

Multistep strong first-order electroweak phase transitions in the inverted type-I 2HDM: Parameter space, gravitational waves, and collider phenomenology

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We investigate the electroweak phase transition (EWPT) within the inverted type-I two-Higgs-doublet model, where the observed 125 GeV Higgs boson is identified as the heavier CP -even scalar H . Through a comprehensive parameter-space scan consistent with current theoretical and experimental constraints, we identify regions supporting strong first-order EWPTs (SFOEWPTs), including multistep transitions. We find that two-step SFOEWPTs occur as frequently as one-step transitions, while three-step transitions can occur, albeit rarely. Crucially, the parameter spaces inducing one-step and two-step transitions are partially yet significantly separated: one-step transitions restrict the charged Higgs mass and $\tan\beta$ to $m_{H^\pm} \in [295, 441]$ GeV and $\tan\beta \in [4.2, 8.8]$, whereas two-step transitions allow $m_{H^\pm} \in [100, 350]$ GeV and $\tan\beta \in [2.5, 45.4]$. Notably, negative values of $\sin(\beta - \alpha)$ arise almost exclusively in one-step scenarios. We present the calculation of gravitational wave (GW) signal-to-noise ratios (SNRs) at Laser Interferometer Space Antenna for multistep EWPTs, finding that detectable GW signals ($\text{SNR} > 10$) predominantly emerge from two-step transitions. Furthermore, we demonstrate that the previously established correlation between the vacuum uplifting measure $\Delta\mathcal{F}_0$ and EWPT strength ξ_c persists only in one-step transitions and breaks down in multistep cases. Finally, we perform a dedicated collider analysis for representative SFOEWPT parameter points at the 1.5 TeV Compact Linear Collider, identifying $e^+e^- \rightarrow H^+H^- \rightarrow W^+W^-hh$ as a promising discovery channel. Enhanced $h \rightarrow \gamma\gamma$ branching ratios for negative $\sin(\beta - \alpha)$ motivate two complementary golden final states, $W^+W^-b\bar{b}\tau^+\tau^-$ and $W^+W^-b\bar{b}\gamma\gamma$, which demonstrate high discovery potential due to negligible Standard Model backgrounds.

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I. INTRODUCTION

The origin of the observed matter-antimatter asymmetry in the Universe remains one of the most compelling open questions in particle physics. This baryon asymmetry is quantitatively expressed by the baryon-to-photon ratio, $n_B/n_\gamma \approx 6 \times 10^{-10}$ [1], a value far exceeding expectations from a matter-antimatter symmetric early Universe.

The necessary conditions for dynamically generating this asymmetry were outlined by Sakharov [2]: baryon number violation, C and CP violation, and a departure from thermal equilibrium.

A strong first-order electroweak phase transition (SFOEWPT) in the early Universe provides a compelling mechanism to satisfy the out-of-equilibrium condition [3,4]. However, for the observed Higgs boson mass of 125 GeV, the Standard Model (SM) predicts a smooth crossover rather than a first-order transition [3–5], precluding the possibility of electroweak baryogenesis [6–9] and motivating the search for physics beyond the SM (BSM).

In addition to its crucial role in baryogenesis, a SFOEWPT can also generate a stochastic gravitational wave (GW) background through processes such as bubble collisions, plasma sound waves, and turbulence [10,11]. These GW signals could fall within the sensitivity range of upcoming space-based interferometers like the Laser Interferometer Space Antenna (LISA) [12–15], offering a

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promising cosmological probe of new physics at the electroweak scale.

Among the well-motivated BSM frameworks capable of inducing a SFOEWPT,¹ the Two-Higgs-Doublet Model (2HDM) remains one of the simplest and most extensively studied [72–78]. The 2HDM extends the SM scalar sector by introducing two $SU(2)_L$ Higgs doublets, Φ_1 and Φ_2 [79], leading to a physical spectrum of five Higgs states: two CP -even scalars (h and H), a CP -odd pseudoscalar (A), and a pair of charged Higgs bosons ($H^\pm$). The doublets Φ_1 and Φ_2 acquire vacuum expectation values (VEVs), v_1 and v_2 , with their ratio $\tan\beta = v_2/v_1$ being a key parameter. To prevent potentially large tree-level flavor-changing neutral currents (FCNCs), a discrete Z_2 symmetry, under which $\Phi_1 \rightarrow \Phi_1$ and $\Phi_2 \rightarrow -\Phi_2$, is typically imposed [80,81]. This leads to four distinct types of Yukawa structures (type I, type II, type X, and type Y).

The 125 GeV Higgs boson observed at the LHC can be identified with either the lighter scalar h (the normal scenario) or the heavier scalar H (the inverted scenario). While most studies of SFOEWPTs in the 2HDM have focused on the normal scenario, the inverted scenario is particularly compelling for achieving a strong phase transition. In the normal scenario, the decoupling limit [82–85], characterized by additional Higgs bosons significantly heavier than 125 GeV, is generally unavoidable, as current and upcoming experiments are unable to probe such heavy scalar masses. This decoupling limit typically disfavors a SFOEWPT because the transition strength ξ_c vanishes as the scalar spectrum becomes increasingly decoupled [86–90], effectively placing an upper bound of around 1 TeV on the BSM Higgs masses [74,90].

In contrast, the inverted scenario inherently avoids a fully decoupled spectrum, since the lighter scalar h necessarily has a mass below 125 GeV. However, in inverted scenarios, not all four types remain viable due to stringent experimental constraints. Specifically, type II and type Y are severely constrained by FCNC processes—most notably from $b \rightarrow s\gamma$ transitions—which typically require charged Higgs boson masses $m_{H^\pm} \gtrsim 580$ GeV [91–94], thus significantly restricting the parameter space compatible with a SFOEWPT. Consequently, in this work, we focus exclusively on the type-I 2HDM, demonstrating that this particular inverted scenario can support a SFOEWPT across a substantial region of the parameter space.

An intriguing and relatively unexplored aspect of SFOEWPTs in the inverted 2HDM is the possibility of multistep phase transitions,² in which the Universe transitions through one or more intermediate metastable vacua

before reaching the final true vacuum [37,39,59,69,98–103]. Multistep transitions are particularly interesting, as they can enhance the strength of individual transition steps [102,104], expand the parameter space compatible with a SFOEWPT [105], and potentially yield multi-peaked GW spectra [102]. In this work, we will explicitly demonstrate that the inverted type-I 2HDM accommodates multistep SFOEWPTs, notably showing that the second transition in a two-step sequence occurs with comparable frequency to conventional one-step transitions.

This naturally raises a crucial question: are the parameter regions supporting one-step and multistep SFOEWPTs distinct, or do they overlap? If these regions are distinct, measurements of model parameters at high-energy colliders could pinpoint the specific nature of the EWPT, thereby establishing a remarkable interplay between microscopic particle dynamics and macroscopic cosmological events. To address this question, a comprehensive and detailed scan of the parameter space, fully consistent with current theoretical and experimental constraints, is required. Through such an extensive parameter-space exploration, we aim to identify viable SFOEWPT parameter regions for each transition type and investigate their combined phenomenological consequences for GW signals and collider observables.

Another important topic closely connected to multistep EWPTs is the Higgs vacuum uplifting, characterized by the measure $\Delta\mathcal{F}_0$ [90,96,106]. Defined explicitly as the difference between $\mathcal{F}_0$ in the 2HDM and SM, where $\mathcal{F}_0$ represents the potential energy difference between the symmetric and broken phases at zero temperature (see Sec. III C for details), the vacuum uplifting measure has been shown to strongly correlate with the phase transition strength ξ_c in conventional one-step EWPTs. Consequently, evaluating the zero-temperature potential difference alone is often sufficient to assess the viability of a strong transition in one-step EWPTs. However, by definition, $\Delta\mathcal{F}_0$ does not capture details associated with intermediate metastable vacua present in multistep EWPTs. Thus, the previously observed correlation between $\Delta\mathcal{F}_0$ and ξ_c might break down for multistep transitions, a possibility that has not yet been thoroughly investigated. Clarifying this issue constitutes another key motivation for the present study.

This paper addresses these open issues by providing a detailed and integrated analysis, with the following novel contributions:

- (1) We perform the first systematic mapping of the parameter space compatible with a SFOEWPT in the inverted type-I 2HDM, including multistep transitions. Based on an extensive numerical scan yielding 2.36×10^6 physically consistent parameter points, we characterize and distinguish the parameter regions associated with one-, two-, and three-step SFOEWPTs.
- (2) We investigate the correlation between the vacuum uplifting measure $\Delta\mathcal{F}_0$ and the EWPT strength ξ_c .

¹An SFOEWPT has been investigated in numerous BSM models, including those with an $SU(2)$ singlet [16–36], those with an extra triplet [37–46], composite Higgs models [47–49], supersymmetric models [50–67], and other frameworks [68–71].

²Multistep EWPTs within the 2HDM framework have primarily been studied in the normal scenario [95–97].

We demonstrate, for the first time, that, although this correlation persists for one-step SFOEWPTs, it breaks down in multistep scenarios.

- (3) We present the first comprehensive calculation of GW signal-to-noise ratios (SNRs) at LISA for multistep SFOEWPTs in the inverted type-I 2HDM. Our analysis reveals that detectable GW signals ($\text{SNR} > 10$) predominantly originate from two-step transitions, even though one-step SFOEWPTs are more common in the parameter space.
- (4) We carry out the first dedicated collider phenomenology analysis of these SFOEWPT-compatible parameter points at the 1.5 TeV Compact Linear Collider (CLIC). We identify the process $e^+e^- \rightarrow H^+H^-$, followed by $H^\pm \rightarrow W^\pm h$, as a promising discovery channel. Notably, we find that the decay $h \rightarrow \gamma\gamma$ is significantly enhanced for negative $s_{\beta-\alpha}$ values, which correlate strongly with one-step SFOEWPTs. Based on this result, we propose two complementary golden final states, $W^+W^-b\bar{b}\tau^+\tau^-$ and $W^+W^-b\bar{b}\gamma\gamma$, ensuring high discovery potential for the inverted type-I 2HDM at the 1.5 TeV CLIC.

These interconnected analyses collectively highlight the rich potential of the inverted type-I 2HDM for generating a SFOEWPT and underscore the critical synergy between early Universe cosmology and collider experiments in the quest to uncover new physics.

The remainder of this paper is organized as follows. Section II provides a review of the type-I 2HDM in the inverted Higgs scenario. In Sec. III, we detail the formalism for the one-loop effective potential at finite temperature, the dynamics of the EWPT, the Higgs vacuum uplifting measure $\Delta\mathcal{F}_0$, and the calculation of GW signals. Section IV presents our numerical scan methodology, the characteristics of the parameter space for multistep SFOEWPTs and their GW signals, and the analysis of the correlation between $\Delta\mathcal{F}_0$ and ξ_c . The collider phenomenology at the 1.5 TeV CLIC is discussed in Sec. V. Finally, Sec. VI summarizes our main findings and presents our conclusions. Appendix contains supplementary details on the calculation of $\Delta\mathcal{F}_0$ in the Higgs basis.

II. REVIEW OF TYPE-I 2HDM IN THE INVERTED HIGGS SCENARIO

The 2HDM extends the SM by introducing two complex $SU(2)_L$ scalar doublet fields, Φ_1 and Φ_2 , both with hypercharge $Y = 1$ under the $Q = T_3 + Y$ convention [79]. These fields are expressed as

$$\Phi_i = \begin{pmatrix} w_i^+ \\ \frac{v_i + \rho_i + i\eta_i}{\sqrt{2}} \end{pmatrix}, \quad (i = 1, 2), \quad (1)$$

where v_1 and v_2 are the VEVs of Φ_1 and Φ_2 , respectively. Their ratio defines a key model parameter, $t_\beta = v_2/v_1$,

where we use $s_x = \sin x$, $c_x = \cos x$, and $t_x = \tan x$ for notational simplicity. The combined VEV, $v = \sqrt{v_1^2 + v_2^2} \approx 246$ GeV, induces spontaneous electroweak symmetry breaking.

To prevent tree-level FCNCs, a discrete Z_2 symmetry is imposed, under which $\Phi_1 \rightarrow \Phi_1$ and $\Phi_2 \rightarrow -\Phi_2$ [80,81]. The scalar potential, respecting CP invariance and this Z_2 symmetry (softly broken), is given by

$$\begin{aligned} V_\Phi(\Phi_1, \Phi_2) = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \text{H.c.}) \\ & + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ & + \frac{1}{2} \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + \text{H.c.}], \end{aligned} \quad (2)$$

where the m_{12}^2 term softly breaks the Z_2 symmetry. This potential yields five physical Higgs bosons: a lighter CP -even scalar h , a heavier CP -even scalar H , a CP -odd pseudoscalar A , and a pair of charged Higgs bosons $H^\pm$.

The interaction eigenstates are related to the mass eigenstates by rotations involving two mixing angles α and β [107]:

$$\begin{aligned} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} &= \mathbb{R}(\alpha) \begin{pmatrix} H \\ h \end{pmatrix}, & \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} &= \mathbb{R}(\beta) \begin{pmatrix} G^0 \\ A \end{pmatrix}, \\ \begin{pmatrix} w_1^\pm \\ w_2^\pm \end{pmatrix} &= \mathbb{R}(\beta) \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}. \end{aligned}$$

Here, G^0 and $G^\pm$ are the Goldstone bosons absorbed to become the longitudinal components of the Z and $W^\pm$ bosons, respectively. The rotation matrix $\mathbb{R}(\theta)$ is defined as

$$\mathbb{R}(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}.$$

A common and convenient set of independent model parameters, often termed the physical basis, is $\{t_\beta, c_{\beta-\alpha}, \bar{M}^2, v, m_h, m_H, m_A, m_{H^\pm}\}$, where $\bar{M}^2 = m_{12}^2/(s_\beta c_\beta)$. The parameters of V_Φ in Eq. (2) can be expressed in terms of these physical parameters. The mass-squared terms m_{11}^2 and m_{22}^2 are determined by the minimization conditions of the potential:

$$\begin{aligned} m_{11}^2 &= s_\beta^2 \bar{M}^2 - \frac{c_\alpha c_{\beta-\alpha}}{2c_\beta} m_H^2 + \frac{s_\alpha s_{\beta-\alpha}}{2c_\beta} m_h^2, \\ m_{22}^2 &= c_\beta^2 \bar{M}^2 - \frac{s_\alpha c_{\beta-\alpha}}{2s_\beta} m_H^2 - \frac{c_\alpha s_{\beta-\alpha}}{2s_\beta} m_h^2. \end{aligned}$$

The quartic couplings λ_i ($i = 1, \dots, 5$) are then given by

$$\begin{aligned}
\lambda_1 &= \frac{1}{c_\beta^2 v^2} (s_\alpha^2 m_h^2 + c_\alpha^2 m_H^2 - s_\beta^2 \bar{M}^2), \\
\lambda_2 &= \frac{1}{s_\beta^2 v^2} (c_\alpha^2 m_h^2 + s_\alpha^2 m_H^2 - c_\beta^2 \bar{M}^2), \\
\lambda_3 &= \frac{1}{v^2} \left[2m_{H^\pm}^2 + \frac{s_{2\alpha}}{s_{2\beta}} (m_H^2 - m_h^2) - \bar{M}^2 \right], \\
\lambda_4 &= \frac{1}{v^2} [m_A^2 - 2m_{H^\pm}^2 + \bar{M}^2], \\
\lambda_5 &= \frac{1}{v^2} [\bar{M}^2 - m_A^2].
\end{aligned}$$

The SM Higgs boson, H_{SM} , is a linear combination of h and H :

$$H_{\text{SM}} = s_{\beta-\alpha} h + c_{\beta-\alpha} H.$$

In the 2HDM, the Higgs boson observed at the LHC with a mass of 125 GeV can be either h (normal scenario) or H (inverted scenario) [108,109]. The inverted scenario is of particular interest as theoretical and experimental constraints impose upper bounds on the BSM Higgs boson masses (typically below ~ 600 GeV), rendering the model verifiable or falsifiable at high-energy colliders. This feature makes the inverted scenario particularly compelling for realizing a SFOEWPT, since strong phase transitions are favored in parameter regions with lower scalar masses [90,96,106]. Motivated by these considerations, this work focuses on the inverted Higgs scenario.

The 2HDM has four variants (type I, type II, type X, and type Y) based on the Z_2 charge assignments to right-handed fermions. In the inverted Higgs scenario, type II and type Y are severely constrained and typically do not offer viable parameter space under the theoretical and experimental limits. This is primarily due to the stringent lower bound on the charged Higgs boson mass ($m_{H^\pm} \gtrsim 580$ GeV) arising from measurements of the inclusive B -meson decay $B \rightarrow X_s \gamma$ [91–94]. In the context of the inverted scenario (where $m_H = 125$ GeV and $m_h < 125$ GeV), such a heavy charged Higgs boson leads to large mass splittings within the scalar spectrum. These large splittings, in turn, severely restrict the allowed parameter space due to theoretical requirements, especially perturbativity and unitarity [110,111]. Therefore, this paper focuses on the type-I 2HDM in the inverted Higgs scenario, hereafter referred to as the “inverted type-I 2HDM” for simplicity. Our specific model configuration is thus

type-I 2HDM with $m_H = 125$ GeV,

which features the following six model parameters:

$$\{t_\beta, c_{\beta-\alpha}, \bar{M}^2, m_h, m_A, m_{H^\pm}\}. \quad (3)$$

The interactions between the Higgs bosons and gauge bosons are described by the Lagrangian term,

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} &= \left(gm_W W_\mu^\dagger W^\mu + \frac{1}{2} g_Z m_Z Z_\mu Z^\mu \right) (s_{\beta-\alpha} h + c_{\beta-\alpha} H) \\
&+ \frac{g}{2} i [W_\mu^+ (c_{\beta-\alpha} h - s_{\beta-\alpha} H) \overleftrightarrow{\partial}^\mu H^- - \text{H.c.}] - \frac{g}{2} [W_\mu^+ H^- \overleftrightarrow{\partial}^\mu A + \text{H.c.}] \\
&+ i \left\{ e A_\mu + \frac{g_Z}{2} (s_W^2 - c_W^2) Z_\mu \right\} H^+ \overleftrightarrow{\partial}^\mu H^- + \frac{g_Z}{2} Z_\mu [c_{\beta-\alpha} A \overleftrightarrow{\partial}^\mu h - s_{\beta-\alpha} A \overleftrightarrow{\partial}^\mu H], \quad (4)
\end{aligned}$$

where $s_W = \sin \theta_W$ (with θ_W denoting the electroweak mixing angle), $g_Z = g/c_W$, and the shorthand $f \overleftrightarrow{\partial}^\mu g$ is defined as $f \overleftrightarrow{\partial}^\mu g \equiv f(\partial^\mu g) - (\partial^\mu f)g$.

The Yukawa Lagrangian is parametrized as

$$\begin{aligned}
\mathcal{L}_{\text{Yuk}} &= - \sum_f \left(\frac{m_f}{v} \xi_f^h \bar{f} f h + \frac{m_f}{v} \kappa_f^H \bar{f} f H - i \frac{m_f}{v} \xi_f^A \bar{f} \gamma_5 f A \right) \\
&- \left\{ \frac{\sqrt{2} V_{ud}}{v} H^+ \bar{u} (m_u \xi_u^A \mathbf{P}_L + m_d \xi_d^A \mathbf{P}_R) d + \frac{\sqrt{2} m_\ell}{v} H^+ \xi_\ell^A \bar{\nu}_L \ell_R + \text{H.c.} \right\}.
\end{aligned}$$

In the type-I 2HDM, the Yukawa coupling modifiers are

$$\kappa_f^H = \frac{s_\alpha}{s_\beta} = c_{\beta-\alpha} - \frac{s_{\beta-\alpha}}{t_\beta}, \quad \xi_f^h = \frac{c_\alpha}{s_\beta} = s_{\beta-\alpha} + \frac{c_{\beta-\alpha}}{t_\beta}, \quad \xi_t^A = -\xi_{b,\tau}^A = \frac{1}{t_\beta}. \quad (5)$$

III. ONE-LOOP EFFECTIVE POTENTIAL, ELECTROWEAK PHASE TRANSITION DYNAMICS, AND GW SIGNALS

A. One-loop effective potential

The dynamics of the EWPT in the early Universe is governed by the effective potential at finite temperature [89,112–114]. This potential is typically evaluated for spatially homogeneous field configurations.³ We therefore replace the quantum fields $\Phi_{1,2}$ [defined in Eq. (1)] with their classical, constant background configurations $\Phi_{1,2}^c$, expressed in terms of three homogeneous real scalar fields w_1 , w_2 , and w_3 as

$$\Phi_1^c = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ w_1 \end{pmatrix}, \quad \Phi_2^c = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ w_2 + iw_3 \end{pmatrix}.$$

The inclusion of w_3 , even in a CP -conserving 2HDM where the zero-temperature vacuum preserves CP symmetry, is essential to account for the possibility of a CP -violating vacuum state or path at finite temperature [74,97,115]. The full one-loop effective potential at a temperature T is then given by the sum of four components,

$$V_{\text{eff}}(\vec{w}, T) = V_{\text{tree}}(\vec{w}) + V_{\text{CW}}(\vec{w}) + V_{\text{CT}}(\vec{w}) + V_T(\vec{w}, T), \quad (6)$$

where $\vec{w} = (w_1, w_2, w_3)$. The tree-level potential $V_{\text{tree}}(\vec{w})$ is obtained by substituting $\Phi_{1,2} \rightarrow \Phi_{1,2}^c$ into the 2HDM potential $V_\Phi(\Phi_1, \Phi_2)$ given in Eq. (2).

