LISA) and are considered promising probes of fundamental physics. Here, we present the first complete Bayesian analysis of EMRI signals in theories with an additional massless scalar, which could arise in an extension of general relativity or of the Standard Model of particle physics. We develop a waveform model accurate at adiabatic order for equatorial eccentric orbits around spinning black holes. Using full Bayesian inference, we forecast LISA's ability to probe the presence of new fundamental fields with EMRI observations.

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I. INTRODUCTION

In the last decade, gravitational wave (GW) astronomy has revolutionized our ability to observe the Universe, offering unique opportunities to test the nature of gravity and search for new fundamental fields in previously unexplored regimes [1–3]. Observations with gravitational wave detectors allow us to probe the highly dynamic and strong-field regime of compact binary coalescences.

Ground-based interferometers have paved the way for searches for new physics beyond general relativity (GR) or the Standard Model [4]. The constraining power of current facilities is limited to a signal-to-noise ratio (SNR) up to 30 [5] and comparable mass binaries, with the most asymmetric system detected so far having a mass ratio of

$q = \mu/M \sim 1/10$, with GW190814 [6–8], where we defined M , μ as the primary and secondary component masses, respectively. The next generation of ground-based detectors, such as the Einstein Telescope [9] and Cosmic Explorer [10,11], along with space-based missions like the Laser Interferometer Space Antenna (LISA) [12], TianQin [13], Taiji [14], and Lunar Gravitational Wave Antenna [15], are expected to observe sources across a broader mass range. Probing fundamental physics is featured prominently in science cases for future detectors [2,3,12,16,17].

LISA is expected to detect gravitational waves from extreme mass ratio inspirals (EMRIs): binary systems composed of stellar-mass compact objects (the secondary), inspiraling into massive black holes at the center of galaxies (the primary). Due to their small mass ratios $q \lesssim 10^{-4}$, EMRIs perform tens of thousands of cycles on highly relativistic trajectories with large inclination and orbital eccentricities within the LISA sensitivity band. The orbit complexity results in gravitational signals with multiple harmonics. The rich harmonic content and the many cycles of EMRI signals allow subpercent parameter measurement precision, rendering EMRIs

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natural laboratories to test gravity and offering a unique opportunity to probe fundamental physics in unprecedented regimes [2]. These same characteristics also make the modeling and generation of EMRI waveforms particularly challenging.

A variety of studies have investigated the scientific potential of such sources to test the nature of black holes (BHs) [18–43], the propagation speed of gravity [44,45], and the existence of new fundamental fields [46–65]. While testing gravity with EMRIs is a key science goal for LISA, calculations of waveform models beyond GR are in their infancy. The majority of works carried out so far have resorted to *ad hoc* modifications of GR templates, adopting hybrid schemes based, for example, on post-Newtonian (PN) expansions [66]. Moreover, such studies have mainly focused on the potential of EMRI observations to identify deviations from the spacetime of the primary, forecasting—in some cases—constraints on such parameters from LISA observations [18,49].

Contrary to the standard lore, which has the EMRI acting as a probe for the spacetime of the primary, Ref. [52] demonstrated that changes from the Kerr metric of the primary can be neglected at leading order in the mass ratio for a large family of theories with scalar fields non-minimally coupled to gravity. It is instead the *scalar charge* of the much lighter secondary that can leave a significant imprint on the GW emission. This is because for massless scalars the scalar charge, if any, is inversely proportional to the square of the mass of the black hole [67–71]. The framework developed in Ref. [52] allows the construction of waveforms that are correct at the leading adiabatic order (first order in the mass ratio) and for which deviations from GR are uniquely determined by the scalar charge of the secondary. This framework has recently been framed into a consistent approach to compute postadiabatic (second order in the mass ratio) waveform corrections to the GR baseline model [71].

This formalism has been exploited to study changes in the GW fluxes for binaries on eccentric equatorial [54] and circular inclined orbits [56,72] for massless scalar fields and for circular equatorial inspirals in the case of massive scalars [55]. Preliminary analyses have also assessed LISA’s capability to infer the measurement precision of the scalar charge using Fisher information matrix calculations [53,55,73–75].

Here, we provide the first implementation of adiabatic waveforms for EMRIs with scalar fields, in eccentric equatorial orbits around spinning BHs. While EMRI formation scenarios are generally expected to be generic, we consider this implementation an important step forward in modeling EMRIs within beyond-GR frameworks. We perform a Bayesian analysis on all the waveform parameters to forecast the power of single EMRI observations to detect the scalar charge and, therefore, probe deviations from general relativity and the Standard Model. The

waveform model and the Bayesian analysis codes are available at [76].

Our results show that a single EMRI is able to constrain the scalar charge of the secondary, with precision of 10% in a theory-agnostic way, i.e., independent of the origin of the scalar field. Moreover, if one selects a specific theory, a constraint of the charge can be converted into a bound on a coupling constant of this theory, which controls deviations from GR. As a characteristic example, we consider here the case of linear-Gauss–Bonnet gravity.

II. EMRIS AND FUNDAMENTAL FIELDS

In this section, we briefly summarize the theoretical approach we use to model EMRIs with scalar fields [52–54,56]. We refer the reader to [71] for a detailed description of the formalism and its extension within a self-force (SF) scheme.

We consider theories with a single massless scalar field ϕ , nonminimally coupled to the metric tensor $\mathbf{g}$, described by the following action:

$$\int d^4x \frac{\sqrt{-g}}{16\pi} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right] + \alpha_c S_c[\mathbf{g}, \phi] + S_m[\mathbf{g}, \phi, \Psi], \quad (1)$$

where R is the Ricci scalar, and g is the metric determinant. S_m describes the dynamics of the matter fields Ψ . The action $\alpha_c S_c$ describes (each of) the scalar field interactions, with the constant α_c having dimensions $(\text{mass})^n$, with $n > 1$. In physical units, this corresponds to interactions that are suppressed by a characteristic energy scale [70]. Varying with respect to the metric and the scalar field, we obtain the equations for the fields,

$$G_{\mu\nu} = 8\pi T_{\mu\nu}^{\text{scal}} + \alpha_c T_{\mu\nu}^c + T_{\mu\nu}^m, \quad \square \phi = T^c + T^m, \quad (2)$$

where $\square = \nabla_\mu \nabla^\mu$, $T_{\mu\nu}^{\text{scal}} = \frac{1}{16\pi} [\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial\phi)^2]$ and

$$T_{\mu\nu}^{c,m} = -\frac{16\pi}{\sqrt{-g}} \frac{\delta S_{c,m}}{\delta g^{\mu\nu}}, \quad T^{c,m} = -\frac{16\pi}{\sqrt{-g}} \frac{\delta S_{c,m}}{\delta \phi}. \quad (3)$$

Since we are considering a massless scalar, we assume that S_c respects shift symmetry, i.e., invariance under $\phi \rightarrow \phi + \text{constant}$. However, our approach can be generalized to light scalars [55]. We focus on EMRIs in which the primary is a BH of mass M and assume that solutions in theories controlled by (1) are continuously connected to GR solutions for $\alpha_c \rightarrow 0$. For shift-symmetric scalars, the scalar charge for black holes, if any [77–82], is fixed in terms of their mass, spin, and the coupling constants of the theory [70]. Hence, the primary mass M and the coupling constant α_c are the only meaningful physical scales of this problem. Their ratio can be expressed as

$$\zeta = \frac{\alpha_c}{M^n} = q^n \frac{\alpha_c}{\mu^n}. \quad (4)$$

Existing bounds already require that $\alpha_c/\mu^n \lesssim \mathcal{O}(1)$ [83], so ζ is order q^n . One can then use q as a single bookkeeping parameter, as for the SF approach in GR.

