

Transition form factors for D/B to $a_0(980)$ from light-cone QCD sum rules

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We apply the light-cone QCD sum rules with chiral currents to compute transition form factors of the semileptonic decay of the charmed and bottom scalar mesons D/B to $a_0(980)l^+\nu_l$ ($l = e, \mu$), where the scalar meson $a_0(980)$ are firstly regarded as two quark states, and contributions from the tetraquark component are then added via proper modulation of four distribution amplitudes considered. The obtained transition form factors and branching fraction are free of the contribution from the light-cone distribution amplitudes at the level of twist-three and two simple relations connecting their form factors are obtained. Our computations indicates that the decay branch fractions are on the margin of the measurements reported by the Beijing Spectrometer III in the pure two-quark scenario and are in good agreement with observations when tetraquark component is considered.

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I. INTRODUCTION

Exclusive semileptonic decays of mesons are essential for determining the couplings of the weak interaction in the Standard Model, owing to their simpler dynamics relative to nonleptonic decays. Until now, extensive research has been conducted on the semileptonic decays of heavy-light mesons to S-wave mesons, from both experimental [1–5] and theoretical [6–16] perspectives. However, semileptonic decays involving P-wave mesons in the final states remain relatively unexplored. Over the past two decades, this has become a topics of growing interest, with several exclusive decays to P-wave mesons being experimentally reported and measured [17–21].

From a theoretical perspective, the form factors (FFs) of a meson play a crucial role in quantitatively describing its exclusive decays. In the case of semileptonic decay of a charmed or bottom (D/B) meson, for instance, the FF can be computed by various methods such as the large energy effective theory (LEET) [22], lattice quantum chromodynamics (LQCD) [23], the QCD sum rule (QCDSR) [24], the heavy quark effective theory (HQET) [25], and the chiral perturbation theory (chPT) [26]. The applicability of these methods is divided into different regions based on the value of the momentum transfer squared, q^2 , of the decay process:

- (1) In the region where $q^2 \rightarrow 0$, the LEET [22] applies.
- (2) In the region where q^2 is relatively large, the LQCD [23] and the QCDSR [24] apply.
- (3) In the region where q^2 is quite large, the HQET [25] and the chPT [26] apply.

Regarding the decay processes of mesons such as the heavy-light mesons $Q\bar{q}$ shown in Fig. 6(a), a notion of transition form factor (TFF) [27,28] is found to be necessary to describe exclusive decays quantitatively. The use of light-cone QCD sum rules (LCSR) without chiral currents [29], commonly applied to correction function, to compute these FFs requires the inclusion of more distribution amplitudes (DAs)—namely, two twist-3 and one twist-2 light-cone DAs.

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This introduction of multiple DAs necessitates a more involved analysis of the LCSR parameters (Borel parameters and the thresholds), as these parameters differ for each DA. Such complexity typically hampers accurate predictions of the transition FFs.

On the other hand, the physical properties of the scalar meson $a_0(980)$ is closely related to its light-cone DA. This meson is notably intriguing in its internal structure among the well-established scalar mesons. Since 2018, many attempts have been made to produce scalar mesons in decays of the D mesons [30–44]. Regarding the meson $a_0(980)$, it is often interpreted as a multi-quark state or a $K\bar{K}$ bound state [45]. In a sense, the semileptonic D decays provide a pristine environment where the $a_0(980)$ is produced by an isovector combination of the up/down quark (u or d) and its antiquark partner [46]. In 2010, a model-independent way was proposed in Ref. [47] to distinguish the quark components of light scalar mesons: a ratio R of partial decay widths is unity for the quark-antiquark (two-quark) picture of the scalar meson, while it is 3 for the four-quark description of the meson. This predictions has been confirmed by the Beijing Spectrometer (BESIII) in 2018 [46] and in 2021 [48].

The purpose of this work is to apply the light-cone QCD sum rule to explore the transition FF associated with semileptonic decay of the charmed and bottom scalar mesons D/B to $a_0(980)l^+\nu_l$ ($l = e, \mu$), where the final state meson $a_0(980)$ is treated as a two-quark state firstly and further regarded to be tetraquark with $s\bar{s}$ insertion. In the two-quark scenario of the $a_0(980)$, the method we use features itself in that it endows with chiral currents in the correlation functions and enable us to find the FFs free of the contributions from twist-3 light-cone DAs of the scalar mesons. Four different light-cone DAs are explored for the scalar meson $a_0(980)$, to find its respective form factors, and thereby compute the decay widths of the semileptonic decays of the D/B mesons. These DAs for the $a_0(980)$ are: (1) the light-cone DAs obtained by QCDSR at $z^2 = 0$ [49], denoted as DA-I; (2) the light-cone DAs obtained by QCDSR with a novel approximation technique in Ref. [50], denoted as DA-II; (3) the light-cone DAs obtained by the basis light-front quantization (BLFQ) method [51–54], denoted as DA-III; and (4) the new light-cone DAs obtained through moments fitted with the DAs with the help of the BLFQ approach, denoted as DA-IV.

The paper is organized as follows: in Sec. II, we describe the flavor wave functions of the $a_0(980)$ and the light-cone DAs of the twist-2 and twist-3. In Sec. III, we discuss the four light-cone DAs obtained with four different methods. Section IV is devoted to computation of the FFs and branching ratios of the semileptonic decays on $D \rightarrow Sl^+\nu_l$ using the LCSR method. In Sec. V, we present numerical analysis of the FFs and branching ratios of the semileptonic decays of the D/B mesons. A comparison is also given among the four predictions and experiments available. In Sec. VI, we extend our computation to the tetraquark case of

final state meson. The paper ends with a summary in Sec. VII.

II. PHYSICAL PROPERTIES OF SCALAR MESONS

Two scalar mesons $f_0(980)$ and $a_0(980)$ were discovered in early 1990s [55–60] with their internal structure unsolved fully. There are two main scenarios about the structure of these two scalar mesons [49,61–73]. One scenario [74–82] suggests them to be the examples of novel mesons which differ only in isospin quantum number: the $f_0(980)$ is the isoscalar state of $\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$, while the $a_0(980)$ is the isovector state of $\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$. The second scenario regards them to be two members of a scalar tetraquark nonet fully composed of light quarks [61,67], being degenerate approximately in mass due to isospin symmetry of the tetraquarks.

The experimentally measured mass of the $a_0(980)$ lies close to the opening of the $K\bar{K}$ channel, to which it strongly couples [83,84], so this scalar meson must contain a large $K\bar{K}$ component in its wave function and might be described by a mixture of a $q\bar{q}$ state and a tetraquark, both with $J^P = 0^+$. For the $q\bar{q}$ component, its flavor wave function is given by

$$\begin{aligned} a_0^0(980) &= \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \\ a_0^-(980) &= d\bar{u}, \\ a_0^+(980) &= u\bar{d}. \end{aligned}$$

For the tetraquark component strongly coupled to the $K\bar{K}$ channel, its flavor wave function takes the form of

$$\begin{aligned} a_0^0(980) &= \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})s\bar{s}, \\ a_0^-(980) &= d\bar{u}s\bar{s}, \\ a_0^+(980) &= u\bar{d}s\bar{s}. \end{aligned}$$

Estimation by QCD factorization [49] indicates that distribution amplitude of the $a_0(980)$ tends to be smaller in the four-quark picture than in the $q\bar{q}$ picture of the $a_0(980)$. This can be understood that it is less likely for a light quark q (thereby $\bar{q}$) to carry a high momentum fraction x (near endpoint $x \sim 1$) in the tetraquark picture than in the two-quark picture because of partial momentum shared by the strange quark-antiquark ($s\bar{s}$) in the later case. Despite that $a_0(980)$ is most likely to be a four-quark bound state, a quantitative prediction based on the tetraquark picture turns out to be quite nontrivial or complex in practice. To avoid this complexity, we regard $a_0(980)$ as a mixture of a pure $q\bar{q}$ state and a tetraquark state given above, and apply the LCSR method, for the time being, to the $q\bar{q}$ component, that is, we compute its FFs based on the two-quark picture

of the $a_0(980)$. Then, we introduce a proper modulation of the DA of the $a_0(980)$ to make smooth transition of the resulted computation to the four-quark scenario. For this, we begin with the light-cone DA of $a_0(980)$ which is made up of $q_2\bar{q}_1$, as follows:

$$\begin{aligned}\langle S(p)|\bar{q}_2(y)\gamma_\mu q_1(x)|0\rangle &= p_\mu \int_0^1 du e^{i(u p \cdot y + \bar{u} p \cdot x)} \phi_S(u, \mu), \\ \langle S(p)|\bar{q}_2(y)q_1(x)|0\rangle &= m_s \int_0^1 du e^{i(u p \cdot y + \bar{u} p \cdot x)} \phi_S^s(u, \mu), \\ \langle S(p)|\bar{q}_2(y)\sigma_{\mu\nu}q_1(x)|0\rangle &= -m_s(p_\mu z_\nu - p_\nu z_\mu) \\ &\quad \times \int_0^1 du e^{i(u p \cdot y + \bar{u} p \cdot x)} \frac{\phi_S^\sigma(u, \mu)}{6},\end{aligned}$$

where ϕ_S is the twist-2 light-cone DA, while ϕ_S^s and ϕ_S^σ are the twist-3 light-cone DAs, $z = y - x$, and $u(\bar{u} = 1 - u)$ is the momentum fraction carried by the quark $q_2(q_1)$ in total momentum of the meson.

