

Rescuing 331 bileptons from the Landau pole

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Among the particles being searched for at the LHC beyond the Standard Model are bileptons, which are doubly charged gauge bosons. Bileptons are predicted by several Standard Model extensions, including the so-called 331 models with $\beta = \sqrt{3}$. The minimal formulation of these models is generally plagued by a low-energy Landau pole, which can undermine current predictions for TeV-scale bileptons. We analyze this issue and investigate the possibility to shift the Landau pole at higher energy scales by extending the troublesome minimal 331 models.

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I. INTRODUCTION

There are several well-known reasons for extending the Standard Model (SM) $SU(3)_c \times SU(2)_L \times U_Y(1)$ group. An interesting possibility is to enlarge the gauge group $SU_L(2)$ to $SU_L(3)$ or more precisely $SU(2)_L \times U_Y(1)$ to $SU(3)_L \times U_X(1)$, where X is a generalized hypercharge. In [1] it was suggested for the first time to embed the three families of fermions into irreducible representations of the gauge group $SU_L(3) \times U_X(1)$. However, the first complete model based on $SU_L(3) \times U_X(1)$, and hereafter named 331 [the first 3 refers to $SU_c(3)$], has been proposed by [2,3]. Quite a large number of works have followed and giving a complete list of all of them is beyond the scope of the present paper [4].

From the phenomenological point of view, 331 models predict the existence of new fermions and gauge bosons beyond the Standard Model (BSM). The gauge properties of the specific 331 model depend on an arbitrary parameter, denoted as β . In the case $\beta = \sqrt{3}$, one extra gauge boson is neutral and two have charges $Q = \pm 1$; we call them Z' , $W'^{\pm}$, respectively. The remaining two extra gauge bosons are doubly charged and are denoted as $Y^{\pm\pm}$. Since they

couple with two standard leptons they are called bileptons. From the perspective of collider physics, the presence of such a bilepton is one of the most interesting features of these 331 models. The existence of bileptons has been actively investigated at LHC, bringing about lower bounds on their masses [5,6]. Several phenomenological studies have explored their possible signatures at LHC (e.g., [7,8]). The downside of 331 models with $\beta = \sqrt{3}$ is the well-known presence of a Landau pole at low-energy scales. Already in the 1950s Landau realized that the QED running coupling diverges—developing a pole—at a finite momentum scale. The presence of such a divergence, known as the Landau pole, is not limited to QED but poses a potential problem for a generic quantum field theory in the perturbative regime. The modern perspective on this issue involves several arguments. One key point is that the Landau pole arises within perturbation theory, which cannot be expected to reliably represent behaviors related to large or even infinite couplings. Real insight can only be gained using nonperturbative approaches. It is not unreasonable to assume that physical observables of a quantum field theory are well behaved even when the coupling diverges at the Landau pole and beyond. This possibility has been explored and several models have been identified where this behavior is realized (see, e.g., [9] and references therein). Another perspective emerges when we examine the energy scales of the Landau pole. In the Standard Model the Landau pole is above the Planck scale, thus it becomes of no practical significance to study the SM in that regime. This leads to view theories with a Landau pole as UV-incomplete or effective theories. The presence of a Landau pole is taken as an evidence of an energy scale at which the

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theory breaks down and new physics must intervene. This interpretation becomes obviously more concerning when the scale of the Landau pole approaches energies accessible to present or near future experiments. Since the running of the couplings changes according to the matter content of the models, it is necessary to investigate SM extensions for the presence of a Landau pole. In the case of 331 models, the scale of the Landau pole depends on the value of the parameter β . Minimal 331 extensions with $\beta = 1/\sqrt{3}$ exhibit a Landau pole in the $U(1)$ coupling at very high energies (much like the Standard Model), while setting $\beta = \sqrt{3}$ lowers the Landau pole to about 4 TeV. Such a low-energy divergence can be troublesome for bilepton detection in the near future by LHC. It is interesting both from the phenomenological and theoretical point of view to investigate how to build extensions of the minimal 331 model with $\beta = \sqrt{3}$ where the Landau pole occurs at higher energy scales [10–13].

In this paper we address the Landau pole problem considering next-to-minimal 331 extensions.

II. GENERAL FEATURES OF 331 MODELS

In 331 models we extend the Standard Model gauge group to $SU(3)_C \times SU(3)_L \times U(1)_X$. The number of the new gauge bosons is fixed by the number of $SU(3)_L$ generators, namely eight [compared to the three of $SU(2)_L$ in the SM]. Since there are three diagonal generators, the hypercharge Y depends on an arbitrary parameter, denoted as β , which defines the gauge properties of the specific 331 model. The electric charge is defined as

$$\hat{Q} = \hat{T}^3 + \beta \hat{T}^8 + X \mathbb{I}, \quad (1)$$

where X is the quantum number associated with $U(1)_X$ and $\mathbb{I}$ is the identity matrix. This expression generalizes the Gell-Mann Nishijima relation, where the X charge is related to the SM hypercharge Y . We have

$$\frac{Y}{2} = \beta T^8 + X \mathbb{I}. \quad (2)$$

We classify 331 extensions according to the value chosen for β .

One of the five new gauge bosons is neutral, independently of the value of β , and it is generally denoted as Z' . The charges of the remaining extra four gauge bosons depend on β .

Other than extra gauge bosons, 331 extensions predict extra fermions, whose charges are also given by Eq. (1) and depend on β . Unlike the gauge sector, their number depends on the matter assignment, which is model dependent. The smallest irreducible representation of $SU_L(3)$ is the triplet. In analogy with the Standard Model, we associate any left-handed quark and lepton family to a $SU_L(3)$ triplet, or antitriplet, representation. A crucial requirement for 331

extensions is gauge anomaly cancellation. It constrains not only the matter content of the theory, but also sets some criteria on how to assign quark and lepton families into triplets and/or antitriplets of the $SU_L(3)$ group.

Let us name N_q and N_ℓ the number of quark and lepton families, respectively. Indicating with $N_q^{3(\bar{3})}$ and $N_\ell^{3(\bar{3})}$ the number of quark and lepton families in the $\mathbf{3}$ ($\bar{\mathbf{3}}$) representations of $SU_L(3)$, we can write

$$\begin{aligned} N_q &= N_q^3 + N_q^{\bar{3}} \\ N_\ell &= N_\ell^3 + N_\ell^{\bar{3}}. \end{aligned} \quad (3)$$

Some general considerations are in order.

