

# Probing the 3 + 1 neutrino model in the SHiP experiment

Ki-Young Choi<sup>1,\*</sup>, Yu Seon Jeong<sup>1</sup>, Sung Hyun Kim<sup>2</sup>, Yeong Gyun Kim<sup>3</sup>, Kang Young Lee<sup>2</sup>,  
Kyong Sei Lee<sup>4</sup>, Byung Do Park<sup>2</sup>, Jong Yoon Sohn<sup>2</sup>, Seong Moon Yoo<sup>1,†</sup> and Chun Sil Yoon<sup>2</sup>

<sup>1</sup>*Department of Physics and Institute of Basic Science, Sungkyunkwan University, Suwon 16419, Korea*

<sup>2</sup>*Department of Physics Education and Research Institute of Natural Science,  
Gyeongsang National University, Jinju 52828, Korea*

<sup>3</sup>*Department of Science Education, Gwangju National University of Education, Gwangju 61204, Korea*

<sup>4</sup>*Center for Extreme Nuclear Matters, Korea University, Seoul 02841, Korea*



(Received 27 June 2025; accepted 29 September 2025; published 28 October 2025)

We present the first comprehensive sensitivity study of the Search for Hidden Particles (SHiP) experiment to sterile neutrino oscillations using all three types of neutrinos, employing a Feldman-Cousins method with a parametric bootstrap to account for systematic uncertainties. In contrast to earlier studies that focused only on  $\nu_\tau$  disappearance, we include both appearance and disappearance channels across all active flavors. We also investigate the dual-baseline configuration, which complements the existing near scattering and neutrino detector (SND) at 27 m with an additional far SND at 120 m. The near SND only configuration can probe mixing parameters of  $|U_{\alpha 4}|^2 \gtrsim 0.1$  near  $\Delta m_{41}^2 \sim 10^3 \text{ eV}^2$ , with a reduction of normalized systematic uncertainties from 20% to 10% improving sensitivity by roughly a factor of two. Moreover, the inclusion of far SND significantly enhances the sensitivity by a factor of 2 to 10 depending on the flavor and the systematic uncertainty. If two active flavors are mixed with a sterile flavor, a cancelation between neutrino appearance and disappearance generates kinks in the sensitivity curves, while those kinks disappear with far SND.

DOI: [10.1103/c7sx-k7dq](https://doi.org/10.1103/c7sx-k7dq)

## I. INTRODUCTION

Neutrino oscillation experiments have shown that neutrinos have masses and mixing among three active flavors [1], which we call the three-flavor model. Long baseline experiments that measure solar neutrinos [2–7], atmospheric neutrinos [8–10], and neutrinos that are produced by accelerators [11–15] probe the three-flavor model from various perspectives, while future experiments such as DUNE [16], Hyper-Kamiokande [17], and ESSnuSB [18] aim to probe it with better precision. Despite these successes, a recent result from IceCube [19] or anomalies observed in LSND [20], MiniBooNE [21], reactor neutrino experiments [22], and gallium experiments [23] could suggest the existence of an additional neutrino flavor state that is *sterile*, meaning it does not participate in weak interactions.

Extensions of the Standard Model of particle physics that include sterile neutrinos may address open questions such

as the origin of the smallness of neutrino masses [24–28], the nature of dark matter [29], and the baryon asymmetry of the Universe [30]. The simplest extension of the Standard Model with additional sterile flavor and mass eigenstate is called the “3 + 1” model [31], which had been suggested to explain the anomalies above. However, global analyses of the 3 + 1 model show a large tension between experiments with anomalies and those with null results [32,33]. Nevertheless, the model remains a useful benchmark to probe sterile neutrinos and potential signals of physics beyond the Standard Model, and it has been investigated by a number of experiments. Long-baseline experiments such as Super-Kamiokande [34], NOvA [35,36], MINOS [37], T2K [38], OPERA [39], and IceCube-DeepCore [40,41] have excluded  $|U_{\alpha 4}|^2 \gtrsim 0.1$  regions with  $\Delta m_{41}^2 \gtrsim 0.1 \text{ eV}^2$ . Reactor neutrino experiments [42–49] constrain  $|U_{e 4}|^2 \gtrsim 0.1$  for  $\Delta m_{41}^2 \lesssim 10 \text{ eV}^2$ , while tritium beta decay experiments [50–55] constrain  $|U_{e 4}|^2 \gtrsim 10^{-2}$  for  $\Delta m_{41}^2 \gtrsim 10^2 \text{ eV}^2$ . Furthermore, future experiments such as Hyper-Kamiokande [56], JUNO [57], DUNE [58,59], and PROSPECT-II [60] will lower the current limit on the 3 + 1 model.

Short baseline experiments, particularly those using fixed-target neutrino sources have also played a crucial role in probing sterile neutrinos [61–70]. Booster Neutrino Beam based experiments SciBooNE [69] and MicroBooNE [70] constrain the  $\Delta m_{41}^2 \gtrsim 1 \text{ eV}^2$  region, with the null result

\*Contact author: [kiyoungchoi@skku.edu](mailto:kiyoungchoi@skku.edu)

†Contact author: [castledoor@skku.edu](mailto:castledoor@skku.edu)

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from MicroBooNE contradicting the  $3 + 1$  model's explanation for the LSND and MiniBooNE anomalies [71]. Also, Super Proton Synchrotron (SPS)-based experiments, such as CDHS [61], NOMAD [64,67], and CHORUS [68], as well as Tevatron-based experiments such as CCFR [62,63] and NuTeV [65] provided constraints on the  $\Delta m_{41}^2 \gtrsim 100 \text{ eV}^2$  region. Their high energy scales generate a detectable flux of tau neutrinos, which is why only a few experiments at the SPS or Tevatron have probed tau neutrino appearance on a short baseline. For higher  $\Delta m_{41}^2$  regions, forward neutrino experiments at the LHC can probe the  $\Delta m_{41}^2 \gtrsim 10^3 \text{ eV}^2$  region [72,73] which is our interest in later contents.

The Search for Hidden Particles (SHiP) experiment has been proposed to search for feebly interacting particles and to perform measurements in neutrino physics at the CERN SPS. [74–77] Like other SPS-based experiments, SHiP could serve as an additional probe of the  $3 + 1$  model as we discussed in our previous study [78]. SHiP will collide 400 GeV SPS protons on a heavy target to produce an intense neutrino beam containing all flavors, including a significant flux of  $\nu_\tau$ . The scattering and neutrino detector (SND) at SHiP, which utilizes an emulsion cloud chamber and a subsequent muon spectrometer for both tracking and calorimetry, is designed to identify charged-current (CC) interactions and distinguish neutrino flavors with high spatial resolution.

This paper presents the first comprehensive sensitivity study of the SHiP experiment to sterile neutrino oscillations using all three types of neutrinos. Our analysis employs statistically rigorous confidence regions from the Feldman-Cousins (FC) method [79] combined with a parametric bootstrap [80] to include contributions from auxiliary measurements [81]. Moreover, unlike our earlier work, this study covers all three active flavors and both appearance and disappearance channels, and uses signal-to-background ratios and detection efficiencies informed by OPERA's  $\nu_e/\nu_\mu$  performance [39,82] and a recent  $\nu_\tau$  identification analysis [83], for a more robust estimation of our sensitivities. In particular, if two active flavors are mixed with a sterile flavor, cancelations between appearance and disappearance may lead to weakened the sensitivities, and we show that the inclusion of a far detector can resolve these cancelations.