We consider only the twist-2 light-cone DA ϕ_S since the twist-3 light-cone DAs vanish, given that chiral currents are employed in correlation function of the LCSR, as mentioned in Sec. I. For simplicity, the higher twist distribution amplitudes in LCSR are ignored in this work.

III. THE DISTRIBUTION AMPLITUDE OF THE $a_0(980)$

Until now, much devotion has been made to explore distribution amplitudes of the scalar mesons; see Refs. [6–16, 26, 47, 85] for a details. In this section, we typically consider four distribution amplitudes of the scalar meson $a_0(980)$, which are obtained via four different methods and correspond to different form factors, respectively, and apply them to compute respective decay widths of the semileptonic decay processes considered.

A. The distribution amplitudes via Gegenbauer polynomial at $z^2 = 0$ (DA-I)

For the meson with quark configuration $\bar{q}q$, one can generally write its light-cone DA as

$$\begin{aligned}\langle S(p)|\bar{q}_\beta(y)q_\alpha(x)|0\rangle &= \frac{1}{4} \int_0^1 du e^{i(u p \cdot y + \bar{u} p \cdot x)} \\ &\quad \times \left\{ \not{p} \phi_S(u) + m_s \left[\phi_S^s(u) - \sigma_{\mu\nu} p^\mu z^\nu \frac{\phi_S^\sigma(u)}{6} \right] \right\}_{\beta\alpha},\end{aligned}\quad (1)$$

where $S(p)$ stands for the scalar meson with four-momenta p . Here, we consider only the twist-2 light-cone DA ϕ_S thanks to the usage of chiral currents, as discussed in more details in Sec. IV B. For computation of the DA, one way is

to expand it in terms of Gegenbauer polynomial [86, 87], as follows:

$$\phi_S(u, \mu) = \bar{f}_S 6u(1-u)[B_0(\mu) + \sum_{m=1} B_m(\mu) C_m^{3/2}(2u-1)],\quad (2)$$

where the Gegenbauer coefficients B_m are given by

$$B_m(\mu) = \frac{1}{\bar{f}_S} \frac{2(2m+3)}{3(m+1)(m+2)} \int_0^1 C_m^{3/2}(2u-1) \phi_S(u, \mu) du,\quad (3)$$

with $C_m^{3/2}(2u-1)$ the Gegenbauer polynomials, and other parameters such as $\bar{f}_S$ are defined by the following relations:

$$\begin{aligned}\langle S(p)|\bar{q}_2\gamma_\mu q_1|0\rangle &= f_S p_\mu, \\ \langle S(p)|\bar{q}_2 q_1|0\rangle &= m_s \bar{f}_S, \\ \mu_s f_S &= \bar{f}_S, \\ \mu_s &= \frac{m_s}{m_2(\mu) - m_1(\mu)}.\end{aligned}$$

Conformal invariance of QCD in chiral limit indicates that the partial waves in the expansion (2) with different conformal spin cannot mix upon renormalization to a first approximation [87]. Consequently, one can renormalize the coefficients B_m multiplicatively [49], as follows:

$$\begin{aligned}B_m(\mu) &= B_m(\mu_0) \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{-(\gamma_{(m)}+4)/b}, \\ \bar{f}_S(\mu) &= \bar{f}_S(\mu_0) \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{4/b},\end{aligned}$$

where the one-loop anomalous dimensions are given by [88, 89]

$$\gamma_{(m)} = C_F \left[1 - \frac{2}{(m+1)(m+2)} + 4 \left(\sum_{j=2}^{m+1} \frac{1}{j} \right) \right],\quad (4)$$

with $C_F = (N_c^2 - 1)/(2N_c)$. At the energy scale $\mu = 1$ GeV associated with the $a_0(980)$ meson, the renormalized decay constant and Gegenbauer coefficients are found to be [49]

$$\begin{aligned}\bar{f}_S(\mu = 1 \text{ GeV}) &= 0.365 \text{ GeV}, \\ B_1(\mu = 1 \text{ GeV}) &= -0.93 \pm 0.10, \quad B_2(\mu = 1 \text{ GeV}) = 0, \\ B_3(\mu = 1 \text{ GeV}) &= 0.14 \pm 0.08, \quad B_4(\mu = 1 \text{ GeV}) = 0.\end{aligned}$$

Generally, the Gegenbauer coefficients given above are related to moments of the scale meson with isospin $I = 1$ through the following relations [29]:

$$\begin{aligned}
 B_0 &= \langle \xi^0 \rangle, \\
 B_1 &= \frac{5}{3} \langle \xi^1 \rangle, \\
 B_2 &= \frac{35}{12} \langle \xi^2 \rangle - \frac{7}{12} \langle \xi^0 \rangle, \\
 B_3 &= \frac{21}{4} \langle \xi^3 \rangle - \frac{9}{4} \langle \xi^1 \rangle, \\
 B_4 &= \frac{77}{8} \langle \xi^4 \rangle - \frac{77}{12} \langle \xi^2 \rangle + \frac{11}{24} \langle \xi^0 \rangle, \\
 &\dots
 \end{aligned} \tag{5}$$

In the approximation $z^2 \simeq 0$, the moments $\langle \xi_S^n \rangle$ can be computed in the QCD sum rule, obtaining [49]

$$\begin{aligned}
 &\langle \xi_S^n \rangle m_S \bar{f}_S^2 e^{-m_s^2/M^2} + \langle \xi_{S'}^n \rangle m_{S'} \bar{f}_{S'}^2 e^{-m_{s'}^2/M^2} \\
 &= -\frac{3}{16\pi^2} M^2 \left(\frac{m_{q_2} + m_{q_1}}{n+2} + \frac{m_{q_2} - m_{q_1}}{n+1} \right) (1 - e^{-\frac{s_0}{M^2}}) + \langle \bar{q}_2 q_2 \rangle + \frac{10n-3}{24} \frac{\langle \bar{q}_2 g_s \sigma \cdot G q_2 \rangle}{M^2} \\
 &+ \frac{n(4n-5)}{36} \frac{\langle g_s^2 G^2 \rangle \langle \bar{q}_2 q_2 \rangle}{M^4} + (-1)^{n+1} \left[-\frac{3}{16\pi^2} M^2 \left(\frac{m_{q_2} + m_{q_1}}{n+2} + \frac{m_{q_2} - m_{q_1}}{n+1} \right) (1 - e^{-\frac{s_0}{M^2}}) \right. \\
 &\left. + \langle \bar{q}_1 q_1 \rangle + \frac{10n-3}{24} \frac{\langle \bar{q}_1 g_s \sigma \cdot G q_1 \rangle}{M^2} + \frac{n(4n-5)}{36} \frac{\langle g_s^2 G^2 \rangle \langle \bar{q}_1 q_1 \rangle}{M^4} \right].
 \end{aligned} \tag{6}$$

At $z^2 = 0$ and the threshold $s_0 = 3.1 \text{ GeV}^2$, one can apply Eqs. (2), (4), and (6) to compute the light-cone DA. The result is shown in Fig. 1.

B. The distribution amplitudes via Gegenbauer polynomial in the new approximation (DA-II)

For the scalar meson with $I = 1$, its moments include not only terms of order z^2 , but also other terms of the same order, such as $x \cdot z$ and x^2 . To compute moments of such a meson, Ref. [50] introduces, differing from that in Eq. (6), a new approximation, by which one can use the following relation to compute the moments:

$$\begin{aligned}
 \langle \xi^n \rangle &= \frac{e^{m_s^2/M^2}}{m_s \bar{f}_s^2} \left\{ \frac{3}{16\pi^2} [3 + (-1)^n + 2n] \frac{1}{(n+2)(n+1)} [(-1)^{n+1} m_1 + m_2] (1 + e^{-\frac{s_0}{M^2}}) \right. \\
 &+ \langle 0 | \bar{q}_2 q_2 | 0 \rangle + (-1)^{n+1} \langle 0 | \bar{q}_1 q_1 | 0 \rangle + \frac{2n+1}{2M^2} [m_2^2 \langle 0 | \bar{q}_2 q_2 | 0 \rangle + (-1)^{n+1} m_1^2 \langle 0 | \bar{q}_1 q_1 | 0 \rangle] \\
 &+ \frac{m_1 m_2}{2M^2} [\langle 0 | \bar{q}_2 q_2 | 0 \rangle + (-1)^{n+1} \langle 0 | \bar{q}_1 q_1 | 0 \rangle] \\
 &- \frac{2n}{3} \frac{1}{M^2} [\langle 0 | g \bar{q}_2 T G \sigma q_2 | 0 \rangle + (-1)^{n+1} \langle 0 | g \bar{q}_1 T G \sigma q_1 | 0 \rangle] \\
 &\left. + \frac{8n\pi}{81} \frac{1}{M^4} [m_1 \langle 0 | \sqrt{\alpha_s} \bar{q}_2 q_2 | 0 \rangle^2 + (-1)^{n+1} m_2 \langle 0 | \sqrt{\alpha_s} \bar{q}_1 q_1 | 0 \rangle^2] \right\}.
 \end{aligned} \tag{7}$$

At the energy scale $\mu = 1 \text{ GeV}$, Eqs. (7) and (5) explicitly give rise to

$$B_1(\mu = 1 \text{ GeV}) = -0.718, \quad B_2(\mu = 1 \text{ GeV}) = 0, \quad B_3(\mu = 1 \text{ GeV}) = -0.011, \quad B_4(\mu = 1 \text{ GeV}) = 0.$$

The ensuing DA of the $a_0(980)$ by Eq. (2) is shown in Fig. 2.

