

Valence quark distribution of light ρ and heavy J/ψ vector mesons in a light-cone quark model

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In this work, we have investigated the valence quark structure through chiral-even vector quark generalized parton distributions (GPDs) of light ρ and heavy J/ψ vector mesons using the light-cone quark model. By solving the quark-quark correlator, we have represented the quark GPDs in the light-front wave function (LFWFs) form. The charge, magnetic, and quadrupole form factors of these particles have been derived from the vector quark GPDs and compared with available theoretical predictions and lattice simulation data. The structure functions corresponding to the Rosenbluth scattering cross section of these mesons have also been calculated and compared with available NJL model predictions. All the results of our calculations are found to follow a similar trend to other model results. We have also calculated the parton distribution functions (PDFs) of these particles in the forward limit of GPDs. The calculated PDFs have also been evolved to 5 GeV² through next to next to leading order Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolutions.

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I. INTRODUCTION

Vector mesons such as the ρ and J/ψ play an important role in advancing our understanding of the Standard Model, offering a versatile testing ground for both theoretical frameworks and experimental techniques. Owing to their differing mass scales and spin-1 character, they enable the exploration of polarization-sensitive effects and tensor structures that are not present in scalar or spin-1/2 systems. As such, these mesons provide a valuable context for assessing the applicability of effective models in describing hadronic phenomena across a wide range of energy scales.

At the low energy scale $\mu^2 \leq 1 \text{ GeV}^2$, the valence quark contributes more compared to gluons than at the high energy scale $\mu^2 \geq 1 \text{ GeV}^2$ due to the confinement. So, it is interesting to explore the valence quark behavior within the low energy quark models like Nambu-Jona-Lasinio model (NJL) [1–3], light-front quark models (LFQM) [4–6], light-front holographic model (LFHM) [7,8], spectator model [9–12], quark-diquark model [13,14], MIT-bag

model [15,16], contact interaction [17,18], lattice simulations [19–21], etc. Using these models, one can understand the internal structure of a hadron through five-dimensional generalized transverse momentum distributions (GTMDs) [22–25], three-dimensional generalized parton distribution functions (GPDs) [26–29], transverse momentum parton distribution functions (TMDs) [30–33], one-dimensional parton distribution functions (PDFs) [34–36], etc. One major issue in hadron physics has been and still is the lack of accurate information about the shapes of the above distribution functions generated from the first principle of quantum chromodynamics (QCD). The transverse and spatial structure of the valence quark inside a hadron can be studied using the TMDs and GPDs. TMDs carry information about the transverse structure of the hadron in the form of transverse ($\mathbf{k}_\perp$) and longitudinal (x) momentum of the constituents with no information about the spatial structure, which can be accessed through GPDs as a functions of momentum transferred between final and initial state ($\Delta_\perp$), longitudinal momentum (x) and skewness parameter (ξ). Elastic form factors (Eform factors), orbital angular momentum, charge radii, pressure distributions, shear distributions, gravitational form factors, single spin asymmetry, etc of hadrons can be accessed through the GPDs [26,37,38]. GPDs can be extracted directly from deeply virtual Compton scattering (DVCS) [39,40] and deeply virtual meson production (DVMP) [41]. TMDs and GPDs are the extended form of one-dimensional PDFs, which carry information about the longitudinal momentum fraction x carried by the constituent from the hadrons. PDFs

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can be extracted experimentally from the deep inelastic scattering process (DIS) [42]. For this work, we have considered light and heavy vector mesons for our calculations. Compared to spin-1/2 nucleons and spin-0 mesons, the vector mesons have higher spin degrees of freedom and more polarizations. These polarizations occur due to the transverse and longitudinal polarizations of spin projections, resulting in the formation of tensor structures of spin-1 mesons.

In this work, we have calculated the chiral-even quark GPDs of spin-1 vector mesons within the light-cone quark model (LCQM) [43–45]. There are a total of eighteen quark GPDs [46,47] (nine are chiral-even and nine are chiral-odd) present at the leading twist compared to eight of spin-1/2 nucleons and two of spin-0 mesons [48]. Out of nine chiral-even quark GPDs, five are unpolarized (vector) and four are polarized (axial vector) quark GPDs for spin-1 vector mesons. In this work, we have constrained our work to the chiral-even vector quark GPDs only. However, the chiral-even axial vector quark GPDs have been studied in NJL model [49] and LFQM [50], previously. These vector quark GPDs have been calculated at zero skewness from the quark-quark correlator and presented in the overlap form of light-front wave functions (LFWFs). Our approach constructs GPDs directly from LFWFs, and therefore gauge invariance is not explicitly ensured. Consequently, the normalization is model-dependent. A fully gauge-invariant treatment would require starting from the Compton amplitude, where normalization arises naturally, as discussed for the pion in the NJL model in Ref. [51]. It is established that the LFWF is Lorentz-invariant and can be represented in terms of the constituents' momentum fractions, which are unaffected by the overall hadron momentum. LFWFs are therefore essential to our knowledge of the hadron structure and the light-front quantization approach to QCD. Thus, it is quite advantageous to learn how to derive LFWFs from a fundamental standpoint. For the numerical calculations, we have considered both light ρ and heavy J/ψ vector mesons for the completeness of our model. Out of five $H_i(x, 0, -\Delta_\perp^2)$ ($i = 1, \dots, 5$) vector quark GPDs, $H_4(x, 0, -\Delta_\perp^2)$ is coming out to be zero. With the remaining vector quark GPDs, the H_1 gives rise to the $F_1(x)$ structure function of the DIS process, while H_5 carries information about the polarized $b_1(x)$ structure function of spin-1 mesons. The vector quark GPDs of spin-1 mesons have been studied in different theoretical models in Refs. [49,52–56], while till now there are no lattice simulations and experimental results available. There are a total of three form factors present at the leading twist vector quark GPDs case, which are charge (G_c), magnetic (G_M), and quadrupole (G_Q) form factors for vector mesons [57]. These form factors encode information about the charge radii, magnetic moment, and quadrupole moment of the mesons. These form factors can be used to calculate the structure functions corresponding to the Rosenbluth cross

section [58]. There have been a wide range of theoretical studies have been reported for these form factors [49,52–55,55,56,59–64] and few lattice simulation studies [65,66], however a lack of experimental studies for both particles. While comparing our results with other models' predictions, we observed that all are in good agreement with them. For completeness of our calculations, we have also predicted the quark PDFs for both particles. In case of spin-1 mesons, there are a total of four PDFs present at the leading twist compared to one for spin-0 mesons and three for spin-1/2 nucleons [67]. These are unpolarized $f_1(x)$, transversity $h_1(x)$, helicity $g_1(x)$, and tensor f_{1LL} quark PDFs. The unpolarized $f_1(x)$ and tensor polarized $f_{1LL}(x)$ quark PDFs can be derived from H_1 and H_5 chiral-even vector quark GPDs at the forward limit $\Delta_\perp = 0$. However, the f_{1LL} PDF is found to be zero for our case. For numerical calculations, we have adopted the LCQM. In this framework, we have adopted the Brodsky-Huang-Leepage (BHL) prescription for the momentum space wave function [68] and spin wave functions by Melosh-Wigner rotation [44,69]. This study may be even useful to understand the processes involving the ρ -meson lepton production [48,70–72] or the process of $\gamma\gamma^* \rightarrow \rho\rho$ [73] and the future Electron-Ion Collider (EIC) experiments [74,75].

The paper is arranged as follows. In Sec. II, we have discussed the LCQM along with the spin and momentum wave function in LF formalism. The derivation of vector quark GPDs from quark-quark correlator has been presented in Sec. III. In this section, we have also derived the different form factors along with structure functions. In Sec. IV, we have derived the quark parton distribution functions for spin-1 vector mesons. We have finally summarized our work in Sec. V.

II. LIGHT-CONE QUARK MODEL

Understanding hadrons as relativistic bound states of partons is a central goal of modern particle physics. The light-cone (LC) formalism provides an elegant approach to this problem, expressing the hadron's wave function as a sum over Fock-states defined by the momentum and spin of its constituents. The hadron wave function based on LC quantization QCD using the multiparticle Fock-state expansion is expressed as [43–45]

$$|M_h(P, S_z)\rangle = \sum_{n, \lambda_j} \int \prod_{j=1}^n \frac{dx_j d^2\mathbf{k}_{\perp j}}{\sqrt{x_j} 16\pi^3} 16\pi^3 \\ \times \delta\left(1 - \sum_{j=1}^n x_j\right) \delta^{(2)}\left(\sum_{j=1}^n \mathbf{k}_{\perp j}\right) \psi_{n/M} \\ \times (x_j, \mathbf{k}_{\perp j}, \lambda_j) |n; \mathbf{k}_j^\perp, \mathbf{k}_{\perp j}, \lambda_j\rangle.$$

Here $|M_h(P, S_z)\rangle$ is the hadron eigenstate with $P = (P^+, P^-, P_\perp)$ and S_z being the hadron's total average momentum and longitudinal spin projection, respectively.

n is the number of constituents inside the hadron. λ_j is the helicity of the j th constituent of the hadron. $\mathbf{k}_j = (\mathbf{k}_j^+, \mathbf{k}_j^-, \mathbf{k}_{j\perp})$ is the momentum of the j th constituent of the hadron. $x_j = P^+/\mathbf{k}_j^+$ is the longitudinal momentum fraction carried by the j th constituent from the hadron. Both the longitudinal momentum fraction and transverse momenta of the constituents of the hadron obey the momentum sum rule

$$\sum_{j=1}^n \mathbf{k}_j = 0, \sum_{j=1}^n x_j = 1. \quad (1)$$

As we are dealing with mesons in this work, so the meson Fock-state can be written in terms of quarks, gluons, and sea-quark degrees of freedom as [68,76,77]

$$|M\rangle = \sum |q\bar{q}\rangle \psi_{q\bar{q}} + \sum |q\bar{q}g\rangle \psi_{q\bar{q}g} + \sum |q\bar{q}gg\rangle \psi_{q\bar{q}gg} + |q\bar{q}(q\bar{q})_{sea}\rangle \psi_{q\bar{q}(q\bar{q})_{sea}} + \dots, \quad (2)$$

where $|M\rangle$ denotes the meson eigenstate. The higher Fock-state contribution is very less compared to $|q\bar{q}\rangle$ leading state for the case of heavy mesons [78]. So for this work, we have not considered the higher Fock-state contribution. The two-particle meson minimal Fock-state can be written in terms of the intrinsic momenta of the quark and antiquark from Eq. (1) as

$$|M(P, S_z)\rangle = \sum_{\lambda_1, \lambda_2} \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)} 2(2\pi)^3} \times \Psi_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) |x, \mathbf{k}_\perp, \lambda_1, \lambda_2\rangle. \quad (3)$$

Here, $\lambda_{1(2)}$ are the helicities of quark (antiquark). $\Psi_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)$ is the LC meson wave function with different spin (S_z) and quark (antiquark) helicity (λ) projections. x and $1-x$ are the momentum fractions carried by the active quark and antiquark, respectively. The LC meson wave function can be expressed as

$$\Psi_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = \mathcal{S}_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) \psi_{q\bar{q}}(x, \mathbf{k}_\perp). \quad (4)$$

Here $\mathcal{S}_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)$ and $\psi_{q\bar{q}}(x, \mathbf{k}_\perp)$ are the spin and momentum space wave functions of the mesons respectively. For the momentum space wave function in Eq. (4), we have adopted the Brodsky-Huang-Lepage (BHL), which is expressed as [44,79]

$$\psi_{q\bar{q}}(x, \mathbf{k}_\perp) = A \exp \left[-\frac{\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}{8\beta^2} \right], \quad (5)$$

here, m_q and $m_{\bar{q}}$ are the quark and antiquark masses. β and A are the harmonic scale parameter and normalization constant of the meson. The quark masses ($m_{q(\bar{q})}$) and β are calculated by fitting with the mass spectra and decay constant of the respective mesons. For this work, we have

adopted the quark mass $m_{u(\bar{d})} = 0.2$ GeV and $m_{c\bar{c}} = 1.68$ GeV for light $\rho(u\bar{d})$ [80] and heavy $J/\psi(c\bar{c})$ vector mesons [33], respectively. The β parameter is taken to be 0.410 and 0.699 for $\rho(u\bar{d})$ and $J/\psi(c\bar{c})$ vector mesons, respectively [33,80]. While the normalization constant is calculated by normalizing the momentum space wave function as

$$\int \frac{dx d^2\mathbf{k}_\perp}{2(2\pi)^3} |\psi_{q\bar{q}}(x, \mathbf{k}_\perp)|^2 = 1. \quad (6)$$