The paper is organized as follows: In Sec. II, we review the  $3 + 1$  sterile neutrino model in the context of SHiP's experimental setup. Section III details the statistical framework and assumptions used to derive sensitivity estimates. Section IV presents our results and compares them with existing constraints. Finally, Sec. V summarizes our findings and discusses future prospects.

## II. THE $3 + 1$ MODEL ON THE SHIP EXPERIMENT

In this section, we review the phenomenological aspects of sterile neutrinos within the context of SHiP.

The weak eigenstates of neutrinos  $|\nu_\alpha\rangle$ , including sterile flavors in general, can be transformed into mass eigenstates  $|\nu_i\rangle$  using the mixing matrix  $U$ ,

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle. \quad (1)$$

The Standard Model of particle physics includes three active neutrino flavors, while various extensions introduce additional sterile flavors. In the  $3 + 1$  model, the mixing matrix becomes a  $4 \times 4$  matrix  $U$  with an additional single sterile flavor. Following the standard parametrization from Ref. [84],  $U$  can be constructed as a product of six rotations,

$$U = \mathcal{U}_{34} \mathcal{V}_{24} \mathcal{V}_{14} \mathcal{U}_{23} \mathcal{V}_{13} \mathcal{U}_{12} \quad (2)$$

where  $\mathcal{U}_{ij}$  and  $\mathcal{V}_{ij}$  are matrices representing real and complex rotations, respectively. For example,  $\mathcal{U}_{12}$  and  $\mathcal{V}_{13}$  are defined as

$$\mathcal{U}_{12} = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathcal{V}_{13} = \begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{-i\delta_{13}} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta_{13} e^{i\delta_{13}} & 0 & \cos \theta_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

Therefore, the mixing matrix  $U$  is composed of six mixing angles  $\{\theta_{12}, \theta_{13}, \theta_{23}, \theta_{14}, \theta_{24}, \theta_{34}\}$  and three charge-parity (CP) phases  $\{\delta_{13}, \delta_{14}, \delta_{24}\}$ . Such parametrization is necessary when neutrino oscillations from the three-flavor model are effective, but it can be simplified for short baseline experiments.

Under the relativistic approximation, if the mass of the extra mass eigenstate is sufficiently large and the baseline is short enough to neglect lighter mass eigenstates, the transition probability from an active flavor  $\alpha$  to another active flavor  $\beta$  can be approximated as

$$P_{\alpha\beta} \approx \left| \delta_{\alpha\beta} - \left( 1 - e^{-i\frac{\Delta m_{41}^2 L}{2E_\nu}} \right) U_{\alpha 4} U_{\beta 4}^* \right|^2, \quad (4)$$

where  $L$  is the baseline, and  $E_\nu$  is the neutrino energy. Introducing the notation  $\sin^2 2\theta_{\alpha\beta} \equiv 4|U_{\alpha 4}|^2(1 - |U_{\alpha 4}|^2)$  for  $\alpha = \beta$  and  $\sin^2 2\theta_{\alpha\beta} \equiv 4|U_{\alpha 4}|^2|U_{\beta 4}|^2$  for  $\alpha \neq \beta$ , Eq. (4) can be simplified into

$$P_{\alpha\beta} = \begin{cases} 1 - \sin^2 2\theta_{\alpha\beta} \sin^2 \left( \frac{\Delta m_{41}^2 L}{4E_\nu} \right) & \text{if } \alpha = \beta \\ \sin^2 2\theta_{\alpha\beta} \sin^2 \left( \frac{\Delta m_{41}^2 L}{4E_\nu} \right) & \text{if } \alpha \neq \beta. \end{cases} \quad (5)$$

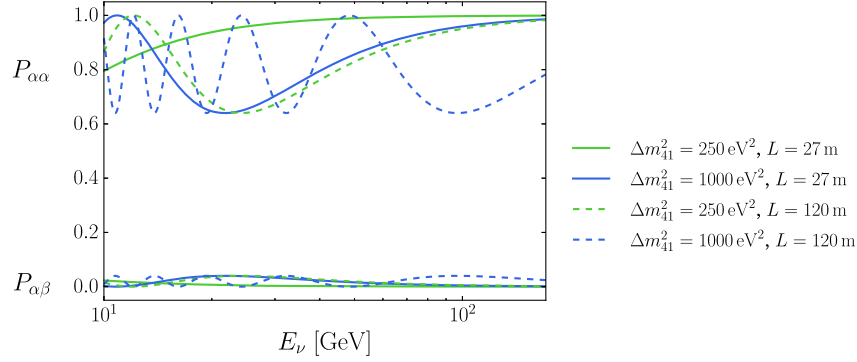


FIG. 1. Appearance (bottom) and disappearance (top) probabilities at 27 m (solid) and 120 m (dashed) baselines for mass-squared differences  $\Delta m^2_{41} = 250 \text{ eV}^2$  (green) and  $\Delta m^2_{41} = 10^3 \text{ eV}^2$  (blue), with mixing parameters  $|U_{\alpha 4}|^2 = |U_{\beta 4}|^2 = 0.1$ .

Thus, under the short baseline approximation, the 3 + 1 model can be parametrized by the mass-squared difference  $\Delta m^2_{41}$  and the mixing parameters  $|U_{\alpha 4}|^2$  while every  $CP$  violating phase can be ignored.

The maximum sensitivity to oscillation is achieved near  $\Delta m^2_{41} \sim 2\pi\langle E_\nu \rangle/L$ , where  $\langle E_\nu \rangle$  denotes the average neutrino energy produced by the proton target. In this paper, following Ref. [78], we refer to the SND with a 27 m baseline as near SND (NSND) and the SND with a 120 m baseline as far SND (FSND). Assuming an average neutrino energy of 30 GeV, NSND is most effective at  $\Delta m^2_{41} \sim 10^3 \text{ eV}^2$ , while FSND is most effective at  $\Delta m^2_{41} \sim 250 \text{ eV}^2$ . Figure 1 depicts probabilities of appearance (bottom) and disappearance (top) at 27 m (solid) and 120 m (dashed) baselines for mass-squared differences  $\Delta m^2_{41} = 250 \text{ eV}^2$  (green) and  $\Delta m^2_{41} = 10^3 \text{ eV}^2$  (blue), with mixing parameters  $|U_{\alpha 4}|^2 = |U_{\beta 4}|^2 = 0.1$ . Note that the total oscillation probability sums to unity when the active-sterile transition probability is included.

### III. METHOD ON SENSITIVITY ESTIMATION

In this section, we present the method for estimating the sensitivity of SHiP under the 3 + 1 model. This sensitivity is determined by identifying the confidence region for the data, assuming that both SHiP and auxiliary measurements observed the most probable outcome under the three-flavor model. To determine the confidence region, we apply the FC method [79] with a parametric bootstrap [80], which constructs a confidence region for a given sample distribution. In real analyses, repeated sampling for SHiP is performed using Monte Carlo (MC) simulation tools such as FairSHiP [77], which is beyond the main focus of our study. Instead, a crude estimation of the probability distribution for the SHiP sample is performed, and a confidence region is determined accordingly.

This section begins with a review of the FC method with a parametric bootstrap to find a confidence region with nuisance parameters. Next, we estimate the probability

distribution of samples and point estimates of nuisance parameters for SHiP.