- (i) The charges of the first two components of the triplet (or antitriplet) must match the charges of the corresponding SM fields. This matching fixes uniquely the X values for the SM fermions as functions of β . The nonstandard fields have the same X values of the other members of the respective (anti)triplet, because of gauge invariance. Hence, each quark (or lepton) family in a $\mathbf{3}$ representation shares the same X value and the same happens for each quark (or lepton) family in a $\bar{\mathbf{3}}$ representation. This constraint does not need to apply when we add triplets (or antitriplets) composed solely of left-handed BSM fermions to the matter content. However, by assumption, we choose to retain this constraint. In analogy to the Standard Model, each nonstandard left-handed quark needs a right-handed component to maintain $SU(3)_C$ vector-like couplings. Obviously, that does not apply to leptons. However, for the anomaly cancellation and in analogy with the SM, we assume that each left-handed nonstandard charged lepton has a right-handed counterpart of the same electric charge. Neutral leptons do not participate to anomaly cancellation, therefore we do not need to assume a right-handed counterpart.

Summarizing, on the matter content we make two assumptions: (i) each $SU(3)_L$ charged fermion has a right-handed counterpart, and (ii) all exotic families are replicas of the standard ones.

Under these assumptions, the requirement of anomaly cancellation implies the relations [6],

$$N_F \equiv N_q = N_\ell, \quad (4)$$

$$N_q^3 = \frac{2N_F - N_\ell^3}{3}, \quad (5)$$

where $N_{q(\ell)}$ is the number of quark (lepton) families.

- (ii) Let us observe that the color sector of the theory, namely $SU_C(3)$, does not change with respect to the SM. BSM quarks (also indicated as exotic) have color as the SM ones. A necessary (but not sufficient)

condition for QCD to be asymptotically free, which constrains 331 models' matter content, is

$$n_{\text{quark}} < \frac{33}{2}. \quad (6)$$

In the SM there are three quark families, each family has two components, then $n_{\text{quark}} = 6$, which respects the limit (6). In $SU_L(3)$, since we are assuming that each left-handed quark family is in a triplet (or antitriplet), we have

$$n_{\text{quark}} = 3N_F. \quad (7)$$

Therefore from relation (6) it follows the limit $N_F \leq 5$. We note that, contrary to what is frequently reported in literature, N_F is not necessarily a multiple of the color number and can be $N_F = 3, 4, 5$ [14].

- (iii) An additional, quite common, assumption is that the exotic gauge bosons of the 331 model have integer charge values, in analogy to the SM ones, which is obtained by setting

$$|\beta| = n/\sqrt{3}, \quad (8)$$

where n is an odd integer number. Another constraint for 331 models is obtained by singling out the coupling with the Z boson and by matching with the corresponding Standard Model coupling. This results in a condition on the gauge couplings of the two theories, which remain real if $|\beta| < 1.8$ holds (see [6] for details). Therefore, models with $n > 3$ are excluded and β is constrained to the values $\beta = \pm 1/\sqrt{3}, \pm\sqrt{3}$.

In the case $|\beta| = 1/\sqrt{3}$, the BSM quarks have SM charges $-1/3$ and $2/3$; there are also three BSM neutral bosons (including Z') and charged BSM boson $W'^{\pm}$. In the case $|\beta| = \sqrt{3}$ the extra quarks have non-SM charges ($4/3$ and $5/3$). The charged BSM bosons $W'^{\pm}$ are accompanied by double charged bosons $Y^{\pm\pm}$, which can decay into a pair of same charge SM leptons, giving an interesting signature for LHC [2,3,6,7]. They are indicated as bileptons, because they do not couple at tree level to SM quark pairs. Therefore the case $|\beta| = \sqrt{3}$ presents more exotic features compared to the case $|\beta| = 1/\sqrt{3}$ and seems more distinctive for LHC detection. In the following sections, we will study extensions of the so-called minimal 331 model with $\beta = \sqrt{3}$.

III. MINIMAL 331 MODEL

We consider 331 models where $\beta = \sqrt{3}$. All 331 models are constrained by relations (4) and (5). In the case $N_F = 3$, one possible choice of representations is

$$N_q^3 = 2, \quad N_q^{\bar{3}} = 1, \quad N_\ell^3 = 0, \quad N_\ell^{\bar{3}} = 3. \quad (9)$$

The $SU_L(3)$ triplet (or antitriplet) has SM fermions occupying the first two components and a BSM fermion the third one. Since according to (9) the lepton multiplets transform as $\bar{3}$ under $SU_L(3)$, they take the general form $(\ell_L, -\nu_L^\ell, \xi_L^\ell)$, where the index ℓ runs on the three families and ξ_L^ℓ is a left-handed BSM lepton. We note that each ξ_L^ℓ has the same electric charge of ℓ_R^c (c stands for conjugate), namely $Q(\xi_L^\ell) = +1$, as well as the same X value, and it belongs to the same 331 representation. The economical identification $\xi_L^\ell \equiv \ell_R^c$ is the assumption which characterizes the so-called *minimal 331 model*. Let us observe that we cannot repeat the assumption in the quark sector, since all BSM quark fields have unconventional electric charges ($-4/3$ or $5/3$). The matter content of the minimal 331 model is summarized in Table III. The gauge symmetry of 331 models is spontaneously broken in two steps, to the Standard Model and the electroweak (EW) symmetries, namely,

$$\begin{aligned} SU_L(3) \times U_X(1) &\xrightarrow{\mu_{331}} SU_L(2) \times U_Y(1) \\ SU_L(2) \times U_Y(1) &\xrightarrow{\mu_{EW}} U_{e.m.} \end{aligned} \quad (10)$$

at the 331 and electroweak scales, μ_{331} and μ_{EW} , respectively. Hence, the matter content is completed by three scalar $SU_L(3)$ triplets χ, ρ, η . They represent the minimal set of Higgs bosons necessary to break the symmetry for the different values of β and X charges in 331 models [17]. Their X values in the 331 minimal model are reported in Table III.