The one-loop Coleman-Weinberg (CW) potential $V_{\text{CW}}(\vec{w})$ in the $\overline{\text{MS}}$ renormalization scheme is given by [116]

$$V_{\text{CW}}(\vec{w}) = \frac{1}{64\pi^2} \sum_i n_i m_i^4(\vec{w}) \left[\ln \left(\frac{m_i^2(\vec{w})}{\mu^2} \right) - c_i \right], \quad (7)$$

where the sum extends over all massive particle species $i = h, H, A, H^\pm, G^0, G^\pm, W^\pm, Z, f$, with f representing the SM fermions. The factor n_i denotes the number of degrees of freedom for each species, being positive for bosons and negative for fermions. Specifically, $n_i = -12$ for each quark, -4 for each charged lepton, 6 for $W^\pm$ bosons, 3 for the Z boson, 2 for $H^\pm$, 2 for $G^\pm$, 1 for G^0 , and 1 for each real neutral scalar (h, H, A).

The field-dependent squared masses, $m_i^2(\vec{w})$, are determined by diagonalizing the field-dependent mass matrices for all particles that couple to the scalar background fields

³For studying the EWPT as a cosmological phase transition driven by scalar field dynamics in thermal equilibrium, it is appropriate to consider the effective potential as a function of spatially homogeneous background fields. This approach is consistent with the homogeneous and isotropic nature of the cosmological background.

$\vec{w}$. Their explicit forms for the 2HDM can be found in, e.g., Ref. [74]. The renormalization scale μ is set to the electroweak scale $v \approx 246$ GeV. The constants c_i 's depend on the particle type in the $\overline{\text{MS}}$ scheme: $c_i = 5/6$ for gauge bosons and $c_i = 3/2$ for scalars and fermions.

Although the CW potential in Eq. (7) incorporates $\overline{\text{MS}}$ subtraction, an on-shell renormalization scheme is adopted by introducing a counterterm potential, $V_{\text{CT}}(\vec{w})$. This ensures that the physical input parameters in Eq. (3) correspond to their intended physical values at the one-loop level, thus preventing shifts in observables. The counterterm potential has the form of

$$V_{\text{CT}}(\vec{w}) = \delta m_{11}^2 \frac{w_1^2}{2} + \delta m_{22}^2 \frac{w_2^2 + w_3^2}{2} - \delta m_{12}^2 w_1 w_2 + \frac{\delta \lambda_1}{8} w_1^4 + \frac{\delta \lambda_2}{8} (w_2^2 + w_3^2)^2 + \frac{\delta \lambda_3 + \delta \lambda_4}{4} w_1^2 (w_2^2 + w_3^2) + \frac{\delta \lambda_5}{4} w_1^2 (w_2^2 - w_3^2). \quad (8)$$

Explicit expressions for these counterterms in the Landau gauge are provided in Ref. [74], where techniques are also discussed for regulating the divergent logarithmic contributions from massless Goldstone bosons, which are characteristic of this gauge choice.

Finally, the thermal correction $V_T(\vec{w}, T)$ [86,117], including daisy resummation for thermal masses [118], is given by

$$V_T(\vec{w}, T) = \frac{T^4}{2\pi^2} \sum_{i_B} n_{i_B} J_B \left(\frac{m_{i_B}^2(\vec{w})}{T^2} \right) + \frac{T^4}{2\pi^2} \sum_{j_F} n_{j_F} J_F \left(\frac{m_{j_F}^2(\vec{w})}{T^2} \right) - \frac{T^4}{12\pi} \sum_{k_L} n_{k_L} \left[\left(\frac{\tilde{m}_{k_L}^2(\vec{w}, T)}{T^2} \right)^{3/2} - \left(\frac{m_{k_L}^2(\vec{w})}{T^2} \right)^{3/2} \right], \quad (9)$$

where the sum over i_B includes all bosonic species (scalars and gauge bosons), j_F includes all fermionic species, and the sum over k_L in the daisy resummation term includes scalars and the longitudinal components of gauge bosons that acquire thermal masses. The thermal functions $J_B(x)$ and $J_F(x)$ are defined by the standard integrals:

$$J_B(x) = \int_0^\infty dk k^2 \ln[1 - e^{-\sqrt{k^2+x}}],$$

$$J_F(x) = \int_0^\infty dk k^2 \ln[1 + e^{-\sqrt{k^2+x}}].$$

The temperature-dependent thermal (Debye) masses, $\tilde{m}_k^2(\vec{w}, T)$, appearing in the daisy resummation term in Eq. (9), are crucial for improving the perturbative expansion at high temperatures and regulating infrared divergences associated with light scalar modes. Explicit expressions for

$\tilde{m}_k^2(\vec{w}, T)$ can be found in the literature such as Refs. [74,119].

B. Thermal history and transition dynamics

The dynamics of the EWPT in the early Universe is governed by the loop-corrected effective potential, $V_{\text{eff}}(w_1, w_2, w_3, T)$. At very high temperatures, long before electroweak symmetry breaking, this potential features a single global minimum at $w_1 = w_2 = w_3 = 0$, corresponding to the electroweak symmetric vacuum. As the Universe cools, additional local minima may develop at nonzero field values. The coordinates of such a minimum evolve as a function of temperature and are denoted by (v_1^T, v_2^T, v_3^T) .

At a particular temperature T_c , one of these broken-phase minima may become degenerate in free energy with the symmetric minimum. This temperature is known as the critical temperature and is defined by the condition:

$$V_{\text{eff}}(0, 0, 0, T_c) = V_{\text{eff}}(v_1^{T_c}, v_2^{T_c}, v_3^{T_c}, T_c). \quad (10)$$

As the temperature continues to decrease below T_c , this broken-phase minimum becomes energetically favored over the symmetric vacuum. If a potential barrier separates this now true (broken) vacuum from the false (symmetric) vacuum, the transition proceeds via thermal tunneling—a hallmark of a *first-order phase transition* (FOEWPT). A key measure of such a transition's strength is the order parameter ξ_c , defined at the critical temperature T_c as

$$\xi_c \equiv \frac{v_c}{T_c}, \quad (11)$$

where $v_c = \sqrt{\sum_{i=1}^3 (v_i^{T_c})^2}$ is the magnitude of the VEV in the broken phase at T_c . These definitions of T_c and ξ_c apply directly to a one-step transition originating from the symmetric phase. However, the EWPT may proceed through multiple steps, involving a sequence of transitions via intermediate metastable vacua before the Universe reaches its final true vacuum [37,39,59,69,98–103]. In such multistep scenarios, the definitions of the critical temperature and order parameter for each individual step are modified, as will be detailed in Sec. IV A.

For successful electroweak baryogenesis, the FOEWPT typically needs to be strong in order to sufficiently suppress sphaleron-induced baryon number washout in the broken phase. An SFOEWPT is often quantified by the condition $\xi_c > 1$. In this work, we adopt a more stringent criterion [86]:

$$\xi_c > 1.3. \quad (12)$$

This choice is motivated by our analysis, which shows that achieving a SNR greater than 10 for detectable GWs at LISA frequently requires large values of ξ_c , often exceeding 2. Thus, imposing $\xi_c > 1.3$ helps isolate regions of the 2HDM

parameter space that are more likely to yield both a SFOEWPT and observable GW signatures.

The actual realization of a FOEWPT, triggered by thermal tunneling, hinges on the nucleation of true vacuum bubbles that are large enough to expand rather than collapse. This occurs at the nucleation temperature T_n , where critical bubbles—those that have just overcome the energy barrier associated with surface tension—become energetically favored to grow.

The nucleation temperature T_n is formally defined by the condition that the probability to nucleate a bubble within one Hubble volume during one Hubble time, Γ/H^4 , equals unity [96,120],

$$\frac{\Gamma(T_n)}{H^4(T_n)} = 1,$$

where $\Gamma(T)$ is approximately given by

$$\Gamma(T) \simeq T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} \exp\left(-\frac{S_3(T)}{T}\right).$$

Here, $S_3(T)$ is the three-dimensional Euclidean action for the $O(3)$ -symmetric bounce solution connecting the false and true vacua,

$$S_3(T) = 4\pi \int_0^\infty dr r^2 \left[\frac{1}{2} \sum_k \left(\frac{dw_k(r)}{dr} \right)^2 + V_{\text{eff}}(\vec{w}(r), T) \right],$$

where $\vec{w}(r)$ with components $w_k(r)$ represents the field profiles along the radial coordinate r for the fields participating in the bounce. These profiles are solutions to the bounce equations,

$$\frac{d^2 w_k}{dr^2} + \frac{2}{r} \frac{dw_k}{dr} = \frac{\partial V_{\text{eff}}(\vec{w}(r), T)}{\partial w_k},$$

subject to the boundary conditions,

$$\lim_{r \rightarrow \infty} \vec{w}(r) = \vec{w}_{\text{false}}, \quad \left. \frac{d\vec{w}(r)}{dr} \right|_{r=0} = \vec{0},$$

where $\vec{w}_{\text{false}}$ denotes the field configuration in the false vacuum. In practice, T_n is often well-approximated by the criterion $S_3(T_n)/T_n \simeq 140$ [120,121].

Following their nucleation, bubbles of the true vacuum expand. As these bubbles grow and coalesce, they eventually percolate, filling space and thereby completing the FOEWPT. The percolation temperature T_p is defined as the temperature at which at least 29% of the false vacuum has tunneled into the true vacuum. This temperature marks the practical completion of the phase transition, with the Universe having largely transitioned out of the symmetric phase into the lower-energy state.

Finally, at zero temperature, the present-day Universe resides in the global minimum of the effective potential.

In the CP -conserving 2HDM, this minimum corresponds to the VEVs:

$$v_1^{T=0} = v_{C\beta}, \quad v_2^{T=0} = v_{S\beta}, \quad v_3^{T=0} = 0.$$

C. Higgs vacuum uplifting and its implications

An important theoretical diagnostic for the nature of the EWPT is the Higgs vacuum uplifting [55,106,122]. This phenomenon is characterized by the difference in zero-temperature vacuum energy density between the 2HDM and the SM. To quantify this, we first express the one-loop effective potential at temperature T as

$$V_{\text{eff}}(\vec{w}, T) = V_0(\vec{w}) + V_T(\vec{w}, T),$$

where the zero-temperature component $V_0(\vec{w})$ is

$$V_0(\vec{w}) = V_{\text{tree}}(\vec{w}) + V_{\text{CW}}(\vec{w}) + V_{\text{CT}}(\vec{w}).$$

The vacuum energy density difference, $\mathcal{F}_T$, between the symmetric phase where $\langle |\vec{w}| \rangle = 0$ and a broken phase where $\langle |\vec{w}| \rangle = v_T$ at temperature T is given by

$$\begin{aligned} \mathcal{F}_T &= V_{\text{eff}}(v_T, T) - V_{\text{eff}}(0, T) \\ &= [V_0(v) - V_0(0)] + [V_0(v_T) - V_0(v) + V_T(v_T, T) \\ &\quad - V_T(0, T)] \\ &\equiv \mathcal{F}_0 + \Delta V_T, \end{aligned} \quad (13)$$

where $\mathcal{F}_0 \equiv V_0(v) - V_0(0)$ is the vacuum energy density difference between the symmetric and broken phases at zero temperature. Since a successful EWPT requires $\mathcal{F}_0 < 0$, we define $\mathcal{F}_0 = -|\mathcal{F}_0|$. The thermal correction term, ΔV_T , typically increases monotonically with temperature.

A key quantity in analyzing vacuum uplifting is the difference between $\mathcal{F}_0$ in the 2HDM and $\mathcal{F}_0$ in the SM,

$$\Delta \mathcal{F}_0 \equiv \mathcal{F}_0^{\text{2HDM}} - \mathcal{F}_0^{\text{SM}}, \quad (14)$$

where $\mathcal{F}_0^{\text{SM}} \approx -1.25 \times 10^8 \text{ GeV}^4$. $\Delta \mathcal{F}_0$ is particularly useful as it is renormalization-scale independent and gauge invariant, providing a robust theoretical measure.

In the 2HDM, $\Delta \mathcal{F}_0$ is given by [106]

$$\begin{aligned} \Delta \mathcal{F}_0 &= \Delta \mathcal{F}_0^{\text{tree}} - \frac{m_{H_{\text{SM}}}^4}{64\pi^2} (3 + \log 2) - \sum_k \frac{m_{0_k}^4}{64\pi^2} \left(\log \frac{|m_{0_k}^2|}{\mu^2} - \frac{1}{2} \right) \\ &\quad + \frac{1}{64\pi^2} \sum_k \frac{1}{4} \left\{ \tilde{I}_k^2 - 2m_k^4 + [(\tilde{I}_k - 2m_k^2)^2 + m_k^2(\tilde{J}_k - \tilde{I}_k)] \log \frac{m_k^2}{\mu^2} \right\}, \end{aligned} \quad (15)$$

where the sum over k includes the scalar states of $H^\pm, A, H, h, G^\pm, G^0$. The expressions for $m_{0_k}^2$ (the scalar squared masses at the origin, i.e., in the symmetric vacuum) and the field-derivative quantities $\tilde{I}_k$ and $\tilde{J}_k$ are detailed in the Appendix. The tree-level contribution $\Delta \mathcal{F}_0^{\text{tree}}$ depends on whether the normal or inverted Higgs scenario is considered:

$$\Delta \mathcal{F}_0^{\text{tree}} = \begin{cases} -c_{\beta-\alpha}^2 \frac{v^2(m_H^2 - m_h^2)}{8}, & \text{if } m_h = 125 \text{ GeV} \quad (\text{normal scenario}); \\ +s_{\beta-\alpha}^2 \frac{v^2(m_H^2 - m_h^2)}{8}, & \text{if } m_H = 125 \text{ GeV} \quad (\text{inverted scenario}). \end{cases} \quad (16)$$

In the normal Higgs scenario, $\Delta \mathcal{F}_0^{\text{tree}}$ is negative definite, while in the inverted Higgs scenario, it is positive definite. In both scenarios, $\Delta \mathcal{F}_0^{\text{tree}}$ vanishes in the Higgs alignment limit where either $s_{\beta-\alpha} = 0$ (normal) or $c_{\beta-\alpha} = 0$ (inverted).

In the Higgs alignment limit of the inverted 2HDM, corresponding to $s_{\beta-\alpha} = 0$, $\Delta \mathcal{F}_0$ simplifies to

$$\begin{aligned} \Delta \mathcal{F}_0|_{s_{\beta-\alpha}=0} &= \frac{1}{64\pi^2} \left[(m_H^2 - 2\bar{M}^2)^2 \left\{ \frac{3}{2} + \frac{1}{2} \log \frac{4m_A m_h m_{H^\pm}^2}{(m_H^2 - 2\bar{M}^2)^2} \right\} \right. \\ &\quad \left. + \frac{1}{2} (m_A^4 + m_h^4 + 2m_{H^\pm}^4) + (m_H^2 - 2\bar{M}^2)(m_A^2 + m_h^2 + 2m_{H^\pm}^2) \right], \end{aligned} \quad (17)$$

which is generally positive definite.

It was first demonstrated through empirical studies in Ref. [106] that, in the 2HDM, the quantity $\Delta \mathcal{F}_0$ exhibits a strong correlation with the strength of the EWPT, as measured by ξ_c . This correlation makes the analysis of

$\Delta \mathcal{F}_0$ an efficient approach since calculations of this zero-temperature quantity is analytically simpler than calculations involving the full thermal effective potential. Specifically, for one-step transitions in the normal Higgs scenario, it was found that a SFOEWPT is typically

guaranteed if

SFOEWPT via one-step with $m_h = 125$ GeV:

$$0.34 \lesssim \frac{\Delta\mathcal{F}_0}{|\mathcal{F}_0^{\text{SM}}|} < 1. \quad (18)$$

The upper bound ensures that the 2HDM electroweak vacuum remains the global minimum at $T = 0$. This range can be reinterpreted as $0 < |\mathcal{F}_0^{2\text{HDM}}| \lesssim 0.66|\mathcal{F}_0^{\text{SM}}|$. This condition implies that for a SFOEWPT, the depth of the true vacuum at $T = 0$ (relative to the symmetric phase) becomes shallower compared to the depth in the SM, meaning that the Higgs vacuum is effectively “uplifted.” Consequently, a more significant uplifting of the Higgs potential at zero temperature tends to correlate with a stronger FOEWPT. In light of this, we will refer to $\Delta\mathcal{F}_0$ as the Higgs vacuum uplifting measure in subsequent discussions.

D. Gravitational wave signal

An SFOEWPT involves the nucleation and expansion of true vacuum bubbles, which subsequently grow, collide, and coalesce, ultimately completing the transition throughout the Universe. These highly energetic processes can leave cosmological imprints, making SFOEWPTs an intriguing target for probing early Universe dynamics [10,123]. If the transition is sufficiently strong, its signatures may be detectable at future space-based interferometers such as LISA, offering a unique cosmological probe of the early Universe [124–126].

The GW spectrum produced in a FOEWPT is characterized by four essential quantities [125–127]: (i) the transition temperature T_* , (ii) the strength of the phase transition α_{GW} , (iii) the inverse duration of the phase transition in Hubble units at the transition temperature, β_{GW}/H_* , and (iv) the bubble wall velocity v_w . Although the notations α and β are commonly used in the literature for the GW spectrum parameters, we adopt α_{GW} and β_{GW} herein to avoid confusion with the mixing angles α and β of the 2HDM.

The first important quantity is the transition temperature T_* , which is the characteristic temperature at which thermal parameters are evaluated for predicting GW signals from a FOEWPT. In this work, we choose T_* to be the percolation temperature T_p .

The second parameter α_{GW} , among the four essential quantities characterizing the GW spectrum [128,129]:

$$\alpha_{\text{GW}} = \frac{\epsilon}{\rho_\gamma},$$

where ϵ is the latent heat released during the phase transition, given by

$$\epsilon(T_*) = \left[-\Delta V_{\text{eff}} + T \frac{\partial \Delta V_{\text{eff}}}{\partial T} \right] \Big|_{T=T_*},$$

where ΔV is the difference between the effective potential of two phases before and after the PT. The radiation energy density is $\rho_\gamma = g_* \pi^2 T_*^4/30$, where $g_* \equiv g_*(T_*)$ is the effective number of relativistic degrees of freedom in the plasma: $g_* \simeq 106.75$ in the SM [126,130–132] and $g_* = 110.75$ in the 2HDMs.

The third key parameter β_{GW}/H_* represents the inverse duration of the phase transition relative to the Hubble time at T_* , and is defined as [133]

$$\frac{\beta_{\text{GW}}}{H_*} \equiv T_* \frac{d}{dT} \left(\frac{S_3}{T} \right) \Big|_{T=T_*},$$

where H_* is the Hubble parameter at temperature T_* ,

$$H_* = T_*^2 \sqrt{\frac{g_* \pi^2}{90 M_{\text{Pl}}^2}},$$

with the reduced Planck mass $M_{\text{Pl}} \approx 2.4 \times 10^{18}$ GeV [133].

The final parameter v_w is the speed of the bubble wall after nucleation, measured in the rest frame of the plasma. This parameter critically influences the GW spectrum by affecting the efficiency of energy transfer from the phase transition to the surrounding plasma, thereby shaping both the peak amplitude and peak frequency of the resulting GW signal [134–143]. Despite promising recent proposals [144,145], a universally accepted prediction for v_w from first principles is still lacking, and its determination remains highly intricate and model dependent. Consequently, in our analysis, we assume a constant terminal bubble wall velocity—a reasonable approximation for nonrunaway transitions where the acceleration phase is brief compared to the bubble lifetime—and thus treat v_w as an input parameter.