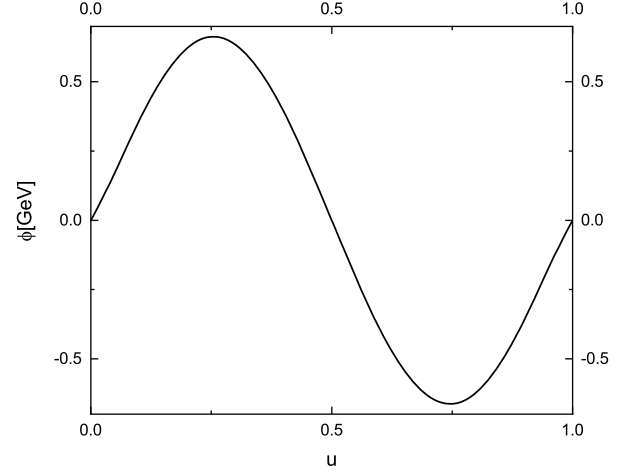


FIG. 1. Distribution amplitudes of the $a_0(980)$ obtained by QCD sum rule at $z^2 = 0$ for DA-I.

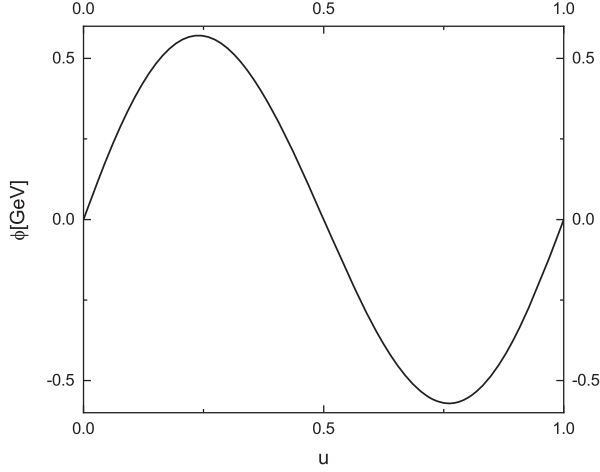


FIG. 2. Distribution amplitudes of the $a_0(980)$ obtained by QCD sum rule at a new approximation for DA-II.

Using a similar method, one can compute the higher moments and the corresponding Gegenbauer coefficients. Writing collectively, the computed results are

$$\begin{aligned} \{\langle \xi^1 \rangle, \langle \xi^3 \rangle, \langle \xi^5 \rangle, \langle \xi^7 \rangle, \dots\}_{\mu=1 \text{ GeV}} \\ = \{-0.431, -0.187, -0.104, -0.066, \dots\}, \\ \{B_1, B_3, B_5, B_7, \dots\}_{\mu=1 \text{ GeV}} \\ = \{-0.718, -0.011, -0.011, -0.009, \dots\}, \end{aligned}$$

which exhibit a notable tendency of convergence associated with n . This is in contrast with that by Eq. (6).

C. The distribution amplitudes based on BLFQ method (DA-III)

In momentum space, the light-front wave function (LFWF) of a meson has the form of [90–101]

$$\Psi_{\{x_i; \vec{p}_\perp; \lambda_i\}}^{N, M_J} = \sum_{n_i; m_i} \psi^N(\{\bar{\alpha}_i\}) \prod_{i=1}^N \phi_{n_i; m_i}(\vec{p}_\perp; b), \quad (8)$$

where $\psi^{N=2}(\{\bar{\alpha}_i\})$ is the component of the mesonic eigenvectors associated with the Fock state $|q\bar{q}\rangle$, and $\psi^{N=3}(\{\bar{\alpha}_i\})$ is the component of the meson eigenvectors associated with $|q\bar{q}g\rangle$, $\{\bar{\alpha}\} \equiv \{x, n, m, \lambda\}$, in which x , n , m , and λ are the longitudinal momentum fraction, and the radial, angular, and spin quantum numbers, respectively, and $\phi_{n_i; m_i}(\vec{p}_\perp; b)$ is the two-dimensional harmonic-oscillator basis function, solved via diagonalizing the full Hamiltonian matrix in the BLFQ method [51–54].

We note that in practical computation, there are two up-limit numbers $N_{\max}$ and K fixing the total numbers of bases of the wave functions ψ^N and $\phi_{n_i; m_i}$, respectively. These two numbers, $N_{\max}$ and K , are introduced to truncate the infinite bases of the respective Fock spaces. The first truncation via

TABLE I. Model parameters for the truncation $N_{\max} = 14$, $K = 15$. All are in units of GeV except g_s , which is dimensionless.

m_q	m_g	b	κ	m_f	g_s
0.39	0.60	0.29	0.65	5.69	1.92

$N_{\max}$ enables us to factorize out the transverse center of mass motion and manifest a natural UV regulator $\Lambda_{\text{UV}} \sim b\sqrt{N_{\max}}$. Given the truncations $N_{\max} = 14$, $K = 15$, one can fix the model parameters so that the ensuing eigenvalues of the full Hamiltonian matrix in the BLFQ method reproduce the measured mass spectra of light mesons. Here, the model parameters include the quark mass m_q , the effective mass of gluon m_g , the harmonic oscillator scale parameter b , the strength of the confinement κ , and the two parameters m_f and g_s (independent of quark mass) in the vertex interaction and coupling constant. We summarize the resulting values of the parameters in Table I.

In the BLFQ method, the light-cone DAs of the scalar meson $q_2\bar{q}_1$ are

$$\langle 0 | \bar{q}_2(z) \gamma^+ q_1(-z) | S(p) \rangle = i p^+ \int_0^1 dx e^{i p^+ z^- (x - \frac{1}{2})} \phi_S(x). \quad (9)$$

In terms of LFWFs, the light-cone DA in Eq. (9) can be solved to be [90,102–110]

$$\phi_S(x) = \bar{f}_S \frac{2\sqrt{2N_C}}{N_F} \int [d^2 p_\perp] \Psi^{\uparrow\downarrow\uparrow\downarrow}(x, p_\perp), \quad (10)$$

with N_F the renormalized constant,

$$N_F = \sum_{s_1 s_2} \int [d^3 p_1] [d^3 p_2] \Psi^{s_1 s_2*}(p_1, p_2) \Psi^{s_1 s_2}(p_1, p_2),$$

where $p_{1,2}$ are the momentum of the quark q_1 and $\bar{q}_2$ and $s_{1,2}$ are the spins of two quarks. Given the LFWFs solved above, one can use Eqs. (10) and (8) to compute the distribution amplitude of the $a_0(980)$. The results are shown Fig. 3.

D. The distribution amplitudes as via Gegenbauer polynomial from BLFQ method (DA-IV)

Assuming distribution amplitudes in DA-III are known rigorously, one can obtain the following relation between moments and the DAs, without any approximation:

$$\langle \xi^n \rangle = \int_0^1 dx (2x - 1)^n \phi_S(x).$$

At the energy scale $\mu = 1 \text{ GeV}$, this enable us to compute the moments and Gegenbauer coefficients. We list the results collectively:

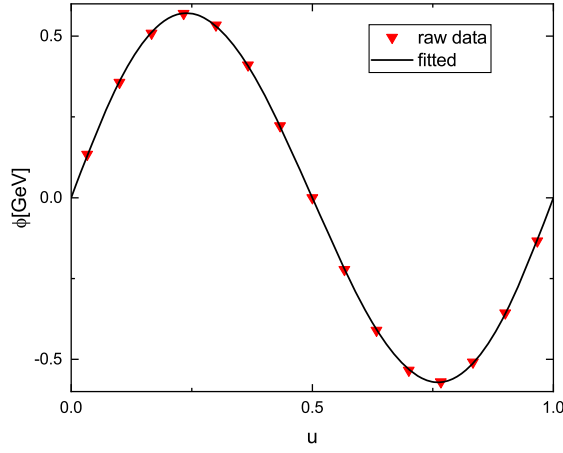


FIG. 3. Distribution amplitudes of the $a_0(980)$ obtained by BLFQ for DA-III.

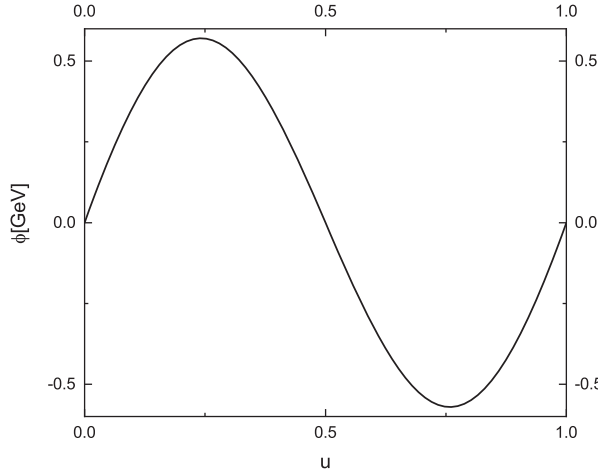


FIG. 4. Distribution amplitudes of the $a_0(980)$ obtained from BLFQ for DA-IV.

$$\begin{aligned} & \{\langle \xi^1 \rangle, \langle \xi^3 \rangle, \langle \xi^5 \rangle, \langle \xi^7 \rangle, \dots\}_{\mu=1 \text{ GeV}} \\ &= \{-0.510, -0.205, -0.109, -0.067, \dots\}, \\ & \{B_1, B_3, B_5, B_7, \dots\}_{\mu=1 \text{ GeV}} \\ &= \{-0.849, 0.073, -0.004, -0.001, \dots\}. \end{aligned}$$

One sees that moments and Gegenbauer coefficients also shows a notable tendency of convergence with respect to n . This follows the distribution amplitude in the BLFQ method. We plot the results in Fig. 4.

