

Flavor and CP symmetries in the standard model effective field theory

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The CP properties of effective operators are closely related to the spacetime and internal symmetries, such as the flavor symmetry, in the standard model effective field theory (SMEFT). In this work, we utilize the flavor symmetry to organize and reduce numbers of independent Wilson coefficients of the SMEFT operators. We classify the dimension-6 and dimension-8 baryon/lepton-number-conserving operators based on their CP properties. The $U(1)^4$ rephasing symmetry is applied to distinguish CP -violating and CP -odd operators, which leads to reduction of the independent CP -violating phases. The $U(3)^5$ and $U(2)^5$ flavor symmetries let us classify the CP -violating phases as the flavor invariants, which can be enumerated by the Hilbert series and obtained explicitly by the Young tensor method. Then after introducing the minimal flavor violation (MFV) hypothesis, we present the flavor structures of the dimension-6 and dimension-8 SMEFT operators under the MFV hypothesis utilizing the spurion method.

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I. INTRODUCTION

The standard model (SM) is a successful theory describing the strong, weak, and electromagnetic interactions with the electroweak unification at the electroweak scale $\Lambda_{EW} \sim 1$ TeV. The SM is a gauge field theory with the gauge group $SU(3)_c \times SU(2)_{EW} \times U(1)_Y$, and the dynamic fields are the gauge bosons of the gauge group, the three-flavor leptons and quarks, and one scalar field, the Higgs. In the past several decades, the search for new particles around 1 TeV has failed, which implies there is a considerable gap from the SM scale to the new physics scale. Under such a circumstance, the effective field theory (EFT) method is beneficial. The EFT with the gauge symmetry $SU(3)_c \times SU(2)_{EW} \times U(1)_Y$ and the SM fields is the standard model effective field theory (SMEFT). Beyond the renormalizable operators, the dimension-5 operators are first written by Weinberg [1], and the complete and independent dimension-6 operators are first

obtained in Refs. [2,3]. Subsequently, the higher-dimensional operators have been obtained in Refs. [4–11].

Generally, there are huge sets of effective operators at higher mass dimensions because of the complicated flavor structures, so it is important to classify and organize these operators systematically. When flavor symmetry is absent, the operators in the SMEFT Lagrangian are all flavor multiples, and an effective organization of these operators is to utilize the repeated fields and the associated symmetric group representations [7,9]. Because the irreducible representations of the symmetric groups correspond to some Young diagrams, the operators of specific representation can be obtained by the corresponding Young symmetrizer applying on the flavor space. In practice, such a decomposition enables one to write down the independent components of the operator in the flavor space by the semistandard Young tableaux.

The CP violation is important in new physics searches beyond the standard model, especially the matter-antimatter asymmetry. Since the CP -violation phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [12,13] is tiny, the new CP -violation phases in the high-dimension operators of the SMEFT should exist to explain the baryon asymmetry of the universe. The CP -violation effects of the SM are highly suppressed, and thus the experiments could be sensitive to the CP violations of the high-dimension operators. The parity (P) is a spacetime transformation,

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while the charge conjugate (C) is related to the outer automorphism of the internal symmetries of the SMEFT, including the gauge symmetry and other global symmetries. The CP -odd operators are of negative eigenvalues under the CP transformations,

$$CP\text{-odd: } \mathcal{O} \xrightarrow{CP} -\mathcal{O}, \quad (1.1)$$

which could be the combinations of the operators involving the fermions. The literature presents the CP -odd operators of the SMEFT at dimension-6 [14] and the part of the CP -odd bosonic operators of the SMEFT at dimension-8 [15,16]. Nevertheless, the CP -odd operators are not necessarily CP -violating, since the CP -violations depend on whether the complex phase of the Wilson coefficients can be absorbed or not by rephasing symmetry. Only when the CP -violating phases in the corresponding Wilson coefficients cannot be eliminated by the field redefinitions, the CP -odd operators are CP -violating ones, indicating that they are invariant under the rephasing symmetry. The Hilbert series has presented the CP -odd operators and the CP -violating operators of the SMEFT up to dimension-14 with the rephasing symmetry added [17], which implies considerable differences between these two kinds of operators.

The assuming flavor symmetry of the SMEFT can reduce the independent parameters and help to organize the effective operators more concisely. At the same time, the flavor symmetry affects the CP violations of the SMEFT since it is an internal symmetry. On the one hand, the flavor symmetry reduces the independent operators, and, on the other hand, different flavor symmetries lead to different rephasing symmetries. Since the rephasing symmetry is a subgroup of the flavor symmetry, it is satisfied automatically and the CP -odd operators are always equivalent to the CP -violating ones. Because the Yukawa interactions in the SM Lagrangian are not flavor universal, it seems the choices of the flavor symmetries are arbitrary. If it is required, the flavor symmetry can generate the SM Yukawa interactions and suppress the nonstandard flavor-violating effects: the choices are quite limited. In this paper, we consider two flavor symmetries, the $U(3)^5$ symmetry and the $U(2)^5$ symmetry. The $U(3)^5$ is the largest flavor symmetry consistent with the SM gauge symmetry, and the single assumption of the $U(3)^5$ symmetry does not affect the CP -violating phases since the rephasing symmetry is the same with the case without the flavor symmetry, as shown in Fig. 1. Nevertheless, the flavor symmetry develops the Wilson coefficients to the spurions covariant under that, and keeps the Lagrangian flavor invariant,

$$\mathcal{L} \supset c f(\mathbf{T}) \cdot \mathcal{O}(\psi), \quad (1.2)$$

where c are complex coefficients and $\mathbf{T}$ are the spurion. The dot “ $\cdot$ ” implies the spurion part $f(\mathbf{T})$, and the operator $\mathcal{O}$

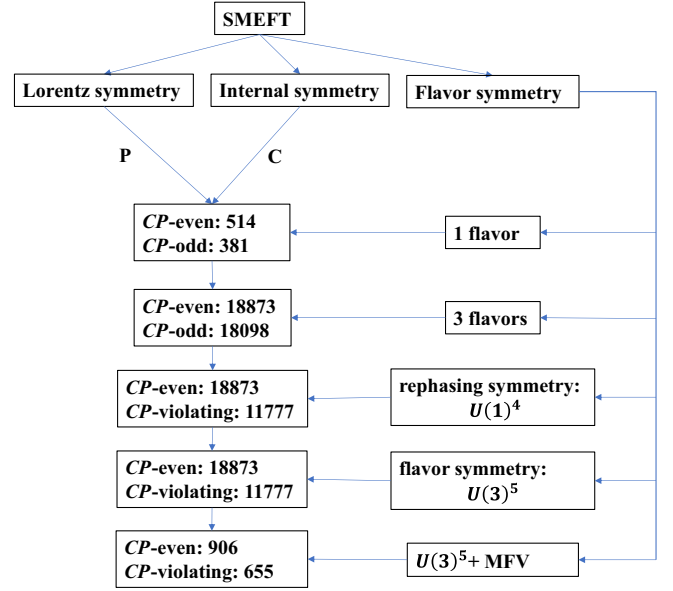


FIG. 1. The logic flow in this paper, where the numbers are of the baryon/lepton-number-conserving dimension-8 operators. The reduction of the CP -violating operators due to the flavor symmetry and the MFV hypothesis is transparent.

composed by the dynamic fields form flavor invariants. Thus the flavor transformations of the Wilson coefficients, which have been developed to the spurions $f(\mathbf{T})$ now, and the operators $\mathcal{O}$ are inverse to each other. Consequently, the assuming flavor symmetry implies that the independent CP -phases of the SMEFT can also be extracted by the flavor invariants [18,19], which claims that the flavor-violating primary invariants linear in the Wilson coefficients correspond to the independent CP -violating phases. Similar arguments are also applied to the seesaw model [20–22] and the SMEFT with sterile neutrinos [23]. The flavor invariants under the $U(3)^5$ symmetry have presented consistent results with the ones obtained from the rephasing symmetry at dimension-6 [17,18]. These flavor invariants form a subset of the primary invariants. The primary invariants are the ones algebraically independent, which means neither themselves nor their products can be expressed by other invariants. The primary invariants correspond to the physical parameters and can be counted by the Hilbert series. In particular, the primary invariants should be distinguished from the basic invariants, which are linearly independent but not algebraically independent. The numbers of the basic invariant usually exceed the numbers of the physical parameters. In practice, we usually construct the basic invariants first and then reduce them to the primary ones. In reality, the construction of the basic invariants is difficult, which exploits the numeric method [18,20–23]. In addition to other methods [24,25], as discussed in Sec. IV, we present a new method utilizing the Young tableaux to construct the basic invariants systematically and analytically.

The flavor symmetry is broken in the SMEFT, and the experiments imply that the flavor violations beyond the leading order (LO) are suppressed. At the LO, the flavor-changing neutral current (FCNC) interactions of the quarks are at the loop level; thus the FCNC interactions are significantly suppressed by the loop integral, the off-diagonal elements of the CKM matrix, and the Glashow-Iliopoulos-Maiani (GIM) mechanics [26] within the SM; and thus they should be sensitive to new physics effects. Nevertheless, the experiments testing the FCNC processes present the consistent results with the SM prediction. For example, the rate of the leptonic decay $K_L^0 \rightarrow \mu^+ \mu^-$, which corresponds to the process $d\bar{s} \rightarrow \mu^+ \mu^-$ at the quark level, is $(6.84 \pm 0.11) \times 10^{-9}$ and is in agreement with the SM prediction [27]. Thus, although the higher-dimension operators present other FCNC processes originating from new physics, their modifications to the SM results should be very small, which constrains the cutoff scale a bound of $O(10^5 \text{ TeV})$ [28]. Nevertheless, the cutoff scale can be pulled down by the minimal flavor-violating (MFV) hypothesis [29–37]. The MFV hypothesis contains two ingredients, the flavor symmetry and a set of symmetry-breaking terms. In the SM, the symmetry-breaking terms are the Yukawa terms, and thus the MFV hypothesis states that all the flavor violation effects in the SMEFT are from the Yukawa terms, which means all the Wilson coefficients are some functions of the Yukawa matrices. The $U(3)^5$ symmetry together with the MFV hypothesis presents strong constraints of the high-dimension flavor-violating contributions, and reduces the independent parameters greatly, while the minimal breaking of the $U(2)^5$ symmetry retains more structures and contains richer phenomena [38–43].

In this paper, we analyze the flavor structures and the CP properties of the SMEFT operators conserving the baryon/lepton numbers of dimension-6 and dimension-8 to organize and classify the large number of operators. We present the SMEFT effective operators of specific CP properties explicitly up to dimension-8, including both the bosonic operators and the fermionic operators. In particular, we consider the three flavors of the fermions. We discuss the relations between the flavor symmetry and the CP properties, and the flavor-invariant description of the CP -violating phases. In particular, we discuss the Young tableaux method to construct the primary invariants. We adopt the $U(3)^5$ symmetry and the MFV hypothesis so that the independent parameters of the effective Lagrangian are reduced. We present the flavor structures of the SMEFT operators of dimension-6 and dimension-8 under the MFV hypothesis. A similar analysis of the dimension-6 operators has been presented in Ref. [44], while the dimension-8 operators have new flavor structures in the classes $\psi^4 \phi^2$ and $\psi^4 \phi D$. For these two classes with new structures, we also present their flavor structures under the $U(2)^5$ symmetry, which retains more operators. To obtain the most

general flavor structures of the operators, we adopt the spurion method, which constructs the flavor-invariant operators via some spurions and has been used in many other theories [45–47]. After the spurions get their vacuum expectation values (VEVs), the MFV operators return to the SMEFT ones. On the other hand, we enumerate the CP -odd operators under the MFV hypothesis, which is equivalent to the CP -violating operators. At the same time, we adopt a specific weak basis, which means the VEVs of the spurions are fixed, the flavor symmetry is broken explicitly, and every operator splits into several SMEFT operators, of which a CP -violating operator also splits into some CP -violating operators sharing the same CP -violating phase. In particular, both the numbers of CP -violating operators before and after the fix of the VEVs are less than the ones without any flavor symmetry. In practice, we perform our analysis on the down basis, and we do not consider the right-handed neutrinos and the baryon/lepton-number violations throughout the discussion. A logic flow of the paper is presented in Fig. 1, where the numbers are of the baryon/lepton-number-conserving dimension-8 operators. Together with Fig. 1, we compare the different assumptions in the flavor sector of the SMEFT as follows:

- (i) When there is no flavor symmetry, the rephasing symmetry $U(1)^4$ emerges at the LO Lagrangian as an accidental symmetry, which acts only on the dynamic fields, and the Wilson coefficients are regarded as general complex numbers.
- (ii) When some flavor symmetry is assumed, the Wilson coefficients are developed to the covariant spurions, and the Lagrangian is formally flavor invariant. The spurions extend the building blocks of the SMEFT, and the Lagrangian takes the form of Eq. (1.2). In general, every distinct operator presents an independent spurion.
- (iii) The MFV hypothesis states the spurions are limited and are the Yukawa matrices, which means the spurion parts in Eq. (1.2) are composed by the three Yukawa matrices,

$$f(\mathbf{T}) \equiv f(\mathbf{Y}_u, \mathbf{Y}_e, \mathbf{Y}_e), \quad (1.3)$$

which generally takes an expansion form, and we have used the bold notation to imply that the Yukawa matrices have been regarded as spurions.

The paper is organized as follows. In Sec. IV, we review the flavor organizations of the effective operators without flavor symmetry. In Sec. III, we review the rephasing symmetry and the CP property of the SMEFT without flavor symmetry. We present the completed operators at dimension-6 and dimension-8 classified by their CP properties. In Sec. IV, we discuss the flavor symmetry and its effects on the CP -violating phases of the SMEFT. In particular, we consider the $U(3)^5$ symmetry and the $U(2)^5$ symmetry and discuss the Young tableaux method applying

on the flavor-invariant descriptions of the CP -violating phases. In Sec. V, we introduce the MFV hypothesis and present the independent flavor structures of the dimension-6 and dimension-8 operators in detail of the SMEFT with the flavor symmetry $U(3)^5$ and $U(2)^5$, respectively. We enumerate the CP -violating operators under the MFV hypothesis and find they are reduced compared to the ones without flavor symmetry. Finally, we conclude this paper in Sec. VI. We present the fermionic operator basis of dimension-6 and dimension-8 in Appendixes A and B, respectively.

II. FLAVOR STRUCTURES OF SMEFT

The SMEFT is an effective theory with the gauge group $SU(3)_c \times SU(2)_{EW} \times U(1)_Y$, of which the associated gauge fields are G , W , and B , respectively, where the first one is the gluon field, while the last two will combine to form the weak gauge bosons Z , $W^\pm$ and the photon A after the breaking of the $SU(2)_{EW}$. In addition to the gauge bosons, there are three-flavor left/right-handed fermions, containing the leptons and quarks, and a scalar field, the Higgs field. The representations under the gauge group of all these fields are presented in Table I. In particular, the right-handed neutrinos are not included in the SMEFT, since it is of trivial representation of the gauge group. In this section, we discuss the flavor structures of the SMEFT when flavor symmetry is absent, and we illustrate an organization of the large number of operators by the repeated fields and the symmetric group.

The LO Lagrangian of the SMEFT is the standard model Lagrangian and can be separated into different sectors,

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{gauge bosons}} + \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}. \quad (2.1)$$

The first two sectors contain the dynamic terms of the gauge bosons and the fermions,

$$\mathcal{L}_{\text{gauge bosons}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^{I\mu\nu}W_{\mu\nu}^I - \frac{1}{4}G^{A\mu\nu}G_{\mu\nu}^A, \quad (2.2)$$

TABLE I. The fields and their representations under the gauge group of the SMEFT.

Field		$SU(3)_c$	$SU(2)_{EW}$	$U(1)_Y$
Gauge boson	G	8	1	0
	W	1	3	0
	B	1	1	0
Fermion	l	1	2	$-\frac{1}{2}$
	e_R	1	1	1
	q	3	2	$\frac{1}{6}$
	u_R	3	1	$\frac{2}{3}$
	d_R	3	1	$-\frac{1}{3}$
	H	1	2	$\frac{1}{2}$

$$\begin{aligned} \mathcal{L}_{\text{fermions}} = & \bar{q}i\gamma^\mu D_\mu q + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R \\ & + \bar{l}i\gamma^\mu D_\mu l + \bar{e}_R i\gamma^\mu D_\mu e_R. \end{aligned} \quad (2.3)$$

The left-handed fermions are $SU(2)_{EW}$ doublets,

$$q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad (2.4)$$

while the right-handed fermions are $SU(2)_{EW}$ singlets. In particular, the summations in the flavor space are left implicitly, which means the fermions in the $\mathcal{L}_{\text{fermion}}$ are flavor triplets

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \quad \psi = q, u_R, d_R, l, e_R, \quad (2.5)$$

and thus each term in the $\mathcal{L}_{\text{fermion}}$ is a short notation of the form

$$\begin{aligned} \bar{\psi}i\gamma^\mu D_\mu \psi = & (\bar{\psi}_1 \quad \bar{\psi}_2 \quad \bar{\psi}_3) i\gamma^\mu D_\mu \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \\ = & \sum_{p=1}^3 \bar{\psi}^p i\gamma^\mu D_\mu \psi_p, \quad \psi = q, u_R, d_R, l, e_R, \end{aligned} \quad (2.6)$$

where p is the flavor index of the fermions.

To generate the mass terms, one more scalar field, the Higgs field $H(x)$, is needed. Its dynamic term and self-interactions are described in the $\mathcal{L}_{\text{Higgs}}$

$$\mathcal{L}_{\text{Higgs}} = D_\mu H^\dagger D^\mu H + V(H), \quad (2.7)$$

and its interactions with the fermions take the form of Yukawa interactions

$$\mathcal{L}_{\text{Yukawa}} = \bar{q} \tilde{H} Y_u u_R + \bar{q} H Y_d d_R + \bar{l} H Y_e e_R + \text{H.c.}, \quad (2.8)$$

where $\tilde{H} = i\sigma^2 H^*$ is its dual field. The Yukawa couplings Y_u , Y_d , and Y_e are general 3×3 complex matrices in the flavor space, and the terms above are implicit for the flavor summations, which can be written explicitly as

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = & \sum_{p,r=1}^3 \bar{q}^p \tilde{H} (Y_u)_p^r u_{Rr} + \sum_{p,r=1}^3 \bar{q}^p H (Y_d)_p^r d_{Rr} \\ & + \sum_{p,r=1}^3 \bar{l}^p H (Y_e)_p^r e_{Rr} + \text{H.c.} \end{aligned} \quad (2.9)$$

Moving on to the high dimension, there are a lot of effective operators of the SMEFT beyond the LO, which

mainly originates from knowing that the fermions are flavor multiples as shown in Eq. (2.5). Because the operators are not necessarily the flavor invariants, when the flavor symmetry is absent, they are generally flavor multiples,

$$\mathcal{L} = \sum_d \frac{c}{\Lambda^{d-4}} \mathcal{O}_{p,r,\dots,s}^d, \quad (2.10)$$

where $p, r, \dots, s$ are the flavor indices ranging from 1 to 3; thus such a term in the Lagrangian presents $3 \times 3 \times \dots \times 3$ operators. For example, each term of the Yukawa interactions at the LO in Eq. (2.8) actually presents $3 \times 3 = 9$ operators, and the four-fermion operator such as

$$\mathcal{O}_{le} = (\bar{l}^p \gamma_\mu l_r)(\bar{e}^s \gamma^\mu e_t) \quad (2.11)$$

at dimension-6 presents $3^4 = 81$ operators in the flavor space.

Beyond the SM, there are repeated fields, and it reduces the number of the operators in the flavor space due to the permutation symmetry in principle. If the repeated fields are bosons, they should be symmetric under the permutation, since their flavor numbers are 1. However, if the repeated fields are fermions, their permutation symmetries are complex because of their nontrivial flavor numbers. We assume there are N repeated fields in an operator, and their flavor number is n_f generally. Because all the fields are of some specific representations of the Lorentz group and the gauge group as shown in Table I, the permutation symmetry of the N repeated fields are determined by the permutation symmetries of the Lorentz group and the gauge group simultaneously,

$$\text{Sym}_{\mathcal{O}} = \text{Sym}_{\text{Lorentz}} \times \text{Sym}_{\text{gauge}} \times \text{Sym}_{\text{statistics}}, \quad (2.12)$$

where the last factor $\text{Sym}_{\text{statistics}}$ is related to the spin statistics of the repeated fields,

$$\text{Sym}_{\text{statistics}} = 1: \text{The repeated field is boson}; \quad (2.13)$$

$$\text{Sym}_{\text{statistics}} = -1: \text{The repeated field is fermion}. \quad (2.14)$$

The permutations of some objects are described by the symmetric groups, and thus the effective operators can be organized by the irreducible representations under the symmetric group of the repeated fields. If there are N repeated fields with flavor number n_f , the operators, as flavor multiples, can be decomposed as the irreducible representations of the symmetric group S_N . If the repeated field is a boson, the flavor number is 1, and only the symmetric representation is nonvanishing. On the contrary, if the repeated field is a fermion, other symmetric group representations could emerge. In general, if the number of the repeated field N exceeds the flavor number n_f , $N > n_f$, the nonvanishing representations of the operators are

truncated by the condition that the number of the asymmetric fields cannot exceed the flavor number n_f .

For example, the dimension-6 operator

$$\mathcal{O}_{quqd}^{(1)} = (\bar{q}^{jp} u_r) \epsilon_{jk} (\bar{q}^{ks} d_t) \quad (2.15)$$

has a pair of repeated quarks, and it can be decomposed into the irreducible representations of the symmetric group S_2 , which correspond to the Young diagrams that

$$[2] = \begin{array}{|c|c|} \hline & \\ \hline \end{array}, \quad [1^2] = \begin{array}{|c|} \hline \\ \hline \end{array}. \quad (2.16)$$

If the flavor number of the quark is 1, $n_f = 1$, only the symmetric $[2]$ representation is nonvanishing, while if the flavor number is 3, $n_f = 3$, both representations are nonvanishing. Thus we can decouple and organize the operators as

$$\mathcal{O}_{quqd}^{(1)} = \mathcal{Y} \left(\begin{array}{|c|c|} \hline p & s \\ \hline \end{array} \right) (\bar{q}^{jp} u_r) \epsilon_{jk} (\bar{q}^{ks} d_t) + \mathcal{Y} \left(\begin{array}{|c|} \hline p \\ \hline s \\ \hline \end{array} \right) (\bar{q}^{jp} u_r) \epsilon_{jk} (\bar{q}^{ks} d_t), \quad (2.17)$$

where $\mathcal{Y}$ is the Young symmetrizer of the corresponding representation.

The organization of the effective operators by the repeated fields and their irreducible representations of the associated symmetric group has been used in the Young tensor constructions of the dimension-8 operators [7] and the dimension-9 operators [9], which present consistent results with the other method such as the Hilbert series method [4]. Besides, the flavor components of the operators are transparent since they can be obtained by the semistandard Young tableaux directly.

III. CP PROPERTY OF SMEFT

In this section, we will discuss the rephasing symmetry of the SMEFT and the associated CP property without flavor symmetry.

A. Charge conjugate and parity

The charge conjugate (C) and the parity (P) are two important discrete symmetries of the elementary particles. Although the effective operators should be invariant under the Lorentz group and the gauge groups, they are not necessary to be invariant under these discrete symmetries, which means CP violation is allowed.

Parity is a spacetime transformation that sends $(t, \vec{x})$ to $(t, -\vec{x})$. If the field is not Lorentz singlet, it would transform under the parity. Thus the parity transformation of a general field $\Phi(t, \vec{x})$ is

$$P\Phi(t, \vec{x})P^{-1} = \eta_P \mathcal{P}\Phi(t, -\vec{x}), \quad (3.1)$$

where η_P is its intrinsic party, satisfying $\eta_P^2 = 1$, and $\mathcal{P}$ is the parity transformation associated with Φ . If Φ is scalar, $\mathcal{P}$ is trivial; otherwise, it is not. For the Dirac spinor field, $\mathcal{P} = \gamma^0$, while for vector field, $\mathcal{P} = \text{diag}(1, -1, -1, -1)$, which is just the spacetime parity transformation.

The charge conjugate interchanges the particle and antiparticle; thus it correlates some field and its complex conjugate,

$$C\Phi(t, \vec{x})C^{-1} = \eta_C \mathcal{C}\Phi^*(t, \vec{x}), \quad (3.2)$$

where similarly η_C is the intrinsic charge, while $\mathcal{C}$ is the charge-conjugate transformation corresponding to Φ . Again, $\mathcal{C}$ is trivial for the scalar field while for others, there is

$$\text{Spinor field: } C\psi(t, \vec{x})C^{-1} = \eta_C i\gamma^0 \gamma^2 \bar{\psi}(t, \vec{x})^T, \quad (3.3)$$

$$\text{Vector field: } CA_\mu(t, \vec{x})C^{-1} = \eta_C A_\mu(t, \vec{x}). \quad (3.4)$$

In particular, the gauge bosons are unchanged beside the phase η_C , because they are of the adjoint representations of the associated gauge groups, which is real. Nevertheless, if the electroweak gauge symmetry is broken spontaneously, the gauge bosons $W^\pm$ transform under the charge conjugate differently as

$$CW_\mu^\pm C^{-1} = \eta_C W_\mu^\mp. \quad (3.5)$$

Because we consider the SMEFT before the symmetry breaking, we stick to the transformation in Eq. (3.4). As discussed above, the parity and charge conjugate are isomorphic to Z_2 groups

$$\{1, P\}, \{1, C\} \cong Z_2, \quad (3.6)$$

which define two outer automorphisms of the Lorentz group $SO(3, 1)$ and the internal group I , respectively [48, 49]. For every element's group, an outer automorphism maps it to other elements. Thus the parity and charge conjugate enlarge the groups by the semidirect product

$$\begin{aligned} SO(3, 1) &\rightarrow O(3, 1) = SO(3, 1) \rtimes \{1, P\}, \\ I &\rightarrow \tilde{I} = I \rtimes \{1, C\}. \end{aligned} \quad (3.7)$$

In particular, if a field is of a specific representation $\mathbf{r}$ of the internal symmetry I , the charge conjugate transforms it to the dual representation $\mathbf{r}^*$. Thus it is natural that the flavor symmetry affects the CP properties of the SMEFT as discussed in Sec. IV.

In summary, the CP transformation is the combination of them, which takes the form

$$\text{Scalar field: } CP\phi(t, \vec{x})CP^{-1} = \eta_{CP}\phi^*(t, -\vec{x}), \quad (3.8)$$

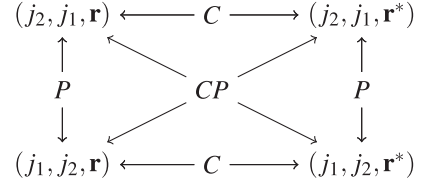


FIG. 2. The actions of C and P on representations of $G = SO(3, 1) \times I$. Here $\mathbf{r}$ is the irreducible representation of I and $\mathbf{r}^*$ is its complex conjugation representation. (j_1, j_2) label the representations of the Lorentz group $SO(3, 1)$.

$$\text{Spinor field: } CP\psi(t, \vec{x})CP^{-1} = \eta_{CP} - i\gamma^0 \gamma^2 \bar{\psi}(t, -\vec{x})^T, \quad (3.9)$$

$$\text{Vector field: } CPA_\mu(t, \vec{x})CP^{-1} = \eta_{CP} \mathcal{P}_\mu^\nu A_\nu(t, -\vec{x}), \quad (3.10)$$

where $\eta_{CP}^2 = 1$. For simplicity we take $\eta_{CP} = 1$. We note the symmetry group of the SMEFT is $G = SO(3, 1) \times I$, where I is the internal symmetry, including the gauge symmetry and some global symmetries, and a field Φ that

$$\Phi(t, \vec{x}) \in (j_1, j_2, \mathbf{r}), \quad (3.11)$$

where $\mathbf{r}$ is an irreducible representation of the internal group I . The C and P transformations of the field Φ are illustrated in Fig. 2.

The composing fields determine the CP property of an operator. If it is invariant under the CP transformation, it is CP -even, and if it becomes its opposite, it is CP -odd,

$$CP\text{-even: } \mathcal{O} \xrightarrow{CP} \mathcal{O}, \quad (3.12)$$

$$CP\text{-odd: } \mathcal{O} \xrightarrow{CP} -\mathcal{O}. \quad (3.13)$$

The CP properties of the pure bosonic operators are simple, since all the operators are real or imaginary, and there are no flavor structures. On the contrary, the CP properties of the fermionic operators are complicated because most of them are complex and there are flavor structures. As shown in Eq. (2.5), all the fermions in the SMEFT are flavor triplets; thus a general bilinear $\psi\Gamma\chi$ is a flavor tensor with nine components,

$$\bar{\psi}\Gamma\chi = \begin{pmatrix} \bar{\psi}_1\Gamma\chi_1 & \bar{\psi}_1\Gamma\chi_2 & \bar{\psi}_1\Gamma\chi_3 \\ \bar{\psi}_2\Gamma\chi_1 & \bar{\psi}_2\Gamma\chi_2 & \bar{\psi}_2\Gamma\chi_3 \\ \bar{\psi}_3\Gamma\chi_1 & \bar{\psi}_3\Gamma\chi_2 & \bar{\psi}_3\Gamma\chi_3 \end{pmatrix}, \quad (3.14)$$

where Γ is any Dirac matrix. After the CP transformation, the bilinear becomes

$$\bar{\psi}\Gamma\chi \xrightarrow{CP} \bar{\chi}\Gamma^C\psi = \begin{pmatrix} \bar{\chi}_1\Gamma^C\psi_1 & \bar{\chi}_1\Gamma^C\psi_2 & \bar{\chi}_1\Gamma^C\psi_3 \\ \bar{\chi}_2\Gamma^C\psi_1 & \bar{\chi}_2\Gamma^C\psi_2 & \bar{\chi}_2\Gamma^C\psi_3 \\ \bar{\chi}_3\Gamma^C\psi_1 & \bar{\chi}_3\Gamma^C\psi_2 & \bar{\chi}_3\Gamma^C\psi_3 \end{pmatrix}, \quad (3.15)$$

where $\Gamma^C = C^{-1}\Gamma^T C$, and in detail

$$1^C = 1, \quad \gamma^{5C} = \gamma^5, \quad (3.16)$$

$$\gamma^{\mu C} = -\gamma^\mu, \quad (3.17)$$

$$(\gamma^5\gamma^\mu)^C = \gamma^5\gamma^\mu, \quad (3.18)$$

$$\sigma^{\mu\nu C} = -\sigma^{\mu\nu}. \quad (3.19)$$

Thus the CP transformation of a bilinear is

$$\bar{\psi}\Gamma\chi \xrightarrow{CP} \bar{\chi}\Gamma^C\psi = \eta_\Gamma \bar{\chi}\Gamma\psi, \quad (3.20)$$

where η_Γ is a factor from Γ according to Eqs. (3.16)–(3.19), so that a fermion bilinear of a specific CP property are the combinations that

$$CP\text{-even: } \bar{\psi}\Gamma\chi + \eta_\Gamma \bar{\chi}\Gamma\psi, \quad (3.21)$$

$$CP\text{-odd: } \bar{\psi}\Gamma\chi - \eta_\Gamma \bar{\chi}\Gamma\psi, \quad (3.22)$$

which contribute to nine CP -even bilinears and nine CP -odd bilinears, according to Eq. (3.14). In particular, if the two fermions ψ and χ are the same, the CP transformation is closed in the bilinear $\bar{\psi}\Gamma\psi$, there are only three CP -odd bilinears from the off-diagonal elements in Eq. (3.14), and there are six CP -even bilinears.

Moving on to the four-fermion operators, which contain two fermion bilinears $(\bar{\psi}\Gamma_1\chi)(\bar{\lambda}\Gamma_2\xi)$, we can get the CP transformation that

$$(\bar{\psi}\Gamma_1\chi)(\bar{\lambda}\Gamma_2\xi) \xrightarrow{CP} \eta_{\Gamma_1}\eta_{\Gamma_2}(\bar{\chi}\Gamma_1\psi)(\bar{\xi}\Gamma_2\lambda), \quad (3.23)$$

so that the combinations of specific CP properties are

$$CP\text{-even: } (\bar{\psi}\Gamma_1\chi)(\bar{\lambda}\Gamma_2\xi) + \eta_{\Gamma_1}\eta_{\Gamma_2}(\bar{\chi}\Gamma_1\psi)(\bar{\xi}\Gamma_2\lambda), \quad (3.24)$$

$$CP\text{-odd: } (\bar{\psi}\Gamma_1\chi)(\bar{\lambda}\Gamma_2\xi) - \eta_{\Gamma_1}\eta_{\Gamma_2}(\bar{\chi}\Gamma_1\psi)(\bar{\xi}\Gamma_2\lambda). \quad (3.25)$$

Suppose the two fermions in a single bilinear are different. In that case, the combinations present 81 CP -even and CP -odd operators, respectively, while if the fermions in the two bilinears are the same, $\psi = \chi$ and $\lambda = \xi$, the CP -transformation is closed, the CP -even operators are reduced to 45, and the CP -odd operators are reduced to 36. In particular, if all four fermions are the same, $\psi = \chi = \lambda = \xi$, the numbers are further reduced to 27 and 18 for the CP -even and CP -odd operators, respectively.

TABLE II. The CP properties of the dimension-6 bosonic operators of the SMEFT.

Class		Operator	C	P
X^3	$\mathcal{O}_G$	$f^{ABC} G_\nu^A G_\rho^{B\nu} G_\mu^C{}^\rho$	+	+
	$\mathcal{O}_{\tilde{G}}$	$f^{ABC} \tilde{G}_\nu^A G_\rho^{B\nu} G_\mu^C{}^\rho$	+	−
	$\mathcal{O}_W$	$\varepsilon^{IJK} W_\nu^I W_\rho^{J\nu} W_\mu^K{}^\rho$	+	+
	$\mathcal{O}_{\tilde{W}}$	$\varepsilon^{IJK} \tilde{W}_\nu^I W_\rho^{J\nu} W_\mu^K{}^\rho$	+	−
$X^2 H^2$	$\mathcal{O}_{HG}$	$(H^\dagger H) G_{\mu\nu}^A G^{A\mu\nu}$	+	+
	$\mathcal{O}_{H\tilde{G}}$	$(H^\dagger H) \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	+	−
	$\mathcal{O}_{HW}$	$(H^\dagger H) W_{\mu\nu}^I W^{I\mu\nu}$	+	+
	$\mathcal{O}_{H\tilde{W}}$	$(H^\dagger H) \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	+	−
	$\mathcal{O}_{HB}$	$(H^\dagger H) B_{\mu\nu} B^{\mu\nu}$	+	+
	$\mathcal{O}_{H\tilde{B}}$	$(H^\dagger H) \tilde{B}_{\mu\nu} B^{\mu\nu}$	+	−
	$\mathcal{O}_{HWB}$	$(H^\dagger \tau^I H) W_{\mu\nu}^I B^{\mu\nu}$	+	−
	$\mathcal{O}_{H\tilde{W}B}$	$(H^\dagger \tau^I H) \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	+	−
$H^4 D^2$	$\mathcal{O}_{H\Box}$	$(H^\dagger H) \Box (H^\dagger H)$	+	+
	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$	+	+
H^6	$\mathcal{O}_H$	$(H^\dagger H)^3$	+	+

In summary, when we note an operator with fermions as $\mathcal{O}$, its CP transformation takes the form

$$\mathcal{O} \xrightarrow{CP} \mathcal{O}^C = \eta_\Gamma \eta_B \mathcal{O}^\dagger, \quad (3.26)$$

where η_Γ is the factor from the Dirac matrices and η_B is the factor from the bosonic fields. The combinations of specific CP properties are

$$CP\text{-even: } \mathcal{O} + \eta_\Gamma \eta_B \mathcal{O}^\dagger, \quad (3.27)$$

$$CP\text{-odd: } \mathcal{O} - \eta_\Gamma \eta_B \mathcal{O}^\dagger. \quad (3.28)$$

In particular, the CP -odd and CP -violating operators are in general not equivalent [17]. The CP -violating operators are the CP -odd ones that are invariant under the rephasing symmetry, which will be discussed next.

B. CP property of effective operators

In this subsection, we classify the effective operators of dimension-6 and dimension-8. Part of the bosonic CP -conserving and CP -violating operators at the dimension-8 have been discussed previously, e.g., [15,16], and in this subsection, we present the complete classification including both the bosonic and the fermionic operators.

1. Dimension-6

Considering the bosonic operators, there are four classes,

$$X^3, \quad X^2 H^2, \quad H^4 D^2, \quad H^6, \quad (3.29)$$

where all the operators are real and CP -odd operators emerge in the classes with the gauge bosons due to the antisymmetric tensor $\varepsilon_{\mu\nu\rho\lambda}$. We list the operators according to their CP properties following the convention in Ref. [3] in Table II.

We present the CP properties of the dimension-6 fermionic operators in Table III, and we present the dimension-6 fermionic operators in Appendix A for convenience. We present the combinations of specific CP properties and the numbers of the CP -even and CP -odd operators from them due to the flavor structures. These results are consistent with the Hilbert series result [17] and the previous discussion [14].

2. Dimension-8

For the bosonic operators, only the ones involving the gauge bosons could be CP -odd at dimension-8. These operators have been counted by the Hilbert series [17] and constructed by the Young tensor method [7]. Next, we list these operators according to their classes in Table IV. The operators involving the anomalous quartic gauge couplings of these results are consistent with Ref. [16]. The operators are presented following the notations in Ref. [7] for writing convenience, and we summarize the notation here for completeness:

- (i) The covariant derivatives are supposed to apply only to the field behind it, for example,

$$\begin{aligned} D_\mu H^\dagger H &= (D_\mu H^\dagger) H, \\ D_\mu H^\dagger D^\mu H &= (D_\mu H^\dagger)(D^\mu H). \end{aligned} \quad (3.30)$$

- (ii) The inner products of two gauge bosons are written in a pair of parenthesis, and the contracting Lorentz indices are dropped, for example,

$$\begin{aligned} (G^A G^B) &= G^{A\mu\nu} G_{\mu\nu}^B, & (G^A W^I) &= G^{A\mu\nu} W_{\mu\nu}^I, \\ (W^I \tilde{W}^J) &= W^{I\mu\nu} \tilde{W}_{\mu\nu}^J. \end{aligned} \quad (3.31)$$

In particular, if the two gauge bosons are the same and both the Lorentz indices and the gauge indices are contracted, we write them in square, for example,

$$W^2 = (W^I W^I) = W^{I\mu\nu} W_{\mu\nu}^I, \quad B^2 = (BB) = B^{\mu\nu} B_{\mu\nu}. \quad (3.32)$$

In summary, we present numbers of the CP -even and the CP -odd bosonic operators at dimension-6 and dimension-8 in Table V. These results do not change when we discuss the flavor symmetries subsequently.

The dimension-8 fermionic operators of specific CP properties can also be obtained according to the discussion about the dimension-6 cases. We present the explicit

TABLE III. The dimension-6 fermionic operators of specific CP properties. The second and the third columns are the CP -even and the CP -odd combinations of the operators, and the last two columns are the numbers considering the flavor components of the combinations. The explicit operators can be found in Appendix A.