This introduces several simplifications in the description of EMRIs.¹ Indeed, by expanding the metric and the scalar field in powers of q ,

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + q h_{\mu\nu}^{(1)} + \dots, \quad \phi = \phi^{(0)} + q \phi^{(1)} + \dots, \quad (5)$$

it was recently shown how to derive a consistent SF formalism that includes postadiabatic corrections to the binary dynamics [71]. In this paper, we focus on the leading EMRI dissipative contribution, which is fully determined by the linear order perturbations $h_{\mu\nu}^{(1)}$ and $\phi^{(1)}$. Equation (4) implies the following: (i) The background spacetime is suitably described by the Kerr metric, with beyond GR deviations being $\mathcal{O}(q^{2n})$ corrections to $g_{\mu\nu}^{(0)}$. (ii) $\phi^{(0)}$ is constant due to the no-hair theorem [77–82] and can be set to zero by a shift. (iii) At adiabatic order, metric and scalar field perturbations are induced by the secondary decouple, leading to a separate set of equations,

$$G^{\alpha\beta}[h_{\alpha\beta}^{(1)}] = 8\pi\mu \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy_p^\alpha}{d\lambda} \frac{dy_p^\beta}{d\lambda} d\lambda, \quad (6)$$

$$\square\phi^{(1)} = -4\pi d\mu \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda, \quad (7)$$

where $G_{\alpha\beta}$ is the Einstein tensor, and $dy_p^\mu/d\lambda$ is the four velocity of the secondary, along its worldline.

Equation (6) is identical to GR, while Eq. (7) determines the scalar field evolution and depends on the scalar charge of the secondary, d , which enters as the only extra EMRI parameter. The solutions for $h_{\mu\nu}^{(1)}$ and $\phi^{(1)}$ allow the energy and angular momentum fluxes to be computed for the gravitational (grav) sector, $(\dot{E}_{\text{grav}}, \dot{L}_{\text{grav}})$, and the scalar sector (scal), $(\dot{E}_{\text{scal}}, \dot{L}_{\text{scal}})$. Beyond GR modifications to the EMRI evolution are *uniquely* controlled by the latter. At this order in mass ratio, the gravitational waveform amplitudes are the same as in GR, whereas the waveform phase is affected by the extra channel of emission. In fact, if the charge is not zero, the secondary plunges faster than in GR. In this work, we consider systems on equatorial eccentric orbits [54], such that $\dot{E}, \dot{L}$ depend on the semilatus rectum p , on the eccentricity e , and on the primary dimensionless spin a .

¹The approach extends to less asymmetric binaries, like intermediate mass ratio inspirals, so long as ζ remains a perturbative parameter.

For a given theory, there is a mapping between d and the coupling constant α_c . This allows constraints on d to be translated into bounds on such coupling. As an example, we consider here linear Gauss Bonnet gravity (GB) [67], for which

$$\alpha_c S_c = \frac{\alpha}{4} \int d^4x \frac{\sqrt{-g}}{16\pi} \phi \mathcal{G}, \quad (8)$$

where $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ is the Gauss Bonnet invariant, $R_{\alpha\beta\mu\nu}, R_{\alpha\beta}$ are the Riemann and Ricci tensors, and α has dimensions of mass squared ($n = 2$). For this theory, $\alpha \simeq 2d\mu^2$ is in geometrized units. Throughout this paper, we use geometrized units; however, following standard practice in the literature, we also express constraints on $\sqrt{\alpha}$ in kilometers by restoring the gravitational constant G_N and light speed c , yielding $\sqrt{\alpha}[\text{km}] = \sqrt{2d\mu}G_N/c^2$ [84]. In Appendix A, we discuss different normalizations of the action present in the literature and obtain the relation between the scalar charge d and the coupling constant $\sqrt{\alpha}$.

III. WAVEFORM MODELING AND DATA ANALYSIS SETUP

We develop a waveform model for EMRIs [85–100] that accounts for the scalar emission and implement it within the FastEMRIWaveform (FEW) package [101–104], which allows for fast generation of EMRI templates on graphics processing units (GPUs).

In the first order in the mass ratio, the orbital evolution is obtained by solving the following system of ordinary differential equations [104]:

$$\frac{dJ}{dt} = \frac{q}{M} f_J, \quad \frac{d\Phi_i}{dt} = \frac{\Omega_i}{M} \quad J = \{p, e\}, \quad (9)$$

where the time t scales with M and $\Omega_{i=r,\phi}$ are the dimensionless fundamental frequencies of the Kerr space-time, and $f_{p,e}$ are the orbital-element fluxes that depend on the BH dimensionless spin a and on the dimensionless orbital elements $\{p, e\}$ [105,106]. Equation (9) can be integrated given the initial conditions for the semilatus rectum p_0 , for the eccentricity e_0 , and for the phases $(\Phi_{\phi 0}, \Phi_{r 0})$ (see Appendix E for a study of the ordinary differential equations' accuracies). The ordinary differential equation is integrated until we reach the separatrix plus a threshold of $0.1M$. We call these plunging orbits.

The orbital-element fluxes, $f_{p,e}$, are written in terms of energy and angular momentum fluxes. For example, for the semilatus rectum, we have

$$f_J = -\frac{M}{q} \left[\frac{\partial J}{\partial E} \dot{E} + \frac{\partial J}{\partial L} \dot{L} \right], \quad J = p, e, \quad (10)$$

$$\dot{E} = \dot{E}_{\text{grav}} + d^2 \dot{E}_{\text{scal}}, \quad \dot{L} = \dot{L}_{\text{grav}} + d^2 \dot{L}_{\text{scal}}, \quad (11)$$

where both the gravitational and scalar fluxes are the sum of the horizon and infinity fluxes and have been computed² using the code of [107,108] and packages of the black hole perturbation toolkit [109], using the mode summation implemented in [110]. Angular momentum and energy fluxes are computed on a 3d grid in (a, p, e) , and interpolated using Chebyshev polynomials [111] (see Appendix B for details).

Once the trajectory is implemented, we can pass it to the augmented analytical kludge (AAK) [112] waveform amplitude model implemented³ in FEW [101,102,113,114]. The AAK waveform is GPU-accelerated and allows the exploitation of the long-wavelength approximation of the LISA response, providing a generic model for investigating tests of GR. The typical waveform generation speed of this new time-domain model is of order 0.1 seconds.

Our trajectory is fully relativistic at adiabatic order, and therefore, the measurement precision of the intrinsic parameters $\Theta_i = (\ln M, \ln \mu, a, p_0, e_0, \Phi_{\phi 0}, \Phi_{r 0}, d)$ is not strongly affected by the choice of the AAK template [102]. The inclusion of postadiabatic corrections and their impact on the scalar charge detectability will be explored in a follow-up work, exploiting recent advancements in parameter estimation for postadiabatic waveforms [87,115].

The AAK amplitudes and the LISA response may, however, affect the accurate reconstruction of the extrinsic parameters Θ_e defined here by the luminosity distance d_L , the polar and azimuthal sky location angles, (θ_S, ϕ_S) , and the polar and azimuthal orientation angles (θ_K, ϕ_K) that determine the orientation of the primary spin (both sets of angles are expressed with respect to the Solar System barycenter reference frame [102]). We have checked using Fisher matrices and Markov Chain Monte Carlo (MCMC) that our results are unchanged when using the full LISA response and an early implementation of the relativistic amplitudes.

To forecast the constraints on the scalar charge with EMRI observations, we sample over all the intrinsic Θ_i and extrinsic parameters Θ_e of a Kerr equatorial eccentric EMRI signal with scalar charge. We obtain the 13-dimensional posterior distribution using the package Eryn [116], which provides a Bayesian inference tool based on MCMC sampling. The technical details about the likelihood, priors, and sampling techniques are extensively discussed in Appendix C. This is the first appearance of a complete Bayesian parameter inference of EMRI systems in these types of orbits.

²Note that the scalar fluxes $\dot{E}_{\text{scal}}$ and $\dot{L}_{\text{scal}}$ derived from the black hole perturbation toolkit and in [107] must be divided by a factor 4, to account for a different normalization of the scalar field used in the action (1).

³In this study, we did not use the relativistic version of FEW since that version of the model is still under revision.

IV. RESULTS

We consider eight different orbital configurations, specified by their component masses, the primary dimensionless spin, the initial eccentricity, and semilatus rectum. We assume binaries evolve in the LISA band, plunging over a period T . This fixes the initial semilatus rectum p_0 . For each system, the luminosity distance is fixed to give $\text{SNR} = 50$. The number of EMRIs detectable by LISA ranges from a few to hundreds per year [20], including tens of events with SNRs between 100 and 1000.