Before ending of this section, we briefly discuss the main features of four distribution amplitudes obtained from the different methods mentioned. We plot the four DAs associated with the respective methods collectively in Fig. 5 for comparison.

Comparing the resulted light-cone DAs in Fig. 5, one can conclude:

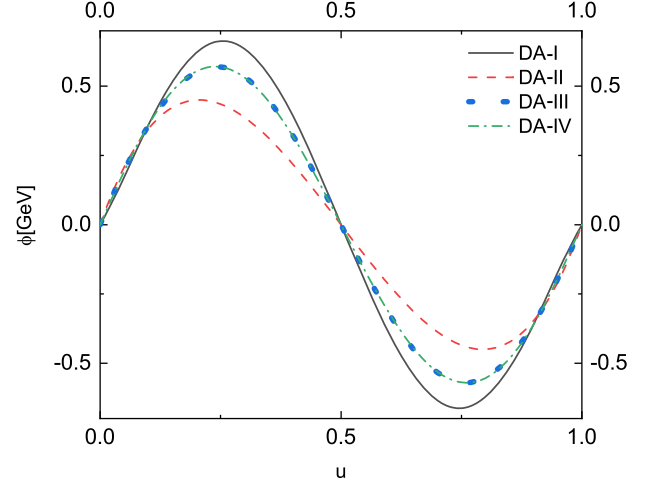


FIG. 5. Distribution amplitudes of the $a_0(980)$ shown as a solid line (DA-I), dashed line (DA-II), dotted line (DA-III), and dashed-dotted line (DA-IV). Two lines corresponding to DA-III and DA-IV overlap exactly.

- (1) The uncertainty of the distribution amplitudes computed via QCD sum rule comes mainly from the approximation on z^2 .
- (2) The distribution amplitudes via the two methods, DA-III and DA-IV, are highly consistent with each other.

IV. FORM FACTORS OF $D/B \rightarrow S$

In the semileptonic decays, one can parametrize the hadron matrix elements, which implicitly encode strong interaction of quarks in hadrons, in terms of the computable FFs. As such, the FF plays an important role in the computation of the decay width. In this section, we employ LCSR to compute a set of (nonvanishing) independent FFs for the semileptonic decays of $D/B \rightarrow S$, which are referred to as $f_+^{D/B \rightarrow S}$, $f_-^{D/B \rightarrow S}$, and $f_T^{D/B \rightarrow S}$. Generally, there are two kind of hadron transition, associated with two decays, which are of the vector and tensor types, for which the parametrizations takes the form [111–114]

$$\begin{aligned} & \langle S(p) | \bar{q}_2 \gamma_\mu \gamma_5 Q | D/B(p+q) \rangle \\ &= -2i f_+^{D/B \rightarrow S}(q^2) p_\mu - i [f_+^{D/B \rightarrow S}(q^2) + f_-^{D/B \rightarrow S}(q^2)] q_\mu, \end{aligned} \quad (11)$$

$$\begin{aligned} & \langle S(p) | \bar{q}_2 \sigma_{\mu\nu} (1 + \gamma_5) q^\nu Q | D/B(p+q) \rangle \\ &= [2p_\mu q^2 - 2q_\mu (p \cdot q)] \frac{-f_T^{D/B \rightarrow S}(q^2)}{m_{D/B} + m_S}, \end{aligned} \quad (12)$$

where $f_{+,-}^{D/B \rightarrow S}(q^2)$ are the transition FFs associated with the transition processes of $Q \rightarrow q_2 \bar{l} \nu_l$, plotted in Fig. 6(a), and $f_T^{D/B \rightarrow S}(q^2)$ is the penguin FF associated with the penguin-induced transition processes of $Q \rightarrow q_2 q_3 \bar{q}_4$, shown in

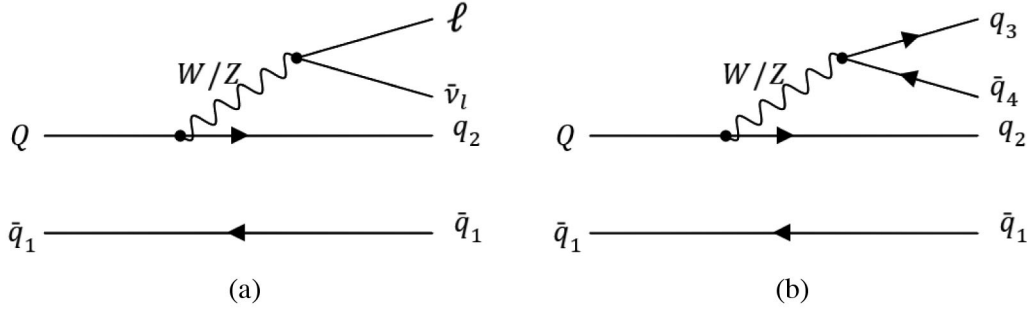


FIG. 6. (a) processes associated with f_+ and f_- . The difference between f_+ and f_- lies in the Lorentz structure. (b) processes associated with f_T .

TABLE II. Chiral currents for different decay processes, where Q stands for b or c quark.

Mode	$J_{i\mu}(x)$	$J_i(0)$
$D/B \rightarrow S$	$J_{1\mu}(x) = \bar{q}_2(x)\gamma_\mu(1 - \gamma_5)Q(x)$ $J_{2\mu}(x) = \bar{q}_2(x)\sigma_{\mu\nu}(1 + \gamma_5)Q(x)$	$J_1(0) = \bar{Q}(0)i(1 - \gamma_5)q_1(0)$ $J_2(0) = \bar{Q}(0)i(1 - \gamma_5)q_1(0)$

Fig. 6(b) (only the lowest order Feynman diagrams are presented here). The variable q stands for the transfer momentum q in transition, which should be distinguished from the same symbol used for the light quarks.

To compute the FFs above with LCSR, we construct two-point correlation functions,

$$\begin{aligned}\Pi_{1\mu}(p, q) &= i \int d^4x e^{iqx} \langle S(p) | T \{ J_{1\mu}(x), J_1(0) \} | 0 \rangle, \\ \Pi_{2\mu}(p, q) &= i \int d^4x e^{iqx} \langle S(p) | T \{ J_{2\mu}(x), J_1(0) \} | 0 \rangle, \quad (13)\end{aligned}$$

where $q = p_{D/B} - p$ stands for the transfer momentum, and $p_{D/B}$ and p are the four-momenta of the initial and final meson states, respectively. Here, $J_{1\mu}(x)$ stand for the chiral currents associated with the vector transition and $J_{2\mu}(x)$ for the chiral currents associated with the tensor transition, as given explicitly in Table II.

According to the LCSR method, one can compute the correction functions given by Eq. (13) separately from two viewpoints and then match ensuing results to extract the information of hadrons. The first viewpoint is phenomenological at the hadronic level, in which one inserts a complete set of meson states between two currents appearing in the correlation function. The second viewpoint happens at the quark level, for which one applies the operator product expansion to reformulate the correlation functions. During the computation via the LCSR method, a complete set of the D/B meson is inserted into the matrix elements in Eq. (13), which correspond to the initial state in the decay process.

A. On the phenomenological side

Inserting the complete set of the initial (D/B meson) states $|i\rangle$ in the correlation functions (13) and isolating the ground-state components, which are referred also as $|D/B\rangle$, in the complete set, one has

$$\begin{aligned}\Pi_{1\mu}(p, q) &= \frac{\langle S(p) | \bar{q}_2 \gamma_\mu (1 \pm \gamma_5) Q | D/B \rangle \langle D/B | \bar{Q} i \gamma_5 q_1 | 0 \rangle}{m_{D/B}^2 - (p + q)^2} + \text{higher states}, \\ \Pi_{2\mu}(p, q) &= \frac{\langle S(p) | \bar{q}_2 \sigma_{\mu\nu} (1 \pm \gamma_5) q^\nu Q | D/B \rangle \langle D/B | \bar{Q} i \gamma_5 q_1 | 0 \rangle}{m_{D/B}^2 - (p + q)^2} + \text{higher states}, \quad (14)\end{aligned}$$

where the hadronic matrix elements $\langle S(p) | \bar{q}_2 \gamma_\mu (1 \pm \gamma_5) Q | D/B \rangle$ and $\langle S(p) | \bar{q}_2 \sigma_{\mu\nu} (1 \pm \gamma_5) q^\nu Q | D/B \rangle$ are to be parametrized by the FFs, as shown in Eqs. (11) and (12).