The ‘‘heavy’’ scalar triplet χ acquires a vacuum expectation value (VEV) $\langle \chi \rangle$, namely,

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ \frac{u}{\sqrt{2}} \end{pmatrix}. \quad (11)$$

It spontaneously breaks the 331 symmetry group down to the SM one at the scale $\mu_{331} \sim u$,

$$SU_L(3) \times U_X(1) \xrightarrow{\mu_{331}} SU_L(2) \times U_Y(1), \quad (12)$$

giving masses to the five BSM gauge bosons. Among them, the bileptons $Y^{\pm\pm}$, doubly charged gauge bosons, are a distinctive feature of this model. They are called *bileptons* because they carry lepton number $L = \pm 2$ and couple only to standard lepton pairs, not to standard quarks.

The mass of the bilepton Y depends on both u and the $SU(2)_L$ coupling g_{2L} , since [18]

$$m_Y \simeq u \frac{g_{2L}}{2}. \quad (13)$$

This relation is derived under the assumption that the VEV scale u is much larger than the VEVs responsible of the SM symmetry breaking (see below).

A lower bound $m_Y \sim 1.3$ TeV at 90% CL for the vector bilepton mass has been derived in [6], recasting the ATLAS search for the doubly charged Higgs bosons in [19]. In the analyses [6], the multilepton cross section of scalar bilepton has been rescaled to the vector one. A similar bound has also been obtained through the use of renormalization group equations [20].

By using the value [21] $g_{2L}(M_Z) = \sqrt{4\pi\alpha_{2L}(M_Z)} \simeq 0.65$ and assuming the lower bound $m_Y \gtrsim 1.3$ TeV, one finds

$$\mu_{331} \sim u \gtrsim 3850 \text{ GeV}. \quad (14)$$

All extra quark fields $D^{1,2}, T$ acquire mass through $\langle \chi \rangle$, therefore their masses are defined by u and by the Yukawa couplings. The latter are free parameters with upper bounds dictated by perturbativity and vacuum stability. It is natural to expect the values of these masses to lie around the u scale or below, with an approximate lower bound at the electro-weak scale. The $SU_L(2) \times U_Y(1)$ is spontaneously broken to $U(1)_{\text{e.m.}}$ by the VEVs of the additional two scalar triplets:

$$\langle \rho \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix}, \quad \langle \eta \rangle = \begin{pmatrix} \frac{v'}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix}, \quad (15)$$

where the VEVs are related to the SM one, v_{EW} , by $v^2 + v'^2 = v_{\text{EW}}^2 = (246 \text{ GeV})^2 \equiv \mu_{\text{EW}}^2$. Since the EW symmetry breaking should occur at a lower scale, we expect the hierarchy $u \gg v, v', v_{\text{EW}}$. We summarize the scalar content of the minimal 331 model in Table I. At this passage, three other generators are broken, which leads us to the three massive SM gauge bosons and one massless gauge boson (the photon).

The scalar sector includes singly and double charged states as well as pseudoscalar and scalar neutral states [18,22–27]. The arbitrariness of the Higgs potential constants adds further complications to its assessment.

A. The Landau pole in the minimal 331 model

The gauge coupling g_i associated to the gauge groups $U(1), SU(2), SU(3)$, or equivalently the couplings $\alpha_i \equiv g_i^2/4\pi$, are not constant and their values depend on the

energy scale μ . In perturbation theory, the expression for the running couplings $\alpha_i(\mu)$ given by the renormalization group in the so-called leading logarithm approximation is

$$\alpha_i(\mu) = \frac{\alpha_i(\mu_0)}{1 - \frac{b_i}{2\pi} \alpha_i(\mu_0) \log(\frac{\mu}{\mu_0})} + \mathcal{O}(\alpha_i^2), \quad (16)$$

where μ_0 is some fixed reference value and b_i is the first coefficient of the perturbative expansion of the β function. When $b_i > 0$, the coupling diverges at some energy scale (Landau pole).

For a gauge theory with a semisimple gauge group and multiplets of fermions and scalars, it is well known that [28,29]

$$b_i = \frac{2}{3} \sum_{\text{fermions}} \text{Tr}(T_a T_a) + \frac{1}{3} \sum_{\text{scalars}} \text{Tr}(T_a T_a) - \frac{11}{3} C_2, \quad (17)$$

where C_2 and T_a are, respectively, the Casimir and the generators of the gauge group. The value of b_i , and therefore the scale of the Landau pole, critically depend on the matter content of the model. More precisely, the dependence is on how many fermions and scalars are active at the energy under scrutiny. Since $C_2 = 0$ for the $U(1)$ gauge group, the b coefficient is positive, hence the $U(1)$ coupling always exhibits a Landau pole. In the case of the SM gauge group $SU_L(2) \times U_Y(1) \times SU_c(3)$, we have

$$\begin{aligned} b_Y &= \frac{20}{9} N_F + \frac{1}{6} N_H \\ b_{2L} &= -\frac{22}{3} + \frac{4}{3} N_F + \frac{1}{6} N_H \\ b_c &= -11 + \frac{4}{3} N_F, \end{aligned} \quad (18)$$

where N_F is the number of families, N_H is the number of Higgs doublets, and the coefficients b_Y, b_{2L}, b_c correspond to the coupling constants $\alpha_Y, \alpha_{2L}, \alpha_c$ of $SU_L(2), U_Y(1)$, and $SU_c(3)$, respectively. We can compare the b_i values in the SM and in the so-called two Higgs doublet model (2HDM), which differs from the SM for the presence of an additional Higgs doublet. We have

$$b_Y = \frac{41}{6} \quad b_{2L} = -\frac{19}{6} \quad b_c = -7 \text{ SM} \quad (19)$$

$$b_Y = 7 \quad b_{2L} = -3 \quad b_c = -7 \text{ 2HDM}. \quad (20)$$

We emphasize that we are not employing the fairly common $SU(5)$ normalization which sets $g_1^2 \equiv (5/3)g_Y^2$ and would lead, for instance, to $b_1 = b_Y 3/5 = 41/10$ in the SM case.

In all 331 models the two scales μ_{331} and μ_{EW} delimitate energy regimes with different symmetries and matter content, both of which affect the running.

TABLE I. Scalar content of minimal 331 model.