Class	Operators combination		Operators number	
	CP -even	CP -odd	CP -even	CP -odd
$\psi^2 H^3 + \text{H.c.}$	$\mathcal{O}_{eH} + \text{H.c.}$	$\mathcal{O}_{eH} - \text{H.c.}$	9	9
	$\mathcal{O}_{uH} + \text{H.c.}$	$\mathcal{O}_{uH} - \text{H.c.}$	9	9
	$\mathcal{O}_{dH} + \text{H.c.}$	$\mathcal{O}_{dH} - \text{H.c.}$	9	9
			27	27
$\psi^2 XH + \text{H.c.}$	$\mathcal{O}_{eW} + \text{H.c.}$	$\mathcal{O}_{eW} - \text{H.c.}$	9	9
	$\mathcal{O}_{eB} + \text{H.c.}$	$\mathcal{O}_{eB} - \text{H.c.}$	9	9
	$\mathcal{O}_{uG} + \text{H.c.}$	$\mathcal{O}_{uG} - \text{H.c.}$	9	9
	$\mathcal{O}_{uW} + \text{H.c.}$	$\mathcal{O}_{uW} - \text{H.c.}$	9	9
	$\mathcal{O}_{uB} + \text{H.c.}$	$\mathcal{O}_{uB} - \text{H.c.}$	9	9
	$\mathcal{O}_{dG} + \text{H.c.}$	$\mathcal{O}_{dG} - \text{H.c.}$	9	9
	$\mathcal{O}_{dW} + \text{H.c.}$	$\mathcal{O}_{dW} - \text{H.c.}$	9	9
	$\mathcal{O}_{dB} + \text{H.c.}$	$\mathcal{O}_{dB} - \text{H.c.}$	9	9
			72	72
$\psi^2 H^2 D$	$\mathcal{O}_{Hl}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Hl}^{(1)} - \text{H.c.}$	6	3
	$\mathcal{O}_{Hl}^{(3)} + \text{H.c.}$	$\mathcal{O}_{Hl}^{(3)} - \text{H.c.}$	6	3
	$\mathcal{O}_{He} + \text{H.c.}$	$\mathcal{O}_{He} - \text{H.c.}$	6	3
	$\mathcal{O}_{Hq}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Hq}^{(1)} - \text{H.c.}$	6	3
	$\mathcal{O}_{Hq}^{(3)} + \text{H.c.}$	$\mathcal{O}_{Hq}^{(3)} - \text{H.c.}$	6	3
	$\mathcal{O}_{Hu} + \text{H.c.}$	$\mathcal{O}_{Hu} - \text{H.c.}$	6	3
	$\mathcal{O}_{Hd} + \text{H.c.}$	$\mathcal{O}_{Hd} - \text{H.c.}$	6	3
	$\mathcal{O}_{Hud} + \text{H.c.}$	$\mathcal{O}_{Hud} - \text{H.c.}$	9	9
			51	30
$(\bar{L}L)(\bar{L}L)$	$\mathcal{O}_{ll} + \text{H.c.}$	$\mathcal{O}_{ll} - \text{H.c.}$	27	18
	$\mathcal{O}_{qq}^{(1)} + \text{H.c.}$	$\mathcal{O}_{qq}^{(1)} - \text{H.c.}$	27	18
	$\mathcal{O}_{qq}^{(3)} + \text{H.c.}$	$\mathcal{O}_{qq}^{(3)} - \text{H.c.}$	27	18
	$\mathcal{O}_{lq}^{(1)} + \text{H.c.}$	$\mathcal{O}_{lq}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{lq}^{(3)} + \text{H.c.}$	$\mathcal{O}_{lq}^{(3)} - \text{H.c.}$	45	36
			171	126
$(\bar{R}R)(\bar{R}R)$	$\mathcal{O}_{ee} + \text{H.c.}$	$\mathcal{O}_{ee} - \text{H.c.}$	21	15
	$\mathcal{O}_{uu} + \text{H.c.}$	$\mathcal{O}_{uu} - \text{H.c.}$	27	18
	$\mathcal{O}_{dd} + \text{H.c.}$	$\mathcal{O}_{dd} - \text{H.c.}$	27	18
	$\mathcal{O}_{eu} + \text{H.c.}$	$\mathcal{O}_{eu} - \text{H.c.}$	45	36
	$\mathcal{O}_{ed} + \text{H.c.}$	$\mathcal{O}_{ed} - \text{H.c.}$	45	36
	$\mathcal{O}_{ud}^{(1)} + \text{H.c.}$	$\mathcal{O}_{ud}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{ud}^{(3)} + \text{H.c.}$	$\mathcal{O}_{ud}^{(3)} - \text{H.c.}$	45	36
			255	195
$(\bar{L}L)(\bar{R}R)$	$\mathcal{O}_{le} + \text{H.c.}$	$\mathcal{O}_{le} - \text{H.c.}$	45	36
	$\mathcal{O}_{lu} + \text{H.c.}$	$\mathcal{O}_{lu} - \text{H.c.}$	45	36
	$\mathcal{O}_{ld} + \text{H.c.}$	$\mathcal{O}_{ld} - \text{H.c.}$	45	36
	$\mathcal{O}_{qe} + \text{H.c.}$	$\mathcal{O}_{qe} - \text{H.c.}$	45	36
	$\mathcal{O}_{qu}^{(1)} + \text{H.c.}$	$\mathcal{O}_{qu}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{qu}^{(8)} + \text{H.c.}$	$\mathcal{O}_{qu}^{(8)} - \text{H.c.}$	45	36
	$\mathcal{O}_{qd}^{(1)} + \text{H.c.}$	$\mathcal{O}_{qd}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{qd}^{(8)} + \text{H.c.}$	$\mathcal{O}_{qd}^{(8)} - \text{H.c.}$	45	36
			360	288
$(\bar{L}R)(\bar{R}L) + \text{H.c.}$	$\mathcal{O}_{ledq} + \text{H.c.}$	$\mathcal{O}_{ledq} - \text{H.c.}$	81	81
			81	81
$(\bar{L}R)(\bar{R}L) + \text{H.c.}$	$\mathcal{O}_{quqd}^{(1)} + \text{H.c.}$	$\mathcal{O}_{quqd}^{(1)} - \text{H.c.}$	81	81
	$\mathcal{O}_{quqd}^{(8)} + \text{H.c.}$	$\mathcal{O}_{quqd}^{(8)} - \text{H.c.}$	81	81
	$\mathcal{O}_{lequ}^{(1)} + \text{H.c.}$	$\mathcal{O}_{lequ}^{(1)} - \text{H.c.}$	81	81
	$\mathcal{O}_{lequ}^{(3)} + \text{H.c.}$	$\mathcal{O}_{lequ}^{(3)} - \text{H.c.}$	81	81
			324	324

TABLE IV. The CP properties of the dimension-6 bosonic operators of the SMEFT.

Class	Operator	C	P	
X^4	$\mathcal{O}_{G^4}^{(1)}$	$(G^A G^B)(G^A G^B)$	+	+
	$\mathcal{O}_{G^4}^{(2)}$	$d^{ACE} d^{BDE} (G^A G^B)(G^C G^D)$	+	+
	$\mathcal{O}_{G^4}^{(3)}$	$f^{ACE} f^{BDE} (G^A G^B)(G^C G^D)$	+	+
	$\mathcal{O}_{G^4}^{(4)}$	$(G^A G^B)(G^A \tilde{G}^B)$	+	-
	$\mathcal{O}_{G^4}^{(5)}$	$d^{ACE} d^{BDE} (G^A G^B)(G^C \tilde{G}^D)$	+	-
	$\mathcal{O}_{G^4}^{(6)}$	$f^{ACE} f^{BDE} (G^A G^B)(G^C \tilde{G}^D)$	+	-
	$\mathcal{O}_{G^4}^{(7)}$	$(G^A \tilde{G}^B)(G^A \tilde{G}^B)$	+	+
	$\mathcal{O}_{G^4}^{(8)}$	$d^{ACE} d^{BDE} (G^A \tilde{G}^B)(G^C \tilde{G}^D)$	+	+
	$\mathcal{O}_{G^4}^{(9)}$	$f^{ACE} f^{BDE} (G^A \tilde{G}^B)(G^C \tilde{G}^D)$	+	+
	$\mathcal{O}_{W^4}^{(1)}$	$(W^I W^I)(W^J W^J)$	+	+
	$\mathcal{O}_{W^4}^{(2)}$	$(W^I W^I)(W^J \tilde{W}^J)$	+	-
	$\mathcal{O}_{W^4}^{(3)}$	$(W^I \tilde{W}^I)(W^J \tilde{W}^J)$	+	+
	$\mathcal{O}_{W^4}^{(4)}$	$(W^I W^J)(W^I W^J)$	+	+
	$\mathcal{O}_{W^4}^{(5)}$	$(W^I W^J)(W^I \tilde{W}^J)$	+	-
	$\mathcal{O}_{W^4}^{(6)}$	$(W^I \tilde{W}^J)(W^I \tilde{W}^J)$	+	+
	$\mathcal{O}_{B^4}^{(1)}$	$(B^2)(B^2)$	+	+
	$\mathcal{O}_{B^4}^{(2)}$	$(B^2)(B\tilde{B})$	+	-
	$\mathcal{O}_{B^4}^{(3)}$	$(B\tilde{B})(B\tilde{B})$	+	+
	$\mathcal{O}_{G^2 W^2}^{(1)}$	$G^2 W^2$	+	+
	$\mathcal{O}_{G^2 W^2}^{(2)}$	$G^2 (W^I \tilde{W}^I)$	+	-
	$\mathcal{O}_{G^2 W^2}^{(3)}$	$(G^A \tilde{G}^A)(W^2)$	+	-
	$\mathcal{O}_{G^2 W^2}^{(4)}$	$(G^A \tilde{G}^A)(W^I \tilde{W}^I)$	+	+
	$\mathcal{O}_{G^2 W^2}^{(5)}$	$(G^A W^I)(G^A W^I)$	+	+
	$\mathcal{O}_{G^2 W^2}^{(6)}$	$(G^A W^I)(G^A \tilde{W}^I)$	+	-
	$\mathcal{O}_{G^2 W^2}^{(7)}$	$(G^A \tilde{W}^I)(G^A \tilde{W}^I)$	+	+
	$\mathcal{O}_{G^2 B^2}^{(1)}$	$G^2 B^2$	+	+
	$\mathcal{O}_{G^2 B^2}^{(2)}$	$G^2 (B\tilde{B})$	+	-
	$\mathcal{O}_{G^2 B^2}^{(3)}$	$(G^A \tilde{G}^A)(B^2)$	+	-
	$\mathcal{O}_{G^2 B^2}^{(4)}$	$(G^A \tilde{G}^A)(B\tilde{B})$	+	+
	$\mathcal{O}_{G^2 B^2}^{(5)}$	$(G^A B)(G^A B)$	+	+
	$\mathcal{O}_{G^2 B^2}^{(6)}$	$(G^A B)(G^A \tilde{B})$	+	-
	$\mathcal{O}_{G^2 B^2}^{(7)}$	$(G^A \tilde{B})(G^A \tilde{B})$	+	+
	$\mathcal{O}_{B^2 W^2}^{(1)}$	$B^2 W^2$	+	+
	$\mathcal{O}_{B^2 W^2}^{(2)}$	$B^2 (W^I \tilde{W}^I)$	+	-
	$\mathcal{O}_{B^2 W^2}^{(3)}$	$(B\tilde{B})(W^2)$	+	-
	$\mathcal{O}_{B^2 W^2}^{(4)}$	$(B\tilde{B})(W^I \tilde{W}^I)$	+	+
	$\mathcal{O}_{B^2 W^2}^{(5)}$	$(BW^I)(BW^I)$	+	+
	$\mathcal{O}_{B^2 W^2}^{(6)}$	$(BW^I)(B\tilde{W}^I)$	+	-
	$\mathcal{O}_{B^2 W^2}^{(7)}$	$(B\tilde{W}^I)(B\tilde{W}^I)$	+	+
	$\mathcal{O}_{BG^3}^{(1)}$	$d^{ABC} (BG^A)(G^B G^C)$	+	+
	$\mathcal{O}_{BG^3}^{(2)}$	$d^{ABC} (BG^A)(G^B \tilde{G}^C)$	+	-
	$\mathcal{O}_{BG^3}^{(3)}$	$d^{ABC} (B\tilde{G}^A)(G^B G^C)$	+	-
	$\mathcal{O}_{BG^3}^{(4)}$	$d^{ABC} (B\tilde{G}^A)(G^B \tilde{G}^C)$	+	+
$X^3 \phi^2$	$\mathcal{O}_{G^3 H^2}^{(1)}$	$f^{ABC} G_{\mu\nu}^A G_{\lambda}^B G_{\nu\lambda}^C H^\dagger H$	+	+
	$\mathcal{O}_{G^3 H^2}^{(2)}$	$f^{ABC} G_{\mu\nu}^A G_{\lambda}^B \tilde{G}_{\nu\lambda}^C H^\dagger H$	+	-
	$\mathcal{O}_{W^3 H^2}^{(1)}$	$\varepsilon^{IJK} W_{\mu\nu}^I W_{\lambda}^J W_{\nu\lambda}^K H^\dagger H$	+	+
	$\mathcal{O}_{W^3 H^2}^{(2)}$	$\varepsilon^{IJK} W_{\mu\nu}^I W_{\lambda}^J \tilde{W}_{\nu\lambda}^K H^\dagger H$	+	-
	$\mathcal{O}_{BW^2 H^2}^{(1)}$	$\varepsilon^{IJK} B_{\mu\nu} W_{\lambda}^I W_{\nu\lambda}^J H^\dagger \tau^K H$	+	+
	$\mathcal{O}_{BW^2 H^2}^{(2)}$	$\varepsilon^{IJK} B_{\mu\nu} W_{\lambda}^I \tilde{W}_{\nu\lambda}^J H^\dagger \tau^K H$	+	-

(Table continued)

TABLE IV. (Continued)

Class	Operator	C	P
$X^2\phi^4$	$\mathcal{O}_{G^2H^4}^{(1)}$	$G^2(H^\dagger H)^2$	+
	$\mathcal{O}_{G^2H^4}^{(2)}$	$G_{\mu\nu}^A \tilde{G}^{A\mu\nu} (H^\dagger H)^2$	+
	$\mathcal{O}_{W^2H^4}^{(1)}$	$W_{\mu\nu}^I W^{J\mu\nu} (H^\dagger \tau^I H)(H^\dagger \tau^J H)$	+
	$\mathcal{O}_{W^2H^4}^{(2)}$	$W^2(H^\dagger H)^2$	+
	$\mathcal{O}_{W^2H^4}^{(3)}$	$W_{\mu\nu}^I \tilde{W}^{J\mu\nu} (H^\dagger \tau^I H)(H^\dagger \tau^J H)$	+
	$\mathcal{O}_{W^2H^4}^{(4)}$	$(W^I \tilde{W}^I)(H^\dagger H)^2$	+
	$\mathcal{O}_{B^2H^4}^{(1)}$	$B^2(H^\dagger H)^2$	+
	$\mathcal{O}_{B^2H^4}^{(2)}$	$B\tilde{B}(H^\dagger H)^2$	+
	$\mathcal{O}_{B^2H^4}^{(3)}$	$B_{\mu\nu} W^{I\mu\nu} (H^\dagger \tau^I H)(H^\dagger H)$	+
	$\mathcal{O}_{BWH^4}^{(1)}$	$B_{\mu\nu} \tilde{W}^{I\mu\nu} (H^\dagger \tau^I H)(H^\dagger H)$	+
	$\mathcal{O}_{BWH^4}^{(2)}$		-
$X^2\phi^2D^2$	$\mathcal{O}_{G^2H^2D^2}^{(1)}$	$G^2(D^\mu H^\dagger D_\mu H)$	+
	$\mathcal{O}_{G^2H^2D^2}^{(2)}$	$(G^A \tilde{G}^A)(D^\mu H^\dagger D_\mu H)$	+
	$\mathcal{O}_{G^2H^2D^2}^{(3)}$	$G_\lambda^A {}^\mu G^{A\nu\lambda} (D_\mu H^\dagger D_\nu H)$	+
	$\mathcal{O}_{G^2H^2D^2}^{(4)}$	$W^2(D^\mu H^\dagger D_\mu H)$	+
	$\mathcal{O}_{W^2H^2D^2}^{(1)}$	$i\epsilon^{IJK} W_\lambda^I {}^\mu W^{J\nu\lambda} (D_\mu H^\dagger \tau^K D_\nu H)$	-
	$\mathcal{O}_{W^2H^2D^2}^{(2)}$	$(W^I \tilde{W}^I)(D^\mu H^\dagger D_\mu H)$	+
	$\mathcal{O}_{W^2H^2D^2}^{(3)}$	$i\epsilon^{IJK} W_\lambda^I {}^\mu \tilde{W}^{J\nu\lambda} (D_\mu H^\dagger \tau^K D_\nu H)$	-
	$\mathcal{O}_{W^2H^2D^2}^{(4)}$	$W_\lambda^I {}^\mu W^{I\nu\lambda} (D_\mu H^\dagger D_\nu H)$	+
	$\mathcal{O}_{W^2H^2D^2}^{(5)}$	$i\epsilon^{IJK} W_\lambda^I {}^\mu \tilde{W}^{J\nu\lambda} (D_\mu H^\dagger \tau^K D_\nu H)$	-
	$\mathcal{O}_{W^2H^2D^2}^{(6)}$	$B^2(D^\mu H^\dagger D_\mu H)$	+
	$\mathcal{O}_{B^2H^2D^2}^{(1)}$	$B\tilde{B}(D^\mu H^\dagger D_\mu H)$	+
	$\mathcal{O}_{B^2H^2D^2}^{(2)}$	$B_\lambda^\mu B^{\nu\lambda} (D_\mu H^\dagger D_\nu H)$	+
	$\mathcal{O}_{B^2H^2D^2}^{(3)}$	$(BW^I)(D^\mu H^\dagger \tau^I D_\mu H)$	+
	$\mathcal{O}_{BWH^2D^2}^{(1)}$	$(B\tilde{W}^I)(D^\mu H^\dagger \tau^I D_\mu H)$	+
	$\mathcal{O}_{BWH^2D^2}^{(2)}$	$iB_\lambda^\mu W^{I\nu\lambda} (D_\nu H^\dagger \tau^I D_\mu H)$	+
	$\mathcal{O}_{BWH^2D^2}^{(3)}$	$iB_\lambda^\mu \tilde{W}^{I\nu\lambda} (D_\nu H^\dagger \tau^I D_\mu H)$	+
	$\mathcal{O}_{BWH^2D^2}^{(4)}$	$iB^{(\mu}{}_\lambda W^{I\nu)\lambda} (D_\nu H^\dagger \tau^I D_\mu H)$	+
	$\mathcal{O}_{BWH^2D^2}^{(5)}$	$iB^{(\mu}{}_\lambda \tilde{W}^{I\nu)\lambda} (D_\nu H^\dagger \tau^I D_\mu H)$	+
	$\mathcal{O}_{BWH^2D^2}^{(6)}$		-
$X\phi^4D^2$	$\mathcal{O}_{WH^4D^2}^{(1)}$	$W^{I\mu\nu} (H^\dagger H)(D_\mu H^\dagger \tau^I D_\nu H)$	+
	$\mathcal{O}_{WH^4D^2}^{(2)}$	$\tilde{W}^{I\mu\nu} (H^\dagger H)(D_\mu H^\dagger \tau^I D_\nu H)$	+
	$\mathcal{O}_{WH^4D^2}^{(3)}$	$W^{I\mu\nu} (H^\dagger \tau^I H)(D_\mu H^\dagger D_\nu H)$	+
	$\mathcal{O}_{WH^4D^2}^{(4)}$	$\tilde{W}^{I\mu\nu} (H^\dagger \tau^I H)(D_\mu H^\dagger D_\nu H)$	+
	$\mathcal{O}_{WH^4D^2}^{(5)}$	$B^{\mu\nu} (H^\dagger H)(D_\mu H^\dagger \tau^I D_\nu H)$	+
	$\mathcal{O}_{BH^4D^2}^{(1)}$	$\tilde{B}^{\mu\nu} (H^\dagger H)(D_\mu H^\dagger \tau^I D_\nu H)$	+
	$\mathcal{O}_{BH^4D^2}^{(2)}$		-
	$\mathcal{O}_{BH^4D^2}^{(3)}$		-
ϕ^8	$\mathcal{O}_{H^8}$	$(H^\dagger H)^3$	+
ϕ^6D^2	$\mathcal{O}_{H^6D^2}^{(1)}$	$(H^\dagger H)\square(H^\dagger H)$	+
	$\mathcal{O}_{H^6D^2}^{(2)}$	$(H^\dagger H) H^\dagger D_\mu H ^2$	+
ϕ^4D^4	$\mathcal{O}_{H^4D^4}^{(1)}$	$(H^\dagger H)\square^2(H^\dagger H)$	+
	$\mathcal{O}_{H^4D^4}^{(2)}$	$H^\dagger D_\mu D_\nu H ^2$	+
	$\mathcal{O}_{H^4D^4}^{(3)}$	$(H^\dagger D_\mu H)^*\square(H^\dagger D^\mu H)$	+

operators in Appendix B following the notations in Ref. [7], and we present their combinations of specific CP properties in Table VI.

C. Rephasing symmetry and CP violation

The CP -odd operators in Tables III and VI are not necessarily CP -violating because of the rephasing symmetry. In this subsection, we introduce the

rephasing symmetry and discuss its relation to the CP properties.

The Yukawa matrices are associated with the masses of the fermions after the spontaneous break of the gauge group $SU(2)_{EW}$, which the VEV of the Higgs field realizes,

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \quad (3.33)$$

TABLE V. The CP properties of the bosonic operators at dimension-6 and dimension-8. The CP -odd operators and the CP -violating operators are equivalent and do not change with the flavor symmetry.

Dimension-6		
Class	Operator number	
	CP -even	CP -odd
X^3	2	2
H^6	1	0
$H^4 D^2$	2	0
$X^2 H^2$	4	4
Total	9	6
15		
Dimension-8		
Class	Operator number	
	CP -even	CP -odd
X^4	26	17
$X^3 H^2$	3	3
$X^2 H^4$	5	5
$X^2 H^2 D^2$	11	7
$X H^4 D^2$	3	3
$H^6 D^2$	2	0
$H^4 D^4$	3	0
H^8	1	0
Total	54	35
89		

where v is the VEV and $h(x)$ is the fluctuation over the VEV. The VEV of the Higgs field makes the gauge group $SU(3)_c \times SU(2)_{EW} \times U(1)_Y$ break to the subgroup $SU(3)_c \times U(1)_{EM}$, where $U(1)_{EM}$ represents the electric charge conservation. At the same time, the Yukawa terms in Eq. (2.8) become

$$\mathcal{L}_{\text{Yukawa}} = (\bar{u}_L M_u u_R) + (\bar{d}_L M_d d_R) + (\bar{e}_L M_e e_R) + \text{H.c.} + \dots, \quad (3.34)$$

where the mass matrices

$$M_\psi = \frac{1}{\sqrt{2}} v Y_\psi, \quad \psi = u, d, e, \quad (3.35)$$

are 3×3 matrices in the flavor space and the ellipse contains the interactions of the fermions and the fluctuation

field $h(x)$. In particular, the neutrinos are massless when the right-handed neutrinos do not exist, $M_\nu = 0$. As mass matrices, the eigenvalues of M_u , M_d , and M_e are real and positive, which means that there exists a set of unitary matrices satisfying

$$U_L^{u\dagger} M_u U_R^u = \text{diag}(m_u, m_c, m_t) = D_u, \quad (3.36)$$

$$U_L^{d\dagger} M_d U_R^d = \text{diag}(m_d, m_s, m_b) = D_d, \quad (3.37)$$

$$U_L^{e\dagger} M_e U_R^e = \text{diag}(m_e, m_\mu, m_\tau) = D_e, \quad (3.38)$$

and the corresponding transformations of the fermions

$$\begin{aligned} u_L^0 &= U_L^{u\dagger} u_L, & d_L^0 &= U_L^{d\dagger} d_L, & u_R^0 &= U_R^{u\dagger} u_R, \\ d_R^0 &= U_R^{d\dagger} d_R, & e_L^0 &= U_L^{e\dagger} e_L, & e_R^0 &= U_R^{e\dagger} e_R, \end{aligned} \quad (3.39)$$

are the mass eigenstates, which form the mass basis. In terms of the mass basis, the Yukawa sector looks like

$$\mathcal{L}_{\text{Yukawa}} = (\bar{u}_L^0 D_u u_R^0) + (\bar{d}_L^0 D_d d_R^0) + (\bar{e}_L^0 D_e e_R^0) + \text{H.c.} + \dots \quad (3.40)$$

At the same time, the charged interactions of the fermions after the spontaneous symmetry breaking are

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} (\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L) W_\mu^+ + \text{H.c.}, \quad (3.41)$$

which can be expressed by the mass basis that

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} (\bar{u}_L^0 V_{CKM} \gamma^\mu d_L^0 + \bar{\nu}_L^0 \gamma^\mu e_L^0) W_\mu^+ + \text{H.c.}, \quad (3.42)$$

where $V_{CKM} = U_L^{u\dagger} U_L^d$ is a unitary matrix in the flavor space called the CKM matrix [12,13]. It means that the three flavors of the quarks are mixed in the mass basis., while for the leptons, the three flavors do not mix.

As a 3×3 unitary matrix, there are generally nine real parameters. However, the field redefinitions can remove some unphysical ones. In principle, the redefinitions by the phases of the six quarks, such as

$$q_p \rightarrow e^{i\alpha_p} q_p, \quad p = 1, 2, 3, \quad \text{and} \quad q = u, d, \quad (3.43)$$

can eliminate five phases; thus there are four physical parameters in the CKM matrix and its standard parametrization [50] is

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (3.44)$$

TABLE VI. The dimension-8 fermionic operators of specific CP properties. The second and the third columns are the CP -even and the CP -odd combinations of the operators, and the last two columns are the numbers considering the flavor components of the combinations. The explicit operators can be found in Appendix B.

Class	Operators combination		Operators number	
	CP -even	CP -odd	CP -even	CP -odd
$\psi^2 \phi^5$	$\mathcal{O}_{quH^5} + \text{H.c.}$	$\mathcal{O}_{quH^5} - \text{H.c.}$	9	9
	$\mathcal{O}_{qdH^5} + \text{H.c.}$	$\mathcal{O}_{qdH^5} - \text{H.c.}$	9	9
	$\mathcal{O}_{leH^5} + \text{H.c.}$	$\mathcal{O}_{leH^5} - \text{H.c.}$	9	9
			27	27
$\psi^2 \phi^4 D$	$\mathcal{O}_{q^2 H^4 D}^{(1,2,4)} + \text{H.c.}$	$\mathcal{O}_{q^2 H^4 D}^{(1,2,4)} - \text{H.c.}$	18	9
	$\mathcal{O}_{q^2 H^4 D}^{(3)} - \text{H.c.}$	$\mathcal{O}_{q^2 H^4 D}^{(3)} + \text{H.c.}$	3	6
	$\mathcal{O}_{u^2 H^4 D} + \text{H.c.}$	$\mathcal{O}_{u^2 H^4 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{d^2 H^4 D} + \text{H.c.}$	$\mathcal{O}_{d^2 H^4 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{l^2 H^4 D}^{(1,2,4)} + \text{H.c.}$	$\mathcal{O}_{l^2 H^4 D}^{(1,2,4)} - \text{H.c.}$	18	9
	$\mathcal{O}_{l^2 H^4 D}^{(3)} - \text{H.c.}$	$\mathcal{O}_{l^2 H^4 D}^{(3)} + \text{H.c.}$	3	6
	$\mathcal{O}_{e^2 H^4 D} + \text{H.c.}$	$\mathcal{O}_{e^2 H^4 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{udH^4 D} + \text{H.c.}$	$\mathcal{O}_{udH^4 D} - \text{H.c.}$	9	9
			69	48
$\psi^2 \phi^3 D^2$	$\mathcal{O}_{quH^3 D^2}^{(1,2)} - \text{H.c.}$	$\mathcal{O}_{quH^3 D^2}^{(1,2)} + \text{H.c.}$	18	18
	$\mathcal{O}_{quH^3 D^2}^{(3-6)} + \text{H.c.}$	$\mathcal{O}_{quH^3 D^2}^{(3-6)} - \text{H.c.}$	36	36
	$\mathcal{O}_{qdH^3 D^2}^{(1,2)} - \text{H.c.}$	$\mathcal{O}_{qdH^3 D^2}^{(1,2)} + \text{H.c.}$	18	18
	$\mathcal{O}_{qdH^3 D^2}^{(3-6)} + \text{H.c.}$	$\mathcal{O}_{qdH^3 D^2}^{(3-6)} - \text{H.c.}$	36	36
	$\mathcal{O}_{leH^3 D^2}^{(1,2)} - \text{H.c.}$	$\mathcal{O}_{leH^3 D^2}^{(1,2)} + \text{H.c.}$	18	18
	$\mathcal{O}_{leH^3 D^2}^{(3-6)} + \text{H.c.}$	$\mathcal{O}_{leH^3 D^2}^{(3-6)} - \text{H.c.}$	36	36
			162	162
$\psi^2 \phi^2 D^3$	$\mathcal{O}_{q^2 H^2 D^3}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{q^2 H^2 D^3}^{(1-4)} - \text{H.c.}$	24	12
	$\mathcal{O}_{u^2 H^2 D^3}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{u^2 H^2 D^3}^{(1-2)} - \text{H.c.}$	12	6
	$\mathcal{O}_{d^2 H^2 D^3}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{d^2 H^2 D^3}^{(1-2)} - \text{H.c.}$	12	6
	$\mathcal{O}_{l^2 H^2 D^3}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{l^2 H^2 D^3}^{(1-4)} - \text{H.c.}$	24	12
	$\mathcal{O}_{e^2 H^2 D^3}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{e^2 H^2 D^3}^{(1-2)} - \text{H.c.}$	12	6
	$\mathcal{O}_{udH^2 D^3}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{udH^2 D^3}^{(1-2)} - \text{H.c.}$	9	9
			93	51
$\psi^2 X \phi^3$	$\mathcal{O}_{GquH^3} - \text{H.c.}$	$\mathcal{O}_{GquH^3} + \text{H.c.}$	9	9
	$\mathcal{O}_{WquH^3}^{(1,2)} - \text{H.c.}$	$\mathcal{O}_{WquH^3}^{(1,2)} + \text{H.c.}$	18	18
	$\mathcal{O}_{BquH^3} - \text{H.c.}$	$\mathcal{O}_{BquH^3} + \text{H.c.}$	9	9
	$\mathcal{O}_{GqdH^3} - \text{H.c.}$	$\mathcal{O}_{GqdH^3} + \text{H.c.}$	9	9
	$\mathcal{O}_{WqdH^3}^{(1,2)} - \text{H.c.}$	$\mathcal{O}_{WqdH^3}^{(1,2)} + \text{H.c.}$	18	18
	$\mathcal{O}_{BqdH^3} - \text{H.c.}$	$\mathcal{O}_{BqdH^3} + \text{H.c.}$	9	9
	$\mathcal{O}_{WleH^3}^{(1,2)} - \text{H.c.}$	$\mathcal{O}_{WleH^3}^{(1,2)} + \text{H.c.}$	18	18
	$\mathcal{O}_{BleH^3} - \text{H.c.}$	$\mathcal{O}_{BleH^3} + \text{H.c.}$	9	9
			99	99

(Table continued)

TABLE VI. (Continued)

Class	Operators combination		Operators number	
	CP -even	CP -odd	CP -even	CP -odd
$\psi^2 X \phi D^2$	$\mathcal{O}_{GquHD^2}^{(1-3)} - \text{H.c.}$	$\mathcal{O}_{GquHD^2}^{(1-3)} + \text{H.c.}$	27	27
	$\mathcal{O}_{WquHD^2}^{(1-3)} - \text{H.c.}$	$\mathcal{O}_{WquHD^2}^{(1-3)} + \text{H.c.}$	27	27
	$\mathcal{O}_{BquHD^2}^{(1-3)} - \text{H.c.}$	$\mathcal{O}_{BquHD^2}^{(1-3)} + \text{H.c.}$	27	27
	$\mathcal{O}_{GqdHD^2}^{(1-3)} - \text{H.c.}$	$\mathcal{O}_{GqdHD^2}^{(1-3)} + \text{H.c.}$	27	27
	$\mathcal{O}_{WqdHD^2}^{(1-3)} - \text{H.c.}$	$\mathcal{O}_{WqdHD^2}^{(1-3)} + \text{H.c.}$	27	27
	$\mathcal{O}_{BqdHD^2}^{(1-3)} - \text{H.c.}$	$\mathcal{O}_{BqdHD^2}^{(1-3)} + \text{H.c.}$	27	27
	$\mathcal{O}_{WleHD^2}^{(1-3)} - \text{H.c.}$	$\mathcal{O}_{WleHD^2}^{(1-3)} + \text{H.c.}$	27	27
	$\mathcal{O}_{BleHD^2}^{(1-3)} - \text{H.c.}$	$\mathcal{O}_{BleHD^2}^{(1-3)} + \text{H.c.}$	27	27
			216	216
$\psi^2 X \phi^2 D$	$\mathcal{O}_{Gq^2 H^2 D}^{(1,4,5,8)} - \text{H.c.}$	$\mathcal{O}_{Gq^2 H^2 D}^{(1,4,5,8)} + \text{H.c.}$	12	24
	$\mathcal{O}_{Gq^2 H^2 D}^{(2,3,6,7)} + \text{H.c.}$	$\mathcal{O}_{Gq^2 H^2 D}^{(2,3,6,7)} - \text{H.c.}$	24	12
	$\mathcal{O}_{Wq^2 H^2 D}^{(1,4,5,8,9,12)} - \text{H.c.}$	$\mathcal{O}_{Wq^2 H^2 D}^{(1,4,5,8,9,12)} + \text{H.c.}$	18	36
	$\mathcal{O}_{Wq^2 H^2 D}^{(2,3,6,7,10,11)} + \text{H.c.}$	$\mathcal{O}_{Wq^2 H^2 D}^{(2,3,6,7,10,11)} - \text{H.c.}$	36	18
	$\mathcal{O}_{Bq^2 H^2 D}^{(1,4,5,8)} - \text{H.c.}$	$\mathcal{O}_{Bq^2 H^2 D}^{(1,4,5,8)} + \text{H.c.}$	12	24
	$\mathcal{O}_{Bq^2 H^2 D}^{(2,3,6,7)} + \text{H.c.}$	$\mathcal{O}_{Bq^2 H^2 D}^{(2,3,6,7)} - \text{H.c.}$	24	12
	$\mathcal{O}_{Gu^2 H^2 D}^{(1,4)} - \text{H.c.}$	$\mathcal{O}_{Gu^2 H^2 D}^{(1,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{Gu^2 H^2 D}^{(2,3)} + \text{H.c.}$	$\mathcal{O}_{Gu^2 H^2 D}^{(2,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{Wu^2 H^2 D}^{(1,4)} - \text{H.c.}$	$\mathcal{O}_{Wu^2 H^2 D}^{(1,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{Wu^2 H^2 D}^{(2,3)} + \text{H.c.}$	$\mathcal{O}_{Wu^2 H^2 D}^{(2,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{Bu^2 H^2 D}^{(1,4)} - \text{H.c.}$	$\mathcal{O}_{Bu^2 H^2 D}^{(1,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{Bu^2 H^2 D}^{(2,3)} + \text{H.c.}$	$\mathcal{O}_{Bu^2 H^2 D}^{(2,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{Gd^2 H^2 D}^{(1,4)} - \text{H.c.}$	$\mathcal{O}_{Gd^2 H^2 D}^{(1,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{Gd^2 H^2 D}^{(2,3)} + \text{H.c.}$	$\mathcal{O}_{Gd^2 H^2 D}^{(2,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{Wd^2 H^2 D}^{(1,4)} - \text{H.c.}$	$\mathcal{O}_{Wd^2 H^2 D}^{(1,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{Wd^2 H^2 D}^{(2,3)} + \text{H.c.}$	$\mathcal{O}_{Wd^2 H^2 D}^{(2,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{Bd^2 H^2 D}^{(1,4)} - \text{H.c.}$	$\mathcal{O}_{Bd^2 H^2 D}^{(1,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{Bd^2 H^2 D}^{(2,3)} + \text{H.c.}$	$\mathcal{O}_{Bd^2 H^2 D}^{(2,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{Wl^2 H^2 D}^{(1,4,5,8,9,12)} - \text{H.c.}$	$\mathcal{O}_{Wl^2 H^2 D}^{(1,4,5,8,9,12)} + \text{H.c.}$	18	36
	$\mathcal{O}_{Wl^2 H^2 D}^{(2,3,6,7,10,11)} + \text{H.c.}$	$\mathcal{O}_{Wl^2 H^2 D}^{(2,3,6,7,10,11)} - \text{H.c.}$	36	18
	$\mathcal{O}_{Bl^2 H^2 D}^{(1,4,5,8)} - \text{H.c.}$	$\mathcal{O}_{Bl^2 H^2 D}^{(1,4,5,8)} + \text{H.c.}$	12	24
	$\mathcal{O}_{Bl^2 H^2 D}^{(2,3,6,7)} + \text{H.c.}$	$\mathcal{O}_{Bl^2 H^2 D}^{(2,3,6,7)} - \text{H.c.}$	24	12
	$\mathcal{O}_{We^2 H^2 D}^{(1,4)} - \text{H.c.}$	$\mathcal{O}_{We^2 H^2 D}^{(1,4)} + \text{H.c.}$	6	12

(Table continued)

TABLE VI. (Continued)

Class	Operators combination		Operators number	
	CP -even	CP -odd	CP -even	CP -odd
$\psi^2 X^2 D$	$\mathcal{O}_{We^2 H^2 D}^{(2,3)} + \text{H.c.}$	$\mathcal{O}_{We^2 H^2 D}^{(2,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{Be^2 H^2 D}^{(1,4)} - \text{H.c.}$	$\mathcal{O}_{Be^2 H^2 D}^{(1,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{Be^2 H^2 D}^{(2,3)} + \text{H.c.}$	$\mathcal{O}_{Be^2 H^2 D}^{(2,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{Gud H^2 D}^{(1)} - \text{H.c.}$	$\mathcal{O}_{Gud H^2 D}^{(1)} + \text{H.c.}$	9	9
	$\mathcal{O}_{Gud H^2 D}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Gud H^2 D}^{(2)} - \text{H.c.}$	9	9
	$\mathcal{O}_{Wud H^2 D}^{(1)} - \text{H.c.}$	$\mathcal{O}_{Wud H^2 D}^{(1)} + \text{H.c.}$	9	9
	$\mathcal{O}_{Wud H^2 D}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Wud H^2 D}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{Bud H^2 D}^{(1)} - \text{H.c.}$	$\mathcal{O}_{Bud H^2 D}^{(1)} + \text{H.c.}$	9	9
	$\mathcal{O}_{Bud H^2 D}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Bud H^2 D}^{(1)} - \text{H.c.}$	9	9
			414	414
	$\mathcal{O}_{G^2 quH}^{(1,2,5)} + \text{H.c.}$	$\mathcal{O}_{G^2 quH}^{(1,2,5)} - \text{H.c.}$	27	27
	$\mathcal{O}_{G^2 quH}^{(3,4)} - \text{H.c.}$	$\mathcal{O}_{G^2 quH}^{(3,4)} + \text{H.c.}$	18	18
	$\mathcal{O}_{W^2 quH}^{(1,3)} + \text{H.c.}$	$\mathcal{O}_{W^2 quH}^{(1,3)} - \text{H.c.}$	18	18
	$\mathcal{O}_{W^2 quH}^{(2)} - \text{H.c.}$	$\mathcal{O}_{W^2 quH}^{(2)} + \text{H.c.}$	9	9
	$\mathcal{O}_{GWquH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{GWquH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{GWquH}^{(2,3)} - \text{H.c.}$	$\mathcal{O}_{GWquH}^{(2,3)} + \text{H.c.}$	18	18
$\psi^2 X^2 \phi$	$\mathcal{O}_{B^2 quH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{B^2 quH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{B^2 quH}^{(2)} - \text{H.c.}$	$\mathcal{O}_{B^2 quH}^{(2)} + \text{H.c.}$	9	9
	$\mathcal{O}_{BGquH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{BGquH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{BGquH}^{(2,3)} - \text{H.c.}$	$\mathcal{O}_{BGquH}^{(2,3)} + \text{H.c.}$	18	18
	$\mathcal{O}_{BWquH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{BWquH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{BWquH}^{(2,3)} - \text{H.c.}$	$\mathcal{O}_{BWquH}^{(2,3)} + \text{H.c.}$	18	18
	$\mathcal{O}_{G^2 qdH}^{(1,2,5)} + \text{H.c.}$	$\mathcal{O}_{G^2 qdH}^{(1,2,5)} - \text{H.c.}$	27	27
	$\mathcal{O}_{G^2 qdH}^{(3,4)} - \text{H.c.}$	$\mathcal{O}_{G^2 qdH}^{(3,4)} + \text{H.c.}$	18	18
	$\mathcal{O}_{W^2 qdH}^{(1,3)} + \text{H.c.}$	$\mathcal{O}_{W^2 qdH}^{(1,3)} - \text{H.c.}$	18	18
	$\mathcal{O}_{W^2 qdH}^{(2)} - \text{H.c.}$	$\mathcal{O}_{W^2 qdH}^{(2)} + \text{H.c.}$	9	9
	$\mathcal{O}_{GWqdH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{GWqdH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{GWqdH}^{(2,3)} - \text{H.c.}$	$\mathcal{O}_{GWqdH}^{(2,3)} + \text{H.c.}$	18	18
	$\mathcal{O}_{B^2 qdH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{B^2 qdH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{B^2 qdH}^{(2)} - \text{H.c.}$	$\mathcal{O}_{B^2 qdH}^{(2)} + \text{H.c.}$	9	9
	$\mathcal{O}_{BGqdH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{BGqdH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{BGqdH}^{(2,3)} - \text{H.c.}$	$\mathcal{O}_{BGqdH}^{(2,3)} + \text{H.c.}$	18	18
	$\mathcal{O}_{BWqdH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{BWqdH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{BWqdH}^{(2,3)} - \text{H.c.}$	$\mathcal{O}_{BWqdH}^{(2,3)} + \text{H.c.}$	18	18