Besides Eqs. (11) and (12), the other matrix element can be parametrized by

$$\langle D/B | \bar{Q} i \gamma_5 q_1 | 0 \rangle = \frac{m_{D/B}^2 f_{D/B}}{m_Q + m_{q_1}}. \quad (15)$$

Substituting Eqs. (11) and (12) into Eq. (14) and taking account of Lorentz structures, one can obtain the phenomenological formulation of two correlation functions. The results are, with the usage of Eq. (15),

$$\begin{aligned}\Pi_{1\mu}(p, q) &= -\frac{2if_+^{D/B \rightarrow S}(q^2)p_\mu + i[f_+^{D/B \rightarrow S}(q^2) + f_-^{D/B \rightarrow S}(q^2)]q_\mu}{m_{D/B}^2 - (p+q)^2} \frac{m_{D/B}^2 f_D}{m_Q + m_{q_1}} + \int_{s_0}^{\infty} \frac{\rho(s)ds}{s - (p+q)^2}, \\ \Pi_{2\mu}(p, q) &= -\frac{q^2 2p_\mu - 2q_\mu(p \cdot q)}{m_{D/B}^2 - (p+q)^2} \frac{if_T^{D/B \rightarrow S}}{m_{D/B} + m_S} \frac{m_{D/B}^2 f_{D/B}}{m_Q + m_{q_1}} + \int_{s_0}^{\infty} \frac{\rho(s)ds}{s - (p+q)^2}.\end{aligned}\quad (16)$$

To avoid the complexity of involving higher excited and continuum spectra on the phenomenological side, it is common to apply quark-hadron duality to simplify integrals involving higher excited states and continuum spectra [89,115–121], as follows:

$$\int_{s_0}^{\infty} ds \frac{\rho(s)}{s - (p+q)^2} \simeq \int_{s_0}^{\infty} ds \frac{1}{\pi} \frac{\text{Im} \Pi_\mu^{\text{pert}}(p, q)}{s - (p+q)^2}, \quad (17)$$

where s_0 is the threshold near the squared mass of the first excited state of D/B mesons, and $\rho(s)$ is the spectral density of the higher excited states and the continuum states.

B. On the theoretical side

At the quark level, which is relevant to the region of the large momentum, the computation of the correlation function will be based on the T-product expansion near the light-cone $x^2 \approx 0$. By contracting quark fields Q and $\bar{Q}$, one has

$$\begin{aligned}\Pi_{1\mu}(p, q) &= i \int d^4x e^{iqx} \langle S(p) | \{ \bar{q}_2(x) \gamma_\mu (1 \pm \gamma_5) S^Q(x, 0) i(1 \pm \gamma_5) q_1(0) \} | 0 \rangle, \\ \Pi_{2\mu}(p, q) &= i \int d^4x e^{iqx} \langle S(p) | \{ \bar{q}_2(x) \sigma_{\mu\nu} (1 \pm \gamma_5) q^\nu S^Q(x, 0) i(1 \pm \gamma_5) q_1(0) \} | 0 \rangle,\end{aligned}\quad (18)$$

where $S^Q(x, 0)$ is the propagator of the Q quark,

$$S^Q(x, 0) = -i \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot x} \frac{\not{k} + m_Q}{m_Q^2 - k^2}. \quad (19)$$

Combining these relations, the correlation function then becomes

$$\begin{aligned}\Pi_{1\mu}(p, q) &= i \int \frac{d^4x d^4k}{(2\pi)^4} \frac{e^{i(q-k)x}}{m_c^2 - k^2} \text{Tr}[\gamma_\mu (1 \pm \gamma_5) (\not{k} + m_Q) (1 \pm \gamma_5)]_{\beta\alpha} \times \langle S(p) | \bar{q}_{2\beta}(x) q_{1\alpha}(0) | 0 \rangle, \\ \Pi_{2\mu}(p, q) &= i \int \frac{d^4x d^4k}{(2\pi)^4} \frac{e^{i(q-k)x}}{m_c^2 - k^2} \text{Tr}[\sigma_{\mu\nu} (1 \pm \gamma_5) q^\nu (\not{k} + m_Q) (1 \pm \gamma_5)]_{\beta\alpha} \times \langle S(p) | \bar{q}_{2\beta}(x) q_{1\alpha}(0) | 0 \rangle.\end{aligned}\quad (20)$$

In the QCDSR method, the nonvanishing matrix elements for scalar mesons are expanded in terms of distribution amplitudes, as shown in Eq. (1).

For the processes of $D/B \rightarrow Sl^+\nu_l$, one can find, upon tracing operation of Eq. (20), that the contribution of twist-3 light cone distribution amplitudes vanishes, left merely with the contribution from the twist-2 LCDA. As a result, the theoretical side of correlation function becomes

$$\Pi_{1\mu} = 2im_Q p_\mu \int_0^1 du \frac{\phi_S(u)}{m_Q^2 - (q+up)^2}, \quad \Pi_{2\mu} = -[2p_\mu q^2 - 2q_\mu(p \cdot q)] \int_0^1 du \frac{\phi_S(u)}{m_Q^2 - (q+up)^2}. \quad (21)$$

C. Form factors

Writing explicitly the correlations in Eq. (16) on the phenomenological side and those given by Eq. (21) on theoretical side and matching two sides, one gets

$$\begin{aligned}
& -\frac{2if_+^{D/B \rightarrow S}(q^2)p_\mu + i[f_+^{D/B \rightarrow S}(q^2) + f_-^{D/B \rightarrow S}(q^2)]q_\mu m_{D/B}^2 f_{D/B}}{m_{D/B}^2 - (p+q)^2} \frac{m_Q^2}{m_Q + m_{q_1}} = 2im_c p_\mu \int_\Delta du \frac{\phi(u)}{m_Q^2 - (q+up)^2}, \\
& -\frac{2[q^2 p_\mu - q_\mu(p \cdot q)]}{m_{D/B}^2 - (p+q)^2} \frac{if_T^{D/B \rightarrow S}}{m_{D/B} + m_S} \frac{m_{D/B}^2 f_{D/B}}{m_Q + m_{q_1}} = -[2p_\mu q^2 - 2q_\mu(p \cdot q)] \int_\Delta du \frac{\phi(u)}{m_Q^2 - (q+up)^2}, \quad (22)
\end{aligned}$$

where Δ is one of the solutions to the algebraic equation $us - m_Q^2 - u(1-u)m_S^2 + (1-u)q^2 = 0$, as follows:

$$\Delta = \frac{\sqrt{(s_0 - m_S^2 - q^2)^2 + 4(m_c^2 - q^2)m_S^2}}{2m_S^2} - \frac{(s_0 - m_S^2 - q^2)}{2m_S^2}. \quad (23)$$

In the framework of LCSR, one has to suppress the contribution of the high excited as well as the continuum states in Eq. (22). To do this, it is common to apply Borel transformation [121] on both sides of Eq. (22). The key relations of the Borel transformation consist of

$$B_{M^2} \frac{1}{m_{D/B}^2 - (q+p)^2} = \frac{1}{M^2} e^{-\frac{m_{D/B}^2}{M^2}}, \quad B_{M^2} \frac{1}{m_Q^2 - (q+up)^2} = \frac{1}{uM^2} e^{-\frac{m_Q^2 + u\bar{u}p^2 - \bar{u}q^2}{uM^2}},$$

where M^2 is the Borel parameter. Using the Borel transformation on Eq. (22), we get the following FFs of $D/B \rightarrow Sl^+\nu_l$ processes:

$$\begin{aligned}
f_+^{D/B \rightarrow S}(q^2) &= -\frac{m_Q + m_{q_1}}{m_{D/B}^2 f_{D/B}} m_Q \int_\Delta \frac{\phi(u)}{u} du e^{FF}, & f_-^{D/B \rightarrow S}(q^2) &= \frac{m_Q + m_{q_1}}{m_{D/B}^2 f_{D/B}} m_Q \int_\Delta \frac{\phi(u)}{u} du e^{FF}, \\
f_T^{D/B \rightarrow S}(q^2) &= -(m_{D/B} + m_S) \frac{m_Q + m_{q_1}}{m_{D/B}^2 f_{D/B}} \int_\Delta \frac{\phi(u)}{u} du e^{FF}, \quad (24)
\end{aligned}$$

where

$$FF := -\frac{1}{uM^2} (m_Q^2 + u\bar{u}p^2 - \bar{u}q^2) + \frac{m_{D/B}^2}{M^2}. \quad (25)$$

From the FFs obtained above, one can find a set of simple relation between them, as follows:

$$f_-^{D/B \rightarrow S}(q^2) = -f_+^{D/B \rightarrow S}(q^2), \quad f_T^{D/B \rightarrow S}(q^2) = \frac{m_{D/B} + m_S}{m_Q} f_+^{D/B \rightarrow S}(q^2). \quad (26)$$

D. Branching fractions

Given the above FFs, we are in the position to compute the differential decay widths of semileptonic decay [16,111,122–134]. For the process $D/B \rightarrow Sl^+\nu_l$, which we address in this work, the LCSR enables us to write the differential decay width $d\Gamma/dq^2$ in terms of the FFs [29], as follows:

$$\begin{aligned}
\frac{d\Gamma(D/B \rightarrow Sl\bar{\nu}_l)}{dq^2} &= \frac{G_F^2 |V_{Qd}|^2}{768\pi^3 m_{D/B}^3} \frac{(q^2 - m_l^2)^2}{q^6} \sqrt{(m_{D/B}^2 + m_S^2 - q^2)^2 - 4m_{D/B}^2 m_S^2} \\
&\times \{ (f_+(q^2))^2 [(q^2 + m_S^2 - m_{D/B}^2)(q^2 + 2m_l^2) - q^2 m_S^2 (4q^2 + 2m_l^2)] \\
&+ 6f_+(q^2)f_-(q^2)q^2 m_l^2 (m_{D/B}^2 + m_S^2 - q^2) + 6(f_-(q^2))^2 q^4 m_l^2 \}, \quad (27)
\end{aligned}$$

where G_F is the Fermi coupling constant and $|V_{Qd}|$ is Cabibbo-Kabayashi-Maskawa matrix element. Integrating q^2 over a range of $m_l^2 \leq q^2 \leq (m_D - m_S)^2$, one can obtain the decay width $\Gamma(D/B \rightarrow Sl^+\nu_l)$ and thereby the decay branching fractions

$$\mathcal{B}(D/B \rightarrow Sl\bar{\nu}_e) = \frac{\Gamma(D/B \rightarrow Sl\bar{\nu}_l)}{\Gamma_{\text{total}}},$$

where Γ_{total} is the total decay width and equal to the inverse lifetime of the D/B mesons, as follows:

$$\Gamma_{\text{total}} = \frac{1}{\tau_{D/B}}.$$

V. NUMERICAL ANALYSES AND DISCUSSIONS

A. Choices of input parameters

In order to apply the relations in Sec. IV to numerically compute the FFs and branching fractions, one has to fix the parameters involved in the processes of semileptonic decay we consider. As numerical inputs, we employ the values of the parameters given in Refs. [49,135,136], which are shown collectively in Table III.

