

Spontaneous spacetime parity breaking without thermal restoration

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We provide strong evidence for spontaneous breaking of spacetime parity persisting to arbitrarily high temperatures in a local, unitary, ultraviolet-complete quantum field theory in $2 + 1$ dimensions. The theory is defined by a renormalization group trajectory, triggered by a relevant deformation of a conformal field theory, consisting of a critical biconical vector model and a free massless Dirac fermion. This deformation couples the fermion to the scalar sector, generating a renormalization group flow that terminates at a nontrivial infrared fixed point described by a conformal Gross-Neveu-Yukawa model and a decoupled critical vector model. By construction, the quantum field theory is parity invariant at zero temperature. However, we show that at sufficiently high temperatures, parity symmetry is spontaneously broken and remains so even in the infinite-temperature limit. Our analysis relies on both, perturbative renormalization group techniques in $4 - \epsilon$ dimensions and functional renormalization group techniques directly in $2 + 1$ dimensions.

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I. INTRODUCTION

According to thermodynamics, a system in equilibrium minimizes its free energy. At high temperatures, entropy typically dominates, favoring disordered phases and restoring symmetry.¹ While commonly expected, this trend is not mandated by thermodynamic laws, which constrain macroscopic behavior but do not uniquely determine the microscopic free energy landscape. As a result, symmetry-breaking phases may occur even as the temperature increases.

Rochelle salt offers a well-known example [1]. Below -18°C , it resides in a high-symmetry orthorhombic phase, while a lower-symmetry monoclinic phase emerges above this temperature. Similarly, in ^3He , the solid phase exhibits higher entropy than the liquid, leading to heating-induced solidification—the Pomeranchuk effect [2]. In both cases,

the system eventually transitions into a phase with higher symmetry at even higher temperatures, again. For example, Rochelle salt transitions to a symmetric phase at $+24^\circ\text{C}$.

Such behavior is rare in relativistic quantum field theories (QFTs). The first theoretical example was constructed by Weinberg [3] who showed that a four-dimensional model with two weakly coupled scalars can exhibit spontaneous symmetry breaking (SSB) persisting at high temperatures. However, this model is ultraviolet (UV) incomplete, and the symmetry pattern becomes uncertain at strong coupling. All of these examples demonstrate symmetry breaking within a finite temperature window but leave the possibility of persistent symmetry breaking (PSB) to arbitrarily high temperatures unresolved.

Recently, relativistic scalar field theories exhibiting PSB of internal symmetries have been constructed in fewer than four dimensions [4–11], with conformal invariance ensuring UV-completeness. Yet many of them suffer from limitations: some are nonlocal [7,8], while others rely on the ϵ -expansion [4,5], raising unitarity concerns [12]—except when $\epsilon = 1$, in which case the perturbative expansion is no longer reliable. A direct calculation in $2 + 1$ dimensions was carried out in [10] using a truncated functional renormalization group (FRG) approach, supporting PSB of a discrete internal symmetry in a local, UV-complete, and unitary QFT [13]. In complementary work, analytically tractable $2 + 1$ -dimensional conformal field theories exhibiting PSB were constructed in [11]. Moreover, it has been shown that lattice models without explicit UV completion can realize PSB [14]. In four dimensions, the construction of such theories is more difficult (see Refs. [15–20]).

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¹In this letter, we focus exclusively on ordinary (zero-form) symmetries. Although the deconfinement transition in QCD famously exhibits opposite behavior, we do not address higher-form symmetries here.

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Here, we propose a candidate for the first known instance of persistent breaking of spacetime parity in a $2 + 1$ -dimensional local, UV-complete relativistic QFT. Our model includes a Dirac fermion and breaks conformal symmetry explicitly. It is defined by a renormalization group (RG) flow between two fixed points: in the UV, a conformal field theory (CFT) of a free Dirac fermion decoupled from a critical biconical vector model, studied via the ϵ -expansion [4,5,21,22] and FRG [10,23]; in the IR, a decoupled pair of known CFTs—the Gross-Neveu-Yukawa (GNY) model and the critical vector model [24]. This flow is triggered by a relevant Yukawa-type deformation.

Parity violation plays a fundamental role in the Standard Model, topological insulators, anyons, and spin liquids. For example, a conventional Quantum Hall effect (QHE) cannot arise in a parity-preserving theory. Breaking internal symmetries, such as Z_2 , alone is therefore insufficient to produce the QHE; violation of a spacetime symmetry is a necessary condition. Parity breaking may also be relevant to early Universe cosmology [25–28], and extensions of our model may provide new insight into such systems.