In the 2HDM, FOEWPTs with nonrunaway bubble expansion produce GWs predominantly from sound waves [146,147].⁴ The GW power spectrum from the dominant sound wave contribution is given by [130–132,148–150]

$$h^2 \Omega_{\text{GW}}(f) \simeq h^2 \Omega_{\text{GW}}^{\text{SW,peak}} \left(\frac{4}{7} \right)^{-\frac{7}{2}} \left(\frac{f}{f_{\text{GW}}^{\text{SW,peak}}} \right)^3 \times \left[1 + \frac{3}{4} \left(\frac{f}{f_{\text{GW}}^{\text{SW,peak}}} \right)^2 \right]^{-\frac{7}{2}}, \quad (19)$$

where $h = 0.674 \pm 0.005$ [151] and $\Omega_{\text{GW}}^{\text{SW,peak}}$ and $f_{\text{GW}}^{\text{SW,peak}}$ are the peak amplitude and peak frequency of the sound wave contribution, respectively.

⁴We scanned the parameter space for SFOEWPT-viable points and computed the corresponding GW spectra. Our results, presented in the next section, show that turbulence contributes negligibly compared to sound waves in the inverted type-I 2HDM.

The peak frequency $f_{\text{GW}}^{\text{SW,peak}}$ is given by

$$f_{\text{GW}}^{\text{SW,peak}} = 8.88 \times 10^{-6} \text{ Hz} \left(\frac{g_*}{100} \right)^{\frac{1}{6}} \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{\beta_{\text{GW}}}{H_*} \right) \frac{1}{\max(v_w, c_s)},$$

where $c_s = 1/\sqrt{3}$ is the speed of sound in the relativistic plasma. Semianalytic expressions for the peak amplitude $h^2 \Omega_{\text{GW}}^{\text{SW,peak}}$ are provided in Refs. [126,129,152] as

$$h^2 \Omega_{\text{GW}}^{\text{SW,peak}} = \begin{cases} 8.75 \times 10^{-6} h^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} \left(\frac{H_*}{\beta_{\text{GW}}} \right)^2 \left(\frac{\kappa_{\text{SW}} \alpha_{\text{GW}}}{1 + \alpha_{\text{GW}}} \right)^{\frac{3}{2}} \max(v_w, c_s)^2, & \text{if } H_* \tau_{\text{sh}} < 1; \\ 2.59 \times 10^{-6} h^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} \left(\frac{H_*}{\beta_{\text{GW}}} \right) \left(\frac{\kappa_{\text{SW}} \alpha_{\text{GW}}}{1 + \alpha_{\text{GW}}} \right)^2 \max(v_w, c_s), & \text{if } H_* \tau_{\text{sh}} \geq 1, \end{cases} \quad (20)$$

where the maximum effective lifetime of the sound wave source is taken to be one Hubble time, $\tau_{\text{sw}} = H_*^{-1}$ [153].

In Eq. (20), τ_{sh} is the shock formation timescale, and $H_* \tau_{\text{sh}}$ is given by

$$H_* \tau_{\text{sh}} = \frac{4\pi^{\frac{1}{3}}}{\sqrt{3}} \left(\frac{H_*}{\beta_{\text{GW}}} \right) \left(\frac{\kappa_{\text{SW}} \alpha_{\text{GW}}}{1 + \alpha_{\text{GW}}} \right)^{-\frac{1}{2}} \max(v_w, c_s).$$

The quantity κ_{SW} is the sound wave efficiency factor, given by [154]

$$\kappa_{\text{SW}} = \begin{cases} \frac{c_s^{11/5} \kappa_A \kappa_B}{(c_s^{11/5} - v_w^{11/5}) \kappa_B + v_w^{6/5} \kappa_A}, & \text{if } v_w < c_s; \\ \kappa_B + (v_w - c_s) \delta \kappa + \frac{(v_w - c_s)^3}{(v_J - c_s)^3} [\kappa_C - \kappa_B - (v_J - c_s) \delta \kappa], & \text{if } c_s < v_w < v_J; \\ \frac{(v_J - 1)^3 v_J^{5/2} v_w^{-5/2} \kappa_C \kappa_D}{[(v_J - 1)^3 - (v_w - 1)^3] v_J^{3/2} \kappa_C + (v_w - 1)^3 \kappa_D}, & \text{if } v_J < v_w, \end{cases} \quad (21)$$

where v_J is the Chapman-Jouguet velocity [154–156], defined as

$$v_J = \frac{1}{1 + \alpha_{\text{GW}}} \left(c_s + \sqrt{\alpha_{\text{GW}}^2 + \frac{2}{3} \alpha_{\text{GW}}} \right).$$

The auxiliary functions κ_X (for $X = A, B, C, D$) and $\delta \kappa$ in Eq. (21) are

$$\begin{aligned} 2\kappa_A &\simeq v_w^{6/5} \frac{6.9 \alpha_{\text{GW}}}{1.36 - 0.037 \sqrt{\alpha_{\text{GW}}} + \alpha_{\text{GW}}}, & \kappa_B &\simeq \frac{\alpha_{\text{GW}}^{2/5}}{0.017 + (0.997 + \alpha_{\text{GW}})^{2/5}}, & \kappa_C &\simeq \frac{\sqrt{\alpha_{\text{GW}}}}{0.135 + \sqrt{0.98 + \alpha_{\text{GW}}}}, \\ \kappa_D &\simeq \frac{\alpha_{\text{GW}}}{0.73 + 0.083 \sqrt{\alpha_{\text{GW}}} + \alpha_{\text{GW}}}, & \delta \kappa &\simeq -0.9 \log \frac{\sqrt{\alpha_{\text{GW}}}}{1 + \sqrt{\alpha_{\text{GW}}}}. \end{aligned}$$

The detectability of GW signals at the future LISA is quantified by the SNR, given by [126]

$$\text{SNR} = \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} df \left[\frac{h^2 \Omega_{\text{GW}}(f)}{h^2 \Omega_{\text{Sens}}(f)} \right]^2}.$$

Here, $\mathcal{T}$ is the experimental observation time in seconds, taken as four years for LISA in this study [12]. The integration limits $f_{\min}$ and $f_{\max}$ define LISA's sensitive frequency range. The term $h^2 \Omega_{\text{GW}}(f)$ is the predicted GW signal (from the 2HDM in this work), and $h^2 \Omega_{\text{Sens}}(f)$ is the nominal sensitivity of the LISA configuration to stochastic

backgrounds [157,158]. A signal is typically considered detectable if $\text{SNR} \gtrsim 10$ [132].

IV. MULTISTEP EWPT: PARAMETER SPACE, VACUUM STRUCTURE, AND GRAVITATIONAL WAVES

A. Scan-based analysis of multistep SFOEWPTs

In this section, we investigate the characteristics of the parameter space that support a SFOEWPT or yield a GW signal detectable at LISA with a SNR greater than 10. To this end, we perform a random scan over the following ranges of physical parameters:

$$m_h \in [30, 120] \text{ GeV}, \quad m_A \in [30, 700] \text{ GeV}, \quad m_{H^\pm} \in [80, 700] \text{ GeV}, \\ s_{\beta-\alpha} \in [-0.5, 0.5], \quad t_\beta \in [1, 50], \quad m_{12}^2 \in [0, 2 \times 10^4] \text{ GeV}^2. \quad (22)$$

Positive values for m_{12}^2 are exclusively scanned because the condition for avoiding a “panic vacuum” [159,160] requires $m_{12}^2 > 0$ [106].

For the parameter points generated within the ranges of Eq. (22), we cumulatively apply three categories of constraints to define distinct sets of points for our analysis. For clarity in the subsequent discussion, we adopt the following terminology:

- (i) *Physical parameter points.* Parameter points that satisfy all the theoretical requirements and current experimental constraints.
- (ii) *SFOEWPT parameter points.* Physical parameter points that additionally satisfy the SFOEWPT condition, $\xi_c > 1.3$.
- (iii) *GW parameter points.* Physical parameter points that also yield a LISA GW signal with a $\text{SNR} > 10$.

Let us begin by outlining the constraints that define the physical parameter points. The theoretical and experimental requirements include:

- (1) vacuum stability [159,161,162];
- (2) the condition that the Higgs potential is bounded from below [163];
- (3) tree-level unitarity in scalar-scalar scattering processes [79,164];
- (4) perturbativity of the Higgs quartic couplings [108];
- (5) consistency with the latest best-fit values for the electroweak oblique parameters (S , T , U) [165], as interpreted within the 2HDM framework [166,167]—the specific values used are $S = -0.04 \pm 0.10$, $T = 0.01 \pm 0.12$, and $U = -0.01 \pm 0.09$ [151], with proper treatment of parameter correlations;
- (6) constraints from FCNC processes at the 95% confidence level, including bounds from $B_{d,s} \rightarrow \mu^+ \mu^-$ [91], $B \rightarrow X_s \gamma$ [91,168–170], $B \rightarrow K^* \gamma$ [171], and $B_s \rightarrow \phi \gamma$ [172];
- (7) limits from direct searches at LEP, Tevatron, and the LHC;
- (8) constraints from Higgs precision measurements.

The theoretical conditions (1)–(5) are evaluated using the publicly available code 2HDMC [173], while the experimental constraints (6)–(8) are implemented using ScannerS2 [174] and HiggsTools1.2 [175].

We briefly comment on the treatment of the Higgs precision measurement constraints. For each 2HDM parameter point that satisfies constraints (1) through (7), we compute the global minimum of the Higgs precision χ^2 , denoted by $\chi_{\min}^2(2\text{HDM})$. We then evaluate, for every point, the difference $\Delta\chi^2 = \chi^2 - \chi_{\min}^2(2\text{HDM})$. Given that the model contains six free parameters, as listed in Eq. (3), we accept parameter points with $\Delta\chi^2 < 12.59$, corresponding to the 95% confidence level for 6 degrees of freedom.

From this comprehensive scan and filtering, we obtained 2.36×10^6 physical parameter points. For these points, we then perform a detailed analysis of the thermal phase transition history. This includes calculations of the one-loop effective potential at finite temperature, the critical temperature T_c , its corresponding VEV v_c , the nucleation temperature T_n (obtained by solving the bounce equation), the percolation temperature T_p , and the resulting GW spectrum. Given the complexity of these calculations, several public packages are available for numerical analysis, such as CosmoTransitions [176], PhaseTracer [153], and beyond the standard model phase transitions (BSMPT) [127,177,178]. We choose BSMPT3.0.7 for our numerical analysis, primarily because it is written in C++, which significantly reduces computation time. Moreover, it is adept at analyzing multi-step EWPTs by tracing multiple temperature-dependent vacuum phases across extended Higgs potentials. The code automatically identifies and characterizes each transition step in the sequence—including associated critical, nucleation, and percolation temperatures—making it particularly powerful for studying complex transition histories involving several vacuum directions.

In our analysis, we used the executable CalcGW to determine the critical temperature T_c , nucleation temperature T_n , percolation temperature T_p , and VEVs at T_c for each transition step. Additionally, we evaluated GW SNRs at LISA by setting $T_* = T_p$.

We retained only parameter points satisfying the following criteria: (i) the diagnostic output STATUS_EWSR equals EW_SYM_RES and (ii) successful (SUCCESS) diagnostics for STATUS_NLO_STABILITY, STATUS_TRACING, STATUS_COEX_PAIRS, STATUS_CRIT, STATUS_BOUNCE_SOL, STATUS_NUCL, STATUS_PERC, STATUS_COMPL, and STATUS_GW. For these points, we subsequently reran the executable MINIMATRACER to verify the absence of vacuum trapping [146,179]. Vacuum trapping is a critical issue in FOEWPT scenarios, in which the Universe remains stuck in a metastable vacuum despite the existence of a deeper true vacuum with $v \approx 246$ GeV at zero temperature. Approximately 5% of the identified SFOEWPT parameter points were found to suffer from vacuum trapping.

We begin by investigating whether the inverted type-I 2HDM can support a SFOEWPT and, if so, whether it proceeds via a one-step, two-step, or higher-order transition pathway. To systematically label individual transitions within an n -step EWPT, we define the notation $\text{PT}_i^{(n\text{-step})}$ as

$\text{PT}_i^{(n\text{-step})}$, the i th transition in an n -step FOEWPT.

To show the evolution through sequential vacuum states in such multistep thermal transitions, Fig. 1 presents

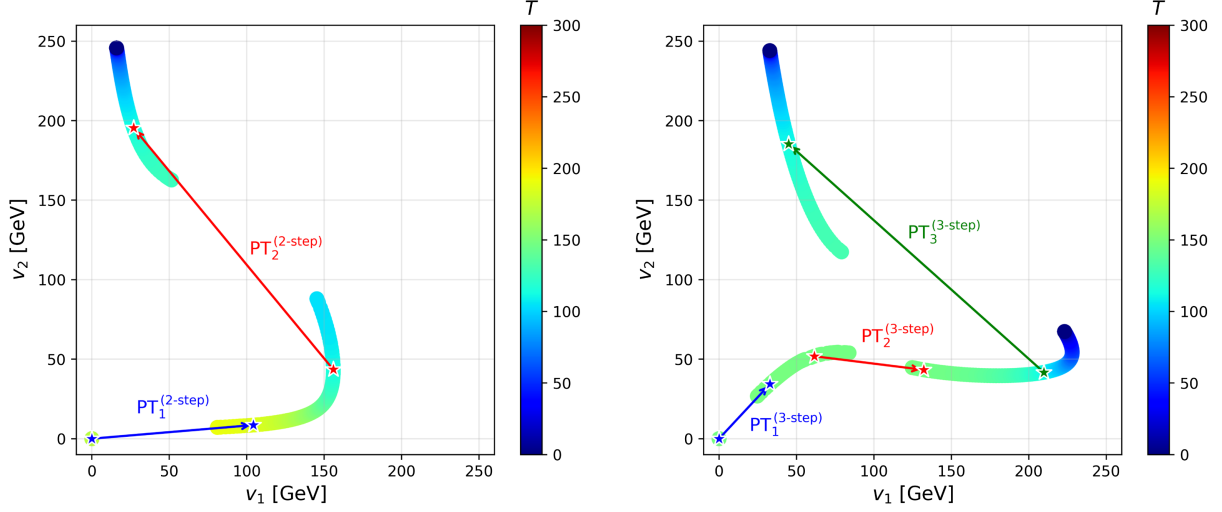


FIG. 1. The evolution of the minima of an effective potential with two relevant scalar field directions, as a function of temperature in units of GeV (indicated by the color scale). The left panel exemplifies a two-step EWPT for the benchmark point $\text{BP}_{2\text{-step}}$, while the right panel presents an example of a three-step EWPT for $\text{BP}_{3\text{-step}}$. Both benchmark points are described in the main text.

representative examples of the VEV evolution in the effective potential (projected onto two relevant field directions) for two-step (left panel) and three-step (right panel) EWPTs. In these plots, the color scale indicates the

temperature. These illustrative multistep EWPTs correspond to the benchmark points $\text{BP}_{2\text{-step}}$ and $\text{BP}_{3\text{-step}}$, which are defined by the following parameters:

$$\begin{aligned}
 \text{BP}_{2\text{-step}}: & \quad m_h = 83.4 \text{ GeV}, & m_A = 194.5 \text{ GeV}, & m_{H^\pm} = 178.9 \text{ GeV}, \\
 & \quad m_{12}^2 = 424.9 \text{ GeV}^2, & t_\beta = 15.4, & s_{\beta-\alpha} = 0.114; \\
 \text{BP}_{3\text{-step}}: & \quad m_h = 88.2 \text{ GeV}, & m_A = 155.4 \text{ GeV}, & m_{H^\pm} = 184.3 \text{ GeV}, \\
 & \quad m_{12}^2 = 1001.7 \text{ GeV}^2, & t_\beta = 7.4, & s_{\beta-\alpha} = 0.212.
 \end{aligned} \tag{23}$$

For transitions originating from a metastable (nonsymmetric) vacuum in a multistep sequence, the definitions of the critical temperature in Eq. (10) and the order parameter ξ_c in Eq. (11) must be generalized. Consider the i th transition in an n -step sequence, $\text{PT}_i^{(n\text{-step})}$. Prior to this transition—but following the completion of the preceding step $\text{PT}_{i-1}^{(n\text{-step})}$ —the Universe is assumed to reside in a global minimum of the effective potential at $(v_{1(i-)}^T, v_{2(i-)}^T)$. Note that we primarily focus on the dynamics within an effective two-field configuration, as the magnitude of v_3^T remains consistently negligible compared to v_1^T and v_2^T across the relevant temperature range (typically suppressed by factors of 10^{-6} or more). As the Universe continues to cool, another local minimum at $(v_{1(i+)}^T, v_{2(i+)}^T)$ may appear and become dynamically relevant.

When the temperature decreases further, these two minima can achieve equal effective potential values. This condition defines the critical temperature T_c for the

specific transition $\text{PT}_i^{(n\text{-step})}$:

$$V_{\text{eff}}(v_{1(i-)}^T, v_{2(i-)}^T, T_c) = V_{\text{eff}}(v_{1(i+)}^T, v_{2(i+)}^T, T_c). \tag{24}$$

The strength of this $\text{PT}_i^{(n\text{-step})}$ transition is then characterized by the order parameter ξ_c ,

$$\xi_c = \frac{\sqrt{(v_{1(i+)}^T - v_{1(i-)}^T)^2 + (v_{2(i+)}^T - v_{2(i-)}^T)^2}}{T_c}, \tag{25}$$

which quantifies the magnitude of the VEV change relative to the critical temperature for that particular step.

It is important to note that both T_c and ξ_c can vary significantly across the individual transitions $\text{PT}_i^{(n\text{-step})}$ within a multistep sequence. This feature is illustrated by the critical temperatures and order parameters associated with each transition stage for the benchmark points given in Eq. (23):

$$\begin{aligned}
\text{BP}_{2\text{-step}}: T_c &= 186.3 \text{ GeV}, \quad \xi_c = 0.562, \quad \text{for } \text{PT}_1^{(2\text{-step})}; \\
T_c &= 115.2 \text{ GeV}, \quad \xi_c = 1.731, \quad \text{for } \text{PT}_2^{(2\text{-step})}; \\
\text{BP}_{3\text{-step}}: T_c &= 148.3 \text{ GeV}, \quad \xi_c = 0.321, \quad \text{for } \text{PT}_1^{(3\text{-step})}; \\
T_c &= 147.4 \text{ GeV}, \quad \xi_c = 0.483, \quad \text{for } \text{PT}_2^{(3\text{-step})}; \\
T_c &= 113.2 \text{ GeV}, \quad \xi_c = 1.930, \quad \text{for } \text{PT}_3^{(3\text{-step})}.
\end{aligned} \tag{26}$$

In both examples, the largest ξ_c occurs at the final stage of the transition sequence. This highlights that focusing solely on the first transition from the symmetric phase may overlook cases where a SFOEWPT emerges only at a later stage in a multistep evolution.

A central question is how many parameter points support a SFOEWPT, and more specifically, which transition step within a multistep EWPT is most likely to be strong. Out of the 2.36×10^6 physically viable parameter points identified in our scan, the number of points that induce a SFOEWPT via a given transition step $\text{PT}_i^{(n\text{-step})}$ is summarized below⁵:

$$\begin{aligned}
N_{\text{SFOEWPT}}^{\text{PT}_1^{(1\text{-step})}} &= 5343, & N_{\text{SFOEWPT}}^{\text{PT}_1^{(2\text{-step})}} &= 200, \\
N_{\text{SFOEWPT}}^{\text{PT}_2^{(2\text{-step})}} &= 4486, & N_{\text{SFOEWPT}}^{\text{PT}_3^{(3\text{-step})}} &= 6.
\end{aligned} \tag{27}$$

All other transition stages, including transitions involving four or more steps, yield zero instances of a SFOEWPT. An interesting finding is that out of the 200 parameter points contributing to $N_{\text{SFOEWPT}}^{\text{PT}_1^{(2\text{-step})}}$ 186 also induce a SFOEWPT via $\text{PT}_2^{(2\text{-step})}$, thereby realizing two successive SFOEWPTs. This implies that only 14 parameter points exhibit a SFOEWPT exclusively through $\text{PT}_1^{(2\text{-step})}$.

Overall, approximately 0.42% of all physical parameter points accommodate a SFOEWPT in at least one transition step. The largest fraction arises from $\text{PT}_1^{(1\text{-step})}$, followed closely by $\text{PT}_2^{(2\text{-step})}$, which exhibits a comparable probability for supporting a SFOEWPT. Although much rarer, the model can also yield a SFOEWPT in a three-step sequence, predominantly at the final step $\text{PT}_3^{(3\text{-step})}$.

Although the numbers of parameter points presented in Eq. (27) are small relative to the total number of physical parameter points, they clearly indicate that a non-negligible region of the parameter space in the inverted type-I 2HDM can support a SFOEWPT. Moreover, it is important to emphasize that Nature selects only a single parameter

point; thus, the overall small percentage does not, by itself, disfavor the model as a viable framework for realizing a SFOEWPT.