The four-vector momenta of the constituent quark (k_1) and antiquark (k_2) in the LC frame are respectively defined as

$$k_1 \equiv \left(xP^+, \frac{\mathbf{k}_\perp^2 + m_q^2}{xP^+}, \mathbf{k}_\perp \right), \quad (7)$$

$$k_2 \equiv \left((1-x)P^+, \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{(1-x)P^+}, -\mathbf{k}_\perp \right), \quad (8)$$

with $M_{q\bar{q}}$ being the invariant mass of the composite meson system defined in terms of its quark mass m_q and antiquark mass $m_{\bar{q}}$ as

$$M_{q\bar{q}}^2 = \frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x}. \quad (9)$$

$\mathcal{S}_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)$ in Eq. (5) is front-form spin wave function derived from the instant form by Melosh-Wigner rotation [44,79,81]. This transformation of instant form state $\Phi(T)$ and front form state $\Phi(F)$ is expressed as

$$\Phi_i^\uparrow(T) = -\frac{[\mathbf{k}_i^R \Phi_i^\downarrow(F) - (\mathbf{k}_i^+ + m_{q(\bar{q})}) \Phi_i^\uparrow(F)]}{\omega_{i(q\bar{q})}}, \quad (10)$$

$$\Phi_i^\downarrow(T) = \frac{[\mathbf{k}_i^L \Phi_i^\uparrow(F) + (\mathbf{k}_i^+ + m_{q(\bar{q})}) \Phi_i^\downarrow(F)]}{\omega_{i(q\bar{q})}}. \quad (11)$$

Here $\Phi(F)$ is a two-component Dirac spinor and $\mathbf{k}_i^{R(L)} = \mathbf{k}_i^1 \pm i\mathbf{k}_i^2$. $\omega_{i(q\bar{q})}$ is defined as $\omega_{i(q\bar{q})} = 1/\sqrt{2\mathbf{k}_i^+ (\mathbf{k}^0 + m_{q(\bar{q})})}$. Now applying different momenta forms from Eqs. (7) and (8) in the Melosh-Wigner rotation, the spin wave function is obtained in the form $\kappa_{S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)$ coefficient as

$$\mathcal{S}_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = \sum_{\lambda_1, \lambda_2} \kappa_{S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) \Phi_1^{\lambda_1}(F) \Phi_2^{\lambda_2}(F). \quad (12)$$

These spin-wave function coefficients satisfy the following normalization relation

$$\sum_{\lambda_1, \lambda_2} \kappa_{S_z}^{F*}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) \kappa_{S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = 1. \quad (13)$$

Similarly, the same spin-wave function can be calculated using the proper vertex chosen for the vector meson [44,69] as

$$\mathcal{S}_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = \bar{u}(k_1, \lambda_1) \left[-\frac{1}{\sqrt{2}\sqrt{M_{q\bar{q}}^2 - (m_q - m_{\bar{q}})^2}} \left(\gamma^\mu - \frac{k_1^\mu - k_2^\mu}{M_{q\bar{q}} + m_q + m_{\bar{q}}} \right) \epsilon(P, S_z) \right] v(k_2, \lambda_2). \quad (14)$$

Here, u and v are the Dirac spinors. In this work, we have only considered the light $\rho(u\bar{d})$ vector and heavy $J/\psi(c\bar{c})$ vector mesons, so $m_q = m_{\bar{q}}$ for this case. $\epsilon(P, S_z)$ is the polarization vector of the vector mesons, which can be expressed for both transverse ($S_z = \pm 1$) and longitudinal polarization ($S_z = 0$) as

$$\begin{aligned} \epsilon(P, S_z = \pm 1) &= (\epsilon_{\pm 1}^+, \epsilon_{\pm 1}^-, \epsilon_{\pm 1}^\perp) = \left(0, \mp \frac{\sqrt{2}P^{R,L}}{P^+}, \mp \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}} \right), \\ \epsilon(P, S_z = 0) &= (\epsilon_0^+, \epsilon_0^-, \epsilon_0^\perp) = \frac{1}{M_{q\bar{q}}} \left(P^+, \frac{P_\perp^2 - M_{q\bar{q}}^2}{P^+}, P_\perp \right). \end{aligned} \quad (15)$$

Here, $P^{R,L} = P_1 \pm iP_2$. Both the above methods give rise to the same form of spin wave function. The spin wave function for vector mesons for different spin projections/polarization and different helicities of quark antiquark [$\lambda = (\uparrow, \downarrow)$] is expressed as, [44] for $S_z = +1$

$$\begin{aligned} \mathcal{S}_{(S_z=+1)}(x, \mathbf{k}_\perp, \uparrow, \uparrow) &= \frac{1}{\omega_{q\bar{q}}} [\mathbf{k}_\perp^2 + (M_{q\bar{q}} + m_q + m_{\bar{q}})((1-x)m_q + xm_{\bar{q}})] \\ \mathcal{S}_{(S_z=+1)}(x, \mathbf{k}_\perp, \uparrow, \downarrow) &= \frac{1}{\omega_{q\bar{q}}} [(\mathbf{k}_1 + i\mathbf{k}_2)(xM_{q\bar{q}} + m_q)] \\ \mathcal{S}_{(S_z=+1)}(x, \mathbf{k}_\perp, \downarrow, \uparrow) &= \frac{1}{\omega_{q\bar{q}}} [-(\mathbf{k}_1 + i\mathbf{k}_2)((1-x)M_{q\bar{q}} + m_{\bar{q}})] \\ \mathcal{S}_{(S_z=+1)}(x, \mathbf{k}_\perp, \downarrow, \downarrow) &= \frac{1}{\omega_{q\bar{q}}} [-(\mathbf{k}_1 + i\mathbf{k}_2)^2]. \end{aligned} \quad (16)$$

For longitudinal spin projection $S_z = 0$, the spin wave function is

$$\begin{aligned} \mathcal{S}_{(S_z=0)}(x, \mathbf{k}_\perp, \uparrow, \uparrow) &= \frac{1}{\sqrt{2}} \frac{1}{\omega_{q\bar{q}}} [(\mathbf{k}_1 - i\mathbf{k}_2)((1-2x)M_{q\bar{q}} + (m_{\bar{q}} - m_q))] \\ \mathcal{S}_{(S_z=0)}(x, \mathbf{k}_\perp, \uparrow, \downarrow) &= \frac{1}{\sqrt{2}} \frac{1}{\omega_{q\bar{q}}} [2\mathbf{k}_\perp^2 + (M_{q\bar{q}} + m_q + m_{\bar{q}})((1-x)m_q + xm_{\bar{q}})] \\ \mathcal{S}_{(S_z=0)}(x, \mathbf{k}_\perp, \downarrow, \uparrow) &= \frac{1}{\sqrt{2}} \frac{1}{\omega_{q\bar{q}}} [2\mathbf{k}_\perp^2 + (M_{q\bar{q}} + m_q + m_{\bar{q}})((1-x)m_q + xm_{\bar{q}})] \\ \mathcal{S}_{(S_z=0)}(x, \mathbf{k}_\perp, \downarrow, \downarrow) &= \frac{1}{\sqrt{2}} \frac{1}{\omega_{q\bar{q}}} [(-\mathbf{k}_1 - i\mathbf{k}_2)((1-2x)M_{q\bar{q}} + (m_{\bar{q}} - m_q))]. \end{aligned} \quad (17)$$

Finally, the spin wave function for transverse spin projection $S_z = -1$

$$\begin{aligned} \mathcal{S}_{(S_z=-1)}(x, \mathbf{k}_\perp, \uparrow, \uparrow) &= \frac{1}{\omega_{q\bar{q}}} [-(\mathbf{k}_1 - i\mathbf{k}_2)^2] \\ \mathcal{S}_{(S_z=-1)}(x, \mathbf{k}_\perp, \uparrow, \downarrow) &= \frac{1}{\omega_{q\bar{q}}} [(\mathbf{k}_1 - i\mathbf{k}_2)((1-x)M_{q\bar{q}} + m_{\bar{q}})] \\ \mathcal{S}_{(S_z=-1)}(x, \mathbf{k}_\perp, \downarrow, \uparrow) &= \frac{1}{\omega_{q\bar{q}}} [-(\mathbf{k}_1 - i\mathbf{k}_2)(xM_{q\bar{q}} + m_q)] \\ \mathcal{S}_{(S_z=-1)}(x, \mathbf{k}_\perp, \downarrow, \downarrow) &= \frac{1}{\omega_{q\bar{q}}} [\mathbf{k}_\perp^2 + (M_{q\bar{q}} + m_q + m_{\bar{q}})((1-x)m_q + xm_{\bar{q}})]. \end{aligned} \quad (18)$$

Here, $\omega_{q\bar{q}} = (M_{q\bar{q}} + m_q + m_{\bar{q}})\sqrt{x(1-x)[M_{q\bar{q}}^2 - (m_q - m_{\bar{q}})^2]}$. The two-particle Fock-state in Eq. (3) can be written in the form of LC wave functions (LCWFs) with all possible helicities of its constituent quark and antiquark for vector mesons as

$$\begin{aligned} |M(P^+, \mathbf{P}_\perp, S_z)\rangle = & \int \frac{dx d^2\mathbf{k}_\perp}{2(2\pi)^3 \sqrt{x(1-x)}} [\Psi_{S_z}(x, \mathbf{k}_\perp, \uparrow, \uparrow) |xP^+, \mathbf{k}_\perp, \uparrow, \uparrow\rangle \\ & + \Psi_{S_z}(x, \mathbf{k}_\perp, \downarrow, \downarrow) |xP^+, \mathbf{k}_\perp, \downarrow, \downarrow\rangle + \Psi_{S_z}(x, \mathbf{k}_\perp, \downarrow, \uparrow) \\ & \times |xP^+, \mathbf{k}_\perp, \downarrow, \uparrow\rangle + \Psi_{S_z}(x, \mathbf{k}_\perp, \uparrow, \downarrow) |xP^+, \mathbf{k}_\perp, \uparrow, \downarrow\rangle]. \end{aligned} \quad (19)$$

III. VECTOR QUARK GPDS AND FORM FACTORS

The amplitudes in DVCS and deep exclusive meson electroproduction (DEMP) on a spin-one target are determined by nonperturbative matrix components that are parametrized in terms of nine chiral-even GPDs for the quark sector. Out of nine, five are (unpolarized) vector quark GPDs. The correlation function for the vector quark GPDs of spin-1 mesons is expressed through the quark-quark correlator as [48,52,54]

$$V_{S'_z, S_z}(x, \xi, -\Delta_\perp^2) = \int \frac{dz^-}{2\pi} e^{ixP_a^+ z^-} \langle M(P'^+, \mathbf{P}'_\perp, S'_z) | \bar{\psi}\left(-\frac{z^-}{2}\right) \gamma^+ \psi\left(\frac{z^-}{2}\right) | M(P^+, \mathbf{P}_\perp, S_z) \rangle. \quad (20)$$