(Table continued)

TABLE VI. (Continued)

Class	Operators combination		Operators number	
	CP -even	CP -odd	CP -even	CP -odd
$\psi^2 X^2 D$	$\mathcal{O}_{G^2 leH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{G^2 leH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{G^2 leH}^{(2)} - \text{H.c.}$	$\mathcal{O}_{G^2 leH}^{(2)} + \text{H.c.}$	9	9
	$\mathcal{O}_{W^2 leH}^{(1,3)} + \text{H.c.}$	$\mathcal{O}_{W^2 leH}^{(1,3)} - \text{H.c.}$	18	18
	$\mathcal{O}_{W^2 leH}^{(2)} - \text{H.c.}$	$\mathcal{O}_{W^2 leH}^{(2)} + \text{H.c.}$	9	9
	$\mathcal{O}_{B^2 leH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{B^2 leH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{B^2 leH}^{(2)} - \text{H.c.}$	$\mathcal{O}_{B^2 leH}^{(2)} + \text{H.c.}$	9	9
	$\mathcal{O}_{BW leH}^{(1)} + \text{H.c.}$	$\mathcal{O}_{BW leH}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{BW leH}^{(2,3)} - \text{H.c.}$	$\mathcal{O}_{BW leH}^{(2,3)} + \text{H.c.}$	18	18
			432	432
	$\mathcal{O}_{G^2 q^2 D}^{(1)} - \text{H.c.}$	$\mathcal{O}_{G^2 q^2 D}^{(1)} + \text{H.c.}$	3	6
	$\mathcal{O}_{G^2 q^2 D}^{(2-5)} + \text{H.c.}$	$\mathcal{O}_{G^2 q^2 D}^{(2-5)} - \text{H.c.}$	24	12
	$\mathcal{O}_{W^2 q^2 D}^{(1)} - \text{H.c.}$	$\mathcal{O}_{W^2 q^2 D}^{(1)} + \text{H.c.}$	3	6
	$\mathcal{O}_{W^2 q^2 D}^{(2-4)} + \text{H.c.}$	$\mathcal{O}_{W^2 q^2 D}^{(2-4)} - \text{H.c.}$	18	9
	$\mathcal{O}_{B^2 q^2 D} + \text{H.c.}$	$\mathcal{O}_{B^2 q^2 D} - \text{H.c.}$	6	3
$\psi^2 X^2 D$	$\mathcal{O}_{G^2 u^2 D}^{(1)} - \text{H.c.}$	$\mathcal{O}_{G^2 u^2 D}^{(1)} + \text{H.c.}$	3	6
	$\mathcal{O}_{G^2 u^2 D}^{(2-5)} + \text{H.c.}$	$\mathcal{O}_{G^2 u^2 D}^{(2-5)} - \text{H.c.}$	24	12
	$\mathcal{O}_{W^2 u^2 D} + \text{H.c.}$	$\mathcal{O}_{B^2 u^2 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{B^2 u^2 D} + \text{H.c.}$	$\mathcal{O}_{B^2 u^2 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{G^2 d^2 D}^{(1)} - \text{H.c.}$	$\mathcal{O}_{G^2 d^2 D}^{(1)} + \text{H.c.}$	3	6
	$\mathcal{O}_{G^2 d^2 D}^{(2-5)} + \text{H.c.}$	$\mathcal{O}_{G^2 d^2 D}^{(2-5)} - \text{H.c.}$	24	12
	$\mathcal{O}_{W^2 d^2 D} + \text{H.c.}$	$\mathcal{O}_{B^2 d^2 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{B^2 d^2 D} + \text{H.c.}$	$\mathcal{O}_{B^2 d^2 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{G^2 l^2 D} + \text{H.c.}$	$\mathcal{O}_{G^2 l^2 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{W^2 l^2 D}^{(1)} - \text{H.c.}$	$\mathcal{O}_{W^2 l^2 D}^{(1)} + \text{H.c.}$	3	6
	$\mathcal{O}_{W^2 l^2 D}^{(2-4)} + \text{H.c.}$	$\mathcal{O}_{W^2 l^2 D}^{(2-4)} - \text{H.c.}$	18	9
	$\mathcal{O}_{B^2 l^2 D} + \text{H.c.}$	$\mathcal{O}_{B^2 l^2 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{G^2 e^2 D} + \text{H.c.}$	$\mathcal{O}_{G^2 e^2 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{W^2 e^2 D} + \text{H.c.}$	$\mathcal{O}_{W^2 e^2 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{B^2 e^2 D} + \text{H.c.}$	$\mathcal{O}_{B^2 e^2 D} - \text{H.c.}$	6	3
	$\mathcal{O}_{GW q^2 D}^{(1,3)} + \text{H.c.}$	$\mathcal{O}_{GW q^2 D}^{(1,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{GW q^2 D}^{(2,4)} - \text{H.c.}$	$\mathcal{O}_{GW q^2 D}^{(2,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{GB q^2 D}^{(1,3)} + \text{H.c.}$	$\mathcal{O}_{GB q^2 D}^{(1,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{GB q^2 D}^{(2,4)} - \text{H.c.}$	$\mathcal{O}_{GB q^2 D}^{(2,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{WB q^2 D}^{(1,3)} + \text{H.c.}$	$\mathcal{O}_{WB q^2 D}^{(1,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{WB q^2 D}^{(2,4)} - \text{H.c.}$	$\mathcal{O}_{WB q^2 D}^{(2,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{GB u^2 D}^{(1,3)} + \text{H.c.}$	$\mathcal{O}_{GB u^2 D}^{(1,3)} - \text{H.c.}$	12	6

(Table continued)

TABLE VI. (Continued)

Class	Operators combination		Operators number	
	CP -even	CP -odd	CP -even	CP -odd
$\psi^4 \phi D$	$\mathcal{O}_{GBu^2D}^{(2,4)} - \text{H.c.}$	$\mathcal{O}_{GBu^2D}^{(2,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{GBd^2D}^{(1,3)} + \text{H.c.}$	$\mathcal{O}_{GBd^2D}^{(1,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{GBd^2D}^{(2,4)} - \text{H.c.}$	$\mathcal{O}_{GBd^2D}^{(2,4)} + \text{H.c.}$	6	12
	$\mathcal{O}_{WBI^2D}^{(1,3)} + \text{H.c.}$	$\mathcal{O}_{WBI^2D}^{(1,3)} - \text{H.c.}$	12	6
	$\mathcal{O}_{WBI^2D}^{(2,4)} - \text{H.c.}$	$\mathcal{O}_{WBI^2D}^{(2,4)} + \text{H.c.}$	6	12
			291	222
	$\mathcal{O}_{q^3uHD}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{q^3uHD}^{(1,2)} - \text{H.c.}$	162	162
	$\mathcal{O}_{q^3uHD}^{(3,6)} - \text{H.c.}$	$\mathcal{O}_{q^3uHD}^{(3,6)} + \text{H.c.}$	324	324
	$\mathcal{O}_{q^3dHD}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{q^3dHD}^{(1,2)} - \text{H.c.}$	162	162
	$\mathcal{O}_{q^3dHD}^{(3,6)} - \text{H.c.}$	$\mathcal{O}_{q^3dHD}^{(3,6)} + \text{H.c.}$	324	324
	$\mathcal{O}_{qu^3HD}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{qu^3HD}^{(1,2)} - \text{H.c.}$	162	162
	$\mathcal{O}_{qu^3HD}^{(3)} - \text{H.c.}$	$\mathcal{O}_{qu^3HD}^{(3)} + \text{H.c.}$	81	81
	$\mathcal{O}_{qu^2dHD}^{(1,4)} - \text{H.c.}$	$\mathcal{O}_{qu^2dHD}^{(1,4)} + \text{H.c.}$	324	324
	$\mathcal{O}_{qu^2dHD}^{(5,6)} + \text{H.c.}$	$\mathcal{O}_{qu^2dHD}^{(5,6)} - \text{H.c.}$	162	162
	$\mathcal{O}_{qud^2HD}^{(1,4)} - \text{H.c.}$	$\mathcal{O}_{qud^2HD}^{(1,4)} + \text{H.c.}$	324	324
	$\mathcal{O}_{qud^2HD}^{(5,6)} + \text{H.c.}$	$\mathcal{O}_{qud^2HD}^{(5,6)} - \text{H.c.}$	162	162
	$\mathcal{O}_{qd^3HD}^{(1,2)} - \text{H.c.}$	$\mathcal{O}_{qd^3HD}^{(1,2)} + \text{H.c.}$	162	162
	$\mathcal{O}_{qd^3HD}^{(3)} + \text{H.c.}$	$\mathcal{O}_{qd^3HD}^{(3)} - \text{H.c.}$	81	81
	$\mathcal{O}_{q^2leHD}^{(1,2,5,6)} - \text{H.c.}$	$\mathcal{O}_{q^2leHD}^{(1,2,5,6)} + \text{H.c.}$	324	324
	$\mathcal{O}_{q^2leHD}^{(3,4)} + \text{H.c.}$	$\mathcal{O}_{q^2leHD}^{(3,4)} - \text{H.c.}$	162	162
	$\mathcal{O}_{qu^2HD}^{(1,2,5,6)} - \text{H.c.}$	$\mathcal{O}_{qu^2HD}^{(1,2,5,6)} + \text{H.c.}$	324	324
	$\mathcal{O}_{qu^2HD}^{(3,4)} + \text{H.c.}$	$\mathcal{O}_{qu^2HD}^{(3,4)} - \text{H.c.}$	162	162
	$\mathcal{O}_{que^2HD}^{(1,3)} - \text{H.c.}$	$\mathcal{O}_{que^2HD}^{(1,3)} + \text{H.c.}$	162	162
	$\mathcal{O}_{que^2HD}^{(2)} + \text{H.c.}$	$\mathcal{O}_{que^2HD}^{(2)} - \text{H.c.}$	81	81
	$\mathcal{O}_{qdl^2HD}^{(1,2,5,6)} - \text{H.c.}$	$\mathcal{O}_{qdl^2HD}^{(1,2,5,6)} + \text{H.c.}$	324	324
	$\mathcal{O}_{qdl^2HD}^{(3,4)} + \text{H.c.}$	$\mathcal{O}_{qdl^2HD}^{(3,4)} - \text{H.c.}$	162	162
	$\mathcal{O}_{qde^2HD}^{(1,3)} - \text{H.c.}$	$\mathcal{O}_{qde^2HD}^{(1,3)} + \text{H.c.}$	162	162
	$\mathcal{O}_{qde^2HD}^{(2)} + \text{H.c.}$	$\mathcal{O}_{qde^2HD}^{(2)} - \text{H.c.}$	81	81
	$\mathcal{O}_{u^2leHD}^{(1,3)} - \text{H.c.}$	$\mathcal{O}_{u^2leHD}^{(1,3)} + \text{H.c.}$	162	162
	$\mathcal{O}_{u^2leHD}^{(2)} + \text{H.c.}$	$\mathcal{O}_{u^2leHD}^{(2)} - \text{H.c.}$	81	81
	$\mathcal{O}_{udleHD}^{(1,3)} - \text{H.c.}$	$\mathcal{O}_{udleHD}^{(1,3)} + \text{H.c.}$	162	162
	$\mathcal{O}_{udleHD}^{(2)} + \text{H.c.}$	$\mathcal{O}_{udleHD}^{(2)} - \text{H.c.}$	81	81
	$\mathcal{O}_{d^2leHD}^{(1,3)} - \text{H.c.}$	$\mathcal{O}_{d^2leHD}^{(1,3)} + \text{H.c.}$	162	162

(Table continued)

TABLE VI. (Continued)

Class	Operators combination		Operators number	
	CP -even	CP -odd	CP -even	CP -odd
$\psi^4 D^2$	$\mathcal{O}_{d^2 leHD}^{(2)} + \text{H.c.}$	$\mathcal{O}_{d^2 leHD}^{(2)} - \text{H.c.}$	81	81
	$\mathcal{O}_{l^3 eHD}^{(1,3)} - \text{H.c.}$	$\mathcal{O}_{l^3 eHD}^{(1,3)} + \text{H.c.}$	162	162
	$\mathcal{O}_{l^3 eHD}^{(2)} + \text{H.c.}$	$\mathcal{O}_{l^3 eHD}^{(2)} - \text{H.c.}$	81	81
	$\mathcal{O}_{le^3 HD}^{(1,2)} - \text{H.c.}$	$\mathcal{O}_{le^3 HD}^{(1,2)} + \text{H.c.}$	108	108
			5454	5454
	$\mathcal{O}_{q^4 D^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{q^4 D^2}^{(1-4)} - \text{H.c.}$	108	72
	$\mathcal{O}_{q^2 u^2 D^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{q^2 u^2 D^2}^{(1-4)} - \text{H.c.}$	180	144
	$\mathcal{O}_{q^2 d^2 D^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{q^2 d^2 D^2}^{(1-4)} - \text{H.c.}$	180	144
	$\mathcal{O}_{u^4 D^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{u^4 D^2}^{(1,2)} - \text{H.c.}$	54	36
	$\mathcal{O}_{u^2 d^2 D^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{u^2 d^2 D^2}^{(1-4)} - \text{H.c.}$	180	144
	$\mathcal{O}_{d^4 D^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{d^4 D^2}^{(1,2)} - \text{H.c.}$	54	36
	$\mathcal{O}_{q^2 udD^2}^{(1-3)} + \text{H.c.}$	$\mathcal{O}_{q^2 udD^2}^{(1-3)} - \text{H.c.}$	243	243
	$\mathcal{O}_{q^2 l^2 D^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{q^2 l^2 D^2}^{(1-4)} - \text{H.c.}$	180	144
	$\mathcal{O}_{q^2 e^2 D^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{q^2 e^2 D^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{u^2 l^2 D^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{u^2 l^2 D^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{u^2 e^2 D^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{u^2 e^2 D^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{d^2 l^2 D^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{d^2 l^2 D^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{d^2 e^2 D^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{d^2 e^2 D^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{quleD^2}^{(1-3)} + \text{H.c.}$	$\mathcal{O}_{quleD^2}^{(1-3)} - \text{H.c.}$	243	243
	$\mathcal{O}_{qdleD^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{qdleD^2}^{(1,2)} - \text{H.c.}$	162	162
	$\mathcal{O}_{l^4 D^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{l^4 D^2}^{(1,2)} - \text{H.c.}$	54	36
	$\mathcal{O}_{l^2 e^2 D^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{l^2 e^2 D^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{e^4 D^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{e^4 D^2}^{(1,2)} - \text{H.c.}$	27	18
			2205	1854
$\psi^4 \phi^2$	$\mathcal{O}_{q^4 H^2}^{(1-3)} + \text{H.c.}$	$\mathcal{O}_{q^4 H^2}^{(1-3)} - \text{H.c.}$	117	90
	$\mathcal{O}_{q^2 u^2 H^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{q^2 u^2 H^2}^{(1-4)} - \text{H.c.}$	180	144
	$\mathcal{O}_{q^2 d^2 H^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{q^2 d^2 H^2}^{(1-4)} - \text{H.c.}$	180	144
	$\mathcal{O}_{u^4 H^2} + \text{H.c.}$	$\mathcal{O}_{u^4 H^2} - \text{H.c.}$	27	18
	$\mathcal{O}_{u^2 d^2 H^2}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{u^2 d^2 H^2}^{(1-2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{d^4 H^2} + \text{H.c.}$	$\mathcal{O}_{d^4 H^2} - \text{H.c.}$	27	18
	$\mathcal{O}'_{q^2 u^2 H^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}'_{q^2 u^2 H^2}^{(1,2)} - \text{H.c.}$	90	90
	$\mathcal{O}'_{q^2 d^2 H^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}'_{q^2 d^2 H^2}^{(1,2)} - \text{H.c.}$	90	90
	$\mathcal{O}_{q^2 udH^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{q^2 udH^2}^{(1-4)} - \text{H.c.}$	324	324
	$\mathcal{O}'_{q^2 udH^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}'_{q^2 udH^2}^{(1,2)} - \text{H.c.}$	162	162
	$\mathcal{O}_{q^2 l^2 H^2}^{(1-5)} + \text{H.c.}$	$\mathcal{O}_{q^2 l^2 H^2}^{(1-5)} - \text{H.c.}$	216	189

(Table continued)

TABLE VI. (Continued)

Class	Operators combination		Operators number	
	CP -even	CP -odd	CP -even	CP -odd
	$\mathcal{O}_{q^2 e^2 H^2}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{q^2 e^2 H^2}^{(1-2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{u^2 l^2 H^2}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{u^2 l^2 H^2}^{(1-2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{u^2 e^2 H^2} + \text{H.c.}$	$\mathcal{O}_{u^2 e^2 H^2} - \text{H.c.}$	45	36
	$\mathcal{O}_{d^2 l^2 H^2}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{d^2 l^2 H^2}^{(1-2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{d^2 e^2 H^2} + \text{H.c.}$	$\mathcal{O}_{d^2 e^2 H^2} - \text{H.c.}$	45	36
	$\mathcal{O}_{quleH^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{quleH^2}^{(1-4)} - \text{H.c.}$	324	324
	$\mathcal{O}'_{quleH^2} + \text{H.c.}$	$\mathcal{O}'_{quleH^2} - \text{H.c.}$	81	81
	$\mathcal{O}_{qdleH^2}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{qdleH^2}^{(1-2)} - \text{H.c.}$	162	162
	$\mathcal{O}'_{qdleH^2} + \text{H.c.}$	$\mathcal{O}'_{qdleH^2} - \text{H.c.}$	162	162
	$\mathcal{O}'_{l^2 udH^2} + \text{H.c.}$	$\mathcal{O}'_{l^2 udH^2} - \text{H.c.}$	81	81
	$\mathcal{O}_{l^4 H^2}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{l^4 H^2}^{(1-2)} - \text{H.c.}$	66	51
	$\mathcal{O}_{l^2 e^2 H^2}^{(1-2)} + \text{H.c.}$	$\mathcal{O}_{l^2 e^2 H^2}^{(1-2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{e^4 H^2} + \text{H.c.}$	$\mathcal{O}_{e^4 H^2} - \text{H.c.}$	21	15
	$\mathcal{O}'_{l^2 e^2 H^2} + \text{H.c.}$	$\mathcal{O}'_{l^2 e^2 H^2} - \text{H.c.}$	45	45
			2895	2622
$\psi^4 X$	$\mathcal{O}_{Gq^4}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{Gq^4}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{Gq^4}^{(3,4)} - \text{H.c.}$	$\mathcal{O}_{Gq^4}^{(3,4)} + \text{H.c.}$	72	90
	$\mathcal{O}_{Wq^4}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{Wq^4}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{Wq^4}^{(3,4)} - \text{H.c.}$	$\mathcal{O}_{Wq^4}^{(3,4)} + \text{H.c.}$	72	90
	$\mathcal{O}_{Bq^4}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{Bq^4}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{Bq^4}^{(3,4)} - \text{H.c.}$	$\mathcal{O}_{Bq^4}^{(3,4)} + \text{H.c.}$	72	90
	$\mathcal{O}_{Gq^2 u^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{Gq^2 u^2}^{(1-4)} - \text{H.c.}$	180	144
	$\mathcal{O}_{Gq^2 u^2}^{(5-8)} - \text{H.c.}$	$\mathcal{O}_{Gq^2 u^2}^{(5-8)} + \text{H.c.}$	144	180
	$\mathcal{O}_{Wq^2 u^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{Wq^2 u^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{Wq^2 u^2}^{(3,4)} - \text{H.c.}$	$\mathcal{O}_{Wq^2 u^2}^{(3,4)} + \text{H.c.}$	72	90
	$\mathcal{O}_{Bq^2 u^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{Bq^2 u^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{Bq^2 u^2}^{(3,4)} - \text{H.c.}$	$\mathcal{O}_{Bq^2 u^2}^{(3,4)} + \text{H.c.}$	72	90
	$\mathcal{O}_{Gq^2 d^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{Gq^2 d^2}^{(1-4)} - \text{H.c.}$	180	144
	$\mathcal{O}_{Gq^2 d^2}^{(5-8)} - \text{H.c.}$	$\mathcal{O}_{Gq^2 d^2}^{(5-8)} + \text{H.c.}$	144	180
	$\mathcal{O}_{Wq^2 d^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{Wq^2 d^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{Wq^2 d^2}^{(3,4)} - \text{H.c.}$	$\mathcal{O}_{Wq^2 d^2}^{(3,4)} + \text{H.c.}$	72	90
	$\mathcal{O}_{Bq^2 d^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{Bq^2 d^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{Bq^2 d^2}^{(3,4)} - \text{H.c.}$	$\mathcal{O}_{Bq^2 d^2}^{(3,4)} + \text{H.c.}$	72	90
	$\mathcal{O}_{Gu^4}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Gu^4}^{(1)} - \text{H.c.}$	45	36

(Table continued)

TABLE VI. (Continued)

Class	Operators combination		Operators number	
	CP -even	CP -odd	CP -even	CP -odd
	$\mathcal{O}_{Gu^4}^{(2)} - \text{H.c.}$	$\mathcal{O}_{Gu^4}^{(2)} + \text{H.c.}$	36	45
	$\mathcal{O}_{Bu^4}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Bu^4}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Bu^4}^{(2)} - \text{H.c.}$	$\mathcal{O}_{Bu^4}^{(2)} + \text{H.c.}$	36	45
	$\mathcal{O}_{Gu^2 d^2}^{(1-4)} + \text{H.c.}$	$\mathcal{O}_{Gu^2 d^2}^{(1-4)} - \text{H.c.}$	180	144
	$\mathcal{O}_{Gu^2 d^2}^{(5-8)} - \text{H.c.}$	$\mathcal{O}_{Gu^2 d^2}^{(5-8)} + \text{H.c.}$	144	180
	$\mathcal{O}_{Bu^2 d^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{Bu^2 d^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{Bu^2 d^2}^{(3,4)} - \text{H.c.}$	$\mathcal{O}_{Bu^2 d^2}^{(3,4)} + \text{H.c.}$	72	90
	$\mathcal{O}_{Gq^2 ud}^{(1-6)} - \text{H.c.}$	$\mathcal{O}_{Gq^2 ud}^{(1-6)} + \text{H.c.}$	486	486
	$\mathcal{O}_{Wq^2 ud}^{(1-3)} - \text{H.c.}$	$\mathcal{O}_{Wq^2 ud}^{(1-3)} + \text{H.c.}$	243	243
	$\mathcal{O}_{Bq^2 ud}^{(1-3)} - \text{H.c.}$	$\mathcal{O}_{Bq^2 ud}^{(1-3)} + \text{H.c.}$	243	243
	$\mathcal{O}_{Gq^2 l^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{Gq^2 l^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{Gq^2 l^2}^{(3,4)} + \text{H.c.}$	$\mathcal{O}_{Gq^2 l^2}^{(3,4)} - \text{H.c.}$	72	90
	$\mathcal{O}_{Wq^2 l^2}^{(1-3)} + \text{H.c.}$	$\mathcal{O}_{Wq^2 l^2}^{(1-3)} - \text{H.c.}$	135	108
	$\mathcal{O}_{Wq^2 l^2}^{(4-6)} + \text{H.c.}$	$\mathcal{O}_{Wq^2 l^2}^{(4-6)} - \text{H.c.}$	108	135
	$\mathcal{O}_{Bq^2 l^2}^{(1,2)} + \text{H.c.}$	$\mathcal{O}_{Bq^2 l^2}^{(1,2)} - \text{H.c.}$	90	72
	$\mathcal{O}_{Bq^2 l^2}^{(3,4)} + \text{H.c.}$	$\mathcal{O}_{Bq^2 l^2}^{(3,4)} - \text{H.c.}$	72	90
	$\mathcal{O}_{Gq^2 e^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Gq^2 e^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Gq^2 e^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Gq^2 e^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Wq^2 e^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Wq^2 e^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Wq^2 e^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Wq^2 e^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Bq^2 e^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Bq^2 e^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Bq^2 e^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Bq^2 e^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Gu^2 l^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Gu^2 l^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Gu^2 l^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Gu^2 l^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Wu^2 l^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Wu^2 l^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Wu^2 l^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Wu^2 l^2}^{(2)} - \text{H.c.}$	36	35
	$\mathcal{O}_{Bu^2 l^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Bu^2 l^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Bu^2 l^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Bu^2 l^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Gu^2 e^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Gu^2 e^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Gu^2 e^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Gu^2 e^2}^{(2)} - \text{H.c.}$	36	35
	$\mathcal{O}_{Bu^2 e^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Bu^2 e^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Bu^2 e^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Bu^2 e^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Gd^2 l^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Gd^2 l^2}^{(1)} - \text{H.c.}$	45	36

(Table continued)

TABLE VI. (Continued)

Class	Operators combination		Operators number	
	CP -even	CP -odd	CP -even	CP -odd
	$\mathcal{O}_{Gd^2 l^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Gd^2 l^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Wd^2 l^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Wd^2 l^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Wd^2 l^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Wd^2 l^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Bd^2 l^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Bd^2 l^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Bd^2 l^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Bd^2 l^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Gd^2 e^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Gd^2 e^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Gd^2 e^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Gd^2 e^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Bd^2 e^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Bd^2 e^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Bd^2 e^2}^{(2)} + \text{H.c.}$	$\mathcal{O}_{Bd^2 e^2}^{(2)} - \text{H.c.}$	36	45
	$\mathcal{O}_{Wl^4}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Wl^4}^{(1)} - \text{H.c.}$	36	27
	$\mathcal{O}_{Wl^4}^{(2)} - \text{H.c.}$	$\mathcal{O}_{Wl^4}^{(2)} + \text{H.c.}$	27	36
	$\mathcal{O}_{Bl^4}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Bl^4}^{(1)} - \text{H.c.}$	18	18
	$\mathcal{O}_{Bl^4}^{(2)} - \text{H.c.}$	$\mathcal{O}_{Bl^4}^{(2)} + \text{H.c.}$	18	18
	$\mathcal{O}_{Be^4}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Be^4}^{(1)} - \text{H.c.}$	9	9
	$\mathcal{O}_{Be^4}^{(2)} - \text{H.c.}$	$\mathcal{O}_{Be^4}^{(2)} + \text{H.c.}$	9	9
	$\mathcal{O}_{Wl^2 e^2}^{(1)} + \text{H.c.}$	$\mathcal{O}_{Wl^2 e^2}^{(1)} - \text{H.c.}$	45	36
	$\mathcal{O}_{Wl^2 e^2}^{(2)} - \text{H.c.}$	$\mathcal{O}_{Wl^2 e^2}^{(2)} + \text{H.c.}$	36	45
	$\mathcal{O}_{Gqule}^{(1-2)} - \text{H.c.}$	$\mathcal{O}_{Gqule}^{(1-2)} + \text{H.c.}$	162	162
	$\mathcal{O}_{Gqule}^{(3)} + \text{H.c.}$	$\mathcal{O}_{Gqule}^{(3)} - \text{H.c.}$	81	81
	$\mathcal{O}_{Wqule}^{(1-2)} - \text{H.c.}$	$\mathcal{O}_{Wqule}^{(1-2)} + \text{H.c.}$	162	162
	$\mathcal{O}_{Wqule}^{(3)} + \text{H.c.}$	$\mathcal{O}_{Wqule}^{(3)} - \text{H.c.}$	81	81
	$\mathcal{O}_{Bqule}^{(1-2)} - \text{H.c.}$	$\mathcal{O}_{Bqule}^{(1-2)} + \text{H.c.}$	162	162
	$\mathcal{O}_{Bqule}^{(3)} + \text{H.c.}$	$\mathcal{O}_{Bqule}^{(3)} - \text{H.c.}$	81	81
	$\mathcal{O}_{Gqdle}^{(1-2)} - \text{H.c.}$	$\mathcal{O}_{Gqdle}^{(1-2)} + \text{H.c.}$	162	162
	$\mathcal{O}_{Wqdle}^{(1-2)} - \text{H.c.}$	$\mathcal{O}_{Wqdle}^{(1-2)} + \text{H.c.}$	162	162
	$\mathcal{O}_{Bqdle}^{(1-2)} - \text{H.c.}$	$\mathcal{O}_{Bqdle}^{(1-2)} + \text{H.c.}$	162	162
			6462	6462

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. The four parameters are the three angles $\theta_{12}, \theta_{13}, \theta_{23}$ and one phase δ . The phase δ is closely related to the CP violations of the SMEFT and will be discussed next.

Return to the field redefinitions in Eq. (3.43), the remaining single redefinition is the universal rephasing of the six quarks,

$$q_p \rightarrow e^{i\alpha} q_p, \quad p = 1, 2, 3, \quad \text{and} \quad q = u, d, \quad (3.45)$$

which is a global symmetry of the SM Lagrangian, called the baryon number symmetry $U(1)_B$. Because the leptons do not mix at the SM Lagrangian, there are three more global symmetries for each flavor of the leptons, respectively, noted as $U(1)_e$, $U(1)_\mu$, and $U(1)_\tau$. We factorize the

flavor-universal one as the lepton number symmetry $U(1)_L$ and formulate the four $U(1)$ symmetries as

$$U(1)^4 = \begin{cases} U(1)_{e-\mu} \times U(1)_{\mu-\tau}, \\ U(1)_B \times U(1)_L \end{cases}, \quad (3.46)$$

where $U(1)_B$ and $U(1)_L$ are flavor universal. $U(1)^4$ is called the rephasing symmetry, which is an accidental symmetry of the SM Lagrangian and could be violated by the high-dimension operators of the SMEFT.

As mentioned before, not all the CP -odd operators are CP -violating, and only the ones that are invariant under the rephasing symmetry in Eq. (3.46) are CP -violating since no field redefinition can be used to eliminate the CP -violating phase in the associated Wilson coefficients.

Since the bosons are trivial under the rephasing symmetry, all the CP -odd bosonic operators are always CP -violating. Considering the LO Lagrangian of the SMEFT, the CP -odd operators emerge only in the charged interactions of the quarks in Eq. (3.42) due to the CKM matrix. Because of the rephasing symmetry, these CP -odd operators are also CP violating, which share the same CP -violating phase, δ . Beyond the LO, the high-dimension operators could break the rephasing symmetry further, and only the ones that are CP -odd and rephasing symmetry conserving are CP -violating. For example, the dimension-6 operator

$$\mathcal{O}_{eH}^p = (H^\dagger H)(\bar{l}^p e_r H) \quad (3.47)$$

presents three CP -odd operators

$$\mathcal{O}_{eH}^p - \text{H.c.}, \quad p < r. \quad (3.48)$$

However, these three operators are not CP -violating, since they are not invariant under the rephasing symmetry in Eq. (3.46). On the contrary, the three CP -odd operators in

$$\mathcal{O}_{uH}^p = (H^\dagger H)(\bar{q}^p u_r H) \quad (3.49)$$

are all CP -violating.

Utilizing the Hilbert series, the numbers of the CP -odd and CP -violating operators are obtained up to dimension-14 [17]. In particular, there are 1422 and 22016 CP -odd operators at dimension-6 and dimension-8, respectively, while there are only 705 and 11777 CP -violating operators at dimension-6 and dimension-8, respectively. The detailed comparison of the dimension-8 operators can be found in Ref. [17].

IV. FLAVOR SYMMETRY OF SMEFT

In the previous section, we have discussed the CP property of the SMEFT without any flavor symmetry. The rephasing symmetry is important for the enumeration of the CP -violating operators, and there are a lot of

CP -violating operators beyond the LO Lagrangian without any flavor symmetry. In this section, we will discuss the situation with some flavor symmetry, especially the non-Abelian symmetry $U(3)^5$ and $U(2)^2$. The flavor invariants and the associated Young tableaux constructions are also discussed.

A. $U(3)^5$ symmetry

Considering the LO Lagrangian of the SMEFT in Eq. (2.1), regardless of the Yukawa sector $\mathcal{L}_{\text{Yukawa}}$, the remaining three sectors³ possess a global symmetry

$$U(3)^5 = U(3)_q \times U(3)_u \times U(3)_d \times U(3)_l \times U(3)_e, \quad (4.1)$$

in the flavor space. It can be further decomposed as

$$U(3)^5 = SU(3)^5 \times U(1)^5, \quad (4.2)$$

where

$$SU(3)^5 = SU(3)_q \times SU(3)_u \times SU(3)_d \times SU(3)_l \times SU(3)_e, \quad (4.3)$$

$$U(1)^5 = U(1)_q \times U(1)_u \times U(1)_d \times U(1)_l \times U(1)_e. \quad (4.4)$$

The five fermions' representations under the $SU(3)^5$ are

$$\begin{aligned} q &\in (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \\ u_R &\in (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \\ d_R &\in (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1}), \\ l &\in (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1}), \\ e_R &\in (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}). \end{aligned} \quad (4.5)$$

The $U(3)^5$ symmetry is the largest flavor symmetry of the SM fermions compatible with the gauge symmetry of the SM Lagrangian [29,51]. Sometimes, it is convenient to assume a subgroup of the $U(3)^5$ as the flavor symmetry, for example, the $U(2)^5$ or some finite groups [52–54]. We will talk about the $U(2)^5$ symmetry later.

The flavor symmetry $U(3)^5$ implies that the SM Lagrangian except the Yukawa interactions $\mathcal{L}_{\text{Yukawa}}$ is invariant under the unitary transformation

$$\begin{aligned} \psi &\rightarrow V_q \psi, & \psi &= u, d, \\ u_R &\rightarrow V_u u_R, & d_R &\rightarrow V_d d_R, \\ \psi &\rightarrow V_l \psi, & \psi &= e, \nu, \\ e_R &\rightarrow V_e e_R, \end{aligned} \quad (4.6)$$

³Actually only the $\mathcal{L}_{\text{fermion}}$ possesses nontrivial flavor structures.

where $(V_q, V_u, V_d, V_l, V_e) \in U(3)^5$. Such transformations are called the flavor rotations, and all the fermion triplets obtained by such transformations are the so-called weak basis. In particular, the mass basis and the weak basis are not equivalent.

Focusing on the weak basis, the $U(3)^5$ transformation induces the redefinitions of the Yukawa interactions,

$$\bar{u}_L M_u u_R \rightarrow \bar{u}'_L (V_q^\dagger M_u V_u) u'_R, \quad (4.7)$$

$$\bar{d}_L M_d d_R \rightarrow \bar{d}'_L (V_q^\dagger M_d V_d) d'_R, \quad (4.8)$$

$$\bar{e}_L M_e e_R \rightarrow \bar{e}'_L (V_l^\dagger M_e V_e) e'_R. \quad (4.9)$$

Although the flavor rotation makes the weak basis arbitrary, there are some convenient choices. For example, if we choose $V_q = U_L^u, V_u = U_R^u$, the mass matrix M_u becomes diagonal, as shown in Eq. (3.36); then we choose $V_d = U_R^d$, and the down-quark mass matrix becomes

$$U_L^{u\dagger} M_d U_R^d = U_L^{u\dagger} U_L^d U_L^d M_d U_R^d = V_{\text{CKM}} D_d, \quad (4.10)$$

according to Eq. (3.37), where $V_{\text{CKM}} = U_L^{u\dagger} U_L^d$ is the CKM matrix [12,13]. Its existence means that when M_u is diagonal by a certain flavor rotation, M_d is generally not, and leads to the quark flavor mixing, which emerges in the charged interactions in Eq. (3.42) under the mass basis. As for the leptons, since the right-handed neutrinos do not exist, the transformation V_l is arbitrary. We choose $V_l = U_L^e, V_e = U_R^e$, and then the lepton mass matrix can always be diagonal

$$U_L^{e\dagger} M_e U_R^e = D_e. \quad (4.11)$$

Thus the lepton flavors do not mix in the LO Lagrangian under the weak basis as well. In summary, $\mathcal{L}_{\text{Yukawa}}$ becomes

$$\mathcal{L}_{\text{Yukawa}} = \bar{u}'_L D_u u'_R + \bar{d}'_L V_{\text{CKM}} D_d d'_R + \bar{e}'_L D_e e'_R + \text{H.c.} \quad (4.12)$$

under this specific weak basis. The up-quark mass matrix is diagonal while the down-quark mass matrix is not; thus this weak basis is called the up-basis. In particular, only the left-handed down-quark d'_L is not of its mass eigenstates in the up-basis. On the other hand, if we require the down-quark mass matrix diagonal, we obtain the down-basis, under which the $\mathcal{L}_{\text{Yukawa}}$ becomes

$$\mathcal{L}_{\text{Yukawa}} = \bar{u}'_L V_{\text{CKM}}^\dagger D_u u'_R + \bar{d}'_L D_d d'_R + \bar{e}'_L D_e e'_R + \text{H.c.}, \quad (4.13)$$

where only the left-handed up-quark u'_L is not the mass eigenstates. In this paper, when some specific weak basis is needed, we use the down-basis consistently. For

convenience, we present the Yukawa interactions of the down-basis before the electroweak symmetry breaking,

$$\mathcal{L}_{\text{Yukawa}} = \bar{q} V_{\text{CKM}}^\dagger Y_u \tilde{H} u_R + \bar{q} Y_d H d_R + \bar{l} Y_e H e_R + \text{H.c.}, \quad (4.14)$$

where $Y_\psi = \frac{\sqrt{2}}{v} D_\psi$ and we have dropped the superscripts of the fermions.

The Yukawa interactions make the flavor symmetry $U(3)^5$ break explicitly. Because the flavor rotations induce the field redefinitions, the specific choice of a weak basis means the exploitation of the field redefinition. For example, in the down-basis, the $U(3)_l \times U(3)_e$ is exploited to diagonalize the lepton Yukawa matrix Y_e with the $3U(1)$ symmetries remaining. The $U(3)_q \times U(3)_u \times U(3)_e$ is exploited to diagonalize the down-quark Yukawa matrix Y_d and generate the CKM matrix of the standard parametrization, with the universal baryon number conservation $U(1)_B$ remaining. Thus the true symmetry is the rephasing symmetry,

$$U(3)^5 = \left\{ \begin{matrix} SU(3)^5 \\ U(1)^5 \end{matrix} \right\} \rightarrow \left\{ \begin{matrix} U(1)_{e-\mu} \times U(1)_{\mu-\tau} \\ U(1)_B \times U(1)_L \end{matrix} \right\} = U(1)^4. \quad (4.15)$$

With the $U(3)^5$ symmetry assumption, the rephasing symmetry $U(1)^4$ emerges as the remaining symmetry of the flavor symmetry after the choice of a specific weak basis.