B. Characteristics of SFOEWPT parameter points in multistep transitions

This subsection investigates the properties of SFOEWPT parameter points. As a primary illustration, Fig. 2 shows the distribution of these points in the $(m_h, m_{H^\pm})$ plane, color-coded by the order parameter ξ_c , across various multistep transition types: $\text{PT}_1^{(1\text{-step})}$ in the upper-left panel, $\text{PT}_1^{(2\text{-step})}$ in the upper-right panel, $\text{PT}_2^{(2\text{-step})}$ in the lower-left panel, and $\text{PT}_3^{(3\text{-step})}$ in the lower-right panel. In each panel, points are layered by ξ_c , with higher values plotted on top to emphasize the stronger transitions.

Note that the results for $\text{PT}_3^{(3\text{-step})}$ are based on a total of 29 parameter points, rather than the smaller sample reported in Eq. (27). Because SFOEWPTs via $\text{PT}_3^{(3\text{-step})}$ were too rare in the general scan (with only six points identified), we performed an additional targeted scan over a narrower parameter range: $m_h \in [70, 90]$ GeV, $m_A \in [110, 250]$ GeV, $m_{H^\pm} \in [160, 240]$ GeV, $s_{\beta-\alpha} \in [0, 0.25]$, $\tan\beta \in [3, 8]$, and $m_{12}^2 \in [700, 1300]$ GeV², generating 4×10^5 additional physical parameter points. This yielded 23 new SFOEWPT points via $\text{PT}_3^{(3\text{-step})}$, bringing the total to 29 for this transition. All of these are included in the lower-right panel of Fig. 2.

In Fig. 2, we also show, for comparison, the full set of physical parameter points as underlying gray scatter points. These illustrate how theoretical requirements and current experimental data constrain the parameter space in the inverted type-I 2HDM. In particular, the mass of the charged Higgs boson $H^\pm$ is bounded from above, with $m_{H^\pm} \lesssim 630$ GeV. It is also notable that the lighter CP -even Higgs boson h can have a mass below $m_{H_{\text{SM}}}/2$, although this region is sparsely populated compared to the region where $m_h > m_{H_{\text{SM}}}/2$. Such points remain viable due to the possibility of a “Higgs-phobic” h , where the H - h - h coupling is sufficiently suppressed to evade stringent constraints from null results in $H_{\text{SM}} \rightarrow hh$ searches [180].

The additional requirement of a SFOEWPT imposes further stringent constraints on the allowed regions for $m_{H^\pm}$ and m_h . A common feature across all transition types is the exclusion of parameter points with $m_h < m_{H_{\text{SM}}}/2$ by the SFOEWPT condition.⁶ Beyond this shared feature, the specific impact of the SFOEWPT requirement on $m_{H^\pm}$ and m_h depends on the transition path. For one-step transitions, $m_{H^\pm}$ is tightly constrained to the range [295, 441] GeV, while m_h spans [62.5, 120] GeV. In contrast, $\text{PT}_2^{(2\text{-step})}$

⁵These results depend on the specific algorithms used to trace the VEVs and identify minima of the effective potential at finite temperature. We employed the default mode in the Multi-Step Phase Transition module of BSMPT, which relies on the MINIMUM_TRACER. Other modes or algorithms may yield different results.

⁶Relaxing the SFOEWPT criterion—for example, to $\xi_c > 1$ —does not significantly alter this outcome.

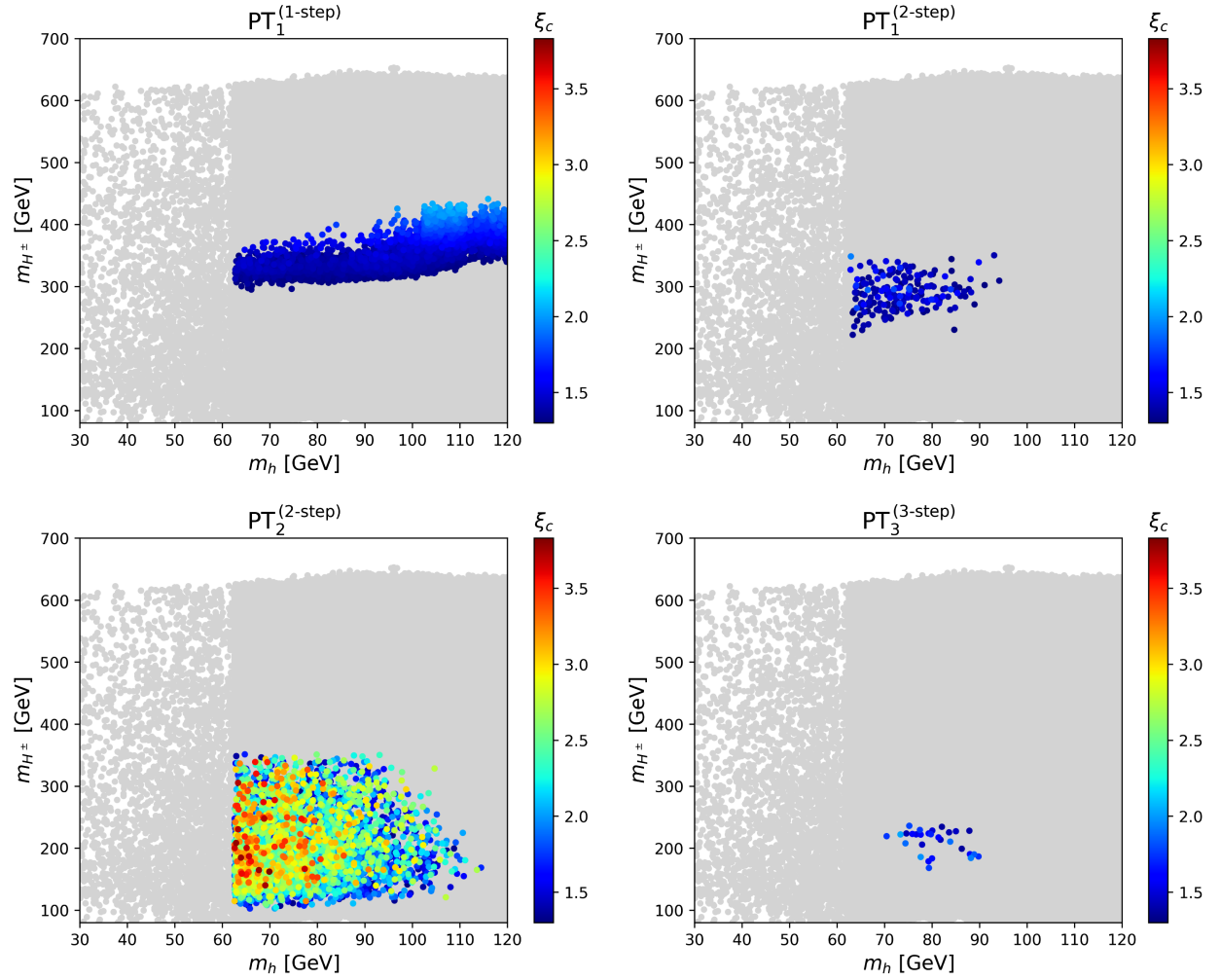


FIG. 2. $m_{H^\pm}$ vs m_h for SFOEWPT parameter points ($\xi_c > 1.3$), with ξ_c indicated by the color scale. Gray points represent all physical parameter points satisfying theoretical and current experimental constraints. The panels display results for $PT_1^{(1\text{-step})}$ (upper left), $PT_1^{(2\text{-step})}$ (upper right), $PT_2^{(2\text{-step})}$ (lower left), and $PT_3^{(3\text{-step})}$ (lower right).

transitions—which make up the second-largest class of SFOEWPT parameter points—permit a broader range, with $m_{H^\pm} \in [100, 350]$ GeV. For the remaining $PT_1^{(2\text{-step})}$ and $PT_3^{(3\text{-step})}$ transitions, the SFOEWPT condition imposes even tighter restrictions on both $m_{H^\pm}$ and m_h .

These results—particularly those concerning the allowed ranges for $m_{H^\pm}$ —provide significant insight into our primary question: are the parameter regions that induce a SFOEWPT via one-step and multistep transitions distinct? The answer, based on our scan, is that they are largely distinct, though a narrow region of overlap exists. To illustrate this, let us focus on the two dominant transition types, $PT_1^{(1\text{-step})}$ and $PT_2^{(2\text{-step})}$. If a charged Higgs boson were observed with a mass $m_{H^\pm} > 351$ GeV, our findings imply that any associated SFOEWPT must have proceeded via a one-step transition. Conversely, if an observation yielded $m_{H^\pm} < 295$ GeV, only a two-step transition could have facilitated the SFOEWPT. In the intermediate

mass window, $295 \text{ GeV} \lesssim m_{H^\pm} \lesssim 351 \text{ GeV}$, both transition types remain possible; a SFOEWPT in this range could arise from either a $PT_1^{(1\text{-step})}$ or a $PT_2^{(2\text{-step})}$ pathway. This overlap, however, is relatively limited compared to the full mass ranges allowed for each transition type, underscoring a partial but meaningful separation in their respective parameter spaces.

Another notable feature revealed in Fig. 2 is that ξ_c varies significantly depending on the specific stage of the multistep transition sequence. Among the transition types considered, $PT_2^{(2\text{-step})}$ stands out by supporting the strongest SFOEWPTs, with ξ_c values reaching up to approximately 3.83. Moreover, for this transition type, parameter points with larger ξ_c values tend to cluster in regions where the lighter CP -even Higgs boson h is also relatively light. In contrast, the other transition stages— $PT_1^{(1\text{-step})}$, $PT_1^{(2\text{-step})}$, and $PT_3^{(3\text{-step})}$ —typically yield more moderate phase transition strengths, with ξ_c values generally below 2.08.

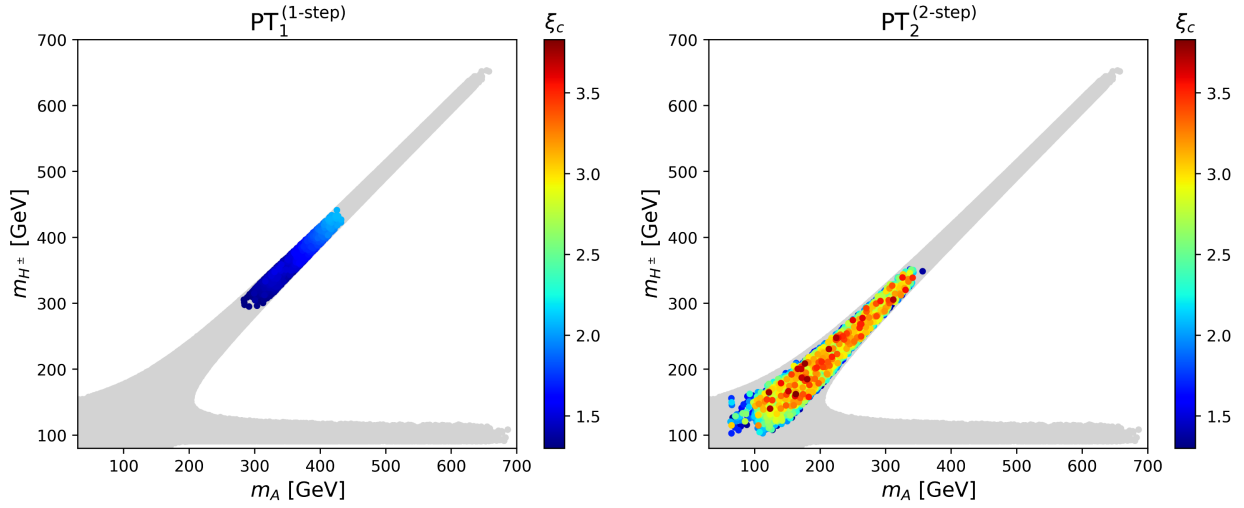


FIG. 3. $m_{H^\pm}$ vs m_A for SFOEWPT parameter points with ξ_c indicated by the color scale. Gray points represent all physical parameter points. The left panel shows results for $PT_1^{(1\text{-step})}$, and the right panel shows results for $PT_2^{(2\text{-step})}$.

To investigate the characteristics of other model parameters, we now focus on two primary stages which can induce SFOEWPT, $PT_1^{(1\text{-step})}$ and $PT_2^{(2\text{-step})}$. Figure 3 presents scatter plots of $m_{H^\pm}$ versus m_A for SFOEWPT parameter points, color-coded by the corresponding order parameter ξ_c . The left panel displays results for $PT_1^{(1\text{-step})}$ and the right panel for $PT_2^{(2\text{-step})}$, with gray points again representing all physical parameter points.

The distribution of physical parameter points (gray) reveals two distinct regions: one where $m_{H^\pm} \sim m_A$ and another where $m_{H^\pm} \sim m_h$. This structure is primarily attributed to constraints from the Peskin-Takeuchi oblique parameters (S , T , U), as BSM Higgs contributions to these parameters tend to be suppressed when at least two of the new scalar masses are degenerate. However, the requirement of a SFOEWPT effectively excludes the second region ($m_{H^\pm} \sim m_h$), favoring a mass degeneracy between $m_{H^\pm}$ and m_A , for both $PT_1^{(1\text{-step})}$ and $PT_2^{(2\text{-step})}$ transitions. Nevertheless, the allowed mass ranges for $m_{H^\pm}$ and m_A vary significantly depending on the transition stage. A SFOEWPT via $PT_1^{(1\text{-step})}$ typically requires relatively heavy charged Higgs bosons and pseudoscalars: $m_{H^\pm} \in [295, 441]$ GeV and $m_A \in [283, 432]$ GeV. In contrast, a SFOEWPT via $PT_2^{(2\text{-step})}$ permits lighter $H^\pm$ and A : $m_{H^\pm} \in [103, 351]$ GeV and $m_A \in [63, 356]$ GeV. Notably, for $PT_2^{(2\text{-step})}$ transitions, it is feasible for the pseudoscalar A to be lighter than the 125 GeV SM-like Higgs boson. Consequently, future observations of $m_{H^\pm}$ and m_A could help distinguish whether a SFOEWPT, if one indeed occurred, proceeded via a one-step or two-step EWPT, except within the narrow overlap region of $m_{H^\pm} \in [295, 351]$ GeV and $m_A \in [283, 356]$ GeV.

Figure 4 shows the distribution of t_β versus $m_{H^\pm}$ for SFOEWPT parameter points, with ξ_c indicated by the color

scale, for the $PT_1^{(1\text{-step})}$ (left) and $PT_2^{(2\text{-step})}$ (right) transitions. For the full set of physical parameter points (shown in gray), the allowed range $t_\beta \in [1.72, 50]$ covers most of the scanned domain. The region $t_\beta \in [1, 1.72]$ is excluded, primarily due to experimental constraints from FCNC measurements, as lower t_β values enhance the charged Higgs Yukawa couplings and lead to sizable contributions to FCNC observables. In contrast, large t_β values remain allowed, as they suppress the BSM Higgs Yukawa couplings [see Eq. (5)], thereby reducing their experimental impact—a well-known feature of the type-I 2HDM.

Interestingly, the SFOEWPT condition imposes a stringent constraint on t_β . For $PT_1^{(1\text{-step})}$ transitions, only a narrow band, $t_\beta \in [4.23, 8.82]$, remains viable. In contrast, $PT_2^{(2\text{-step})}$ transitions accommodate a much broader range, with t_β values extending up to 45.3. However, high- t_β points (e.g., $t_\beta > 30$) appear with lower density and are confined to regions with relatively light charged Higgs bosons. A mild correlation between t_β and $m_{H^\pm}$ is also observed in the $PT_2^{(2\text{-step})}$ case, where heavier $H^\pm$ masses are generally associated with smaller t_β values. Although the EWPT strength ξ_c does not show a strong direct dependence on t_β , points with $\xi_c > 3$ are predominantly located in the region $t_\beta \lesssim 20$.

We now examine the characteristics of t_β versus $s_{\beta-\alpha}$ for SFOEWPT parameter points, as illustrated in Fig. 5. The left panel shows results for the $PT_1^{(1\text{-step})}$ transition, and the right panel shows results for $PT_2^{(2\text{-step})}$, with color codes indicating the corresponding ξ_c values. Gray points represent the full set of physical parameter points. In the type-I 2HDM, sizable deviations from the Higgs alignment limit are permitted by current theoretical and experimental constraints, with $s_{\beta-\alpha}$ spanning the range $[-0.460, 0.314]$.

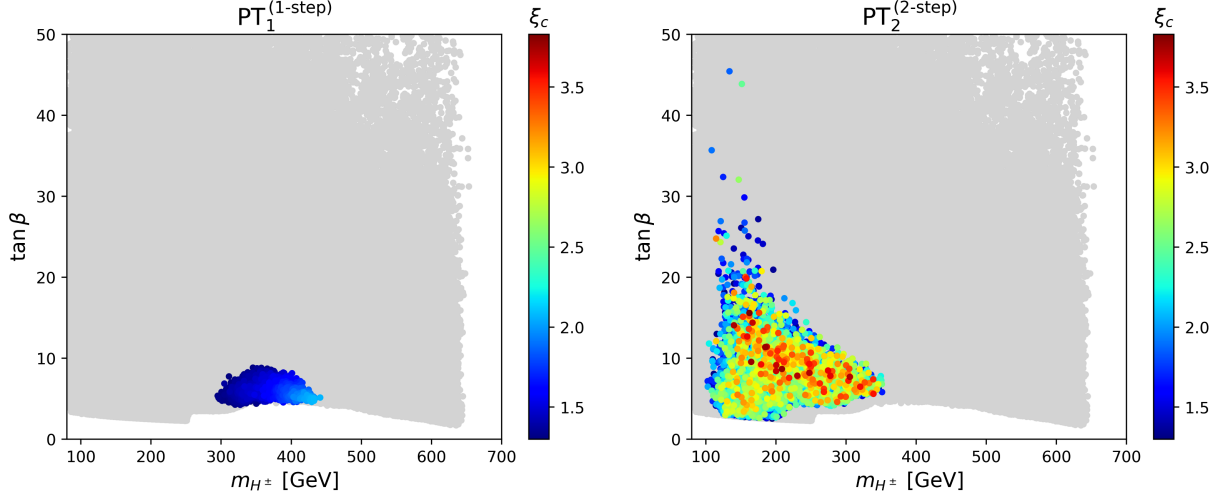


FIG. 4. t_β versus $m_{H^\pm}$ for SFOEWPT parameter points, with the color code of ξ_c . Gray points represent all physical parameter points. The left panel shows results for $PT_1^{(1\text{-step})}$, and the right panel shows results for $PT_2^{(2\text{-step})}$.

Imposing the SFOEWPT condition substantially narrows the allowed range of $s_{\beta-\alpha}$. Most notably, the preferred values differ significantly between $PT_1^{(1\text{-step})}$ and $PT_2^{(2\text{-step})}$ transitions. For $PT_1^{(1\text{-step})}$ transitions, the viable range is reduced to $s_{\beta-\alpha} \in [-0.22, 0.30]$, with negative values comprising approximately 44% of the SFOEWPT parameter points. Although more constrained, sizable deviations from the alignment limit remain allowed. This behavior in one-step SFOEWPTs is closely tied to the vacuum uplifting measure $\Delta\mathcal{F}_0$. In such transitions, larger ξ_c values correlate with more positive $\Delta\mathcal{F}_0$ [see Eq. (18)]. In the inverted scenario, the tree-level contribution $\Delta\mathcal{F}_0^{\text{tree}}$ [see Eq. (16)] favors nonzero $s_{\beta-\alpha}$ to enhance $\Delta\mathcal{F}_0$, thus promoting departures from alignment. However, $|s_{\beta-\alpha}|$ cannot be arbitrarily large; excessively positive $\Delta\mathcal{F}_0^{\text{tree}}$ pushes the

total $\Delta\mathcal{F}_0$ beyond the vacuum stability threshold $|\mathcal{F}_0^{\text{SM}}|$, leading to an unwanted metastable vacuum at zero temperature. One-loop contributions to $\Delta\mathcal{F}_0$ in the alignment limit [Eq. (17)] are also strictly positive and reinforce this behavior.

In contrast, SFOEWPTs proceeding via $PT_2^{(2\text{-step})}$ exhibit a strong preference for positive $s_{\beta-\alpha}$ values. As shown in the right panel of Fig. 5, negative $s_{\beta-\alpha}$ values make up only about 1% of the SFOEWPT parameter points for $PT_2^{(2\text{-step})}$. Although not explicitly shown, similar trends are observed for $PT_1^{(2\text{-step})}$ and $PT_3^{(3\text{-step})}$ transitions, where negative $s_{\beta-\alpha}$ values comprise approximately 3% and 0% of the respective SFOEWPT samples.