Scalar	$SU_L(3)$	$U_X(1)$
χ	3	+1
ρ	3	0
η	3	-1

The SM gauge group dictates the running behavior of the gauge couplings g_Y, g_{2L} from the electroweak scale $\mu_{EW} \sim M_Z$ up to the μ_{331} scale. Above μ_{331} the heavy degrees of freedom become on shell. The different matter content and the unbroken 331 gauge symmetry lead the running of g_X, g_{3L} above the μ_{331} scale. The matching condition at μ_{331} is given by [18]

$$g_{2L}(\mu_{331}) = g_{3L}(\mu_{331}),$$

$$\frac{1}{g_X^2(\mu_{331})} = \frac{1}{6} \left(\frac{1}{g_Y^2(\mu_{331})} - \beta^2 \frac{1}{g_L^2(\mu_{331})} \right). \quad (21)$$

It yields the value of g_X^2 at the scale μ_{331} as a function of g_Y^2 and g_L^2 at the same scale. The latter values are obtained by running α_Y and α_L to $\mu_0 = \mu_{331}$ from $\mu_{EW} \sim M_Z$, given the measured values [21]

$$\alpha_Y^{-1}(M_Z) \simeq 98.341,$$

$$\alpha_{2L}^{-1}(M_Z) \simeq 29.589,$$

$$\alpha_c^{-1}(M_Z) \simeq 8.42. \quad (22)$$

The matching condition (21) gives the dependence of $\alpha_X^{-1}(\mu_{331})$ on μ_{331} , which is clearly governed by the running of α_Y and α_L from $\mu_{EW} \sim M_Z$ to μ_{331} . The scale μ_{331} satisfies both the experimental lower bound (14) and the condition $\alpha_X^{-1}(\mu_{331}) \geq 0$, but it is otherwise free.

To determine the running behavior of α_X^{-1} , and hence its dependency on the matching scale μ_{331} , we need to clearly define the matter content both above and below μ_{331} . In the scalar sector we expect that at least one of the three Higgs doublets has mass $\simeq \mu_{331}$ and contributes to the running only above the μ_{331} scale. We assume that the heavy Higgs, the exotic gauge fields W', Z', Y , and the exotic fermions D^1, D^2, T have masses $\simeq \mu_{331}$; hence, they are not active below that scale. One of the other Higgs doublets can be identified with the SM Higgs, leaving two alternatives for the remaining Higgs doublet:

- (i) “heavy,” with mass around μ_{331} . Only the SM Higgs contributes to the b_i -coefficients below μ_{331} .
- (ii) “light,” with mass between the electroweak scale and μ_{331} . The second Higgs doublet affects the running of the couplings below μ_{331} , mimicking the running of the 2HDM in that region.

In Fig. 1 we show $\alpha_X^{-1}(\mu_{331})$ in case (i), dot-dashed line, and (ii), continuous line. A Landau pole arises when $\alpha_X^{-1}(\mu_{331})$ becomes zero, which occurs at $\mu_{331} = 3620$ GeV [case (i)] and at $\mu_{331} = 3908$ GeV [case (ii)]. These values set the upper limit of the 331 breaking scale $\mu_{331}^{\max}$. Thus, we have

$$(i) \quad \mu_{331} \lesssim \mu_{331}^{\max} = 3620 \text{ GeV.}$$

$$(ii) \quad \mu_{331} \lesssim \mu_{331}^{\max} = 3908 \text{ GeV.}$$

Given the lower bound (14), namely $\mu_{331} \gtrsim 3850$ GeV, case (i) is excluded by experimental data.

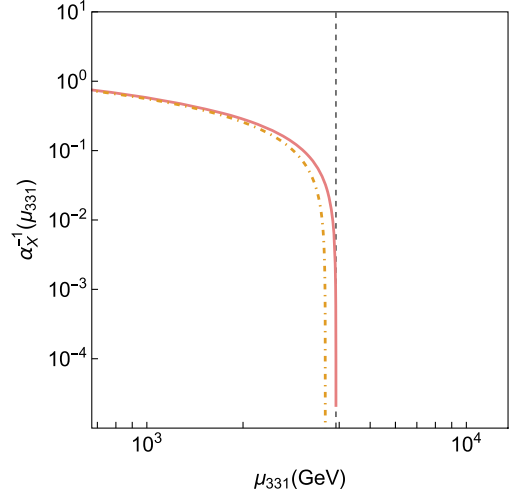


FIG. 1. We report the dependence of α_X^{-1} on the energy scale μ_{331} . The dot-dashed and continuous lines correspond, respectively, to the cases where one and two Higgs doublets are on shell below μ_{331} . The vertical dashed line represents the Landau pole at 3908 GeV.

The only possibility remains case (ii), which assumes two Higgs of mass lower than μ_{331} . The scale μ_{331} is constrained in the narrow range

$$3850 \text{ GeV} \lesssim \mu_{331} \lesssim 3908 \text{ GeV.} \quad (23)$$

According to Eqs. (13) and (14), the bilepton mass cannot exceed $\mu_{331}^{\max}$.

At the upper limit $\mu_{331}^{\max}$, α_X^{-1} hits the Landau pole. Let us observe that, by definition of the mixing angle θ_W , we can write

$$\alpha_Y^{-1}(M_Z) = (1 - \sin^2 \theta_W) \alpha_{em}^{-1}(M_Z). \quad (24)$$

In literature, a different value for the maximum value (Landau pole) is sometimes reported (about 4300 GeV). We can trace the origin of this discrepancy to the numerical value of $\alpha_{em}^{-1}(M_Z)$ used in Eq. (24).

Let us now consider the range (23) and set the 331 breaking scale at the lower edge, namely $\mu_{331} = 3850$ GeV. The running of $\alpha_X^{-1}(\mu)$ at energy scales $\mu > \mu_{331}$ is determined by its starting value $\alpha_X^{-1}(3850 \text{ GeV})$, which in turn depend on β . This running also depends on the values of the b_i parameters in Eq. (17), which are

$$b_X = 22, \quad b_{3L} = -13/2, \quad b_c = -7. \quad (25)$$

Below μ_{331} , the b_i parameters are given by Eq. (20). In Fig. 2 we plot the running of the coupling constants, above and below μ_{331} . It is apparent the dependence on the matching conditions (21). While α_{2L}^{-1} run smoothly to α_{3L}^{-1} across μ_{331} , there is a gap, which depends on β , between the values $\alpha_Y^{-1}(\mu_{331})$ and $\alpha_X^{-1}(\mu_{331})$. In the case $\beta = \pm\sqrt{3}$,