B. $U(2)^5$ symmetry

Cases are different for a subgroup of the $U(3)^5$, for example, the $U(2)^5$ symmetry. Noticing the mass gaps between the first two particles and the last particle in a flavor triplet, we define the first two particles to form a doublet, while the last one is a singlet,

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \rightarrow \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} + \psi_3, \quad \psi = q, u_R, d_R, l, e_R. \quad (4.16)$$

Under such an arrangement, the general $U(3)^5$ is released to the subgroup

$$U(2)^5 \in U(3)^5, \quad \text{with } U(2)^5 = U(2)_q \times U(2)_u \times U(2)_d \times U(2)_l \times U(2)_e, \quad (4.17)$$

and the fermions' representations are

$$q \in (2, 1, 1, 1, 1), \quad (4.18)$$

$$u_R \in (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \quad (4.19)$$

$$d_R \in (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}), \quad (4.20)$$

$$l \in (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1}), \quad (4.21)$$

$$e_R \in (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}), \quad (4.22)$$

$$q_3, u_{R3}, d_{R3}, l_3, e_{R3} \in (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}). \quad (4.23)$$

The reduction of the symmetry group implies more abundant phenomena. According to the Yukawa interactions in Eq. (2.8), we parametrize the Yukawa matrices as

$$Y_{u/d} = y_{t/b} \begin{pmatrix} \Delta_{u/d} & x_{t/b} V_q \\ 0 & 1 \end{pmatrix}, \quad Y_e = y_\tau \begin{pmatrix} \Delta_e & x_l V_l \\ 0 & 1 \end{pmatrix}, \quad (4.24)$$

where $y_{\tau,t,b}$ and $x_{\tau,t,b}$ are free coefficients expected to be of order $O(1)$. The $\Delta_{u/d/e}$ are 2×2 matrices, while V_q and V_l are dimension-2 vectors. Writing the two-component fermions as

$$\begin{aligned} Q &= \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, & \mathcal{U} &= \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, & \mathcal{D} &= \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}, \\ \mathcal{L} &= \begin{pmatrix} l_1 \\ l_2 \end{pmatrix}, & \mathcal{E} &= \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}, \end{aligned} \quad (4.25)$$

the Yukawa interactions become

$$\begin{aligned} \mathcal{L}_{\text{Yukawa sector}} &= y_t \bar{Q}_L \Delta_u \mathcal{U}_R + y_t \bar{l}_L t_R + y_t x_t \bar{Q}_L V_q t_R + \text{H.c.} \\ &+ y_b \bar{Q}_L \Delta_d \mathcal{D}_R + y_b \bar{l}_L b_R + y_b x_b \bar{Q}_L V_q b_R + \text{H.c.} \\ &+ y_\tau \bar{\mathcal{L}}_L \Delta_e \mathcal{E}_R + y_\tau \bar{\tau}_L \tau_R + y_\tau x_\tau \bar{\mathcal{L}}_L V_l \tau_R + \text{H.c.} \end{aligned} \quad (4.26)$$

Next, we discuss the exploitations of the flavor symmetry and the resultant rephasing symmetry. Starting with the lepton sector with the symmetry $U(2)_l \times U(2)_e$, the vector V_l can be reduced to

$$V_l = e^{i\phi_l} \begin{pmatrix} 0 \\ \varepsilon_l \end{pmatrix}, \quad (4.27)$$

by the exploitation of the $U(2)_l$, with a remaining $U(1)$ symmetry. Then the fix of $U(2)_e$ reduces the $U(2)_l \times U(2)_e$ to a single $U(1)$, and the matrix Δ_e becomes a real matrix,

$$\Delta_e = O_e^T \begin{pmatrix} \delta'_e & 0 \\ 0 & \delta_e \end{pmatrix}, \quad \text{where } O_e = \begin{pmatrix} \cos \theta_e & \sin \theta_e \\ -\sin \theta_e & \cos \theta_e \end{pmatrix}. \quad (4.28)$$

Nevertheless the remaining $U(1)$ is eliminated by the mix of the lepton doublet $\mathcal{L}$ and the τ since it is required to be invariant under that. Thus there is no remaining symmetry for the leptons. A similar argument applies to the quark sector, the flavor symmetry is eliminated completely, and the parametrizations of the Yukawa matrices are

$$\begin{aligned} V_q &= e^{i\phi_q} \begin{pmatrix} 0 \\ \varepsilon_q \end{pmatrix}, & \Delta_u &= U_u^\dagger \begin{pmatrix} \delta'_u & 0 \\ 0 & \delta_u \end{pmatrix}, \\ \Delta_d &= U_d^\dagger \begin{pmatrix} \delta'_d & 0 \\ 0 & \delta_d \end{pmatrix}, \end{aligned} \quad (4.29)$$

where

$$U_q = \begin{pmatrix} \cos \theta_q & \sin \theta_q e^{i\alpha_q} \\ -\sin \theta_q e^{-i\alpha_q} & \cos \theta_q \end{pmatrix}, \quad q = u, d. \quad (4.30)$$

In conclusion, there is no rephasing symmetry of the $U(2)^5$ flavor symmetry.

The $U(2)^5$ symmetry is more phenomenologically abundant than the $U(3)^5$ symmetry since it presents fewer constraints [38–43]. Even in the LO Lagrangian, the new phenomenology appears as, for example, the mixing of the leptons in Eq. (4.26), which is absent in the $U(3)^5$ symmetry.

C. CP property with flavor symmetry

Although the flavor symmetries, including the $U(3)^5$, $U(2)^5$, and others, are broken in the SMEFT, they could be exact in the UV theory. To complement this symmetry-breaking pattern, the spurion method is convenient. In this method, some spurions are needed to keep the flavor symmetry formally, and the SMEFT operators are obtained by the VEVs of the spurions.

Assuming a specific flavor symmetry, the spurions $\mathbf{T}$ should be some scalar fields covariant under the flavor group, and the terms in the Lagrangian should be the invariant composed by the spurions and the other fields under the flavor group,

$$\mathcal{L} \supset c f(\mathbf{T}) \cdot \mathcal{O}(\psi), \quad (4.31)$$

where c is the Wilson coefficient, $f(\mathbf{T})$ is a flavor tensor composed by the spurions, $\mathcal{O}(\psi)$ is an operator composed by the other dynamic fields, and the dot ‘ $\cdot$ ’ between them means the composition of the invariant of them.

Focus on the LO Lagrangian, because the Yukawa interactions are the only terms breaking the flavor symmetry, the Yukawa matrices should be prompted to the spurions. For the $U(3)^5$ and the $U(2)^5$ flavor symmetries, the spurions and their representations at the LO Lagrangian are

$$U(3)^5: \begin{cases} \mathbf{Y}_u \in (3, \bar{3}, 1, 1, 1) \\ \mathbf{Y}_d \in (3, 1, \bar{3}, 1, 1) \\ \mathbf{Y}_e \in (1, 1, 1, 3, \bar{3}) \end{cases}, \quad U(2)^5: \begin{cases} \Delta_u \in (2, \bar{2}, 1, 1, 1) \\ \Delta_d \in (2, 1, \bar{2}, 1, 1) \\ \Delta_e \in (1, 1, 1, 2, \bar{2}) \\ V_q \in (2, 1, 1, 1, 1) \\ V_l \in (1, 1, 1, 2, 1) \end{cases}. \quad (4.32)$$

For the high-dimension operators, there are more independent spurions in principle. For example, the dimension-8 Lagrangian contains the terms such as

$$\begin{aligned} \mathcal{L} \supset & C_{u^2 H^2 D} (\overline{u^p} \gamma^\mu u_r) (H^\dagger \overleftrightarrow{D}_\mu H) (H^\dagger H) \\ & + C_{quH^5} (\overline{q^p} \tilde{H} u_r) (H^\dagger H)^2 \\ & + C_{q^4 D^2}^{(1)} (\overline{q^p} \gamma^\mu q_r) D^2 (\overline{q^s} \gamma_\mu q_t), \end{aligned} \quad (4.33)$$

where the coefficients can be developed to the spurions. Under the $U(3)^5$ symmetry, the representations of these spurions can be assigned as

$$C_{u^2 H^2 D} \rightarrow \mathbf{C}_{u^2 H^2 D} \in (\mathbf{1}, \mathbf{3} \times \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \quad (4.34)$$

$$C_{quH^5} \rightarrow \mathbf{C}_{quH^5} \in (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \quad (4.35)$$

$$C_{q^4 D^2}^{(1)} \rightarrow \mathbf{C}_{q^4 D^2}^{(1)} \in (\mathbf{3} \times \bar{\mathbf{3}} \times \mathbf{3} \times \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}). \quad (4.36)$$

According to the previous discussion, the effective operators of the SMEFT can be classified into the CP -even and CP -odd sectors according to the CP transformations of the fields, and the CP -odd operators that are rephasing invariant are CP -violating. As shown before, the CP -odd and the CP -violating operators are not equivalent when the flavor symmetry is absent; for example, the Hilbert series gives that there are 1422 CP -odd operators at dimension-6, but only 705 of them are CP -violating [17].

The cases are different for a specific flavor symmetry. Because the flavor symmetry is internal, it affects the CP properties. First, the flavor symmetry constrains the independent Wilson coefficients; thus the independent operators are reduced in principle. Second, as a subgroup, the rephasing symmetry depends on flavor symmetry. Besides, the CP -odd and the CP -violating operators are equivalent since the flavor symmetry guarantees the rephasing symmetries. This statement is general: for any specific symmetry, the CP -violating operators and the CP -odd operators are equivalent since the rephasing symmetry is satisfied automatically as a subgroup of the flavor symmetry. Considering the $U(3)^5$ and the $U(2)^5$ symmetries discussed before, for the $U(3)^5$ symmetry, the rephasing symmetry $U(1)^4$ is a subgroup, so the flavor symmetry guarantees the rephasing symmetry, while for the $U(2)^5$

symmetry, the rephasing symmetry is trivial and is satisfied automatically.

This means the single assumption of $U(3)^5$ flavor symmetry does not affect the CP violations of the SMEFT, since it is the maximal flavor symmetry and its break pattern is fixed by the Yukawa matrices. Nevertheless, the cases could be different for the other flavor symmetries such as the $U(2)^5$ [38–43].

The assumption of the $U(3)^5$ symmetry presents another way to express the CP -violating phases by the flavor invariants linear in the Wilson coefficients. As discussed before, although the aforementioned rephasing symmetry acts only on the fields and exits without any flavor symmetry, while the $U(3)^5$ symmetry is supposed to be on the Lagrangian including both the Wilson coefficients and the fields, the equivalence between them is realized by the fact that under the flavor symmetry, the Wilson coefficients become the spurions transforming according to the associated operators so that the invariants linear in them correspond to the operators. In particular, the Hilbert series is used in both the methods, and we compare them in the following:

- (i) Adopting the rephasing symmetry, the Hilbert series is utilized [17] to count the independent CP -odd operators satisfying the rephasing symmetry. The Hilbert series of the CP -odd operators can be obtained by the extension of the Lorentz group and the internal group with the automorphism defined by the parity and the charge conjugate, which takes the form

$$\mathcal{H}_{CP\text{-odd}} = \frac{1}{2}(\mathcal{H}_+ + \mathcal{H}_-), \quad (4.37)$$

where $\mathcal{H}_\pm$ are the Hilbert series on the different branches separated by the CP transformation [55–57].

- (ii) The Hilbert series can also be used to count the flavor invariants, which are part of the primary invariants. For some building blocks q , the Hilbert series takes the form

$$\mathcal{H}(q) = \frac{N(q)}{D(q)}, \quad (4.38)$$

where the denominator takes the form

$$D(q) = \prod_{r=1}^p (1 - q^{d_r}). \quad (4.39)$$

The number of the factors, p , is equal to the number of the primary invariants. Although the Hilbert series can be used to count their numbers, there is no systematic method to construct them explicitly.

In the rest of this section, we will discuss the flavor invariants of the dimension-8 operators and introduce the Young tensor method to construct the invariants.

D. Flavor invariant description

The assumption of the flavor symmetry implies that the flavor-invariant description of the CP -violating phases would be beneficial. Because the CP -violating observables are invariant under the flavor symmetry, they could be expressed in a flavor-invariant way. Actually, the single CP -violating phase δ at LO can be expressed by the Jarlskog invariant [58–60] as

$$J_4 = \text{Im}(\text{Tr}[Y_u Y_u^\dagger, Y_d Y_d^\dagger]^3). \quad (4.40)$$

Beyond the LO, the high-dimension Wilson coefficients C are included. The CP -violating primary invariants linear in the Wilson coefficients correspond to the CP -violating operators counted by the Hilbert series with the rephasing symmetry added. We can argue this equivalence by three aspects:

Linearity and CP violations. If we specify a weak basis, and suppose the parameters in Yukawa matrices of dimension-4 are θ , the parameters of the high-dimension Wilson coefficients are λ_i , the linearity means the invariants I take an expansion form

$$I(Y, C) \rightarrow I(\theta, \lambda) = \sum_i f(\theta) \lambda_i, \quad (4.41)$$

and the CP -violations mean the parameters λ_i are complex, which correspond to the CP -odd operators.

Invariance. The flavor invariance $U(3)^5$ guarantees the rephasing symmetry $U(1)^4$, which means that when a specific weak basis is adopted, every term in Eq. (4.41) is invariant under the rephasing symmetry.

Primary. A primary invariant cannot be expressed by the secondary invariants that are factorized. A secondary invariant I' takes the form

$$I'(Y, C) \rightarrow I'(\theta, \lambda) = g(\theta) \sum_i f(\theta) \lambda_i, \quad (4.42)$$

after the specification of a weak basis, which is nothing but a primary invariant multiplied by an overall factor $g(\theta)$, and thus is equivalent to the primary one.

Thus the flavor invariants composed by the Wilson coefficients and the CP -odd operators satisfying the rephasing symmetry are equivalent.

The application of the flavor-invariant description to the dimension-6 operators with the $U(3)^5$ flavor symmetry gets 699 CP -violating flavor invariants [18], which is consistent with the Hilbert series result utilizing the flavor symmetry.⁴ Considering the interference between the dimension-4 and the dimension-6 operators, even more CP -violating invariants emerge [19].

⁴The total of 699 is the number of the CP -violating fermionic operators obtained by the 705 minus the 6 CP -violating bosonic operators.

The flavor-invariant description also applies to the dimension-8 operators. For example, the Wilson coefficient $C_{u^2 H^2 D}$ of a dimension-8 operator

$$\mathcal{O}_{u^2 H^2 D} = (\bar{u}^p \gamma^\mu u_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H) \quad (4.43)$$

is a spurion in the flavor space, which transforms as

$$C_{u^2 H^2 D} \rightarrow V_u C_{u^2 H^2 D} V_u^\dagger, \quad V_u \in SU(3)_u. \quad (4.44)$$

As discussed before, three components of these operators are CP -odd, and they satisfy the rephasing symmetry and thus are CP violating. The three CP -violating phases in the coefficient $C_{u^2 H^2 D}$ can be expressed by the flavor invariant that

$$\begin{aligned} L_{1100}(Y_u C_{u^2 H^2 D} Y_u^\dagger), & \quad L_{2200}(Y_u C_{u^2 H^2 D} Y_u^\dagger), \\ L_{1122}(Y_u C_{u^2 H^2 D} Y_u^\dagger), & \end{aligned} \quad (4.45)$$

where

$$\begin{aligned} L_{abcd}(C) &= \text{Im} \text{Tr}(X_u^a X_d^b X_u^c X_d^d C), \\ a, b, c, d &= 0, 1, 2, \quad \text{and} \quad a \neq c, \quad b \neq d, \end{aligned} \quad (4.46)$$

with $X_u = Y_u Y_u^\dagger, X_d = Y_d Y_d^\dagger$. The vanishing of the three invariants is the sufficient and necessary condition for the vanishing of the CP violations. On the other hand, the coefficient of the complex operator

$$\mathcal{O}_{quH^5} = (\bar{q}^p \tilde{H} u_r)(H^\dagger H)^2 \quad (4.47)$$

presents nine CP -violating phases, which correspond to the flavor invariants that

$$\begin{aligned} L_{0000}(C_{quH^5} Y_u^\dagger), & \quad L_{1000}(C_{quH^5} Y_u^\dagger), & \quad L_{0100}(C_{quH^5} Y_u^\dagger), \\ L_{1100}(C_{quH^5} Y_u^\dagger), & \quad L_{0110}(C_{quH^5} Y_u^\dagger), & \quad L_{2200}(C_{quH^5} Y_u^\dagger), \\ L_{0220}(C_{quH^5} Y_u^\dagger), & \quad L_{1220}(C_{quH^5} Y_u^\dagger), & \quad L_{0122}(C_{quH^5} Y_u^\dagger). \end{aligned} \quad (4.48)$$

For the four-fermion operators, similar invariants exist. For example, we consider the real operator

$$\mathcal{O}_{q^4 D^2}^{(1)} = (\bar{q}^p \gamma^\mu q_r) D^2 (\bar{q}^s \gamma_\mu q_t). \quad (4.49)$$

As mentioned before, its coefficient $C_{q^4 D^2}$ presents 18 CP -violating phases, which can also be expressed by the invariants. We define

$$\begin{aligned} \text{Tr}_A(M^{(1)}, M^{(2)}, C) &= M^{(1)r}_p M^{(1)t}_s C_{rt}^{ps}, \\ \text{Tr}_B(M^{(1)}, M^{(2)}, C) &= M^{(1)}_p M^{(1)}_s C_{rt}^{ps}, \end{aligned} \quad (4.50)$$

and

$$A_{efgh}^{abcd}(C) = \text{Im Tr}_A(X_u^a X_d^b X_u^c X_d^d, X_u^e X_d^f X_u^g X_d^h, C), \quad (4.51)$$

$$B_{efgh}^{abcd}(C) = \text{Im Tr}_B(X_u^a X_d^b X_u^c X_d^d, X_u^e X_d^f X_u^g X_d^h, C), \quad (4.52)$$

and then the 18 flavor invariants are

$$\begin{aligned} & A_{1100}^{0000}(C_{q^4 D^2}), & A_{1100}^{1000}(C_{q^4 D^2}), & A_{1100}^{0100}(C_{q^4 D^2}), \\ & A_{2200}^{0000}(C_{q^4 D^2}), & A_{1100}^{1100}(C_{q^4 D^2}), & A_{1100}^{0200}(C_{q^4 D^2}), \\ & A_{2200}^{0100}(C_{q^4 D^2}), & A_{1122}^{0000}(C_{q^4 D^2}), & A_{2200}^{1100}(C_{q^4 D^2}), \\ & A_{1122}^{1000}(C_{q^4 D^2}), & A_{1122}^{0100}(C_{q^4 D^2}), & A_{1122}^{1100}(C_{q^4 D^2}), \\ & A_{2200}^{1200}(C_{q^4 D^2}), & B_{1100}^{0000}(C_{q^4 D^2}), & B_{1100}^{0100}(C_{q^4 D^2}), \\ & B_{2100}^{0200}(C_{q^4 D^2}), & A_{1122}^{1200}(C_{q^4 D^2}), & B_{1200}^{1000}(C_{q^4 D^2}). \end{aligned} \quad (4.53)$$

Actually, the primary invariance can be constructed analytically via the Young tableaux, which will be discussed in the next subsection.

E. Flavor invariants construction

The primary invariants are important since they are algebraically independent and correspond to the physical parameters, which are the ones that cannot be removed by the field redefinitions. The identification of the primary invariants is difficult. As discussed before, the Hilbert series method can enumerate the primary invariants [61], but it is difficult to construct them explicitly. The common method is to construct an overcomplete set of the invariants and reduce them by finding their relations called syzygies. Thus it is useful to take the plethystic logarithm (PL) of the Hilbert series $\mathcal{H}(q)$. The $PL(\mathcal{H}(q))$ is an expansion of the building blocks q and encodes the important information that the leading positive terms give the numbers and the orders of the basic invariants, and the leading negative terms give the orders of their relations called syzygies.

The basic invariants are generally linearly independent but not algebraically independent. After the exploitation of the syzygies, they are reduced to the primary invariants. For example, we consider the group tensors composed of the two distinct matrices X_u and X_d of $U(3)$ adjoint representation. The PL of the corresponding Hilbert series is

$$\begin{aligned} PL(\mathcal{H}(X_u, X_d)) &= (X_u + X_d) + (X_u^2 + X_d^2 + X_u X_d) \\ &\quad + (X_u^3 + X_u^2 X_d + X_u X_d^2 + X_d^3) \\ &\quad + X_u^2 X_d^2 + X_u^3 X_d^3 - X_u^6 X_d^6, \end{aligned} \quad (4.54)$$

which means there are 11 basic invariants, while the one of order $X_u^3 X_d^3$ is not primary due to a syzygy of order

$X_u^6 X_d^6$. Thus there are ten primary invariants. In practice, the syzygies are obtained by numeric method, and the $PL(\mathcal{H}(q))$ is instructive.

Actually, even the basic invariants are difficult to find. In this subsection, we present the Young tensor method as an analytic and systematic method to find the basic invariants and expect to pave the way to the primary ones.

As discussed in Ref. [62], the symmetrization relations between the tensors of the Young tableaux method are equivalent to the Cayley-Hamilton relations. In principle, the primary invariants correspond to the group tensors that cannot be factorized, for example, f^{ABC} , d^{ABC} , $d^{ABE} d^{CDE}$ cannot be factorized, while $\delta^{AB} \delta^{CD}$ is factorized. These tensors can be obtained by the Young tableaux and after the symmetrization of the tensors via the Young symmetrizers, the relations among them can be used to find the primary invariants. This method is systematic and we outline the basic steps here:

- (i) Utilize the Young tableaux to obtain the independent tensors at each order. These tensors are interpreted from the Young tableaux, which can be constructed from the outer product such as

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} \dots \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array} \subset \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} \otimes \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} \otimes \dots \quad (4.55)$$

- (ii) Assign different repetitions and use the associated Young symmetrizers to obtain the relations among the symmetrized tensors. These relations generally mix the factorized and the unfactorized tensors. The independent unfactorized tensors correspond to the primary invariants.

Next, we consider the primary invariants composed of the building blocks X_u , X_d in terms of their orders. The PL has been presented in Eq. (4.54).

- (i) Order-1: The Young tensor method presents no constraints, and thus there is one primary invariant for each of X_u and X_d ,

$$\text{Tr}(X_u), \quad \text{Tr}(X_d). \quad (4.56)$$

- (ii) Order-2: There is only one unfactorized tensor δ^{AB} , which is symmetric, and thus there is one primary invariant for each of X_u^2 , X_d^2 and $X_u X_d$,

$$\text{Tr}(X_u^2), \quad \text{Tr}(X_d^2), \quad \text{Tr}(X_u X_d). \quad (4.57)$$

- (iii) Order-3: The two tensors at this order are the totally symmetric tensor d^{ABC} and the totally antisymmetric tensor f^{ABC} . Since there are only two distinct building blocks, there must be repetition. The symmetrization of the repeated building blocks implies that only d^{ABC} should be reserved. Thus

there is one primary invariant for each of $X_u^3, X_d^3, X_u^2 X_d$, and $X_d^2 X_u$,

$$\text{Tr}(X_u^3), \quad \text{Tr}(X_d^3), \quad \text{Tr}(X_u^2 X_d), \quad \text{Tr}(X_u X_d^2). \quad (4.58)$$

(iv) Order-4: The Young tableaux present eight independent tensors

$$\begin{aligned} \mathcal{B}_1 &= d^{ABE} d^{CDE}, & \mathcal{B}_2 &= d^{ABE} f^{CDE}, \\ \mathcal{B}_3 &= f^{ABE} f^{CDE}, & \mathcal{B}_4 &= \delta^{AB} \delta^{CD}, \\ \mathcal{B}_5 &= f^{ABE} d^{CDE}, & \mathcal{B}_6 &= \delta^{AC} \delta^{BD}, \\ \mathcal{B}_7 &= d^{ACE} d^{BDE}, & \mathcal{B}_8 &= d^{ACE} f^{BDE}. \end{aligned} \quad (4.59)$$

Suppose there is only one kind of building block, X_u or X_d ; the tensors should be symmetrized by the symmetrizer of S_4 group $\mathcal{Y}_{[4]}$, which takes the form that

$$\mathcal{Y}_{[4]} = \begin{pmatrix} \frac{1}{9} & 0 & -\frac{1}{9} & \frac{2}{27} & 0 & \frac{4}{27} & \frac{2}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & -\frac{1}{3} & \frac{2}{9} & 0 & \frac{4}{9} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & -\frac{1}{3} & \frac{2}{9} & 0 & \frac{4}{9} & \frac{2}{3} & 0 \\ \frac{1}{9} & 0 & -\frac{1}{9} & \frac{2}{27} & 0 & \frac{4}{27} & \frac{2}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (4.60)$$

There are only four nonvanishing tensors after symmetrization, and it is obvious that they are linear to each other. Denoting the tensor after symmetrization, which are usually polynomials, by

$$\mathcal{B} \xrightarrow{\text{Symmetrization}} \mathcal{Y}(\mathcal{B}), \quad (4.61)$$

their linear relation can be expressed as

$$\mathcal{Y}(\mathcal{B}_4) = \mathcal{Y}(\mathcal{B}_6) = 3\mathcal{Y}(\mathcal{B}_1) = 3\mathcal{Y}(\mathcal{B}_7) \quad (4.62)$$

or

$$\begin{aligned} \mathcal{Y}(\delta^{AB} \delta^{CD}) &= \mathcal{Y}(\delta^{AC} \delta^{BD}) = 3\mathcal{Y}(d^{ABE} d^{CDE}) \\ &= 3\mathcal{Y}(d^{ACE} d^{BDE}). \end{aligned} \quad (4.63)$$

The original unfactorized $\mathcal{B}_1$ and $\mathcal{B}_7$ are identical with the factorized ones $\mathcal{B}_4$ and $\mathcal{B}_6$ after the symmetrization, and correspond to no primary tensor. Thus there exists no primary invariant in this case.

If we assign the first two building blocks X_u and the last two building blocks X_d , the eight tensors must be symmetrized by the Young symmetrizer that

$$\mathcal{Y}_{[2]}\mathcal{Y}_{[2]} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{6} & 0 & \frac{2}{3} & 1 & 0 \\ -\frac{1}{3} & 0 & -\frac{1}{6} & \frac{1}{9} & 0 & \frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (4.64)$$

It is a rank-three matrix, and its application presents a relation among the symmetrized tensors that

$$\begin{aligned} \mathcal{Y}(d^{ACE} d^{BDE}) &= -\frac{1}{2}\mathcal{Y}(d^{ABE} d^{CDE}) + \frac{1}{6}\mathcal{Y}(\delta^{AB} \delta^{CD}) \\ &\quad + \frac{1}{3}\mathcal{Y}(\delta^{AC} \delta^{BD}). \end{aligned} \quad (4.65)$$

This relation can be interpreted as the two unfactorized tensors $\mathcal{Y}(d^{ACE} d^{BDE})$ and $\mathcal{Y}(d^{ABE} d^{CDE})$ are not independent, and they are related by the other two factorized tensors $\mathcal{Y}(\delta^{AB} \delta^{CD})$ and $\mathcal{Y}(\delta^{AC} \delta^{BD})$. Thus there is only one primary invariant composed by two X_u and X_d , which corresponds to the tensor $\mathcal{Y}(d^{ABE} d^{CDE})$, and can be interpreted as the trace

$$\mathcal{Y}(d^{ABE} d^{CDE}) \rightarrow \text{Tr}(X_u^2 X_d^2). \quad (4.66)$$

The cases where there are three repeated building blocks present, the same symmetrizer

$$\mathcal{Y}_{[3]} = \mathcal{Y}_{[4]} \quad (4.67)$$

thus presents no primary invariant.

(v) Order-5: There are 32 tensors, and we present the relations among the symmetrized tensors, rather than listing all the tensors here:

$$\begin{aligned} X_u^5, X_d^5: \mathcal{Y}(d^{ABC} \delta^{DE}) &= 3\mathcal{Y}(d^{ACF} d^{BFG} d^{DEG}) \\ &= 3\mathcal{Y}(d^{ACF} d^{DFG} d^{BEG}) \\ &= \mathcal{Y}(d^{BDE} \delta^{AC}) \\ &= 3\mathcal{Y}(d^{ABF} d^{CFG} d^{DEG}) \\ &= 3\mathcal{Y}(d^{ABF} d^{DFG} d^{CEG}) \\ &\quad + 3\mathcal{Y}(d^{ADF} d^{BFG} d^{CEG}), \end{aligned} \quad (4.68)$$

$$\begin{aligned}
X_u^4 X_d, X_d^4 X_u: \mathcal{Y}(d^{ABC} \delta^{DE}) &= \mathcal{Y}(d^{ABC} \delta^{DE}), \\
\mathcal{Y}(d^{ACF} d^{BFG} d^{DEG}) &= \mathcal{Y}(d^{ACF} d^{BEG} d^{DFG}) = \frac{1}{3} \mathcal{Y}(d^{BDE} \delta^{AC}) = \mathcal{Y}(d^{ABF} d^{DEG} d^{CFG}) \\
&= \mathcal{Y}(d^{ABF} d^{CEG} d^{DFG}) = \mathcal{Y}(d^{ADF} d^{BFG} d^{CEG}),
\end{aligned} \tag{4.69}$$

$$\begin{aligned}
X_u^3 X_d^2, X_u^2 X_d^3: \mathcal{Y}(d^{ACF} d^{BFG} d^{DEG}) &= \mathcal{Y}(d^{ABF} d^{CFG} d^{DEG}) = \frac{1}{3} \mathcal{Y}(d^{BDE} \delta^{AC}), \\
\mathcal{Y}(d^{ACF} d^{BEG} d^{DFG}) &= \mathcal{Y}(d^{ABF} d^{DFG} d^{CEG}), \\
\mathcal{Y}(d^{ACF} f^{BEG} f^{DFG}) &= \mathcal{Y}(d^{ABF} f^{CEG} f^{DFG}), \\
\mathcal{Y}(d^{ABC} \delta^{DE}) &= 3\mathcal{Y}(d^{ACF} d^{BEG} d^{DFG}) - \mathcal{Y}(d^{ACF} f^{BEG} f^{DFG}), \\
\mathcal{Y}(d^{ADF} d^{BFG} d^{CEG}) &= \mathcal{Y}(d^{ACF} d^{BFG} d^{DEG}) + \frac{1}{6} \mathcal{Y}(d^{ACF} f^{BEG} f^{DFG}).
\end{aligned} \tag{4.70}$$

There is no independent unfactorized tensor, and thus there is no primary invariant in this order. Thus we obtain all the ten primary invariants composed of the two building blocks X_u and X_d .

For the invariants linear in the high-dimension Wilson coefficients C , the arguments are similar. We fix a building block as the Wilson coefficient C , and other building blocks are X_u and X_d dependently to form invariants. Following the steps above, the primary invariants can be obtained according to the building block repetitions order by order in principle. For example, we consider a simple case where a high-dimension coefficient C is included, which transforms in the same way as the X_u, X_d . As discussed before, the Young tensor method presents no constraints up to order-3, and the resultant basic invariants are

$$\text{Tr}(C), \quad \text{Tr}(X_u C), \quad \text{Tr}(X_d C), \quad \text{Tr}(X_u^2 C), \quad \text{Tr}(X_u X_d C), \quad \text{Tr}(X_d^2 C). \tag{4.71}$$

Considering order-4, it has been shown that there is no basic invariant when there are three repeated building blocks. Thus we consider only the case where there are exactly two repeated building blocks; for example, we assign the first two building blocks that are repeated, where the Young symmetrizer is

$$\mathcal{Y}_{[2]} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{6} & 0 & \frac{2}{3} & 1 & 0 \\ -\frac{1}{3} & 0 & -\frac{1}{6} & \frac{1}{9} & 0 & \frac{2}{9} & \frac{1}{3} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{4.72}$$

which is a rank-four matrix, and thus there are four independent tensors after the symmetrization. However, there are relations that

$$\mathcal{Y}(\delta^{AC} \delta^{BD}) = -\frac{3}{2} \mathcal{Y}(d^{ACE} d^{BDE}), \tag{4.73}$$

$$\mathcal{Y}(d^{ACE} f^{BDE}) = -\frac{1}{2} \mathcal{Y}(d^{ABE} f^{CDE}), \tag{4.74}$$

and thus there are only two independent unfactorized tensors, $d^{ABE} d^{CDE}$ and $d^{ABE} f^{CDE}$, which correspond to four basic invariants

$$\begin{aligned} \text{Tr}(X_u X_d^2 C), & \quad \text{Tr}(X_u^2 X_d C), \\ \text{Tr}(X_d X_u^2 C), & \quad \text{Tr}(X_d^2 X_u C). \end{aligned} \quad (4.75)$$

For the higher orders, we can perform the algorithm repeatedly and find all the basic invariants.

V. MINIMAL FLAVOR VIOLATION HYPOTHESIS

In the point of EFT, all the Wilson coefficients of the effective operators are independent, and up to now, the complete set of the SMEFT operators up to dimension-12 has been constructed [1–3,5–7,9,11]. However, the Wilson coefficients are constrained by the experiments. To be consistent with the experiment results involving the FCNC processes and not push the new physics scale too high, the MFV hypothesis is introduced, which states that the Yukawa matrices are the only flavor-symmetry-breaking sources.

Specifically, we consider a dimension- d effective operator $\mathcal{O}_{pr,\dots,s}^d$, where $pr,\dots,s$ are its flavor indices. The MFV hypothesis implies that such an operator appears in the effective Lagrangian in the form that

$$\mathcal{L} \supset \frac{c}{\Lambda^{d-4}} f(Y_u, Y_d, Y_e)^{pr,\dots,s} \mathcal{O}_{pr,\dots,s}^d, \quad (5.1)$$

where c is the Wilson coefficient and $f(Y_u, Y_d, Y_e)^{pr,\dots,s}$ is a polynomial composed of the Yukawa matrices. For example, the dimension-4 Yukawa interactions satisfy

$$f_\psi(Y_u, Y_d, Y_e) = Y_\psi, \quad \psi = u, d, e, \quad (5.2)$$

and $c = 1$.

Generally, the polynomial is a series

$$\begin{aligned} f(Y_u, Y_d, Y_e) &= f_0 + O(Y_\psi) + O(Y_\psi^2) + \dots, \\ \psi &= u, d, e, \end{aligned} \quad (5.3)$$

and thus a power-counting scheme is needed. Because the Yukawa matrices are proportional to the fermion masses, their hierarchy can help to define the expansion parameters. Adopting the down-basis, the three Yukawa matrices take the form

$$Y_u \propto y_t V_{\text{CKM}}^\dagger \text{diag}\left(\frac{y_u}{y_t}, \frac{y_c}{y_t}, 1\right) \sim y_t, \quad (5.4)$$

$$Y_d \propto y_b \text{diag}\left(\frac{y_d}{y_b}, \frac{y_s}{y_b}, 1\right) \sim y_b, \quad (5.5)$$

$$Y_e \propto y_\tau \text{diag}\left(\frac{y_e}{y_\tau}, \frac{y_\mu}{y_\tau}, 1\right) \sim y_\tau, \quad (5.6)$$

and

$$\frac{y_\tau}{y_t} \approx \frac{y_b}{y_t} \approx O(10^{-3}), \quad (5.7)$$

where we have set $\frac{\sqrt{2}}{v} m = y$, and thus we can define the expansion parameters as

$$\frac{Y_u}{y_t} \sim 1, \quad \frac{Y_d}{y_t}, \frac{Y_e}{y_t} \sim 10^{-3}. \quad (5.8)$$

Consequently, it looks like the Y_u can be any power in the expansion, but it is false since the ring generated of it is finite-generated [63]. In this paper, we usually expand to $O(Y_u)$ or $O(Y_u^2)$ instead of any power of it [64,65], since the terms with more powers of Y_u can be absorbed by field redefinitions [66].

Under the MFV hypothesis, the effective operators can be classified in terms of the Yukawa matrices they couple, which has been done for the dimension-6 operators in Ref. [44], and in the rest of this section, we will summarize the result and focus on the same analysis for the dimension-8 operators. In particular, we present the MFV operators up to 10^{-3} according to Eq. (5.8).

A. Dimension-6 MFV operators

As discussed before, the most efficient way to construct MFV effective operators is to promote the Yukawa matrices to the dynamic degrees of freedom

$$Y_\psi \rightarrow \mathbf{Y}_\psi(x), \quad \psi = u, d, e, \quad (5.9)$$

called spurions, which transform under the global $U(3)^5$ symmetry as

$$\mathbf{Y}_u \in (3, \bar{3}, 1, 1, 1), \quad (5.10)$$

$$\mathbf{Y}_d \in (3, 1, \bar{3}, 1, 1), \quad (5.11)$$

$$\mathbf{Y}_e \in (1, 1, 1, 3, \bar{3}), \quad (5.12)$$

and to require the operators to be invariant under the $U(3)^5$. Such management assumes that the underlying UV theory shares the same flavor symmetry $U(3)^5$. After the fields $\mathbf{Y}_\psi(x)$ are frozen and take their VEVs, these operators become the SMEFT ones.

We adopt the Warsaw basis [3] of the dimension-6 operators, and the explicit operators are presented in Appendix A. Apart from the pure boson sectors, there are four fermionic sectors, $\psi^2 H^3$, $\psi^2 XH$, $\psi^2 H^2 D$, and ψ^4 . Next, we will discuss the MFV operators of these classes separately, and a similar discussion can be found in Ref. [44].

- (i) There are nine flavorful operators in the class $\psi^2 H^2 D$, and only $\mathcal{O}_{Hud}$ is complex. For the real ones, the expansion of the coefficients is

$$f(\mathbf{Y})_p^r = \delta_p^r + f_2(\mathbf{Y}_\psi^\dagger \mathbf{Y}_\psi)_p^r. \quad (5.13)$$

For $\mathcal{O}_{Hl}^{(1,3)}$, $\mathcal{O}_{He}$, and $\mathcal{O}_{Hd}$, the expansion is truncated at δ_p^r , and thus these MFV operators are diagonal, while for $\mathcal{O}_{Hq}^{(1,3)}$ and $\mathcal{O}_{Hu}$, the second term is preserved,

$$\mathcal{O}_{Hl}^{(1,3)}, \mathcal{O}_{He}, \mathcal{O}_{Hd}: f(\mathbf{Y})_p^r = \delta_p^r, \quad (5.14)$$

$$\mathcal{O}_{Hq}^{(1,3)}: f(\mathbf{Y})_p^r = \delta_p^r + f_2(\mathbf{Y}_u \mathbf{Y}_u^\dagger)_p^r, \quad (5.15)$$

$$\mathcal{O}_{Hu}: f(\mathbf{Y})_p^r = \delta_p^r + f_2(\mathbf{Y}_u^\dagger \mathbf{Y}_u)_p^r. \quad (5.16)$$

As for the complex one $\mathcal{O}_{Hud}$, it vanishes at leading order and only the quadratic term survives,

$$\mathcal{O}_{Hud}: f(\mathbf{Y})_p^r = (\mathbf{Y}_u^\dagger \mathbf{Y}_d)_p^r, \quad (5.17)$$

where we follow the convention that the coefficient of the first term in the expansion is always absorbed in the overall number c in Eq. (5.1).

- (ii) The operators in the classes $\psi^2 H^3$ and $\psi^2 XH$ are similar to the dimension-4 Yukawa interactions, and thus the linear term in the expansion is included: $f(\mathbf{Y}) = \mathbf{Y}_\psi + \dots$, $\psi = u, d, e$. Besides, the coefficients of the operators containing down-quark $d(x)$ can be expanded to the next order; thus, we have

$$f(\mathbf{Y}) = \mathbf{Y}_d + f_3(\mathbf{Y}_u \mathbf{Y}_u^\dagger) \mathbf{Y}_d + \dots, \quad (5.18)$$

for the operators $\mathcal{O}_{dH}$, $\mathcal{O}_{dG}$, $\mathcal{O}_{dW}$, and $\mathcal{O}_{dB}$.