This yields a valuable phenomenological insight: if future measurements or global fits favor a negative value

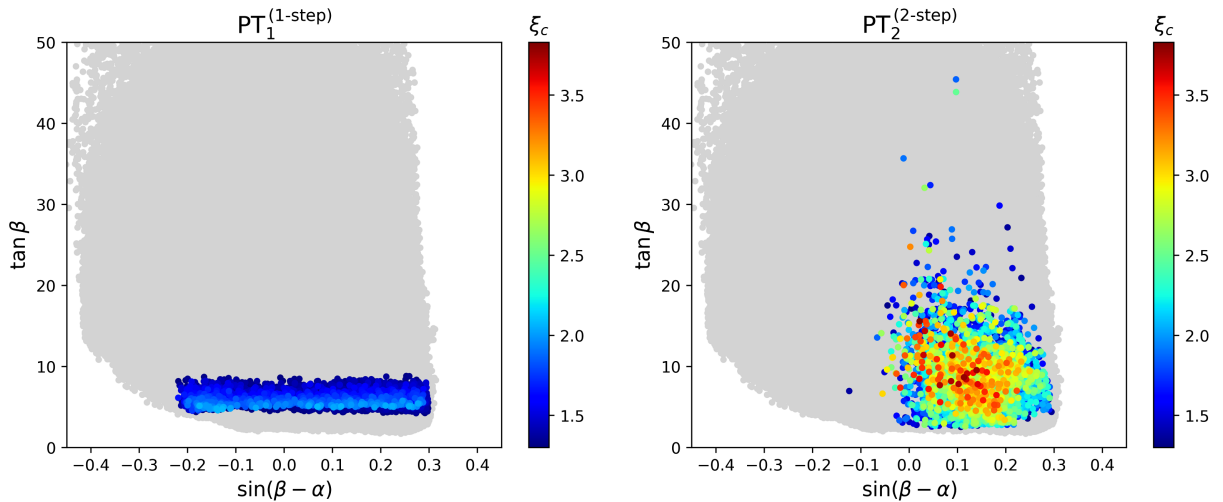


FIG. 5. t_β versus $s_{\beta-\alpha}$ for SFOEWPT parameter points, with $\xi_c (> 1.3)$ indicated by the color scale. Gray points represent all physical parameter points. The left panel shows results for $PT_1^{(1\text{-step})}$ and the right panel for $PT_2^{(2\text{-step})}$.

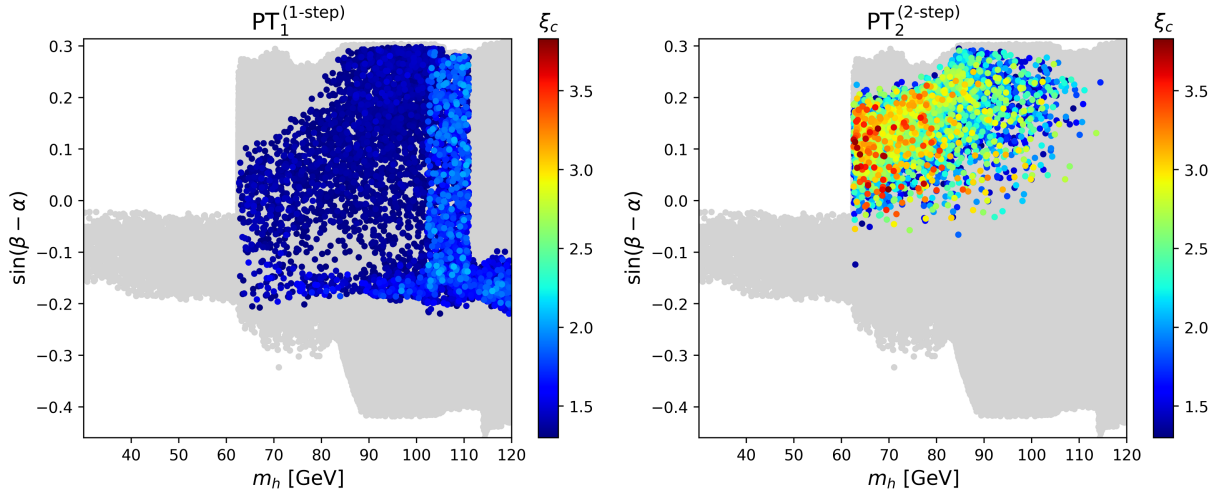


FIG. 6. $s_{\beta-\alpha}$ versus m_h for SFOEWPT parameter points, with ξ_c indicated by the color scale. Gray points represent all physical parameter points. The left panel shows results for $PT_1^{(1\text{-step})}$ and the right panel for $PT_2^{(2\text{-step})}$.

of $s_{\beta-\alpha}$, then a SFOEWPT—if it indeed occurred—likely proceeded via a one-step transition. However, the converse does not hold; a one-step SFOEWPT does not imply negative $s_{\beta-\alpha}$, as $PT_1^{(1\text{-step})}$ transitions also accommodate positive values of $s_{\beta-\alpha}$.

Finally, we present an interesting correlation between $s_{\beta-\alpha}$ and m_h for SFOEWPT points in Fig. 6, with ξ_c as the color code. Gray points again denote all physical parameter points. The left panel shows results for $PT_1^{(1\text{-step})}$, and the right panel shows results for $PT_2^{(2\text{-step})}$. For $PT_1^{(1\text{-step})}$, as m_h increases from approximately 62.5 up to around 85 GeV, the allowed range of $s_{\beta-\alpha}$ broadens. For $m_h \in [85, 100]$ GeV, nearly the entire SFOEWPT-allowed range of $s_{\beta-\alpha} \in [-0.22, 0.30]$ is populated. More strikingly,

for m_h in the $[110, 120]$ GeV range, only negative values, $s_{\beta-\alpha} \in [-0.22, -0.10]$, are found to accommodate SFOEWPT. This implies that if h is relatively heavy a SFOEWPT via a one-step transition in this model requires a sizable deviation from the Higgs alignment limit, specifically toward negative $s_{\beta-\alpha}$.

For SFOEWPTs via $PT_2^{(2\text{-step})}$, the distribution of parameter points in the $s_{\beta-\alpha}$ versus m_h plane exhibits a relatively simple structure, with points clustering in a compact region without any notable features or peculiar shapes. An interesting observation is that for $m_h \gtrsim 100$ GeV the Higgs alignment limit is no longer maintained in this case.

Let us summarize the allowed regions of the SFOEWPT parameter space for the $PT_1^{(1\text{-step})}$ and $PT_2^{(2\text{-step})}$ transitions:

$$\begin{aligned}
 &\text{SFOEWPT via } PT_1^{(1\text{-step})}: \xi_c \in [1.30, 2.08], \\
 &\quad m_h \in [62.7, 120.0] \text{ GeV}, \quad m_A \in [283.4, 431.9] \text{ GeV}, \\
 &\quad m_{H^\pm} \in [295.1, 441.3] \text{ GeV}, \quad m_{12}^2 \in [0.077, 2549] \text{ GeV}^2, \\
 &\quad t_\beta \in [4.2, 8.8], \quad s_{\beta-\alpha} \in [-0.221, 0.30]; \\
 &\text{SFOEWPT via } PT_2^{(2\text{-step})}: \xi_c \in [1.30, 3.83], \\
 &\quad m_h \in [62.5, 114.4] \text{ GeV}, \quad m_A \in [63.2, 356.4] \text{ GeV}, \\
 &\quad m_{H^\pm} \in [102.7, 351.4] \text{ GeV}, \quad m_{12}^2 \in [29.0, 2775] \text{ GeV}^2, \\
 &\quad t_\beta \in [2.5, 45.4], \quad s_{\beta-\alpha} \in [-0.12, 0.30].
 \end{aligned} \tag{28}$$

We emphasize that the SFOEWPT parameter points are not uniformly distributed across these ranges, as evident from the scatter plots presented in this subsection.

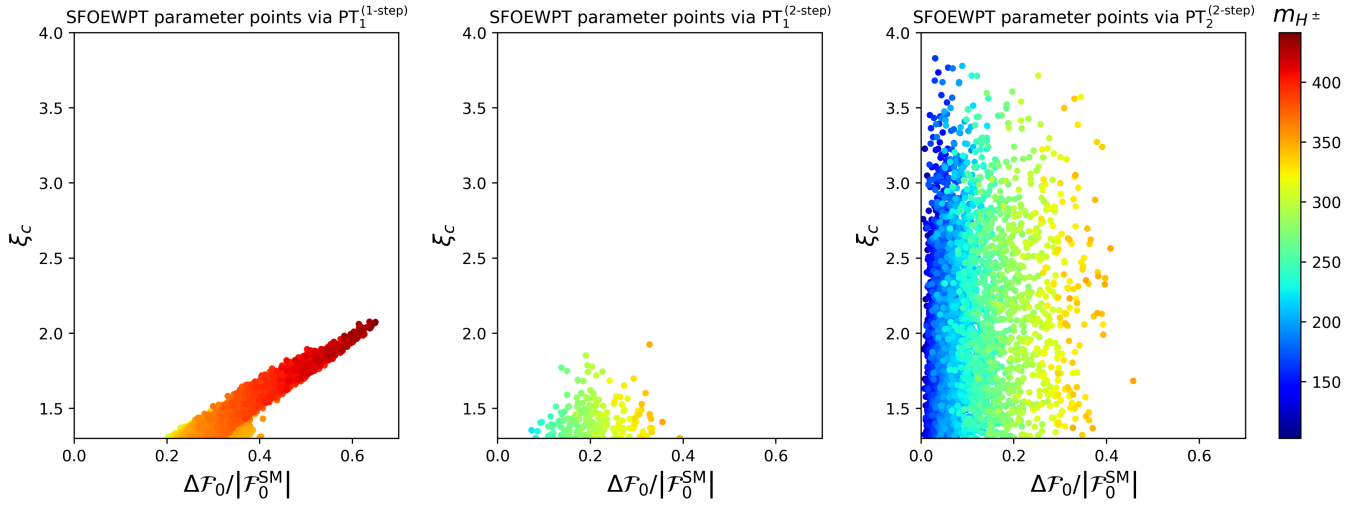


FIG. 7. ξ_c versus $\Delta\mathcal{F}_0/|\mathcal{F}_0^{\text{SM}}|$ for SFOEWPT parameter points with the color code of $m_{H^\pm}$ in units of GeV. The results of $\text{PT}_1^{(1\text{-step})}$, $\text{PT}_1^{(2\text{-step})}$, and $\text{PT}_2^{(2\text{-step})}$ are presented in the left, middle, and right panels, respectively.

C. Correlation between vacuum uplifting and ξ_c in multistep transitions

In Sec. III C, we discussed a well-established feature of SFOEWPTs in one-step transitions within the normal Higgs scenario: the vacuum uplifting measure $\Delta\mathcal{F}_0$, which represents the contrast between the 2HDM and SM in terms of the vacuum energy density difference between symmetric and broken phases at zero temperature, is strongly correlated with the FOEWPT strength ξ_c . In this section, we examine whether such a correlation persists in the inverted Higgs scenario, particularly in the context of multistep EWPTs.

In Fig. 7, we show the correlation between ξ_c and $\Delta\mathcal{F}_0/|\mathcal{F}_0^{\text{SM}}|$ for SFOEWPT parameter points, with $m_{H^\pm}$ indicated by the color scale. The results for the $\text{PT}_1^{(1\text{-step})}$, $\text{PT}_1^{(2\text{-step})}$, and $\text{PT}_2^{(2\text{-step})}$ transitions are displayed in the left, middle, and right panels, respectively.

For the one-step transition, a clear positive correlation is observed: larger values of $\Delta\mathcal{F}_0$ generally correspond to stronger phase transitions, i.e., larger ξ_c . In the inverted type-I scenario, the requirement of $\xi_c > 1.3$ constrains the vacuum uplifting measure to the approximate range of

$$\begin{aligned} &\text{SFOEWPT via one-step with } m_H \\ &= 125 \text{ GeV: } 0.20 \lesssim \frac{\Delta\mathcal{F}_0}{|\mathcal{F}_0^{\text{SM}}|} \lesssim 0.65. \end{aligned}$$

This result indicates that the degree of vacuum uplifting required to realize a SFOEWPT in the inverted scenario differs from that in the normal scenario [cf. Eq. (18)]. In particular, the allowance of smaller $\Delta\mathcal{F}_0/|\mathcal{F}_0^{\text{SM}}|$ values suggests that even a 2HDM vacuum with a potential depth relatively close to that of the SM vacuum can still support a SFOEWPT.

However, the direct correlation between ξ_c and $\Delta\mathcal{F}_0/|\mathcal{F}_0^{\text{SM}}|$ largely disappears in multistep EWPTs. This behavior is illustrated in the middle and right panels of Fig. 7, corresponding to the $\text{PT}_1^{(2\text{-step})}$ and $\text{PT}_2^{(2\text{-step})}$ transitions, respectively. For both cases, no clear correlation is observed; for a fixed value of $\Delta\mathcal{F}_0/|\mathcal{F}_0^{\text{SM}}|$, the order parameter ξ_c can span a wide range while still satisfying the SFOEWPT condition. This lack of correlation is particularly pronounced in the $\text{PT}_2^{(2\text{-step})}$ transition.

Instead, a different pattern emerges in the multistep scenarios: a correlation between $\Delta\mathcal{F}_0/|\mathcal{F}_0^{\text{SM}}|$ and $m_{H^\pm}$, where larger $\Delta\mathcal{F}_0/|\mathcal{F}_0^{\text{SM}}|$ values tend to be associated with heavier $m_{H^\pm}$. This trend can be partly understood from the analytic structure of $\Delta\mathcal{F}_0$ in the Higgs alignment limit, as discussed in connection with Eq. (17).

In summary, our investigation of the vacuum uplifting measure $\Delta\mathcal{F}_0$ and its correlation to the EWPT strength ξ_c in the inverted Higgs scenario reveals qualitatively distinct behaviors between one-step and multistep transitions. For one-step transitions, a positive correlation is confirmed, although the viable range of $\Delta\mathcal{F}_0/|\mathcal{F}_0^{\text{SM}}|$ differs from that in the normal Higgs scenario. In contrast, for multistep transitions, this correlation breaks down entirely; the strength of the phase transition ξ_c shows no clear dependence on $\Delta\mathcal{F}_0/|\mathcal{F}_0^{\text{SM}}|$. As a result, in multistep scenarios, evaluating the strength of the FOEWPT requires a full computation of the one-loop effective potential at finite temperature to determine the critical temperature and the corresponding order parameter.

D. Characteristics of the gravitational wave parameter space

In the preceding subsections, we have demonstrated that the inverted type-I 2HDM accommodates a sizable region

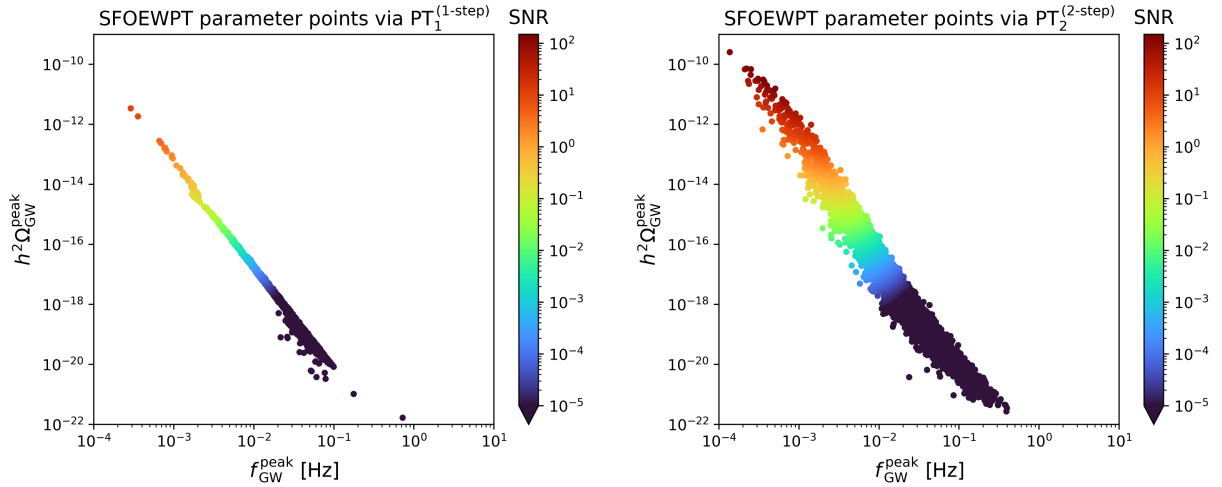


FIG. 8. $h^2\Omega_{\text{GW}}^{\text{SW,peak}}$ versus $f_{\text{GW}}^{\text{SW,peak}}$ for SFOEWPT parameter points, with the color code indicating the SNR (for a four-year LISA mission). Results shown are for the dominant sound wave contributions. The left and right panels present results for $\text{PT}_1^{(1\text{-step})}$ and $\text{PT}_2^{(2\text{-step})}$ transitions, respectively. The bubble wall velocity is fixed at $v_w = 0.95$.

of parameter space capable of supporting a SFOEWPT. One of the most compelling phenomenological implications of such a transition is the potential generation of stochastic GWs. In this subsection, we compute the resulting GW spectrum—focusing on the peak amplitude, peak frequency, and the corresponding SNR at LISA—using BSMPT version 3 [127]. Parameter points with $\text{SNR} > 10$ are identified as GW parameter points, and we analyze their key features to elucidate the structure of the associated GW parameter space.

A crucial input for determining whether $\text{SNR} > 10$ is the bubble wall velocity. In our analysis, we adopt $v_w = 0.95$, following a widely used convention in studies of gravitational-wave signals from cosmological first-order electroweak phase transitions. This value serves as a standard benchmark for nearly luminal, nonrunaway bubble-wall velocities in forecasting analyses [126,132,181–184].

Figure 8 displays the peak amplitude $h^2\Omega_{\text{GW}}^{\text{SW,peak}}$ versus the peak frequency $f_{\text{GW}}^{\text{SW,peak}}$ for SFOEWPT parameter points, focusing on the dominant GW contributions from sound waves. The color scale represents the SNR computed for a four-year LISA mission. The left panel shows results for the $\text{PT}_1^{(1\text{-step})}$ transition, while the right panel corresponds to $\text{PT}_2^{(2\text{-step})}$. A common inverse correlation between $h^2\Omega_{\text{GW}}^{\text{SW,peak}}$ and $f_{\text{GW}}^{\text{SW,peak}}$ is observed across both transition types: higher peak frequencies generally correspond to lower peak amplitudes. However, the degree of correlation varies. The $\text{PT}_1^{(1\text{-step})}$ transitions exhibit a tighter correlation, manifested as a narrower distribution, compared to the broader spread seen in the $\text{PT}_2^{(2\text{-step})}$ case.

The SNR values at LISA, illustrated by the color map in Fig. 8, span a broad range—from well below 10^{-5} to $\mathcal{O}(10^2)$. The number of GW parameter points satisfying the

$\text{SNR} > 10$ criterion varies significantly across different transition types. Out of the 2.36×10^6 physical parameter points obtained from the random scan over the ranges specified in Eq. (22), the number of GW parameter points for each transition stage is

$$v_w = 0.95: N_{\text{GW}}^{\text{PT}_1^{(1\text{-step})}} = 1, \quad N_{\text{GW}}^{\text{PT}_1^{(2\text{-step})}} = 0, \\ N_{\text{GW}}^{\text{PT}_2^{(2\text{-step})}} = 114, \quad N_{\text{GW}}^{\text{PT}_3^{(3\text{-step})}} = 1. \quad (29)$$

Including the results of the targeted scan for $\text{PT}_3^{(3\text{-step})}$ (which yielded a total of 29 SFOEWPT parameter points as shown in Fig. 2), the number of GW parameter points for this transition slightly increases to $N_{\text{GW}}^{\text{PT}_3^{(3\text{-step})}} = 2$.

This distribution is noteworthy: although one-step transitions account for the largest share of SFOEWPT parameter points [see Eq. (27)], they rarely produce GW signals detectable by LISA. In contrast, $\text{PT}_2^{(2\text{-step})}$ transitions stand out as the most productive source of potentially observable GW signals in our scan.