We first focus on *agnostic* forecasts of new fundamental fields, assessing LISA's ability to constrain the scalar charge. To this aim, we study the case in which the injected signal is modeled in GR, i.e., assuming $d = 0$, while the recovery template includes the scalar charge. This setup allows us to investigate the upper bound (or constraint) on d , which we define as given by the upper 95% credible interval of the corresponding marginalized posterior.

Figure 1 shows histograms of the marginalized posteriors of the scalar charge for the EMRIs we considered (see Fig. 6 of Appendix C for the full posterior).

Bounds on d are tighter for large eccentricity and primary dimensionless spin. For fixed component masses and evolution time, doubling the eccentricity from $e_0 = 0.2$ to $e_0 = 0.4$ yields a 10% stronger bound on d (cf. systems 6 and 7 in Fig. 1). We find the same level of improvement when increasing the BH dimensionless spin from $a = 0.8$ to $a = 0.95$ (systems 5 and 6). However, the precision

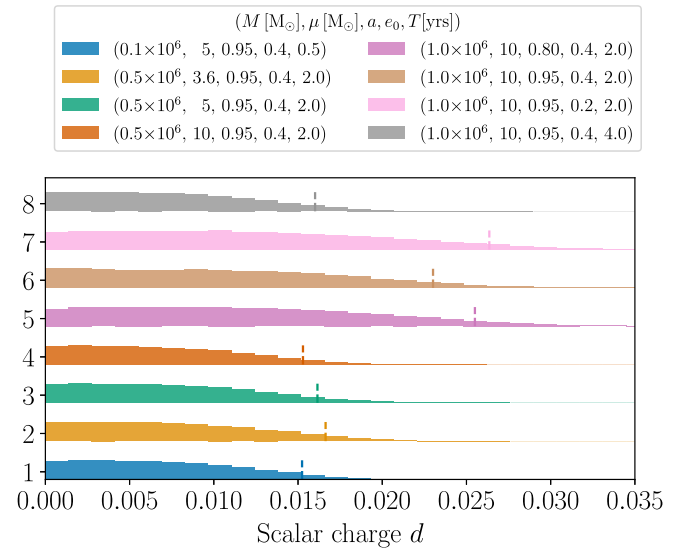


FIG. 1. Histograms of posterior samples of the scalar charge inferred by LISA observations of EMRIs with different orbital configurations of central black hole mass M and dimensionless spin a , compact object mass μ , initial eccentricity e_0 , and time to plunge T . The colored vertical dashed lines show the one-sided 95% credible interval of the distribution. All EMRI systems are characterized by an SNR of 50.

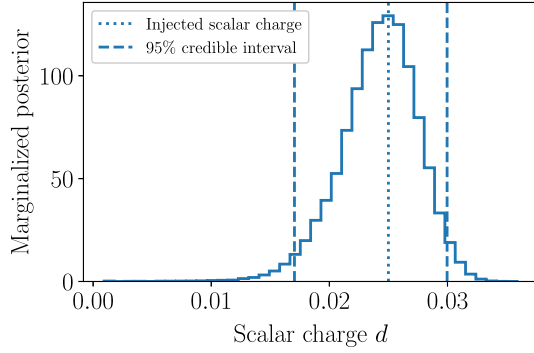


FIG. 2. Marginalized posterior distribution of an EMRI system with scalar charge $d = 0.025$ and source parameters $M = 10^5 M_\odot$, $\mu = 5 M_\odot$, $a = 0.95$, $e_0 = 0.4$, $T = 2$ yrs, and $\text{SNR} = 50$. The estimated median and 95% credible interval are $0.0244^{+0.006}_{-0.007}$.

might vary differently for lower eccentricity binaries, see for instance, Fig. 9 of [74].

We also find that increasing the mass ratio provides narrower posteriors. Assuming $\mu = 10 M_\odot$, if we reduce the primary mass by a factor of two, we obtain a bound on d that is $\sim 50\%$ tighter (systems 4 and 6). This is primarily because a less asymmetric system plunges faster. For a fixed evolution time, such binaries have larger initial orbital separations, where the effect of scalar emission is stronger. To illustrate this point, we analyze a system with component masses $M = 10^5 M_\odot$ and $\mu = 5 M_\odot$ (system 1). This system has the largest initial semilatus rectum $p_0 \approx 16$ and, although we consider only the last half a year before the plunge for computational reasons, it yields the best 95% upper bound on d , $d_{95\%} = 0.015$. Fixing the intrinsic source parameters, the measurement precision improves for EMRIs evolving over a longer timescale. For a $10^6 M_\odot + 10 M_\odot$ system, doubling of T improves the constraint on d by 60% and increases the number of cycles from 1.1×10^5 to 1.7×10^5 and the semilatus rectum from $p_0 = 8.34$ to $p_0 = 10$.

We now explore the case in which *both* the injected and the recovery waveforms have a nonvanishing scalar charge. We inject a signal with $d = 0.025$, consistent with the upper bound from GW230529 ($d \approx 0.035$), and study constraints on the charge for a $10^5 M_\odot + 5 M_\odot$ EMRI, with the same orbital parameters as system 1 in Fig. 1. This binary provides a measurement of the charge accurate to $\sim 10\%$, with median and 95% credible intervals of $d = 0.0244^{+0.006}_{-0.007}$. The marginalized posterior of d for this system is shown in Fig. 2.

For the same system, we also explore the impact of ignoring the scalar charge and fitting the data with a GR template (see Appendix F for further details). We find that the GR waveform recovers the injected signal with $2 - 3\sigma$ biases in the source intrinsic parameters, i.e., its masses and spins. While such systematic errors are large compared to

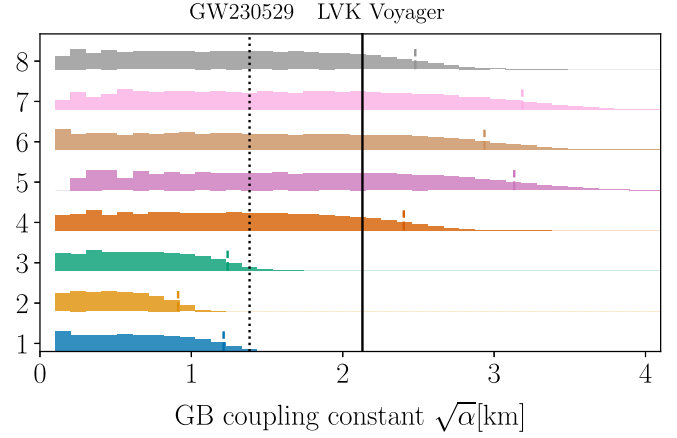


FIG. 3. Posterior distribution of the Gauss Bonnet coupling mapped from the scalar charge constraints (see Fig. 1). The black dotted line in the right panel shows the bound [117] from the observation of the gravitational wave event GW230529 [7], while the black solid line shows the best forecasted bound for the LVK Voyager configuration obtained in [118]. We use a different normalization with respect to [117,118] (see Appendix A).

the size of the posterior, they can be considered small for astrophysically motivated studies. For instance, the central black hole dimensionless spin a estimated with a GR template is shifted by $\approx 4 \times 10^{-5}$ from the injected value $a = 0.95$.

To illustrate how a bound in d can be converted to a constraint on a specific theory, we now consider GB gravity defined by the action in Eq. (8). The forecasted constraints on the coupling constant are shown in Fig. 3. An interesting feature is that the strongest bound for $\sqrt{\alpha}$ comes from system 2, while the strongest bound for d came from system 1. This is because the relation between d and α involves the mass of the secondary.