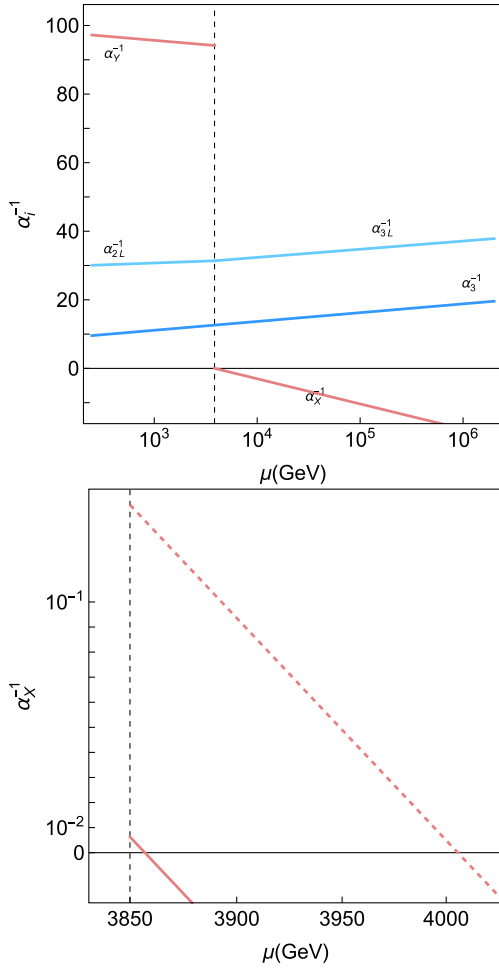


FIG. 2. Above: running of α_i^{-1} with the energy scale μ . The vertical dashed line is the arbitrary 331 breaking scale μ_{331} . Below: enlargement of α_X^{-1} . We compare with the multi-Higgs case with $N_\rho = 4$ (dashed line) considered in Sec. IV.

we have $\alpha_X^{-1}(3850 \text{ GeV}) = 0.006$, which leads to $\alpha_X^{-1}(\mu) = 0$ (Landau pole) at $\mu \sim 3858 \text{ GeV}$, not too far above μ_{331} (see the right side of Fig. 2). This is not unexpected, given the bounds (23) on μ_{331} and the diverging behavior of the $U_X(1)$ coupling above that scale. In the case $\beta = \pm 1/\sqrt{3}$, we would have had a higher starting point, $\alpha_X^{-1}(3850 \text{ GeV}) = 13.951$, and a similar slope (because of the same b_i), leading to a higher value for the Landau pole. Let us say in passing that another common way to show the presence of the Landau pole for 331 extensions is through the weak mixing angle θ_W . It can be shown [18] that for $\beta = \sqrt{3}$ the following relation holds:

$$\frac{\alpha_X}{\alpha_{2L}} = \frac{6 \sin^2 \theta_W}{1 - 4 \sin^2 \theta_W}, \quad (26)$$

which obviously links the divergence of α_X to $\sin^2 \theta_W \rightarrow 1/4$, when α_{2L} is finite. Thus, one can use

the running of $\sin^2 \theta_W$ to study the Landau pole behavior since

$$\sin^2 \theta_W = \frac{g_X^2}{6g_L^2 + 4g_X^2}. \quad (27)$$

The presence of a Landau pole at low energies poses a significant challenge, particularly in light of the stringent bounds on μ_{331} (hence on the bilepton's mass). The running in the $SU_L(3) \times U_X(1)$ sector is short lived in terms of energy range.

The mass values of bileptons, given by $m_Y \sim \mu_{331}/3$, is close to the Landau pole. There is a narrowing gap between the areas where current experiments can look for bileptons and where the perturbation theory ceases to be accurate.

In order to rescue bileptons in 331 models, it becomes necessary to move beyond the minimal 331 model. Several options can be considered, such as the following:

- (1) assuming light masses for the D^1, D^2, T extra quarks;
- (2) relaxing the condition $\xi_L^\ell = \ell_R^c$ and including exotic leptons;
- (3) extending the scalar sector; and
- (4) enlarging the number of families.

We have found that the cases (1) and (2) do not provide a viable solution—we find in both cases that the Landau pole is again around 4 TeV. In the following sections we discuss the remaining two cases.

IV. EXTENDING THE SCALAR SECTOR

Let us consider extending the scalar sector of the minimal 331 model through the inclusion of $SU_L(3)$ triplets, with the aim of shifting the maximal value for μ_{331} . Since $m_Y \sim \mu_{331}$, the shift towards a higher value implies the possibility to have higher bilepton masses in a regime which is still perturbatively valid.

The light scalar triplet ρ is neutral under $U_X(1)$ (see Table III), hence the addition of an arbitrary number of ρ -like triplets leaves b_X invariant. This argument does not hold for the other weak b_i coefficients. In the minimal 331 model, we add an arbitrary number of ρ -like scalar triplets N_ρ which correspond to extra Higgs $SU_L(2)$ doublets and contribute to the running between μ_{EW} and μ_{331} . The b_i parameters responsible of the running couplings below μ_{331} become [see Eqs. (18) and (20)]

$$b_Y = 7 + \frac{N_\rho}{6}, \quad b_{2L} = -3 + \frac{N_\rho}{6}. \quad (28)$$

At the maximum, N_ρ can be $\tilde{N}_\rho = 18$, since at that value b_{2L} changes sign producing a new divergence in the coupling g_L (at lowest order). When the b_{2L} value moves from (20) to (28), $g_L(\mu)$ increases for every fixed μ between the EW scale and μ_{331} . Above μ_{331} , Eq. (25) becomes

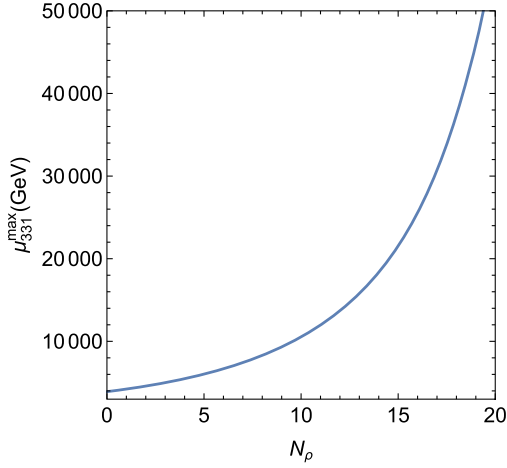


FIG. 3. Maximal value of the 331 breaking scale as a function of the number of Higgs doublets.

$$b_X = 22, \quad b_{3L} = -\frac{13}{2} + \frac{N_\rho}{6}, \quad (29)$$

which leave $g_X(\mu)$ invariant. The matching condition (21) gives the dependence of $\alpha_X^{-1}(\mu_{331})$ on the scale μ_{331} . Even if the running of α_X^{-1} above μ_{331} does not change, the different starting point pushes the maximal value of μ_{331} ($\mu_{331}^{\max}$) at higher energies. It can be seen explicitly in the right-hand side of Fig. 2. The dependence of $\mu_{331}^{\max}$ on N_ρ is shown in Fig. 3.