We note that the choices of the threshold and the Borel parameter are crucial to find the reasonable results within the LCSR. Normally, the threshold is chosen as squared mass of the first excited state of the D or B meson. However, the such-chosen values of the threshold are not universal for different processes and have to be determined individually. Since FFs should not depend on Borel parameter, one normally constraints the range of the Borel parameter so that

- (1) The contributions from the higher excited and continuum states are less than 30%.
- (2) The dependence of the FF upon the Borel parameters is weak.

Given the two constraints listed above, one can numerically search and determine the corresponding thresholds and Borel windows for the considered semileptonic decay

processes. As such, one is able to use four different methods (DA-I, DA-II, DA-III, and DA-IV) to determine distribution amplitudes of the scalar meson $a_0(980)$ and thereby one can compute respective form factors for the semileptonic decay we consider. The obtained results for the transition FFs for the decay $D^0 \rightarrow a_0^-(980)$ are plotted in Figs. 7 and 8.

One sees from Fig. 7 that f_+ is stable with variation of M^2 within the range of $1.5 \text{ GeV}^2 < M^2 < 2.2 \text{ GeV}^2$. The dependence of the transition FFs for the decay $D^0 \rightarrow a_0^-(980)$ on the threshold s_0 is plotted in Fig. 8. Following Ref [138], we take the threshold s_0 to vary in the range $5 \text{ GeV}^2 \leq s_0 < 5.3 \text{ GeV}^2$, which is somehow smaller than the measured threshold $s_0 = 6.5 \text{ GeV}^2$ reported in experiments [137].

Given the parameter windows for LCSR, one can fix the FFs at $q^2 = 0$ and list them in Table IV, where the obtained results are compared to that by other methods cited for the decays $D/B \rightarrow Sl^+\nu_l$. Further, in Fig. 9, we show dependence of the FF on the transferred momentum q^2 with $m_l^2 \leq q^2 \leq (m_D - m_S)^2$ for the $D \rightarrow S$ at $s_0 = 5.18 \text{ GeV}^2$ and $M^2 = 1.72 \text{ GeV}^2$. As indicated from Figs. 8 and 9 the FFs are not sensitive to the threshold s_0 and the momentum transfer q^2 .

B. Branching fractions

Application of the FFs obtained in Sec. VA and the relations in Sec. IV leads to respective decay widths and corresponding branching fractions of the semileptonic decay processes considered in this work. In Figs. 10 and 11, we show the differential decay width versus transfer momentum q^2 .

Integrating over a range of $m_l^2 \leq q^2 \leq (m_D - m_S)^2$, we obtain the decay widths and thereby the decay branching

TABLE III. Values of the parameters for FFs and branching fractions [49,136,137].

Method	$m_{a_0(980)}$ [GeV]	$\tilde{f}_M$ [GeV]	B_1 ($\mu = 1 \text{ GeV}$)	B_3 ($\mu = 1 \text{ GeV}$)
DA-I			-0.93	0.14
DA-II			-0.72	-0.01
DA-III	0.980	0.365		
DA-IV			-0.85	0.07
m_{D^0} [GeV]	m_{D^+} [GeV]	$ V_{cd} $	τ_{B^+} [10^{-12} s]	τ_{B^0} [10^{-12} s]
1.865	1.870	0.220	1.638 ± 0.004	1.519 ± 0.004
m_b [GeV]	m_c [GeV]	$ V_{bu} $	f_D [GeV]	f_B [GeV]
4.18	1.27	0.004	0.2054	0.19
G_F [GeV^{-2}]	m_e [GeV]	m_μ [GeV]	τ_{D^+} [10^{-12} s]	τ_{D^0} [10^{-13} s]
1.1664×10^{-5}	0.511×10^{-3}	0.106	1.040 ± 0.007	4.101 ± 0.015
m_{B^0} [GeV]	m_{B^+} [GeV]	m_u [GeV]	m_d [GeV]	...
5.280	5.280	2.16×10^{-3}	4.67×10^{-3}	...

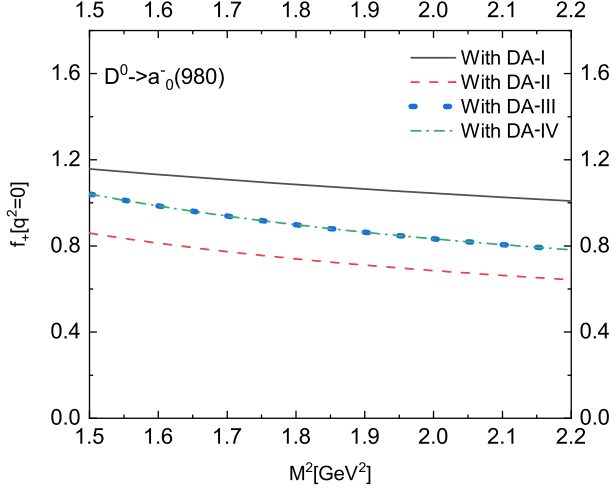


FIG. 7. Transition FF for $D^0 \rightarrow a_0^-(980)$ as a function of Borel parameter M^2 at threshold $s_0 = 5.18 \text{ GeV}^2$ and $q^2 = 0$. The four lines—solid (DA-I), dashed (DA-II), dotted (DA-III), and dashed-dotted (DA-IV)—correspond to the four distribution amplitudes given by different methods, with two of the lines (DA-III and DA-IV) overlapping exactly.

fractions, with the numerical results shown in Table V. The branching fractions computed with different methods and their comparisons are plotted in Fig. 12 for $D^0 \rightarrow a_0^-(980)e^+\nu_e$, Fig. 13 for $D^+ \rightarrow a_0^0(980)e^+\nu_e$, and Fig. 14 for $D^+ \rightarrow a_0^0(980)\mu^+\nu_\mu$. The measurement by BESIII [46] yields the ratio of two partial widths, $\Gamma(D^0 \rightarrow a_0^-(980)e^+\nu_e)/\Gamma(D^+ \rightarrow a_0^0(980)e^+\nu_e)$, to be $2.03 \pm 0.95 \pm 0.06$, which is in consistent with our

TABLE IV. Form factors at $q^2 = 0$ for the transition $D/B \rightarrow S$ compared with those via other theoretical methods cited.

Processes	$D \rightarrow a_0(980)$		$B \rightarrow a_0(980)$	
	f_+	f_-	f_+	f_-
LCSR with DA-I	1.03	-1.03	0.66	-0.66
LCSR with DA-II	0.77	-0.77	0.53	-0.53
LCSR with DA-III	0.93	-0.93	0.61	-0.61
LCSR with DA-IV	0.93	-0.93	0.61	-0.61
QCDSR [139]	...	...	0.56	-0.56
pQCD [126]	...	...	0.39	...
LCSR [29]	1.75	0.31	...	...
The covariant confining quark model (CCQM) [42]	0.55	...	...	...

predictions for this ratio. For the decay processes of $D^{0(+)}B^{0(+)} \rightarrow a_0^{-(0)}(980)l^+\nu_l$ ($l = e, \mu$) in Table V, one can easily show that our predictions of branching fractions are well consistent with the measured values by BESIII [46] within the error range, where we have used the following branching ratio [129]:

$$\mathcal{B}(a_0(980) \rightarrow \eta\pi) = 0.845 \pm 0.017.$$

Note that the difference of branching fractions between our and other theoretical methods comes mainly from the differences of the FF $f_+(q^2)$, which makes the main contribution to the differential decay width.

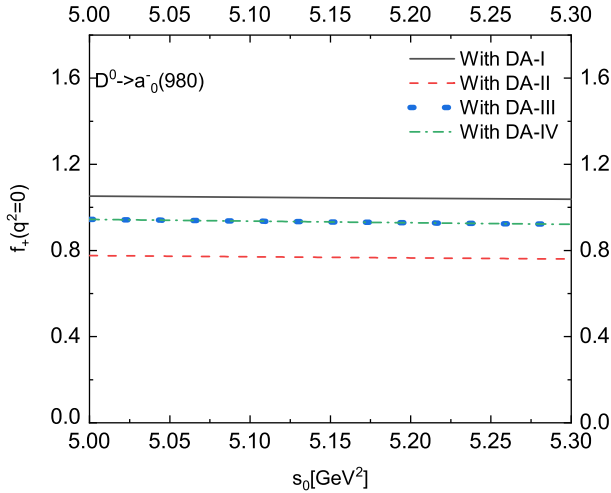


FIG. 8. Transition FF for $D^0 \rightarrow a_0^-(980)$ versus the threshold s_0 at the Borel parameter $M^2 = 1.72 \text{ GeV}^2$ and $q^2 = 0$. The four lines—solid (DA-I), dashed (DA-II), dotted (DA-III), and dashed-dotted (DA-IV)—correspond to the four distribution amplitudes given by different methods, with two of the lines (DA-III and DA-IV) overlapping exactly.