Before discussing the characteristics of the GW parameter points, we briefly examine how the predicted SNRs change with a different bubble wall velocity. Since recent hydrodynamic studies suggest typical values around $v_w \sim 0.6$ for some benchmark models [185–187], we have repeated the analysis with $v_w = 0.6$. Out of the 2.36×10^6 physical parameter points, the numbers of GW parameter points with $\text{SNR} > 10$ are

$$v_w = 0.6: N_{\text{GW}}^{\text{PT}_1^{(1\text{-step})}} = 8, \quad N_{\text{GW}}^{\text{PT}_1^{(2\text{-step})}} = 0, \\ N_{\text{GW}}^{\text{PT}_2^{(2\text{-step})}} = 128, \quad N_{\text{GW}}^{\text{PT}_3^{(3\text{-step})}} = 2. \quad (30)$$

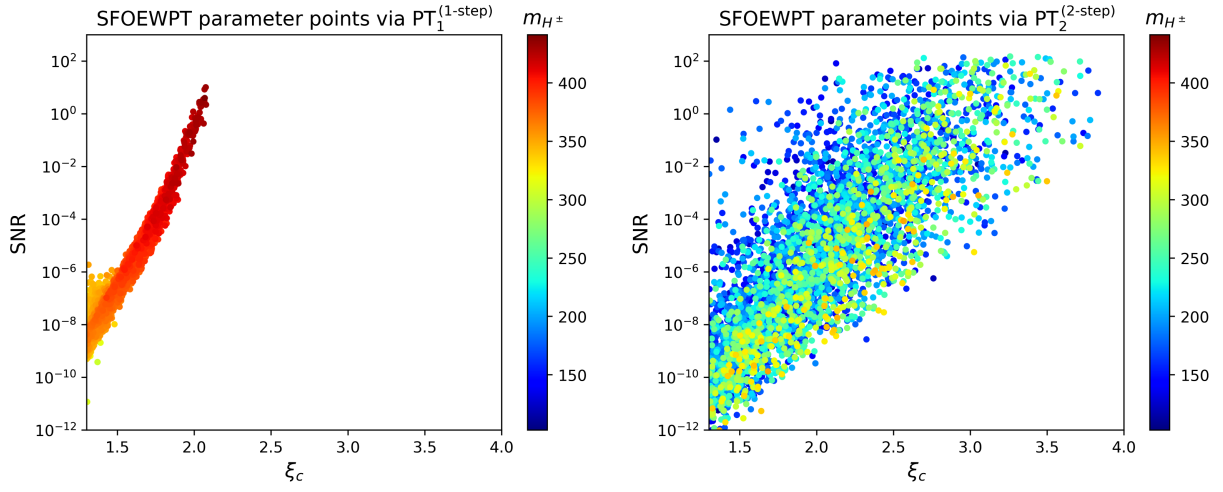


FIG. 9. GW SNR at LISA versus ξ_c for SFOEWPT parameter points, with the color code indicating $m_{H^\pm}$ (in units of GeV). The left and right panels present results for $PT_1^{(1\text{-step})}$ and $PT_2^{(2\text{-step})}$ transitions, respectively. The bubble wall velocity is fixed at $v_w = 0.95$.

Comparing with the $v_w = 0.95$ case in Eq. (29), we observe that adopting $v_w = 0.6$ increases the number of GW parameter points. The most pronounced change appears in the $PT_1^{(1\text{-step})}$ case, where the number of GW points rises eightfold. In contrast, the $PT_2^{(2\text{-step})}$ transition stage—which contributes the largest share of GW parameter points—shows only a modest increase of about 10%. Importantly, the qualitative characteristics of the GW parameter points at $v_w = 0.6$ remain very similar to those at $v_w = 0.95$. For this reason, in what follows, we focus primarily on the case $v_w = 0.95$.

We next examine the correlation between the GW SNR at LISA and the phase transition strength ξ_c . A strong correlation between these quantities would be especially valuable; the detection of a high-SNR signal could serve as a robust probe of the underlying electroweak phase

transition dynamics. Such a constraint on ξ_c would, in turn, help to restrict the viable parameter space and improve our ability to discriminate between competing new physics scenarios. Figure 9 shows the SNR versus ξ_c for SFOEWPT parameter points, with the charged Higgs mass $m_{H^\pm}$ indicated by the color scale. The left panel corresponds to $PT_1^{(1\text{-step})}$ transitions, while the right panel displays results for $PT_2^{(2\text{-step})}$.

For one-step transitions, a clear positive correlation is observed: larger values of ξ_c generally correspond to higher SNRs. This suggests that stronger phase transitions are more likely to produce GW signals detectable by LISA. In contrast, no such strong correlation is evident for $PT_2^{(2\text{-step})}$ transitions. Even near $\xi_c \approx 2$, the associated SNR values span several orders of magnitude, from $\mathcal{O}(10^{-10})$ to $\mathcal{O}(1)$. Nonetheless, all GW parameter points with $\text{SNR} > 10$

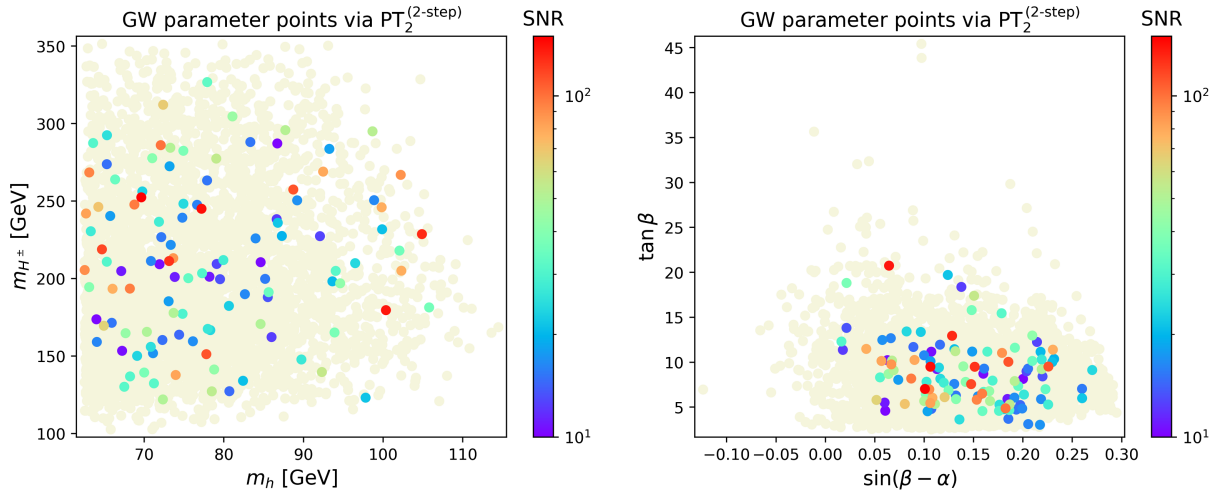


FIG. 10. $m_{H^\pm}$ vs m_h for GW parameter points (SFOEWPT points with $\text{SNR} > 10$) for $PT_2^{(2\text{-step})}$ transition, with the color code indicating SNR. SFOEWPT parameter points that do not satisfy $\text{SNR} > 10$ are denoted by beige points. The bubble wall velocity is fixed at $v_w = 0.95$.

satisfy $\xi_c > 2$, indicating that a sufficiently strong phase transition is still a necessary condition for GW signal detectability. The charged Higgs mass $m_{H^\pm}$ shows no significant correlation with SNR in either transition type. In summary, our results highlight a robust correlation between ξ_c and SNR in one-step transitions, while this relationship appears to be absent or substantially weakened in multistep scenarios.

Finally, we investigate whether the condition $\text{SNR} > 10$ imposes additional constraints on the parameter space of the inverted type-I 2HDM. Figure 10 displays the distribution of parameter points satisfying $\text{SNR} > 10$ for $\text{PT}_2^{(2\text{-step})}$ transitions—the only transition stage yielding a substantial number of such GW parameter points. SNR values are indicated by the color scale. The left panel shows the $(m_h, m_{H^\pm})$ plane, while the right panel presents $(s_{\beta-\alpha}, t_\beta)$. For comparison, SFOEWPT parameter points with $\text{SNR} \leq 10$ are shown in beige.

Strikingly, the requirement of $\text{SNR} > 10$ does not significantly alter or further restrict the allowed parameter space. Rather, the high-SNR points largely overlap with the broader SFOEWPT regions previously identified for the $\text{PT}_2^{(2\text{-step})}$ transitions.

V. COLLIDER PHENOMENOLOGY AT THE 1.5 TeV CLIC

A. Golden channels of the SFOEWPT parameter space at the 1.5 TeV CLIC

The preceding sections identified parameter regions within the inverted type-I 2HDM that can induce a SFOEWPT and potentially generate GW signals detectable at LISA. These regions, which are significantly more constrained than the full physical parameter space, provide compelling targets for experimental investigation.

This section explores the phenomenological signatures associated with these SFOEWPT parameter points at high-energy colliders.⁷ A primary motivation is that a discovery at a collider would offer crucial supporting evidence, directly linking macroscopic cosmological events such as the EWPT to specific microscopic dynamics. Conversely, null results from collider searches can help exclude portions of the SFOEWPT parameter space in the inverted type-I 2HDM.

Among various high-energy colliders, we select the 1.5 TeV stage of the CLIC [188]. CLIC is CERN's proposed linear electron-positron collider, with the 1.5 TeV stage designed to extend the physics reach beyond its initial 380 GeV phase. CLIC employs advanced technologies, including a two-beam acceleration scheme in which a separate high-current drive beam generates radio-

frequency power at 12 GHz to feed the main accelerating structures. These structures operate with accelerating gradients up to 100 MV/m. The 1.5 TeV configuration features a 29 km linac and recombination systems capable of producing 100 A current pulses. The design emphasizes modularity, allowing for the reuse of infrastructure from the 380 GeV stage and enabling a cost-effective upgrade to higher energies. The combination of a clean experimental environment and high center-of-mass energy makes the 1.5 TeV CLIC stage particularly well suited for probing the SFOEWPT parameter space of the inverted type-I 2HDM, where $m_{H^\pm}$ and m_A are bounded from above by approximately 440 and 430 GeV, respectively [see Eq. (28)].

Let us first examine the most efficient production channels for BSM Higgs bosons relevant to the SFOEWPT parameter space. For the lighter CP -even Higgs boson h , potential production modes include Higgsstrahlung ($e^+e^- \rightarrow Z^* \rightarrow Zh$), WW fusion ($e^+e^- \rightarrow \nu_e \bar{\nu}_e h$), ZZ fusion ($e^+e^- \rightarrow e^+e^- h$) and associated production with top quark pairs ($e^+e^- \rightarrow t\bar{t}h$). However, these processes are typically suppressed in or near the Higgs alignment limit, where the V - V - h coupling ($V = W, Z$) becomes negligible, or in the large t_β regime, where the Yukawa coupling modifier ξ_f^h is significantly reduced.

Given these limitations, we shift our focus to production channels involving $H^\pm$ and A , specifically the pair production process $e^+e^- \rightarrow H^+H^-$ and the associated production $e^+e^- \rightarrow Ah$. The cross section for $e^+e^- \rightarrow H^+H^-$, mediated by γ and Z exchange, depends predominantly on the charged Higgs mass $m_{H^\pm}$, as detailed in Eq. (4). The $e^+e^- \rightarrow Ah$ channel, proceeding through an s -channel Z boson, has a coupling strength governed by the Z - A - h vertex, which is proportional to $c_{\beta-\alpha}$.

Figure 11 shows the calculated cross sections for the two production modes across the SFOEWPT parameter space. The left panel displays $\sigma(e^+e^- \rightarrow H^+H^-)$ as a function of $m_{H^\pm}$, while the right panel shows $\sigma(e^+e^- \rightarrow Ah)$ as a function of m_A , with points color-coded by $c_{\beta-\alpha}$. Both channels can yield sizable production rates at the 1.5 TeV CLIC, typically at or above $\mathcal{O}(1)$ fb. However, the $e^+e^- \rightarrow H^+H^-$ process generally results in significantly larger cross sections and, importantly, depends almost exclusively on $m_{H^\pm}$. For these reasons, we focus on charged Higgs pair production as the primary channel for probing the SFOEWPT parameter space in the inverted type-I 2HDM.

We now turn to the decay patterns of $H^\pm$ within the SFOEWPT parameter space. The charged Higgs boson $H^\pm$ predominantly decays via $H^\pm \rightarrow W^\pm h$. This behavior is illustrated in Fig. 12, which shows the branching ratio $\text{Br}(H^\pm \rightarrow W^\pm h)$ as a function of $m_{H^\pm}$, with m_A indicated by the color scale. For heavier pseudoscalar masses ($m_A \gtrsim 200$ GeV), the $H^\pm \rightarrow W^\pm h$ channel is overwhelmingly dominant, with branching ratios typically exceeding 80%. For lighter m_A , however, a small subset of SFOEWPT parameter points exhibits a reduced $\text{Br}(H^\pm \rightarrow W^\pm h)$,

⁷We focus on the SFOEWPT parameter space, as the absence of a detectable GW signal does not diminish the importance of identifying BSM models capable of supporting a SFOEWPT.

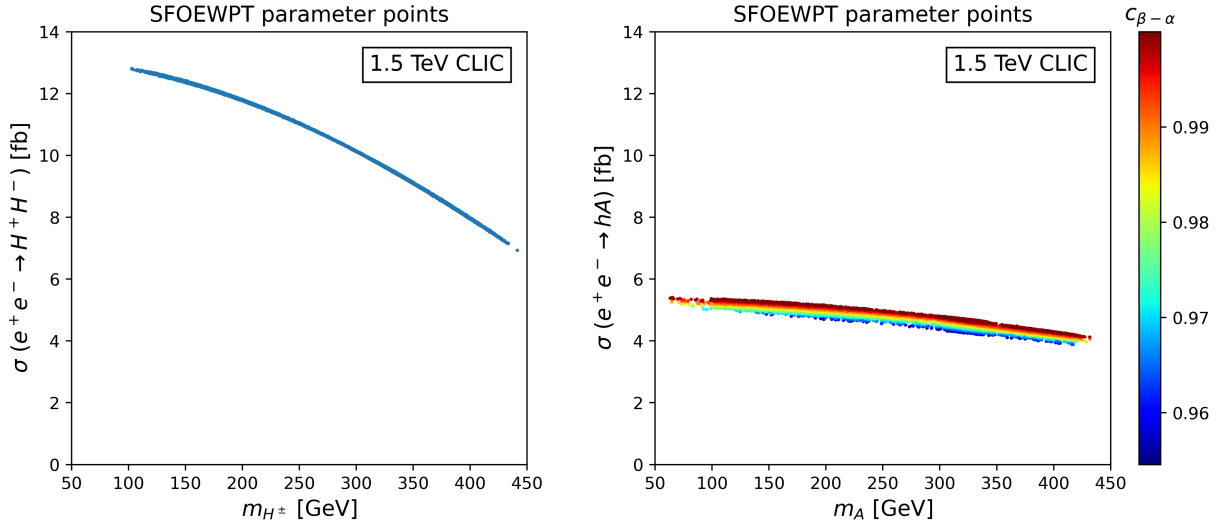


FIG. 11. Production cross sections at the 1.5 TeV CLIC for $e^+e^- \rightarrow H^+H^-$ (left panel, as a function of $m_{H^\pm}$) and $e^+e^- \rightarrow Ah$ (right panel, as a function of m_A , color coded by $c_{\beta-\alpha}$) over the SFOEWPT parameter space.

which can be as low as approximately 10%; this reduction is primarily due to the competing decay $H^\pm \rightarrow W^\pm A$ becoming kinematically accessible. Nonetheless, for the vast majority of SFOEWPT parameter points, the $H^\pm \rightarrow W^\pm h$ mode remains dominant, motivating our focus on the following golden channel for collider studies:

$$e^+e^- \rightarrow H^+H^- \rightarrow W^+W^-hh.$$

The final-state signatures of this golden channel are dictated by the subsequent decays of the lighter CP -even Higgs boson h , whose branching ratios depend sensitively on the underlying model parameters. The dominant decay modes typically involve fermion pairs, governed by the Yukawa couplings of h , which scale universally with the

factor ξ_f^h and are proportional to the corresponding fermion masses in the type-I 2HDM. Additional relevant decay channels include WW^* and ZZ^* , which become significant at larger m_h —near the respective kinematic thresholds—and for larger values of $|s_{\beta-\alpha}|$, since the hVV ($V = W^\pm, Z$) coupling scales with $s_{\beta-\alpha}$.

Another potentially significant decay channel for h is $h \rightarrow \gamma\gamma$. The partial decay width is given by [189]

$$\Gamma(h \rightarrow \gamma\gamma) = \frac{\alpha_{\text{em}}^2 m_h^3}{256\pi^3 v^2} \left| \sum_f N_C^f Q_f^2 \xi_f^h A_{1/2}^{\mathcal{H}}(\tau_f) + s_{\beta-\alpha} A_1^{\mathcal{H}}(\tau_W) + \frac{v^2}{2m_{H^\pm}^2} \hat{g}_{hH^+H^-} A_0^{\mathcal{H}}(\tau_{H^\pm}) \right|^2, \quad (31)$$

where N_C^f and Q_f are the color factor and electric charge of fermion f , respectively; ξ_f^h is defined in Eq. (5); and $\tau_i = m_h^2/(4m_i^2)$. The loop functions $A_j^{\mathcal{H}}(\tau)$ are given by [189]

$$\begin{aligned} A_{1/2}^{\mathcal{H}}(\tau) &= \frac{2}{\tau^2} [\tau + (\tau - 1)f(\tau)], \\ A_1^{\mathcal{H}}(\tau) &= -\frac{1}{\tau^2} [2\tau^2 + 3\tau + 3(2\tau - 1)f(\tau)], \\ A_0^{\mathcal{H}}(\tau) &= -\frac{1}{\tau^2} [\tau - f(\tau)], \end{aligned}$$

where the function $f(\tau)$ is

$$f(\tau) = \begin{cases} \arcsin^2 \sqrt{\tau} & \text{if } \tau \leq 1; \\ -\frac{1}{4} \left[\ln \left(\frac{\sqrt{\tau} + \sqrt{\tau-1}}{\sqrt{\tau} - \sqrt{\tau-1}} \right) - i\pi \right]^2 & \text{if } \tau > 1. \end{cases}$$

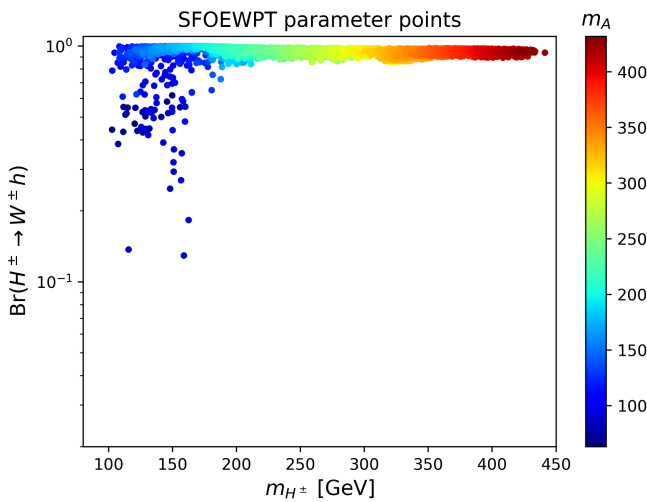


FIG. 12. Branching ratio of the decay mode $H^\pm \rightarrow W^\pm h$ as a function of $m_{H^\pm}$ across the SFOEWPT parameter space. The color code denotes m_A in units of GeV.

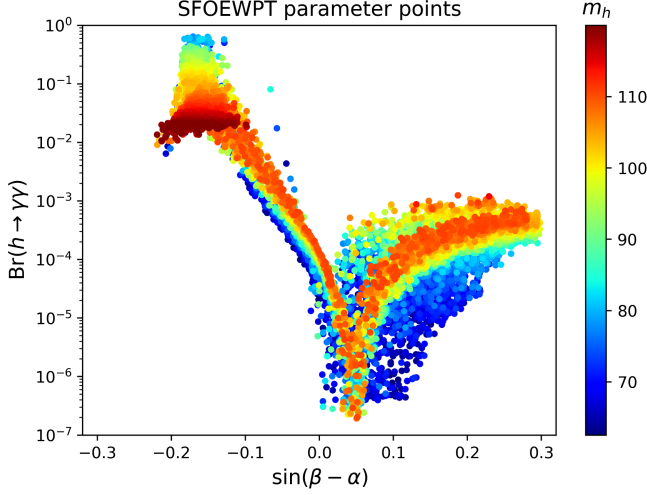


FIG. 13. $\text{Br}(h \rightarrow \gamma\gamma)$ versus $s_{\beta-\alpha}$ for SFOEWPT parameter points. The color code denotes m_h in units of GeV. Parameter points are sorted by m_h , with smaller m_h values plotted underneath.

The normalized coupling $\hat{g}_{hH^+H^-}$ in Eq. (31) is given by [190]

$$\hat{g}_{hH^+H^-} = \frac{1}{v^2} \left[s_{\beta-\alpha} \{ m_h^2 + 2(m_{H^\pm}^2 - \bar{M}^2) \} + c_{\beta-\alpha} \frac{1 - t_\beta^2}{t_\beta} (m_h^2 - \bar{M}^2) \right].$$

Interestingly, $\text{Br}(h \rightarrow \gamma\gamma)$ exhibits a strong sensitivity to $s_{\beta-\alpha}$, particularly its sign. Figure 13 displays $\text{Br}(h \rightarrow \gamma\gamma)$ as a function of $s_{\beta-\alpha}$ across the SFOEWPT parameter points, with m_h indicated by the color scale (points with smaller m_h are plotted beneath those with larger m_h). These branching ratios are computed using 2HDMC [173].