Here, $V_{S'_z, S_z}(x, \xi, -\Delta_\perp^2)$ is the light-front (LF) overlap correlation function between the initial and final state meson. P and P' are the four-momenta of the initial and final state of the meson having spin projections S_z and S'_z , respectively. $P_a = (P' + P)/2$ is the average momentum of the meson. $\Delta(\Delta^+, \Delta^-, \Delta_\perp) = P' - P$ is the momentum transferred between the final and initial meson states. The skewness variable is denoted by $\xi = \frac{\Delta^+}{2P_a^+}$. However, for this work, we have not considered the longitudinal momentum transfer between the initial and final state mesons. So, the $\xi = 0$ as $\Delta^+ = 0$ in this case. Consequently, the correlator can be decomposed to five vector quark GPDs of the form $V_{S'_z, S_z}(x, \xi = 0, -\Delta_\perp^2)$ as [53,54]

$$\begin{aligned} V_{S'_z, S_z}(x, 0, -\Delta_\perp^2) = & -(\epsilon'^* \cdot \epsilon) H_1(x, 0, -\Delta_\perp^2) + \frac{(\epsilon \cdot n)(\epsilon' \cdot P) + (\epsilon' \cdot n)(\epsilon \cdot P)}{P \cdot n} H_2(x, 0, -\Delta_\perp^2) \\ & - 2 \frac{(\epsilon \cdot P)(\epsilon'^* \cdot P)}{M_{q\bar{q}}^2} H_3(x, 0, -\Delta_\perp^2) + \frac{(\epsilon \cdot n)(\epsilon' \cdot P) - (\epsilon'^* \cdot n)(\epsilon \cdot P)}{P \cdot n} H_4(x, 0, -\Delta_\perp^2) \\ & + \left\{ M_{q\bar{q}}^2 \frac{(\epsilon \cdot n)(\epsilon' \cdot n)}{(P \cdot n)^2} + \frac{1}{3} (\epsilon'^* \cdot \epsilon) \right\} H_5(x, 0, -\Delta_\perp^2), \end{aligned} \quad (21)$$

with $\epsilon \equiv \epsilon(P, S_z)$ and $\epsilon' \equiv \epsilon(P', S'_z)$ are polarization vector of the initial and final state meson, which are discussed in Eq. (15). The four-vector notations used in the above equation are expressed in the Berit frame as [54,69]

$$n = (0, \sqrt{2}, 0, 0), \quad (22)$$

$$\Delta = (0, 0, \Delta_\perp, 0), \quad (23)$$

$$P = \frac{M_{q\bar{q}}}{\sqrt{2}} \left(\sqrt{1+\eta}, \sqrt{1+\eta}, -\frac{\Delta_\perp}{M_{q\bar{q}}\sqrt{2}}, 0 \right), \quad (24)$$

$$P' = \frac{M_{q\bar{q}}}{\sqrt{2}} \left(\sqrt{1+\eta}, \sqrt{1+\eta}, \frac{\Delta_\perp}{M_{q\bar{q}}\sqrt{2}}, 0 \right). \quad (25)$$

Here, $\eta = \frac{\Delta_\perp^2}{4M_{q\bar{q}}^2}$. Consequently, the LF overlap correlation function $V_{S'_z, S_z}(x, \xi, -\Delta_\perp^2)$ obey the parity and time reversal invariance as [48,54,82]

$$V_{S'_z, S_z}(x, 0, -\Delta_\perp^2) = (-1)^{S'_z - S_z} V_{-S'_z, -S_z}(x, 0, -\Delta_\perp^2), \quad (26)$$

$$V_{S'_z, S_z}(x, \xi, -\Delta_\perp^2) = (-1)^{S'_z - S_z} V_{S'_z, S_z}(x, -\xi, -\Delta_\perp^2). \quad (27)$$

Now, applying the above parity and time-reversal condition to calculate the unpolarized quark GPDs at zero skewness, we are left with four independent $V_{S'_z, S_z}(x, \xi, -\Delta_\perp^2)$ functions in the form of different spin projections, which are $V_{0,0}(x, 0, -\Delta_\perp^2)$, $V_{0,1}(x, 0, -\Delta_\perp^2)$, $V_{1,1}(x, 0, -\Delta_\perp^2)$, and $V_{1,-1}(x, 0, -\Delta_\perp^2)$. Now reversing Eq. (21), by using the meson momentum and polarization vector, the unpolarized GPDs are found to be

$$\begin{aligned} H_1(x, 0, -\Delta_\perp^2) = & \frac{1}{3} [V_{0,0}(x, 0, -\Delta_\perp^2) - 2(\eta - 1)V_{1,1}(x, 0, -\Delta_\perp^2) \\ & + 2\sqrt{2}\eta V_{1,0}(x, 0, -\Delta_\perp^2) + 2V_{1,-1}(x, 0, -\Delta_\perp^2)], \end{aligned} \quad (28)$$

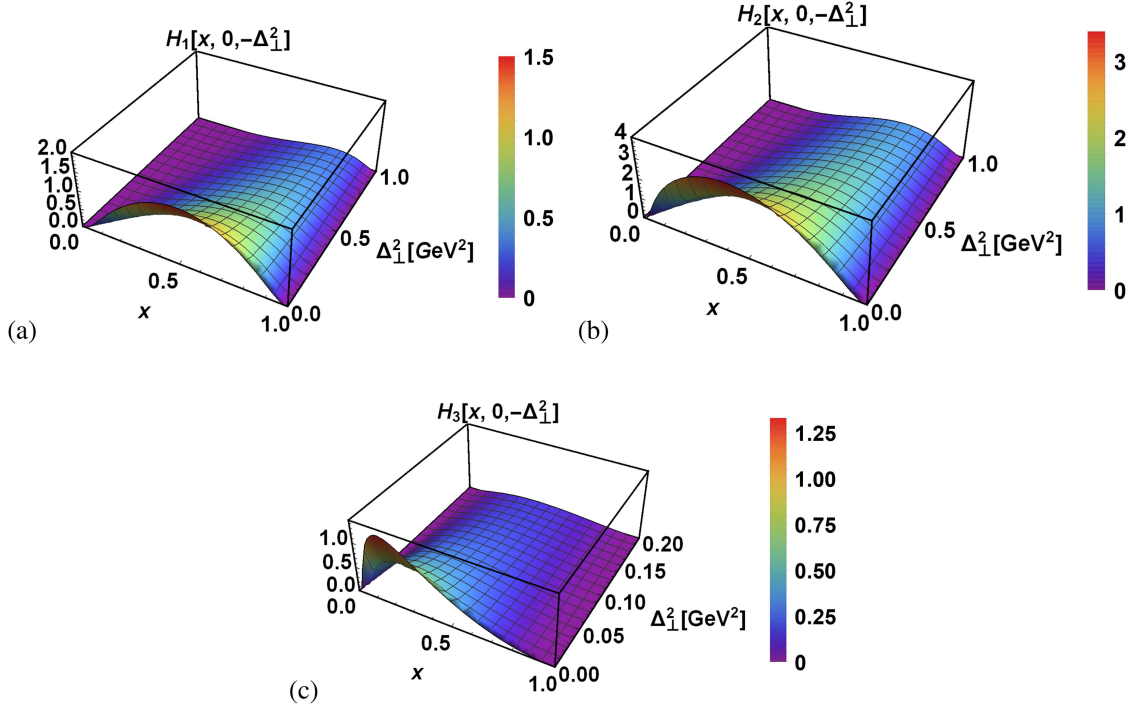


FIG. 1. The three-dimensional quark GPDs have been plotted with respect to longitudinal momentum fraction (x) and transverse momentum transferred $\Delta_\perp^2$ (GeV^2) for ρ -meson in (a) for $H_1(x, 0, -\Delta_\perp^2)$, (b) for $H_2(x, 0, -\Delta_\perp^2)$, and (c) for $H_3(x, 0, -\Delta_\perp^2)$.

$$H_2(x, 0, -\Delta_\perp^2) = 2V_{1,1}(x, 0, -\Delta_\perp^2) - \frac{2}{\sqrt{2}\eta} V_{1,0}(x, 0, -\Delta_\perp^2), \quad (29)$$

$$H_3(x, 0, -\Delta_\perp^2) = -\frac{V_{1,-1}(x, 0, -\Delta_\perp^2)}{\eta}, \quad (30)$$

$$H_4(x, 0, -\Delta_\perp^2) = 0, \quad (31)$$

$$H_5(x, 0, -\Delta_\perp^2) = V_{0,0}(x, 0, -\Delta_\perp^2) - (1 + 2\eta)V_{1,1} + 2\sqrt{2}\tau V_{1,0}(x, 0, -\Delta_\perp^2) - V_{1,-1}(x, 0, -\Delta_\perp^2), \quad (32)$$

The LF overlap correlation function $V_{S'_z, S_z}(x, 0, -\Delta_\perp^2)$ can be written in the overlap form of LFWFs of the total meson wave function as

$$V_{S'_z, S_z}(x, 0, -\Delta_\perp^2) = \sum_{\lambda_1, \lambda_2} \int \frac{d^2 \mathbf{k}_\perp}{2(2\pi)^3} \Psi_{S'_z}^*(x, \mathbf{k}'_\perp, \lambda_1, \lambda_2) \times \Psi_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2). \quad (33)$$

Here, $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\Delta_\perp$ and $\mathbf{k}_\perp$ are the transverse momentum of the quark of the final and initial meson. H_4 quark GPD for our case is found to be zero similar to NJL model, DS-BSE approach, BLFQ, and LFQM.

The behavior of these vector quark GPDs for ρ -meson have been provided through three-dimensional plots in Fig. 1 as a function of longitudinal momentum fraction x and squared momentum transferred $\Delta_\perp^2$. $H_1(x, 0, -\Delta_\perp^2)$ quark GPD shows a smooth, positive distributions all over, and symmetric behavior around $x = 0.5$ at $\Delta_\perp = 0$ under $x \leftrightarrow (1-x)$ as shown in Fig. 1(a), reflecting the equal constituent quark masses of ρ -meson. A similar kind of positive distributions of $H_1(x, 0, -\Delta_\perp^2)$ can also be seen in LFQM [52] and NJL model [49]. The symmetric behavior at $\Delta_\perp^2 = 0$ of our result can also be seen in NJL model also. The distribution nearly vanishes around the end points $x = 0$ and $x = 1$. As the $\Delta_\perp^2$ increases, the distribution $H_1(x, 0, -\Delta_\perp^2)$ behavior suppresses and become zero after $\Delta_\perp \geq 1.2 \text{ GeV}$. The $H_2(x, 0, -\Delta_\perp^2)$ quark GPD has a higher peak distribution compared to other quark GPDs as shown in Fig. 1. It has more of a sharp asymmetric behavior around $x \approx 0.6-0.7$ with larger amplitude. The $H_2(x, 0, -\Delta_\perp^2)$ exhibits a stronger suppression and a more noticeable shift of the peak toward higher x as $\Delta_\perp^2$ increases. The $H_2(x, 0, -\Delta_\perp^2)$ and $H_3(x, 0, -\Delta_\perp^2)$ GPDs show positive distribution in all the regions of x and $\Delta_\perp$ as like NJL model [49]. In case of LFQM, the H_2 shows positive distribution, while H_3 has both positive and negative distribution, which contradicts our model results. The $H_5(x, 0, -\Delta_\perp^2)$ quark GPDs have a lower peak amplitude distribution compared to others and have zero distribution at $\Delta_\perp = 0$. The terms $V_{0,0}$ and $V_{1,1}$ shows the

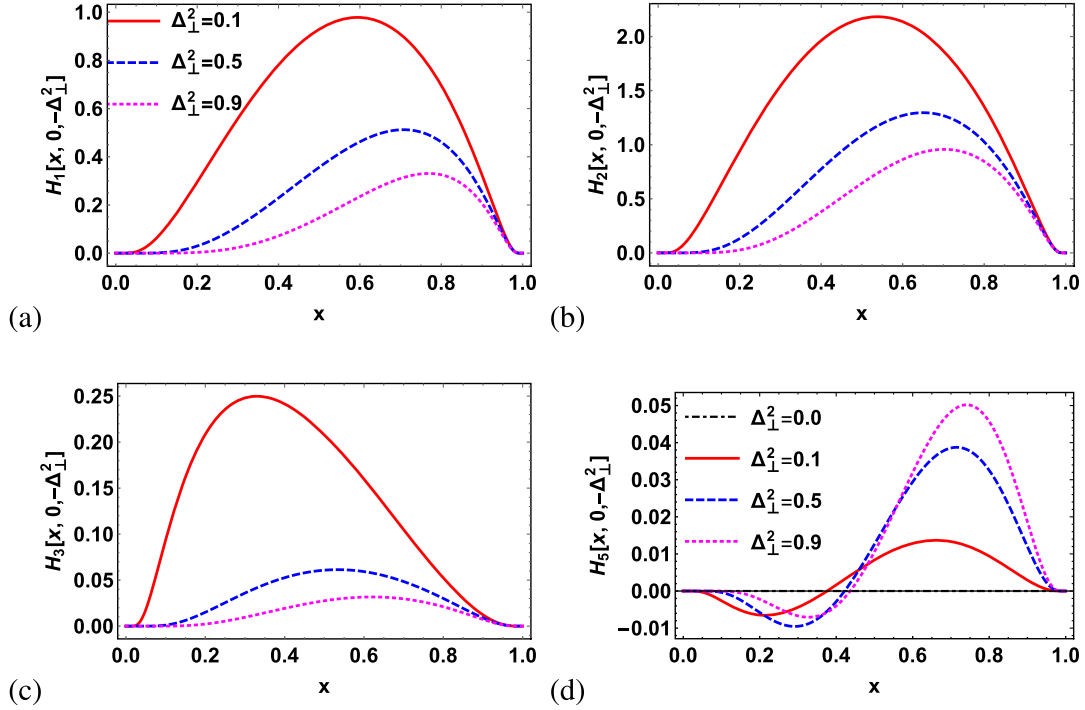


FIG. 2. The (a) $H_1(x, 0, -\Delta_\perp^2)$, (b) $H_2(x, 0, -\Delta_\perp^2)$, (c) $H_3(x, 0, -\Delta_\perp^2)$, and (d) $H_5(x, 0, -\Delta_\perp^2)$ quark GPDs of ρ -meson have been plotted with respect to longitudinal momentum fraction (x) at fixed value of $\Delta_\perp^2 = 0, 0.1, 0.2$, and 0.9 GeV^2 .