II. MODEL

Consider a d -dimensional Euclidean field theory involving a real pseudo-scalar field χ , a Dirac fermion ψ_i transforming in the fundamental representation of $U(N_f)$, and a real scalar field $\phi = (\phi_1, \dots, \phi_{N_1})$ transforming as a vector under $O(N_1)$. The action is given by [29]

$$S = \int d^d x \left\{ \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} (\partial_\mu \chi)^2 + \bar{\psi} \not{\partial} \psi + \frac{\lambda_\phi \mu^\epsilon}{8} (\phi^2)^2 + \frac{\lambda_\chi \mu^\epsilon}{8} \chi^4 + \frac{\lambda_{\phi\chi} \mu^\epsilon}{4} \phi^2 \chi^2 + h \mu^{\epsilon/2} \chi \bar{\psi} \psi \right\}, \quad (1)$$

where $d = 4 - \epsilon$ with $0 < \epsilon \leq 1$, and μ is a mass scale so that all dimensionless couplings are relevant deformations of the Gaussian fixed point. In $d = 2 + 1$ dimensions, this model possesses an $O(N_1) \times U(N_f) \times \mathcal{P}$ symmetry, where $\mathcal{P}$ stands for parity,

$$\mathcal{P}: x_1 \rightarrow -x_1, \quad \chi \rightarrow -\chi, \quad \psi \rightarrow \gamma^1 \psi, \quad \bar{\psi} \rightarrow -\bar{\psi} \gamma^1. \quad (2)$$

Fractional dimensions with $0 < \epsilon < 1$ are treated via analytic continuation. For later convenience, we define $N_2 = d_\gamma N_f$, where d_γ is the dimension of the spinor representation.

By construction, this field theory admits a rich structure of multicritical fixed points. In $d = 2 + 1$ dimensions, these fixed points are strongly coupled. Some of the associated

CFTs are well-known and extensively studied in the literature. For example, one finds a fixed point corresponding to a decoupled Dirac fermion ($h = 0$) and a critical vector model exhibiting enhanced $O(N + 1)$ symmetry [24]. Furthermore, setting $\lambda_{\phi\chi} = 0$ decouples the field ϕ from the other two fields, χ and ψ . The resulting RG flow terminates at a nontrivial IR fixed point described by a decoupled product of two familiar CFTs: the GNY model and the critical vector model. This particular fixed point plays an important role in our construction; we will refer to it as CFT_{IR} .

The existence of critical GNY and scalar vector models in $d = 2 + 1$ dimensions has been confirmed by various complementary methods, including the large- N expansion, FRG, ϵ -expansion, conformal bootstrap, as well as lattice and Monte Carlo simulations (see e.g., Refs. [30–42]).

Another interesting CFT in (1) is located on the hypersurface in coupling space defined by $h = 0$. On this hypersurface, the Dirac field decouples from the scalar sector, while the remaining couplings can be tuned to the critical values corresponding to the conformal biconical vector model with $\lambda_{\phi\chi}^* < 0$ for sufficiently large N_1 , which has a stable potential. This CFT has been studied using large- N and ϵ -expansion techniques [4–6,21,22], as well as via the FRG approach directly in $d = 2 + 1$ [10,23]. We will henceforth refer to the direct product of this CFT and a free massless Dirac fermion as CFT_{UV} .

The QFT of interest is defined by deforming CFT_{UV} with a Yukawa interaction controlled by the coupling h in Eq. (1). The action of the resulting theory is given by

$$S' = S_{\text{UV}} + \int d^d x \bar{\psi} \not{\partial} \psi + h \mu^{\epsilon/2} \int d^d x \chi \bar{\psi} \psi, \quad (3)$$

where S_{UV} denotes the action of the conformal biconical vector model characterized by $\lambda_{\phi\chi}^* < 0$. As we will argue below, this deformation is relevant and induces an RG flow that drives the theory toward CFT_{IR} in the infrared limit. We note that in the ϵ -expansion approach, the RG relevant mass terms are already chosen at their critical values, and no additional fine-tuning is required. This will be different in the FRG approach.

The model is strongly coupled in $2 + 1$ dimensions. Therefore, in the next section, we employ numerical FRG methods to demonstrate that S' flows to CFT_{IR} in the deep infrared. In the remainder of this section, we assume $\epsilon \ll 1$ and establish this connection within the framework of the ϵ -expansion.