Selecting a specific theory also allows for a comparison between bounds from EMRIs and bounds from other systems. The analysis of the event GW230529 [7,117,119] yielded $\sqrt{\alpha}_{95\%} = 1.4$ km, which is a few percent larger than the EMRI constraint obtained with systems 1, 2, and 3. Interestingly, the forecasted best constraint on $\sqrt{\alpha}$ for LVK Voyager is larger (see extremal bound from Fig. 21 of [118]). The initial observational frequency of GW230529 is $f = 20$ Hz $\approx 10^{-4} M_\odot$, and the total system mass is $M_{\text{tot}} = 5.1 M_\odot$, and its $\text{SNR} = 11.1$ [7]. The dimensionless velocity of GW230529 is $v = (\pi M_{\text{tot}} f)^{1/3} \approx 0.117$, whereas the same initial velocity for the lowest total mass EMRI system, $10^5 M_\odot + 5 M_\odot$, is $v = (\Omega_\phi)^{1/3} \approx 0.23$ (see Appendix D for details on the velocity definitions). This means that GW230529 is in a weaker field compared to the EMRI systems we considered. In fact, since scalar emission becomes more significant at lower frequencies due to its leading dipole character, we expect that considering nonplunging EMRIs with an initial dimensionless velocity of $v \approx 0.1$ and $p_0 \approx 60$ would

further strengthen EMRI constraints. For the same reason, intermediate-mass ratio inspirals with sufficiently small secondaries could provide even tighter constraints on fundamental fields. This is because their observed inspiral can begin in much weaker field regions, i.e., at larger orbital separations.

V. DISCUSSION

In this work, we produced the first ready-to-use fully relativistic EMRI waveforms in theories of gravity that include a massless scalar field and perform the first Bayesian analysis of EMRI signals in this context. The phase of our waveforms is modeled at the adiabatic order within the flux accuracy and includes a single extra parameter, the scalar charge of the EMRI secondary. This is the only parameter needed to capture the effect of the scalar field at this order [52].

Our study provides Bayesian methods and gravitational wave models to investigate the fundamental physics potential of single EMRI systems in future LISA observations. These are two key objectives outlined in the LISA definition study report [12] and the fundamental physics white paper [3]. We also provide an approximate comparison of EMRI constraints on agnostic PN deviations in the phase in Appendix D, where we find that EMRIs provide three orders of magnitude improvement compared to current detectors.

We forecast the expected bound on the scalar charge of the secondary, d , in a theory-agnostic manner. Bounds on d can be translated to bounds on the coupling constant(s) or a particular theory. We have demonstrated this for a specific example, that of linear-Gauss-Bonnet gravity. Focusing on a specific theory allowed us to compare our forecast with existing bounds and forecasts for the next generation of GW detectors for this particular theory. One of the caveats of this comparison is that our exploration of the parameter space is not exhaustive. As such, it is not clear if the EMRI parameters we have considered are the ones that would yield the most stringent bounds for either d or for the coupling constant of some particular theory. We plan to address this question in future work.

There are several ways in which this work can be extended. Our waveform model can be extended in several directions. Realistic EMRIs are expected to follow generic, inclined orbits [56]. Studies of EMRIs with new fundamental fields evolving on eccentric and inclined orbits are underway (see [120] for circular inclined results). Inclusion of postadiabatic corrections, along the lines of Ref. [71], is also essential to include known GR effects that enter at postadiabatic order, such as the secondary spin [115], and to assess how new fundamental physics can affect the waveform at this order. Our approach can also be generalized to capture massive scalars [55] and more general vector/tensor fields [65]. Finally, the presence of environmental effects in realistic EMRI signals might affect the

detectability of the scalar charge through parameter correlations [121], so further joint studies are required. Future studies will need to incorporate generic waveforms and more comprehensive modeling of environmental and beyond-GR effects.

Our Bayesian analysis could be extended to account for the presence of multiple sources, as expected in the LISA data stream. Future gravitational wave inference pipelines will need to explore how tests of fundamental physics can be performed within a global fit framework [122–125]. In particular, it will be important to assess how the presence of additional sources affects the constraints obtained in single-source scenarios such as the one presented here. More importantly, these pipelines must evaluate how the detectability of deviations from general relativity may be influenced by the global fit procedure.

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The author contributions statement describes the contributions of individual authors referred to by their initials, and, in doing so, all authors agree to be accountable for the content of the work (see [129] for more information and [130] for the taxonomy of contributor roles). L. S.: Conceptualization, Data Curation, Formal Analysis, Investigation, Methodology, Project Administration, Resources, Software, Supervision, Validation, Visualization, Writing—Original Draft. S. B.: Data Curation, Investigation, Resources, Software, Writing—Review and Editing. A. M.: Conceptualization,

Methodology, Writing—Original Draft, Writing—Review and Editing, Supervision, Project Administration. T. P. S.: Conceptualization, Methodology, Writing—Original Draft, Writing—Review and Editing, Supervision, Project Administration. N. W.: Data Curation, Software, Writing—Review and Editing. M. V. d. M.: Methodology, Software, Writing—Review and Editing. A. J. K. C.: Conceptualization, Methodology, Writing—Review and Editing. O. B.: Investigation, Cross-Validation (MCMC & FM studies), Data curation, Writing—Review and Editing. J. G.: Methodology, Writing—Review and Editing, Supervision.

APPENDIX A: NORMALIZATION OF THE ACTION

We provide here details on the mapping between constraints on the Gauss Bonnet coupling parameter inferred in this paper and from current and future ground-based detectors.

To compare bounds on the Gauss Bonnet coupling α , we need to take into account the different normalizations considered in the literature for the actions (1) and (8). In particular, constraints derived from current GW observations [131] and by the network of future detectors [118] are obtained by assuming the following action for GB gravity:

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{\kappa} - \frac{1}{2} (\nabla \bar{\phi})^2 + \bar{\alpha}_{\text{GB}} \bar{\phi} \mathcal{G} \right], \quad (\text{A1})$$

where $\kappa = 16\pi$, and the scalar field $\bar{\phi}$ and constant $\bar{\alpha}_{\text{GB}}$ are related to the action considered in this work,

$$S = \int d^4x \frac{\sqrt{-g}}{\kappa} \left[R - \frac{1}{2} (\nabla \phi)^2 + \frac{\alpha}{4} \phi \mathcal{G} \right], \quad (\text{A2})$$

via the relations $\bar{\phi} = \phi \kappa^{-1/2}$ and $\alpha = 4\kappa^{1/2} \bar{\alpha}_{\text{GB}}$. Using this last equation, we provide the conversion of the bounds presented in the literature to our conventions. The constraint available for GB gravity of [131], $\sqrt{\bar{\alpha}_{\text{GB}}} \simeq 1.18$ km, translates to $\sqrt{\alpha} = 4\pi^{1/4} \sqrt{\bar{\alpha}_{\text{GB}}} \simeq 6.3$ km and the constraints from GW230529 $\sqrt{\bar{\alpha}_{\text{GB}}} \simeq 0.260$ km $\rightarrow \sqrt{\alpha_{\text{GB}}} \simeq 1.4$ km [117]. Similarly, for the projected bound by LIGO O8, $\sqrt{\bar{\alpha}_{\text{GB}}} \simeq 0.4$ km (see Fig. 21 of [118]), we obtain $\sqrt{\alpha} \simeq 2.1$ km.

The map between the scalar charge and the GB coupling has been obtained in [84], using the action

$$S_J = \int \frac{\sqrt{-g}}{\kappa} d^4x [R - 2(\nabla \tilde{\phi})^2 + \tilde{\alpha} \tilde{\phi} \mathcal{G}]. \quad (\text{A3})$$

In this setup, at the leading order in $\tilde{\alpha}$, the scalar charge is given by $\tilde{d} = \tilde{\alpha}/(2\mu^2)$. Passing to the normalization, we use in Eq. (A2), $\tilde{d} = d/2$ and $\tilde{\alpha} = \alpha/2$, which leaves the relation $d = d(\alpha)$ unchanged.

APPENDIX B: FLUX INTERPOLATION

Energy and angular momentum fluxes are interpolated over a three-dimensional grid constructed using 13 Chebyshev-Gauss-Lobatto (CGL) nodes for the eccentricity $e \in [0.0, 0.5]$ and the primary dimensionless spin $a \in [-0.99, 0.99]$. Rather than using the semilatus rectum, we find it more convenient to introduce the variable

$$u = (1 + e) \left[\frac{\Omega_\phi(a, p, e)}{\Omega_\phi(a, \frac{p_{\text{sep}}(a, e)}{1+e}, e)} \right]^{2/3}, \quad (\text{B1})$$

where $p_{\text{sep}}(a, e)$ is the separatrix for Kerr spacetime, and negative spins correspond to retrograde orbits. We compute u on 17 CGL nodes within $u \in [0.08, 0.97]$. Therefore, we calculate scalar and gravitational fluxes on a total of $13 \times 13 \times 17 = 2873$ grid points.