contribution of S-wave as there is no increment in orbital angular momentum ($L_z = 0$), whereas the contribution of the P-wave is shown by the presence of $V_{1,0}$ term indicating the increment in the angular momentum by unity ($L_z = \pm 1$). Additionally, the term $V_{1,-1}$ contributes to the presence of D-wave as the orbital angular momentum is raised by two units ($L_z = \pm 2$). Consequently, the quark GPDs $H_{1(5)}(x, 0, -\Delta_\perp^2)$ include the superposition of S, P, and D-wave, whereas only $H_2(x, 0, -\Delta_\perp^2)$ contributes S and P-wave, and $H_5(x, 0, -\Delta_\perp^2)$ only contains D-wave. In Figs. 2 and 3, we have plotted the quark GPDs of ρ -meson with respect to x at fixed values of $\Delta_\perp$ and with respect to $\Delta_\perp$ at fixed values of x , respectively. We observed that the distribution decreases with x with an increase in momentum transferred between the states for all the quark GPDs except H_5 . For the case H_5 , we found that the distribution shows both negative and positive behavior, and with an increase in $\Delta_\perp$, the positive distribution increases. This kind of behavior shows up to $\Delta_\perp^2 = 20 \text{ GeV}^2$, then the distribution becomes zero. A similar kind of behavior is also seen with an increase in x also.

To have an idea about the heavy quark structure, we have also studied the vector quark GPDs of the heavy charmonia J/ψ vector meson. The quark GPDs of J/ψ -meson have been plotted with respect to x and $\Delta_\perp^2$ through three-dimensional and two-dimensional plots in Figs. 4–6. All the quark GPDs are found to have maximum distributions around $x = 0.5$ and shows $x \leftrightarrow (1 - x)$ symmetry.

The distributions of quark GPDs decrease with an increase in $\Delta_\perp$, as like ρ -meson case. The H_5 quark GPDs show only positive behavior for the case of J/ψ -meson. We observed that $H_{1,2,3}$ quark GPDs of our results have a similar kind of trend with BLFQ results [53]. Overall, we found that the momentum fraction carried by the quark decreases with an increase in momentum transferred to the final state meson.

All these vector quark GPDs have also been plotted in impact parameter space by taking the Fourier transform of $\Delta_\perp$ as

$$H_i(x, 0, b_\perp^2) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta_\perp} H_i(x, 0, -\Delta_\perp^2), \quad (34)$$

These vector quark GPDs at impact parameter space have been studied using the three-dimensional plots in Fig. 7 for both light ρ and heavy J/ψ vector mesons. All the quark GPDs are found to have positive distributions with a sharply decreasing behavior with maximum distributions at $b_\perp = 0$.

A. Form factors

There are a total of three electromagnetic form factors $[F_i(Q^2)] (i = 1, 2, 3)$ present at the leading twist spin-1 vector mesons case. These form factors can be calculated through both solving the quark-quark correlation function and from vector quark GPDs. These Lorentz invariant form

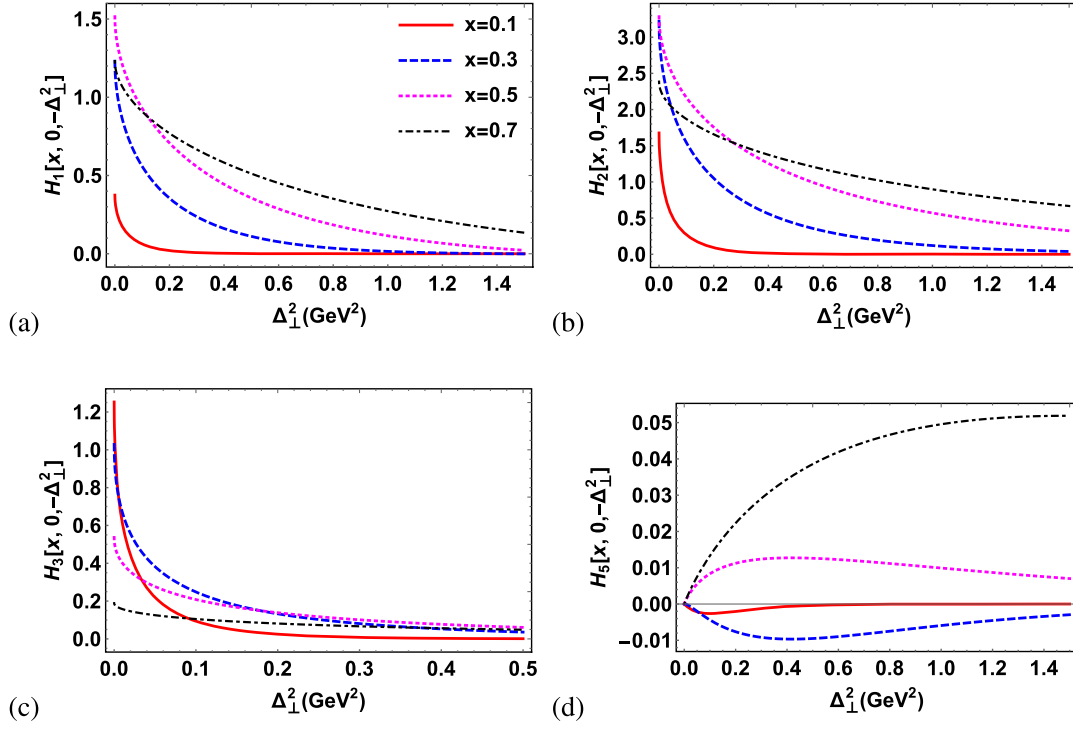


FIG. 3. The (a) $H_1(x, 0, -\Delta_\perp^2)$, (b) $H_2(x, 0, -\Delta_\perp^2)$, (c) $H_3(x, 0, -\Delta_\perp^2)$, and (d) $H_5(x, 0, -\Delta_\perp^2)$ quark GPDs of ρ -meson have been plotted with respect to $\Delta_\perp^2$ at fixed value of longitudinal momentum fraction $x = 0.1, 0.3, 0.5$, and 0.7 .

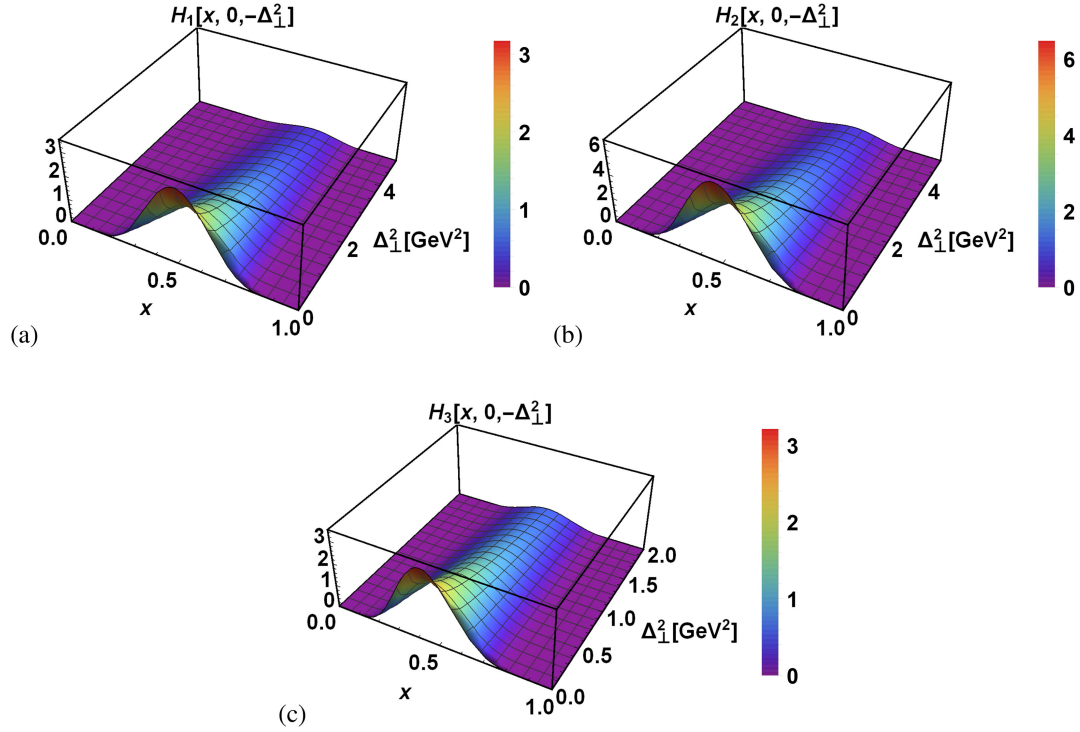


FIG. 4. The three-dimensional quark GPDs have been plotted with respect to longitudinal momentum fraction (x) and transverse momentum transferred $\Delta_\perp^2$ (GeV^2) for J/ψ -meson in (a) for $H_1(x, 0, -\Delta_\perp^2)$, (b) for $H_2(x, 0, -\Delta_\perp^2)$, and (c) for $H_3(x, 0, -\Delta_\perp^2)$.

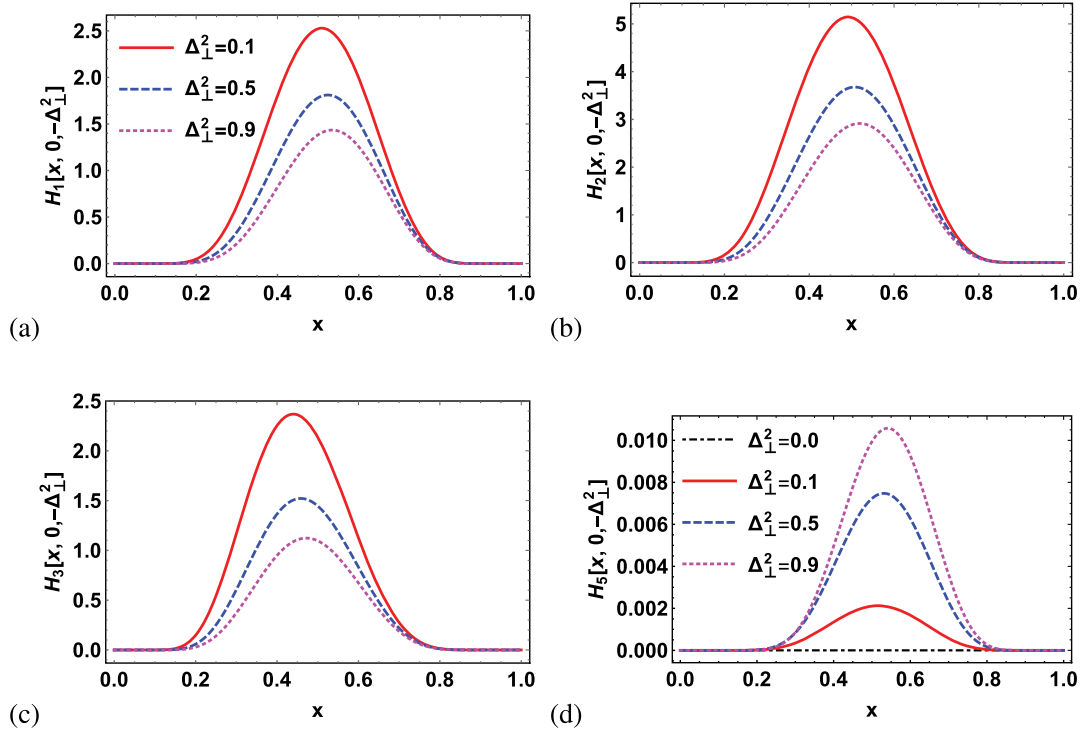


FIG. 5. The (a) $H_1(x, 0, -\Delta_\perp^2)$, (b) $H_2(x, 0, -\Delta_\perp^2)$, (c) $H_3(x, 0, -\Delta_\perp^2)$, and (d) $H_5(x, 0, -\Delta_\perp^2)$ quark GPDs of J/ψ -meson have been plotted with respect to longitudinal momentum fraction (x) at fixed value of $\Delta_\perp^2 = 0.1, 0.2$, and 0.9 GeV^2 .