#### A. Statistical method

In this Subsection, we introduce the statistical methods that we employed to estimate the sensitivities. The confidence region with significance level  $\alpha \equiv 1 - \text{CL}$  using the FC method is defined as follows:

$$\text{CR}(\mathbf{D}, \boldsymbol{\varphi}; \alpha) = \{\boldsymbol{\theta} | \mathbf{p}(\boldsymbol{\theta}; \mathbf{D}, \boldsymbol{\varphi}) > \alpha\}, \quad (6)$$

where  $\mathbf{D}$  represents the data from a SND,  $\boldsymbol{\varphi}$  is the point estimate of the nuisance parameters  $\boldsymbol{\phi}$  obtained from auxiliary measurements, and  $\mathbf{p}(\boldsymbol{\theta}; \mathbf{D}, \boldsymbol{\varphi})$  denotes the p-value for the model parameters  $\boldsymbol{\theta}$  given  $\mathbf{D}$  and  $\boldsymbol{\varphi}$ . Ideally, repeated sampling for auxiliary measurements should be performed to follow the FC method rigorously, although this is often not the case in practice. Therefore, we utilize a parametric bootstrap [80] for nuisance parameters, treating  $\boldsymbol{\varphi}$  as a substitute for the data of auxiliary measurements.

The p-value is defined as

$$1 - \mathbf{p}(\boldsymbol{\theta}; \mathbf{D}, \boldsymbol{\varphi}) = \int_0^{\lambda(\mathbf{D}, \boldsymbol{\varphi}, \boldsymbol{\theta})} \rho(\lambda' | \boldsymbol{\theta}, \boldsymbol{\varphi}) d\lambda', \quad (7)$$

where  $\rho(\lambda' | \boldsymbol{\theta}, \boldsymbol{\varphi})$  is the probability density function of the test statistic  $\lambda'$  for given  $\boldsymbol{\theta}$  and  $\boldsymbol{\varphi}$ , which is determined through repeated sampling of  $\mathbf{D}$  and  $\boldsymbol{\varphi}$ . If we denote the samples of  $\mathbf{D}$  and  $\boldsymbol{\varphi}$  as  $\mathbf{O}$  and  $\boldsymbol{\varphi}'$ , the probability of the test statistic is equal to

$$\rho(\lambda' | \boldsymbol{\theta}, \boldsymbol{\varphi}) = \int \delta[\lambda' - \lambda(\mathbf{O}, \boldsymbol{\varphi}', \boldsymbol{\theta})] \rho(\mathbf{O}, \boldsymbol{\varphi}' | \boldsymbol{\theta}, \boldsymbol{\varphi}) d\mathbf{O} d\boldsymbol{\varphi}', \quad (8)$$

where  $\delta(x - y)$  is a Dirac-delta distribution and  $\rho(\mathbf{O}, \boldsymbol{\varphi}' | \boldsymbol{\theta}, \boldsymbol{\varphi})$  is the sample distribution of SHiP and its auxiliary measurements.

The sample distribution in this paper is written as,

$$\rho(\mathbf{O}, \boldsymbol{\varphi}' | \boldsymbol{\theta}, \boldsymbol{\phi}) = \prod_{\alpha, i} \text{Pois}(O_{\alpha, i} | \mu_{\alpha, i}) \times \rho(\boldsymbol{\varphi}' | \boldsymbol{\phi}), \quad (9)$$

where  $\mu_{\alpha, i}$  is the mean number of events at the  $i$ th bin of the  $\alpha$  flavor CC deep inelastic scattering (DIS) channel, which is a function of  $\boldsymbol{\theta}$  and  $\boldsymbol{\phi}$ ,  $O_{\alpha, i} \in \mathbf{O}$  is the number of events of the sample at the  $i$ th bin of  $\alpha$  flavor CC DIS channel,  $\text{Pois}(x | \mu)$  is a Poisson distribution of  $x$  for mean  $\mu$ , and  $\rho(\boldsymbol{\varphi}' | \boldsymbol{\phi})$  is the probability density function of point estimates of  $\boldsymbol{\phi}$ , which determines the systematic uncertainty of SHiP. For the test statistic, the profile likelihood ratio [85,86] is chosen, which is defined as follows:

$$\lambda(\mathbf{O}, \boldsymbol{\varphi}, \boldsymbol{\theta}) \equiv -2 \log \frac{\max_{\boldsymbol{\phi}} \rho(\mathbf{O}, \boldsymbol{\varphi} | \boldsymbol{\theta}, \boldsymbol{\phi})}{\max_{\boldsymbol{\theta}, \boldsymbol{\phi}} \rho(\mathbf{O}, \boldsymbol{\varphi} | \boldsymbol{\theta}, \boldsymbol{\phi})}. \quad (10)$$

However, to perform repeated sampling, one must choose specific values of  $\boldsymbol{\theta}$  and  $\boldsymbol{\phi}$  while the definition of the p-value does not specify  $\boldsymbol{\phi}$ . Several variations of the FC method, such as the Berger-Boos method [87] or the Highland-Cousins method [88] are proposed to solve the issue. In this paper, we use the profiled FC method, which selects the value  $\boldsymbol{\phi}$  that maximizes  $\rho(\mathbf{D}, \boldsymbol{\varphi} | \boldsymbol{\theta}, \boldsymbol{\phi})$  during repeated sampling. [81,89].

## B. Estimation of SHiP data

In this Subsection, we estimate the number of CC DIS events at SHiP and the probability distribution of SHiP samples. If event classification were perfect,  $\mu_{\alpha, i}$  would equal the mean number of actual CC DIS events with the corresponding flavor. However, due to imperfections in event classification, some CC DIS events could be excluded by the criteria of the corresponding channel or even classified as a different flavor, and events from other sources could be classified as CC DIS events. Therefore, only a portion of actual CC DIS events with corresponding flavor contributes as a signal, while contributions from other sources constitute the background, which can be written as follows:

$$\mu_{\alpha, i} \equiv s_{\alpha, i} + b_{\alpha, i}, \quad (11)$$

where  $s_{\alpha, i}$  and  $b_{\alpha, i}$  are the signal and the background at the  $i$ th bin of the  $\alpha$  flavor CC DIS channel. Here, we estimate the signal  $s_{\alpha, i}$  using the formula below:

$$s_{\alpha, i} = \sum_{\beta}^{e, \mu, \tau} (1 + \phi_{\beta} + \phi_{\beta, i}) \int_{E_{\text{start}, i}}^{E_{\text{end}, i}} dE_{\nu} \int_{L_0}^{L_0 + L_d} dL \\ \times \int_0^{\infty} dE_{\text{rec}} \epsilon_{\alpha} P_{\beta\alpha} \frac{d^2 N_{\beta}}{dE_{\nu} dL} \frac{\sigma_{\nu\alpha A}}{\sigma_{\nu\beta A}} \rho(E_{\text{rec}} | E_{\nu}), \quad (12)$$

where parameters are defined as:

- (i)  $\phi_{\beta} \in \boldsymbol{\phi}$  and  $\phi_{\beta, i} \in \boldsymbol{\phi}$  are overall and shape nuisance parameters for the neutrino flux with flavor  $\beta$ ,
- (ii)  $N_{\beta}$  is the number of neutrinos with flavor  $\beta$  that pass through a SND under the three-flavor hypothesis,
- (iii) each  $\epsilon_{\alpha}$  and  $\sigma_{\nu\alpha A}$  are a detection efficiency and the scattering cross section of neutrino CC DIS events with flavor  $\alpha$ ,
- (iv)  $L_0$  is a distance between the proton target and the detector,
- (v)  $L_d$  is a length of the detector,
- (vi)  $E_{\text{rec}}$  is a reconstructed energy of the neutrino that is estimated by the data from a SND,
- (vii)  $[E_{\text{start}, i}, E_{\text{end}, i}]$  is the range of reconstructed energy for  $i$ th bin, and
- (viii) and  $\rho(E_{\text{rec}} | E_{\nu})$  is a probability density of  $E_{\text{rec}}$  for the given  $E_{\nu}$ , which is chosen as a Gaussian distribution with the standard deviation  $\sigma = 0.2E_{\nu}$  following the analysis of Ref. [90].