Before interpolation, we normalize the fluxes by their leading order contribution in a post-Newtonian expansion. We construct 4 Chebyshev interpolants for the gravitational and scalar energy and angular momentum fluxes. Assuming the fluxes are smooth functions of the variables on our interpolation domain, the accuracy of the Chebyshev interpolation should converge exponentially with the number of grid points. Consequently, we can estimate the interpolation “aliasing” error from the magnitude of the coefficients of the highest order Chebyshev polynomials in each direction. The interpolated quantities are the dimensionless scaled fluxes $\dot{\tilde{E}} = \dot{E}/q^2$, $\dot{\tilde{L}}_r = \dot{L}_r/(q^2 M)$, and their Chebyshev errors σ are shown in Table I.

If we write the total energy flux as

$$\dot{\tilde{E}} = \frac{32}{5p^5} \left(\frac{5p^5}{32} \dot{\tilde{E}}_{\text{grav}} + \frac{5p^5}{32} d^2 \dot{\tilde{E}}_{\text{scal}} \right), \quad (\text{B2})$$

we can estimate the size of the scalar charge that is comparable to the error in the gravitational fluxes,

$$\frac{5p^5}{32} \dot{\tilde{E}}_{\text{scal}} d^2 > \sigma_{\text{grav}} \rightarrow 0.12 d^2 > \sigma_{\text{grav}}, \quad (\text{B3})$$

where we inserted $p^4 \dot{\tilde{E}}_{\text{scal}} = 0.3$ which is a typical value across the grid for $p = 10$. This gives a relation between the

TABLE I. Chebyshev interpolation of the energy and angular momentum fluxes and the estimates of their absolute interpolation errors.

Interpolated expression	Abs. error
$\frac{5}{32} p^5 \dot{\tilde{E}}_{\text{grav}}$	7.5×10^{-4}
$\frac{5}{32} p^{7/2} \dot{\tilde{L}}_{\text{grav}}$	7.1×10^{-4}
$p^4 \dot{\tilde{E}}_{\text{scal}}$	2.3×10^{-4}
$p^{5/2} \dot{\tilde{L}}_{\text{scal}}$	2.2×10^{-4}

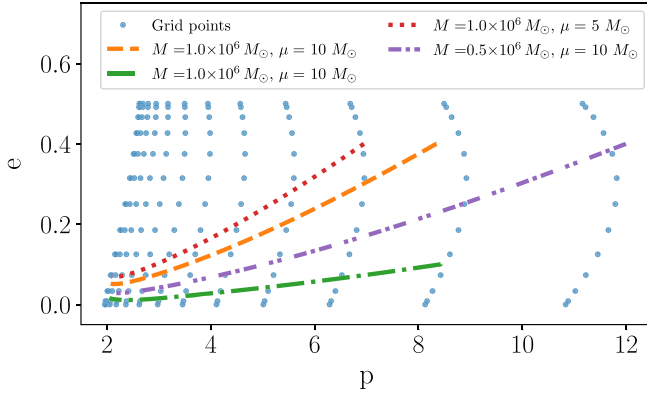


FIG. 4. Trajectory evolution of semilatus rectum and eccentricity for four EMRIs with different component masses and $a = 0.95$. For reference, we show the location of the grid points used for interpolation in the (p, e) plane for a constant dimensionless spin slice at $a = 0.9562665680261776$, where the largest value of p reached for this dimensionless spin is approximately $p \approx 30$.

scalar charge and the Chebyshev interpolation. For values of $d = 0.01$, we obtain a constraint on the error of the order 10^{-5} , which is one order of magnitude smaller than what we obtained with our interpolation scheme. This does not invalidate our work, as we treat our interpolated fluxes as the true fluxes and use them consistently for injection and recovery. However, this does highlight the need for denser flux grids and accurate interpolation schemes. The production of dense self-force grids with accurate interpolation methods is a current major challenge for extending the FastEMRIWaveforms package to fully generic orbits in Kerr spacetimes.

Using the Gremlin code for flux calculations evaluated at $a = 0.95$, $p = 10.1930405906075$, and $e = 0.4081632653061225$, we obtain a flux value for $\frac{5}{32} p^5 \ddot{E}_{\text{grav}}$ which differs from our interpolation by absolute and relative errors of 3.8×10^{-4} , 3.6×10^{-4} , respectively. This is compatible within our estimated interpolation error. Given the recent work [104], we compared the fluxes at $p = 10.0$, $e = 0.4$, $a = 0.5$ and find a relative difference of 7×10^{-5} which can be compared to Fig. 5 of [104].

Having the fluxes in hand, we can obtain the right hand side of the Eq. (9) and then use the FEW package to obtain the EMRI trajectory. As an example, we show in Fig. 4 the trajectories in the $p-e$ plane for binaries with $a = 0.95$ in GR, i.e., setting the scalar charge to zero, and different EMRI masses. For reference, we also show the Chebyshev grid points at $a = 0.9562665680261776$.

APPENDIX C: DATA ANALYSIS SETUP

The posterior distributions presented in this work are obtained using MCMC sampling with the package Eryn [116,132]. To run the MCMC, we need to specify the priors $p(\Theta)$ and the likelihood function

TABLE II. Prior distributions on the waveform parameters used for MCMC posterior sampling.

Parameter	Priors ($\delta = 0.01$)
$\ln M/M_\odot$	Uniform $[\ln M^*(1-\delta), \ln M^*(1+\delta)]$
$\ln \mu/M_\odot$	Uniform $[\ln \mu^*(1-\delta), \ln \mu^*(1+\delta)]$
a	Uniform $[a^*(1-\delta), 0.98]$
p_0	Uniform $[p_0^*(1-\delta), p_0^*(1+\delta)]$
e_0	Uniform $[e_0^*(1-\delta), e_0^*(1+\delta)]$
d_L [Gpc]	Power law $[0.01, 10.0]$
$\cos \theta_S$	Uniform $[-0.99999, 0.99999]$
ϕ_S	Uniform $[0.0, 2\pi]$
$\cos \theta_K$	Uniform $[-0.99999, 0.99999]$
ϕ_K	Uniform $[0.0, 2\pi]$
Φ_{ϕ_0}	Uniform $[0.0, \pi]$
Φ_{r_0}	Uniform $[0.0, 2\pi]$
Λ	Uniform $[-0.6, 0.6]$

$$\ln p(s|\Theta) = -\frac{1}{2} \langle s - h(\Theta) | s - h(\Theta) \rangle, \quad (\text{C1})$$

where we defined the inner product between two GW templates $h_1(t)$ and $h_2(t)$ [133],

$$\langle h_1(t) | h_2(t) \rangle = 4\text{Re} \int_0^\infty \frac{\tilde{h}_1^*(f) \tilde{h}_2(f)}{S_n(f)} df. \quad (\text{C2})$$

The noise spectral density $S_n(f)$ of LISA is taken from [134] and assumed to be known. The tilde denotes the Fourier transform of the waveform, and the symbol $*$ denotes complex conjugation. The Fourier transform and likelihood evaluation are performed on GPUs using cupy [135]. Before passing to the frequency space, we taper $h(t)$ with a Tukey window. The parameter that controls the magnitude of the sinusoidal lobes of the window has been fixed to $\alpha = 0.005$ [136]. The sampling interval was

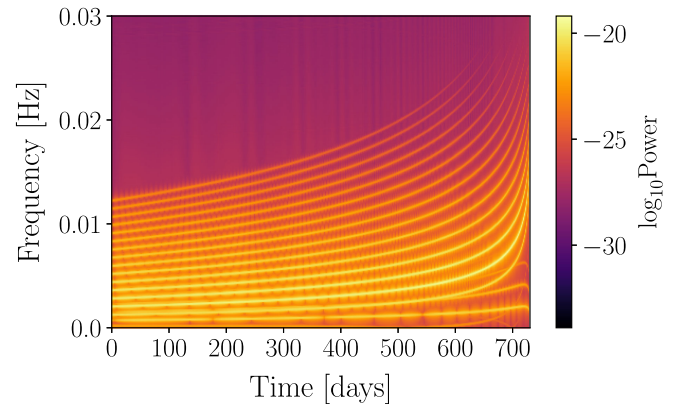


FIG. 5. Spectrogram of the gravitational wave signal output of the AAK waveform obtained from an EMRI system with parameters $M = 10^6 M_\odot$, $\mu = 10 M_\odot$, $e_0 = 0.4$, $d = 0.0025$, $p_0 = 8.3$, $T = 2$ yrs. The different color bands represent the different harmonics, and their color intensity represents their power.