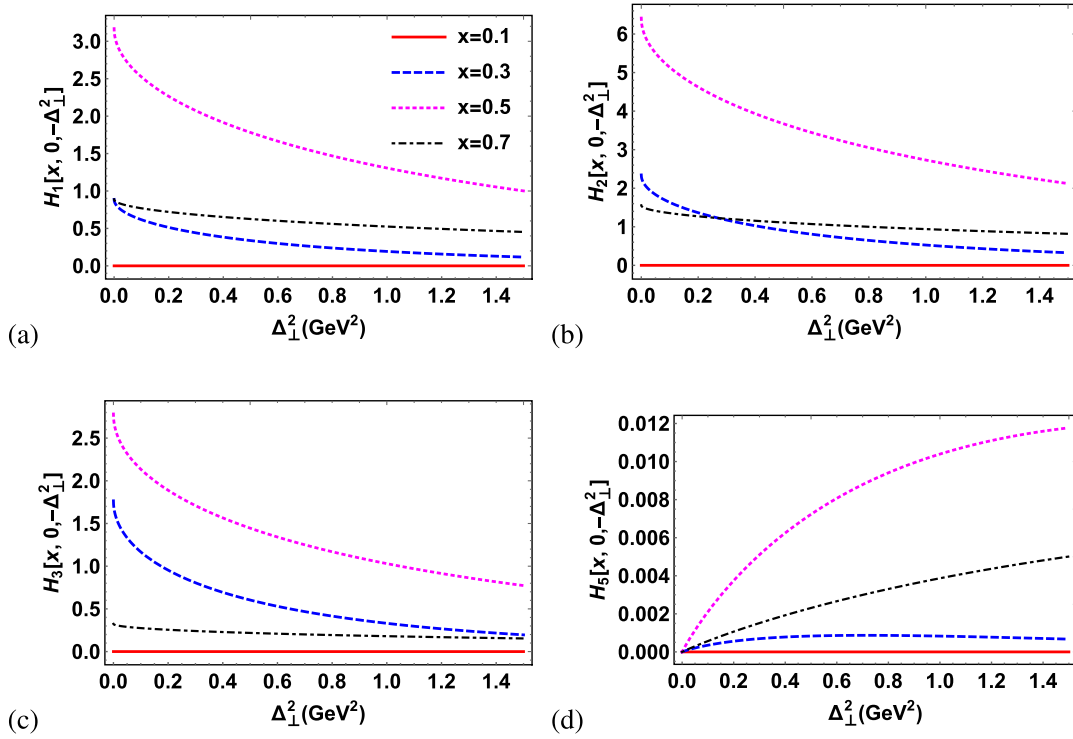


FIG. 6. The (a) $H_1(x, 0, -\Delta_\perp^2)$, (b) $H_2(x, 0, -\Delta_\perp^2)$, (c) $H_3(x, 0, -\Delta_\perp^2)$, and (d) $H_5(x, 0, -\Delta_\perp^2)$ quark GPDs of J/ψ -meson have been plotted with respect to $\Delta_\perp^2$ at fixed value of longitudinal momentum fraction $x = 0.1, 0.3, 0.5$, and 0.7 .

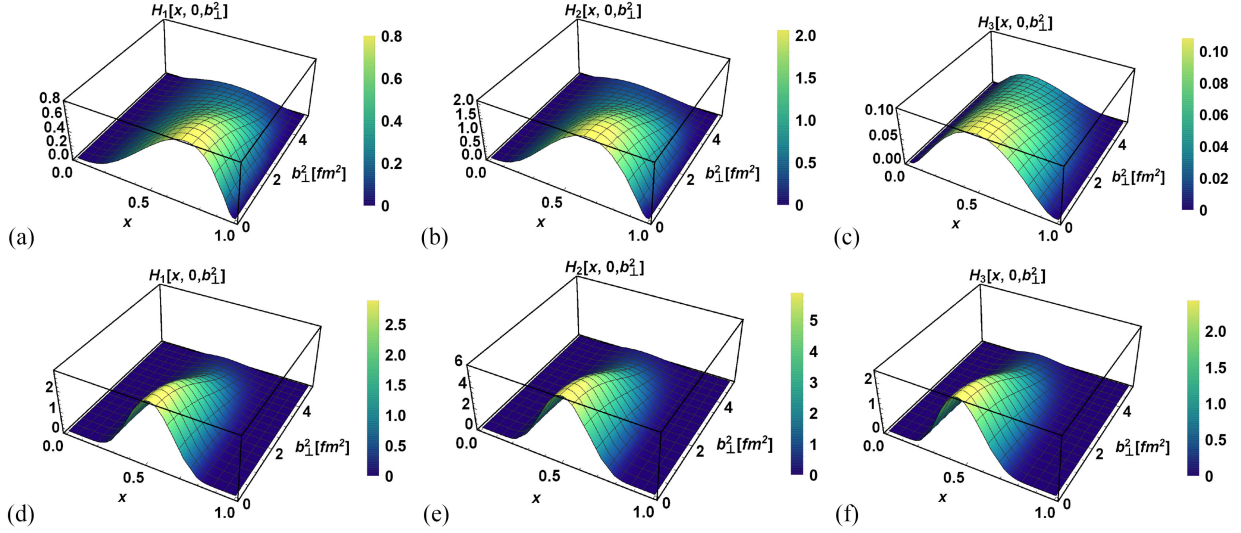


FIG. 7. The $H_1(x, 0, b_\perp^2)$, $H_2(x, 0, b_\perp^2)$, and $H_3(x, 0, b_\perp^2)$ quark GPDs have been plotted with respect to $b_\perp^2$ (fm^2) and longitudinal momentum fraction x . In (a)–(c) correspond to the ρ -meson with H_1 , H_2 , and H_3 , respectively; (d)–(f) show the same distributions for the J/ψ -meson.

factors are defined by the matrix elements of the J^μ current sandwiched between initial $|M(P^+, \mathbf{P}_\perp, S_z)\rangle$ and final $\langle M(P'^+, \mathbf{P}'_\perp, S'_z)|$ states as follows [52,57]

$$\begin{aligned} & \langle M(P'^+, S'_z) | J^\mu | M(P^+, \mathbf{P}_\perp, S_z) \rangle \\ &= -\epsilon' \cdot \epsilon (P + P')^\mu F_1(Q^2) + (\epsilon' n \cdot \epsilon' - \epsilon' n \cdot \epsilon) F_2(Q^2) \\ &+ \frac{(\epsilon' \cdot n)(\epsilon \cdot n)}{2M_v^2} (P + P')^\mu F_3(Q^2), \end{aligned} \quad (35)$$

where $Q^2 = \Delta^2 = -\Delta_\perp^2$. However, these form factors can be calculated from unpolarized quark GPDs as

$$\int_{-1}^1 dx H_i(x, 0, -\Delta_\perp^2) = F_i(Q^2), \quad (i = 1, 2, 3), \quad (36)$$

$$\int_{-1}^1 dx H_i(x, \xi, -\Delta_\perp^2) = 0, \quad (i = 4, 5). \quad (37)$$

Both calculations give rise to equal form factor results. These form factors have been plotted with respect to

Q^2 GeV^2 in Fig. 8 for both ρ and J/ψ vector mesons. All the form factors are found to have positive distributions and have a maximum distribution at $Q^2 = 0$. It is observed that $F_2(Q^2)$ has higher distributions compared to other form factors for both particles. These form factors can be used to calculate the charge, magnetic, and quadrupole form factors of vector mesons. These can be calculated as

$$\begin{aligned} G_C(Q^2) &= \left(1 + \frac{2}{3}\eta\right) F_1(Q^2) + \frac{2}{3}\eta F_2(Q^2) \\ &+ \frac{2}{3}\eta(1 + \eta) F_3(Q^2), \end{aligned} \quad (38)$$

$$G_M(Q^2) = F_2(Q^2), \quad (39)$$

$$G_Q(Q^2) = F_1(Q^2) + F_2(Q^2) + (1 + \eta) F_3(Q^2). \quad (40)$$

These form factors have been plotted for the light ρ -meson and heavy J/ψ -meson in Figs. 9 and 10 with respect to Q^2 GeV^2 , respectively. All the form factors of ρ -meson

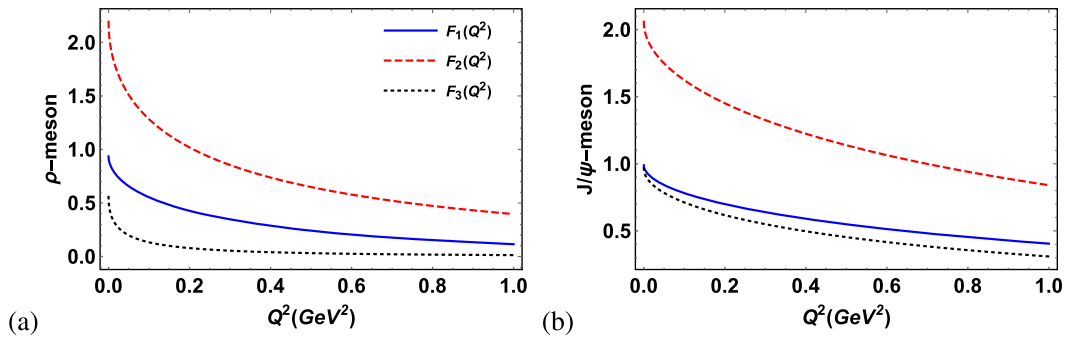


FIG. 8. The elastic form factors $F_1(Q^2)$, $F_2(Q^2)$, and $F_3(Q^2)$ have been plotted with respect to Q^2 for (a) light ρ and (b) heavy J/ψ vector meson.