Summarizing, the addition of ρ -like scalar triplets makes it possible to push $\mu_{331}^{\max}$ to higher energies, which allows accommodating a wider range of mass values for the bileptons and enlarging the energy range of validity (above μ_{331}) of 331 extensions. We highlight that the case with $N_H = 2 + N_\rho = 6$ can arise from an E_6 inspired framework. Indeed the 331 model can be embedded into E_6 as (see, e.g., [30])

$$\begin{aligned} E_6 &\rightarrow SU(2) \times SU(6) \\ &\rightarrow SU_c(3) \times SU_L(3) \times U(1). \end{aligned} \quad (30)$$

It is possible to show that the only colorless irreducible representations of $SU(6)$ are **6** and **84**. These contain exactly six Higgs $SU_L(2)$ doublets. When $N_H = 6$ we have $\mu_{331}^{\max} \sim 5500$ GeV.

There are other ways to extend the scalar sector. So far, we have considered one, two, and many light Higgs $SU_L(2)$ doublets that are embedded into $SU_L(3)$ triplets. The triplets represent minimal configurations for the scalar sector. Maintaining the natural assumption that fermions transform either as the representation **3** or $\bar{\mathbf{3}}$ under $SU_L(3)$, gauge invariance also allows the possibility of scalar sextets (see, e.g., [17,31,32]). It has to be noted that, should we choose to evade this assumption, e.g., including octet leptons [33] or scalar leptoquarks assigned to the representation **8** of

$SU_L(3)$ [12], a wider choice of representations for the scalar sector would be available, and thus further ways to move the Landau pole.

Let us now extend the scalar sector of the minimal 331 with a sextet S of zero X charge, which contributes to break spontaneously the 331 symmetry. Its presence allows the addition of a renormalizable [34] Majorana neutrino mass term [26],

$$\mathcal{L}_Y = Y^{ij} \bar{L}_i^c S L_j, \quad (31)$$

where L_i ($i = 1, 2, 3$) are the left-handed lepton $SU_L(3)$ triplets. Since the sextet representation of $SU(3)$ decomposes into representations of the $SU(2)$ subgroup as $\mathbf{6} \rightarrow \mathbf{3} \oplus \mathbf{2} \oplus \mathbf{1}$, below μ_{331} we have one triplet and one isodoublet and one scalar. In order to give a Majorana mass to neutrinos in the SM through the type II seesaw mechanism, one needs a $SU_L(2)$ scalar triplet to couple with lepton $SU_L(2)$ doublets. Following [12], we assume the $SU_L(2)$ doublet and triplet contained in the sextet S to be light, while the singlet is heavy. The b_i coefficients of Eq. (18) are modified as

$$\begin{aligned} b_Y &= \frac{20}{9} N_F + \frac{1}{6} N_H + N_T \\ b_{2L} &= -\frac{22}{3} + \frac{4}{3} N_F + \frac{1}{6} N_H + \frac{2}{3} N_T. \end{aligned}$$

We remind that N_H is the number of (light) $SU_L(2)$ Higgs doublets and N_T is the number of (light) $SU_L(2)$ Higgs triplets. In this case we have $N_H = 3$ and $N_T = 1$ and therefore

$$b_Y = \frac{49}{6}, \quad b_{2L} = -\frac{13}{6}, \quad (32)$$

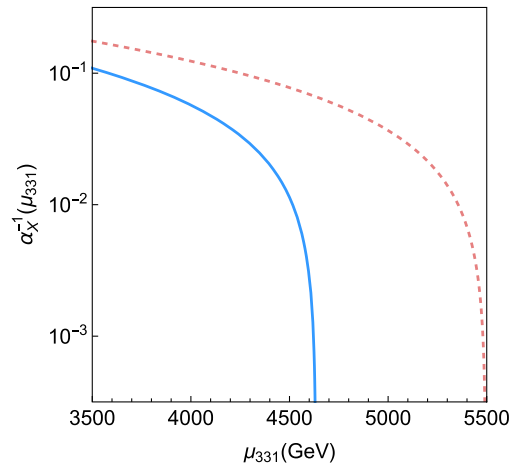


FIG. 4. Running of α_X^{-1} in the sextet extension for different values of the mass of the doublet and triplet: electroweak scale (dashed line), 600 GeV (continuous line).

while b_X is unchanged. Using these b_i parameters, we obtain a value for the scale $\mu_{331}^{\max} \sim 5500$ GeV as shown in Fig. 4 with the dashed line. However, if we assume that the new fields belonging to the sextet participate to the running starting from an intermediate scale, let us say $\mu \sim 600$ GeV, then $\mu_{331}^{\max} \sim 4630$ GeV, see Fig. 4 (continuous line).

V. FOURTH FAMILY EXTENSION MODEL

As pointed out in Sec. III A, the Landau pole problem can be mitigated by enlarging the number of families. Here we extend the minimal model by adding a fourth generation in both the lepton and quark sector. It has been shown [35–37] that the Standard Model with a sequential fourth generation (SM4), where the fourth family is an exact replica of the standard ones, is ruled out by data global fits. However, these conclusions rely on the assumption of one single Higgs doublet and they do not apply to cases, like the present one, where we have more (see, e.g., [38–40]).

In 331 models, anomaly cancellation requires that the matter assignment satisfies Eqs. (3) and (4). Since Eq. (5) also holds, the only possible configuration with $N_F = 4$ is

$$N_q^3 = 2, \quad N_{\bar{q}}^3 = 2, \quad N_l^3 = 2, \quad N_{\bar{l}}^3 = 2. \quad (33)$$

The matter content is reported in Table IV. We observe that this fourth family model includes exotic fermions beyond those present in the minimal 331 model (see Table III), namely quarks b', t', T^2 and leptons τ', ν_τ, E^4 . Here we assume that b', t', τ', ν_τ are light with masses below μ_{331} , while T^2, E^4 are heavy with masses which are proportional to the χ VEV.