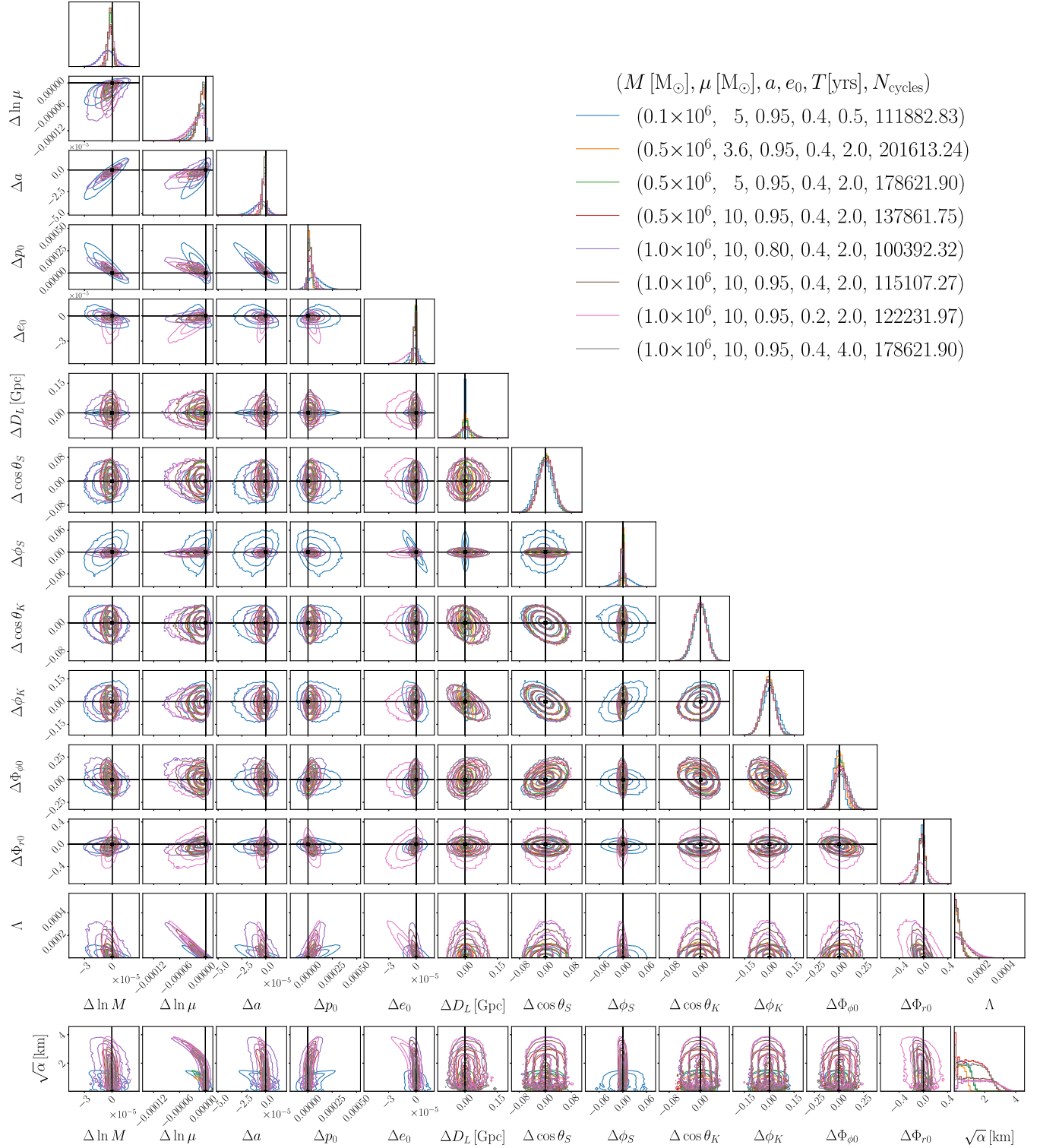


FIG. 6. Posterior distribution of EMRI injections with different orbital configurations. The posteriors are centered around the injected parameters. Diagonal and off-diagonal plots provide marginalized and 2D-joint posteriors, respectively. Contour lines in off-diagonal panels identify the 1, 2, 3- σ Gaussian credible contours of each distribution. The upper corner plot shows the posterior distribution output of the MCMC analysis in the parameters of Table II. The lower corner shows how the list row of the upper corner plot transforms when mapping to the coupling $\sqrt{\alpha}$. In the legend, we provide the system parameters and the number of orbital cycles performed N_{cycles} . A downloadable version of these plots can be found [138].

adjusted for different black hole masses M to avoid aliasing.

The full parameter space of the AAK model is given in Table II. The response of the detector to the signal is described by the two data channels $h_I(t; \Theta)$ and $h_{II}(t; \Theta)$. The full log-likelihood is given by the sum of the log-likelihood for each channel. For reference, we show in Fig. 5 the spectrogram of h_I for a system with parameters $M = 10^6 M_\odot$, $\mu = 10 M_\odot$, $e_0 = 0.4$, $d = 0.0025$, $p_0 = 8.3$, $T = 2$ yrs.

We assume flat priors for all parameters apart from the luminosity distance, which is assumed to follow a power-law distribution with slope -2 in the range $[0.01, 10.0]$ Gpc. A summary of the priors we consider is given in Table II. We restrict the prior in Φ_{ϕ_0} to π and not to 2π because there is an exact degeneracy every π . The prior choice of power law with slope -2 for the luminosity distance is motivated by the fact that we found some chains getting stuck at large unphysical values of d_L . This was caused by chains getting stuck on secondary modes of the likelihood. Our choices allow for better sampling efficiency without affecting the results. For parameters that are typically constrained with high precision, i.e., (M, μ, a, p_0, e_0) , we center the priors around the true injected values Θ_{true} . In all runs, we find that the posterior support of all parameters is much tighter than the assumed prior.

Here, we provide a motivation for sampling in $\Lambda = d^2$. The deviation due to the scalar emission enters the fluxes and the dephasing from vacuum as d^2 (see Fig. 8 and discussion in Appendix E). This means that the waveform difference between a scalar charge template and a vacuum one is approximately $h(d \neq 0) - h(d = 0) \approx h(d = 0)e^{i\Delta\Phi}$ where the dephasing $\Delta\Phi$ is a small number proportional d^2 . Therefore, the log-likelihood becomes approximately Gaussian in d^2 . For this reason, we sample in the parameter $\Lambda = d^2$ with a uniform prior $\mathcal{U}_{[-0.6, 0.6]}$ and then select the samples with $\Lambda > 0$. Sampling in Λ , both positive and negative, allows us to obtain near-Gaussian posteriors, which can be sampled more efficiently with MCMC methods and the chosen proposals. We also verified that sampling Λ with a uniform positive prior would not change the posterior. However, we found that this would require running the MCMC for longer to accumulate a larger number of effective samples. We use two proposals to efficiently sample: the stretch move [132] and an adaptive metropolis move that jumps along the eigen directions of the covariance matrix. We sample the posteriors using 26 walkers, monitoring the integrated autocorrelation time τ as a function of the iteration. We assume that the estimator for τ is reliable when it plateaus below the line given by $N_{\text{it}}/50$ for N_{it} the number of iterations (see [137] for further details). We show in the upper corner plot of Fig. 6 the full posterior distribution used for the constraints obtained in Figs. 1 and 3.