- (iii) For the class ψ^4 , we assume the general operators take the form $(\bar{\psi}_1^p \Gamma_1 \psi_{2r})(\bar{\psi}_3^s \Gamma_2 \psi_{4t})$, where p, r, s, t are flavor indices. It should be noted that when there are repeated fields among the four fermions, not all the terms in the expansion are independent. If the four fermions are the same, $(\bar{\psi}^p \Gamma_1 \psi_r)(\bar{\psi}^s \Gamma_2 \psi_t)$, where $\psi = q, u, d, l, e$, the leading terms are generally $\delta_p^r \delta_s^t + f_0' \delta_p^t \delta_s^r$, but for the right-handed lepton e , the Fierz identity implies

$$(\bar{\psi}^p \gamma_\mu \psi_r)(\bar{\psi}^s \gamma^\mu \psi_t) = (\bar{\psi}^s \gamma_\mu \psi_r)(\bar{\psi}^p \gamma^\mu \psi_t), \quad (5.19)$$

and thus the second tensor vanishes. Only if $\psi = q, u$ is the next order preserved, $f_2 \delta_p^r (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_s^t + f_2' \delta_s^r (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_p^t$ or $f_2 \delta_p^r (\mathbf{Y}_u^\dagger \mathbf{Y}_u)_s^t + f_2' \delta_s^r (\mathbf{Y}_u^\dagger \mathbf{Y}_u)_p^t$. Thus we obtain

$$\mathcal{O}_{ll}, \mathcal{O}_{dd}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r \delta_s^t + f_0' \delta_p^t \delta_s^r, \quad (5.20)$$

$$\mathcal{O}_{qq}^{(1,3)}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r \delta_s^t + f_0' \delta_p^t \delta_s^r + f_2 \delta_p^r (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_s^t + f_2' \delta_s^r (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_p^t, \quad (5.21)$$

$$\mathcal{O}_{ee}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r \delta_s^t, \quad (5.22)$$

$$\mathcal{O}_{uu}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r \delta_s^t + f_0' \delta_p^t \delta_s^r + f_2 \delta_p^r (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_s^t + f_2' \delta_s^r (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_p^t. \quad (5.23)$$

If there are only two different pairs of repeated fields, $(\bar{\psi}_1^p \Gamma_1 \psi_{1r})(\bar{\psi}_2^s \Gamma_2 \psi_{2t})$, the leading order in the expansion is $\delta_p^r \delta_s^t$, and if $\psi_1/\psi_2 = q, u$, the second order survives. Thus we have

$$\mathcal{O}_{lq}^{(1,3)}, \mathcal{O}_{qe}, \mathcal{O}_{qd}^{(1,8)}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r \delta_s^t + f_2 \delta_s^t (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_p^r, \quad (5.24)$$

$$\mathcal{O}_{eu}, \mathcal{O}_{ud}^{(1,8)}, \mathcal{O}_{lu}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r \delta_s^t + f_2 \delta_s^t (\mathbf{Y}_u^\dagger \mathbf{Y}_u)_p^r, \quad (5.25)$$

$$\mathcal{O}_{ed}, \mathcal{O}_{le}, \mathcal{O}_{ld}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r \delta_s^t, \quad (5.26)$$

$$\mathcal{O}_{qu}^{(1,8)}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r \delta_s^t + f_2 \delta_s^t (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_p^r + f_2' \delta_p^r (\mathbf{Y}_u^\dagger \mathbf{Y}_u)_s^t + f_2'' (\mathbf{Y}_u^\dagger)_p^t (\mathbf{Y}_u)_s^r. \quad (5.27)$$

If there are only two repeated fields, there are two operators $\mathcal{O}_{quqd}^{(1,8)}$, and their coefficients are

$$\mathcal{O}_{quqd}^{(1,8)}: f(\mathbf{Y})_{ps}^{rt} = (\mathbf{Y}_u)_p^r (\mathbf{Y}_d)_s^t. \quad (5.28)$$

Finally, if there is no repeated field, there are three operators, $\mathcal{O}_{lequ}^{(1,3)}$, $\mathcal{O}_{ledq}$, and the first two's coefficients are

$$\mathcal{O}_{lequ}^{(1,3)}: f(\mathbf{Y})_{ps}^{rt} = (\mathbf{Y}_e)_p^r (\mathbf{Y}_u)_s^t, \quad (5.29)$$

where the last one's coefficient is zero up to $O(10^{-3})$.

In particular, the operators discussed above are all lepton/baryon-number conserving. Actually, the operators violating the lepton/baryon number are eliminated under the MFV hypothesis, since all the Yukawa matrices carry no lepton/baryon number.

Under the MFV hypothesis, both the CP -even and the CP -odd operators are reduced, because the Yukawa matrices are prompted to be spurions, and the independent parameters are reduced. We summarize the CP -even and CP -violating operators of dimension-6 in Table VII, where n_Y is the number of the operators when the Yukawa matrices are regarded as the spurions, and n_Y is the number of the operators when the Yukawa matrices take their VEVs, which is taken as the down-basis in this paper. In particular, we present the CP -even and the CP -violating operators without any flavor symmetry in the column n .

As discussed before, the CP -odd operators are equivalent to the CP -violating ones, and thus there are 26 CP -violating operators when the Yukawa matrices are regarded as spurions, which is the number of the independent CP -violating phases under the MFV hypothesis, and there are 327 CP -violating operators when the spurions take their VEVs, which is generally larger than the number of the independent CP -violating phases. In particular, the number of the CP -violating phases is reduced compared to 705 without any flavor symmetry.

B. Dimension-8 MFV operators

The same analysis can be applied to the dimension-8 operators of the SMEFT. Consequently, new flavor structures appear at dimension-8, whose MFV coefficients need more attention. Next, we will present the coefficient expansion of them by the different classes, and such an analysis provides a classification of the dimension-8 operators. In this paper, we adopt the explicit operators of Ref. [7] but reformulate them more concisely so that their flavor structures are more transparent. The explicit operators are presented in Appendix B.

Current operators. As shown previously the classes of the current operators can be divided into two parts, the first contains the Yukawa-like operators composed of scalar bilinears, including

$$\begin{aligned} &\psi^2 \phi^5, \quad \psi^2 \phi^3 D^2, \quad \psi^2 X \phi^3, \\ &\psi^2 X \phi D^2, \quad \psi^2 X^2 \phi, \end{aligned} \quad (5.30)$$

and the second is composed of vector bilinears, including

$$\psi^2 \phi^4 D, \quad \psi^2 \phi^2 D^3, \quad \psi^2 X \phi^2 D, \quad \psi^2 X^2 D. \quad (5.31)$$

For the first part, all the coefficient expansions take the form

$$f(\mathbf{Y})_p^r = (\mathbf{Y}_\psi)_p^r, \quad \psi = u, d, e, \quad (5.32)$$

but for the ones containing the down-quark $d(x)$, it can be expanded to the next order,

$$f(\mathbf{Y})_p^r = (\mathbf{Y}_d)_p^r + f_3(\mathbf{Y}_u \mathbf{Y}_u^\dagger)(\mathbf{Y}_d)_p^r. \quad (5.33)$$

For the second part, the coefficients of the real operators are diagonal,

$$f(\mathbf{Y})_p^r = \delta_p^r, \quad (5.34)$$

while for the ones containing the left quarks or the up-quark $u(x)$, the coefficients can be expanded to the next order, $f_2(\mathbf{Y}_u \mathbf{Y}_u^\dagger)_p^r$ or $f_2(\mathbf{Y}_u^\dagger \mathbf{Y}_u)_p^r$. For the complex operators containing the bilinear $(\bar{u} \Gamma d)$, the coefficients are

$$f(\mathbf{Y})_p^r = (\mathbf{Y}_u^\dagger \mathbf{Y}_d)_p^r + \dots \quad (5.35)$$

Four-fermion operators. There are four classes:

$$\psi^4 D^2, \quad \psi^4 \phi D, \quad \psi^4 \phi^2, \quad \psi^4 X. \quad (5.36)$$

The classes $\psi^4 D^2$ and $\psi^4 X$ share the same flavor structures with the dimension-6 class ψ^4 , and thus their coefficients can be read from Eqs. (5.20) to (5.29) according to the field content, while for the remaining two classes $\psi^4 \phi^2$ and $\psi^4 \phi D$, new flavor structures appear.

(i) Class $\psi^4 \phi^2$: Because of the two Higgs fields, new complex operators appear,

$$\begin{aligned} \mathcal{O}_{q^2 u^2 H^2}^{(1)} &= (\bar{q}^p \tilde{H} u_r)(\bar{q}^s \tilde{H} u_t), & \mathcal{O}_{q^2 u^2 H^2}^{(2)} &= (\bar{q}^p \tilde{H} \lambda^A u_r)(\bar{q}^s \tilde{H} \lambda^A u_t), \\ \mathcal{O}_{q^2 d^2 H^2}^{(1)} &= (\bar{q}^p \tilde{H} d_r)(\bar{q}^s \tilde{H} d_t), & \mathcal{O}_{q^2 d^2 H^2}^{(2)} &= (\bar{q}^p \tilde{H} \lambda^A d_r)(\bar{q}^s \tilde{H} \lambda^A d_t), \\ \mathcal{O}_{l^2 e^2 H^2} &= (\bar{l}^p H e_r)(\bar{l}^s H e_t), & \mathcal{O}_{q^2 u d H^2}^{(1)} &= (\bar{q}^p \tilde{H} u_r)(\bar{d}^s H^\dagger q_t), \\ \mathcal{O}_{q^2 u d H^2}^{(2)} &= (\bar{q}^p \tilde{H} \lambda^A u_r)(\bar{d}^s H^\dagger \lambda^A q_t), & \mathcal{O}_{l^2 u d H^2} &= (\bar{l}^p H \gamma^\mu \tilde{H}^\dagger l_r)(\bar{u}^s \gamma_\mu d_t), \\ \mathcal{O}_{q d l e H^2}^{(1)} &= (\bar{q}^p H d_r)(\bar{l}^s H e_t), & \mathcal{O}_{q d l e H^2}^{(2)} &= (\bar{q}^p H \sigma^{\mu\nu} d_r)(\bar{l}^s H \sigma_{\mu\nu} e_t), \\ \mathcal{O}_{q u l e H^2} &= (\bar{q}^p \tilde{H} u_r)(\bar{e}^s H^\dagger l_t), \end{aligned} \quad (5.37)$$

where the quadratic terms dominate all the coefficients. Up to $O(10^{-3})$, only $\mathcal{O}_{q^2 u^2 H^2}^{(1,2)}$, $\mathcal{O}_{q^2 u d H^2}^{(1,2)}$, $\mathcal{O}_{l^2 u d H^2}$, and $\mathcal{O}_{q u l e H^2}$ are nonzero,

$$\mathcal{O}_{q^2 u^2 H^2}^{(1,2)}: f(\mathbf{Y})_{ps}^{rt} = (\mathbf{Y}_u)_p^r (\mathbf{Y}_u)_s^t + f_2'(\mathbf{Y}_u)_p^t (\mathbf{Y}_u)_s^r, \quad (5.38)$$

$$\mathcal{O}_{q^2 u d H^2}^{(1,2)}: f(\mathbf{Y})_{ps}^{rt} = (\mathbf{Y}_u)_p^r (\mathbf{Y}_d)_s^t + f_2' \delta_p^t (\mathbf{Y}_d^\dagger \mathbf{Y}_u)_s^r, \quad (5.39)$$

TABLE VII. The CP properties of the SMEFT dimension-6 effective operators, where n corresponds to the operators without flavor symmetry, n_Y corresponds to the operators with the spurions $\mathbf{Y}_\psi$, and n_Y corresponds to the cases when $\mathbf{Y}_\psi$'s become matrices Y_ψ 's. The CP -odd bosonic operators can also be found in Table V.

Class	n		n_Y		n_Y	
	CP -even	CP -violating	CP -even	CP -violating	CP -even	CP -violating
Bosonic sectors	9	6	9	6	9	6
$\psi^2 H^3$	21	21	4	4	21	21
$\psi^2 XH$	60	60	11	11	60	60
$\psi^2 H^2 D$	42	21	11	1	36	15
ψ^4	774	597	50	4	447	231
Total	906	705	85	26	567	327
	1611		111		894	

$$\mathcal{O}'_{\ell^2 u d H^2}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r (\mathbf{Y}_u^\dagger \mathbf{Y}_d)_s^t, \quad (5.40)$$

$$\mathcal{O}'_{qule H^2}: f(\mathbf{Y})_{ps}^{rt} = (\mathbf{Y}_u)_p^r (\mathbf{Y}_e^\dagger)_s^t, \quad (5.41)$$

$$\mathcal{O}_{q^2 d^2 H^2}^{(1,2)}, \mathcal{O}_{\ell^2 e^2 H^2}, \mathcal{O}_{qdle H^2}^{(1,2)}: f(\mathbf{Y})_{ps}^{rt} = 0. \quad (5.42)$$

- (ii) Class $\psi^4 \phi D$: All the operators of this class possess different flavor structures from the dimension-6 ones because the single Higgs field carries nonzero hypercharge. Next, we will discuss their coefficients in terms of the repeated fields.

If there are three repeated fields, the coefficient expansion depends on whether the operator contains the down-quark $d(x)$, if so, the expansion can take cubic terms,

$$\begin{aligned} \mathcal{O}_{q^3 d HD}^{(1-6)}: f(\mathbf{Y})_{ps}^{rt} = & \delta_p^r (\mathbf{Y}_d)_s^t + f_1' \delta_s^r (\mathbf{Y}_d)_p^t \\ & + f_3^{(1)} (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_p^r (\mathbf{Y}_d)_s^t \\ & + f_3^{(2)} (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_s^r (\mathbf{Y}_d)_p^t \\ & + f_3^{(3)} \delta_p^r (\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_d)_s^t \\ & + f_3^{(4)} \delta_s^r (\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_d)_p^t, \end{aligned} \quad (5.43)$$

$$\begin{aligned} \mathcal{O}_{qd^3 HD}^{(1-6)}: f(\mathbf{Y})_{ps}^{rt} = & (\mathbf{Y}_d)_p^r \delta_s^t + f_1' \delta_s^r (\mathbf{Y}_d)_p^t \\ & + f_3^{(1)} (\mathbf{Y}_d)_p^r (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_s^t \\ & + f_3^{(2)} (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_s^r (\mathbf{Y}_d)_p^t \\ & + f_3^{(3)} (\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_d)_p^r \delta_s^t \\ & + f_3^{(4)} \delta_s^r (\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_d)_p^t; \end{aligned} \quad (5.44)$$

otherwise, only the linear terms are reserved,

$$\mathcal{O}_{q^3 u HD}^{(1-6)}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r (\mathbf{Y}_u)_s^t + \delta_s^r (\mathbf{Y}_u)_p^t, \quad (5.45)$$

$$\mathcal{O}_{qu^3 HD}^{(1-6)}: f(\mathbf{Y})_{ps}^{rt} = (\mathbf{Y}_u)_p^r \delta_s^t + \delta_s^r (\mathbf{Y}_u)_p^t, \quad (5.46)$$

$$\mathcal{O}_{\ell^3 e HD}^{(1-3)}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r (\mathbf{Y}_e)_s^t + \delta_s^r (\mathbf{Y}_e)_p^t, \quad (5.47)$$

$$\mathcal{O}_{\ell e^3 HD}^{(1,2)}: f(\mathbf{Y})_{ps}^{rt} = (\mathbf{Y}_e)_p^r \delta_s^t + \delta_s^r (\mathbf{Y}_e)_p^t. \quad (5.48)$$

When there are two repeated fields if the repeated ones are left-handed quark $q(x)$ or right-handed up-quark $u(x)$, the expansions are

$$\begin{aligned} \mathcal{O}_{qu^2 d HD}^{(1-6)}: f(\mathbf{Y})_{ps}^{rt} = & (\mathbf{Y}_d)_p^r \delta_s^t + f_3 (\mathbf{Y}_d)_p^r (\mathbf{Y}_u^\dagger \mathbf{Y}_u)_s^t \\ & + f_3' (\mathbf{Y}_u)_p^t (\mathbf{Y}_u^\dagger \mathbf{Y}_d)_s^r \\ & + f_3'' (\mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_d)_p^r \delta_s^t, \end{aligned} \quad (5.49)$$

$$\mathcal{O}_{u^2 \ell e HD}^{(1-3)}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r (\mathbf{Y}_e)_s^t + f_3 (\mathbf{Y}_u^\dagger \mathbf{Y}_u)_p^r (\mathbf{Y}_e)_s^t, \quad (5.50)$$

$$\mathcal{O}_{q^2 \ell e HD}^{(1-6)}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r (\mathbf{Y}_e)_s^t + f_3 (\mathbf{Y}_u \mathbf{Y}_u^\dagger)_p^r (\mathbf{Y}_e)_s^t; \quad (5.51)$$

otherwise, the expansions are dominant by the linear terms

$$\mathcal{O}_{qu d^2 HD}^{(1-6)}, \mathcal{O}_{qu \ell^2 HD}^{(1-6)}, \mathcal{O}_{qu e^2 HD}^{(1-3)}: f(\mathbf{Y})_{ps}^{rt} = (\mathbf{Y}_u)_p^r \delta_s^t, \quad (5.52)$$

$$\mathcal{O}_{qd \ell^2 HD}^{(1-6)}, \mathcal{O}_{qd e^2 HD}^{(1-3)}: f(\mathbf{Y})_{ps}^{rt} = (\mathbf{Y}_d)_p^r \delta_s^t, \quad (5.53)$$

$$\mathcal{O}_{d^2 \ell e HD}^{(1-3)}: f(\mathbf{Y})_{ps}^{rt} = \delta_p^r (\mathbf{Y}_e)_s^t. \quad (5.54)$$

Thus all the MFV operators of the (lepton/baryon-number-conserving) dimension-8 operators are presented. Apart from the same flavor structures with the dimension-6 case, new structures appear in the classes $\psi^4 \phi D$ and $\psi^4 \phi^2$.

TABLE VIII. The CP properties of the SMEFT dimension-8 effective operators, where n corresponds to the operators without flavor symmetry, n_Y corresponds to the operators with the spurions $\mathbf{Y}_\psi$, and n_Y corresponds to the cases when $\mathbf{Y}_\psi$'s become matrices Y_ψ 's. The CP -odd bosonic operators can also be found in Table V.

Class	n		n_Y		n_Y	
	CP -even	CP -violating	CP -even	CP -violating	CP -even	CP -violating
Bosonic sectors	54	35	54	35	54	35
$\psi^2\phi^5$	27	21	4	4	21	21
$\psi^2\phi^4D$	69	33	14	4	54	21
$\psi^2\phi^3D^2$	162	126	24	24	126	126
$\psi^2\phi^2D^3$	93	33	21	1	63	21
$\psi^2X\phi^3$	99	81	15	15	81	81
$\psi^2X\phi^2D$	414	330	86	86	258	258
$\psi^2X\phi D^2$	216	180	33	33	180	180
$\psi^2X^2\phi$	432	372	67	67	372	372
ψ^2X^2D	291	183	62	28	186	117
ψ^4D^2	2205	1086	98	6	840	462
$\psi^4\phi^2$	2895	1566	94	18	1032	561
$\psi^4\phi D$	5454	3387	175	175	2559	2559
ψ^4X	6462	4344	159	159	2043	2043
Total	18873	11777	906	655	7869	6857
	30650		1561		14726	

We summarize the result in Table VIII. Similarly, the number of the CP -violating phases (655) is reduced compared to the number without flavor symmetry (11777).

C. $U(2)^5$ symmetry

Adopting the MFV hypothesis, the effective operators of the $U(2)^5$ symmetry can also be obtained by the spurion technique. Different from the $U(3)^5$ symmetry, each Yukawa matrix can contribute more than 1 spurion. We list the spurions and their representations as follows:

$$\Delta_u \in (2, \bar{2}, 1, 1, 1), \quad (5.55)$$

$$\Delta_d \in (2, 1, \bar{2}, 1, 1), \quad (5.56)$$

$$\Delta_e \in (1, 1, 1, 2, \bar{2}), \quad (5.57)$$

$$V_q \in (2, 1, 1, 1, 1), \quad (5.58)$$

$$V_l \in (1, 1, 1, 2, 1), \quad (5.59)$$

where Δ 's are called the tensor spurions and V 's are called the vector spurions. Thus five spurions are needed in the $U(2)^5$ flavor symmetry, and their power-counting results are different [44],

$$\Delta_u, \Delta_d, \Delta_e \sim 10^{-2}, \quad V_q, V_l \sim 10^{-1}. \quad (5.60)$$

Compared to the $U(3)^5$ flavor symmetry, there are two differences in the effective operators under the $U(2)^5$ symmetry. First, the existing operators in the $U(3)^5$

symmetry would split into the ones that are independent individually; second, the vanishing operators in the $U(3)^5$ symmetry may exist. Because most of the flavor structures of the dimension-6 and dimension-8 operators are the same, we only discuss the effective operators of the classes $\psi^4\phi D$ and $\psi^4\phi^2$ here, since other cases are the same as the dimension-6 cases and can be found in Ref. [44]

1. $\psi^4\phi^2$ class

The operators with the new flavor structures have been listed in Eq. (5.37). Considering the operators $\mathcal{O}'_{q^2u^2H^2}^{(1,2)}$ as an example, every single operator splits into several ones in the $U(2)^5$ symmetry. Including the spurions and truncating at 10^{-4} according to the power counting in Eq. (5.60), we list all the operators in terms of their powers,

$$\mathcal{O}'_{q^2u^2H^2}^{(1,2)}:$$

$$1: a_1 \epsilon_{ps} \epsilon^{rt} (\bar{q}^p u_r) (\bar{q}^s u_t) + a_2 (\bar{q}^3 u_3) (\bar{q}^3 u_3), \quad (5.61)$$

$$10^{-1}: b_1 (V_q)_p (\bar{q}^p u_3) (\bar{q}^3 u_3), \quad (5.62)$$

$$10^{-2}: c_1 (V_q)_p (V_q)_s (\bar{q}^p u_3) (\bar{q}^s u_3) \quad (5.63)$$

$$+ d_1 (\Delta_u)_p^r (\bar{q}^p u_r) (\bar{q}^3 u_3) \\ + d_2 (\Delta_u)_p^r (\bar{q}^p u_3) (\bar{q}^3 u_r), \quad (5.64)$$

$$10^{-3}: e_1 (\Delta_u)_p^r (V_q)_s (\bar{q}^p u_r) (\bar{q}^s u_3) \\ + e_2 (\Delta_u)_s^r (V_q)_p (\bar{q}^p u_r) (\bar{q}^s u_3), \quad (5.65)$$

where a_i, b_i, c_i, d_i, e_i 's are the Wilson coefficients, and for simplicity, we have dropped the quantities other than the fermions in each bilinear. In particular, the operator

$$a_1 \varepsilon_{ps} \varepsilon^{rt} (\bar{q}^p u_r) (\bar{q}^s u_t) \quad (5.66)$$

is of power 1, which is composed of antisymmetric tensor ε and is absent in the $U(3)^5$ symmetry.

The effective operators of $\mathcal{O}'_{q^2 d^2 H^2}, \mathcal{O}'_{l^2 e^2 H^2}, \mathcal{O}'_{qdleH^2}$ under the $U(2)^5$ symmetry can be obtained similarly

$$\mathcal{O}'_{q^2 d^2 H^2}: \quad 1: a_1 \varepsilon_{ps} \varepsilon^{rt} (\bar{q}^p d_r) (\bar{q}^s d_t) + a_2 (\bar{q}^3 d_3) (\bar{q}^3 d_3), \quad (5.67)$$

$$10^{-1}: b_1 (V_q)_p (\bar{q}^p d_3) (\bar{q}^3 d_3), \quad (5.68)$$

$$10^{-2}: c_1 (V_q)_p (V_q)_s (\bar{q}^p d_3) (\bar{q}^s d_3) \quad (5.69)$$

$$+ d_1 (\Delta_d)_p^r (\bar{q}^p d_r) (\bar{q}^3 d_3) \\ + d_2 (\Delta_d)_p^r (\bar{q}^p d_3) (\bar{q}^3 d_r), \quad (5.70)$$

$$10^{-3}: e_1 (\Delta_d)_p^r (V_q)_s (\bar{q}^p d_r) (\bar{q}^s d_3) \\ + e_2 (\Delta_d)_s^r (V_q)_p (\bar{q}^p d_r) (\bar{q}^s d_3), \quad (5.71)$$

$$\mathcal{O}'_{l^2 e^2 H^2}: \quad 1: a_1 \varepsilon_{ps} \varepsilon^{rt} (\bar{l}^p e_r) (\bar{l}^s e_t) + a_2 (\bar{l}^3 e_3) (\bar{l}^3 e_3), \quad (5.72)$$

$$10^{-1}: b_1 (V_l)_p (\bar{l}^p e_3) (\bar{l}^3 e_3), \quad (5.73)$$

$$10^{-2}: c_1 (V_l)_p (V_l)_s (\bar{l}^p e_3) (\bar{l}^s e_3) \quad (5.74)$$

$$+ d_1 (\Delta_e)_p^r (\bar{l}^p e_r) (\bar{l}^3 e_3) \\ + d_2 (\Delta_e)_p^r (\bar{l}^p e_3) (\bar{l}^3 e_r), \quad (5.75)$$

$$10^{-3}: e_1 (\Delta_e)_p^r (V_l)_s (\bar{l}^p e_r) (\bar{l}^s e_3) \\ + e_2 (\Delta_e)_s^r (V_l)_p (\bar{l}^p e_r) (\bar{l}^s e_3), \quad (5.76)$$

$$\mathcal{O}'_{qdleH^2}: \quad 1: a_1 (\bar{q}^3 d_3) (\bar{l}^3 e_3), \quad (5.77)$$

$$10^{-1}: b_1 (V_q)_p (\bar{q}^p d_3) (\bar{l}^3 e_3) + b_2 (V_l)_p (\bar{q}^3 d_3) (\bar{l}^p e_3), \quad (5.78)$$

$$10^{-2}: c_1 (V_q)_p (V_l)_s (\bar{q}^p d_3) (\bar{l}^s e_3) \quad (5.79)$$

$$+ d_1 (\Delta_d)_p^r (\bar{q}^p d_r) (\bar{l}^3 e_3) \\ + d_2 (\Delta_e)_s^t (\bar{q}^3 d_3) (\bar{l}^s e_t), \quad (5.80)$$

$$10^{-3}: e_1 (\Delta_d)_p^r (V_l)_s (\bar{q}^p d_r) (\bar{l}^s e_3) \\ + e_2 (\Delta_e)_s^t (V_l)_p (\bar{q}^p d_3) (\bar{l}^s e_t). \quad (5.81)$$

Thus the absent operators $\mathcal{O}'_{q^2 d^2 H^2}, \mathcal{O}'_{l^2 e^2 H^2}, \mathcal{O}'_{qdleH^2}$ under the $U(3)^5$ symmetry are nonvanishing under the $U(2)^5$ symmetry.

For the operators $\mathcal{O}'_{q^2 udH^2}$ there are

$$\mathcal{O}'_{q^2 udH^2}: \quad 1: a_1 (\bar{q}^3 u_3) (\bar{d}^3 q_3) + a_2 \delta_p^r (\bar{q}^p u_3) (\bar{d}^3 q_r), \quad (5.82)$$

$$10^{-1}: b_1 (V_q)_p (\bar{q}^p u_3) (\bar{d}^3 q_3) \\ + b_2 (V_q^\dagger)^r (\bar{q}^3 u_3) (\bar{d}^3 q_r), \quad (5.83)$$

$$10^{-2}: c_1 (V_q)_p (V_q^\dagger)^r (\bar{q}^p u_3) (\bar{d}^3 q_r) \quad (5.84)$$

$$+ d_1 (\Delta_u)_p^r (\bar{q}^p u_r) (\bar{d}^3 q_3) \\ + d_2 (\Delta_d)_s^t (\bar{q}^3 u_3) (\bar{d}^s q_t), \quad (5.85)$$

$$10^{-3}: e_1 (\Delta_u)_p^r (V_q^\dagger)^t (\bar{q}^p u_r) (\bar{d}^3 q_t) \\ + e_2 \delta_p^t (V_q^\dagger \Delta_u)^r (\bar{q}^p u_r) (\bar{d}^3 q_t) \\ + e_3 (\Delta_d^\dagger)_s^t (V_q)_p (\bar{q}^p u_3) (\bar{d}^s q_t) \\ + e_4 \delta_p^t (\Delta_d^\dagger V_q)_s (\bar{q}^p u_3) (\bar{d}^s q_t), \quad (5.86)$$

while for the $\mathcal{O}'_{l^2 udH^2}$, the operators are

$$\mathcal{O}'_{l^2 udH^2}: \quad 1: a_1 (\bar{l}^3 l_3) (\bar{u}^3 d_3) + a_2 \delta_p^r (\bar{l}^p l_r) (\bar{u}^3 d_3), \quad (5.87)$$

$$10^{-1}: b_1 (V_l)_p (\bar{l}^p l_3) (\bar{u}^3 d_3) + b_2 (V_l^\dagger)^r (\bar{l}^3 l_r) (\bar{u}^3 d_3), \quad (5.88)$$

$$10^{-2}: c_1 (V_l)_p (V_l^\dagger)^r (\bar{l}^p l_r) (\bar{u}^3 d_3), \quad (5.89)$$

$$10^{-3}: d_1 \delta_p^r (\Delta_u^\dagger V_q)_s (\bar{l}^p l_r) (\bar{u}^s d_3) \\ + d_2 \delta_p^r (V_q^\dagger \Delta_d)^t (\bar{l}^p l_r) (\bar{u}^3 d_t) \\ + d_3 (\Delta_u^\dagger V_q)_s (\bar{l}^3 l_3) (\bar{u}^s d_3) \\ + d_4 (V_q^\dagger \Delta_d)^t (\bar{l}^3 l_3) (\bar{u}^3 d_t). \quad (5.90)$$

2. $\psi^4 \phi D$ class

Because of the nonvanishing hypercharge of the Higgs field, the flavor structures of all the operators of the $\psi^4 \phi D$ class are different from the dimension-6 cases.

First, we notice the operators $\mathcal{O}'_{q^3 dHD}, \mathcal{O}'_{q^3 dHD}, \mathcal{O}'_{l^3 eHD}$ are similar since there are three repeated left-handed fermions, so their flavor structures are similar, and we present explicitly the effective operators of $\mathcal{O}'_{q^3 dHD}$ under the $U(2)^5$ symmetry here,

$$\mathcal{O}_{\bar{q}^3 dHD}^{(1-6)} : \\ 1: a_1(\bar{q}^3 q_3)(\bar{q}^3 u_3) + a_2 \delta_p^r(\bar{q}^p q_r)(\bar{q}^3 u_3) + a_3 \delta_p^r(\bar{q}^3 q_r)(\bar{q}^p u_3) + a_4 \epsilon_{ps}(\bar{q}^p q_3)(\bar{q}^s u_3), \quad (5.91)$$

$$10^{-1}: b_1(V_q)_p(\bar{q}^p q_3)(\bar{q}^3 u_3) + b_2(V_q)_p(\bar{q}^3 q_3)(\bar{q}^p u_3) + b_3(V_q^\dagger)^r(\bar{q}^3 q_r)(\bar{q}^3 u_3) \\ + b_4(V_q)_p \delta_s^r(\bar{q}^p q_r)(\bar{q}^s u_3) + b_5(V_q)_p \delta_s^r(\bar{q}^s q_r)(\bar{q}^p u_3) + b_6(V_q^\dagger)^r \epsilon_{ps}(\bar{q}^p q_r)(\bar{q}^s u_3), \quad (5.92)$$

$$10^{-2}: c_1(V_q)_p(V_q^\dagger)^r(\bar{q}^p q_r)(\bar{q}^3 u_3) + c_2(V_q)_p(V_q^\dagger)^r(\bar{q}^3 q_r)(\bar{q}^p u_3) + c_3(V_q)_p(V_q)_s(\bar{q}^p q_3)(\bar{q}^s u_3) \quad (5.93)$$

$$+ d_1(\Delta_u)_s^t(\bar{q}^3 q_3)(\bar{q}^s u_t) + d_2(\Delta_u)_p^t(\bar{q}^p q_3)(\bar{q}^3 u_t) + d_3(V_q)_p(V_q^\dagger)^r(V_q)_s(\bar{q}^p q_r)(\bar{q}^s u_3) \\ + d_4(\Delta_u)_s^t \delta_p^r(\bar{q}^p q_r)(\bar{q}^s u_t) + d_5(\Delta_u)_p^t \delta_s^r(\bar{q}^p q_r)(\bar{q}^s u_t), \quad (5.94)$$

$$10^{-3}: e_1 \delta_p^r(V_q^\dagger \Delta_u)^t(\bar{q}^p q_r)(\bar{q}^3 u_t) + e_2(\Delta_u)_p^t(V_u^\dagger)^r(\bar{q}^p q_r)(\bar{q}^3 u_t) + e_3(\Delta_u)_s^t(V_q)_p(\bar{q}^p q_s)(\bar{q}^s u_t) \\ + e_4(\Delta_u)_p^t(V_q)_s(\bar{q}^p q_s)(\bar{q}^s u_t) + e_5 \delta_s^r(V_q^\dagger \Delta_u)^t(\bar{q}^3 q_r)(\bar{q}^s u_t) + e_6(\Delta_u)_s^t(V_q^\dagger)^r(\bar{q}^3 q_r)(\bar{q}^s u_t) \\ + e_7(V_q^\dagger \Delta_u)^t(\bar{q}^3 q_3)(\bar{q}^3 u_t). \quad (5.95)$$

The other two cases can be obtained by the spurion replacements. Second, the operators $\mathcal{O}_{\bar{q}d^3HD}^{(1-6)}$, $\mathcal{O}_{\bar{q}u^3HD}^{(1-6)}$, $\mathcal{O}_{\bar{l}e^3HD}^{(1-3)}$ have the same flavor structures since there are three repeated right-handed fermions. We present the explicit operators of only one of them, $\mathcal{O}_{\bar{q}u^3HD}^{(1-6)}$,

$$\mathcal{O}_{\bar{q}u^3HD}^{(1-6)} : \\ 1: a_1 \delta_p^r(\bar{d}^p d_r)(\bar{q}^3 d_3) + a_2 \delta_p^t(\bar{d}^p d_3)(\bar{q}^3 d_t) + a_3(\bar{d}^3 d_3)(\bar{q}^3 d_3) + a_4 \epsilon^{rt}(\bar{d}^3 d_r)(\bar{q}^3 d_t), \quad (5.96)$$

$$10^{-1}: b_1 \delta_p^r(V_q)_s(\bar{d}^p d_r)(\bar{q}^s d_3) + b_2 \delta_p^t(V_q)_s(\bar{d}^p d_3)(\bar{q}^s d_t) + b_3 \epsilon^{rt}(V_q)_s(\bar{d}^3 d_r)(\bar{q}^s d_t) \\ + b_4(V_q)_s(\bar{d}^3 d_3)(\bar{q}^s d_3), \quad (5.97)$$

$$10^{-2}: c_1 \delta_p^r(\Delta_d)_s^t(\bar{d}^p d_r)(\bar{q}^s d_t) + c_2 \delta_p^t(\Delta_d)_s^r(\bar{d}^p d_r)(\bar{q}^s d_t) + c_3(\Delta_d)_s^t(\bar{d}^3 d_3)(\bar{q}^s d_t) \\ + c_4(\Delta_d)_s^r(\bar{d}^3 d_r)(\bar{q}^s d_3), \quad (5.98)$$

$$10^{-3}: d_1 \delta_p^r(V_q^\dagger \Delta_d)^t(\bar{d}^p d_r)(\bar{q}^3 d_t) + d_2 \delta_p^t(V_q^\dagger \Delta_d)^r(\bar{d}^p d_r)(\bar{q}^3 d_t) + d_3(\Delta_d^\dagger V_q)_p(\bar{d}^p d_3)(\bar{q}^3 d_3) \\ + d_4(\Delta_d V_q^\dagger)^r(\bar{d}^3 d_r)(\bar{q}^3 d_3) + d_5(V_q^\dagger \Delta_d)^t(\bar{d}^3 d_3)(\bar{q}^3 d_t). \quad (5.99)$$

Next, we identify the operators $\mathcal{O}_{\bar{q}u^2dHD}^{(1-6)}$, $\mathcal{O}_{\bar{q}u^2HD}^{(1-6)}$, $\mathcal{O}_{\bar{u}^2leHD}^{(1-3)}$, $\mathcal{O}_{\bar{d}^2leHD}^{(1-3)}$, $\mathcal{O}_{\bar{q}ue^2HD}^{(1-3)}$, $\mathcal{O}_{\bar{e}^2qdHD}^{(1-3)}$ are of the same flavor structures, and thus we present only the operators of $\mathcal{O}_{\bar{q}u^2dHD}^{(1-6)}$ explicitly here,

$$\mathcal{O}_{\bar{q}u^2dHD}^{(1-6)} : \\ 1: a_1 \delta_p^r(\bar{u}^p u_r)(\bar{q}^3 d_3) + a_2(\bar{u}^3 u_3)(\bar{q}^3 d_3), \quad (5.100)$$

$$10^{-1}: b_1 \delta_p^r(V_q)_s(\bar{u}^p u_r)(\bar{q}^s d_3) + b_2(V_q)_s(\bar{u}^3 u_3)(\bar{q}^s d_3), \quad (5.101)$$

$$10^{-2}: c_1 \delta_p^r(\Delta_d)_s^t(\bar{u}^p u_r)(\bar{q}^s d_t) + c_2(\Delta_d)_s^t(\bar{u}^3 u_3)(\bar{q}^s d_t) + c_3(\Delta_u)_s^r(\bar{u}^3 u_r)(\bar{q}^s d_3), \quad (5.102)$$

$$10^{-3}: d_1 \delta_p^r(V_q^\dagger \Delta_d)^t(\bar{u}^p u_r)(\bar{q}^3 d_t) + d_2(V_q^\dagger \Delta_d)^t(\bar{u}^3 u_3)(\bar{q}^3 d_t) + d_3(\Delta_u^\dagger V_q)_p(\bar{u}^p u_3)(\bar{q}^3 d_3) \\ + d_4(V_q^\dagger \Delta_u)^r d_1(\bar{u}^3 u_r)(\bar{q}^3 d_3). \quad (5.103)$$

Finally, the operators $\mathcal{O}_{q^2leHD}^{(1-6)}$, $\mathcal{O}_{l^2quHD}^{(1-6)}$, $\mathcal{O}_{qdl^2HD}^{(1-6)}$ are of the same flavor structures under the $U(2)^5$ flavor symmetry, and thus we present the operators of $\mathcal{O}_{q^2leHD}^{(1-6)}$ here,

$$\mathcal{O}_{q^2leHD}^{(1-6)} : \quad 1: a_1 \delta_p^r (\bar{q}^p q_r) (\bar{l}^s e_3) + a_2 (\bar{q}^3 q_3) (\bar{l}^s e_3), \quad (5.104)$$

$$10^{-1}: b_1 \delta_p^r (V_l)_s (\bar{q}^p q_r) (\bar{l}^s e_3) + b_2 (V_l)_s (\bar{q}^3 q_3) (\bar{l}^s e_3) \\ + b_3 (V_l)_p (\bar{q}^p q_3) (\bar{l}^s e_3) + b_4 (V_l^\dagger)^r (\bar{q}^3 q_r) (\bar{l}^s e_3), \quad (5.105)$$

$$10^{-2}: c_1 (V_q)_p (V_q^\dagger)^r (\bar{q}^p q_r) (\bar{l}^s e_3) \\ + c_2 (V_q)_p (V_l)_s (\bar{q}^p q_3) (\bar{l}^s e_3) \\ + c_3 (V_q^\dagger)^r (V_l)_s (\bar{q}^3 q_r) (\bar{l}^s e_3), \quad (5.106)$$

$$10^{-3}: d_1 (V_q)_p (V_q^\dagger)^r (V_l)_s (\bar{q}^p q_r) (\bar{l}^s e_3) \\ + d_2 \delta_p^r (\Delta_e)_s (\bar{q}^p q_r) (\bar{l}^s e_t) + d_3 (\Delta_e)_r (\bar{q}^3 q_3) (\bar{l}^s e_t) \quad (5.107)$$

$$+ e_1 \delta_p^r (V_l^\dagger \Delta_e)^t (\bar{q}^p q_r) (\bar{l}^s e_t) + e_2 (V_l^\dagger \Delta_e)^t (\bar{q}^3 q_3) (\bar{l}^s e_t) \\ + e_3 (V_q)_p (\Delta_e)_s (\bar{q}^p q_3) (\bar{l}^s e_t) \\ + e_4 (V_q^\dagger)^r (\Delta_e)_s (\bar{q}^s q_r) (\bar{l}^s e_t). \quad (5.108)$$

In summary, the $U(2)^5$ as a subgroup of $U(3)^5$ is a less-constrained flavor symmetry; thus the involving operators are more abundant than the $U(3)^5$ case, which means richer phenomenologies. Besides, there are two advantages of the $U(2)^5$ symmetry compared to the $U(3)^5$: first, the split of the fermion triplet makes the power-counting more precise; second, more spurions make the flavor-changed process occur also in the leptonic sector, which is banned in the $U(3)^5$ symmetry. No matter the $U(3)^5$ or $U(2)^5$ symmetry, we adopt the MFV hypothesis, which is consistent with the observations and protects the cutoff scale of the SMEFT from being pushed too high.