For positive $s_{\beta-\alpha}$, the $h \rightarrow \gamma\gamma$ decay mode is generally negligible, with branching ratios typically below approximately 10^{-3} . In contrast, for negative $s_{\beta-\alpha}$, $\text{Br}(h \rightarrow \gamma\gamma)$ can be significantly enhanced, reaching values as high as $\sim 80\%$. This enhancement for negative $s_{\beta-\alpha}$ arises primarily from the suppression of $h \rightarrow b\bar{b}$ decays. Since the Yukawa coupling modifier is given by $\xi_f^h = s_{\beta-\alpha} + c_{\beta-\alpha}/t_\beta$ [see Eq. (5)], a negative $s_{\beta-\alpha}$ can induce a cancellation with the $c_{\beta-\alpha}/t_\beta$ term, leading to a suppressed ξ_f^h and hence a reduced $\Gamma(h \rightarrow f\bar{f})$. This suppression, in turn, boosts the relative importance of $h \rightarrow \gamma\gamma$. Given that only one-step SFOEWPTs accommodate negative $s_{\beta-\alpha}$ values (see Fig. 5), an observed enhancement in $\text{Br}(h \rightarrow \gamma\gamma)$ could serve as an indirect probe favoring one-step SFOEWPT realizations.

We now examine the complete decay pattern of the lighter CP -even Higgs boson h . First, Figure 14 shows two cases: the exact alignment limit with $s_{\beta-\alpha} = 0$ (left panel) and a moderate positive deviation with $s_{\beta-\alpha} = 0.2$ (right panel). The branching ratios of h are plotted as a function of m_h . We consider $t_\beta = 5$ (solid lines) and $t_\beta = 7$ (dashed lines), setting $m_A = m_{H^\pm} = 300$ GeV and $\bar{M} = m_h$. The condition $\bar{M} = m_h$ is imposed to avoid theoretical restrictions on t_β [90,106,191].

For both $s_{\beta-\alpha} = 0$ and $s_{\beta-\alpha} = 0.2$, the dominant decay mode of h is into $b\bar{b}$, followed by $\tau^+\tau^-$. The third most significant decay mode is $c\bar{c}$ for $m_h \lesssim 76$ GeV and gg for $76 \text{ GeV} \lesssim m_h \lesssim 110$ GeV. The fermionic branching ratios show little dependence on t_β , as evidenced by the close overlap of solid and dashed lines. The $h \rightarrow WW^*$ channel is absent in the exact Higgs alignment limit ($s_{\beta-\alpha} = 0$) but becomes sizable when $m_h \gtrsim 105$ GeV and $s_{\beta-\alpha} = 0.2$. At larger t_β , the suppression of fermionic decays (due to the $c_{\beta-\alpha}/t_\beta$ term in ξ_f^h) enhances the relative importance of $h \rightarrow WW^*$.

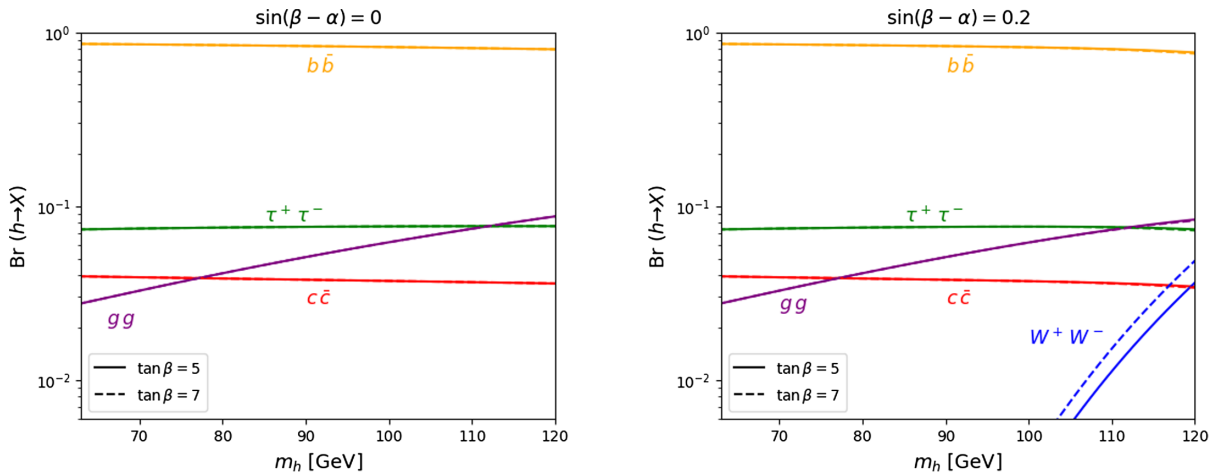


FIG. 14. Branching ratios of various decay modes of the lighter CP -even Higgs boson h as a function of m_h for $s_{\beta-\alpha} = 0$ (left panel) and $s_{\beta-\alpha} = 0.2$ (right panel). Results are shown for $t_\beta = 5$ (solid lines) and $t_\beta = 7$ (dashed lines), with $m_A = m_{H^\pm} = 300$ GeV and $\bar{M} = m_h$.

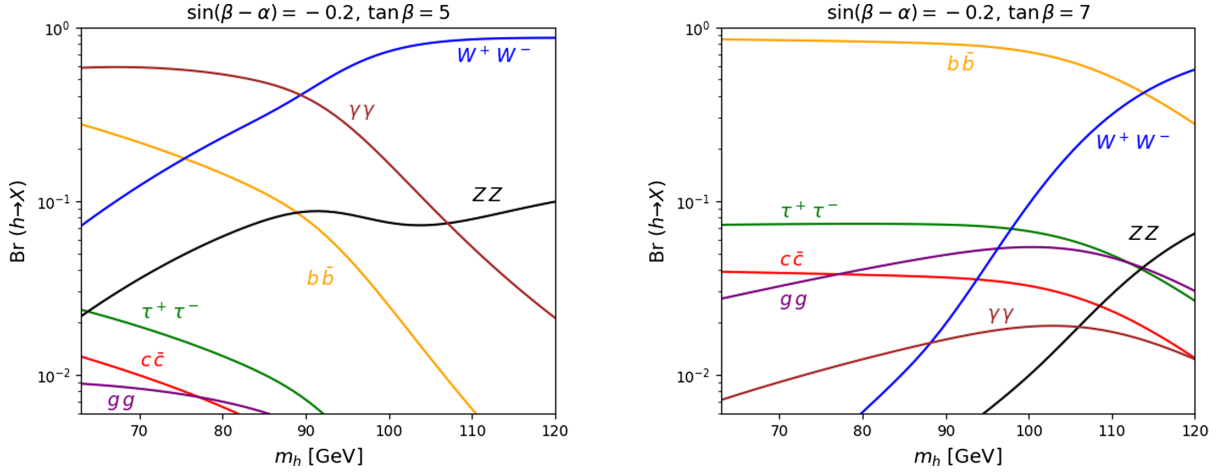


FIG. 15. For negative $s_{\beta-\alpha} = -0.2$, branching ratios of various decay modes of the lighter CP -even Higgs boson h as a function of m_h . Results are shown for $t_\beta = 5$ (left panel) and $t_\beta = 7$ (right panel), with $m_A = m_{H^\pm} = 300$ GeV and $\tilde{M} = m_h$.

For negative $s_{\beta-\alpha}$, the decay pattern of h changes considerably compared to the positive or aligned cases. Figure 15 presents the branching ratios for $s_{\beta-\alpha} = -0.2$. Here, results are shown for $t_\beta = 5$ (left panel) and $t_\beta = 7$ (right panel), assuming $m_A = m_{H^\pm} = 300$ GeV and $\tilde{M} = m_h$. For $t_\beta = 5$ and $m_h \lesssim 90$ GeV, $h \rightarrow \gamma\gamma$ can become the dominant decay mode, followed by $h \rightarrow b\bar{b}$, $h \rightarrow \tau^+\tau^-$, $h \rightarrow c\bar{c}$, and $h \rightarrow gg$. This dramatic enhancement of $h \rightarrow \gamma\gamma$ arises from a near cancellation in the Yukawa coupling modifier ξ_f^h (leading to $\xi_f^h \approx -0.004$ for $s_{\beta-\alpha} = -0.2$ and $t_\beta = 5$, for example), which drastically suppresses fermionic decays. For heavier h in the range $90 \text{ GeV} \lesssim m_h \lesssim 120$ GeV, the dominant decay mode shifts to WW^* , with $h \rightarrow \gamma\gamma$ and $h \rightarrow ZZ^*$ also contributing significantly.

For $t_\beta = 7$ (right panel of Fig. 15), $h \rightarrow b\bar{b}$ generally remains dominant for $m_h \lesssim 90$ GeV, followed by $\tau^+\tau^-$ and $c\bar{c}/gg$. Although $h \rightarrow \gamma\gamma$ is typically subdominant in this case, its branching ratio can still reach a few percent. When $m_h \gtrsim 90$ GeV, WW^* and ZZ^* modes become important, while $\text{Br}(h \rightarrow \gamma\gamma)$ can persist at the $\mathcal{O}(10^{-2})$ level. In summary, the $h \rightarrow \gamma\gamma$ decay mode attains a significantly enhanced branching ratio—potentially reaching several percent or more—if $s_{\beta-\alpha} < 0$, providing a distinctive feature of such scenarios.

Considering the dominant $H^\pm \rightarrow W^\pm h$ decay and the characteristic decay patterns of h (which depend on the sign of $s_{\beta-\alpha}$), we propose the following two final states to probe the SFOEWPT parameter space of the inverted type-I 2HDM at CLIC:

$$e^+e^- \rightarrow H^+H^- \rightarrow W^+W^-hh \rightarrow W^+W^-b\bar{b}\tau^+\tau^-, \quad (32)$$

$$e^+e^- \rightarrow H^+H^- \rightarrow W^+W^-hh \rightarrow W^+W^-b\bar{b}\gamma\gamma. \quad (33)$$

The process in Eq. (33) is particularly interesting for targeting the negative $s_{\beta-\alpha}$ case, which predominantly corresponds to one-step SFOEWPT scenarios.

B. Signal-to-background analysis

In this subsection, we perform a detector-level signal-to-background analysis for the proposed golden channels defined in Eqs. (32) and (33). A key aspect of evaluating the discovery potential of the 1.5 TeV CLIC is the precise estimation of SM backgrounds for the complex final states $W^+W^-b\bar{b}\tau^+\tau^-$ and $W^+W^-b\bar{b}\gamma\gamma$.

However, direct computation of the SM cross sections for these final states using public Monte Carlo tools—such as MadGraph5_aMC@NLO [192] or CalcHEP [193]—is often practically infeasible. These simulations can fail to converge even after extensive run times (exceeding ten days), primarily due to the enormous number of Feynman diagrams contributing to such high-multiplicity processes. For example, the $W^+W^-b\bar{b}\gamma\gamma$ final state involves 4308 tree-level diagrams, which significantly complicates event generation.

To estimate the SM background contributions, we instead compute parton-level cross sections for simplified reference processes with fewer final-state particles: $e^+e^- \rightarrow W^+W^-b\bar{b}$ and $e^+e^- \rightarrow W^+W^-b\bar{b}\gamma$. These background estimates are obtained by including all relevant Feynman diagrams at the matrix-element level, thereby accounting for both on-shell and off-shell $t\bar{t}$ contributions in addition to the nonresonant $W^+W^-b\bar{b}$ and $W^+W^-b\bar{b}\gamma$ processes. Applying representative kinematic cuts (for b -quarks: $p_T^b > 20$ GeV, $|\eta_b| < 2.5$, and $m_{b\bar{b}} < m_h$ ⁸; for photons in the second process: $p_T^\gamma > 10$ GeV and $|\eta_\gamma| < 2.5$), we obtain the following cross sections:

⁸The cut $m_{b\bar{b}} < m_h$ is primarily intended to enhance signal characteristics and will be further justified in the subsequent signal selection analysis.

TABLE I. Representative BPs from the SFOEWPT parameter space, illustrating model parameters, key branching ratios (BRs) of the lighter CP -even Higgs boson h , the associated transition stage, and the corresponding value of ξ_c . All masses ($m_{H^\pm}$, m_h , m_A , $\bar{M}$) are given in GeV.

BP	$m_{H^\pm}$	m_h	m_A	$s_{\beta-\alpha}$	t_β	$\bar{M}$	$\text{Br}(h \rightarrow b\bar{b})$	$\text{Br}(h \rightarrow \gamma\gamma)$	Stage	ξ_c
BP-1	104.6	71.3	107.5	0.022	11.8	72.0	8.42×10^{-1}	3.00×10^{-6}	$\text{PT}_2^{(2\text{-step})}$	2.23
BP-2	199.1	70.4	168.7	0.151	4.5	68.6	8.42×10^{-1}	3.50×10^{-5}	$\text{PT}_2^{(2\text{-step})}$	1.31
BP-3	199.4	101.2	174.3	0.184	12.6	95.9	8.02×10^{-1}	6.39×10^{-4}	$\text{PT}_2^{(2\text{-step})}$	1.69
BP-4	301.3	71.2	300.7	0.037	8.6	51.2	8.42×10^{-1}	3.00×10^{-6}	$\text{PT}_2^{(2\text{-step})}$	1.47
BP-5	309.2	97.7	313.5	0.217	9.9	90.4	8.09×10^{-1}	5.07×10^{-4}	$\text{PT}_2^{(2\text{-step})}$	2.80
BP-6	345.5	78.0	343.8	-0.171	5.7	52.6	3.52×10^{-2}	6.42×10^{-1}	$\text{PT}_1^{(1\text{-step})}$	1.41
BP-7	351.1	79.8	338.7	0.200	6.8	61.4	8.33×10^{-1}	1.82×10^{-4}	$\text{PT}_2^{(2\text{-step})}$	2.57
BP-8	395.1	102.7	402.5	0.004	5.8	85.6	8.06×10^{-1}	9.10×10^{-5}	$\text{PT}_1^{(1\text{-step})}$	1.77
BP-9	401.8	95.6	399.2	-0.186	5.6	70.6	1.90×10^{-1}	2.02×10^{-1}	$\text{PT}_1^{(1\text{-step})}$	1.77
BP-10	403.1	119.8	393.6	-0.178	4.6	102.6	1.53×10^{-1}	1.86×10^{-2}	$\text{PT}_1^{(1\text{-step})}$	1.77

$$\begin{aligned}\sigma_{\text{SM}}(e^+e^- \rightarrow W^+W^-b\bar{b})|_{\text{cuts}} &\simeq 6.79 \text{ fb}, \\ \sigma_{\text{SM}}(e^+e^- \rightarrow W^+W^-b\bar{b}\gamma)|_{\text{cuts}} &\simeq 0.782 \text{ fb}.\end{aligned}\quad (34)$$

We expect the full SM background cross section for $e^+e^- \rightarrow W^+W^-b\bar{b}\tau^+\tau^-$ to be substantially smaller than $\sigma_{\text{SM}}(e^+e^- \rightarrow W^+W^-b\bar{b})$. This suppression arises from additional electroweak coupling factors (e.g., proportional to α_{em}^2 for the $\tau^+\tau^-$ pair) and further phase-space suppression due to the increased final-state multiplicity. An analogous suppression is expected when comparing $e^+e^- \rightarrow W^+W^-b\bar{b}\gamma\gamma$ to the simpler process $e^+e^- \rightarrow W^+W^-b\bar{b}\gamma$.

It is therefore reasonable to expect the cross sections for the full SM background processes $e^+e^- \rightarrow W^+W^-b\bar{b}\tau^+\tau^-$ and $e^+e^- \rightarrow W^+W^-b\bar{b}\gamma\gamma$ to lie well below 1 ab. Given the anticipated total integrated luminosity of 4 ab^{-1} at the 1.5 TeV CLIC, the expected number of background events is significantly less than 1, rendering these backgrounds effectively negligible for our signal analysis.

A key question then arises: how should the signal significance be computed when the expected number of background events (N_{bg}) is negligible? Consider the general case in which N_{tot} events are observed, given a SM background prediction for N_{bg} (assuming, for simplicity, no uncertainty on N_{bg}). Since $N_{\text{tot}} = N_{\text{sg}} + N_{\text{bg}}$, where N_{sg} denotes the number of signal events, the significance $\mathcal{S}$ based on a Poisson counting likelihood is given by [194]

$$\mathcal{S} = \sqrt{2 \left\{ N_{\text{tot}} \ln \frac{N_{\text{tot}}}{N_{\text{bg}}} - N_{\text{sg}} \right\}}. \quad (35)$$

In scenarios with a sufficiently large N_{bg} (typically $\gtrsim 20$), where the Gaussian approximation to the Poisson distribution is valid, this expression simplifies to

$$\mathcal{S}|_{\text{Gaussian}} \approx \frac{N_{\text{sg}}}{\sqrt{N_{\text{bg}}}}.$$

However, when N_{bg} is small, the Gaussian approximation becomes unreliable. This is because it incorrectly assumes symmetric fluctuations in the background, whereas the Poisson distribution that governs background-only expectations is inherently asymmetric at low event counts—especially in the tails, which are critical for p -value estimation. In such cases, the Poisson-based likelihood formula in Eq. (35) is theoretically more accurate.

Then, a complication arises when N_{bg} approaches or equals zero. Specifically, if $N_{\text{bg}} \rightarrow 0$, the logarithmic term in Eq. (35) causes the significance $\mathcal{S}$ to diverge. The expression becomes ill defined for $N_{\text{bg}} = 0$. Given these considerations—and in light of our earlier estimate indicating negligible background contributions for the proposed golden channels—we choose to present only the expected number of signal events (N_{sg}), rather than quoting significance values that would be ill defined or potentially misleading in the $N_{\text{bg}} \approx 0$ regime.

To efficiently explore the SFOEWPT parameter space in our phenomenological study, we select ten representative benchmark points (BPs), summarized in Table I. The table lists the model parameters, the key branching ratios of the lighter CP -even Higgs boson h , the stage of electroweak phase transition, and the corresponding value of ξ_c . Among these, BP-6, BP-8, BP-9, and BP-10 realize a SFOEWPT via a one-step transition, while the remaining points correspond to $\text{PT}_2^{(2\text{-step})}$. Notably, BP-6, BP-9, and BP-10 feature negative values of $s_{\beta-\alpha}$.

For these benchmark points, we conduct a detector-level signal analysis. Event generation is performed using `MadGraph5_aMC@NLO`. Parton showering and hadronization are handled by `Pythia8.309`[195], followed by fast detector simulation using `Delphes3.5.0` [196] with the

delphes_card_CLICdet_Stage2.tcl configuration. Jet clustering is carried out using the exclusive Valencia⁹ algorithm [197,198], as implemented in FastJet [199]. The algorithm is configured with standard parameters: jet radius $R = 0.5$ and energy/angular weighting parameters $\beta = \gamma = 1$, which define the distance metric used in clustering [198].

Let us first present the signal analysis for the $hh \rightarrow b\bar{b}\tau^+\tau^-$ mode, focusing on the final state,

$$e^+e^- \rightarrow H^+H^- \rightarrow W^+W^-hh \rightarrow \ell^\pm \nu jjb\bar{b}\tau^+\tau^-,$$

where $\ell^\pm = e^\pm, \mu^\pm$. This final state comprises one charged lepton, missing energy attributed to the neutrino (reconstructable due to the clean initial state at CLIC), two light-flavor jets, two b -tagged jets, and a pair of tau leptons. For convenience, we refer to this as the $bb\tau\tau$ mode in the discussion that follows.

Accurate reconstruction of this final state critically depends on efficient b tagging and τ tagging. A jet is identified as a b jet if a B hadron with $p_T > 5$ GeV is found within a cone of $\Delta R = 0.3$ around the jet axis. We implement b -tagging efficiencies and mistagging rates based on the default configuration in delphes_card_CLICdet_Stage2.tcl, which offers three fixed values for the b -tagging efficiency: $P_{b \rightarrow b} = 50\%, 70\%, 90\%$. For each chosen $P_{b \rightarrow b}$, the corresponding mistagging rates—dependent on jet kinematics such as energy and pseudorapidity—are automatically determined. In our analysis, we adopt $P_{b \rightarrow b} = 70\%$. Tau leptons are identified through their hadronic decays, denoted as τ_h , which appear as narrow jets with a small number of charged tracks [200–202]. Tau-tagging and mistagging rates are also p_T dependent, and we follow the default parametrization provided in delphes_card_CLICdet_Stage2.tcl for their implementation.