The one-loop beta functions are derived in the Supplemental Material [43],

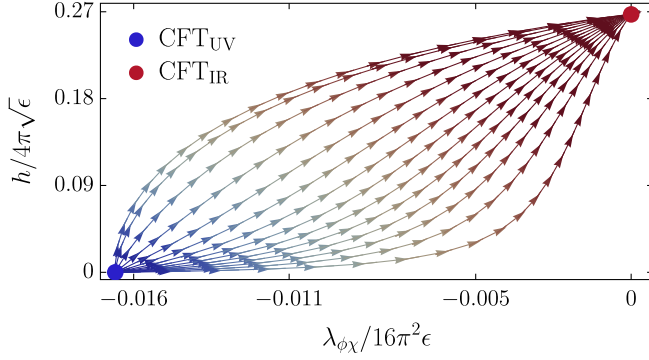


FIG. 1. RG flow from the ϵ -expansion for $N_1 = 100$, $N_2 = 8$, shown in the $h - \lambda_{\phi\chi}$ coupling plane. Within this approximation, most deformations of CFT_{UV} with $h \neq 0$ flow to CFT_{IR} , which serves as an RG sink with no relevant directions. Trajectories flow into CFT_{IR} but not out, i.e., it is an IR stable fixed point.

$$\begin{aligned}\beta_{\lambda_\phi} &= -\epsilon\lambda_\phi + \frac{N_1 + 8}{16\pi^2}\lambda_{\phi\chi}^2 + \frac{\lambda_{\phi\chi}^2}{16\pi^2}, \\ \beta_{\lambda_{\phi\chi}} &= -\epsilon\lambda_{\phi\chi} + \frac{N_2}{16\pi^2}\lambda_{\phi\chi}h^2 + \frac{3}{16\pi^2}\lambda_\chi\lambda_{\phi\chi} + \frac{\lambda_{\phi\chi}^2}{4\pi^2} \\ &\quad + \frac{N_1 + 2}{16\pi^2}\lambda_\phi\lambda_{\phi\chi}, \\ \beta_{\lambda_\chi} &= -\epsilon\lambda_\chi + \frac{1}{8\pi^2}\left(\frac{9}{2}\lambda_\chi^2 + \frac{N_1}{2}\lambda_{\phi\chi}^2 + N_2\lambda_\chi h^2 - 2N_2h^4\right), \\ \beta_{h^2} &= -\epsilon h^2 + \frac{N_2 + 6}{16\pi^2}h^4.\end{aligned}\quad (4)$$

For $N_1 \geq 11$, one can verify the existence of a weakly coupled fixed point characterized by $h_* = 0$ and $\lambda_{\phi\chi}^* < 0$, which we denote as CFT_{UV} . At this point, the anomalous dimension of χ is of order $\mathcal{O}(\epsilon^2)$, indicating that the Yukawa deformation $\sim h$ of CFT_{UV} is relevant. It induces an RG flow that drives the theory toward the CFT_{IR} in the infrared limit. Figure 1 illustrates typical RG flows associated with Eqs. (3) and (4). In the next section, we confirm this observation directly in $d = 2 + 1$.

III. FRG ANALYSIS IN 2+1D

Consider a microscopic action $S_\Lambda[\Phi]$, which is a functional of a collective field variable $\Phi(x)$ defined at some UV scale Λ . Within the FRG [44,45], we define the *effective average action* $\Gamma_k[\Phi]$ by the path integral

$$e^{-\Gamma_k[\Phi]} = \int D\hat{\Phi} e^{-S_\Lambda[\Phi + \hat{\Phi}] + \int d^d x \frac{\delta \Gamma_k[\Phi]}{\delta \Phi} \hat{\Phi} - \Delta S_k[\hat{\Phi}]}. \quad (5)$$

In the above, $\Phi(x)$ is a classical field configuration and $\hat{\Phi}(x)$ is the quantum field integrated over in the path integral. We further introduced the regulator insertion ΔS_k acting as an IR regulator, i.e., in the path integral, it suppresses modes below the energy scale k . Thus, the

average action $\Gamma_k[\Phi]$ contains all the quantum fluctuations of the theory above the energy scale $k \in [0, \Lambda]$ and can be viewed as an effective action for field averages over k^{-d} volume.