VI. CONCLUSION

The flavor symmetry can be used to organize and classify the large number of operators of the SMEFT. When the flavor symmetries are absent, the operators are organized by their flavor structures and CP properties. The flavor structures can be decomposed into irreducible representations of the symmetric groups, which are closely related to the Lorentz and gauge structures of the operators. The CP violation of the SMEFT is important in the research of new physics. In general, the CP -odd operators are not equivalent to CP -violating operators, and only the ones invariant under the rephasing symmetry are CP -violating, since there is no field redefinition

to eliminate the complex phases of the associated Wilson coefficients. We present the complete CP -even and CP -odd operators at dimension-6 and dimension-8, including both the bosonic operators and the fermionic operators.

The flavor symmetries such as $U(3)^5$ and $U(2)^5$ can reduce the independent parameters in principle, and affect the CP violation in two aspects. First, the flavor symmetries reduce the independent operators, and second, the flavor symmetries change the rephasing symmetry. Nevertheless, since the rephasing symmetry is a subgroup of the flavor symmetry, it is guaranteed by the flavor symmetry; thus the CP -odd and the CP -violating operators are always equivalent with flavor symmetry. However, the single assumption of the $U(3)^5$ symmetry does not change the CP -violating phases. At the same time, the subgroup $U(2)^5$ could be different. At the same time, adding flavor symmetries, the flavor-invariant method can also be used to express the CP -violating phases by the flavor invariants. As discussed in this paper, the tensors obtained by the Young tableaux are equivalent to the traces obtained by the Cayley-Hamilton relations. Thus the flavor invariants can be obtained by the Young tensor method analytically, which can be applied to the constructions of the high-dimension flavor invariants of the SMEFT.

To reduce the parameters and classify the operators furthermore, we consider the MFV hypothesis to be consistent with the experiment results. We analyze the flavor structures and the CP properties of the SMEFT operators at dimension-6 and dimension-8 with the flavor symmetries $U(3)^5$. We use the spurion method to present their flavor structures explicitly. In dimension-6, we find 26 independent CP -violating phases, which is smaller than the 705 without flavor symmetry. In dimension-8, there are new flavor structures in the classes $\psi^4 \phi^2$ and $\psi^4 \phi D$. We find 655 independent CP -violating phases, and it is also smaller than the 11777 without the flavor symmetry. For the two classes $\psi^4 \phi^2$ and $\psi^4 \phi D$ with the new flavor structures in dimension-8, we also present their flavor structures assuming the $U(2)^5$ symmetry and its minimal breaking, which include more operators than the $U(3)^5$ symmetry.

The flavor symmetry and the MFV hypothesis are used to reduce and classify the effective operators of the SMEFT, and their relation to the CP violations presented in this paper is general. The Young tensor method can be used to obtain the flavor invariants of the SMEFT to extract the CP -violating phases in a basis-invariant way. The analysis here can be extended to other flavor symmetries, e.g., [19,23,67–72]. Based on the analysis, we can classify and organize the large number of SMEFT operators systematically, and reduce the parameters in the related experiments.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: DIMENSION-6 OPERATORS

Here the dimension-6 SMEFT operators are adopted from Ref. [3], and only the baryon-number and lepton-number-conserving fermionic operators are presented. In particular, we have simplified the operators by writing the right-handed fermions as

$$u_R, d_R, e_R \rightarrow u, d, e. \quad (\text{A1})$$

1: $\psi^2 H^3 + \text{H.c.}$	
$\mathcal{O}_{eH}$	$(H^\dagger H)(\bar{l}^p e_r H)$
$\mathcal{O}_{uH}$	$(H^\dagger H)(\bar{q}^p u_r \tilde{H})$
$\mathcal{O}_{dH}$	$(H^\dagger H)(\bar{q}^p d_r H)$
2: $\psi^2 XH + \text{H.c.}$	
$\mathcal{O}_{eW}$	$(\bar{l}^p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$
$\mathcal{O}_{eB}$	$(\bar{l}^p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
$\mathcal{O}_{uG}$	$(\bar{q}^p \sigma^{\mu\nu} \lambda^A u_r) \tilde{H} G_{\mu\nu}^A$
$\mathcal{O}_{uW}$	$(\bar{q}^p \sigma^{\mu\nu} u_r) \tau^I \tilde{H} W_{\mu\nu}^I$
$\mathcal{O}_{uB}$	$(\bar{q}^p \sigma^{\mu\nu} u_r) \tilde{H} B_{\mu\nu}$
$\mathcal{O}_{dG}$	$(\bar{q}^p \sigma^{\mu\nu} \lambda^A d_r) H G_{\mu\nu}^A$
$\mathcal{O}_{dW}$	$(\bar{q}^p \sigma^{\mu\nu} d_r) \tau^I H W_{\mu\nu}^I$
$\mathcal{O}_{dB}$	$(\bar{q}^p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$
3: $\psi^2 H^2 D$	
$\mathcal{O}_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}^p \gamma^\mu l_r)$
$\mathcal{O}_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}^p \tau^I \gamma^\mu l_r)$
$\mathcal{O}_{He}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}^p \gamma^\mu e_r)$
$\mathcal{O}_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}^p \gamma^\mu q_r)$
$\mathcal{O}_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}^p \tau^I \gamma^\mu q_r)$
$\mathcal{O}_{Hu}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}^p \gamma^\mu u_r)$
$\mathcal{O}_{Hd}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}^p \gamma^\mu d_r)$
$\mathcal{O}_{Hud}$	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}^p \gamma^\mu d_r)$

4: $(\bar{L}L)(\bar{L}L)$	
$\mathcal{O}_{ll}$	$(\bar{l}^p \gamma_\mu l_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}^p \gamma_\mu q_r)(\bar{q}^s \gamma^\mu q_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}^p \gamma_\mu \tau^I q_r)(\bar{q}^s \gamma^\mu \tau^I q_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{q}^p \gamma_\mu q_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{q}^p \gamma_\mu \tau^I q_r)(\bar{l}^s \gamma^\mu \tau^I l_t)$
4: $(\bar{R}R)(\bar{R}R)$	
$\mathcal{O}_{ee}$	$(\bar{e}^p \gamma_\mu e_r)(\bar{e}^s \gamma^\mu e_t)$
$\mathcal{O}_{uu}$	$(\bar{u}^p \gamma_\mu u_r)(\bar{u}^s \gamma^\mu u_t)$
$\mathcal{O}_{dd}$	$(\bar{d}^p \gamma_\mu d_r)(\bar{d}^s \gamma^\mu d_t)$
$\mathcal{O}_{eu}$	$(\bar{u}^p \gamma_\mu u_r)(\bar{e}^s \gamma^\mu e_t)$
$\mathcal{O}_{ed}$	$(\bar{d}^p \gamma_\mu d_r)(\bar{e}^s \gamma^\mu e_t)$
$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}^p \gamma_\mu u_r)(\bar{d}^s \gamma^\mu d_t)$
$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}^p \gamma_\mu \lambda^A u_r)(\bar{d}^s \gamma^\mu \lambda^A d_t)$
4: $(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{le}$	$(\bar{l}^p \gamma_\mu l_r)(\bar{e}^s \gamma^\mu e_t)$
$\mathcal{O}_{lu}$	$(\bar{u}^p \gamma_\mu u_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{ld}$	$(\bar{d}^p \gamma_\mu d_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{qe}$	$(\bar{q}^p \gamma_\mu q_r)(\bar{e}^s \gamma^\mu e_t)$
$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}^p \gamma_\mu q_r)(\bar{u}^s \gamma^\mu u_t)$
$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}^p \gamma_\mu \lambda^A q_r)(\bar{u}^s \gamma^\mu \lambda^A u_t)$
$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}^p \gamma_\mu q_r)(\bar{d}^s \gamma^\mu d_t)$
$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}^p \gamma_\mu \lambda^A q_r)(\bar{d}^s \gamma^\mu \lambda^A d_t)$
4: $(\bar{L}R)(\bar{R}L) + \text{H.c.}$	
$\mathcal{O}_{ledq}$	$(\bar{d}^p q_{jr})(\bar{l}^{is} e_t)$
4: $(\bar{L}R)(\bar{L}R) + \text{H.c.}$	
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}^{jp} u_r) \varepsilon_{jk} (\bar{q}^{ks} d_t)$
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}^{jp} \lambda^A u_r) \varepsilon_{jk} (\bar{q}^{ks} \lambda^A d_t)$
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{q}^{kp} u_r) \varepsilon_{jk} (\bar{l}^{js} e_t)$
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{q}^{kp} \sigma^{\mu\nu} u_r) \varepsilon_{jk} (\bar{l}^{js} \sigma_{\mu\nu} e_t)$

APPENDIX B: DIMENSION-8 OPERATORS

The dimension-8 operators here are the reformulations of the ones obtained in Ref. Similar to the dimension-6 case, only the lepton-number- and baryon-number-conserving fermionic operators are presented, and the same simplification is adopted,

$$u_R, d_R, e_R \rightarrow u, d, e. \quad (\text{B1})$$

1: $\psi^2 \phi^5$					
$\mathcal{O}_{quH^5}$					$(\bar{q}^p \tilde{H} u_r)(H^\dagger H)^2$
$\mathcal{O}_{qdH^5}$					$(\bar{q}^p H d_r)(H^\dagger H)^2$
$\mathcal{O}_{leH^5}$					$(\bar{l}^p H e_r)(H^\dagger H)^2$
2: $\psi^2 \phi^4 D$					
$\mathcal{O}_{q^2 H^4 D}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\mathcal{O}_{l^2 H^4 D}^{(1)}$			$(\bar{l}^p \gamma^\mu l_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$
$\mathcal{O}_{q^2 H^4 D}^{(2)}$	$(\bar{q}^p \tau^I \gamma^\mu q_r)(H^\dagger \tau^I H)(H^\dagger \overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{l^2 H^4 D}^{(2)}$			$(\bar{l}^p \tau^I \gamma^\mu l_r)(H^\dagger \tau^I H)(H^\dagger \overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{q^2 H^4 D}^{(3)}$	$(\bar{q}^p \tau^I \gamma^\mu q_r)(H^\dagger \tau^I H) D_\mu (H^\dagger H)$	$\mathcal{O}_{l^2 H^4 D}^{(3)}$			$(\bar{l}^p \tau^I \gamma^\mu l_r)(H^\dagger \tau^I H) D_\mu (H^\dagger H)$
$\mathcal{O}_{q^2 H^4 D}^{(4)}$	$(\bar{q}^p \tau^I \gamma^\mu q_r)(H^\dagger \tau^I \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\mathcal{O}_{l^2 H^4 D}^{(4)}$			$(\bar{l}^p \tau^I \gamma^\mu l_r)(H^\dagger \tau^I \overleftrightarrow{D}_\mu H)(H^\dagger H)$
$\mathcal{O}_{u^2 H^4 D}$	$(\bar{u}^p \gamma^\mu u_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\mathcal{O}_{e^2 H^4 D}$			$(\bar{e}^p \gamma^\mu e_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$
$\mathcal{O}_{d^2 H^4 D}$	$(\bar{d}^p \gamma^\mu d_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\mathcal{O}_{udH^4 D}$			$(\bar{u}^p \gamma^\mu d_r)(\tilde{H}^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$
3: $\psi^2 \phi^3 D^2$					
$\mathcal{O}_{quH^3 D^2}^{(1)}$	$(\bar{q}^p D^\mu \tilde{H} u_r)(H^\dagger \overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{qdH^3 D^2}^{(1)}$	$(\bar{q}^p D^\mu H d_r)(H^\dagger \overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{leH^3 D^2}^{(1)}$	$(\bar{l}^p D^\mu H e_r)(H^\dagger \overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{quH^3 D^2}^{(2)}$	$(\bar{q}^p D_\mu \tilde{H} \sigma^{\mu\nu} u_r)(H^\dagger \overleftrightarrow{D}_\nu H)$	$\mathcal{O}_{qdH^3 D^2}^{(2)}$	$(\bar{q}^p D_\mu H \sigma^{\mu\nu} d_r)(H^\dagger \overleftrightarrow{D}_\nu H)$	$\mathcal{O}_{leH^3 D^2}^{(2)}$	$(\bar{l}^p D_\mu H \sigma^{\mu\nu} e_r)(H^\dagger \overleftrightarrow{D}_\nu H)$
$\mathcal{O}_{quH^3 D^2}^{(3)}$	$(\bar{q}^p \tilde{H} u_r) D^2 (H^\dagger H)$	$\mathcal{O}_{qdH^3 D^2}^{(3)}$	$(\bar{q}^p H d_r) D^2 (H^\dagger H)$	$\mathcal{O}_{leH^3 D^2}^{(3)}$	$(\bar{l}^p H e_r) D^2 (H^\dagger H)$
$\mathcal{O}_{quH^3 D^2}^{(4)}$	$(\bar{q}^p \tilde{H} \sigma^{\mu\nu} u_r)(D_\mu H^\dagger D_\nu H)$	$\mathcal{O}_{qdH^3 D^2}^{(4)}$	$(\bar{q}^p H \sigma^{\mu\nu} d_r)(D_\mu H^\dagger D_\nu H)$	$\mathcal{O}_{leH^3 D^2}^{(4)}$	$(\bar{l}^p H \sigma^{\mu\nu} e_r)(D_\mu H^\dagger D_\nu H)$
$\mathcal{O}_{quH^3 D^2}^{(5)}$	$(\bar{q}^p D^\mu \tilde{H} u_r) D_\mu (H^\dagger H)$	$\mathcal{O}_{qdH^3 D^2}^{(5)}$	$(\bar{q}^p D^\mu H d_r) D_\mu (H^\dagger H)$	$\mathcal{O}_{leH^3 D^2}^{(5)}$	$(\bar{l}^p D^\mu H e_r) D_\mu (H^\dagger H)$
$\mathcal{O}_{quH^3 D^2}^{(6)}$	$(\bar{q}^p D_\mu \tilde{H} \sigma^{\mu\nu} u_r) D_\nu (H^\dagger H)$	$\mathcal{O}_{qdH^3 D^2}^{(6)}$	$(\bar{q}^p D_\mu H \sigma^{\mu\nu} d_r) D_\nu (H^\dagger H)$	$\mathcal{O}_{leH^3 D^2}^{(6)}$	$(\bar{l}^p D_\mu H \sigma^{\mu\nu} e_r) D_\nu (H^\dagger H)$
4: $\psi^2 \phi^2 D^3$					
$\mathcal{O}_{q^2 H^2 D^3}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r) D^2 (H^\dagger \overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{u^2 H^2 D^3}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r) D^2 (H^\dagger \overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{l^2 H^2 D^3}^{(1)}$	$(\bar{l}^p \gamma^\mu l_r) D^2 (H^\dagger \overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{q^2 H^2 D^3}^{(2)}$	$(\bar{q}^p \gamma_\mu \overleftrightarrow{D}_\nu q_r)(D^\mu H^\dagger D^\nu H)$	$\mathcal{O}_{u^2 H^2 D^3}^{(2)}$	$(\bar{u}^p \gamma_\mu \overleftrightarrow{D}_\nu u_r)(D^\mu H^\dagger D^\nu H)$	$\mathcal{O}_{l^2 H^2 D^3}^{(2)}$	$(\bar{l}^p \gamma_\mu \overleftrightarrow{D}_\nu l_r)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_{q^2 H^2 D^3}^{(3)}$	$(\bar{q}^p \tau^I \gamma^\mu q_r) D^2 (H^\dagger \tau^I \overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{d^2 H^2 D^3}^{(1)}$	$(\bar{d}^p \gamma^\mu d_r) D^2 (H^\dagger \overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{l^2 H^2 D^3}^{(3)}$	$(\bar{l}^p \tau^I \gamma^\mu l_r) D^2 (H^\dagger \tau^I \overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{q^2 H^2 D^3}^{(4)}$	$(\bar{q}^p \tau^I \gamma_\mu \overleftrightarrow{D}_\nu q_r)(D^\mu H^\dagger \tau^I D^\nu H)$	$\mathcal{O}_{d^2 H^2 D^3}^{(2)}$	$(\bar{d}^p \gamma_\mu \overleftrightarrow{D}_\nu d_r)(D^\mu H^\dagger D^\nu H)$	$\mathcal{O}_{l^2 H^2 D^3}^{(4)}$	$(\bar{l}^p \tau^I \gamma_\mu \overleftrightarrow{D}_\nu l_r)(D^\mu H^\dagger \tau^I D^\nu H)$
$\mathcal{O}_{e^2 H^2 D^3}^{(1)}$	$(\bar{e}^p \gamma^\mu e_r) D^2 (H^\dagger \overleftrightarrow{D}_\mu H)$	$\mathcal{O}_{e^2 H^2 D^3}^{(2)}$	$(\bar{e}^p \gamma_\mu \overleftrightarrow{D}_\nu e_r)(D^\mu H^\dagger D^\nu H)$	$\mathcal{O}_{udH^2 D^3}$	$(\bar{u}^p \gamma^\mu \overleftrightarrow{D}_\nu d_r)(D_\mu \tilde{H}^\dagger D_\nu H)$

5: $\psi^2 X \phi^3$			
$\mathcal{O}_{GquH^3}$	$(\bar{q}^p \tilde{H} \lambda^A \sigma^{\mu\nu} u_r)(H^\dagger H) G_{\mu\nu}^A$	$\mathcal{O}_{WqdH^3}^{(2)}$	$(\bar{q}^p \tau^I H \sigma^{\mu\nu} d_r)(H^\dagger H) W_{\mu\nu}^I$
$\mathcal{O}_{WquH^3}^{(1)}$	$(\bar{q}^p H \sigma^{\mu\nu} u_r)(H^\dagger \tau^I \tilde{H}) W_{\mu\nu}^I$	$\mathcal{O}_{BqdH^3}$	$(\bar{q}^p H \sigma^{\mu\nu} d_r)(H^\dagger H) B_{\mu\nu}$
$\mathcal{O}_{WquH^3}^{(2)}$	$(\bar{q}^p \tau^I \tilde{H} \sigma^{\mu\nu} u_r)(H^\dagger H) W_{\mu\nu}^I$	$\mathcal{O}_{WleH^3}^{(1)}$	$(\bar{l}^p H \sigma^{\mu\nu} e_r)(H^\dagger \tau^I H) W_{\mu\nu}^I$
$\mathcal{O}_{BquH^3}$	$(\bar{q}^p \tilde{H} \sigma^{\mu\nu} u_r)(H^\dagger H) B_{\mu\nu}$	$\mathcal{O}_{WleH^3}^{(2)}$	$(\bar{l}^p \tau^I H \sigma^{\mu\nu} e_r)(H^\dagger H) W_{\mu\nu}^I$
$\mathcal{O}_{GqdH^3}$	$(\bar{q}^p H \lambda^A \sigma^{\mu\nu} d_r)(H^\dagger H) G_{\mu\nu}^A$	$\mathcal{O}_{BleH^3}$	$(\bar{l}^p H \sigma^{\mu\nu} e_r)(H^\dagger H) B_{\mu\nu}$
$\mathcal{O}_{WqdH^3}^{(1)}$	$(\bar{q}^p H \sigma^{\mu\nu} d_r)(H^\dagger \tau^I H) W_{\mu\nu}^I$		

6: $\psi^2 X \phi D^2$					
$\mathcal{O}_{GquHD^2}^{(1)}$	$(\bar{q}^p D^\mu D_\nu \tilde{H} G_{\mu\lambda} \sigma^{\nu\lambda} u_r)$	$\mathcal{O}_{BquHD^2}^{(3)}$	$(\bar{q}^p D^\nu \tilde{H} \overleftrightarrow{D}^\mu u_r) B_{\mu\nu}$	$\mathcal{O}_{BqdHD^2}^{(2)}$	$(\bar{q}^p D^\mu D_\nu H \sigma^{\nu\lambda} d_r) \tilde{B}_{\mu\lambda}$
$\mathcal{O}_{GquHD^2}^{(2)}$	$(\bar{q}^p D^\mu D_\nu \tilde{H} \tilde{G}_{\mu\lambda} \sigma^{\nu\lambda} u_r)$	$\mathcal{O}_{GqdHD^2}^{(1)}$	$(\bar{q}^p D^\mu D_\nu H G_{\mu\lambda} \sigma^{\nu\lambda} d_r)$	$\mathcal{O}_{BqdHD^2}^{(3)}$	$(\bar{q}^p D^\nu H \overleftrightarrow{D}^\mu d_r) B_{\mu\nu}$
$\mathcal{O}_{GquHD^2}^{(3)}$	$(\bar{q}^p D^\nu \tilde{H} G_{\mu\nu} \overleftrightarrow{D}^\mu u_r)$	$\mathcal{O}_{GqdHD^2}^{(2)}$	$(\bar{q}^p D^\mu D_\nu H \tilde{G}_{\mu\lambda} \sigma^{\nu\lambda} d_r)$	$\mathcal{O}_{WleHD^2}^{(1)}$	$(\bar{l}^p W_{\mu\lambda} D^\mu D_\nu H \sigma^{\nu\lambda} e_r)$
$\mathcal{O}_{WquHD^2}^{(1)}$	$(\bar{q}^p W_{\mu\lambda} D^\mu D_\nu \tilde{H} \sigma^{\nu\lambda} u_r)$	$\mathcal{O}_{GqdHD^2}^{(3)}$	$(\bar{q}^p D^\nu H G_{\mu\nu} \overleftrightarrow{D}^\mu d_r)$	$\mathcal{O}_{WleHD^2}^{(2)}$	$(\bar{l}^p \tilde{W}_{\mu\lambda} D^\mu D_\nu H \sigma^{\nu\lambda} e_r)$
$\mathcal{O}_{WquHD^2}^{(2)}$	$(\bar{q}^p \tilde{W}_{\mu\lambda} D^\mu D_\nu \tilde{H} \sigma^{\nu\lambda} u_r)$	$\mathcal{O}_{WqdHD^2}^{(1)}$	$(\bar{q}^p W_{\mu\lambda} D^\mu D_\nu H \sigma^{\nu\lambda} d_r)$	$\mathcal{O}_{WleHD^2}^{(3)}$	$(\bar{l}^p W_{\mu\nu} D^\nu H \overleftrightarrow{D}^\mu e_r)$
$\mathcal{O}_{WquHD^2}^{(3)}$	$(\bar{q}^p W_{\mu\nu} D^\nu \overleftrightarrow{H} D^\mu u_r)$	$\mathcal{O}_{WqdHD^2}^{(2)}$	$(\bar{q}^p \tilde{W}_{\mu\lambda} D^\mu D_\nu H \sigma^{\nu\lambda} d_r)$	$\mathcal{O}_{BleHD^2}^{(1)}$	$(\bar{l}^p D^\mu D_\nu H \sigma^{\nu\lambda} e_r) B_{\mu\lambda}$
$\mathcal{O}_{BquHD^2}^{(1)}$	$(\bar{q}^p D^\mu D_\nu \tilde{H} \sigma^{\nu\lambda} u_r) B_{\mu\lambda}$	$\mathcal{O}_{WqdHD^2}^{(3)}$	$(\bar{q}^p W_{\mu\nu} D^\nu H \overleftrightarrow{D}^\mu d_r)$	$\mathcal{O}_{BleHD^2}^{(2)}$	$(\bar{l}^p D^\mu D_\nu H \sigma^{\nu\lambda} e_r) \tilde{B}_{\mu\lambda}$
$\mathcal{O}_{BquHD^2}^{(2)}$	$(\bar{q}^p D^\mu D_\nu \tilde{H} \sigma^{\nu\lambda} u_r) \tilde{B}_{\mu\lambda}$	$\mathcal{O}_{BqdHD^2}^{(1)}$	$(\bar{q}^p D^\mu D_\nu H \sigma^{\nu\lambda} d_r) B_{\mu\lambda}$	$\mathcal{O}_{BleHD^2}^{(3)}$	$(\bar{l}^p D^\nu H \overleftrightarrow{D}^\mu e_r) B_{\mu\nu}$

7: $\psi^4 \phi D$					
$\mathcal{O}_{q^3uHD}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{q}^s \tilde{H} \overleftrightarrow{D}^\mu u_t)$	$\mathcal{O}_{qud^2HD}^{(3)}$	$(\bar{q}^p D_\mu \tilde{H} \sigma^{\mu\nu} u_r)(\bar{d}^s \gamma_\nu d_t)$	$\mathcal{O}_{qdl^2HD}^{(2)}$	$(\bar{q}^{ip} u_r)(\bar{l}^s D_\mu H \gamma^\mu l_{it})$
$\mathcal{O}_{q^3uHD}^{(2)}$	$(\bar{q}^{ip} \gamma^\mu H^\dagger q_r) \varepsilon_{ij} (\bar{q}^{js} \overleftrightarrow{D}^\mu u_t)$	$\mathcal{O}_{qud^2HD}^{(4)}$	$(\bar{q}^p D_\mu \tilde{H} \lambda^A \sigma^{\mu\nu} u_r)(\bar{d}^s \lambda^A \gamma_\nu d_t)$	$\mathcal{O}_{qdl^2HD}^{(3)}$	$(\bar{q}^p D_\mu H \sigma^{\mu\nu} d_r)(\bar{l}^s \gamma_\nu l_t)$
$\mathcal{O}_{q^3uHD}^{(3)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{q}^s D_\mu \tilde{H} u_t)$	$\mathcal{O}_{qud^2HD}^{(5)}$	$(\bar{q}^p \tilde{H} \sigma^{\mu\nu} u_r)(\bar{d}^s \gamma_\nu \overleftrightarrow{D}^\mu d_t)$	$\mathcal{O}_{qdl^2HD}^{(4)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} d_r)(\bar{l}^s D_\mu H \gamma_\nu l_{it})$
$\mathcal{O}_{q^3uHD}^{(4)}$	$(\bar{q}^{ip} \gamma^\mu D_\mu H^\dagger q_r) \varepsilon_{ij} (\bar{q}^{js} u_t)$	$\mathcal{O}_{qud^2HD}^{(6)}$	$(\bar{q}^p \tilde{H} \lambda^A \sigma^{\mu\nu} u_r)(\bar{d}^s \lambda^A \gamma_\nu \overleftrightarrow{D}^\mu d_t)$	$\mathcal{O}_{qdl^2HD}^{(5)}$	$(\bar{q}^p H \sigma^{\mu\nu} \overleftrightarrow{D}^\mu d_r)(\bar{l}^s \gamma_\nu l_t)$
$\mathcal{O}_{q^3uHD}^{(5)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r)(\bar{q}^s D_\mu \tilde{H} \lambda^A u_t)$	$\mathcal{O}_{qd^3HD}^{(1)}$	$(\bar{q}^p D_\mu H d_r)(\bar{d}^s \gamma^\mu d_t)$	$\mathcal{O}_{qdl^2HD}^{(6)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} \overleftrightarrow{D}^\mu d_r)(\bar{l}^s H \gamma_\nu l_{it})$
$\mathcal{O}_{q^3uHD}^{(6)}$	$(\bar{q}^{ip} \lambda^A \gamma^\mu D_\mu H^\dagger q_r) \varepsilon_{ij} (\bar{q}^{js} \lambda^A u_t)$	$\mathcal{O}_{qd^3HD}^{(2)}$	$(\bar{q}^p D_\mu H \lambda^A d_r)(\bar{d}^s \gamma^\mu \lambda^A d_t)$	$\mathcal{O}_{qde^2HD}^{(1)}$	$(\bar{q}^p D_\mu H d_r)(\bar{e}^s \gamma^\mu e_t)$
$\mathcal{O}_{q^3dHD}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{q}^s \overleftrightarrow{H} \overleftrightarrow{D}^\mu d_t)$	$\mathcal{O}_{qd^3HD}^{(3)}$	$(\bar{q}^p \overleftrightarrow{H} \overleftrightarrow{D}^\mu d_r)(\bar{d}^s \gamma^\mu d_t)$	$\mathcal{O}_{qde^2HD}^{(2)}$	$(\bar{q}^p D_\mu H \sigma^{\mu\nu} d_r)(\bar{e}^s \gamma_\nu e_t)$
$\mathcal{O}_{q^3dHD}^{(2)}$	$(\bar{q}^p H \gamma^\mu q_{ir})(\bar{q}^{is} \overleftrightarrow{D}^\mu d_t)$	$\mathcal{O}_{q^2leHD}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{l}^s D_\mu H e_t)$	$\mathcal{O}_{qde^2HD}^{(3)}$	$(\bar{q}^p H \sigma^{\mu\nu} \overleftrightarrow{D}^\mu d_r)(\bar{e}^s \gamma_\nu e_t)$
$\mathcal{O}_{q^3dHD}^{(3)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{q}^s D_\mu H d_t)$	$\mathcal{O}_{q^2leHD}^{(2)}$	$(\bar{q}^p D^\mu H \gamma^\mu q_{ir})(\bar{l}^s e_t)$	$\mathcal{O}_{u^2leHD}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{l}^s D_\mu H e_t)$
$\mathcal{O}_{q^3dHD}^{(4)}$	$(\bar{q}^p D_\mu H \gamma^\mu q_{ir})(\bar{q}^{is} d_t)$	$\mathcal{O}_{q^2leHD}^{(3)}$	$(\bar{q}^p \gamma_\nu q_r)(\bar{l}^s D_\mu H \sigma^{\mu\nu} e_t)$	$\mathcal{O}_{u^2leHD}^{(2)}$	$(\bar{u}^p \gamma_\nu u_r)(\bar{l}^s D_\mu H \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{q^3dHD}^{(5)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r)(\bar{q}^s D_\mu H \lambda^A d_t)$	$\mathcal{O}_{q^2leHD}^{(4)}$	$(\bar{q}^p D_\mu H \gamma_\nu q_{ir})(\bar{l}^s \sigma^{\mu\nu} e_t)$	$\mathcal{O}_{u^2leHD}^{(3)}$	$(\bar{u}^p \gamma_\nu \overleftrightarrow{D}^\mu u_r)(\bar{l}^s H \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{q^3dHD}^{(6)}$	$(\bar{q}^p D_\mu H \lambda^A \gamma^\mu q_{ir})(\bar{q}^{is} \lambda^A d_t)$	$\mathcal{O}_{q^2leHD}^{(5)}$	$(\bar{q}^p \gamma_\nu \overleftrightarrow{D}^\mu q_r)(\bar{l}^s H \sigma^{\mu\nu} e_t)$	$\mathcal{O}_{udleHD}^{(1)}$	$(\bar{d}^p \gamma^\mu u_r)(\bar{l}^s D_\mu \tilde{H} e_t)$
$\mathcal{O}_{qu^3HD}^{(1)}$	$(\bar{q}^p \tilde{H} \overleftrightarrow{D}^\mu u_r)(\bar{u}^s \gamma^\mu u_t)$	$\mathcal{O}_{q^2leHD}^{(6)}$	$(\bar{q}^p H \gamma_\nu \overleftrightarrow{D}^\mu q_{ir})(\bar{l}^s \sigma^{\mu\nu} e_t)$	$\mathcal{O}_{udleHD}^{(2)}$	$(\bar{d}^p \gamma_\mu u_r)(\bar{l}^s D_\mu \tilde{H} \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{qu^3HD}^{(2)}$	$(\bar{q}^p \tilde{H} \lambda^A \overleftrightarrow{D}^\mu u_r)(\bar{u}^s \lambda^A \gamma^\mu u_t)$	$\mathcal{O}_{qu^2HD}^{(1)}$	$(\bar{q}^p D_\mu \tilde{H} u_r)(\bar{l}^s \gamma^\mu l_t)$	$\mathcal{O}_{udleHD}^{(3)}$	$(\bar{d}^p \gamma_\mu \overleftrightarrow{D}^\nu u_r)(\bar{l}^s \tilde{H} \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{qu^3HD}^{(3)}$	$(\bar{q}^p D_\mu \tilde{H} u_r)(\bar{u}^s \gamma^\mu u_t)$	$\mathcal{O}_{qu^2HD}^{(2)}$	$(\bar{q}^{ip} u_r) \varepsilon_{ij} (\bar{l}^s \gamma^\mu D_\mu H^\dagger l_t)$	$\mathcal{O}_{d^2leHD}^{(1)}$	$(\bar{d}^p \gamma^\mu d_r)(\bar{l}^s D_\mu H e_t)$
$\mathcal{O}_{qu^2dHD}^{(1)}$	$(\bar{q}^p D_\mu H d_r)(\bar{u}^s \gamma^\mu u_t)$	$\mathcal{O}_{qu^2HD}^{(3)}$	$(\bar{q}^p D_\mu \tilde{H} \sigma^{\mu\nu} u_r)(\bar{l}^s \gamma_\nu l_t)$	$\mathcal{O}_{d^2leHD}^{(2)}$	$(\bar{d}^p \gamma_\nu d_r)(\bar{l}^s D_\mu H \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{qu^2dHD}^{(2)}$	$(\bar{q}^p D_\mu H \lambda^A d_r)(\bar{u}^s \lambda^A \gamma^\mu u_t)$	$\mathcal{O}_{qu^2HD}^{(4)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} u_r) \varepsilon_{ij} (\bar{l}^s \gamma_\nu D_\mu H^\dagger l_t)$	$\mathcal{O}_{d^2leHD}^{(3)}$	$(\bar{d}^p \gamma_\nu \overleftrightarrow{D}^\mu d_r)(\bar{l}^s H \sigma^{\mu\nu} e_t)$

(Table continued)

(Continued)

7: $\psi^4 \phi D$					
$\mathcal{O}_{qu^2 dHD}^{(3)}$	$(\bar{q}^p D_\mu H \sigma^{\mu\nu} d_r)(\bar{u}^s \gamma_\nu u_t)$	$\mathcal{O}_{qu^2 dHD}^{(5)}$	$(\bar{q}^p \tilde{H} \sigma^{\mu\nu} \tilde{D}_\mu u_r)(\bar{l}^s \gamma_\nu l_t)$	$\mathcal{O}_{\tilde{F}eHD}^{(1)}$	$(\bar{l}^p \gamma^\mu l_r)(\bar{s} D_\mu H e_t)$
$\mathcal{O}_{qu^2 dHD}^{(4)}$	$(\bar{q}^p D_\mu H \lambda^A \sigma^{\mu\nu} d_r)(\bar{u}^s \lambda^A \gamma_\nu u_t)$	$\mathcal{O}_{qu^2 dHD}^{(6)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} \tilde{D}_\mu u_r) \varepsilon_{ij} (\bar{l}^j s \gamma_\nu H^\dagger l_t)$	$\mathcal{O}_{\tilde{F}eHD}^{(2)}$	$(\bar{l}^p D_\mu H \gamma^\mu l_r)(\bar{l}^s e_t)$
$\mathcal{O}_{qu^2 dHD}^{(5)}$	$(\bar{q}^p H \sigma^{\mu\nu} d_r)(\bar{u}^s \gamma_\mu \tilde{D}_\nu u_t)$	$\mathcal{O}_{qu^2 dHD}^{(1)}$	$(\bar{q}^p D_\mu \tilde{H} u_r)(\bar{e}^s \gamma^\mu e_t)$	$\mathcal{O}_{\tilde{F}eHD}^{(3)}$	$(\bar{l}^p \gamma_\nu \tilde{D}_\mu l_r)(\bar{l}^s H \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{qu^2 dHD}^{(6)}$	$(\bar{q}^p H \lambda^A \sigma^{\mu\nu} d_r)(\bar{u}^s \lambda^A \gamma_\mu \tilde{D}_\nu u_t)$	$\mathcal{O}_{qu^2 dHD}^{(2)}$	$(\bar{q}^p D_\mu \tilde{H} \sigma^{\mu\nu} u_r)(\bar{e}^s \gamma_\nu e_t)$	$\mathcal{O}_{le^3 HD}^{(1)}$	$(\bar{l}^p D_\mu H e_r)(\bar{e}^s \gamma^\mu e_t)$
$\mathcal{O}_{qu d^2 HD}^{(1)}$	$(\bar{q}^p D_\mu \tilde{H} u_r)(\bar{d}^s \gamma^\mu d_t)$	$\mathcal{O}_{qu d^2 HD}^{(3)}$	$(\bar{q}^p \tilde{H} \sigma^{\mu\nu} \tilde{D}_\mu u_r)(\bar{e}^s \gamma_\nu e_t)$	$\mathcal{O}_{le^3 HD}^{(2)}$	$(\bar{l}^p H \sigma^{\mu\nu} \tilde{D}_\mu e_r)(\bar{e}^s \gamma_\nu e_t)$
$\mathcal{O}_{qu d^2 HD}^{(2)}$	$(\bar{q}^p D_\mu \tilde{H} \lambda^A u_r)(\bar{d}^s \lambda^A \gamma^\mu d_t)$	$\mathcal{O}_{qu d^2 HD}^{(1)}$	$(\bar{q}^p D_\mu H d_r)(\bar{l}^s \gamma^\mu l_t)$		