In this study, we calculate the expected event rates for a total exposure of  $2 \times 10^{20}$  protons on target (POT), corresponding to a five-year operation of SHiP. Also, we use a total of seven energy bins with a logarithmic scale according to the reconstructed energy. Each energy bin starts at the endpoint of the previous energy bin and ends at 1.5 times its starting energy, with the first energy bin beginning at 10 GeV. Additionally, to ensure that  $s_{\alpha, i}$  remains positive, the domain of each nuisance parameter is chosen as  $(-0.5, \infty)$ , and the probability density function of the point estimate of nuisance parameters is chosen as

$$\rho(\boldsymbol{\varphi} | \boldsymbol{\phi}) = \prod_j \mathcal{N}_{(-0.5, \infty)}(\varphi_j | \phi_j, \sigma_{\text{sys}, j}), \quad (13)$$

where  $\mathcal{N}_{(a, b)}(x | \mu, \sigma)$  is a truncated normal distribution with the mean value  $\mu$  and the standard deviation  $\sigma$  on the domain of  $(a, b)$ , and  $\varphi_j$  and  $\phi_j$  are  $j$ th components of  $\boldsymbol{\varphi}$  and  $\boldsymbol{\phi}$ . In general cases, covariance terms between different nuisance parameters are included in systematic uncertainties. In this study, to simplify the situation, we ignore any covariance terms inside  $\rho(\boldsymbol{\varphi} | \boldsymbol{\phi})$  and  $\sigma_{\text{sys}, j}$  are chosen to be equal to the normalized systematic uncertainty  $\sigma_{\text{norm}}$ , as the biggest contribution of the uncertainty comes from the neutrino flux. The current uncertainty of the tau neutrino flux that is generated from the proton target is  $\sim 40\%$  [91], while DsTau could improve the uncertainty around 10%. [92] We perform our sensitivity analysis using two benchmark values for the normalized systematic uncertainty:  $\sigma_{\text{norm}} = 10\%$  and  $\sigma_{\text{norm}} = 20\%$ , where 20% value represents a conservative scenario, which allows for residual uncertainties or contributions from systematic sources that are not yet fully constrained. In contrast, the 10% value represents an optimistic scenario that assumes the flux uncertainty is fully reduced by the DsTau and remains the dominant source of error.

The estimated number of neutrino CC DIS events is given by repeated sampling of SHiP, while background and detection efficiencies are estimated by testing particle identification methods on MC samples from repeated sampling. Instead of rigorous repetitive sampling of SHiP, we import the number of CC DIS events from the recent report of SHiP [77] assuming that the differential number of events is constant with respect to the distance. To estimate the background and detection efficiencies of CC DIS events, we use results from OPERA [39,82] for electron and muon neutrino channels, as the detector technologies of OPERA and the SND at SHiP are similar. For the  $\nu_e$  channel, the detection efficiency is based on Ref. [82], which ranges from 25% to 40% depending on the neutrino energy. For the  $\nu_\mu$  channel, Ref. [39] reports an average efficiency of 46%. For the  $\nu_\tau$  channel, on the other hand, we use the analysis of Ref. [83] with the criterion  $\phi_{\tau-h} \geq 2.6$  rad where  $\phi_{\tau-h}$  is the angle between the  $\tau$  candidate track and the hadronic shower direction in the plane perpendicular to the neutrino beam. The detection efficiencies are therefore chosen as follows:

$$\begin{aligned}\varepsilon_e &= 30\%, \\ \varepsilon_\mu &= 40\%, \\ \varepsilon_\tau &= 10\%,\end{aligned}\tag{14}$$

where we choose conservative values compared to reported values for electron and muon, to compensate with any energy dependencies of them.

Regarding background contributions, we assume electron and muon neutrino CC DIS channels contain negligible background as OPERA achieved purities of  $\sim 96\%$  and  $99.5\%$  [39], respectively. For the tau neutrino channel, the source of background is dominated by muon neutrino CC events that generate charm mesons while outgoing muons are unidentified. In this study, we choose a background of the tau neutrino CC DIS channel with an energy-independent signal-to-background ratio  $R_{s/b} = 2$  for a three-flavor model that is consistent with the analysis of Ref. [83] for the criterion  $\phi_{\tau-h} \geq 2.6$  rad. Therefore,  $b_{\tau,i}$  can be written as

$$\begin{aligned}b_{\tau,i} &= (1 + \phi_\mu + \phi_{\mu,i}) \int_{E_{\text{start},i}}^{E_{\text{end},i}} dE_\nu \int_{L_0}^{L_0+L_d} dL \\ &\times \int_0^\infty dE_{\text{rec}} R_{s/b}^{-1} \varepsilon_\tau P_{\mu\mu} \frac{d^2 N_\tau}{dE_\nu dL} \rho(E_{\text{rec}}|E_\nu),\end{aligned}\tag{15}$$

where we ignore any background contribution from muon neutrino appearance as they are negligible compared with the signal contribution from tau neutrino appearance. Note that detection efficiencies and purities of each channel can be enhanced by recent advancements in computing power

and machine learning [93], which is not considered in this paper.

To calculate the contribution of neutrino appearance, we consider the ratio of neutrino CC DIS cross section between two flavors, following Ref. [91]. The ratio of neutrino CC DIS cross section remains the same for antineutrinos, as the ratio depends on the mass of the lepton and the energy of the incoming neutrino. We also ignore the difference between  $\sigma_{\nu_e A}$  and  $\sigma_{\nu_\mu A}$  since the masses of the electron and muon are negligible in our energy regime.

Finally, gathering more data from FSND would increase the sensitivity of the 3 + 1 model due to its longer baseline compared to NSND. For simplicity, we assume that the signal and the background of neutrino CC DIS events at FSND follow the same formula as those at NSND, using the same nuisance parameters, while the signal and the background at FSND are multiplied by the overall ratio  $R_{F/N}$  to reflect scaling due to the detector configuration. Also, we choose  $L_0 = 27$  m and  $L_d = 2.6$  m for NSND, and  $L_0 = 120$  m and  $L_d = 2.6$  m are chosen for FSND. In reality, uncertainties due to the properties of detectors reduce the correlation between NSND and FSND, which may contribute to a reduction in the sensitivity compared to our estimation.