In the case of our field content $\Phi = (\phi, \chi, \psi, \bar{\psi})$, we choose the regulator insertion bilinear in the fields

$$\Delta S_k[\Phi] = \int \frac{d^d q}{(2\pi)^d} \left\{ \frac{1}{2} \phi(-q) R_k^\phi(q) \phi(q) + \frac{1}{2} \chi(-q) R_k^\chi(q) \chi(q) + \bar{\psi}(q) R_k^\psi(q) \psi(q) \right\}, \quad (6)$$

where $R_k(q)$ is large for $q < k$ and vanishes for $q \gtrsim k$. For example, choosing a sharp IR regulator makes the relation of $\Gamma_{k \rightarrow \Lambda}$ to the microscopic action obvious and shows that Γ_k interpolates between the microscopic action in the UV and the full quantum effective action in the IR, i.e., $\Gamma_{k=\Lambda}[\Phi] = S_\Lambda[\Phi]$, $\Gamma_{k=0}[\Phi] = \Gamma[\Phi]$ [46].

The dependence of Γ_k on k is governed by the Wetterich equation [44],

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{STr}([\Gamma_k^{(2)}[\Phi] + R_k]^{-1} \partial_t R_k), \quad (7)$$

where $t = \ln \frac{k}{\Lambda}$ is the FRG time, the supertrace is taken over spacetime degrees of freedom, field, and spinor indices with a minus sign for the fermionic sector. The inverse average propagator $\Gamma_k^{(2)}$ is the second functional derivative of Γ_k ,

$$(\Gamma_k^{(2)}[\Phi])_{p,q;ij} = \frac{\bar{\delta}}{\delta \Phi_i(-p)} \Gamma_k[\Phi] \frac{\bar{\delta}}{\delta \Phi_j(q)}, \quad (8)$$

with $\Phi(p) = (\phi(p), \chi(p), \psi(p), \bar{\psi}(-p))$. The regulator matrix R_k is the second functional derivative of $\Delta S_k[\Phi]$ with respect to the field arguments. The Wetterich equation (7) is an exact identity. However, in general cases, it cannot be solved exactly, and an appropriate truncation scheme for the average action is needed to limit the number of coupled differential equations.

We choose our truncation scheme to be the leading order in the derivative expansion, known as the extended local potential approximation (LPA'),

$$\begin{aligned}\Gamma_k[\Phi] &= \int d^d x \left\{ \frac{Z_{\phi,k}}{2} (\partial\phi)^2 + \frac{Z_{\chi,k}}{2} (\partial\chi)^2 + U_k[\rho_\phi, \rho_\chi] \right. \\ &\quad \left. + Z_{\psi,k} \bar{\psi} \not{\partial} \psi + h_k \chi \bar{\psi} \psi \right\}.\end{aligned}\quad (9)$$

This truncation includes the scale-dependent wave-function renormalizations $Z_{\phi/\chi/\psi,k}$, scale-dependent Yukawa coupling h_k , and scale-dependent effective potential U_k as a function of global symmetry invariants $\rho_\phi = \frac{1}{2} \phi_i \phi_i$ and

TABLE I. Fixed-point data of CFT_{UV} and CFT_{IR} . Displayed are the critical couplings and positive scaling exponents θ_i 's. In particular, θ_h corresponds to the pure Yukawa h -direction. CFT_{IR} data is split into the two decoupled sectors for clarity. CFT_{UV} is in the SSB-SSB regime, where both $\kappa_{\phi/\chi} > 0$, while $m_{\phi/\chi}^2 = 0$ and CFT_{IR} is in the SSB-SYM regime, where $\kappa_\phi, m_\chi^2 > 0$ and $\kappa_\chi = m_\phi^2 = 0$.

SSB-SSB	κ_ϕ	κ_χ	λ_ϕ	λ_χ	$\lambda_{\phi\chi}$	h	θ_1	θ_2	θ_3	θ_h
CFT_{UV}	1.93	0.26	0.29	0.31	-0.29	0	1.99	1.01	0.95	0.50
SSB-SYM	κ_ϕ	λ_ϕ	θ	m_χ^2	λ_χ	h	θ	$\lambda_{\phi\chi}$		
CFT_{IR}	1.68	0.29	1.01	0.40	5.42	1.51	0.98	0		

$\rho_\chi = \frac{1}{2}\chi^2$. For brevity, we omit the k subscripts further on. Close to four dimensions, the above ansatz is able to reproduce the one-loop results from the ϵ -expansion for certain regulator choices (see, e.g., Ref. [47]). This scheme was successfully used in many works (see e.g., Refs. [10,48–52]).