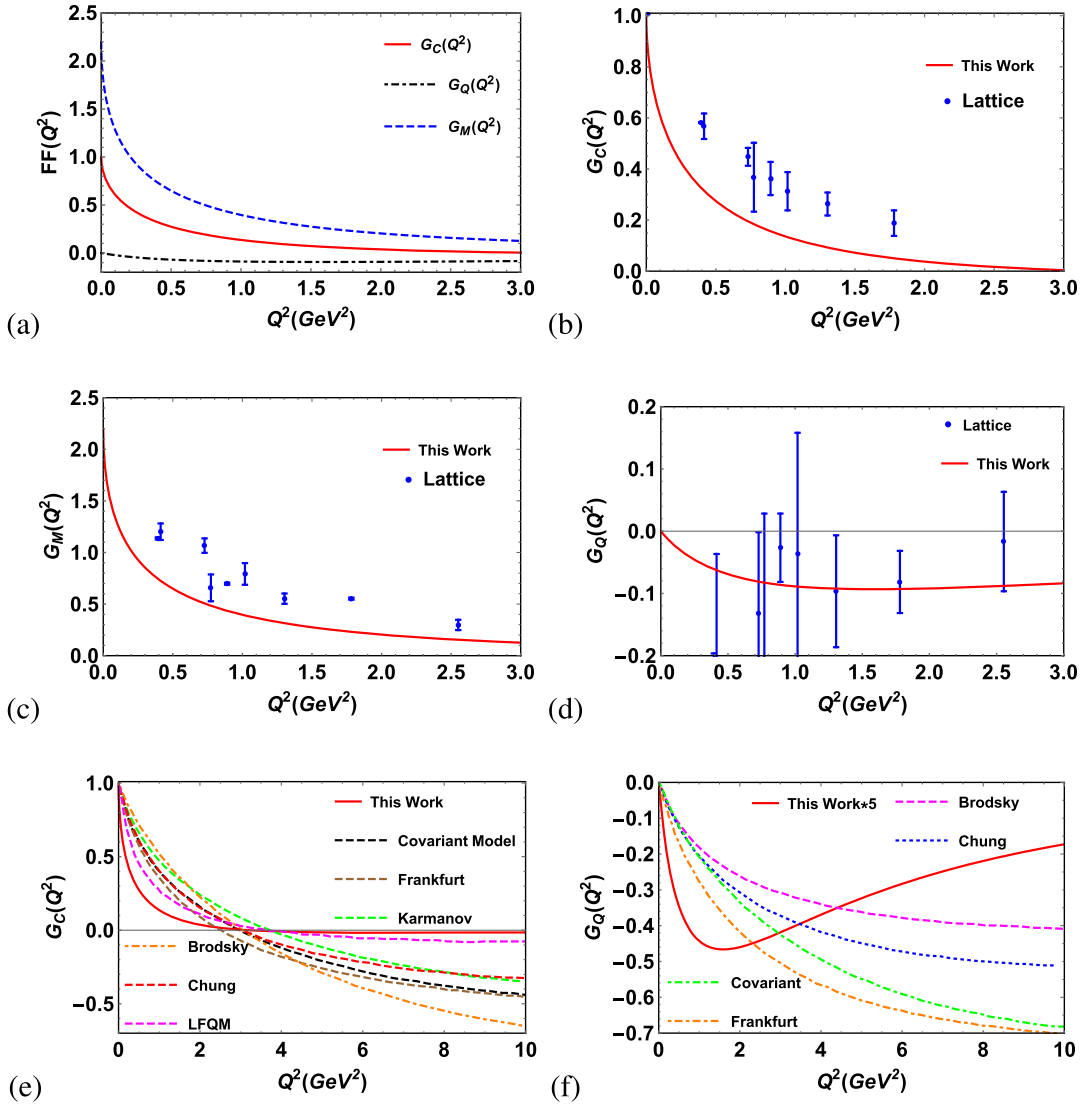


FIG. 9. (a) The charge [$G_C(Q^2)$], magnetic [$G_M(Q^2)$], and quadrupole [$G_Q(Q^2)$] form factors have been plotted with respect to Q^2 for ρ -meson. The charge [$G_C(Q^2)$], magnetic [$G_M(Q^2)$], and quadrupole [$G_Q(Q^2)$] form factors have been compared with available lattice simulations data [83] in (b), (c), and (d), respectively. The charge $G_C(Q^2)$ and quadrupole $G_Q(Q^2)$ form factors have been compared with available theoretical models [55,59–63].

have been compared with lattice simulation results [83] up to $Q^2 = 3 \text{ GeV}^2$ and found to have a similar trend but lower distributions than theirs, except G_Q . At $Q^2 = 0$, the G_C , G_M , and G_Q are found to be unity, 2.1, and zero for both particles, respectively. For a better understanding of these form factors, we have compared our results with theoretical model results [55,59–63] up to $Q^2 = 10 \text{ GeV}^2$ in Figs. 9(e) and 9(f). The charge G_C and magnetic G_M form factors have found to be have similar kind of trend with NJL model [49,84], LFQM [52], DS-BSE [54], contact interaction [85], AdS/QCD model [64], LFCQM [55], BLFQ [53] and other models [59–63] results, but have a lower distributions compared to them. However, the quadrupole form factor G_Q has the opposite kind of behavior at low Q^2 compared to Refs. [49,52,54,63,84,85], while it has

the same kind of trend with Refs. [53,60–62] as shown in Fig. 9(f). The charge form factor G_C shows negative distribution after $Q^2 \geq 4 \text{ GeV}^2$ as like NJL model [49], LFCQM [55] and other models [59–62] as shown in Fig. 9(e). We have also observed that the heavier J/ψ -meson has higher form factor distributions compared to the light ρ -meson as shown in Fig. 10. We have also observed that the asymptotic limit of QCD reported in Refs. [61,85] is strictly followed by our model, which reads

$$G_C(Q^2) : G_M(Q^2) : G_Q(Q^2) \xrightarrow{Q^2 \rightarrow \infty} 1 - \frac{2\eta}{3} : 2 : -1. \quad (41)$$

At zero momentum transfer $Q^2 = 0$, one can find the charge, magnetic moment (μ_p), and quadrupole moment Q_p of the vector mesons from these form factors as

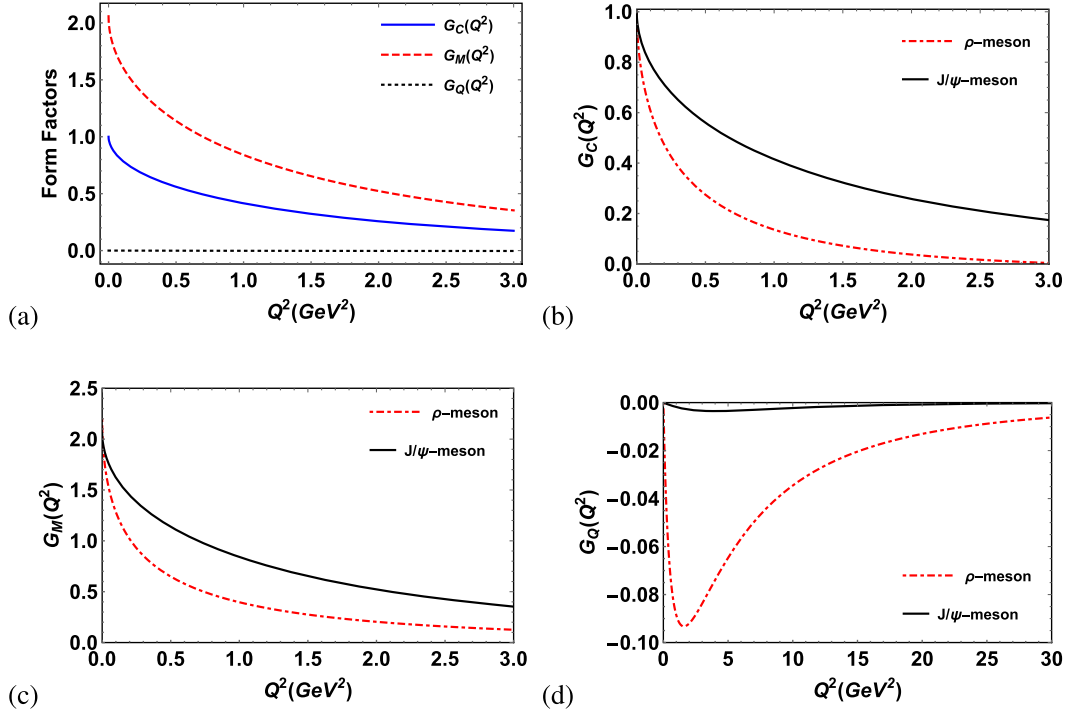


FIG. 10. (a) The charge [$G_C(Q^2)$], magnetic [$G_M(Q^2)$], and quadrupole [$G_Q(Q^2)$] form factors have been plotted with respect to Q^2 for J/ψ -meson. The charge [$G_C(Q^2)$], magnetic [$G_M(Q^2)$], and quadrupole [$G_Q(Q^2)$] form factors of J/ψ been compared with ρ -meson form factors in (b), (c), and (d), respectively.

$$eG_C(Q^2=0) = e, \quad G_M(Q^2=0) = \mu_p, \\ G_Q(Q^2=0) = Q_p. \quad (42)$$

Here, e is the static charge. μ_p and Q_p are in the units of Bohr magneton $e/2M_{\rho(J/\psi)}$ and $e/M_{\rho(J/\psi)}^2$. Here, $M_{\rho(J/\psi)}$ is the physical mass of ρ and J/ψ -mesons. The charge radii $\sqrt{\langle r_c^2 \rangle}$ of the respective vector mesons can be calculated using the fundamental radius equation as

$$\langle r_c^2 \rangle = \frac{-6}{G_C(0)} \frac{\partial G_C(Q^2)}{\partial Q^2} \Big|_{Q^2 \rightarrow 0}. \quad (43)$$

However, the $G_C(0)$ is one. The value of charge radii $\sqrt{\langle r_c^2 \rangle}$, magnetic moment μ_p , and quadrupole moment Q_p have been presented in Table I for both particles along with the comparison with available predictions [66,85–89]. We observe that our charge radii are slightly higher than those of other results. μ_p and Q_p are in good agreement with them.

Further, using $G_C(Q^2)$, $G_M(Q^2)$, and $G_Q(Q^2)$ form factors, one can define the conserving [$G_{++}^+(Q^2)$ and $G_{00}^+(Q^2)$] and nonconserving [$G_{-+}^+(Q^2)$ and $G_{0+}^+(Q^2)$] helicity matrix element as

$$G_{++}^+(Q^2) = \frac{1}{1+\eta} \left(G_C(Q^2) + \eta G_M(Q^2) + \frac{\eta}{3} G_Q(Q^2) \right), \quad (44)$$

$$G_{00}^+(Q^2) = \frac{1}{1+\eta} \left((1-\eta)G_C(Q^2) + 2\eta G_M(Q^2) - \frac{2\eta}{3}(1+2\eta)G_Q(Q^2) \right), \quad (45)$$

$$G_{0+}^+(Q^2) = -\frac{\sqrt{2\eta}}{1+\eta} \left(G_C(Q^2) - \frac{1}{2}(1-\eta)G_M(Q^2) + \frac{\eta}{3}G_Q(Q^2) \right), \quad (46)$$

TABLE I. The charge radii $\sqrt{\langle r_c^2 \rangle}$ fm, magnetic moment μ_p (in units of Bohr magneton), and quadrupole moment Q_p fm ^{2} of light ρ and heavy J/ψ vector mesons have been compared with available data [66,85–89].

		$\sqrt{\langle r_c^2 \rangle}$ fm	μ_p	Q_p
$\rho(u\bar{d})$	Our result	0.95	2.19	-0.023
	Ref. [86]	0.73	2.01	-0.026
	Ref. [89]	0.82	2.07	-0.045
	Ref. [66]	0.82	2.48	-0.070
	Ref. [85]	0.56	0.75	...
	Ref. [88]	1.12	2.54	...
$J/\psi(c\bar{c})$	Our result	0.69	2.05	-0.006
	Ref. [85]	0.350	2.03	...
	Ref. [86]	0.24	2.12	...
	Ref. [87]	0.066	2.10	...

