

Unified coupled-channel description for the five near-threshold structures $\psi(3770)$, $G(3900)$, $R(3760)$, $R(3780)$ and $R(3810)$ from e^+e^- annihilation

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The recent observation of multiple near-threshold structures in e^+e^- annihilation—including $\psi(3770)$, $G(3900)$, $R(3760)$, $R(3780)$, and $R(3810)$ —reveals limitations in existing models of charmonium and exotic hadrons. In this paper, we propose a unified coupled-channel description that simultaneously incorporates all five near-threshold structures using parameters constrained by hadron spectroscopy. Our model accurately reproduces the line shapes in both $D\bar{D}$ and nonopen-charm hadron (nOCH) channels, resolves the asymmetric profile of $\psi(3770)$, and produces the $G(3900)$ bump. We identify the $R(3780)$ as the dominant manifestation of $\psi(3770)$, attribute $R(3760)$ to $D\bar{D} \rightarrow$ nOCH rescattering and suggest that $R(3810)$ arises from coupling between the $\psi(1D)$ state and the $h_c\pi\pi$ channel. The significant non- $D\bar{D}$ decay of $\psi(3770)$ is explained by its near-threshold position. This analysis provides a coherent, unquenched framework centered on a charmonium $\psi(1D)$ core for near-threshold phenomena and offers a predictive approach extendable to bottomonium systems and future data from Belle II.

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Introduction. Since 2003, the discovery of numerous new hadronic states in experiments has spurred a renaissance in hadron spectroscopy (see review articles [1–6]). These observations are closely linked to our understanding of nonperturbative aspects of strong interactions—a central challenge in modern particle physics. Among these, charmoniumlike XYZ states have attracted particular interest. A common feature of these states is their proximity to various hadronic thresholds. A notable example is the $X(3872)$ [7], often described as a “superstar” due to its location near the $D\bar{D}^*$ threshold, which has garnered widespread attention in the field. In fact, near-threshold phenomena—exemplified by the deuteron in nuclear physics [8] and Feshbach resonances in cold atoms [9]—are popular in whole physics and reflect fundamental dynamical mechanisms.

In recent years, enhanced experimental precision has enabled detailed measurements of the cross sections for e^+e^- annihilation into both open-charm and noncharm final states, unveiling a range of structures near open-charm thresholds. These include the well-established charmonium state $\psi(3770)$, as well as the newly reported resonances $G(3900)$, $R(3760)$, $R(3780)$, and $R(3810)$ (see Table I). As the first charmonium state above the open-charm threshold, the $\psi(3770)$ displays a number of anomalous properties. Though traditionally assigned to the 1^3D_1 $c\bar{c}$ states, the central value of the measured mass of the $\psi(3770)$ deviates from the predictions of various potential models [10–12]. Also, it exhibits a significant nonopen-charm hadron (nOCH) decay fraction of approximately 15% [13,14], in contrast to the nearly negligible nonopen-charm hadron partial widths of the $\psi(3686)$ and J/ψ . Beyond its anomalous mass and decay behaviors, the $\psi(3770)$ line shape deviates from a standard Breit-Wigner distribution—a discrepancy that has led to proposal that two distinct structures might be present [15]. In addition to the structures near $\psi(3770)$, a resonancelike feature denoted as $G(3900)$ has been observed just above the $\psi(3770)$ peak in the $e^+e^- \rightarrow D\bar{D}$ cross section [16–19]. In response to these novel near-threshold phenomena, theoretical attempts

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TABLE I. The structures observed in e^+e^- annihilation near the $D\bar{D}$ threshold.

Structure	$\psi(3770)$	$G(3900)$	$R(3760)$, $R(3780)$ and $R(3810)$
Channel	$D\bar{D}$	$D\bar{D}$ [16–19]	nOCH/hadrons [27,28]

have been made from various perspectives, such as exotic hadronic states [20] and hadron scattering theory [21–26]. Recently, both the hadronic cross section [27] and the nonopen-charm hadron cross section [28] unveil intricate structures in the 3.7–3.82 GeV region, which are well described by introducing three near-threshold structures: the $R(3760)$, $R(3780)$, and $R(3810)$. Clearly, this indicates a redundancy of near-threshold structures, as they cannot be comprehensively described under either the conventional charmonium or exotic hadron state frameworks.