We define the dimensionless effective potential $u \equiv k^{-d}U$ and its derivatives $u^{(l,m)} \equiv \partial_{\bar{\rho}_\phi}^l \partial_{\bar{\rho}_\chi}^m u$ in terms of the dimensionless and renormalized $\bar{\rho}_{\phi/\chi} \equiv Z_{\phi/\chi} k^{2-d} \rho_{\phi/\chi}$. Likewise, dimensionless and renormalized Yukawa coupling is given by $\bar{h} \equiv k^{d/2-2} Z_\chi^{-1/2} Z_\psi^{-1} h$.

Employing the linear regulator [53], we deduce from (7) the FRG flow of all the couplings of the truncated average action (9). Finite temperature is introduced by compactifying the time direction on a circle of radius $\beta = \frac{1}{T}$. The flow of the dimensionless effective potential at finite temperature reads

$$\begin{aligned} \partial_t u = & -du + (d-2+\eta_\phi)\bar{\rho}_\phi u^{(1,0)} + (d-2+\eta_\chi)\bar{\rho}_\chi u^{(0,1)} \\ & + I_R(\omega_\chi, \omega_\phi, \omega_{\phi\chi})S_\phi(\tau) + (N_1-1)I_G(u^{(1,0)})S_\phi(\tau) \\ & + I_R(\omega_\phi, \omega_\chi, \omega_{\phi\chi})S_\chi(\tau) - N_2 I_F(\omega_\psi)S_\psi(\tau), \end{aligned} \quad (10)$$

where $\eta_i = -\partial_t \ln Z_i$ are anomalous dimensions, and $S_i(\tau) = s_i(\tau) - \frac{\eta_i}{d+1+\delta_{i,B}} \hat{s}_i(\tau)$ with $\tau \equiv 2\pi T/k$, $i \in \{\phi, \chi, \psi\}$, $B = \{\phi, \chi\}$. Thermal functions s_i , $\hat{s}_i$ are normalized to unity at $T = 0$. Arguments of the $I_{R,G,F}$ threshold functions read: $\omega_\phi \equiv u_k^{(1,0)} + 2\bar{\rho}_\phi u_k^{(2,0)}$, $\omega_\chi \equiv u_k^{(0,1)} + 2\bar{\rho}_\chi u_k^{(0,2)}$, $\omega_{\phi\chi} \equiv 2\sqrt{\bar{\rho}_\phi \bar{\rho}_\chi} u^{(1,1)}$, and $\omega_\psi = 2\bar{\rho}_\chi \bar{h}^2$. The first line of (10) arises from the classical and anomalous scalings, while subsequent lines represent contributions from fluctuations of the radial ϕ -mode, $N_1 - 1$ transverse ϕ -modes, the radial χ -mode, and fermionic modes, respectively.

The flow of the Yukawa coupling at finite temperature reads

$$\begin{aligned} \partial_t \bar{h}^2 = & (d-4+\eta_\chi+2\eta_\psi)\bar{h}^2 + 8v_d \left(\bar{h}^3 l_{(1,1)}^{\chi\chi} - 2\omega_\psi \bar{h}^3 l_{(2,1)}^{\chi\chi} \right. \\ & \left. - \bar{h} \sqrt{\omega_\psi} \left(\omega_{\chi\chi} l_{(1,2)}^{\chi\chi} + \omega_{\chi\phi\phi} l_{(1,2)}^{\phi\phi} - 2\omega_{\chi\phi} l_{(1,2)}^{\phi\chi} \right) \right), \end{aligned} \quad (11)$$

where $v_d^{-1} = 2^{d+1} \pi^d \Gamma(\frac{d}{2})$, the l threshold functions are functions of d , τ , η_i , and ω 's, ω_{ijk} are algebraic-differential functions of u . All explicit expressions omitted here,

including the anomalous dimensions, can be found in Supplemental Material [43].