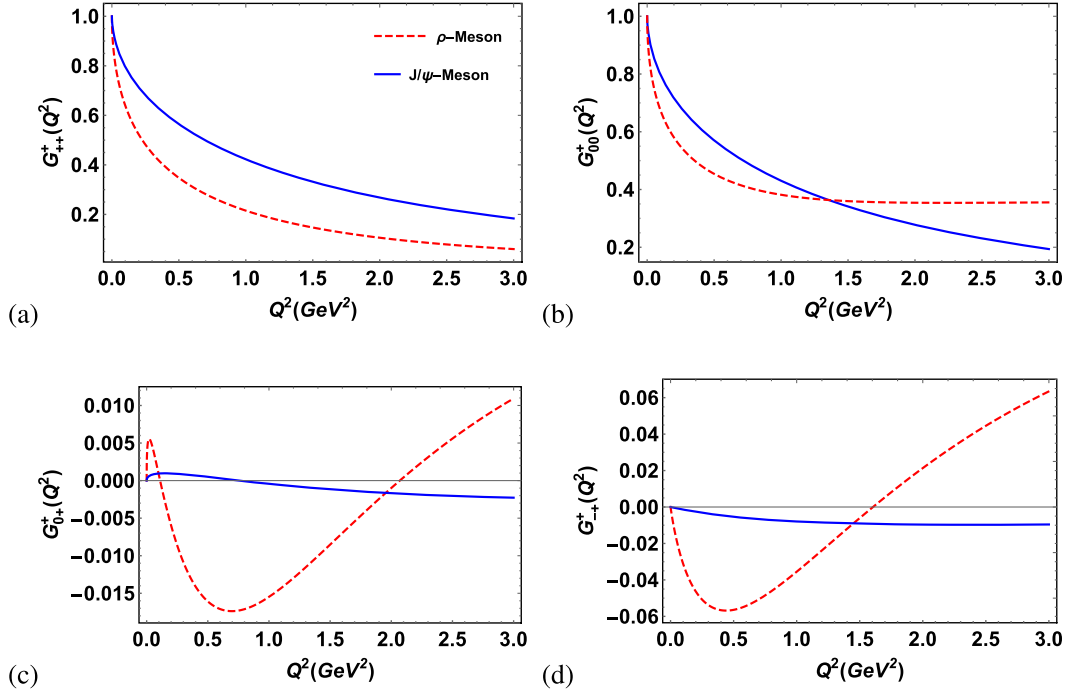


FIG. 11. The helicity-conserving matrix elements $G_{++}^+(Q^2)$ and $G_{00}^+(Q^2)$ have been plotted with respect to Q^2 in (a) and (b), respectively, for both ρ and J/ψ -mesons. The helicity-flip matrix element with one unit $G_{0+}^+(Q^2)$ and two units $G_{-+}^+(Q^2)$ of helicity-flip has been plotted in (c) and (d), respectively, for both ρ and J/ψ -mesons.

$$G_{-+}^+(Q^2) = \frac{\eta}{1+\eta} \left(G_C(Q^2) - G_M(Q^2) - \left(1 + \frac{2\eta}{3} \right) G_Q(Q^2) \right). \quad (47)$$

Here, $+$ $(-)$ are denoted for $+1(-1)$ spin projections. These matrix elements have been plotted in Fig. 11 for both the mesons. Both the conserving matrix elements G_{++}^+ and G_{00}^+ shows positive behavior and found to be unity at $Q^2 = 0$. A similar kind of results have also been seen in NJL model [84] and LFQM [52]. The nonconserving $[G_{0+}^+(Q^2)$ and $G_{-+}^+(Q^2)]$ helicity matrix elements show both positive and negative distributions. The G_{0+}^+ elements have negative distribution in between $0.1 \leq Q^2 \leq 2.1$ GeV², while G_{-+}^+ shows negative distributions around $0 \leq Q^2 \leq 1.7$ GeV². In the case of LFQM [52] and NJL model [84], the nonconserving G_{-+}^+ matrix element shows only a negative

distribution. Another thing to notice is that the nonconserving matrix elements have lower distributions compared to the conserving matrix elements.

The important reason to study these $G_C(Q^2)$, $G_M(Q^2)$, and $G_Q(Q^2)$ form factors is that they can be directly connected with the Rosenbluth cross section [58] for elastic electron scattering on a target of arbitrary spin in the laboratory frame through $A(Q^2)$ and $B(Q^2)$ structure function along with tensor polarization $T_{20}(Q^2, \phi)$. The $A(Q^2)$, $B(Q^2)$, and $T_{20}(Q^2, \phi)$ can be calculated from $G_C(Q^2)$, $G_M(Q^2)$, and $G_Q(Q^2)$ as [84,90]

$$A(Q^2) = G_C^2(Q^2) + 2/3\eta G_M^2(Q^2) + 2(2/3)^2\eta^2 G_Q^2(Q^2), \quad (48)$$

$$B(Q^2) = \frac{4}{3}\eta(1+\eta)G_M^2, \quad (49)$$

$$T_{20}(Q^2, \phi) = -\eta \frac{\sqrt{2}\frac{4}{3}\eta G_Q^2(Q^2) + 4G_Q(Q^2)G_C(Q^2) + (1/2 + (1 + \eta \tan^2 \frac{\phi}{2}))G_M^2(Q^2)}{A(Q^2) + B(Q^2)\tan^2 \frac{\phi}{2}}. \quad (50)$$

Using these, the Rosenbluth cross section of ρ and J/ψ mesons can be calculated as [58,84]

$$\frac{d\sigma}{d(-Q^2)} = \frac{4\pi\alpha^2}{(-Q^2)^2} \left[\left(1 + \frac{(-Q^2)s}{(s - M_{\rho(J/\psi)}^2)^2} \right) A(Q^2) - \frac{M_{\rho(J/\psi)}^2(-Q^2)}{(s - M_{\rho(J/\psi)}^2)^2} B(Q^2) \right]. \quad (51)$$

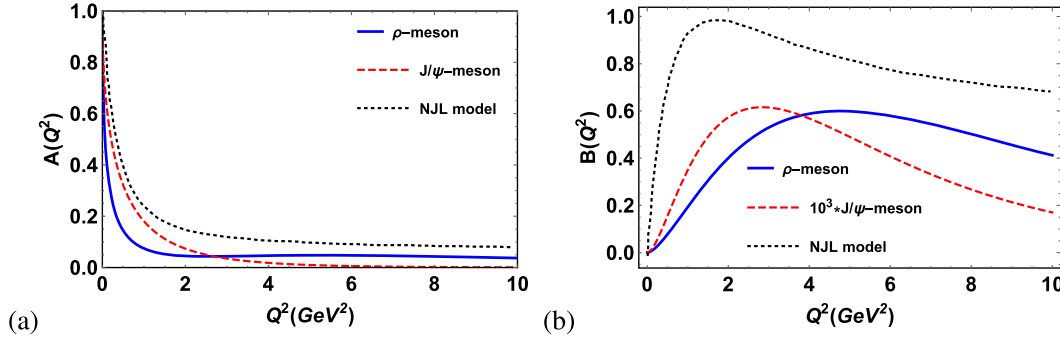


FIG. 12. (a) The $A(Q^2)$ and (b) $B(Q^2)$ structure functions as functions of Q^2 for light ρ (solid blue) and heavy J/ψ (dashed red) vector mesons along with a comparison with the NJL model (dotted black).

As there is no experimental data available for $A(Q^2)$ and $B(Q^2)$ structure functions, so we have compared our results with NJL model calculations [84] and found that to have similar behavior as shown in Fig. 12. The $A(Q^2)$ structure function saturates after $Q^2 = 1.5$ GeV², as like NJL model and LFQM [57]. Like form factor results, the structure functions also have a similar kind of distribution with lesser magnitudes. For the case of heavy J/ψ -vector meson, the $B(Q^2)$ structure function is very low compared to the ρ -meson. However, both the structure and function obey the sum rule

$$A(Q^2 = 0) = 1, \quad B(Q^2 = 0) = 0. \quad (52)$$

IV. SPIN-1 PARTON DISTRIBUTION FUNCTIONS

Without knowing the parton's inherent transverse momentum, the PDFs encode the longitudinal momentum distribution and the polarization carried by the partons at $P = P'$ and $\Delta_\perp = 0$. For the case of spin-1 vector mesons, there are a total of four quark PDFs present at the leading twist, which are unpolarized $f_1(x)$, transversity polarized $h_1(x)$, helicity $g_1(x)$, and tensor polarized $f_{1LL}(x)$ quark PDFs [91]. From unpolarized GPDs, we can only derive quark $f_1(x)$ and $f_{1LL}(x)$ PDFs. For polarized quark $g_1(x)$ and $h_1(x)$ PDFs, we have considered the possible LFWFs. These quark PDFs can be calculated using the overlap form of LFWFs and from GPDs limit as [33,49,67,92]

$$f_1(x) = H_1(x, 0, 0),$$

$$= \int \frac{d^2 \mathbf{k}_\perp}{2(2\pi)^3} \frac{1}{3} \sum_{\lambda_1, \lambda_2} (|\Psi_{S_z=0}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)|^2 + |\Psi_{S_z=1}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)|^2 + |\Psi_{S_z=-1}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)|^2), \quad (53)$$

$$f_{1LL}(x) = H_5(x, 0, 0)$$

$$= b_1(x)$$

$$= \int \frac{d^2 \mathbf{k}_\perp}{2(2\pi)^3} \sum_{\lambda_1, \lambda_2} (|\Psi_{S_z=0}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)|^2 - (|\Psi_{S_z=1}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)|^2 + |\Psi_{S_z=-1}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)|^2)), \quad (54)$$

$$h_1(x) = \int \frac{d^2 \mathbf{k}_\perp}{2(2\pi)^3} \frac{1}{2\sqrt{2}} \sum_{\lambda_1, \lambda_2} (\Psi_{S_z=1}^*(x, \mathbf{k}_\perp, \uparrow, \lambda_2) \Psi_{S_z=0}(x, \mathbf{k}_\perp, \downarrow, \lambda_2) + \Psi_{S_z=0}^*(x, \mathbf{k}_\perp, \downarrow, \lambda_2) \Psi_{S_z=1}(x, \mathbf{k}_\perp, \uparrow, \lambda_2)$$

$$+ \Psi_{S_z=-1}^*(x, \mathbf{k}_\perp, \downarrow, \lambda_2) \Psi_{S_z=0}(x, \mathbf{k}_\perp, \uparrow, \lambda_2) + \Psi_{S_z=0}^*(x, \mathbf{k}_\perp, \uparrow, \lambda_2) \Psi_{S_z=-1}(x, \mathbf{k}_\perp, \downarrow, \lambda_2)), \quad (55)$$

$$g_1(x) = \int \frac{d^2 \mathbf{k}_\perp}{2(2\pi)^3} \frac{1}{2} \sum_{\lambda_1, \lambda_2} (|\Psi_{S_z=1}(x, \mathbf{k}_\perp, \uparrow, \lambda_2)|^2 - |\Psi_{S_z=1}(x, \mathbf{k}_\perp, \downarrow, \lambda_2)|^2$$

$$+ |\Psi_{S_z=-1}(x, \mathbf{k}_\perp, \downarrow, \lambda_2)|^2 - |\Psi_{S_z=-1}(x, \mathbf{k}_\perp, \uparrow, \lambda_2)|^2). \quad (56)$$

The explicit form of these quark PDFs have been given in the Appendix. Even though this work does not explicitly analyze the chiral-odd GPDs, we make use of their known overlap forms to include the transversity h_1 and helicity

g_1 PDFs. These are presented to offer a more comprehensive understanding of the internal spin structure of spin-1 mesons. The tensor polarized f_{1LL} carries information about the b_1 structure function of the vector mesons [93].

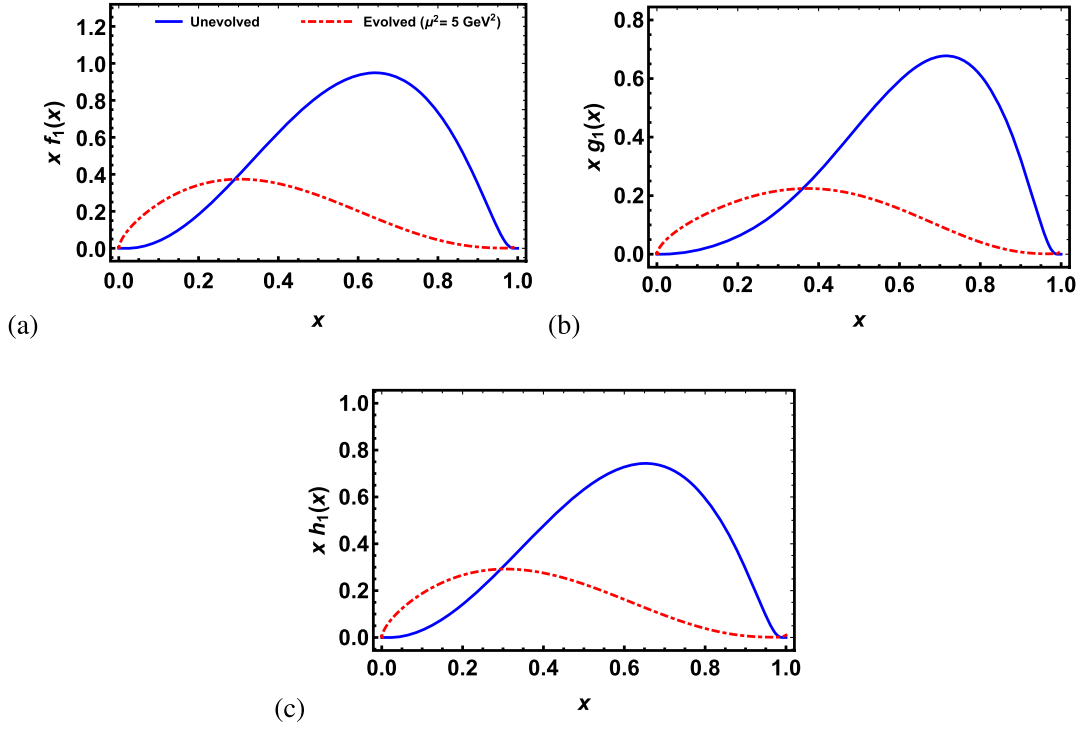


FIG. 13. The unpolarized $xf_1(x)$, helicity $xg_1(x)$, and transversity polarized $xh_1(x)$ quark PDFs have been plotted with respect to longitudinal momentum fraction x for light ρ -meson at model scale $\mu_0^2 = 0.19 \text{ GeV}^2$ (blue solid line) and at scale $\mu^2 = 5 \text{ GeV}^2$ (red dot dashed line) in (a), (b) and (c), respectively.