8: $\psi^2 X \phi^2 D$					
$\mathcal{O}_{Gq^2 H^2 D}^{(1)}$	$(\bar{q}^p G_{\mu\nu} \gamma^\nu q_r) D^\mu (H^\dagger H)$	$\mathcal{O}_{Gu^2 H^2 D}^{(1)}$	$(\bar{u}^p G_{\mu\nu} \gamma^\nu u_r) D^\mu (H^\dagger H)$	$\mathcal{O}_{Wl^2 H^2 D}^{(5)}$	$(\bar{l}^p W_{\mu\nu} \tau^I \gamma^\nu l_r) D^\mu (H^\dagger \tau^I H)$
$\mathcal{O}_{Gq^2 H^2 D}^{(2)}$	$(\bar{q}^p G_{\mu\nu} \gamma^\nu q_r) (H^\dagger \tilde{D}^\mu H)$	$\mathcal{O}_{Gu^2 H^2 D}^{(2)}$	$(\bar{u}^p G_{\mu\nu} \gamma^\nu u_r) (H^\dagger \tilde{D}^\mu H)$	$\mathcal{O}_{Wl^2 H^2 D}^{(6)}$	$(\bar{l}^p W_{\mu\nu} \tau^I \gamma^\nu l_r) (H^\dagger \tau^I \tilde{D}^\mu H)$
$\mathcal{O}_{Gq^2 H^2 D}^{(3)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \gamma^\nu q_r) D^\mu (H^\dagger H)$	$\mathcal{O}_{Gu^2 H^2 D}^{(3)}$	$(\bar{u}^p \tilde{G}_{\mu\nu} \gamma^\nu u_r) D^\mu (H^\dagger H)$	$\mathcal{O}_{Wl^2 H^2 D}^{(7)}$	$(\bar{l}^p \tilde{W}_{\mu\nu} \tau^I \gamma^\nu l_r) D^\mu (H^\dagger \tau^I H)$
$\mathcal{O}_{Gq^2 H^2 D}^{(4)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \gamma^\nu q_r) (H^\dagger \tilde{D}^\mu H)$	$\mathcal{O}_{Gu^2 H^2 D}^{(4)}$	$(\bar{u}^p \tilde{G}_{\mu\nu} \gamma^\nu u_r) (H^\dagger \tilde{D}^\mu H)$	$\mathcal{O}_{Wl^2 H^2 D}^{(8)}$	$(\bar{l}^p \tilde{W}_{\mu\nu} \tau^I \gamma^\nu l_r) (H^\dagger \tau^I \tilde{D}^\mu H)$
$\mathcal{O}_{Gq^2 H^2 D}^{(5)}$	$(\bar{q}^p G_{\mu\nu} \tau^I \gamma^\nu q_r) D^\mu (H^\dagger \tau^I H)$	$\mathcal{O}_{Wu^2 H^2 D}^{(1)}$	$(\bar{u}^p \gamma^\nu u_r) D^\mu (H^\dagger W_{\mu\nu} H)$	$\mathcal{O}_{Wl^2 H^2 D}^{(9)}$	$(\bar{l}^p \tau^I W_{\mu\nu} \gamma^\nu l_r) D^\mu (H^\dagger \tau^I H)$
$\mathcal{O}_{Gq^2 H^2 D}^{(6)}$	$(\bar{q}^p G_{\mu\nu} \tau^I \gamma^\nu q_r) (H^\dagger \tau^I \tilde{D}^\mu H)$	$\mathcal{O}_{Wu^2 H^2 D}^{(2)}$	$(\bar{u}^p \gamma^\nu u_r) (H^\dagger W_{\mu\nu} \tilde{D}^\mu H)$	$\mathcal{O}_{Wl^2 H^2 D}^{(10)}$	$(\bar{l}^p \#3 l_r) \tau^I W_{\mu\nu} \gamma^\nu (H^\dagger \tau^I \tilde{D}^\mu H)$
$\mathcal{O}_{Gq^2 H^2 D}^{(7)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \tau^I \gamma^\nu q_r) D^\mu (H^\dagger \tau^I H)$	$\mathcal{O}_{Wu^2 H^2 D}^{(3)}$	$(\bar{u}^p \gamma^\nu u_r) D^\mu (H^\dagger \tilde{W}_{\mu\nu} H)$	$\mathcal{O}_{Wl^2 H^2 D}^{(11)}$	$(\bar{l}^p \tau^I \tilde{W}_{\mu\nu} \gamma^\nu l_r) D^\mu (H^\dagger \tau^I H)$
$\mathcal{O}_{Gq^2 H^2 D}^{(8)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \tau^I \gamma^\nu q_r) (H^\dagger \tau^I \tilde{D}^\mu H)$	$\mathcal{O}_{Wu^2 H^2 D}^{(4)}$	$(\bar{u}^p \gamma^\nu u_r) (H^\dagger \tilde{W}_{\mu\nu} \tilde{D}^\mu H)$	$\mathcal{O}_{Wl^2 H^2 D}^{(12)}$	$(\bar{l}^p \tau^I \tilde{W}_{\mu\nu} \gamma^\nu l_r) (H^\dagger \tau^I \tilde{D}^\mu H)$
$\mathcal{O}_{Wq^2 H^2 D}^{(1)}$	$(\bar{q}^p \gamma^\nu q_r) D^\mu (H^\dagger W_{\mu\nu} H)$	$\mathcal{O}_{Bu^2 H^2 D}^{(1)}$	$(\bar{u}^p \gamma^\nu u_r) D^\mu (H^\dagger H) B_{\mu\nu}$	$\mathcal{O}_{Bl^2 H^2 D}^{(1)}$	$(\bar{l}^p \gamma^\nu l_r) D^\mu (H^\dagger H) B_{\mu\nu}$
$\mathcal{O}_{Wq^2 H^2 D}^{(2)}$	$(\bar{q}^p \gamma^\nu q_r) (H^\dagger W_{\mu\nu} \tilde{D}^\mu H)$	$\mathcal{O}_{Bu^2 H^2 D}^{(2)}$	$(\bar{u}^p \gamma^\nu u_r) (H^\dagger \tilde{D}^\mu H) B_{\mu\nu}$	$\mathcal{O}_{Bl^2 H^2 D}^{(2)}$	$(\bar{l}^p \gamma^\nu l_r) (H^\dagger \tilde{D}^\mu H) B_{\mu\nu}$
$\mathcal{O}_{Wq^2 H^2 D}^{(3)}$	$(\bar{q}^p \gamma^\nu q_r) D^\mu (H^\dagger \tilde{W}_{\mu\nu} H)$	$\mathcal{O}_{Bu^2 H^2 D}^{(3)}$	$(\bar{u}^p \gamma^\nu u_r) D^\mu (H^\dagger H) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bl^2 H^2 D}^{(3)}$	$(\bar{l}^p \gamma^\nu l_r) D^\mu (H^\dagger H) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Wq^2 H^2 D}^{(4)}$	$(\bar{q}^p \gamma^\nu q_r) (H^\dagger \tilde{W}_{\mu\nu} \tilde{D}^\mu H)$	$\mathcal{O}_{Bu^2 H^2 D}^{(4)}$	$(\bar{u}^p \gamma^\nu u_r) (H^\dagger \tilde{D}^\mu H) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bl^2 H^2 D}^{(4)}$	$(\bar{l}^p \gamma^\nu l_r) (H^\dagger \tilde{D}^\mu H) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Wq^2 H^2 D}^{(5)}$	$(\bar{q}^p \tau^I W_{\mu\nu} \gamma^\nu q_r) D^\mu (H^\dagger \tau^I H)$	$\mathcal{O}_{Gd^2 H^2 D}^{(1)}$	$(\bar{d}^p G_{\mu\nu} \gamma^\nu d_r) D^\mu (H^\dagger H)$	$\mathcal{O}_{Bl^2 H^2 D}^{(5)}$	$(\bar{l}^p \tau^I \gamma^\nu l_r) D^\mu (H^\dagger \tau^I H) B_{\mu\nu}$
$\mathcal{O}_{Wq^2 H^2 D}^{(6)}$	$(\bar{q}^p \tau^I W_{\mu\nu} \gamma^\nu q_r) (H^\dagger \tau^I \tilde{D}^\mu H)$	$\mathcal{O}_{Gd^2 H^2 D}^{(2)}$	$(\bar{d}^p G_{\mu\nu} \gamma^\nu d_r) (H^\dagger \tilde{D}^\mu H)$	$\mathcal{O}_{Bl^2 H^2 D}^{(6)}$	$(\bar{l}^p \tau^I \gamma^\nu l_r) (H^\dagger \tau^I \tilde{D}^\mu H) B_{\mu\nu}$
$\mathcal{O}_{Wq^2 H^2 D}^{(7)}$	$(\bar{q}^p \tau^I \tilde{W}_{\mu\nu} \gamma^\nu q_r) D^\mu (H^\dagger \tau^I H)$	$\mathcal{O}_{Gd^2 H^2 D}^{(3)}$	$(\bar{d}^p \tilde{G}_{\mu\nu} \gamma^\nu d_r) D^\mu (H^\dagger H)$	$\mathcal{O}_{Bl^2 H^2 D}^{(7)}$	$(\bar{l}^p \tau^I \gamma^\nu l_r) D^\mu (H^\dagger \tau^I H) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Wq^2 H^2 D}^{(8)}$	$(\bar{q}^p \tau^I \tilde{W}_{\mu\nu} \gamma^\nu q_r) (H^\dagger \tau^I \tilde{D}^\mu H)$	$\mathcal{O}_{Gd^2 H^2 D}^{(4)}$	$(\bar{d}^p \tilde{G}_{\mu\nu} \gamma^\nu d_r) (H^\dagger \tilde{D}^\mu H)$	$\mathcal{O}_{Bl^2 H^2 D}^{(8)}$	$(\bar{l}^p \tau^I \gamma^\nu l_r) (H^\dagger \tau^I \tilde{D}^\mu H) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Wq^2 H^2 D}^{(9)}$	$(\bar{q}^p W_{\mu\nu} \tau^I \gamma^\nu q_r) D^\mu (H^\dagger \tau^I H)$	$\mathcal{O}_{Wd^2 H^2 D}^{(1)}$	$(\bar{d}^p \gamma^\nu d_r) D^\mu (H^\dagger W_{\mu\nu} H)$	$\mathcal{O}_{We^2 H^2 D}^{(1)}$	$(\bar{e}^p \gamma^\nu e_r) D^\mu (H^\dagger W_{\mu\nu} H)$
$\mathcal{O}_{Wq^2 H^2 D}^{(10)}$	$(\bar{q}^p W_{\mu\nu} \tau^I \gamma^\nu q_r) (H^\dagger \tau^I \tilde{D}^\mu H)$	$\mathcal{O}_{Wd^2 H^2 D}^{(2)}$	$(\bar{d}^p \gamma^\nu d_r) (H^\dagger W_{\mu\nu} \tilde{D}^\mu H)$	$\mathcal{O}_{We^2 H^2 D}^{(2)}$	$(\bar{e}^p \gamma^\nu e_r) (H^\dagger W_{\mu\nu} \tilde{D}^\mu H)$
$\mathcal{O}_{Wq^2 H^2 D}^{(11)}$	$(\bar{q}^p \tilde{W}_{\mu\nu} \tau^I \gamma^\nu q_r) D^\mu (H^\dagger \tau^I H)$	$\mathcal{O}_{Wd^2 H^2 D}^{(3)}$	$(\bar{d}^p \gamma^\nu d_r) D^\mu (H^\dagger \tilde{W}_{\mu\nu} H)$	$\mathcal{O}_{We^2 H^2 D}^{(3)}$	$(\bar{e}^p \gamma^\nu e_r) D^\mu (H^\dagger \tilde{W}_{\mu\nu} H)$
$\mathcal{O}_{Wq^2 H^2 D}^{(12)}$	$(\bar{q}^p \tilde{W}_{\mu\nu} \tau^I \gamma^\nu q_r) (H^\dagger \tau^I \tilde{D}^\mu H)$	$\mathcal{O}_{Wd^2 H^2 D}^{(4)}$	$(\bar{d}^p \gamma^\nu d_r) (H^\dagger \tilde{W}_{\mu\nu} \tilde{D}^\mu H)$	$\mathcal{O}_{We^2 H^2 D}^{(4)}$	$(\bar{e}^p \gamma^\nu e_r) (H^\dagger \tilde{W}_{\mu\nu} \tilde{D}^\mu H)$
$\mathcal{O}_{Bq^2 H^2 D}^{(1)}$	$(\bar{q}^p \gamma^\nu q_r) D^\mu (H^\dagger H) B_{\mu\nu}$	$\mathcal{O}_{Bd^2 H^2 D}^{(1)}$	$(\bar{d}^p \gamma^\nu d_r) D^\mu (H^\dagger H) B_{\mu\nu}$	$\mathcal{O}_{Be^2 H^2 D}^{(1)}$	$(\bar{e}^p \gamma^\nu e_r) D^\mu (H^\dagger H) B_{\mu\nu}$
$\mathcal{O}_{Bq^2 H^2 D}^{(2)}$	$(\bar{q}^p \gamma^\nu q_r) (H^\dagger \tilde{D}^\mu H) B_{\mu\nu}$	$\mathcal{O}_{Bd^2 H^2 D}^{(2)}$	$(\bar{d}^p \gamma^\nu d_r) (H^\dagger \tilde{D}^\mu H) B_{\mu\nu}$	$\mathcal{O}_{Be^2 H^2 D}^{(2)}$	$(\bar{e}^p \gamma^\nu e_r) (H^\dagger \tilde{D}^\mu H) B_{\mu\nu}$
$\mathcal{O}_{Bq^2 H^2 D}^{(3)}$	$(\bar{q}^p \gamma^\nu q_r) D^\mu (H^\dagger H) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bd^2 H^2 D}^{(3)}$	$(\bar{d}^p \gamma^\nu d_r) D^\mu (H^\dagger H) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Be^2 H^2 D}^{(3)}$	$(\bar{e}^p \gamma^\nu e_r) D^\mu (H^\dagger H) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Bq^2 H^2 D}^{(4)}$	$(\bar{q}^p \gamma^\nu q_r) (H^\dagger \tilde{D}^\mu H) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bd^2 H^2 D}^{(4)}$	$(\bar{d}^p \gamma^\nu d_r) (H^\dagger \tilde{D}^\mu H) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Be^2 H^2 D}^{(4)}$	$(\bar{e}^p \gamma^\nu e_r) (H^\dagger \tilde{D}^\mu H) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Bq^2 H^2 D}^{(5)}$	$(\bar{q}^p \tau^I \gamma^\nu q_r) D^\mu (H^\dagger \tau^I H) B_{\mu\nu}$	$\mathcal{O}_{Wl^2 H^2 D}^{(1)}$	$(\bar{l}^p \gamma^\nu l_r) D^\mu (H^\dagger W_{\mu\nu} H)$	$\mathcal{O}_{GudH^2 D}^{(1)}$	$(\bar{u}^p G_{\mu\nu} \gamma^\nu d_r) D^\mu (\tilde{H}^\dagger H)$
$\mathcal{O}_{Bq^2 H^2 D}^{(6)}$	$(\bar{q}^p \tau^I \gamma^\nu q_r) (H^\dagger \tau^I \tilde{D}^\mu H) B_{\mu\nu}$	$\mathcal{O}_{Wl^2 H^2 D}^{(2)}$	$(\bar{l}^p \gamma^\nu l_r) (H^\dagger W_{\mu\nu} \tilde{D}^\mu H)$	$\mathcal{O}_{GudH^2 D}^{(2)}$	$(\bar{u}^p \tilde{G}_{\mu\nu} \gamma^\nu d_r) D^\mu (\tilde{H}^\dagger H)$
$\mathcal{O}_{Bq^2 H^2 D}^{(7)}$	$(\bar{q}^p \tau^I \gamma^\nu q_r) D^\mu (H^\dagger \tau^I H) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Wl^2 H^2 D}^{(3)}$	$(\bar{l}^p \gamma^\nu l_r) D^\mu (H^\dagger \tilde{W}_{\mu\nu} H)$	$\mathcal{O}_{WudH^2 D}^{(1)}$	$(\bar{u}^p \gamma^\nu d_r) D^\mu (\tilde{H}^\dagger W_{\mu\nu} H)$

(Table continued)

(Continued)

8: $\psi^2 X \phi^2 D$					
$\mathcal{O}_{Bq^2 H^2 D}^{(8)}$	$(\bar{q}^P \tau^I \gamma^\nu q_r)(H^\dagger \tau^I \overleftrightarrow{D}^\mu H) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Wl^2 H^2 D}^{(4)}$	$(\bar{l}^P \gamma^\nu l_r)(H^\dagger \tilde{W}_\mu \overleftrightarrow{D}^\mu H)$	$\mathcal{O}_{Wud H^2 D}^{(2)}$	$(\bar{u}^P \gamma^\nu d_r) D^\mu (\tilde{H}^\dagger \tilde{W}_\mu H)$
				$\mathcal{O}_{Bud H^2 D}^{(1)}$	$(\bar{u}^P \gamma^\nu d_r) D^\mu (\tilde{H}^\dagger H) B_{\mu\nu}$
				$\mathcal{O}_{Bud H^2 D}^{(2)}$	$(\bar{u}^P \gamma^\nu d_r) D^\mu (\tilde{H}^\dagger H) \tilde{B}_{\mu\nu}$

9: $\psi^2 X^2 \phi$					
$\mathcal{O}_{G^2 quH}^{(1)}$	$(\bar{q}^P \tilde{H} \{G_{\mu\nu}, G^{\mu\nu}\} u_r)$	$\mathcal{O}_{BWquH}^{(1)}$	$(\bar{q}^P W_\mu \tilde{H} u_r) B^{\mu\nu}$	$\mathcal{O}_{BGqdH}^{(1)}$	$(\bar{q}^P H G_\mu d_r) B^{\mu\nu}$
$\mathcal{O}_{G^2 quH}^{(2)}$	$(\bar{q}^P \tilde{H} u_r) \langle G_{\mu\nu} G^{\mu\nu} \rangle$	$\mathcal{O}_{BWquH}^{(2)}$	$(\bar{q}^P \tilde{W}_\mu \tilde{H} u_r) B^{\mu\nu}$	$\mathcal{O}_{BGqdH}^{(2)}$	$(\bar{q}^P H \tilde{G}_\mu d_r) B^{\mu\nu}$
$\mathcal{O}_{G^2 quH}^{(3)}$	$(\bar{q}^P \tilde{H} \{ \tilde{G}_{\mu\nu}, G^{\mu\nu} \} u_r)$	$\mathcal{O}_{BWquH}^{(3)}$	$(\bar{q}^P W_\mu^\lambda \tilde{H} \sigma^{\nu\lambda} u_r) B_{\mu\nu}$	$\mathcal{O}_{BGqdH}^{(3)}$	$(\bar{q}^P H G_\mu^\lambda \sigma^{\nu\lambda} d_r) B_{\mu\nu}$
$\mathcal{O}_{G^2 quH}^{(4)}$	$(\bar{q}^P \tilde{H} u_r) \langle \tilde{G}_{\mu\nu} G^{\mu\nu} \rangle$	$\mathcal{O}_{G^2 qdH}^{(1)}$	$(\bar{q}^P H \{G_{\mu\nu}, G^{\mu\nu}\} d_r)$	$\mathcal{O}_{BWqdH}^{(1)}$	$(\bar{q}^P W_\mu H d_r) B^{\mu\nu}$
$\mathcal{O}_{G^2 quH}^{(5)}$	$(\bar{q}^P \tilde{H} [G_{\mu\nu}, G_\lambda^\mu] \sigma^{\nu\lambda} u_r)$	$\mathcal{O}_{G^2 qdH}^{(2)}$	$(\bar{q}^P H d_r) \langle G_{\mu\nu} G^{\mu\nu} \rangle$	$\mathcal{O}_{BWqdH}^{(2)}$	$(\bar{q}^P \tilde{W}_\mu H d_r) B^{\mu\nu}$
$\mathcal{O}_{W^2 quH}^{(1)}$	$(\bar{q}^P \tilde{H} u_r) \langle W_{\mu\nu} W^{\mu\nu} \rangle$	$\mathcal{O}_{G^2 qdH}^{(3)}$	$(\bar{q}^P H \{ \tilde{G}_{\mu\nu}, G^{\mu\nu} \} d_r)$	$\mathcal{O}_{BWqdH}^{(3)}$	$(\bar{q}^P W_\mu^\lambda H \sigma^{\nu\lambda} d_r) B_{\mu\nu}$
$\mathcal{O}_{W^2 quH}^{(2)}$	$(\bar{q}^P \tilde{H} u_r) \langle \tilde{W}_{\mu\nu} W^{\mu\nu} \rangle$	$\mathcal{O}_{G^2 qdH}^{(4)}$	$(\bar{q}^P H d_r) \langle \tilde{G}_{\mu\nu} G^{\mu\nu} \rangle$	$\mathcal{O}_{G^2 leH}^{(1)}$	$(\bar{l}^P H e_r) \langle G_{\mu\nu} G^{\mu\nu} \rangle$
$\mathcal{O}_{W^2 quH}^{(3)}$	$(\bar{q}^P [W_{\mu\nu}, W_\lambda^\mu] \tilde{H} \sigma^{\nu\lambda} u_r)$	$\mathcal{O}_{G^2 qdH}^{(5)}$	$(\bar{q}^P H [G_{\mu\nu}, G_\lambda^\mu] \sigma^{\nu\lambda} d_r)$	$\mathcal{O}_{G^2 leH}^{(2)}$	$(\bar{l}^P H e_r) \langle \tilde{G}_{\mu\nu} G^{\mu\nu} \rangle$
$\mathcal{O}_{GWquH}^{(1)}$	$(\bar{q}^P W_\mu \tilde{H} G^{\mu\nu} u_r)$	$\mathcal{O}_{W^2 qdH}^{(1)}$	$(\bar{q}^P H d_r) \langle W_{\mu\nu} W^{\mu\nu} \rangle$	$\mathcal{O}_{W^2 leH}^{(1)}$	$(\bar{l}^P H e_r) \langle W_{\mu\nu} W^{\mu\nu} \rangle$
$\mathcal{O}_{GWquH}^{(2)}$	$(\bar{q}^P \tilde{W}_\mu \tilde{H} G^{\mu\nu} u_r)$	$\mathcal{O}_{W^2 qdH}^{(2)}$	$(\bar{q}^P H d_r) \langle \tilde{W}_{\mu\nu} W^{\mu\nu} \rangle$	$\mathcal{O}_{W^2 leH}^{(2)}$	$(\bar{l}^P H e_r) \langle \tilde{W}_{\mu\nu} W^{\mu\nu} \rangle$
$\mathcal{O}_{GWquH}^{(3)}$	$(\bar{q}^P W_\mu^\lambda \tilde{H} G_{\mu\nu} \sigma^{\nu\lambda} u_r)$	$\mathcal{O}_{W^2 qdH}^{(3)}$	$(\bar{q}^P [W_{\mu\nu}, W_\lambda^\mu] H \sigma^{\nu\lambda} d_r)$	$\mathcal{O}_{W^2 leH}^{(3)}$	$(\bar{l}^P [W_{\mu\nu}, W_\lambda^\mu] \sigma^{\nu\lambda} H e_r)$
$\mathcal{O}_{B^2 quH}^{(1)}$	$(\bar{q}^P \tilde{H} u_r) B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{GWqdH}^{(1)}$	$(\bar{q}^P W_\mu H G^{\mu\nu} d_r)$	$\mathcal{O}_{B^2 leH}^{(1)}$	$(\bar{l}^P H e_r) B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{B^2 quH}^{(2)}$	$(\bar{q}^P \tilde{H} u_r) \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{W^2 qdH}^{(2)}$	$(\bar{q}^P \tilde{W}_\mu H G^{\mu\nu} d_r)$	$\mathcal{O}_{B^2 leH}^{(2)}$	$(\bar{l}^P H e_r) \tilde{B}_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{BGquH}^{(1)}$	$(\bar{q}^P \tilde{H} G_{\mu\nu} u_r) B^{\mu\nu}$	$\mathcal{O}_{GWqdH}^{(3)}$	$(\bar{q}^P W_\mu^\lambda H G_{\mu\nu} \sigma^{\nu\lambda} d_r)$	$\mathcal{O}_{BWleH}^{(1)}$	$(\bar{l}^P W_{\mu\nu} H e_r) B^{\mu\nu}$
$\mathcal{O}_{BGquH}^{(2)}$	$(\bar{q}^P \tilde{H} \tilde{G}_{\mu\nu} u_r) B^{\mu\nu}$	$\mathcal{O}_{B^2 qdH}^{(1)}$	$(\bar{q}^P H d_r) B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{BWleH}^{(2)}$	$(\bar{l}^P \tilde{W}_{\mu\nu} H e_r) B^{\mu\nu}$
$\mathcal{O}_{BGquH}^{(3)}$	$(\bar{q}^P \tilde{H} G_\mu^\lambda \sigma^{\nu\lambda} u_r) B_{\mu\nu}$	$\mathcal{O}_{B^2 qdH}^{(2)}$	$(\bar{q}^P H d_r) \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{BWleH}^{(3)}$	$(\bar{l}^P W_\mu^\lambda H \sigma^{\nu\lambda} e_r) B^{\mu\nu}$

10: $\psi^2 X^2 D$					
$\mathcal{O}_{G^2 q^2 D}^{(1)}$	$(\bar{q}^P [G_\mu^\nu, G_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu q_r)$	$\mathcal{O}_{G^2 d^2 D}^{(3)}$	$(\bar{d}^P [G_\mu^\nu, \tilde{G}_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu d_r)$	$\mathcal{O}_{GBq^2 D}^{(2)}$	$(\bar{q}^P \tilde{G}_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu q_r) B^{\nu\lambda}$
$\mathcal{O}_{G^2 q^2 D}^{(2)}$	$(\bar{q}^P [\tilde{G}_\mu^\nu, G_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu q_r)$	$\mathcal{O}_{G^2 d^2 D}^{(4)}$	$(\bar{d}^P \{G_\mu^\nu, G_\lambda^\nu\} \gamma^\lambda \overleftrightarrow{D}_\mu d_r)$	$\mathcal{O}_{GBq^2 D}^{(3)}$	$(\bar{q}^P G_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu q_r) B^{\mu\lambda}$
$\mathcal{O}_{G^2 q^2 D}^{(3)}$	$(\bar{q}^P [G_\mu^\nu, \tilde{G}_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu q_r)$	$\mathcal{O}_{G^2 d^2 D}^{(5)}$	$(\bar{d}^P \gamma^\lambda \overleftrightarrow{D}_\mu d_r) \langle G_\mu^\nu G_\lambda^\nu \rangle$	$\mathcal{O}_{GBq^2 D}^{(4)}$	$(\bar{q}^P \tilde{G}_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu q_r) B^{\mu\lambda}$
$\mathcal{O}_{G^2 q^2 D}^{(4)}$	$(\bar{q}^P \{G_\mu^\nu, G_\lambda^\nu\} \gamma^\lambda \overleftrightarrow{D}_\mu q_r)$	$\mathcal{O}_{W^2 d^2 D}$	$(\bar{d}^P \gamma^\lambda \overleftrightarrow{D}_\mu d_r) \langle W_\mu^\nu W_\lambda^\nu \rangle$	$\mathcal{O}_{WBq^2 D}^{(1)}$	$(\bar{q}^P W_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu q_r) B^{\nu\lambda}$
$\mathcal{O}_{G^2 q^2 D}^{(5)}$	$(\bar{q}^P \gamma^\lambda \overleftrightarrow{D}_\mu q_r) \langle G_\mu^\nu G_\lambda^\nu \rangle$	$\mathcal{O}_{B^2 d^2 D}$	$(\bar{d}^P \gamma^\lambda \overleftrightarrow{D}_\mu d_r) B_\mu^\nu B_\lambda^\nu$	$\mathcal{O}_{WBq^2 D}^{(2)}$	$(\bar{q}^P \tilde{W}_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu q_r) B^{\nu\lambda}$
$\mathcal{O}_{W^2 q^2 D}^{(1)}$	$(\bar{q}^P [W_\mu^\nu, W_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu q_r)$	$\mathcal{O}_{G^2 l^2 D}$	$(\bar{l}^P \gamma^\lambda \overleftrightarrow{D}_\mu l_r) \langle G_\mu^\nu G_\lambda^\nu \rangle$	$\mathcal{O}_{WBq^2 D}^{(3)}$	$(\bar{q}^P W_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu q_r) B^{\mu\lambda}$
$\mathcal{O}_{W^2 q^2 D}^{(2)}$	$(\bar{q}^P [\tilde{W}_\mu^\nu, W_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu q_r)$	$\mathcal{O}_{W^2 l^2 D}^{(1)}$	$(\bar{l}^P [W_\mu^\nu, W_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu l_r)$	$\mathcal{O}_{WBq^2 D}^{(4)}$	$(\bar{q}^P \tilde{W}_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu q_r) B^{\mu\lambda}$
$\mathcal{O}_{W^2 q^2 D}^{(3)}$	$(\bar{q}^P [W_\mu^\nu, \tilde{W}_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu q_r)$	$\mathcal{O}_{W^2 l^2 D}^{(2)}$	$(\bar{l}^P [\tilde{W}_\mu^\nu, W_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu l_r)$	$\mathcal{O}_{GBu^2 D}^{(1)}$	$(\bar{u}^P G_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu u_r) B^{\nu\lambda}$
$\mathcal{O}_{W^2 q^2 D}^{(4)}$	$(\bar{q}^P \gamma^\lambda \overleftrightarrow{D}_\mu q_r) \langle W_\mu^\nu W_\lambda^\nu \rangle$	$\mathcal{O}_{W^2 l^2 D}^{(3)}$	$(\bar{l}^P [W_\mu^\nu, \tilde{W}_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu l_r)$	$\mathcal{O}_{GBu^2 D}^{(2)}$	$(\bar{u}^P \tilde{G}_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu u_r) B^{\nu\lambda}$
$\mathcal{O}_{B^2 q^2 D}$	$(\bar{q}^P \gamma^\lambda \overleftrightarrow{D}_\mu q_r) B_\mu^\nu B_\lambda^\nu$	$\mathcal{O}_{B^2 l^2 D}^{(4)}$	$(\bar{l}^P \gamma^\lambda \overleftrightarrow{D}_\mu l_r) \langle W_\mu^\nu W_\lambda^\nu \rangle$	$\mathcal{O}_{GBu^2 D}^{(3)}$	$(\bar{u}^P G_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu u_r) B^{\mu\lambda}$
$\mathcal{O}_{G^2 u^2 D}^{(1)}$	$(\bar{u}^P [G_\mu^\nu, G_\lambda^\nu] \gamma^\lambda \overleftrightarrow{D}_\mu u_r)$	$\mathcal{O}_{B^2 l^2 D}$	$(\bar{l}^P \gamma^\lambda \overleftrightarrow{D}_\mu l_r) B_\mu^\nu B_\lambda^\nu$	$\mathcal{O}_{GBu^2 D}^{(4)}$	$(\bar{u}^P \tilde{G}_\mu^\lambda \gamma_\nu \overleftrightarrow{D}_\mu u_r) B^{\mu\lambda}$

(Table continued)

(Continued)

10: $\psi^2 X^2 D$					
$\mathcal{O}_{G^2 u^2 D}^{(2)}$	$(\bar{u}^p [\tilde{G}^\mu_\nu, G^\nu_\lambda] \gamma^\lambda \vec{D}_\mu u_r)$	$\mathcal{O}_{G^2 e^2 D}$	$(\bar{e}^p \gamma^\lambda \vec{D}_\mu e_r) \langle G^\mu_\nu G^\nu_\lambda \rangle$	$\mathcal{O}_{GB d^2 D}^{(1)}$	$(\bar{d}^p G^\mu_{\lambda\gamma_\nu} \vec{D}_\mu d_r) B^{\nu\lambda}$
$\mathcal{O}_{G^2 u^2 D}^{(3)}$	$(\bar{u}^p [G^\mu_\nu, \tilde{G}^\nu_\lambda] \gamma^\lambda \vec{D}_\mu u_r)$	$\mathcal{O}_{W^2 e^2 D}$	$(\bar{e}^p \gamma^\lambda \vec{D}_\mu e_r) \langle W^\mu_\nu W^\nu_\lambda \rangle$	$\mathcal{O}_{GB d^2 D}^{(2)}$	$(\bar{d}^p \tilde{G}^\mu_{\lambda\gamma_\nu} \vec{D}_\mu d_r) B^{\nu\lambda}$
$\mathcal{O}_{G^2 u^2 D}^{(4)}$	$(\bar{u}^p \{G^\mu_\nu, G^\nu_\lambda\} \gamma^\lambda \vec{D}_\mu u_r)$	$\mathcal{O}_{B^2 e^2 D}$	$(\bar{e}^p \gamma^\lambda \vec{D}_\mu e_r) B^\mu_\nu B^\nu_\lambda$	$\mathcal{O}_{GB d^2 D}^{(3)}$	$(\bar{d}^p G^\nu_{\lambda\gamma_\nu} \vec{D}_\mu d_r) B^{\mu\lambda}$
$\mathcal{O}_{G^2 u^2 D}^{(5)}$	$(\bar{u}^p \gamma^\lambda \vec{D}_\mu u_r) \langle G^\mu_\nu G^\nu_\lambda \rangle$	$\mathcal{O}_{GW q^2 D}^{(1)}$	$(\bar{q}^p G^\mu_{\lambda\gamma_\nu} W^{\nu\lambda} \vec{D}_\mu q_r)$	$\mathcal{O}_{GB d^2 D}^{(4)}$	$(\bar{d}^p \tilde{G}^\nu_{\lambda\gamma_\nu} \vec{D}_\mu d_r) B^{\mu\lambda}$
$\mathcal{O}_{W^2 u^2 D}$	$(\bar{u}^p \gamma^\lambda \vec{D}_\mu u_r) \langle W^\mu_\nu W^\nu_\lambda \rangle$	$\mathcal{O}_{GW q^2 D}^{(2)}$	$(\bar{q}^p \tilde{G}^\mu_{\lambda\gamma_\nu} W^{\nu\lambda} \vec{D}_\mu q_r)$	$\mathcal{O}_{WBI^2 D}^{(1)}$	$(\bar{l}^p W^\mu_{\lambda\gamma_\nu} \vec{D}_\mu l_r) B^{\nu\lambda}$
$\mathcal{O}_{B^2 u^2 D}$	$(\bar{u}^p \gamma^\lambda \vec{D}_\mu u_r) B^\mu_\nu B^\nu_\lambda$	$\mathcal{O}_{GW q^2 D}^{(3)}$	$(\bar{q}^p G^\nu_{\lambda\gamma_\nu} W^{\mu\lambda} \vec{D}_\mu q_r)$	$\mathcal{O}_{WBI^2 D}^{(2)}$	$(\bar{l}^p \tilde{W}^\mu_{\lambda\gamma_\nu} \vec{D}_\mu l_r) B^{\nu\lambda}$
$\mathcal{O}_{G^2 d^2 D}^{(1)}$	$(\bar{d}^p [G^\mu_\nu, G^\nu_\lambda] \gamma^\lambda \vec{D}_\mu d_r)$	$\mathcal{O}_{GW q^2 D}^{(4)}$	$(\bar{q}^p \tilde{G}^\nu_{\lambda\gamma_\nu} W^{\mu\lambda} \vec{D}_\mu q_r)$	$\mathcal{O}_{WBI^2 D}^{(3)}$	$(\bar{l}^p W^\nu_{\lambda\gamma_\nu} \vec{D}_\mu l_r) B^{\mu\lambda}$
$\mathcal{O}_{G^2 d^2 D}^{(2)}$	$(\bar{d}^p [\tilde{G}^\mu_\nu, G^\nu_\lambda] \gamma^\lambda \vec{D}_\mu d_r)$	$\mathcal{O}_{GB q^2 D}^{(1)}$	$(\bar{q}^p G^\mu_{\lambda\gamma_\nu} \vec{D}_\mu q_r) B^{\nu\lambda}$	$\mathcal{O}_{WBI^2 D}^{(4)}$	$(\bar{l}^p \tilde{W}^\nu_{\lambda\gamma_\nu} \vec{D}_\mu l_r) B^{\mu\lambda}$

11: $\psi^4 D^2$					
$\mathcal{O}_{q^4 D^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r) D^2 (\bar{q}^s \gamma_\mu q_t)$	$\mathcal{O}_{d^4 D^2}^{(1)}$	$(\bar{d}^p \gamma^\mu d_r) D^2 (\bar{d}^s \gamma_\mu d_t)$	$\mathcal{O}_{d^2 l^2 D^2}^{(2)}$	$(\bar{d}^p \gamma^\mu \vec{D}_\nu d_r) (\bar{l}^s \gamma_\nu \vec{D}_\mu l_t)$
$\mathcal{O}_{q^4 D^2}^{(2)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r) D^2 (\bar{q}^s \lambda^A \gamma_\mu q_t)$	$\mathcal{O}_{d^4 D^2}^{(2)}$	$(\bar{d}^p \gamma^\mu \vec{D}_\nu d_r) (\bar{d}^s \gamma_\nu \vec{D}_\mu d_t)$	$\mathcal{O}_{d^2 e^2 D^2}^{(1)}$	$(\bar{d}^p \gamma^\mu d_r) D^2 (\bar{e}^s \gamma_\mu e_t)$
$\mathcal{O}_{q^4 D^2}^{(3)}$	$(\bar{q}^p \gamma^\mu \vec{D}_\nu q_r) (\bar{q}^s \gamma_\nu \vec{D}_\mu q_t)$	$\mathcal{O}_{q^2 ud D^2}^{(1)}$	$(\bar{q}^{ip} d_r) \varepsilon_{ij} D^2 (\bar{q}^{js} u_t)$	$\mathcal{O}_{d^2 e^2 D^2}^{(2)}$	$(\bar{d}^p \gamma^\mu \vec{D}_\nu d_r) (\bar{e}^s \gamma_\nu \vec{D}_\mu e_t)$
$\mathcal{O}_{q^4 D^2}^{(4)}$	$(\bar{q}^p \lambda^A \gamma^\mu \vec{D}_\nu q_r) (\bar{q}^s \lambda^A \gamma_\nu \vec{D}_\mu q_t)$	$\mathcal{O}_{q^2 ud D^2}^{(2)}$	$(\bar{q}^{ip} \lambda^A d_r) \varepsilon_{ij} D^2 (\bar{q}^{js} \lambda^A u_t)$	$\mathcal{O}_{qule D^2}^{(1)}$	$(\bar{q}^{ip} u_r) \varepsilon_{ij} D^2 (\bar{l}^{js} e_t)$
$\mathcal{O}_{q^2 u^2 D^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r) D^2 (\bar{u}^s \gamma_\mu u_t)$	$\mathcal{O}_{q^2 ud D^2}^{(3)}$	$D_\mu (\bar{q}^{ip} \vec{D}_\nu d_r) \varepsilon_{ij} (\bar{q}^{js} \sigma^{\mu\nu} u_t)$	$\mathcal{O}_{qule D^2}^{(2)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} u_r) \varepsilon_{ij} D^2 (\bar{l}^{js} \sigma_{\mu\nu} e_t)$
$\mathcal{O}_{q^2 u^2 D^2}^{(2)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r) D^2 (\bar{u}^s \lambda^A \gamma_\mu u_t)$	$\mathcal{O}_{q^2 l^2 D^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r) D^2 (\bar{l}^s \gamma_\mu l_t)$	$\mathcal{O}_{qule D^2}^{(3)}$	$D_\mu (\bar{q}^{ip} \vec{D}_\nu u_r) \varepsilon_{ij} (\bar{l}^{js} \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{q^2 u^2 D^2}^{(3)}$	$(\bar{q}^p \gamma^\mu \vec{D}_\nu q_r) (\bar{u}^s \gamma_\nu \vec{D}_\mu u_t)$	$\mathcal{O}_{q^2 l^2 D^2}^{(2)}$	$(\bar{q}^p \tau^I \gamma^\mu q_r) D^2 (\bar{l}^s \tau^I \gamma_\mu l_t)$	$\mathcal{O}_{qdl e D^2}^{(1)}$	$(\bar{q}^{ip} d_r) D^2 (\bar{e}^s l_{it})$
$\mathcal{O}_{q^2 u^2 D^2}^{(4)}$	$(\bar{q}^p \lambda^A \gamma^\mu \vec{D}_\nu q_r) (\bar{u}^s \lambda^A \gamma_\nu \vec{D}_\mu u_t)$	$\mathcal{O}_{q^2 l^2 D^2}^{(3)}$	$(\bar{q}^p \gamma^\mu \vec{D}_\nu q_r) (\bar{l}^s \gamma_\nu \vec{D}_\mu l_t)$	$\mathcal{O}_{qdl e D^2}^{(2)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} d_r) D_\mu (\bar{e}^s \vec{D}_\nu l_{it})$
$\mathcal{O}_{q^2 d^2 D^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r) D^2 (\bar{d}^s \gamma_\mu d_t)$	$\mathcal{O}_{q^2 l^2 D^2}^{(4)}$	$(\bar{q}^p \tau^I \gamma^\mu \vec{D}_\nu q_r) (\bar{l}^s \tau^I \gamma_\nu \vec{D}_\mu l_t)$	$\mathcal{O}_{l^4 D^2}^{(1)}$	$(\bar{l}^p \gamma^\mu l_r) D^2 (\bar{l}^s \gamma_\mu l_t)$
$\mathcal{O}_{q^2 d^2 D^2}^{(2)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r) D^2 (\bar{d}^s \lambda^A \gamma_\mu d_t)$	$\mathcal{O}_{q^2 e^2 D^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r) D^2 (\bar{e}^s \gamma_\mu e_t)$	$\mathcal{O}_{l^4 D^2}^{(2)}$	$(\bar{l}^p \gamma^\mu \vec{D}_\nu l_r) (\bar{l}^s \gamma_\nu \vec{D}_\mu l_t)$
$\mathcal{O}_{q^2 d^2 D^2}^{(3)}$	$(\bar{q}^p \gamma^\mu \vec{D}_\nu q_r) (\bar{d}^s \gamma_\nu \vec{D}_\mu d_t)$	$\mathcal{O}_{q^2 e^2 D^2}^{(2)}$	$(\bar{q}^p \gamma^\mu \vec{D}_\nu q_r) (\bar{e}^s \gamma_\nu \vec{D}_\mu e_t)$	$\mathcal{O}_{l^2 e^2 D^2}^{(1)}$	$(\bar{l}^p \gamma^\mu l_r) D^2 (\bar{e}^s \gamma_\mu e_t)$
$\mathcal{O}_{q^2 d^2 D^2}^{(4)}$	$(\bar{q}^p \lambda^A \gamma^\mu \vec{D}_\nu q_r) (\bar{d}^s \lambda^A \gamma_\nu \vec{D}_\mu d_t)$	$\mathcal{O}_{u^2 l^2 D^2}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r) D^2 (\bar{l}^s \gamma_\mu l_t)$	$\mathcal{O}_{l^2 e^2 D^2}^{(2)}$	$(\bar{l}^p \gamma^\mu \vec{D}_\nu l_r) (\bar{e}^s \gamma_\nu \vec{D}_\mu e_t)$
$\mathcal{O}_{u^4 D^2}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r) D^2 (\bar{u}^s \gamma_\mu u_t)$	$\mathcal{O}_{u^2 l^2 D^2}^{(2)}$	$(\bar{u}^p \gamma^\mu \vec{D}_\nu u_r) (\bar{l}^s \gamma_\nu \vec{D}_\mu l_t)$	$\mathcal{O}_{e^4 D^2}^{(1)}$	$(\bar{e}^p \gamma^\mu e_r) D^2 (\bar{e}^s \gamma_\mu e_t)$
$\mathcal{O}_{u^4 D^2}^{(2)}$	$(\bar{u}^p \gamma^\mu \vec{D}_\nu u_r) (\bar{u}^s \gamma_\nu \vec{D}_\mu u_t)$	$\mathcal{O}_{u^2 e^2 D^2}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r) D^2 (\bar{e}^s \gamma_\mu e_t)$	$\mathcal{O}_{e^4 D^2}^{(2)}$	$(\bar{e}^p \gamma^\mu \vec{D}_\nu e_r) (\bar{e}^s \gamma_\nu \vec{D}_\mu e_t)$
$\mathcal{O}_{u^2 d^2 D^2}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r) D^2 (\bar{d}^s \gamma_\mu d_t)$	$\mathcal{O}_{u^2 e^2 D^2}^{(2)}$	$(\bar{u}^p \gamma^\mu \vec{D}_\nu u_r) (\bar{e}^s \gamma_\nu \vec{D}_\mu e_t)$		
$\mathcal{O}_{u^2 d^2 D^2}^{(2)}$	$(\bar{u}^p \lambda^A \gamma^\mu u_r) D^2 (\bar{d}^s \lambda^A \gamma_\mu d_t)$	$\mathcal{O}_{d^2 l^2 D^2}^{(1)}$	$(\bar{d}^p \gamma^\mu d_r) D^2 (\bar{l}^s \gamma_\mu l_t)$		
$\mathcal{O}_{u^2 d^2 D^2}^{(3)}$	$(\bar{u}^p \gamma^\mu \vec{D}_\nu u_r) (\bar{d}^s \gamma_\nu \vec{D}_\mu d_t)$				
$\mathcal{O}_{u^2 d^2 D^2}^{(4)}$	$(\bar{u}^p \lambda^A \gamma^\mu \vec{D}_\nu u_r) (\bar{d}^s \lambda^A \gamma_\nu \vec{D}_\mu d_t)$				