We expand the effective dimensionless potential locally around its running minimum $(\kappa_\phi, \kappa_\chi) := (\rho_\phi, \rho_\chi)|_{\min U_k}$. In the IR limit $k \rightarrow 0$, κ_ϕ and κ_χ are the vacuum expectation values $\frac{1}{2}\langle\phi\rangle^2$ and $\frac{1}{2}\langle\chi\rangle^2$, respectively. At a given FRG scale k , we distinguish four different regimes depending on the location of the U_k minima: (1) SYM-SYM: $\kappa_\phi = \kappa_\chi = 0$, (2) SYM-SSB: $\kappa_\phi = 0, \kappa_\chi > 0$, (3) SSB-SYM: $\kappa_\phi > 0, \kappa_\chi = 0$, and (4) SSB-SSB: $\kappa_\phi > 0, \kappa_\chi > 0$. At $k = 0$, these regimes respectively correspond to the $O(N_1) \times \mathbb{Z}_2$, $O(N_1)$, $O(N_1 - 1) \times \mathbb{Z}_2$, and $O(N_1 - 1)$ unbroken symmetries of the theory in the bosonic sector.

The effective potential is expanded up to total polynomial order $n_{\max}$, known as the LPA'(2n_{max}) truncation,

$$u(\bar{\rho}_\phi, \bar{\rho}_\chi) = \sum_{n+m \leq n_{\max}} \frac{\bar{\lambda}_{nm}}{n!m!} (\bar{\rho}_\phi - \bar{\kappa}_\phi)^n (\bar{\rho}_\chi - \bar{\kappa}_\chi)^m, \quad (12)$$

with $\bar{\lambda}_{1,0} \equiv \bar{m}_\phi^2 \geq 0$ if $\bar{\kappa}_\phi = 0$ (SYM) and $\bar{\lambda}_{1,0} = 0$ if $\bar{\kappa}_\phi > 0$ (SSB); similarly for $\bar{\lambda}_{0,1}$ and $\bar{\kappa}_\chi$. For each of the four regimes, using (10), one gets the β^{FRG} -functions of the $\bar{\kappa}_\phi$, $\bar{\kappa}_\chi$ and $\bar{\lambda}_{nm}$ couplings (see Supplemental Material [43]).

IV. 2+1D FLOW

Model (1) admits two interacting CFTs, denoted by CFT_{UV} and CFT_{IR} . CFT_{UV} is the product of the well-established biconical fixed point of the $O(N_1) \times \mathbb{Z}_2$ model and a decoupled free massless fermion. CFT_{IR} is the decoupled product of the critical $O(N_1)$ vector model and the critical Gross-Neveu-Yukawa model, both well known in the literature. In the following, we restrict to $d = 3$, $T = 0$, $N_1 = 100$, and $N_2 = 8$ with all the computations performed within the nonperturbative FRG framework in the LPA'8 truncation (9) and (12). Large N_1 provides rapid convergence of the biconical and vector models in the LPA' and LPA truncations [10,23,54]. The anomalous dimensions of the critical biconical model scale as $1/N_1$ at large N_1 , yielding small numerical values of $\eta_\phi = 0.00241$ and $\eta_\chi = 0.00243$ for $N_1 = 100$, which indicates a well-controlled derivative expansion. We further choose $N_2 = 8$ for better comparability with literature and its relevance for condensed matter systems (see, e.g.,

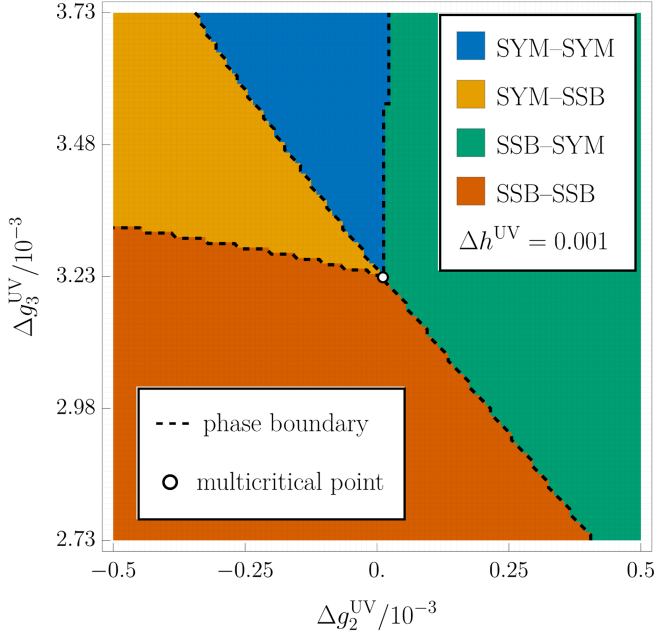


FIG. 2. Infrared phases of deformed CFT_{UV} . The relevant deformation Δh is fixed, while Δg_2 and Δg_3 are varied. The deformation flowing to the multicritical point can be reduced by a factor of 10 and still flow to the vicinity of the multicritical point, confirming the smallness of CFT_{UV} deformation.