Figure 2 shows histograms of the expected number of CC DIS events per energy bin after a five-year operation for each electron (left), muon (center), and tau (right) flavor channel in NSND (up) and FSND with  $R_{F/N} = 100\%$  (down). Colored regions show contributions of  $e$  (red),  $\mu$  (green), and  $\tau$  (blue) flavor neutrino flux on CC DIS event channels under the 3 + 1 model with  $\Delta m_{41}^2 = 1000$  eV<sup>2</sup> and  $|U_{e4}|^2 = |U_{\mu4}|^2 = |U_{\tau4}|^2 = 0.1$ . For example, colored regions of the top left plot show the expected number of electron neutrino CC DIS events, where each red, green, and blue part indicates the contribution of each electron, muon, and tau neutrino that are oscillated into electron neutrino via the 3 + 1 model. Grey regions in  $\tau$  flavor channels indicate background contributions under the same  $\Delta m_{41}^2$  and mixing parameters, assuming that all backgrounds consist of muon neutrinos. Expected numbers of events are derived by Eqs. (12) and (15), while every nuisance parameter is assumed to be zero. For comparison, we also draw expected numbers of events and background for the three-flavor model with null nuisance parameters as black solid lines and black dashed lines, which will be used as a data to find a sensitivity. For electron and muon flavor channels, backgrounds are assumed as zero as they were negligible in OPERA. The other FSND scenarios considered in this analysis, with  $R_{F/N} = 10\%$  and  $0\%$ , correspond to a simple scaling of bottom-row plots by factors of 0.1 and 0, respectively.

Notably, neutrino disappearance on tau flavor is compensated by the appearance from other flavors, which indicates that in specific mixing parameters, there could

$$\Delta m_{41}^2 = 1000 \text{ eV}^2, |U_{e4}|^2 = |U_{\mu4}|^2 = |U_{\tau4}|^2 = 0.1$$

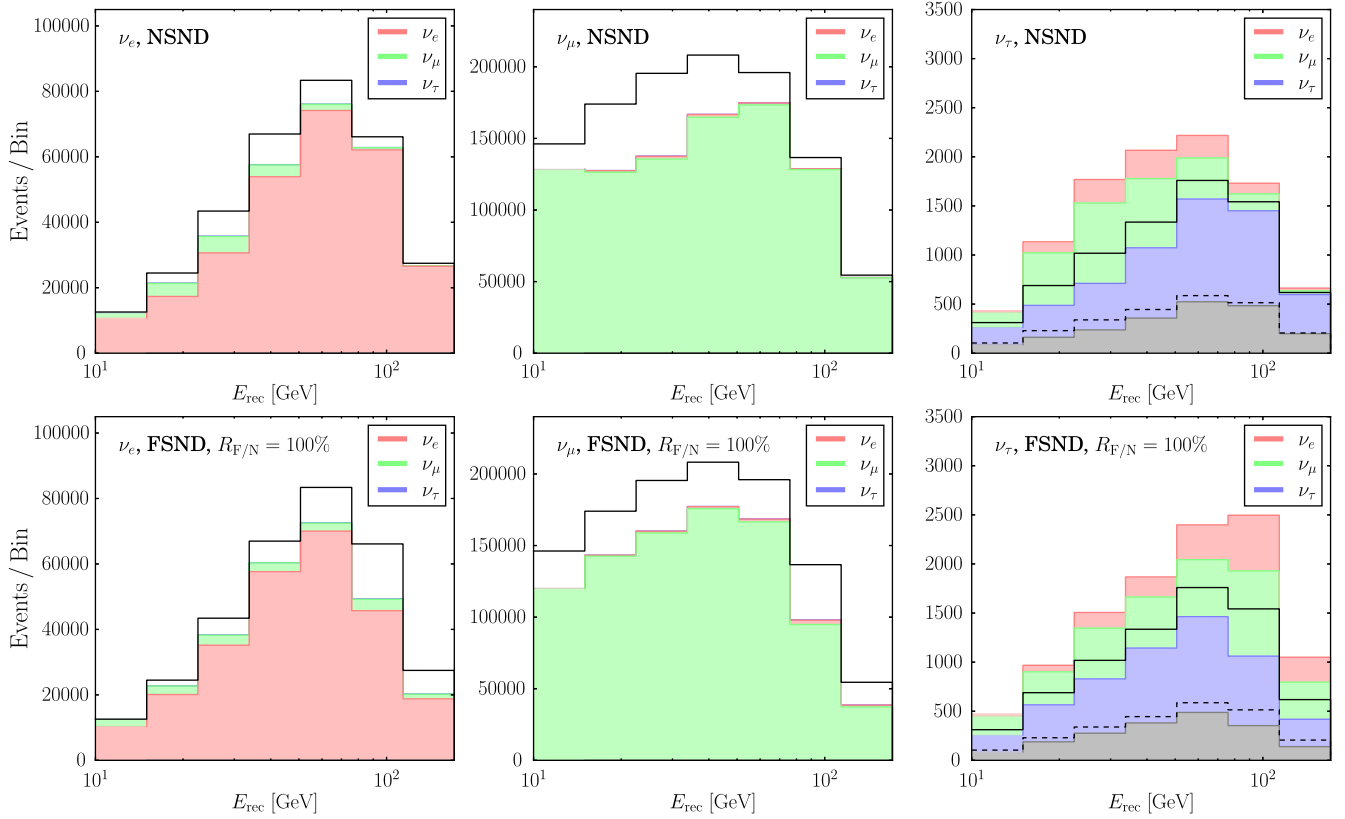


FIG. 2. Histograms showing the expected number of CC DIS events per energy bin after a five-year operation (corresponding to  $2 \times 10^{20}$  POT), for each electron (left), muon (center), and tau (right) flavor channel in NSND (up) and FSND with  $R_{F/N} = 100\%$  (down), assuming null nuisance parameters. Colored regions correspond to the contributions of  $e$  (red),  $\mu$  (green), and  $\tau$  (blue) flavor neutrino flux under the  $3 + 1$  model with  $\Delta m_{41}^2 = 1000 \text{ eV}^2$  and  $|U_{e4}|^2 = |U_{\mu4}|^2 = |U_{\tau4}|^2 = 0.1$ , following Eq. (12). Gray regions in the  $\tau$  flavor channels indicate background contributions under the same  $\Delta m_{41}^2$  and mixing parameters, following Eq. (15). For comparison, solid and dashed black lines represent the corresponding expected number of events and background under the standard three-flavor model. For electron and muon flavor channels, backgrounds are assumed as zero as they were negligible in OPERA.

be a cancelation between the appearance and the disappearance if sterile neutrinos are mixed with two or more active flavors. We will examine the impact of this cancelation effect in detail in the next Section.

#### IV. ESTIMATED SENSITIVITIES OF SHIP ON THE $3 + 1$ MODEL

In this Section, we present the estimated sensitivities of SHiP to the  $3 + 1$  model under various combinations of the normalized systematic uncertainty and the FSND contribution. To evaluate the sensitivity, we compute 90% CL confidence regions in the parameter spaces, assuming that the observed number of CC DIS events coincides with the mean expectation under the three-flavor model with zero nuisance parameters. First, we consider scenarios in which only a single flavor is mixed with sterile neutrinos, so that only disappearance can be observed. In these cases, the sensitivities are presented on the  $(\Delta m_{41}^2, |U_{\alpha 4}|^2)$  plane for each active flavor  $\alpha$ . Next, we consider scenarios in

which two flavors are mixed with sterile neutrinos, and the corresponding sensitivities are shown on the  $(|U_{\alpha 4}|^2, |U_{\beta 4}|^2)$  plane at fixed values of  $\Delta m_{41}^2$  with reference values of 50  $\text{eV}^2$ , 250  $\text{eV}^2$ , 1000  $\text{eV}^2$ , and 5000  $\text{eV}^2$ . Note that each 250  $\text{eV}^2$  and 1000  $\text{eV}^2$  are the optimal choice of  $\Delta m_{41}^2$  for NSND and FSND, while cases with 50  $\text{eV}^2$  and 5000  $\text{eV}^2$  show how the contours evolve away from optimal points.