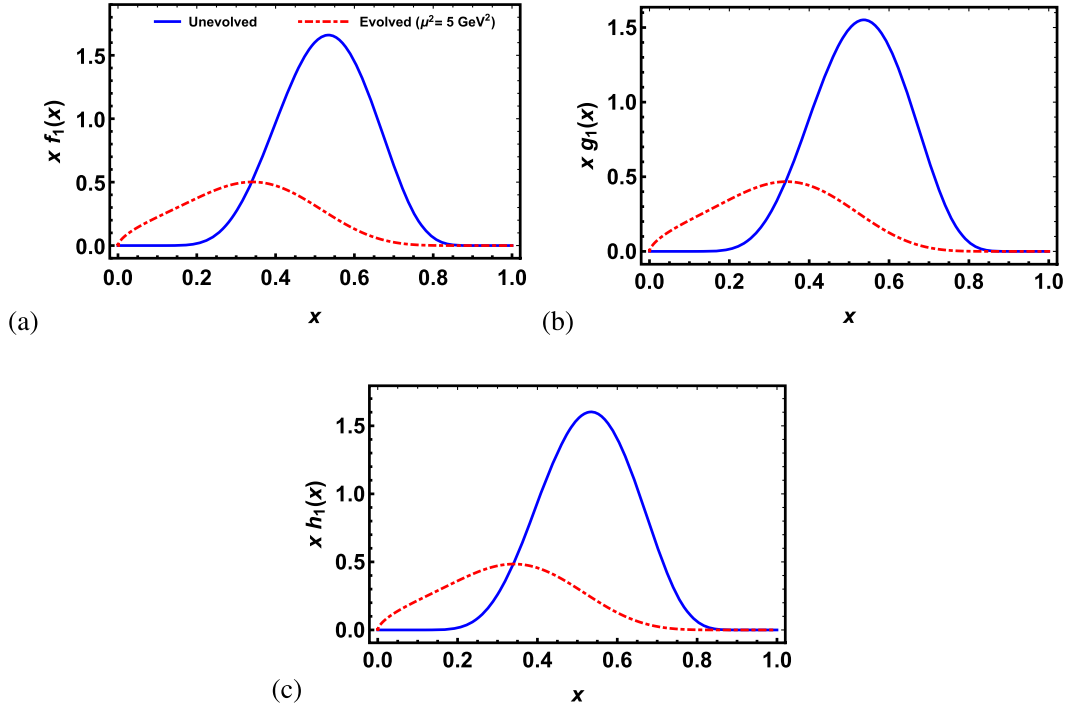


FIG. 14. The unpolarized $xf_1(x)$, helicity $xg_1(x)$, and transversity polarized $xh_1(x)$ quark PDFs have been plotted with respect to longitudinal momentum fraction x for heavy J/ψ -meson at model scale $\mu_0^2 = 0.19 \text{ GeV}^2$ (blue solid line) and at scale $\mu^2 = 5 \text{ GeV}^2$ (red dot dashed line) in (a), (b) and (c), respectively.

TABLE II. The average longitudinal momentum fractions $\langle x^{(0)} \rangle$ and $\langle x^{(1)} \rangle_{\text{evolved}}$ for both ρ and J/ψ vector mesons.

PDFs	$\langle x^{(0)} \rangle$		$\langle x^{(1)} \rangle$		$\langle x^{(1)} \rangle_{\text{evolved}}$	
	ρ	J/ψ	ρ	J/ψ	ρ	J/ψ
$f_1(x)$	1	1	0.5	0.5	0.199	0.210
$g_1(x)$	0.559	0.932	0.314	0.467	0.126	0.190
$h_1(x)$	0.779	0.966	0.392	0.483	0.157	0.203

However, in this work, the $f_{1LL}(x)$ is found to be zero. These PDFs obey the PDF sum rule

$$\int f_1(x)dx = 1, \quad \int f_{1LL}(x)dx = \int b_1(x)dx = 0. \quad (57)$$

These quark PDFs at model scale ($\mu^2 = 0.19 \text{ GeV}^2$) as well as at scale $\mu^2 = 5 \text{ GeV}^2$ have been plotted with respect to the longitudinal momentum fraction x in Figs. 13 and 14 for both light ρ and heavy J/ψ vector mesons, respectively. This evolution is carried out by next to next to leading order Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [94–96]. All the quark PDFs are found to have positive distributions and maximum distributions after $x = 0.5$. Looking into ρ -meson quark PDFs, we observe that unpolarized $f_1(x)$ have higher distributions compared to $g_1(x)$ and $h_1(x)$ quark PDFs. Similar observations can also be seen at $\mu^2 = 5 \text{ GeV}^2$. The ρ -meson quark PDFs have been explored in NJL model [97,98], DS-BSE approach [54], LFHM [80] etc. Our results are found to have a similar kind of positive distribution to the above models. The tensor f_{1LL} quark PDF is found to be zero for both particles. Due to the heavy quark masses of the J/ψ -meson, the quark PDFs show a sharp distribution around $x = 0.5$ and show symmetry $x \leftrightarrow (1-x)$. We observed that there is no distribution of quark PDFs below $x \leq 0.2$ and above $x \geq 0.8$ for J/ψ -meson. In LFHM [33], DS-BSE approach [54], BLFQ [99], and in Ref. [100] have reported similar results and behavior as the present work. The positivity constraints on the PDFs mentioned in Refs. [33,80,97] of spin-1 mesons is also verified in our calculations. The average momentum fraction of order n (Mellin moments) carried by the quark PDFs can be calculated as

$$\langle x^n \rangle = \frac{\int dx x^n \text{PDFs}}{\int dx \text{PDFs}}. \quad (58)$$

The $\langle x^n \rangle$ ($n = 0, 1$) and $\langle x^{(1)} \rangle_{\text{evolved}}$ of both particles has been presented in Table II. We observed that the unpolarized $f_1(x)$ carries the highest average momenta among other PDFs. Furthermore, the result for the evolved Mellin moment $\langle x^{(1)} \rangle_{\text{evolved}}$ is found to be have similar results to that given in Ref. [101].

V. CONCLUSION

In this work, we have evaluated the chiral-even vector quark generalized parton distributions (GPDs) of spin-1 vector ρ and J/ψ mesons at zero skewness, in the light-cone quark model (LCQM). We have employed the Brodsky-Huang-Lepage (BHL) prescription as the momentum space wave function, and the spin part of the wave function is obtained from the Melosh-Wigner rotation.

For spin-1 vector mesons, the vector quark GPDs have been calculated using the correlator $V_{S'_z, S_z}$, which is an overlap of the initial and final state of the light-front wave function. Out of five vector GPDs, $H_4(x, 0, -\Delta_\perp^2)$ is coming out to be zero. We have observed the three-dimensional behavior of the quark GPDs $H_{1(2,3,4,5)}(x, 0, -\Delta_\perp^2)$ with transverse momentum transferred and longitudinal momentum fraction. The vector quark GPDs of J/ψ mesons were analyzed to have a sharper peak in x than that of ρ mesons, which can be attributed to the active quark carrying more momentum at lower x for ρ mesons. Furthermore, the symmetric distribution of $H_1(x, 0, -\Delta_\perp^2)$ as a function of x and $\Delta_\perp^2$ for both the vector mesons due to identical quark content, along with a contribution of superpositioned S, P and D-waves. However, the distribution $H_3(x, 0, -\Delta_\perp^2)$ has a contribution of D-wave and $H_2(x, 0, -\Delta_\perp^2)$ with the contribution from S and P-wave. Also, the behavior of these vector quark GPDs is observed at finite values of x and $\Delta_\perp^2$. Additionally, we have examined the chiral-even vector quark GPDs in impact parameter space, which may be accomplished by taking the GPD's Fourier transform in momentum space.

In light of the progress, the electromagnetic form factors (EM form factors) have also been extracted from the respective GPDs, which are further used to calculate the charge $G_C(Q^2)$, magnetic $G_M(Q^2)$ and quadrupole $G_Q(Q^2)$ form factors for both ρ and J/ψ mesons. Our results have also been compared with the lattice stimulation and other theoretical models, showing a similar trend. Subsequently, physical observables such as the charge radii, magnetic moment, and quadrupole moment for both the vector mesons have also been computed and compared with the available data. We have also predicted the $A(Q^2)$ and $B(Q^2)$ structure functions for the Rosenbluth cross section in terms of charge, magnetic, and quadrupole form factors. Furthermore, we have also evaluated the four parton distribution functions (PDFs), namely, unpolarized $f_1(x)$, helicity $g_1(x)$, transversity $h_1(x)$ and tensor $f_{1LL}(x)$ PDFs of ρ and J/ψ mesons in LCQM at leading twist. Additionally, we have also evolved our PDFs from our model scale to higher scale μ^2 for both the vector mesons.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX

The explicit form of quark PDFs are

$$f_1(x) = \int \frac{d^2\mathbf{k}_\perp}{3(16\pi^3)} [((\mathbf{k}_\perp(1-2x)M_{q\bar{q}})^2 + (2\mathbf{k}_\perp^2 + (M_{q\bar{q}} + 2m)m)^2) + 2((\mathbf{k}_\perp^2(M_{q\bar{q}} + 2m)m) + \mathbf{k}_\perp^2(xM_{q\bar{q}} + m) + (-\mathbf{k}_\perp(1-x)M_{q\bar{q}} + m)^2 + \mathbf{k}_\perp^4)] \frac{|\psi_{q\bar{q}}(x, \mathbf{k}_\perp)|^2}{\omega_{q\bar{q}}^2}, \quad (\text{A1})$$

$$g_1(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} [(\mathbf{k}_\perp^2 + (M_{q\bar{q}} + 2m)m)^2 + (\mathbf{k}_\perp(xM_{q\bar{q}} + m))^2 - \mathbf{k}_\perp^4 - (\mathbf{k}_\perp((1-x)M_{q\bar{q}} + m))^2] \frac{|\psi_{q\bar{q}}(x, \mathbf{k}_\perp)|^2}{\omega_{q\bar{q}}^2}, \quad (\text{A2})$$

$$h_1(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[((2\mathbf{k}_\perp^2 + (M_{q\bar{q}} + 2m)m)(\mathbf{k}_\perp^2 + (M_{q\bar{q}} + 2m)m)) + \left(\frac{1}{2} \mathbf{k}_\perp^2(1-2x)(1-x)M_{q\bar{q}}^2 \right) - \left(\frac{1}{2} (xM_{q\bar{q}} + m)(1-2x)M_{q\bar{q}} \right) \right] \frac{|\psi_{q\bar{q}}(x, \mathbf{k}_\perp)|^2}{\omega_{q\bar{q}}^2}, \quad (\text{A3})$$

$$f_{1ll} = 0. \quad (\text{A4})$$

Here, we have considered $m_q = m_{\bar{q}} = m$.

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