As far as we know, experimental constraints on the sequential fourth family extending minimal 331 models are not available yet. Model-dependent bounds in different frameworks are given in [41,42]. Some insight can be gained by considering limits from vectorlike quark searches. In particular, we have $m_{\tau'} > 700$ GeV [43] and $m_{b'} > 1390 \div 1570$ GeV [44] from the CMS collaboration (the range is due to the variation of branching ratios) and $m_{\tau'} > 715 \div 950$ GeV [45] and $m_{b'} > 1000 \div 2000$ GeV [46] from the ATLAS collaboration. A combined CMS and ATLAS study [47] reports exclusion limits for masses below about 1500 GeV of both b' and t' . We remark again that the previous limits cannot be directly applied to our case. In CMS and ATLAS analyses, the lepton universality is not required, while it is mandatory in a sequential fourth family model.

We note that requiring unitarity at very high energies in fourth generation models suggests a bound on the BSM fermion masses that is $m'_f \lesssim 500$ GeV [48]. However, as reported in [37] this bound should not be viewed as a strict limit but simply as an indication that strong dynamics takes place. Moreover, simulation of a strong Yukawa coupling regime on the lattice found $m'_f \lesssim 700$ GeV [49]. Therefore

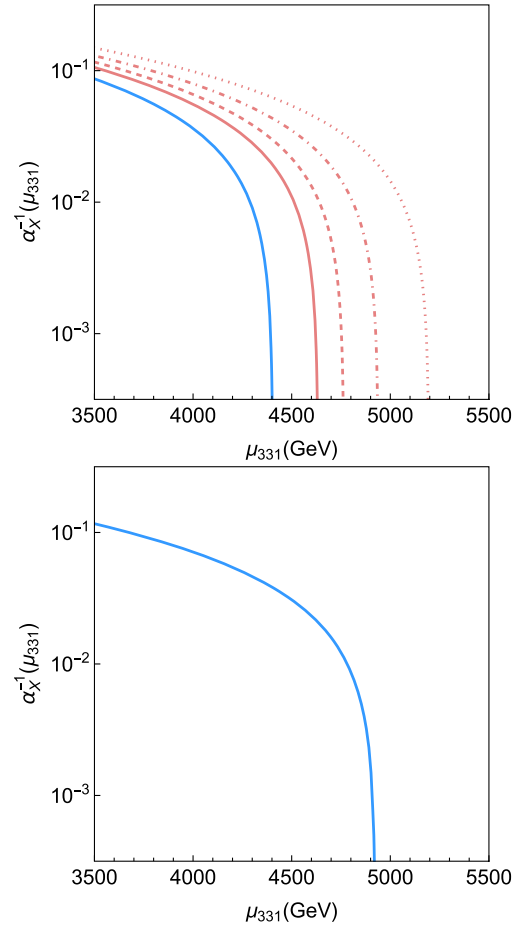


FIG. 5. Running of α_X^{-1} with the energy scale μ_{331} . Top plot: fourth family model for different values of the mass scale m_{NP} : 1500 GeV (dotted), 1000 GeV (dot-dashed), 700 GeV (dashed), 500 GeV (continuous). Bottom plot: fourth family model + sextet with $m_{NP} \sim 700$ GeV.

hereafter we will consider the special scale $m_{NP} \sim 700$ GeV as a benchmark case.

For the sake of simplicity, we assume b', t', T^2 to have the same mass m_{NP} and we consider e.g. the cases where $m_{NP} = 500, 700, 1000, 1500$ GeV. We proceed in two steps: we run $\alpha_X^{-1}(\mu)$ from the electroweak breaking scale to m_{NP} using the b_i parameters (20). Then we run it above m_{NP} using the following b_i parameters:

$$b_Y = \frac{83}{9}, \quad b_{2L} = -\frac{5}{3} \quad \text{for } \mu < \mu_{331}, \quad (34)$$

$$b_X = \frac{134}{3}, \quad b_{3L} = -\frac{31}{6} \quad \text{for } \mu > \mu_{331}. \quad (35)$$

By using these parameters, we determine the dependence of α_X^{-1} on μ_{331} for different values of the mass scale m_{NP} . This is shown in the upper plot of Fig. 5. In the benchmark case $m_{NP} \sim 700$ GeV, we find $\mu_{331}^{\max} \sim 4840$ GeV.

TABLE II. Summary of b parameters and corresponding values of the Landau pole for different 331 models and expected bilepton mass: minimal 331, only sextet, extra ρ -like triplets ($N_H = 6$), fourth family, and fourth family + sextet with $m_{NP} \sim 700$ GeV.

	b_Y	$-b_{2L}$	$\mu_{\max}$ (GeV)	m_Y (GeV)
Minimal	7	3	3908	1270
Sextet	49/6	13/6	4531	1472
$N_H = 6$	47/6	13/6	5500	1788
Fourth family	83/9	5/3	4840	1573
Fourth family + sextet	187/18	5/6	5912	1921

As a final step we also include the scalar sextet (see Sec. IV), obtaining $\mu_{331}^{\max} = 5912$ GeV as shown in the lower plot of Fig. 5.

In Table II, we compare the different extensions of the minimal 331 models we examined. We report the values of b_i parameters in the energy range between the electroweak and the 331 symmetry breaking scales, the values of $\mu_{331}^{\max}$ and the bileptons' masses estimated at $\mu_{331}^{\max}$, according to Eq. (13).

VI. CONCLUSIONS

The 331 extensions of the Standard Model are characterized by a free parameter β . By setting $\beta = \sqrt{3}$, 331 models predict doubly charged gauge bosons that mainly decay into pairs of same-sign leptons and are therefore dubbed bileptons. Their production and decay yield distinctive signatures that enable direct searches at colliders.

The minimal realizations of 331 extensions with $\beta = \sqrt{3}$ are plagued by the presence of a Landau pole around 4 TeV, which significantly constrains their range of perturbative validity. The bilepton mass is predicted to be very close to 1300 GeV, a value that will soon be either observed or ruled out by the LHC. Assuming the worst-case scenario of no detection, we investigate whether bileptons remain viable within the 331 framework. To answer this question, we address the issue of the Landau pole at approximately 4 TeV and explore possible ways to shift the pole to higher energy scales. This would enlarge the range over which the spontaneous symmetry breaking of the 331 model can occur and thus rescue bileptons, allowing for higher mass values.