For the $bb\tau\tau$ mode, we establish the following selection criteria:

- (i) *Lepton multiplicity.* Require exactly one charged lepton (electron or muon) with transverse momentum $p_T > 10$ GeV and pseudorapidity $|\eta| < 2.5$.
- (ii) *Jet multiplicity.* Using the “exclusive” Valencia jet algorithm with the total jet number fixed to 6, we require exactly two b -tagged jets, two τ_h -tagged jets, and two light-flavor jets, each with $p_T > 20$ GeV and $|\eta| < 2.5$.
- (iii) *Invariant mass cuts.* Impose upper bounds on the invariant masses of the b -jet pair and the τ_h -jet pair: $m_{b\bar{b}} < m_h$ and $m_{\tau_h\tau_h} < m_h$.

⁹The Valencia algorithm is particularly well suited for high-energy lepton colliders, as it effectively mitigates the impact of initial-state radiation and beam-induced backgrounds by reconstructing beam jets explicitly. We adopt the exclusive mode, which halts jet clustering once the desired number of jets is reached—appropriate here given the known jet multiplicity.

We opt for upper-only cuts on $m_{b\bar{b}}$ and $m_{\tau_h\tau_h}$ rather than symmetric mass windows centered around m_h . Our analysis of the reconstructed invariant mass distributions reveals that both $m_{b\bar{b}}$ and $m_{\tau_h\tau_h}$ exhibit significant skewness toward lower values, rather than peaking sharply at m_h . For b jets, this distortion arises from neutrinos produced in semi-leptonic B -hadron decays, finite jet energy resolution, and final-state radiation escaping the jet cone [203,204]. Similarly, the $m_{\tau_h\tau_h}$ distribution is affected by neutrinos in all τ decays, the narrow and low-multiplicity nature of τ_h jets, calorimeter resolution effects, and final-state radiation.

For the $hh \rightarrow b\bar{b}\gamma\gamma$ mode, we consider the process

$$e^+e^- \rightarrow H^+H^- \rightarrow W^+W^-hh \rightarrow \ell^\pm \nu jjb\bar{b}\gamma\gamma.$$

The final state consists of one charged lepton, missing energy, two light-flavor jets, two b -tagged jets, and a pair of photons. As in the $bb\tau\tau$ case, we refer to this final state as the $bb\gamma\gamma$ mode.

The selection criteria for the $bb\gamma\gamma$ mode are as follows:

- (i) *Lepton multiplicity.* Require exactly one charged lepton (electron or muon) with $p_T > 10$ GeV and $|\eta| < 2.5$.
- (ii) *Jet multiplicity.* Using the exclusive Valencia jet algorithm with the total number of jets fixed to 4, require exactly two b -tagged jets and two light-flavor jets, each with $p_T > 20$ GeV and $|\eta| < 2.5$.
- (iii) *Photon multiplicity.* Require exactly two photons with $p_T > 10$ GeV and $|\eta| < 2.5$.
- (iv) *Invariant mass cuts.* Impose upper bounds on the invariant masses: $m_{b\bar{b}} < m_h$ and $m_{\gamma\gamma} < m_h$.

Table II presents the expected number of signal events at the 1.5 TeV CLIC with a total luminosity of 4 ab⁻¹ for the ten benchmark points in the two golden channels, after all selection cuts. These counts directly reflect the observational potential at CLIC, given our earlier arguments that the SM backgrounds are expected to be negligible.

Several benchmark points show significant yields in the $bb\tau\tau$ mode. Notably, BP-1 and BP-3 predict the highest yields with approximately 68.5 events each. Other points, such as BP-2 (46.6 events), BP-5 (42.4 events), BP-4 (38.6 events), BP-8 (34.1 events), and BP-7 (29.6 events), also yield tens of events, indicating good prospects for observation. Conversely, BP-6, BP-9, and BP-10 show very few events in this channel (<2 events), making their detection challenging via this mode.

The $bb\gamma\gamma$ mode exhibits a complementary pattern to the $bb\tau\tau$ channel. The benchmark points that yield strong signals in the $bb\tau\tau$ mode—such as BP-1, BP-2, BP-3, and BP-4—produce very few or no events in the $bb\gamma\gamma$ channel. Conversely, BP-9 (54.2 events) and BP-6 (36.1 events), which were weak in the $bb\tau\tau$ mode, predict substantial event yields in the $bb\gamma\gamma$ channel. This stark contrast highlights the importance of analyzing both final states to ensure broader coverage of the SFOEWPT parameter space.

TABLE II. Number of expected signal events for the process $e^+e^- \rightarrow H^+H^-$ leading to the $bb\tau\tau$ and $bb\gamma\gamma$ final states (defined in text) at the 1.5 TeV CLIC with an integrated luminosity of 4 ab^{-1} . These counts are for the benchmark points detailed in Table I and after applying the selection criteria.

Number of signal events for $e^+e^- \rightarrow H^+H^-$ at the 1.5 TeV CLIC with $\mathcal{L}_{\text{tot}} = 4 \text{ ab}^{-1}$					
Benchmark	$bb\tau\tau$ mode	$bb\gamma\gamma$ mode	Benchmark	$bb\tau\tau$ mode	$bb\gamma\gamma$ mode
BP-1	68.49	0.00	BP-6	0.06	36.14
BP-2	46.64	0.05	BP-7	29.64	0.26
BP-3	68.50	0.98	BP-8	34.10	0.12
BP-4	38.55	0.00	BP-9	1.53	54.16
BP-5	42.38	0.75	BP-10	1.17	4.35

From the perspective of observational prospects, most benchmark points yield a substantial number of signal events—typically tens—in at least one of the two proposed golden channels. Specifically, BP-1, BP-2, BP-3, BP-4, BP-5, BP-7, and BP-8 show strong potential in the $bb\tau\tau$ mode, while BP-6 and BP-9 are particularly promising in the $bb\gamma\gamma$ mode. Even BP-10, which yields the fewest events overall, predicts a combined total of approximately 5.5 signal events across both channels. Assuming negligible background, these signal yields are generally sufficient to indicate strong discovery potential and enable detailed phenomenological studies at the 1.5 TeV CLIC.

In conclusion, the projected signal event counts demonstrate that the 1.5 TeV CLIC, with an integrated luminosity of 4 ab^{-1} , possesses significant discovery potential for the SFOEWPT parameter space within the inverted type-I 2HDM. While the $bb\tau\tau$ mode appears broadly promising for a number of benchmark points, the $bb\gamma\gamma$ channel provides crucial complementarity, essential for enhancing the overall discovery potential. Furthermore, the varying signal strengths and channel preferences across different benchmark points suggest that observations (or nonobservations) in these channels could play a vital role in characterizing and potentially differentiating between various SFOEWPT scenarios within the model.

Final comments on the HL-LHC prospects are in order. Although the HL-LHC program is much more certain than CLIC, it is not as advantageous for probing the full set of SFOEWPT parameter points. Focusing on our target channel—the pair production of charged Higgs bosons—the production cross section at $\sqrt{s} = 14 \text{ TeV}$ decreases rapidly as the charged Higgs mass grows. Specifically, we find $\sigma(pp \rightarrow H^+H^-) \simeq 2.2 \times 10^2 \text{ fb}$ for $m_{H^\pm} = 100 \text{ GeV}$, $\sigma \simeq 3.8 \text{ fb}$ for $m_{H^\pm} = 300 \text{ GeV}$, and $\sigma \simeq 6.8 \times 10^{-1} \text{ fb}$ for $m_{H^\pm} = 450 \text{ GeV}$. This steep falloff originates from the rapidly decreasing parton distribution functions (PDFs) at large momentum fractions in hadronic collisions. As a result, the HL-LHC loses sensitivity to heavier charged Higgs bosons.

In contrast, at a lepton collider such as CLIC, the pair-production cross section does not suffer from this strong PDF suppression. As shown in Fig. 11, the cross section at

$\sqrt{s} = 1.5 \text{ TeV}$ for $m_{H^\pm} = 450 \text{ GeV}$ retains about 53% of the value for $m_{H^\pm} = 100 \text{ GeV}$. This scaling behavior makes CLIC much more effective for probing the heavy charged Higgs sector.

Backgrounds also pose an additional limitation at the HL-LHC. The dominant irreducible backgrounds to our final states $W^+W^-b\bar{b}\tau^+\tau^-$ and $W^+W^-b\bar{b}\gamma\gamma$ arise from $pp \rightarrow t\bar{t}Z(\rightarrow \tau^+\tau^-)$ and $pp \rightarrow t\bar{t}\gamma\gamma$, respectively. Their next-to-leading-order cross sections are $\sigma(pp \rightarrow t\bar{t}Z) \times \text{Br}(Z \rightarrow \tau^+\tau^-) \simeq 31 \text{ fb}$ and $\sigma(pp \rightarrow t\bar{t}\gamma\gamma) \simeq 7.2 \text{ fb}$ [205]. By contrast, at CLIC the absence of QCD-driven top-pair production renders such backgrounds negligible.

For these two reasons—the rapid PDF suppression of heavy $H^\pm$ pair production and the sizable hadronic backgrounds—we conclude that the 1.5 TeV CLIC provides a more advantageous environment for the collider study of the entire SFOEWPT parameter space. Nevertheless, a comprehensive signal-to-background analysis at the HL-LHC remains an important and complementary task, which we leave for future work.

VI. CONCLUSION

In this work, we have performed a comprehensive investigation of the electroweak phase transition within the inverted type-I two-Higgs-doublet model, in which the observed 125 GeV Higgs boson corresponds to the heavier CP -even scalar H . Motivated by the critical role of a strong first-order electroweak phase transition in enabling electroweak baryogenesis, as well as by its associated gravitational wave, we systematically explored the viable parameter space under current theoretical and experimental constraints. We provided a detailed characterization of multistep electroweak phase transition dynamics, evaluated GW signals, and assessed collider discovery prospects.

Our analysis offers several novel insights into the electroweak phase transition within the inverted type-I 2HDM. First, we have systematically mapped—for the first time—the parameter space compatible with SFOEWPTs, explicitly including multistep transitions. Using the notation $\text{PT}_i^{(n\text{-step})}$ to denote the i th transition in an n -step sequence, we demonstrated that SFOEWPTs can occur not only via

conventional one-step pathways but also prominently through $\text{PT}_2^{(2\text{-step})}$, and occasionally via $\text{PT}_3^{(3\text{-step})}$.

A key finding is that parameter regions supporting one-step and two-step SFOEWPTs display distinct characteristics, despite limited overlap. For instance, the charged Higgs mass $m_{H^\pm}$ in one-step scenarios is tightly constrained to approximately $[295, 441]$ GeV, whereas $\text{PT}_2^{(2\text{-step})}$ transitions allow a broader and lighter range, $m_{H^\pm} \in [100, 350]$ GeV. Similarly, one-step transitions prefer a narrower range of $\tan\beta$ ($t_\beta \in [4.2, 8.8]$), while $\text{PT}_2^{(2\text{-step})}$ transitions accommodate a significantly wider range ($t_\beta \in [2.5, 45.4]$).

We also presented the first detailed calculation of the GW signal-to-noise ratio (SNR) at LISA for SFOEWPTs in this model, explicitly addressing multistep transitions. Although one-step SFOEWPTs are more abundant across the parameter space, points generating detectable GW signals ($\text{SNR} > 10$) arise predominantly from $\text{PT}_2^{(2\text{-step})}$ transitions. This underscores the importance of considering multistep dynamics when assessing GW detectability in extended scalar sectors.

Additionally, we analyzed the correlation between the vacuum uplifting measure $\Delta\mathcal{F}_0$ and the electroweak phase transition strength parameter ξ_c . We confirmed the clear positive correlation for one-step transitions in the inverted scenario, which was established for one-step transitions in the normal scenario. Crucially, however, we found that this correlation breaks down entirely for multistep transitions, demonstrating that $\Delta\mathcal{F}_0$ cannot serve as a universal proxy for the phase transition strength. A full finite-temperature analysis thus remains essential whenever intermediate metastable vacua appear.

Finally, we conducted the first collider phenomenology study targeting parameter points compatible with SFOEWPTs in the inverted type-I 2HDM, focusing on their discovery potential at the proposed 1.5 TeV CLIC. We identified charged Higgs pair production, $e^+e^- \rightarrow H^+H^-$, followed by $H^\pm \rightarrow W^\pm h$, as a particularly promising discovery channel due to its sizable cross section and no dependence on scalar mixing parameters at tree level.

An important phenomenological highlight is the significant enhancement of the diphoton decay $h \rightarrow \gamma\gamma$ at negative $s_{\beta-\alpha}$, strongly correlated with one-step SFOEWPT scenarios. Leveraging this feature, we proposed two complementary golden final states, $W^+W^-b\bar{b}\tau^+\tau^-$ and $W^+W^-b\bar{b}\gamma\gamma$,

both characterized by high-multiplicity final states and negligible SM backgrounds, thus ensuring high discovery potential at the 1.5 TeV CLIC.

In summary, our work has revealed a distinctive phenomenology of SFOEWPTs in the inverted type-I 2HDM. We have mapped the viable parameter space, clarified the breakdown of the vacuum uplifting correlation in multistep transitions, highlighted the importance of multistep dynamics for GW signatures, and identified collider channels with excellent discovery prospects. Together, these results illustrate the deep interplay between early Universe cosmology and high-energy experiments in probing physics beyond the Standard Model.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: VACUUM UPLIFTING MEASURE $\Delta\mathcal{F}_0$ IN THE HIGGS BASIS

For the vacuum energy difference between the symmetric and broken phases at *zero temperature*, it is more useful to use the Higgs basis of the 2HDM [85], where two Higgs doublet fields H_1 and H_2 are defined by

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}.$$

Then, H_2 does not develop nonzero VEV, yielding

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+h_1+iG_0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{h_2+iA}{\sqrt{2}} \end{pmatrix},$$

where G^+ and G_0 are the Goldstone bosons to be absorbed into the SM W^+ and Z , respectively. The tree-level potential in the Higgs basis becomes

$$\begin{aligned} V_{\text{tree}}(H_1, H_2) = & \bar{\mu}_1^2 |H_1|^2 + \bar{\mu}_2^2 |H_2|^2 - \bar{\mu}^2 [H_1^\dagger H_2 + \text{H.c.}] + \frac{\bar{\lambda}_1}{2} |H_1|^4 + \frac{\bar{\lambda}_2}{2} |H_2|^4 + \bar{\lambda}_3 |H_1|^2 |H_2|^2 + \bar{\lambda}_4 |H_1^\dagger H_2|^2 \\ & + \frac{\bar{\lambda}_5}{2} [(H_1^\dagger H_2)^2 + \text{H.c.}] + \bar{\lambda}_6 [|H_1|^2 H_1^\dagger H_2 + \text{H.c.}] + \bar{\lambda}_7 [|H_2|^2 H_1^\dagger H_2 + \text{H.c.}]. \end{aligned} \quad (\text{A1})$$

Let us present the parameters in Eq. (A1) in terms of our physical parameters $\{t_\beta, c_{\beta-\alpha}, M^2, v, m_h, m_H, m_A, m_{H^\pm}\}$.

Using a simplified notation of $\mathcal{A}_{\alpha\beta}$ as

$$\mathcal{A}_{\alpha\beta} = s_{\beta-\alpha} - c_{\beta-\alpha}(t_\beta - t_\beta^{-1}),$$

the mass squared parameters are

$$\begin{aligned}\bar{\mu}_1^2 &= -\frac{1}{2}[c_{\beta-\alpha}^2 m_H^2 + s_{\beta-\alpha}^2 m_h^2], \\ \bar{\mu}_2^2 &= M^2 - \frac{m_H^2}{2} + \frac{m_H^2 - m_h^2}{2} s_{\beta-\alpha} \mathcal{A}_{\alpha\beta}, \\ \bar{\mu}^2 &= -\frac{1}{2}(m_H^2 - m_h^2) s_{\beta-\alpha} c_{\beta-\alpha}.\end{aligned}$$

The modified quartic couplings are

$$\begin{aligned}\bar{\lambda}_1 &= -\frac{2\bar{\mu}_1^2}{v^2}, \\ \bar{\lambda}_2 &= \frac{1}{v^2}[m_h^2 + (m_H^2 - M^2)(t_\beta - t_\beta^{-1})^2 + (m_{H_0}^2 - m_h^2)(1 - \mathcal{A}_{\alpha\beta}^2)], \\ \bar{\lambda}_3 &= 2\frac{m_{H^\pm}^2 - \bar{\mu}_2^2}{v^2}, \\ \bar{\lambda}_4 &= \frac{1}{v^2}[m_A^2 - 2m_{H^\pm}^2 + m_h^2 + (m_H^2 - m_h^2)s_{\beta-\alpha}^2], \\ \bar{\lambda}_5 &= \frac{1}{v^2}[-m_A^2 + m_h^2 + (m_H^2 - m_h^2)s_{\beta-\alpha}^2], \\ \bar{\lambda}_6 &= \frac{2\bar{\mu}^2}{v^2}, \\ \bar{\lambda}_7 &= \frac{1}{v^2}[(M^2 - m_H^2)(t_\beta - t_\beta^{-1}) - (m_{H_0}^2 - m_h^2)c_{\beta-\alpha}\mathcal{A}_{\alpha\beta}].\end{aligned}$$

In the 2HDM, the vacuum energy difference between the 2HDM and the SM is

$$\begin{aligned}\Delta\mathcal{F}_0 &= \Delta\mathcal{F}_0^{\text{tree}} - \frac{m_{H_{\text{SM}}}^4}{64\pi^2}(3 + \log 2) - \sum_k \frac{m_{0k}^4}{64\pi^2} \left(\log \frac{|m_{0k}^2|}{\mu^2} - \frac{1}{2} \right) \\ &\quad + \frac{1}{64\pi^2} \sum_k \frac{1}{4} \left\{ \tilde{I}_k^2 - 2m_k^4 + [(\tilde{I}_k - 2m_k^2)^2 + m_k^2(\tilde{J}_k - \tilde{I}_k)] \log \frac{m_k^2}{\mu^2} \right\},\end{aligned}\tag{A2}$$

where $k = H^\pm, A, H, h, G, G^\pm$. The m_{0k}^2 is the squared mass at the origin of the field configuration space, which are in the inverted Higgs scenario

$$\begin{aligned}m_{0h}^2 &= m_{0A}^2 = m_{0H^\pm}^2 = \bar{\mu}_2^2, \\ m_{0H}^2 &= m_{0G}^2 = m_{0G^\pm}^2 = \bar{\mu}_1^2.\end{aligned}$$

The required mass derivatives of $\tilde{I}_k$ and $\tilde{J}_k$ in Eq. (A2) are given by

$$\begin{aligned}
\tilde{I}_G &= m_h^2 + c_{\beta-\alpha}^2(m_H^2 - m_h^2), \\
\tilde{I}_{H^\pm} &= 2m_{H^\pm}^2 + c_{\beta-\alpha}^2 m_h^2 + s_{\beta-\alpha}^2 m_H^2 - [2M^2 - (m_H^2 - m_h^2)\mathcal{B}_{\alpha\beta}], \\
\tilde{J}_{H^\pm} &= \tilde{I}_{H^\pm} + 2c_{\beta-\alpha}^2 s_{\beta-\alpha}^2 \frac{(m_H^2 - m_h^2)^2}{m_{H^\pm}^2}, \\
\tilde{I}_A &= \tilde{I}_{H^\pm} - 2m_{H^\pm}^2 + 2m_A^2, \\
\tilde{J}_A &= \tilde{I}_A + 2c_{\beta-\alpha}^2 s_{\beta-\alpha}^2 \frac{(m_H^2 - m_h^2)^2}{m_A^2}, \\
\tilde{I}_h &= 3m_h^2 - c_{\beta-\alpha}^2 [2M^2 - (m_H^2 - m_h^2)\mathcal{B}_{\alpha\beta}], \\
\tilde{J}_h &= \tilde{I}_h - \frac{2c_{\beta-\alpha}^2 s_{\beta-\alpha}^2}{(m_H^2 - m_h^2)} [2M^2 - (m_H^2 - m_h^2)\mathcal{B}_{\alpha\beta}]^2, \\
\tilde{I}_{H_0} &= 3m_H^2 - s_{\beta-\alpha}^2 [2M^2 - (m_H^2 - m_h^2)\mathcal{B}_{\alpha\beta}], \\
\tilde{J}_{H_0} &= \tilde{I}_{H_0} + \frac{2c_{\beta-\alpha}^2 s_{\beta-\alpha}^2}{(m_H^2 - m_h^2)} [2M^2 - (m_H^2 - m_h^2)\mathcal{B}_{\alpha\beta}]^2,
\end{aligned}$$

where $\mathcal{B}_{\alpha\beta} = s_\alpha c_\alpha / (s_\beta c_\beta)$.

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