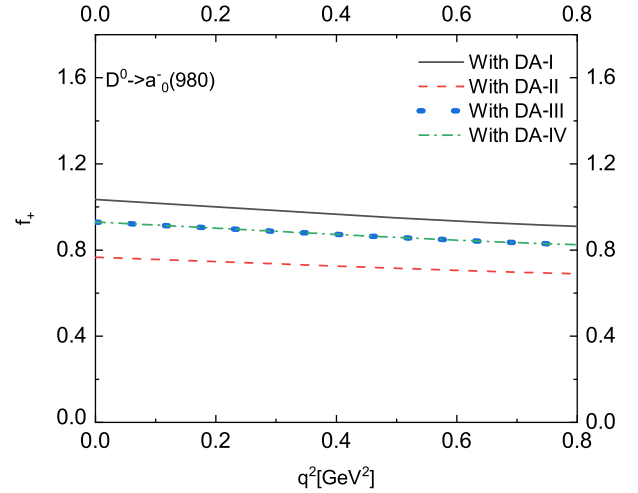


FIG. 9. FFs of $D^0 \rightarrow a_0^-(980)$ decay as a function of q^2 at Borel parameter $M^2 = 1.72 \text{ GeV}^2$ and the threshold $s_0 = 5.18 \text{ GeV}^2$. The four lines—solid (DA-I), dashed (DA-II), dotted (DA-III), and dashed-dotted (DA-IV)—correspond to the four distribution amplitudes given by different methods, with two of the lines (DA-III and DA-IV) overlapping exactly.

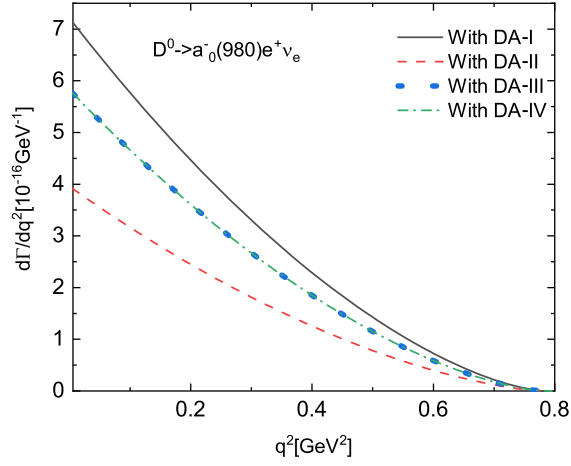


FIG. 10. Differential decay width of $D^0 \rightarrow a_0(980)e^+\nu_e$ for $m_e^2 \leq q^2 \leq (m_D - m_{a_0(980)})^2$ by four methods. The four lines—solid (DA-I), dashed (DA-II), dotted (DA-III), and dashed-dotted (DA-IV)—correspond to the four distribution amplitudes given by different methods, with two of the lines (DA-III and DA-IV) overlapping exactly.

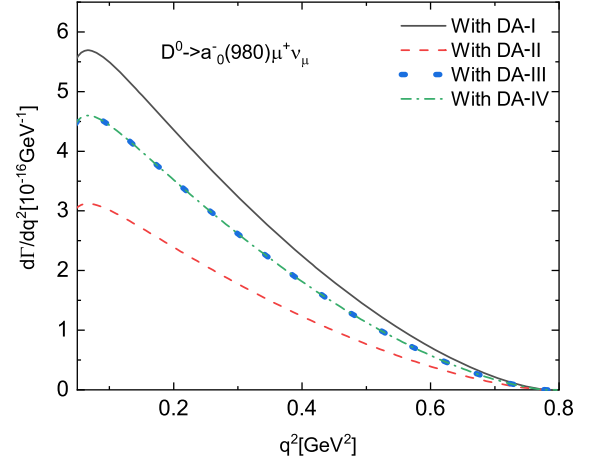


FIG. 11. $D^0 \rightarrow a_0(980)\mu^+\nu_\mu$ differential decay width at $m_\mu^2 \leq q^2 \leq (m_D - m_{a_0(980)})^2$ by four methods. The solid (DA-I), dashed (DA-II), dotted (DA-III), and dashed-dotted (DA-IV) lines correspond to the four distribution amplitudes given by different methods, with two of the lines (DA-III and DA-IV) overlapping exactly.

TABLE V. Branching fractions (in unit of 10^{-4}) in this work compared with that via other theoretical methods cited as well as the experimental values for $D \rightarrow Sl^+\nu_l$.

Process	Method	Branching fractions	$\mathcal{B}(D^{0(+)} \rightarrow Sl\bar{\nu}_e) \times \mathcal{B}(S \rightarrow \eta\pi)$
$D^0 \rightarrow a_0(980)e^+\nu_e$	LCSR with DA-I	1.35	1.14
	LCSR with DA-II	0.74	0.6
	LCSR with DA-III	1.02	0.86
	LCSR with DA-IV	1.02	0.86
	CCQM [42]	1.68 ± 0.15	1.84
	LCSR [29]	$4.08^{+1.37}_{-1.22}$	4.56
	BESIII [46]	...	$1.33^{+0.33}_{-0.29}$
$D^0 \rightarrow a_0(980)\mu^+\nu_\mu$	LCSR with DA-I	1.20	1.01
	LCSR with DA-II	0.66	0.55
	LCSR with DA-III	0.97	0.82
	LCSR with DA-IV	0.97	0.82
	CCQM [42]	1.63 ± 0.14	1.84
$D^+ \rightarrow a_0(980)e^+\nu_e$	LCSR with DA-I	1.77	1.49
	LCSR with DA-II	0.94	0.79
	LCSR with DA-III	1.29	1.08
	LCSR with DA-IV	1.29	1.08
	CCQM [42]	2.18 ± 0.38	1.84
	LCSR [29]	$5.40^{+1.78}_{-1.59}$	4.56
$D^+ \rightarrow a_0(980)\mu^+\nu_\mu$	BESIII [46]	...	$1.66^{+0.81}_{-0.66}$
	LCSR with DA-I	1.51	1.29
	LCSR with DA-II	0.83	0.70
	LCSR with DA-III	1.22	1.04
	LCSR with DA-IV	1.22	1.04
	CCQM [42]	2.12 ± 0.37	1.84
	BESIII [46]	...	$1.66^{+0.81}_{-0.66}$

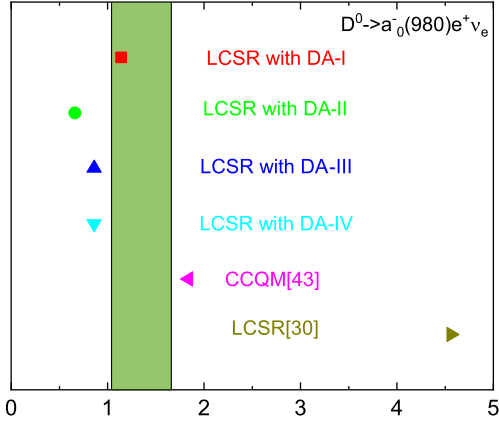


FIG. 12. Comparison of branching fractions for $D^0 \rightarrow a_0^-(980)e^+\nu_e$. The green band corresponds to the measurement of BESIII [46].

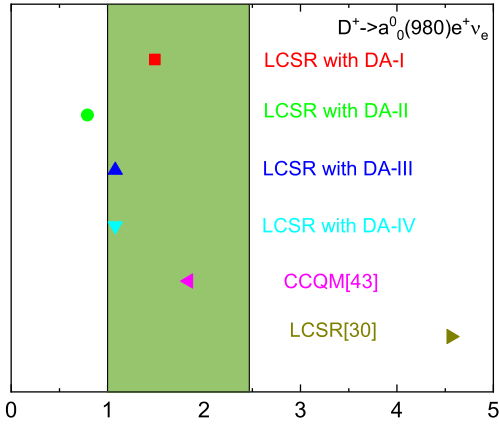


FIG. 13. Comparison of branching fractions for $D^+ \rightarrow a_0^0(980)e^+\nu_e$. The green band corresponds to the measurement of BESIII [46].

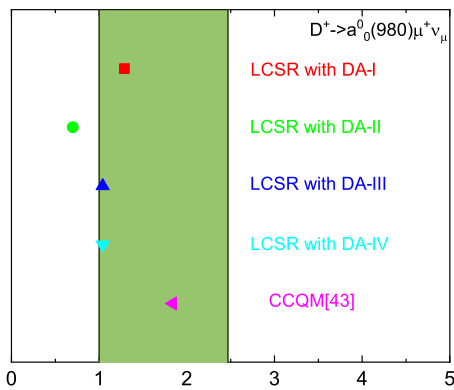


FIG. 14. Comparison of branching fractions for $D^+ \rightarrow a_0^0(980)\mu^+\nu_\mu$. The green band corresponds to the measurement of BESIII [46].