We begin by analyzing the running of the gauge coupling constants in the minimal 331 model, considering its matter content and the associated scales of spontaneous symmetry breaking for both the 331 and electroweak sectors. We find that, to comply with experimental constraints, this model must behave as a two Higgs doublet model in the energy range between μ_{EW} and μ_{331} , with both Higgs bosons acquiring mass within this range. Unfortunately, we also find that it is not possible to shift the Landau pole by varying the free parameters, namely μ_{331} and the masses of the exotic quarks. This remains true even in the so-called

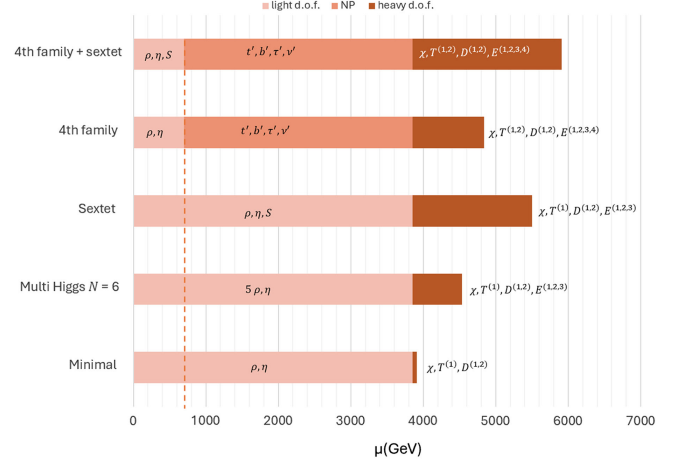


FIG. 6. Benchmark models running report.

next-to-minimal 331 model, which includes additional nonstandard leptons.

Viable solutions require extending the scalar sector and/or adding a sequential fourth family of nonstandard quarks and leptons. The running below μ_{331} of the $U_Y(1)$ and $SU(2)_L$ coupling constants changes upon supplementing the minimal model with ρ -like Higgs triplets [corresponding to extra Higgs $SU_L(2)$ doublets]. Although this procedure is constrained by bounds on the behavior of the $SU_L(2)$ coupling constant, the addition of scalar triplets of this kind extends the valid energy range, which in turn allows for a wider spectrum of bilepton masses, e.g., $m_Y \sim 1788$ GeV if $N_H = 6$. Notably, the case with $N_H = 6$ can originate from an E_6 -inspired framework.

Another scalar sector extension analyzed in this work includes an $SU_L(3)$ sextet. This case, which allows the introduction of a mass term for Majorana neutrinos, predicts $m_Y \sim 1472$ GeV.

Finally, we have studied an extension of the fermionic sector involving a sequential fourth family with two Higgs doublets. By making conservative assumptions on the mass of the fourth family based on recent experimental results, one manages to shift the Landau pole, and get $m_Y \sim 1573$ GeV with an intermediate mass scale of 700 GeV. For the sake of completeness, we also study this model including the scalar sextet, obtaining the higher value $m_Y \sim 1921$ GeV at the same intermediate scale.

We summarize our results in Table II and Fig. 6. Extending the minimal 331 models increases the Landau pole, yet this increase remains within the range of experimental values attainable at LHC in the near future.

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DATA AVAILABILITY

The data that support the findings of this article are openly available in Refs. [19,21,41–47].

APPENDIX: TABLES OF MATTER CONTENT OF THE MODELS

TABLE III. Matter content of the minimal 331 model with $\beta = \sqrt{3}$.

		Q	$SU_L(3)$	$U_X(1)$
Quarks L	u_L	2/3	3	-1/3
	d_L	-1/3		
	D_L^1	-4/3		
	c_L	2/3	3	-1/3
	s_L	-1/3		
	D_L^2	-4/3		
	b_L	-1/3	$\bar{3}$	2/3
	$-t_L$	2/3		
	T_L	5/3		
Leptons L	ℓ_L	-1	$\bar{3}$	0
	$-\nu_L^\ell$	0		
	ℓ_R^c	+1		
Quarks R	u_R, c_R	2/3	1	2/3
	d_R, s_R	-1/3		-1/3
	D_R^1, D_R^2	-4/3		-4/3
	b_R	-1/3		-1/3
	t_R	2/3		2/3
	T_R	5/3		5/3

TABLE IV. Matter content of the 331 model with fourth family.

		Q	$SU_L(3)$	$U_X(1)$	$U_Y(1)$
Quarks L	u_L	2/3	3	-1/3	1/6
	d_L	-1/3			1/6
	D_L^1	-4/3			-4/3
	c_L	2/3	3	-1/3	1/6
	s_L	-1/3			1/6
	D_L^2	-4/3			-4/3
	b_L	-1/3	$\bar{3}$	2/3	1/6
	$-t_L$	2/3			1/6
	T_L^1	5/3			5/3
	b_L'	-1/3	$\bar{3}$	2/3	1/6
	$-t_L'$	2/3			1/6
	T_L^2	5/3			5/3
Leptons L	ν_{eL}	0	3	-1	-1/2
	e_L	-1			-1/2
	E_L^1	-2			-2
	$\nu_{\mu L}$	0	3	-1	-1/2
	μ_L	-1			-1/2
	E_L^2	-2			-2
	τ_L	-1	$\bar{3}$	0	-1/2
	$-\nu_{\tau L}$	0			-1/2
	E_L^3	+1			+1
	τ_L'	-1	$\bar{3}$	0	-1/2
	$-\nu_{\tau L}'$	0			-1/2
	E_L^4	+1			+1
Quarks R	u_R, c_R	2/3	1	2/3	
	d_R, s_R	-1/3		-1/3	
	D_R^1, D_R^2	-4/3		-4/3	
	b_R, b_R'	-1/3		-1/3	
	t_R, t_R'	2/3		2/3	
	T_R^1, T_R^2	5/3		5/3	
Leptons R	$\nu_{eR}, \nu_{\mu R}$	0	1	0	
	e_R, μ_R	-1		-1	
	E_R^1, E_R^2	-2		-2	
	$\nu_{\tau R}, \nu_{\tau R}'$	0		0	
	τ_R, τ_R'	-1		-1	
	E_R^3, E_R^4	1		1	

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