Every figure in this Section is drawn following the rules below. First, a Fisher  $z$  transformation is applied to the p-value to reduce interpolation errors. Different settings for the normalized systematic uncertainty are distinguished by line styles: cases with  $\sigma_{\text{norm}} = 10\%$  are depicted with dashed lines, while those with  $\sigma_{\text{norm}} = 20\%$  are shown with solid lines. For the FSND contributions, NSND-only scenarios (i.e.,  $R_{F/N} = 0\%$ , corresponding to the current design of SHiP) are represented by blue lines, cases with  $R_{F/N} = 10\%$  are drawn in purple, and those with  $R_{F/N} = 100\%$  are drawn in red. Also, for comparison, we include

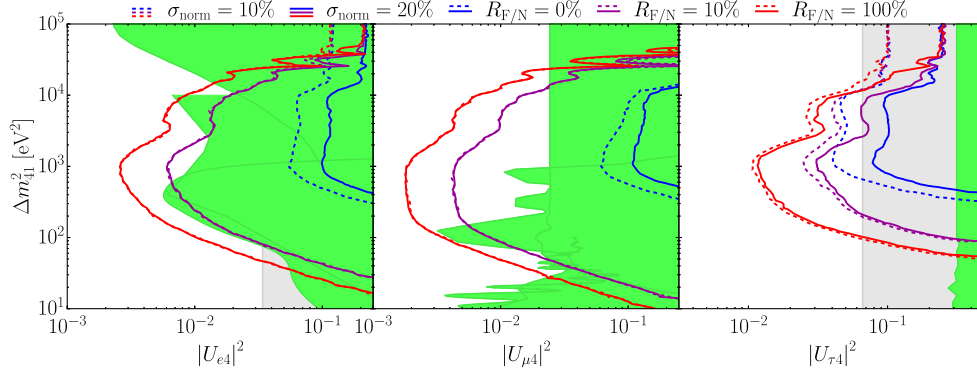


FIG. 3. Expected sensitivities (90% CL) of SHiP after a five-year operation (corresponding to  $2 \times 10^{20}$  POT) in the  $(|U_{\alpha 4}|^2, \Delta m_{41}^2)$  plane, assuming zero mixing with flavors other than  $\alpha$ . Sensitivities are shown for  $\sigma_{\text{norm}} = 10\%$  (dashed) and  $\sigma_{\text{norm}} = 20\%$  (solid), while  $R_{F/N}$  is varied as 0% (blue, NSND only), 10% (purple), and 100% (red). Green regions are disfavored by past experiments: for electron flavor, constraints are from tritium beta decay experiments [52,54,55]; for muon flavor, constraints are from CCFR [62], MiniBooNE + SciBooNE [69], and MINOS [37]; and for tau flavor, the only constraint is from NOvA [36]. Grey regions indicate constraints from different parameter spaces: the limit on  $|U_{e4}|^2$  is derived from Super-Kamiokande [34], while the limit on  $|U_{\tau 4}|^2$  is derived from IceCube-DeepCore [41].

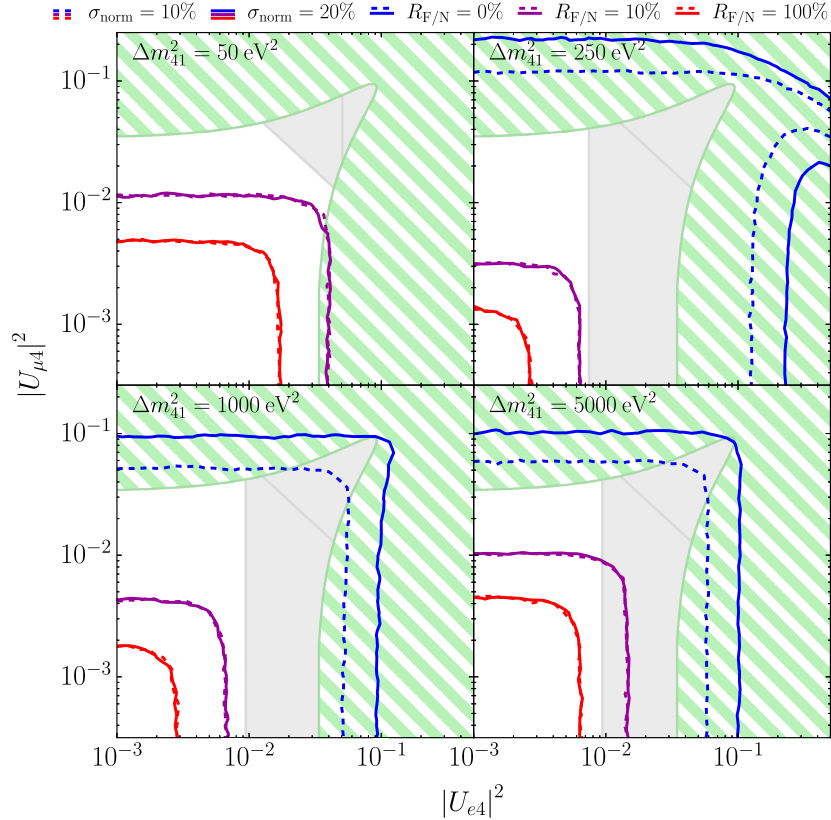


FIG. 4. Expected sensitivities (90% CL) of SHiP after a five-year operation in the  $(|U_{e4}|^2, |U_{\mu 4}|^2)$  plane for fixed  $\Delta m_{41}^2 = 50, 250, 1000, 5000$  eV<sup>2</sup>, assuming  $|U_{\tau 4}|^2 = 0$ . The same line styles and color conventions as in Fig. 3 are used, while the constraint from Super-Kamiokande [34] is drawn in light green to indicate the additional assumption  $\delta_{14} = \delta_{24} = 0$ . Each grey region shows the constraints from different parameter spaces: regions with vertical boundaries show the limit on  $|U_{e4}|^2$  from tritium beta decay experiments [52,54,55], and regions with diagonal boundaries show the limit on  $\sin^2 2\theta_{e\mu}$  from MicroBooNE [70].

constraints from past experiments that are consistent with the three-flavor model. We do not include results that reported a preference for the  $3 + 1$  model, such as IceCube [19] and ANTARES [94]. Constraints on the same parameter space are shown as green regions, or as light green hatched regions if constraints are given with additional assumptions. Because confidence regions in different parameter spaces cannot be directly compared, we show constraints from different parameter spaces as grey regions, where we include such regions only if the limit holds regardless of the values of the other  $3 + 1$  model parameters. Finally, every constraint from past experiments has 90% CL, except tritium beta decay experiments, which have 95% CL.

### A. Single flavor mixing with sterile neutrino

Figure 3 shows the expected sensitivities (90% CL) of the SHiP experiment in the  $(|U_{\alpha 4}|^2, \Delta m_{41}^2)$  plane after a five-year operation, assuming zero mixing with flavors other than  $\alpha$ . Sensitivities are shown for  $\sigma_{\text{norm}} = 10\%$  (dashed) and  $\sigma_{\text{norm}} = 20\%$  (solid), while  $R_{F/N}$  is varied as 0% (blue, NSND only), 10% (purple), and 100% (red). Green regions are disfavored by past experiments: for electron flavor, constraints are from tritium beta decay experiments [52,54,55]; for muon flavor, constraints are

from CCFR [62], MiniBooNE + SciBooNE [69], and MINOS [37]; and for tau flavor, the only constraint is from NOvA [36]. Constraints from MINOS [37] and NOvA [36] are extended to larger  $\Delta m_{41}^2$  since they provide constraints at sufficiently large  $\Delta m_{41}^2$  where oscillations are averaged on both near and far detectors. Grey regions indicate the constraints from different parameter spaces: the limit on  $|U_{e4}|^2$  is derived from Super-Kamiokande [34] in the  $(|U_{e4}|^2, |U_{\mu 4}|^2)$  plane assuming  $\delta_{24} = 0$ , while the limit on  $|U_{\tau 4}|^2$  is derived from IceCube-DeepCore [41] in the  $(|U_{\mu 4}|^2, |U_{\tau 4}|^2)$  plane.