Ref. [55]). In particular, for the chosen N_1 and N_2 , our values of critical exponents and anomalous dimensions for the biconical, vector, and GNY critical models are in reasonable agreement with the recent literature [24,39–41,56] (see Supplemental Material [43] where the results of comparison are summarized). Table I summarizes a number of critical coupling values and positive scaling exponents of the two CFTs. Higher order truncations up to LPA'12 do not yield any significant improvement in numerical precision. Scaling exponents θ_i 's are minus the eigenspectrum of the stability matrix $B_{ij} = \partial\beta_i/\partial\lambda_j|_{\lambda=\lambda^*}$ evaluated at the fixed point. We denote g_i to be the corresponding eigendirection in the space of couplings, and Δg_i as the size of the deformation from the fixed point along this direction.

We note that within the functional RG approach, various regulator functions can be employed, and numerical values of couplings as well as critical exponents are influenced by these choices. The fixed points and critical exponents considered in our work have been well-established using the functional RG within different truncations and regulator functions [37,57], and they are also consistent with other methods (see references in [37,57]). Consequently, we can consider their existence and essential qualitative features as robust properties of the model under investigation.

We choose to deform CFT_{UV} along its three relevant directions g_2 , g_3 , and h , triggering an RG flow that drives the theory away from CFT_{UV} toward a different point in theory space. Figure 2 shows the IR phases of the deformed

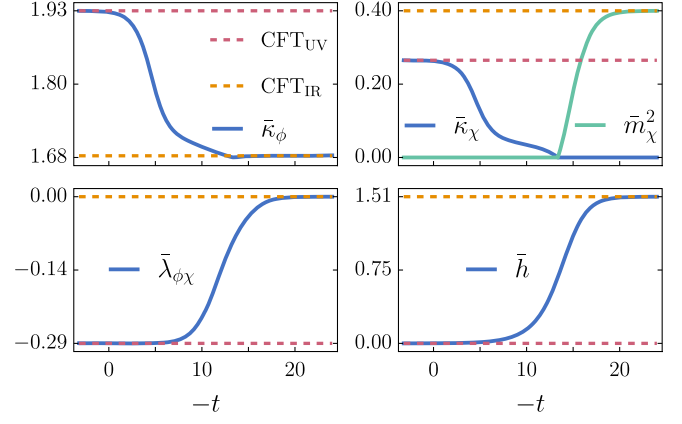


FIG. 3. FRG flow connecting CFT_{UV} and CFT_{IR} within the LPA'8 truncation. Dashed lines correspond to the critical coupling values. At $t = 0$, CFT_{UV} is deformed by Δg_2 , Δg_3 , and Δh relevant deformations (see Supplemental Material [43] for details). At $t \approx -13.4$, the flow switches from the SSB-SSB to the SSB-SYM regime, consistent with the fixed point data from Table I.

theories. The phase diagram clearly shows a multicritical point where all four phases meet. In combination with our fixed-point analysis, we can conclude that this point is a CFT. Indeed, it corresponds to CFT_{IR} , which is a decoupled product of two critical models. By tuning their respective relevant deformations, one is undergoing the spontaneous symmetry-breaking transition in the ϕ field and the other in the χ field. Figure 3 shows a concrete example of the RG flow between CFT_{UV} and CFT_{IR} .

Our QFT model is defined by specifying an energy scale Λ and selecting an interior point along the RG trajectory connecting two CFTs, thereby fixing dimensionless couplings at this scale. Within the ϵ -expansion, turning on solely the relevant coupling h suffices to define the flow. In the $2 + 1$ dimensional FRG analysis, the construction of the RG trajectory requires turning on, alongside h , a specific combination of other relevant deformations, e.g., Δg_2 and Δg_3 .

V. THERMAL EFFECTS

The action (3) preserves parity. At zero temperature, the effective potential of our construction is governed by CFT_{IR} , placing the system in a conformally invariant vacuum. This vacuum corresponds to the direct product of the critical GNY model and the critical vector model, both of which respect parity. Consequently, at $T = 0$, the theory is manifestly invariant under $\mathcal{P}$.