VI. EXTENDING THE TFF TO TETRAQUARK

The DA is known to quantify the longitudinal momentum fraction carried by valence quarks (or partons) within a meson in the infinite-momentum frame, encoding essential nonperturbative information about its light-cone wave function. For conventional two-quark ($q\bar{q}$) configurations of a scalar meson, its distribution amplitude exhibits a typical double extremum, which substantially implies the underlying symmetry of the momentum carried by constituent (quark q) and anticonstituent ($\bar{q}$) in meson. The meson $a_0(980)$ typically conforms to this feature, two extremum of its DA happening at the symmetric positions of the center $u_c = 0.5$, namely, $u \sim 0.25$ and 0.75 , as explicitly demonstrated in Figs. 1–5.

In Ref. [49], the DAs of the light mesons are analyzed with QCD factorization and is found to undergo some deformation, that is, the DA of a light meson made of tetraquark (with $q\bar{q}$ insertion) manifests a suppression near the endpoints $u = 0$ and $u = 1$ and enhances around the midregion, relative to that for the two-quark mesons. This may be explained by tendency of momentum sharing that, in a tetraquark, constituent (e.g., quark q or antiquark $\bar{q}$) of tetraquark is hard to carry a high momentum (near $u = 1$ or $u = 0$) but easy to have an averaged momentum due to sharing of the momentum among all constituents.

Given the above features in DA, it is very useful to introduce, for simplicity, an extra multiplicative “modulation function,” to parametrize the additional effect of a tetraquark component mixed in the $a_0(980)$ wave function. One simple function which demonstrates the deformation mentioned above is $1 + 0.5(1 - e^{-k\frac{(u-0.25)^2(u-0.75)^2}{0.02}})$, with prefactor k as a parameter. We multiply this function to the four DAs (modulate them) that we have used in Secs. II–V, and choose the deformed DAs as the DA inputs for the tetraquarks with $s\bar{s}$ insertion. Given four modulated DAs, denoted also as DA-I through DA-IV, we apply the LCSR endowed with chiral currents to recompute the FFs of the same semileptonic decays of the B/D mesons, as well for the different values of k . The fixed DAs are shown with dashed and dotted lines in Figs. 1–4. Furthermore, using these modulated DAs as input, we recompute the transition form factors and branching fractions for the semileptonic decays. The results for the branching fractions across a range of k values are summarized in Table VI.

From Table VI, one observes a continuous transition from DA of the (original) pure $q\bar{q}$ to that of tetraquark mixed configuration and ensuing branching fractions of the B/D decays which increase monotonically with k . The obtained branching fractions computed with different methods and their comparisons are plotted in Fig. 15 for $D^0 \rightarrow a_0^-(980)e^+\nu_e$, Fig. 16 for $D^+ \rightarrow a_0^0(980)e^+\nu_e$, and Fig. 17 for $D^+ \rightarrow a_0^0(980)\mu^+\nu_\mu$. From these figures, one sees that a considerable improvement is achieved for extended computation of the TFFs and decay rates applied

TABLE VI. Branching fractions (in units of 10^{-4}) in this work compared for different values of k for $\mathcal{B}(D \rightarrow a_0(980)l\nu_l) \times \mathcal{B}(a_0 \rightarrow \eta\pi)$, with experiment included for comparison.

Method	Value of k	$D^0 \rightarrow a_0^-(980)e^+\nu_e$	$D^0 \rightarrow a_0^-(980)\mu^+\nu_\mu$	$D^+ \rightarrow a_0^0(980)e^+\nu_e$	$D^+ \rightarrow a_0^0(980)\mu^+\nu_\mu$
DA-I	$k = 0$	1.14	1.01	1.49	1.29
	$k = 1$	1.29	1.25	1.69	1.46
	$k = 1.3$	1.33	1.28	1.73	1.50
	$k = 2$	1.40	1.35	1.82	1.58
	$k = 7$	1.66	1.60	2.17	1.88
DA-II	$k = 0$	0.66	0.55	0.79	0.70
	$k = 1$	0.78	0.65	0.86	0.83
	$k = 1.3$	0.80	0.67	0.89	0.86
	$k = 2$	0.85	0.71	0.94	0.91
	$k = 7$	1.01	0.85	1.12	1.08
DA-III/IV	$k = 0$	0.86	0.82	1.08	1.04
	$k = 1$	1.00	0.94	1.24	1.20
	$k = 1.3$	1.02	0.97	1.28	1.23
	$k = 2$	1.08	1.02	1.34	1.30
	$k = 7$	1.29	1.22	1.61	1.55
Exp.	BESIII [46]	$1.33^{+0.33}_{-0.29}$	...	$1.66^{+0.81}_{-0.66}$	$1.66^{+0.81}_{-0.66}$

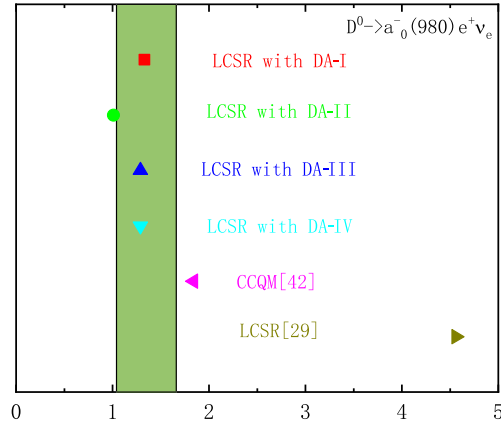


FIG. 15. Branching fractions for $D^0 \rightarrow a_0^-(980)e^+\nu_e$ with modulated DAs featuring a four-quark component, with $k = 1.3$ for DA-I and $k = 7$ for DA II–IV. The green band corresponds to the measurements of BESIII [46].

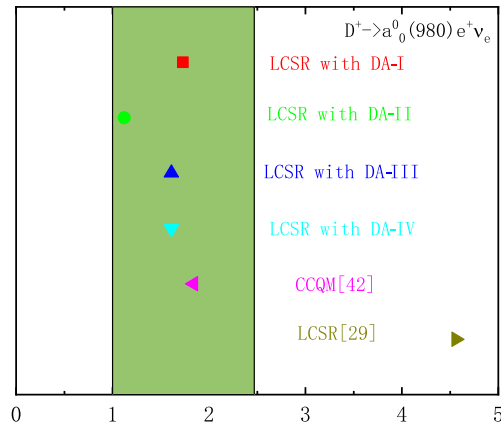


FIG. 16. Branching fractions for $D^+ \rightarrow a_0^0(980)e^+\nu_e$ with modulated DAs featuring a four-quark component, with $k = 1.3$ for DA I and $k = 7$ for DA II–IV. The green band corresponds to the measurements of BESIII [46].

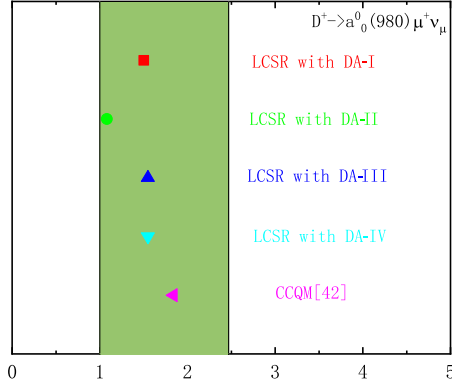


FIG. 17. Branching fractions for $D^+ \rightarrow a_0^0(980)\mu^+\nu_\mu$ with modulated DAs featuring a four-quark component, with $k = 1.3$ for DA-I and $k = 7$ for DA II–IV. The green band corresponds to the measurements of BESIII [46].

to the tetraquark-dominated meson, in the sense that it captures the main features of the BESIII measurements. By the way, our analysis gives strong support for the dominant four-quark structure of the $a_0(980)$, which agrees with its measured strong coupling to the $K\bar{K}$.

VII. SUMMARY

In this work, we systematically study the semileptonic decay process of $D/B \rightarrow a_0 l^+ \nu_l$ firstly in the two-quark picture and then in the four-quark pictures of the final state meson $a_0(980)$, with emphasis on the decay form factors that exclusively incorporate the contributions of the leading distribution amplitudes. We eliminate the contributions arising from twist-3 light-cone DAs with the help of the light-cone QCD sum rules with chiral currents and find a simple relationships among the form factors associated with the process of $D/B \rightarrow Sl^+\nu_l$. An explicit expression of the form factors for $D/B \rightarrow Sl^+\nu_l$ is presented in terms of q^2 . The obtained FFs are utilized to predict the branching fractions of the relevant semileptonic decay process and to compare with those obtained via the LCSR without chiral currents [29] and via other methods. Our numerical computations for the pure $q\bar{q}$ picture suggest that the DAs in the four methods yields nearly same transition FFs and decay rates, with that by the LCSR with DA-II smaller by 20%–30%. The obtained branching fractions are tested against with the observed data of BESIII, with the agreement achieved only marginally for the two-quark picture.

We further extend our analysis for the two-quark meson to that of tetraquark configuration with the help of proper shape modulation of the DAs of the $a_0(980)$. The DA of the two-quark meson is deformed to become more steep as that

for tetraquark should be under the modulation. As a results, the transition FFs and branching fraction for the D/B decays are recomputed with the improved DAs via proper modulation and found to be in good agreement with the central data of the experiment by BESIII. This behavior provides evidence for a significant four-quark component in the $a_0(980)$, lending further support to its interpretation as a state strongly coupled to the $K\bar{K}$ channel. This gives further support for the dominant four-quark structure of the $a_0(980)$, which is indicated by its measured strong coupling to the $K\bar{K}$. We hope future experiments by updated Beijing Electron Positron Collider (BEPC) and other faculties test our predictions of the TFFs and decay branching fractions in this work.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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