In the NSND-only case, each flavor probes roughly the  $|U_{\alpha 4}|^2 \gtrsim 0.1$  region near  $\Delta m_{41}^2 \sim 10^3 \text{ eV}^2$ , and reducing  $\sigma_{\text{norm}}$  from 20% to 10% improves the sensitivity by approximately a factor of 2. Also, the sensitivities are similar across all flavors in this case because the statistical uncertainty is negligible compared with the systematic uncertainty. The sensitivity presented here is stronger than that in our previous study [78] since a parametric bootstrap for nuisance parameters was not applied previously.

When including FSND, the sensitivity to neutrino disappearance is unchanged with respect to  $\sigma_{\text{norm}}$  for  $\Delta m_{41}^2 \lesssim 10^4 \text{ eV}^2$ . In regions where oscillations are averaged (i.e.,  $\Delta m_{41}^2 \gtrsim 10^4 \text{ eV}^2$ ), the sensitivity depends solely on  $\sigma_{\text{norm}}$ . For scenarios with  $R_{F/N} = 10\%$ , the sensitivities

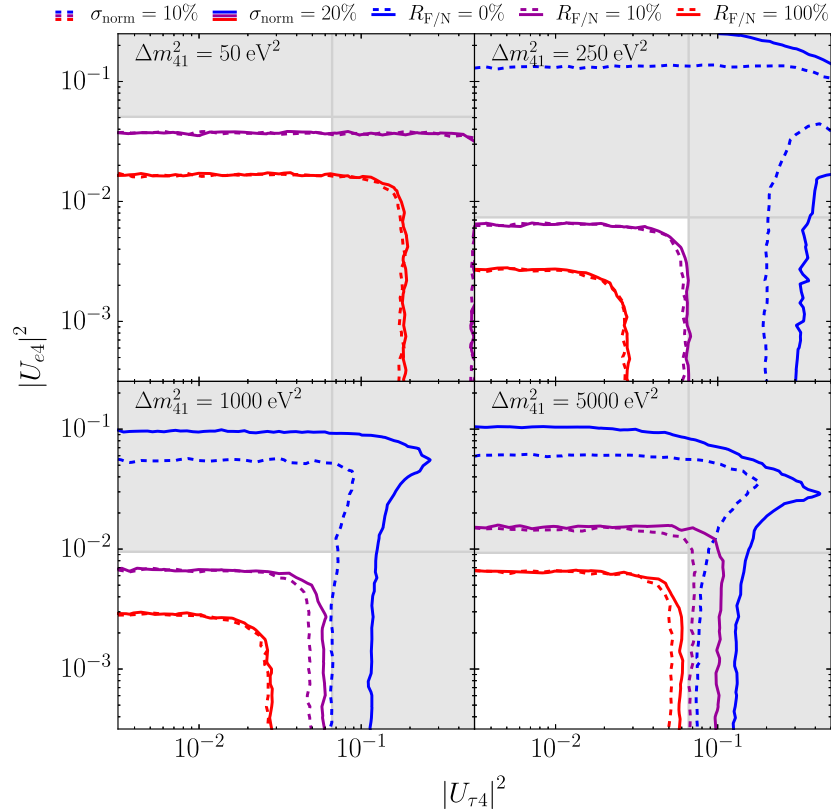


FIG. 5. Same as Fig. 4, but in the  $(|U_{\tau 4}|^2, |U_{e4}|^2)$  plane assuming  $|U_{\mu 4}|^2 = 0$ . The limit on  $|U_{e4}|^2$  is from tritium beta decay experiments [52,54,55] and Super-Kamiokande [34], the limit on  $|U_{\tau 4}|^2$  is from IceCube-DeepCore [41].

to the electron and muon neutrino mixing parameters are enhanced by roughly a factor of 10 near  $\Delta m_{41}^2 = 10^3 \text{ eV}^2$  compared to NSND-only cases, while for the tau flavor the improvement is by a factor of 2 to 3, depending on  $\sigma_{\text{norm}}$ . Increasing  $R_{F/N}$  from 10% to 100% further enhances the sensitivity by approximately a factor of 2.3 in the relevant regions.

Note that sensitivities of cases with FSND are independent of  $\sigma_{\text{norm}}$  if neutrino oscillation is not averaged, which indicates that the cross-check with multiple baselines could distinguish the oscillation effect from the fluctuations due to systematic uncertainty. If the neutrino oscillation is averaged in both near and far detectors, however, sensitivity is only dependent on  $\sigma_{\text{norm}}$ . This is because under our assumption on systematic uncertainty, neutrino oscillations and nuisance parameters cannot be measured independently in these cases so that there is no more merit on FSND. Also, Wilks's theorem with two degrees of freedom is satisfied on the nonaveraged regions of electron and tau flavor mixing cases, while for muon flavor mixing cases, the number of effective degrees of freedom is approximately 4.

Comparing with other constraints, the NSND-only sensitivities are generally not competitive, except for the tau flavor at  $\sigma_{\text{norm}} = 10\%$ , where the sensitivity exceeds the limit from IceCube-DeepCore [41]. In contrast, scenarios

with  $R_{F/N} \geq 10\%$  can probe regions beyond the reach of past experiments. Near  $\Delta m_{41}^2 \sim 10^3 \text{ eV}^2$ , the sensitivities with  $R_{F/N} = 100\%$  are up to approximately ten times stronger for the muon flavor and seven times stronger for the tau flavor than the existing constraints, while for the electron flavor most of the parameter space accessible with  $R_{F/N} = 10\%$  is already constrained by beta decay experiments. Even with  $R_{F/N} = 100\%$ , the sensitivity of SHiP is only about three times stronger than the constraints from tritium beta decay experiments.

## B. Two-flavor mixing with sterile neutrino

Figures 4–6 show the expected sensitivities after a five-year operation in the  $(|U_{\alpha 4}|^2, |U_{\beta 4}|^2)$  plane for fixed  $\Delta m_{41}^2 = 50, 250, 1000, 5000 \text{ eV}^2$ . As before, sensitivities are drawn using dashed (for  $\sigma_{\text{norm}} = 10\%$ ) or solid (for  $\sigma_{\text{norm}} = 20\%$ ) lines with the color coding according to  $R_{F/N}$ . We draw the constraints from IceCube-DeepCore [41] on the  $(|U_{\tau 4}|^2, |U_{\mu 4}|^2)$  plane as a green, and constraints from Super-Kamiokande [34] on  $(|U_{e 4}|^2, |U_{\mu 4}|^2)$  and  $(|U_{\tau 4}|^2, |U_{\mu 4}|^2)$  planes are drawn as a light green hatched region to indicate the additional assumption  $\delta_{14} = \delta_{24} = 0$ . Limits on  $|U_{e 4}|^2$  at Figs. 4 and 5 are from tritium beta decay experiments [52,54,55], and the limit on  $|U_{\tau 4}|^2$  at