This situation changes at finite temperature. The effective potential is computed by evaluating the path integral of the QFT on a cylinder whose radius is set by the inverse temperature. Consequently, the temperature—or equivalently, the size of the cylinder—sets the dominant energy scale, and naturally plays the role of the RG scale μ at which the theory is probed.

Such a perspective aligns with the framework of critical phenomena, where T controls the relevance of various deformations under coarse-graining (e.g., masses and Yukawa couplings in our model). This motivates the identification $\mu \sim T$, which we adopt in the following. A similar idea appears in studies of c -theorems [58–60], where one considers a QFT on a sphere [59,61–63]: the sphere’s radius acts as an RG scale, interpolating between UV and IR fixed points as the radius varies from zero to infinity.

In our case, increasing T can be mapped to RG flow trajectories that run into the UV fixed point. At sufficiently high temperatures, the system becomes smeared over the excited states of CFT_{UV} , deformed by a small, relevant Yukawa coupling h . This coupling decreases with increasing T , and can be tuned to arbitrarily small values. Consequently, the phase structure of the theory is governed entirely by perturbation theory around the high-temperature limit of CFT_{UV} [64]. As shown in [4,5,10], the strongly coupled CFT_{UV} exhibits large thermal expectation values $\langle \chi \rangle_T \neq 0$ at high temperatures. Perturbative corrections cannot drive this condensate to zero. This leads to spontaneous breaking of $\mathcal{P}$ -symmetry at sufficiently high but finite temperature and generates a mass for the Dirac fermion.

The mechanism of spontaneous parity breaking in our construction becomes particularly transparent within the ϵ -expansion. Sufficiently close to $3 + 1$ dimensions, the theory remains weakly coupled along the entire RG flow. This allows for a perturbative computation of the effective potential at any point along the RG trajectory. At finite temperatures, to leading order in the couplings, the effective potential is given by the classical potential supplemented by thermally induced masses [43],

$$V_{\text{eff}}(\phi, \chi) = \frac{\lambda_\phi}{8}(\phi^2)^2 + \frac{\lambda_\chi}{8}\chi^4 + \frac{\lambda_{\phi\chi}}{4}\chi^2\phi^2 + \frac{1}{2}m_\phi^2(T)\phi^2 + \frac{1}{2}m_\chi^2(T)\chi^2, \quad (13)$$

where

$$m_\phi^2(T) = \frac{T^2}{24}[\lambda_\phi(N_1 + 2) + \lambda_{\phi\chi}],$$

$$m_\chi^2(T) = \frac{T^2}{24}(3\lambda_\chi + N_1\lambda_{\phi\chi} + N_2h^2). \quad (14)$$

The thermal masses vanish at zero temperature, and the theory resides in the global minimum characterized by $\langle \chi \rangle = \langle \phi \rangle = 0$, with the couplings fixed at the critical values defined by CFT_{IR} . As the temperature increases, the thermal masses and couplings evolve. At sufficiently high temperatures, the Yukawa coupling h becomes arbitrarily small, while the remaining couplings flow toward

those of the conformal biconical vector model, with $\lambda_{\phi\chi}^* < 0$. Using (4), one can verify that at these temperatures, $m_\chi^2(T) < 0$ for $N_1 \geq 18$. Consequently, $\langle \chi \rangle_T \neq 0$, resulting in a nontrivial mass for the Dirac field and indicating spontaneous parity breaking. A similar observation can be inferred from the FRG approach [43].

To conclude, we note that while in $2 + 1\text{D}$, PSB must involve discrete symmetries due to the Coleman-Mermin-Wagner-Hohenberg theorem [65–67], no such constraint exists in $3 + 1\text{D}$. Thus, identifying 4D QFTs with finitely many fields that exhibit persistent breaking of continuous symmetries remains a compelling open problem.¹ A better understanding of PSB mechanisms is essential for both conceptual and practical progress. A first step in this direction was taken in [14], and further insight may arise from holographic approaches [68–74].

Note that the transition in our model is incompatible with the standard notion of a crossover, which *analytically* connects two regimes without changing the symmetry realization. A crossover typically occurs only when there is no exact symmetry distinction between the two regimes. In contrast, our model possesses an exact space-time parity symmetry. Consequently, the low-temperature phase is characterized by a *unique* parity-invariant vacuum, whereas the high-temperature phase exhibits *two* degenerate parity-breaking minima of the effective potential. The change in the vacuum structure across the transition precludes an analytic interpolation between the two phases in the thermodynamic limit. Therefore, the transition must be a genuine phase transition rather than a crossover.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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