12: $\psi^4 \phi^2$					
$\mathcal{O}_{q^4 H^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r) (\bar{q}^s H \gamma_\mu H^\dagger q_t)$	$\mathcal{O}_{q^2 d^2 H^2}^{(2)}$	$(\bar{q}^p H \lambda^A d_r) (\bar{q}^s H \lambda^A d_t)$	$\mathcal{O}_{d^2 l^2 H^2}^{(2)}$	$(\bar{d}^p \gamma^\mu d_r) (\bar{l}^s \gamma_\mu l_t) (H^\dagger H)$
$\mathcal{O}_{q^4 H^2}^{(2)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r) (\bar{q}^s H \lambda^A \gamma_\mu H^\dagger q_t)$	$\mathcal{O}_{q^2 ud H^2}^{(1)}$	$(\bar{q}^p \tilde{H} u_r) (\bar{q}^s H d_t)$	$\mathcal{O}_{d^2 e^2 H^2}$	$(\bar{d}^p \gamma^\mu d_r) (\bar{e}^s \gamma_\mu e_t) (H^\dagger H)$
$\mathcal{O}_{q^4 H^2}^{(3)}$	$(\bar{q}^p \gamma^\mu q_r) (\bar{q}^s \gamma_\mu q_t) (H^\dagger H)$	$\mathcal{O}_{q^2 ud H^2}^{(2)}$	$(\bar{q}^p \lambda^A \tilde{H} u_r) (\bar{q}^s \lambda^A H d_t)$	$\mathcal{O}_{qule H^2}^{(1)}$	$(\bar{q}^{ip} u_r) \varepsilon_{ij} (\bar{l}^{js} e_t) (H^\dagger H)$
$\mathcal{O}_{q^2 u^2 H^2}^{(1)}$	$(\bar{q}^p \tau^I \gamma^\mu q_r) (\bar{u}^s H^\dagger \gamma_\mu H u_t)$	$\mathcal{O}_{q^2 ud H^2}^{(3)}$	$(\bar{q}^{ip} u_r) \varepsilon_{ij} (\bar{q}^{js} d_t) (H^\dagger H)$	$\mathcal{O}_{qule H^2}^{(2)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} u_r) \varepsilon_{ij} (\bar{l}^{js} \sigma_{\mu\nu} e_t) (H^\dagger H)$

(Table continued)

(Continued)

12: $\psi^A \phi^2$					
$\mathcal{O}_{q^2 u^2 H^2}^{(2)}$	$(\bar{q}^p \lambda^A \tau^l \gamma^\mu q_r)(\bar{u}^s H^\dagger \lambda^A \gamma_\mu H u_t)$	$\mathcal{O}_{q^2 udH^2}^{(4)}$	$(\bar{q}^{ip} \lambda^A u_r) \varepsilon_{ij} (\bar{q}^{js} \lambda^A d_t) (H^\dagger H)$	$\mathcal{O}_{quleH^2}^{(3)}$	$(\bar{q}^p \tilde{H} u_r)(\bar{l}^s H e_t)$
$\mathcal{O}_{q^2 u^2 H^2}^{(3)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{u}^s \gamma_\mu u_t)(H^\dagger H)$	$\mathcal{O}_{q^2 udH^2}^{(1)}$	$(\bar{q}^p \tilde{H} u_r)(\bar{d}^s H^\dagger q_t)$	$\mathcal{O}_{quleH^2}^{(4)}$	$(\bar{q}^p \tilde{H} \sigma^{\mu\nu} u_r)(\bar{l}^s H \sigma_{\mu\nu} e_t)$
$\mathcal{O}_{q^2 u^2 H^2}^{(4)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r)(\bar{u}^s \lambda^A \gamma_\mu u_t)(H^\dagger H)$	$\mathcal{O}_{q^2 udH^2}^{(2)}$	$(\bar{q}^p \tilde{H} \lambda^A u_r)(\bar{d}^s H^\dagger \lambda^A q_t)$	$\mathcal{O}'_{quleH^2}$	$(\bar{q}^p \tilde{H} u_r)(\bar{e}^s H^\dagger l_t)$
$\mathcal{O}_{q^2 d^2 H^2}^{(1)}$	$(\bar{q}^p \tau^l \gamma^\mu q_r)(\bar{d}^s H^\dagger \gamma_\mu H d_t)$	$\mathcal{O}_{q^2 l^2 H^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{l}^s H \gamma_\mu H^\dagger l_t)$	$\mathcal{O}_{qdleH^2}^{(1)}$	$(\bar{q}^p H d_r)(\bar{e}^s H^\dagger l_t)$
$\mathcal{O}_{q^2 d^2 H^2}^{(2)}$	$(\bar{q}^p \lambda^A \tau^l \gamma^\mu q_r)(\bar{d}^s H^\dagger \lambda^A \gamma_\mu H d_t)$	$\mathcal{O}_{q^2 l^2 H^2}^{(2)}$	$(\bar{q}^{ip} \gamma^\mu H^\dagger q_r)(\bar{l}^s H \gamma_\mu l_{it})$	$\mathcal{O}_{qdleH^2}^{(2)}$	$(\bar{q}^{ip} d_r)(\bar{e}^s l_{it})(H^\dagger H)$
$\mathcal{O}_{q^2 d^2 H^2}^{(3)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{d}^s \gamma_\mu d_t)(H^\dagger H)$	$\mathcal{O}_{q^2 l^2 H^2}^{(3)}$	$(\bar{q}^p H \gamma^\mu q_{ir})(\bar{l}^{is} \gamma_\mu H^\dagger l_t)$	$\mathcal{O}'_{qdleH^2}^{(1)}$	$(\bar{q}^p H d_r)(\bar{l}^s H e_t)$
$\mathcal{O}_{q^2 d^2 H^2}^{(4)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r)(\bar{d}^s \lambda^A \gamma_\mu d_t)(H^\dagger H)$	$\mathcal{O}_{q^2 l^2 H^2}^{(4)}$	$(\bar{q}^p \tau^l \gamma^\mu q_r)(\bar{l}^s \tau^l \gamma_\mu l_t)(H^\dagger H)$	$\mathcal{O}_{qdleH^2}^{(2)}$	$(\bar{q}^p H \sigma^{\mu\nu} d_r)(\bar{l}^s H \sigma_{\mu\nu} e_t)$
$\mathcal{O}_{u^4 H^2}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{u}^s \gamma_\mu u_t)(H^\dagger H)$	$\mathcal{O}_{q^2 l^2 H^2}^{(5)}$	$(\bar{q}^p H \gamma^\mu H^\dagger q_r)(\bar{l}^s \gamma_\mu l_t)$	$\mathcal{O}'_{l^2 udH^2}$	$(\bar{l}^p H \gamma^\mu \tilde{H}^\dagger l_r)(\bar{u}^s \gamma_\mu d_t)$
$\mathcal{O}_{u^2 d^2 H^2}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{d}^s \gamma_\mu d_t)(H^\dagger H)$	$\mathcal{O}_{q^2 e^2 H^2}^{(1)}$	$(\bar{q}^p H \gamma^\mu H^\dagger q_r)(\bar{e}^s \gamma_\mu e_t)$	$\mathcal{O}_{l^4 H^2}^{(1)}$	$(\bar{l}^p H \gamma^\mu H^\dagger l_r)(\bar{l}^s \gamma_\mu l_t)$
$\mathcal{O}_{u^2 d^2 H^2}^{(2)}$	$(\bar{u}^p \lambda^A \gamma^\mu u_r)(\bar{d}^s \lambda^A \gamma_\mu d_t)(H^\dagger H)$	$\mathcal{O}_{q^2 e^2 H^2}^{(2)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{e}^s \gamma_\mu e_t)(H^\dagger H)$	$\mathcal{O}_{l^4 H^2}^{(2)}$	$(\bar{l}^p \gamma^\mu l_r)(\bar{l}^s \gamma_\mu l_t)(H^\dagger H)$
$\mathcal{O}_{d^4 H^2}$	$(\bar{d}^p \gamma^\mu d_r)(\bar{d}^s \gamma_\mu d_t)(H^\dagger H)$	$\mathcal{O}_{u^2 l^2 H^2}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{l}^s H \gamma_\mu H^\dagger l_t)$	$\mathcal{O}_{l^2 e^2 H^2}^{(1)}$	$(\bar{l}^p H \gamma^\mu H^\dagger l_r)(\bar{e}^s \gamma_\mu e_t)$
$\mathcal{O}_{q^2 u^2 H^2}^{(1)}$	$(\bar{q}^p \tilde{H} u_r)(\bar{q}^s \tilde{H} u_t)$	$\mathcal{O}_{u^2 l^2 H^2}^{(2)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{l}^s \gamma_\mu l_t)(H^\dagger H)$	$\mathcal{O}_{l^2 e^2 H^2}^{(2)}$	$(\bar{l}^p \gamma^\mu l_r)(\bar{e}^s \gamma_\mu e_t)(H^\dagger H)$
$\mathcal{O}_{q^2 u^2 H^2}^{(2)}$	$(\bar{q}^p \tilde{H} \lambda^A u_r)(\bar{q}^s \tilde{H} \lambda^A u_t)$	$\mathcal{O}_{u^2 e^2 H^2}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{e}^s \gamma_\mu e_t)(H^\dagger H)$	$\mathcal{O}_{e^4 H^2}$	$(\bar{e}^p \gamma^\mu e_r)(\bar{e}^s \gamma_\mu e_t)(H^\dagger H)$
$\mathcal{O}_{q^2 d^2 H^2}^{(1)}$	$(\bar{q}^p H d_r)(\bar{q}^s H d_t)$	$\mathcal{O}_{d^2 l^2 H^2}^{(1)}$	$(\bar{d}^p \gamma^\mu d_r)(\bar{l}^s H \gamma_\mu H^\dagger l_t)$	$\mathcal{O}'_{l^2 e^2 H^2}$	$(\bar{l}^p H e_r)(\bar{l}^s H e_t)$

13: $\psi^4 X$					
$\mathcal{O}_{Gq^4}^{(1)}$	$(\bar{q}^p G_{\mu\nu} \gamma^\mu q_r)(\bar{q}^s \gamma^\nu q_t)$	$\mathcal{O}_{Wq^2 d^2}^{(1)}$	$(\bar{q}^p W_{\mu\nu} \gamma^\mu q_r)(\bar{d}^s \gamma^\nu d_t)$	$\mathcal{O}_{Wq^2 ud}^{(3)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} u_r)(W_{\mu\nu})^k_{ij} \varepsilon_{jk} (\bar{q}^{js} d_t)$
$\mathcal{O}_{Gq^4}^{(2)}$	$(\bar{q}^p G_{\mu\nu} \tau^l \gamma^\mu q_r)(\bar{q}^s \tau^l \gamma^\nu q_t)$	$\mathcal{O}_{Wq^2 d^2}^{(2)}$	$(\bar{q}^p \lambda^A W_{\mu\nu} \gamma^\mu q_r)(\bar{d}^s \lambda^A \gamma^\nu d_t)$	$\mathcal{O}_{Bq^2 ud}^{(1)}$	$(\bar{q}^{ip} u_r) \varepsilon_{ij} (\bar{q}^{js} \sigma^{\mu\nu} d_t) B_{\mu\nu}$
$\mathcal{O}_{Gq^4}^{(3)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \gamma^\mu q_r)(\bar{q}^s \gamma^\nu q_t)$	$\mathcal{O}_{Wq^2 d^2}^{(3)}$	$(\bar{q}^p \tilde{W}_{\mu\nu} \gamma^\mu q_r)(\bar{d}^s \gamma^\nu d_t)$	$\mathcal{O}_{Bq^2 ud}^{(2)}$	$(\bar{q}^{ip} \lambda^A u_r) \varepsilon_{ij} (\bar{q}^{js} \lambda^A \sigma^{\mu\nu} d_t) B_{\mu\nu}$
$\mathcal{O}_{Gq^4}^{(4)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \tau^l \gamma^\mu q_r)(\bar{q}^s \tau^l \gamma^\nu q_t)$	$\mathcal{O}_{Wq^2 d^2}^{(4)}$	$(\bar{q}^p \lambda^A \tilde{W}_{\mu\nu} \gamma^\mu q_r)(\bar{d}^s \lambda^A \gamma^\nu d_t)$	$\mathcal{O}_{Bq^2 ud}^{(3)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} u_r) \varepsilon_{ij} (\bar{q}^{js} d_t) B_{\mu\nu}$
$\mathcal{O}_{Wq^4}^{(1)}$	$(\bar{q}^p W_{\mu\nu} \gamma^\mu q_r)(\bar{q}^s \gamma^\nu q_t)$	$\mathcal{O}_{Bq^2 d^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{d}^s \gamma^\nu d_t) B_{\mu\nu}$	$\mathcal{O}_{Gq^2 l^2}^{(1)}$	$(\bar{q}^p G_{\mu\nu} \gamma^\nu q_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{Wq^4}^{(2)}$	$(\bar{q}^p \lambda^A W_{\mu\nu} \gamma^\mu q_r)(\bar{q}^s \lambda^A \gamma^\nu q_t)$	$\mathcal{O}_{Bq^2 d^2}^{(2)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r)(\bar{d}^s \lambda^A \gamma^\nu d_t) B_{\mu\nu}$	$\mathcal{O}_{Gq^2 l^2}^{(2)}$	$(\bar{q}^p G_{\mu\nu} \tau^l \gamma^\nu q_r)(\bar{l}^s \tau^l \gamma^\mu l_t)$
$\mathcal{O}_{Wq^4}^{(3)}$	$(\bar{q}^p \tilde{W}_{\mu\nu} \gamma^\mu q_r)(\bar{q}^s \gamma^\nu q_t)$	$\mathcal{O}_{Bq^2 d^2}^{(3)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{d}^s \gamma^\nu d_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Gq^2 l^2}^{(3)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \gamma^\nu q_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{Wq^4}^{(4)}$	$(\bar{q}^p \lambda^A \tilde{W}_{\mu\nu} \gamma^\mu q_r)(\bar{q}^s \lambda^A \gamma^\nu q_t)$	$\mathcal{O}_{Bq^2 d^2}^{(4)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r)(\bar{d}^s \lambda^A \gamma^\nu d_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Gq^2 l^2}^{(4)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \tau^l \gamma^\nu q_r)(\bar{l}^s \tau^l \gamma^\mu l_t)$
$\mathcal{O}_{Bq^4}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{q}^s \gamma^\nu q_t) B_{\mu\nu}$	$\mathcal{O}_{Gu^4}^{(1)}$	$(\bar{u}^p G_{\mu\nu} \gamma^\mu u_r)(\bar{u}^s \gamma^\nu u_t)$	$\mathcal{O}_{Wq^2 l^2}^{(1)}$	$(\bar{q}^p W_{\mu\nu} \gamma^\nu q_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{Bq^4}^{(2)}$	$(\bar{q}^p \tau^l \gamma^\mu q_r)(\bar{q}^s \tau^l \gamma^\nu q_t) B_{\mu\nu}$	$\mathcal{O}_{Gu^4}^{(2)}$	$(\bar{u}^p \tilde{G}_{\mu\nu} \gamma^\mu u_r)(\bar{u}^s \gamma^\nu u_t)$	$\mathcal{O}_{Wq^2 l^2}^{(2)}$	$(\bar{q}^p \tau^l \gamma^\nu q_r)(\bar{l}^s W_{\mu\nu} \tau^l \gamma^\mu l_t)$
$\mathcal{O}_{Bq^4}^{(3)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{q}^s \gamma^\nu q_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bu^4}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{u}^s \gamma^\nu u_t) B_{\mu\nu}$	$\mathcal{O}_{Wq^2 l^2}^{(3)}$	$(\bar{q}^p \gamma^\nu q_r)(\bar{l}^s W_{\mu\nu} \gamma^\mu l_t)$
$\mathcal{O}_{Bq^4}^{(4)}$	$(\bar{q}^p \tau^l \gamma^\mu q_r)(\bar{q}^s \tau^l \gamma^\nu q_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bu^4}^{(2)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{u}^s \gamma^\nu u_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Wq^2 l^2}^{(4)}$	$(\bar{q}^p \tilde{W}_{\mu\nu} \gamma^\nu q_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{Gq^2 u^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{u}^s G_{\mu\nu} \gamma^\nu u_t)$	$\mathcal{O}_{Gd^4}^{(1)}$	$(\bar{d}^p G_{\mu\nu} \gamma^\mu d_r)(\bar{d}^s \gamma^\nu d_t)$	$\mathcal{O}_{Wq^2 l^2}^{(5)}$	$(\bar{q}^p \tau^l \gamma^\nu q_r)(\bar{l}^s \tilde{W}_{\mu\nu} \tau^l \gamma^\mu l_t)$
$\mathcal{O}_{Gq^2 u^2}^{(2)}$	$(\bar{q}^p G_{\mu\nu} \lambda^A \gamma^\mu q_r)(\bar{u}^s \lambda^A \gamma^\nu u_t)$	$\mathcal{O}_{Gd^4}^{(2)}$	$(\bar{d}^p \tilde{G}_{\mu\nu} \gamma^\mu d_r)(\bar{d}^s \gamma^\nu d_t)$	$\mathcal{O}_{Wq^2 l^2}^{(6)}$	$(\bar{q}^p \gamma^\nu q_r)(\bar{l}^s \tilde{W}_{\mu\nu} \gamma^\mu l_t)$
$\mathcal{O}_{Gq^2 u^2}^{(3)}$	$(\bar{q}^p \lambda^A G_{\mu\nu} \gamma^\mu q_r)(\bar{u}^s \lambda^A \gamma^\nu u_t)$	$\mathcal{O}_{Bd^4}^{(1)}$	$(\bar{d}^p \gamma^\mu d_r)(\bar{d}^s \gamma^\nu d_t) B_{\mu\nu}$	$\mathcal{O}_{Bq^2 l^2}^{(1)}$	$(\bar{q}^p \gamma^\nu q_r)(\bar{l}^s \gamma^\mu l_t) B_{\mu\nu}$
$\mathcal{O}_{Gq^2 u^2}^{(4)}$	$(\bar{q}^p G_{\mu\nu} \gamma^\mu q_r)(\bar{u}^s \gamma^\nu u_t)$	$\mathcal{O}_{Bd^4}^{(2)}$	$(\bar{d}^p \gamma^\mu d_r)(\bar{d}^s \gamma^\nu d_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bq^2 l^2}^{(2)}$	$(\bar{q}^p \tau^l \gamma^\nu q_r)(\bar{l}^s \tau^l \gamma^\mu l_t) B_{\mu\nu}$
$\mathcal{O}_{Gq^2 u^2}^{(5)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{u}^s \tilde{G}_{\mu\nu} \gamma^\nu u_t)$	$\mathcal{O}_{Gu^2 d^2}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{d}^s G_{\mu\nu} \gamma^\nu d_t)$	$\mathcal{O}_{Bq^2 l^2}^{(3)}$	$(\bar{q}^p \gamma^\nu q_r)(\bar{l}^s \gamma^\mu l_t) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Gq^2 u^2}^{(6)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \lambda^A \gamma^\mu q_r)(\bar{u}^s \lambda^A \gamma^\nu u_t)$	$\mathcal{O}_{Gu^2 d^2}^{(2)}$	$(\bar{u}^p G_{\mu\nu} \lambda^A \gamma^\mu u_r)(\bar{d}^s \lambda^A \gamma^\nu d_t)$	$\mathcal{O}_{Bq^2 l^2}^{(4)}$	$(\bar{q}^p \tau^l \gamma^\nu q_r)(\bar{l}^s \tau^l \gamma^\mu l_t) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Gq^2 u^2}^{(7)}$	$(\bar{q}^p \lambda^A \tilde{G}_{\mu\nu} \gamma^\mu q_r)(\bar{u}^s \lambda^A \gamma^\nu u_t)$	$\mathcal{O}_{Gu^2 d^2}^{(3)}$	$(\bar{u}^p \lambda^A G_{\mu\nu} \gamma^\mu u_r)(\bar{d}^s \lambda^A \gamma^\nu d_t)$	$\mathcal{O}_{Gq^2 e^2}^{(1)}$	$(\bar{q}^p G_{\mu\nu} \gamma^\mu q_r)(\bar{e}^s \gamma^\nu e_t)$

(Table continued)

(Continued)

13: $\psi^4 X$					
$\mathcal{O}_{Gq^2 u^2}^{(8)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \gamma^\mu q_r)(\bar{u}^s \gamma^\nu u_t)$	$\mathcal{O}_{Gu^2 d^2}^{(4)}$	$(\bar{u}^p G_{\mu\nu} \gamma^\mu u_r)(\bar{d}^s \gamma^\nu d_t)$	$\mathcal{O}_{Gq^2 e^2}^{(2)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \gamma^\mu q_r)(\bar{e}^s \gamma^\nu e_t)$
$\mathcal{O}_{Wq^2 u^2}^{(1)}$	$(\bar{q}^p W_{\mu\nu} \gamma^\mu q_r)(\bar{u}^s \gamma^\nu u_t)$	$\mathcal{O}_{Gu^2 d^2}^{(5)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{d}^s \tilde{G}_{\mu\nu} \gamma^\nu d_t)$	$\mathcal{O}_{Wq^2 e^2}^{(1)}$	$(\bar{q}^p W_{\mu\nu} \gamma^\mu q_r)(\bar{e}^s \gamma^\nu e_t)$
$\mathcal{O}_{Wq^2 u^2}^{(2)}$	$(\bar{q}^p \lambda^A W_{\mu\nu} \gamma^\mu q_r)(\bar{u}^s \lambda^A \gamma^\nu u_t)$	$\mathcal{O}_{Gu^2 d^2}^{(6)}$	$(\bar{u}^p \tilde{G}_{\mu\nu} \lambda^A \gamma^\mu u_r)(\bar{d}^s \lambda^A \gamma^\nu d_t)$	$\mathcal{O}_{Wq^2 e^2}^{(2)}$	$(\bar{q}^p \tilde{W}_{\mu\nu} \gamma^\mu q_r)(\bar{e}^s \gamma^\nu e_t)$
$\mathcal{O}_{Wq^2 u^2}^{(3)}$	$(\bar{q}^p \tilde{W}_{\mu\nu} \gamma^\mu q_r)(\bar{u}^s \gamma^\nu u_t)$	$\mathcal{O}_{Gu^2 d^2}^{(7)}$	$(\bar{u}^p \lambda^A \tilde{G}_{\mu\nu} \gamma^\mu u_r)(\bar{d}^s \lambda^A \gamma^\nu d_t)$	$\mathcal{O}_{Bq^2 e^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{e}^s \gamma^\nu e_t) B_{\mu\nu}$
$\mathcal{O}_{Wq^2 u^2}^{(4)}$	$(\bar{q}^p \lambda^A \tilde{W}_{\mu\nu} \gamma^\mu q_r)(\bar{u}^s \lambda^A \gamma^\nu u_t)$	$\mathcal{O}_{Gu^2 d^2}^{(8)}$	$(\bar{u}^p \tilde{G}_{\mu\nu} \gamma^\mu u_r)(\bar{d}^s \gamma^\nu d_t)$	$\mathcal{O}_{Bq^2 e^2}^{(2)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{e}^s \gamma^\nu e_t) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Bq^2 u^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{u}^s \gamma^\nu u_t) B_{\mu\nu}$	$\mathcal{O}_{Bu^2 d^2}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{d}^s \gamma^\nu d_t) B_{\mu\nu}$	$\mathcal{O}_{Gu^2 l^2}^{(1)}$	$(\bar{u}^p G_{\mu\nu} \gamma^\nu u_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{Bq^2 u^2}^{(2)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r)(\bar{u}^s \lambda^A \gamma^\nu u_t) B_{\mu\nu}$	$\mathcal{O}_{Bu^2 d^2}^{(2)}$	$(\bar{u}^p \lambda^A \gamma^\mu u_r)(\bar{d}^s \lambda^A \gamma^\nu d_t) B_{\mu\nu}$	$\mathcal{O}_{Gu^2 l^2}^{(2)}$	$(\bar{u}^p \tilde{G}_{\mu\nu} \gamma^\nu u_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{Bq^2 u^2}^{(3)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{u}^s \gamma^\nu u_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bu^2 d^2}^{(3)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{d}^s \gamma^\nu d_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Wu^2 l^2}^{(1)}$	$(\bar{u}^p \gamma^\nu u_r)(\bar{l}^s W_{\mu\nu} \gamma^\mu l_t)$
$\mathcal{O}_{Bq^2 u^2}^{(4)}$	$(\bar{q}^p \lambda^A \gamma^\mu q_r)(\bar{u}^s \lambda^A \gamma^\nu u_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bu^2 d^2}^{(4)}$	$(\bar{u}^p \lambda^A \gamma^\mu u_r)(\bar{d}^s \lambda^A \gamma^\nu d_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Wu^2 l^2}^{(2)}$	$(\bar{u}^p \gamma^\nu u_r)(\bar{l}^s \tilde{W}_{\mu\nu} \gamma^\mu l_t)$
$\mathcal{O}_{Gq^2 d^2}^{(1)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{d}^s G_{\mu\nu} \gamma^\nu d_t)$	$\mathcal{O}_{Gq^2 ud}^{(1)}$	$(\bar{q}^p G_{\mu\nu} u_r)(\bar{q}^s \sigma^{\mu\nu} d_t)$	$\mathcal{O}_{Bu^2 l^2}^{(1)}$	$(\bar{u}^p \gamma^\nu u_r)(\bar{l}^s \gamma^\mu l_t) B_{\mu\nu}$
$\mathcal{O}_{Gq^2 d^2}^{(2)}$	$(\bar{q}^p G_{\mu\nu} \lambda^A \gamma^\mu q_r)(\bar{d}^s \lambda^A \gamma^\nu d_t)$	$\mathcal{O}_{Gq^2 ud}^{(2)}$	$(\bar{q}^p G_{\mu\nu} \lambda^A u_r)(\bar{q}^s \lambda^A \sigma^{\mu\nu} d_t)$	$\mathcal{O}_{Bu^2 l^2}^{(2)}$	$(\bar{u}^p \gamma^\nu u_r)(\bar{l}^s \gamma^\mu l_t) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Gq^2 d^2}^{(3)}$	$(\bar{q}^p \lambda^A G_{\mu\nu} \gamma^\mu q_r)(\bar{d}^s \lambda^A \gamma^\nu d_t)$	$\mathcal{O}_{Gq^2 ud}^{(3)}$	$(\bar{q}^p \lambda^A G_{\mu\nu} u_r)(\bar{q}^s \lambda^A \sigma^{\mu\nu} d_t)$	$\mathcal{O}_{Gu^2 e^2}^{(1)}$	$(\bar{u}^p G_{\mu\nu} \gamma^\mu u_r)(\bar{e}^s \gamma^\nu e_t)$
$\mathcal{O}_{Gq^2 d^2}^{(4)}$	$(\bar{q}^p G_{\mu\nu} \gamma^\mu q_r)(\bar{d}^s \gamma^\nu d_t)$	$\mathcal{O}_{Gq^2 ud}^{(4)}$	$(\bar{q}^p u_r)(\bar{q}^s G_{\mu\nu} \sigma^{\mu\nu} d_t)$	$\mathcal{O}_{Gu^2 e^2}^{(2)}$	$(\bar{u}^p \tilde{G}_{\mu\nu} \gamma^\mu u_r)(\bar{e}^s \gamma^\nu e_t)$
$\mathcal{O}_{Gq^2 d^2}^{(5)}$	$(\bar{q}^p \gamma^\mu q_r)(\bar{d}^s \tilde{G}_{\mu\nu} \gamma^\nu d_t)$	$\mathcal{O}_{Gq^2 ud}^{(5)}$	$(\bar{q}^p \sigma^{\mu\nu} u_r)(\bar{q}^s G_{\mu\nu} d_t)$	$\mathcal{O}_{Bu^2 e^2}^{(1)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{e}^s \gamma^\nu e_t) B_{\mu\nu}$
$\mathcal{O}_{Gq^2 d^2}^{(6)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \lambda^A \gamma^\mu q_r)(\bar{d}^s \lambda^A \gamma^\nu d_t)$	$\mathcal{O}_{Gq^2 ud}^{(6)}$	$(\bar{q}^p G_{\mu\nu} \lambda^A \sigma^{\mu\nu} u_r)(\bar{q}^s \lambda^A d_t)$	$\mathcal{O}_{Bu^2 e^2}^{(2)}$	$(\bar{u}^p \gamma^\mu u_r)(\bar{e}^s \gamma^\nu e_t) \tilde{B}_{\mu\nu}$
$\mathcal{O}_{Gq^2 d^2}^{(7)}$	$(\bar{q}^p \lambda^A \tilde{G}_{\mu\nu} \gamma^\mu q_r)(\bar{d}^s \lambda^A \gamma^\nu d_t)$	$\mathcal{O}_{Wq^2 ud}^{(1)}$	$(\bar{q}^{ip} u_r)(W_{\mu\nu})_i^k \varepsilon_{jk} (\bar{q}^{js} \sigma^{\mu\nu} d_t)$	$\mathcal{O}_{Gd^2 l^2}^{(1)}$	$(\bar{d}^p G_{\mu\nu} \gamma^\nu d_r)(\bar{l}^s \gamma^\mu l_t)$
$\mathcal{O}_{Gq^2 d^2}^{(8)}$	$(\bar{q}^p \tilde{G}_{\mu\nu} \gamma^\mu q_r)(\bar{d}^s \gamma^\nu d_t)$	$\mathcal{O}_{Wq^2 ud}^{(2)}$	$(\bar{q}^{ip} \lambda^A u_r)(W_{\mu\nu})_i^k \varepsilon_{jk} (\bar{q}^{js} \lambda^A \sigma^{\mu\nu} d_t)$	$\mathcal{O}_{Gd^2 l^2}^{(2)}$	$(\bar{d}^p \tilde{G}_{\mu\nu} \gamma^\nu d_r)(\bar{l}^s \gamma^\mu l_t)$

13: $\psi^4 X$			
$\mathcal{O}_{Wd^2 l^2}^{(1)}$	$(\bar{d}^p \gamma^\nu d_r)(\bar{l}^s W_{\mu\nu} \gamma^\mu l_t)$	$\mathcal{O}_{Gqule}^{(1)}$	$(\bar{q}^{ip} G_{\mu\nu} \sigma^{\mu\nu} u_r) \varepsilon_{ij} (\bar{l}^{js} e_t)$
$\mathcal{O}_{Wd^2 l^2}^{(2)}$	$(\bar{d}^p \gamma^\nu d_r)(\bar{l}^s \tilde{W}_{\mu\nu} \gamma^\mu l_t)$	$\mathcal{O}_{Gqule}^{(2)}$	$(\bar{q}^{ip} G_{\mu\nu} u_r) \varepsilon_{ij} (\bar{l}^{js} \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{Bd^2 l^2}^{(1)}$	$(\bar{d}^p \gamma^\nu d_r)(\bar{l}^s \gamma^\mu l_t) B_{\mu\nu}$	$\mathcal{O}_{Gqule}^{(3)}$	$(\bar{q}^{ip} G^{\mu\nu} \sigma_{\nu}{}^\rho u_r) \varepsilon_{ij} (\bar{l}^{js} \sigma_{\mu\rho} e_t)$
$\mathcal{O}_{Bd^2 l^2}^{(2)}$	$(\bar{d}^p \gamma^\nu d_r)(\bar{l}^s \gamma^\mu l_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Wqule}^{(1)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} u_r) \varepsilon_{jk} (W_{\mu\nu})_i^k (\bar{l}^{js} e_t)$
$\mathcal{O}_{Gd^2 e^2}^{(1)}$	$(\bar{d}^p G_{\mu\nu} \gamma^\mu d_r)(\bar{e}^s \gamma^\nu e_t)$	$\mathcal{O}_{Wqule}^{(2)}$	$(\bar{q}^{ip} u_r) \varepsilon_{jk} (W_{\mu\nu})_i^k (\bar{l}^{js} \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{Gd^2 e^2}^{(2)}$	$(\bar{d}^p \tilde{G}_{\mu\nu} \gamma^\mu d_r)(\bar{e}^s \gamma^\nu e_t)$	$\mathcal{O}_{Wqule}^{(3)}$	$(\bar{q}^{ip} \sigma_{\nu}{}^\rho u_r) \varepsilon_{jk} (W_{\mu\nu})_i^k (\bar{l}^{js} \sigma^{\mu\rho} e_t)$
$\mathcal{O}_{Bd^2 e^2}^{(1)}$	$(\bar{d}^p \gamma^\mu d_r)(\bar{e}^s \gamma^\nu e_t) B_{\mu\nu}$	$\mathcal{O}_{Bqule}^{(1)}$	$(\bar{q}^{ip} \sigma^{\mu\nu} u_r) \varepsilon_{ij} (\bar{l}^{js} e_t) B_{\mu\nu}$
$\mathcal{O}_{Bd^2 e^2}^{(2)}$	$(\bar{d}^p \gamma^\mu d_r)(\bar{e}^s \gamma^\nu e_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bqule}^{(2)}$	$(\bar{q}^{ip} u_r) \varepsilon_{ij} (\bar{l}^{js} \sigma^{\mu\nu} e_t) B_{\mu\nu}$
$\mathcal{O}_{Wl^4}^{(1)}$	$(\bar{l}^p W_{\mu\nu} \gamma^\mu l_r)(\bar{l}^s \gamma^\nu l_t)$	$\mathcal{O}_{Bqule}^{(3)}$	$(\bar{q}^{ip} \sigma_{\nu}{}^\rho u_r) \varepsilon_{ij} (\bar{l}^{js} \sigma_{\mu\rho} e_t) B^{\mu\nu}$
$\mathcal{O}_{Wl^4}^{(2)}$	$(\bar{l}^p \tilde{W}_{\mu\nu} \gamma^\mu l_r)(\bar{l}^s \gamma^\nu l_t)$	$\mathcal{O}_{Gqdle}^{(1)}$	$(\bar{d}^p G_{\mu\nu} \sigma^{\mu\nu} q_{ir})(\bar{l}^{js} e_t)$
$\mathcal{O}_{Bl^4}^{(1)}$	$(\bar{l}^p \gamma^\mu l_r)(\bar{l}^s \gamma^\nu l_t) B_{\mu\nu}$	$\mathcal{O}_{Gqdle}^{(2)}$	$(\bar{d}^p G_{\mu\nu} q_{ir})(\bar{l}^{js} \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{Bl^4}^{(2)}$	$(\bar{l}^p \gamma^\mu l_r)(\bar{l}^s \gamma^\nu l_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Gqdle}^{(1)}$	$(\bar{d}^p \sigma^{\mu\nu} q_{ir})(W_{\mu\nu})_j^i (\bar{l}^{js} e_t)$
$\mathcal{O}_{Be^4}^{(1)}$	$(\bar{e}^p \gamma^\mu e_r)(\bar{e}^s \gamma^\nu e_t) B_{\mu\nu}$	$\mathcal{O}_{Gqdle}^{(2)}$	$(\bar{d}^p q_{ir})(W_{\mu\nu})_j^i (\bar{l}^{js} \sigma^{\mu\nu} e_t)$
$\mathcal{O}_{Be^4}^{(2)}$	$(\bar{e}^p \gamma^\mu e_r)(\bar{e}^s \gamma^\nu e_t) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{Bqdle}^{(1)}$	$(\bar{d}^p \sigma^{\mu\nu} q_{ir})(\bar{l}^{js} e_t) B_{\mu\nu}$
$\mathcal{O}_{Wl^2 e^2}^{(1)}$	$(\bar{l}^p W_{\mu\nu} \gamma^\mu l_r)(\bar{e}^s \gamma^\nu e_t)$	$\mathcal{O}_{Bqdle}^{(2)}$	$(\bar{d}^p q_{ir})(\bar{l}^{js} \sigma^{\mu\nu} e_t) B_{\mu\nu}$
$\mathcal{O}_{Wl^2 e^2}^{(2)}$	$(\bar{l}^p \tilde{W}_{\mu\nu} \gamma^\mu l_r)(\bar{e}^s \gamma^\nu e_t)$		
$\mathcal{O}_{Bl^2 e^2}^{(1)}$	$(\bar{l}^p \gamma^\mu l_r)(\bar{e}^s \gamma^\nu e_t) B_{\mu\nu}$		
$\mathcal{O}_{Bl^2 e^2}^{(2)}$	$(\bar{l}^p \gamma^\mu l_r)(\bar{e}^s \gamma^\nu e_t) \tilde{B}_{\mu\nu}$		

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