To obtain a bound on this specific theory from the posteriors on $\Lambda = d^2$ and $\ln \mu$, we use

$$\sqrt{\alpha} = \sqrt{2}\mu\sqrt{d} = \sqrt{2}\mu\Lambda^{1/4}, \quad (\text{C3})$$

and the determinant of the Jacobian of such transformation

$$d\sqrt{\alpha} = \left| \frac{\partial \sqrt{\alpha}}{\partial \Lambda} \right| d\Lambda \propto \mu \Lambda^{-3/4} d\Lambda. \quad (\text{C4})$$

Similarly, one can obtain the bound on the scalar charge d . In the lower corner plot of Fig. 6, we can see how the posterior samples are mapped to the coupling $\sqrt{\alpha}$. The posteriors are non-Gaussian and have long tails. This is not only due to the quartic root relation between $\sqrt{\alpha}$ and Λ but also due to the strong correlations of Λ with the secondary mass μ . In fact, the deviations from vacuum in the fluxes and phases are both proportional to μ and Λ . This demonstrates the importance of sampling in Λ instead of $\sqrt{\alpha}$. Sampling in the latter requires longer iterations to reach convergence and to resolve the tails of the “banana-shaped” distributions.

APPENDIX D: MAPPING TO AGNOSTIC BOUNDS

To compare the potential of EMRIs to constrain deviations from general relativity for agnostic parametrization, we provide a comparison in terms of the parametrized post-Einsteinian (ppE) formalism or flexible theory independent formalism [15, 139–146]. We consider the system with $10^6 M_\odot + 10 M_\odot$ solar masses, initial eccentricity $e_0 = 0.4$, and time to plunge $T = 2$ years and run a GR inspiral. We obtain the gravitational energy fluxes $\dot{E}_{\text{grav}}$ from the samples obtained, evaluated at the start of the inspiral. If we subtract the median value from this set of samples, and divide by the median, we obtain a set of samples representing the fractional deviation from the expected energy dissipation in GR, that are consistent with the posterior. These can be related to ppE deviations by writing

$$\Delta \dot{E} / \dot{E}_{\text{grav}} = B v^{2n}, \quad (\text{D1})$$

where $v = (\pi(M + \mu)f)^{1/3}$ with f frequency of the $(l, n_\phi, n_r) = (2, 2, 0)$ and n the post-Newtonian (PN) order. For EMRIs we can approximate $v = \Omega_\phi^{1/3}$. The quantity B can be mapped to an “agnostic” deviation in the waveform phase $\delta\varphi$ at different PN orders using the formalism described in [144] (see Eqs. (9)–(11) and (19)–(28) [144] for the mapping $B \rightarrow \delta\varphi$). We provide constraints on $\delta\varphi$ obtained with this procedure as “GR mapping” in Fig. 7. Constraints are 95% upper limits obtained from the posterior on $\delta\varphi$.

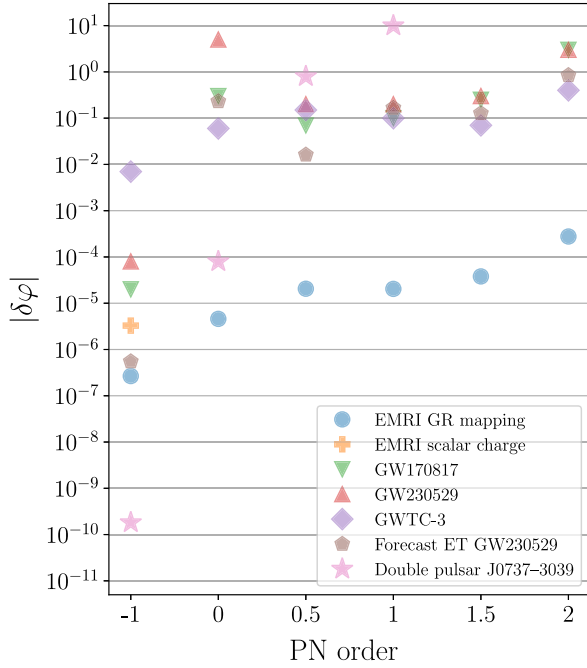


FIG. 7. Comparison of the constraints on the phase deviation at different PN orders. The EMRI constraints are obtained from mapping the posterior distribution of a system with parameters $M = 10^6 M_\odot$, $\mu = 10 M_\odot$, $a = 0.95$, $e_0 = 0.4$, $T = 2$ yrs, and $\text{SNR} = 50$ into the phase deviation at different PN orders (blue dots, EMRI GR mapping). For the case of -1 PN order we use the posterior distribution obtained from an EMRI embedded in a scalar field (orange cross) as done in Fig. 1. We show the current constraints obtained from the gravitational wave events GW170817 (down green triangle) [147], GW230529 (up red triangle) [119], and using the GW transient catalogs of the third observing run (GWTC-3) (violet rombo) [4]. Brown pentagon markers correspond to forecasts on $\delta\phi$ obtained for a GW230529-like binary observed by ET (see main text). The double pulsar constraints obtained from PSR J0737-3039 [148] are shown with pink star markers.

We remark that this analysis is approximate for three reasons. First, post-Newtonian expansions do not provide a good description of the evolution of EMRIs. Second, this mapping only considers deviations in the waveforms that can be described by changes in the GR parameters. While any GR deviation causing such a change would definitely not be detectable, larger deviations could also be undetectable, since they are clearly strongly correlated with changes in the EMRI parameters. In that sense, this should be considered an optimistic bound. We note that this method cannot assess the detectability of components of the deviation that change the waveforms in ways that are orthogonal to the GR waveform space, but we expect these to be subdominant. Third, we are mapping an eccentric inspiral to a quasi-circular ppE parametrization.

For comparison, we also provide the mapping between the agnostic approach and the scalar charge at the -1 PN order, which would correspond to the leading contribution

of our full adiabatic scalar emission. In this case, the parameter B is given by [149]

$$B = d^2 \Omega_\phi^{2/3} \dot{E}_{\text{scal}} / \dot{E}_{\text{grav}}. \quad (\text{D2})$$

In Fig. 7, we show the constraints obtained using the GR mapping at different PN orders for the EMRI configuration we considered (blue dots). The scalar charge mapping shows degradation of one order of magnitude (orange cross) at the -1 PN order. This demonstrates the importance of correlations that are not taken into account in the GR mapping.

The constraints obtained from the gravitational wave events GW170817 [147], GW230529 [119], and using the GW transient catalogs of the third observing run (GWTC-3) [4] are also shown, and these are a few orders of magnitude larger than the EMRI constraints. This is expected since EMRIs complete $10^4 - 10^5$ cycles during the observation, which is two to three orders of magnitude than the number of cycles typically observed in a merger seen by ground-based detectors, leading to correspondingly higher measurement precision. However, better phase constraints do not necessarily imply tighter bounds on the coupling, as this map depends on the specific theory considered. This is the case for GB gravity for which constraints improve for lighter objects, as in the case of GW230529 (see vertical line Fig. 3).

For comparison, we compute bounds on $\delta\phi$ forecasted for a third-generation ground-based detector like the Einstein Telescope (ET). Constraints are derived through a Fisher matrix approach, analysing the inspiral phase of a binary BH system with the same properties as GW230529 (the source parameters are fixed to the median values reported in [150]). For the analysis, we consider a TaylorF2 waveform model, integrated between 3 Hz and the Schwarzschild ISCO.⁴ The waveform depends on seven parameters, $(\mathcal{M}, \eta, t_c, \phi_c, \chi_s, \chi_a, \delta\phi)$, where $\mathcal{M}$ and η are the chirp mass and the symmetric mass ratio, (t_c, ϕ_c) the time and phase at the coalescence, and $\chi_{s,a} = (\chi_1 \pm \chi_2)/2$ combinations of the individual spin components. The phase shift enters the frequency-domain template as $\tilde{h}(f) = A e^{i\varphi_{\text{GR}(f)}} e^{i\psi(f)}$, where

$$\varphi_{\text{GR}} = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128} (\pi \mathcal{M} f)^{-5/3} \sum_{i=0}^9 \varphi_i (\pi M f)^{i/3}, \quad (\text{D3})$$

is the GR phase (see Appendix A of [151] for the explicit form of the PN coefficients φ_i) and

⁴Constraints obtained using the maximum frequency at the Kerr ISCO do not change significantly.

$$\psi = \frac{3}{128\eta^{n/5}} \varphi_n \delta\varphi_n (\pi\mathcal{M}f)^{(n-5)/3} \quad \text{if } \varphi_n \neq 0, \quad (\text{D4})$$

$$\psi = \frac{3}{128\eta^{n/5}} \delta\varphi_n (\pi\mathcal{M}f)^{(n-5)/3} \quad \text{if } \varphi_n = 0. \quad (\text{D5})$$

For the Fisher analysis, we average over the angles that define the source position in the sky and consider Gaussian priors on $\chi_{1,2}$, centered around the injected values, and with unit width. Finally, we assume for ET a single L-shaped detector with 15 km arm length [152].