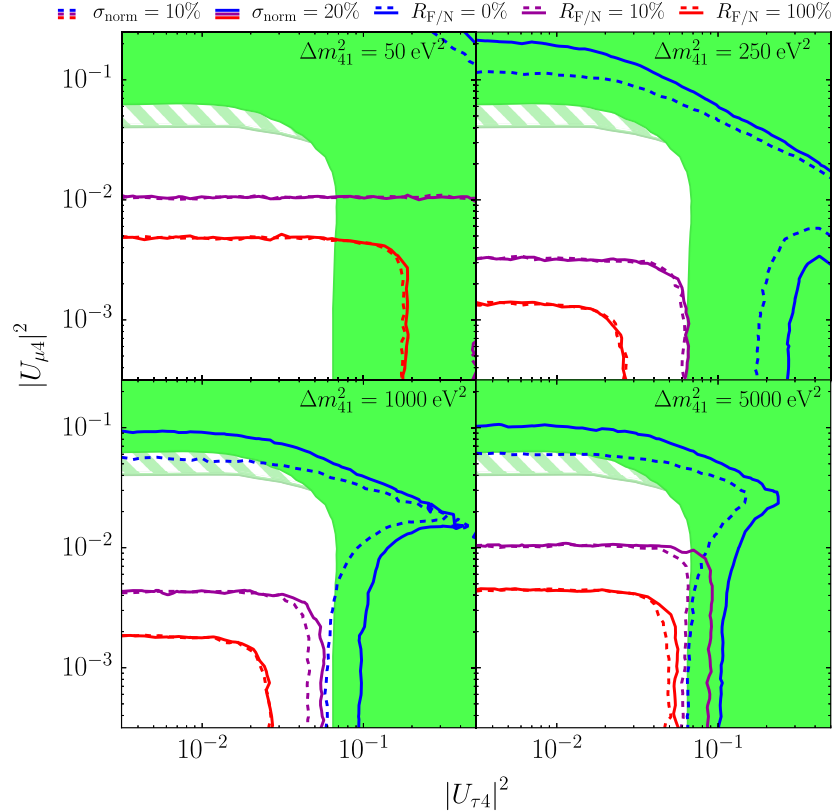


FIG. 6. Same as Fig. 4, but in the  $(|U_{\tau 4}|^2, |U_{\mu 4}|^2)$  plane assuming  $|U_{e 4}|^2 = 0$ . IceCube-DeepCore [41] is shown in green, while the constraint from Super-Kamiokande [34] is drawn as a light green hatched region to indicate the additional assumption  $\delta_{24} = 0$ .

Fig. 5 comes from IceCube-DeepCore [41] where the limit is unchanged on varying  $|U_{e4}|^2$  as the contribution of the electron neutrino CC event is negligible. Finally, the neutrino appearance limit from MicroBooNE [70] is shown as diagonal grey regions in Fig. 4, while other neutrino appearance limits are not shown because they could be affected by cancelation effects from neutrino disappearance.

A distinct feature of NSND-only cases is that kinks or discontinuities are shown on the shape of sensitivities, due to a cancelation between appearance and disappearance as we mentioned in Fig. 2. Although reducing  $\sigma_{\text{norm}}$  from 20% to 10% improves the sensitivity by approximately a factor of 2, discontinuities and kinks remains.

With FSND, the kinks or discontinuities in the sensitivities disappear, which suggests that cross-checks with multiple baselines could distinguish between the cancelation in the  $3 + 1$  model and a null result under the three-flavor model. The enhancement factors due to FSND are similar to those in the single-flavor cases on Fig. 3, with approximately a factor of 10 improvement for  $|U_{e4}|^2$  and  $|U_{\mu4}|^2$ , and a factor of 2 improvement for  $|U_{\tau4}|^2$  at  $\Delta m_{41}^2 = 1000 \text{ eV}^2$ . Increasing  $R_{F/N}$  from 10% to 100% further enhances the sensitivities by roughly 2.3 times. Finally, under our assumption on systematic uncertainty, the sensitivities do not depend on  $\sigma_{\text{norm}}$ , which we already saw in Fig. 3.

When compared with existing constraints, only a small portion of the unprobed region on the  $(|U_{\tau4}|^2, |U_{\mu4}|^2)$  plane can be constrained in the NSND-only case with  $\sigma_{\text{norm}} = 10\%$  at  $\Delta m_{41}^2 = 1000 \text{ eV}^2$ , whereas the single-flavor scenario in Fig. 3 probes the  $|U_{\tau4}|^2 \gtrsim 4 \times 10^{-2}$  region under the same conditions. In contrast, the FSND-enhanced sensitivities can reach  $|U_{\alpha4}|^2 \lesssim 10^{-2}$  for the electron and muon flavors, probing parameter space that is unprobed by past experiments.

## V. CONCLUSION

In this paper, we have investigated the potential of SHiP as a probe of the  $3 + 1$  model by estimating sensitivities on various perspectives. To find sensitivities, we used the FC method and a parametric bootstrap for nuisance parameters through the probability of their point estimates. As a follow-up study of Ref. [78], we also evaluate the effect of FSND in full three-flavor mixing.

Our results show that on NSND-only cases, SHiP could probe  $|U_{\alpha4}|^2 \gtrsim 0.1$  regions near  $\Delta m_{41}^2 \sim 10^3 \text{ eV}^2$  if  $\sigma_{\text{norm}} = 20\%$ . In scenarios that sterile neutrinos are mixed with two active flavors, kinks and discontinuities of sensitivities are generated due to a cancelation between appearance and disappearance. Reducing  $\sigma_{\text{norm}}$  from 20% to 10% improves the sensitivity by approximately a factor of 2 while reducing  $\sigma_{\text{norm}}$  could not remove any kinks and discontinuities in two flavor mixing scenarios. The inclusion of FSND further enhances the sensitivities, with approximately a factor of 10 for electron and muon flavors, with a factor of 2 to 3 for tau flavors depending on  $\sigma_{\text{norm}}$ . Also, increasing  $R_{F/N}$  from 10% to 100% further enhances the sensitivity by a factor of 2.3 in all flavors in regions where neutrino oscillations are not averaged.

Compared with existing constraints, the NSND-only configuration does not exceed previous bounds, except for tau flavor mixing when  $\sigma_{\text{norm}} = 10\%$ . However, the inclusion of FSND significantly improves the sensitivity. Near  $\Delta m_{41}^2 \sim 10^3 \text{ eV}^2$ , with  $R_{F/N} = 100\%$ , the sensitivity for the muon flavor mixing parameter is a factor of 10 stronger than the current limit, and for the tau flavor, the sensitivity is roughly a factor of 7 stronger. For the electron flavor, however, the SHiP sensitivity only surpasses the tritium beta decay constraints by a factor of 3.

Taken together, we show that the multibaseline approach gives constraints that are independent of nuisance parameters under the ideal condition, even though sensitivities of NSND can be changed depending on the uncertainty of nuisance parameters. A cancelation between appearance and disappearance is also ineffective under the dual baseline scenarios as we have seen in Sec. IV B. Such benefits could help probing sterile neutrinos with more complicated models.

## ACKNOWLEDGMENTS

The authors were supported by the National Research Foundation of Korea (NRF) grant funded by the Korean government (MEST) Grants No. 2021R1A2C2011003, No. RS-2022-NR069214, and No. RS-2025-00555834.

## DATA AVAILABILITY

The data that support the findings of this article are openly available [95].

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