We also show in Fig. 7 the constraint on dipole emission, inferred from observations of the double pulsar PSR J0737–3039 [148]. Such binaries evolve in a low-dynamical regime with $v \approx 2 \times 10^{-3}$ and provide the tightest bound on the 1PN phase deviation $\delta\varphi$.

APPENDIX E: ACCURACY OF THE TRAJECTORY INTEGRATION

Gravitational wave observations constrain the frequency evolution of the waveform with great precision. Therefore, it is key to check that the phase evolution is not affected by systematic errors, which could influence the parameter reconstruction. We study the accuracy of the numerical integration of the ordinary differential equations (ODEs) (9). These ODEs are solved using [153] with adaptive step size `gsl_odeiv2_step_rk8pd` provided by [154].

First, we cross-checked the phase evolution of the implementation against *Mathematica* on four test trajectories. Then, we investigated the difference of the final phase of an inspiral in GR with an ODE absolute error of 10^{-14} with respect to the final phase of an inspiral with a given scalar charge d and various different ODE absolute errors. We show the results in Fig. 8 for a system with $M = 10^6 M_\odot$, $\mu = 10 M_\odot$, $a = 0.95$, $p_0 = 8.343242843079224$, $e_0 = 0.4$ until its plunge. For small scalar charges, $d < 3 \times 10^{-6}$, the phase difference is determined by the ODE solver's noise floor. On the contrary, the value of the phase difference is independent of the ODE error for large scalar charges $d > 3 \times 10^{-6}$, and it follows the d^2 scaling expected by an expansion of the

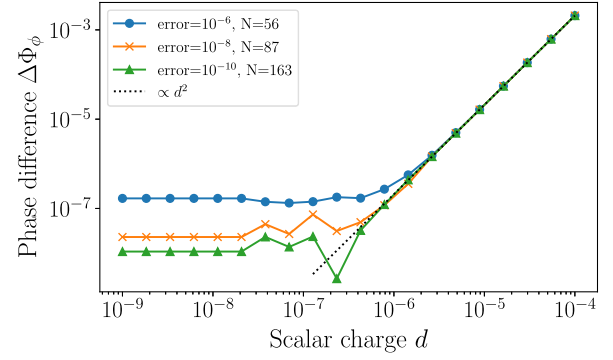


FIG. 8. Difference in the final phase between an EMRI evolution obtained with a nonzero scalar charge and various ODE errors (see legend) and an EMRI evolution in GR with an ODE error of 10^{-11} . For reference, we provide the number of points N taken from the ODE integrator.

phase difference for small d . This expansion is a good approximation up to $d \approx 1$. The ODE error adopted in this work was 5×10^{-10} .

APPENDIX F: SYSTEMATIC BIAS DUE TO NONZERO SCALAR CHARGE

In this appendix, we provide further details on the systematic bias that could potentially affect EMRI analyses due to the mismatch between a GR recovery template and a signal with a nonzero scalar charge. This is critical as the charge can influence the binary phase evolution and bias parameter estimation. To investigate such bias, we inject an EMRI signal with $d = 0.025$ and source parameters $M = 10^5 M_\odot$, $\mu = 5 M_\odot$, $a = 0.95$, $e_0 = 0.4$, $T = 2$ yrs. We analyse the data with two templates: (i) a GR waveform and (ii) a waveform model in which the scalar charge is free to vary. The posterior distributions on the EMRI parameters are shown in Fig. 9. We find 2/3 sigma systematic biases in the intrinsic parameters recovered with the GR template. Instead, extrinsic parameters present biases smaller than 1 sigma, and are less affected by the presence of the scalar charge. The best log-likelihood point obtained with the GR template is $\ln p(s|\Theta) \approx -9$.

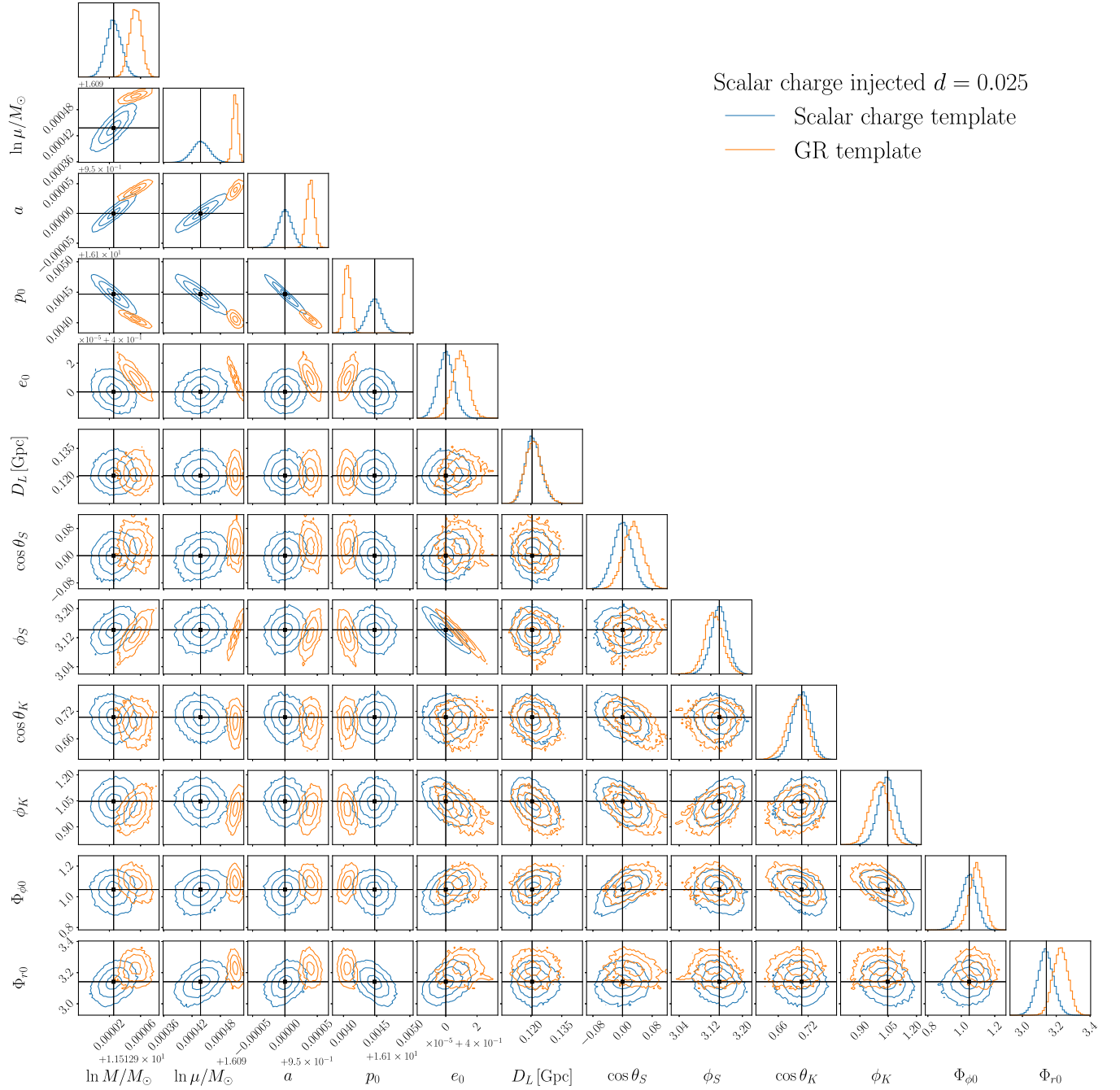


FIG. 9. Posterior distribution of an EMRI system with scalar charge $d = 0.025$ and source parameters $M = 10^5 M_\odot$, $\mu = 5 M_\odot$, $a = 0.95$, $e_0 = 0.4$, $T = 2$ yrs, and $\text{SNR} = 50$. The scalar charge template (blue) correctly recovers the injected parameters, whereas the GR template is biased due to the nonzero scalar charge $d = 0.025$. A downloadable version of this plot can be found [here](#).

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