Facing this challenge, we propose in this paper a unified coupled-channel description for the five near-threshold structures— $\psi(3770)$, $G(3900)$, $R(3760)$, $R(3780)$, and $R(3810)$. Our approach is based on a coupled-channel framework with most input parameters constrained by hadron spectroscopy studies, ensuring a self-consistent and robust theoretical model. Our model accurately reproduces the line shapes of the cross sections for e^+e^- annihilation into both $D\bar{D}$ and nonopen-charm hadron final states. It successfully captures the asymmetric line shape of the $\psi(3770)$ and the resonancelike structure denoted as $G(3900)$. Our analysis identifies the $R(3780)$ as the $\psi(3770)$ resonance, attributes the $R(3760)$ to $D\bar{D} \rightarrow$ nOCH rescattering dynamics, and suggests that the $R(3810)$ may stem from coupling between the $\psi(1D)$ state and the $h_c\pi\pi$ channel. The sizable nonopen-charm hadron branching fraction of the $\psi(3770)$ is naturally explained by its mass lying very close to the $D\bar{D}$ threshold.

Understanding these near-threshold phenomena marks a shift from a quenched to an unquenched picture in high-precision hadron spectroscopy. Moreover, the coupled-channel framework presented here is not limited to the charmonium sector; it can be directly applied to e^+e^- annihilation into open-bottom and nonopen-bottom hadrons. This extension represents a promising direction for future research, particularly with the high-statistics data expected from Belle II.

The $\psi(3770)$ and $G(3900)$ in the $D\bar{D}$ channel. Our starting point is a coupled-channel framework that incorporates the bare $c\bar{c}$ core, hadron-hadron kinematics, and the transition interaction between the bare $c\bar{c}$ states and various hadronic channels. The Hamiltonian is expressed as

$$H = H_{c\bar{c}} + H_{c\bar{c} \leftrightarrow \text{hadrons}} + H_{\text{hadrons}}. \quad (1)$$

The $H_{c\bar{c}}$ can be modeled using a conventional potential model [11], with the $c\bar{c}$ eigenstate $|\psi_n\rangle$ denoted as bare

states. The hadronic Hamiltonian H_{hadrons} include kinematics and interaction of hadronic channels,

$$H_{\text{hadrons}} = \sum_i \left(m_i^{\text{th}} + \frac{p_i^2}{2\mu_i} \right) + \sum_{i,j} V_{ij}^{\text{direct}}(\mathbf{p}, \mathbf{p}'). \quad (2)$$

When restricted to the space of hadron state $\{|i; \mathbf{p}\rangle\}$, the transition term $H_{c\bar{c} \leftrightarrow \text{hadrons}}$ effectively induces an s -channel potential between hadron states

$$V_{ij}^s(\mathbf{p}, \mathbf{p}') = \sum_n \frac{f_{in}(\mathbf{p}) f_{nj}(\mathbf{p}')}{E - E_n}, \quad (3)$$

where $f_{n,i}(\mathbf{p}) \equiv \langle \psi_n | H_{c\bar{c} \leftrightarrow \text{hadrons}} | i; \mathbf{p} \rangle$ denotes the transition matrix element between the n th bare $c\bar{c}$ state and the i th hadronic channel. E_n represents the bare mass of the $c\bar{c}$ state, which is shifted to a complex physical mass $E - i\Gamma/2$ for the observed resonance after incorporating coupled-channel dynamics. The total interaction is

$$V_{ij} = V_{ij}^s + V_{ij}^{\text{direct}}, \quad (4)$$

In the following analysis, we neglect the direct hadron-hadron interaction V_{ij}^{direct} unless explicitly required, for the following reasons. First, e^+e^- predominantly couple to vector $c\bar{c}$ states, making the $c\bar{c}$ -hadron coupling (V_{ij}^s) essential for modeling production, while the direct hadron-hadron interaction V_{ij}^{direct} only serves as a modification to the production cross section. Second, the explicit form of V_{ij}^{direct} is generally more model-dependent and its effect is relatively difficult to account for.

Since the s -channel potential is separable, the corresponding Lippmann-Schwinger equation can be solved exactly, yielding the T -matrix

$$T_{ij}(E; \mathbf{p}, \mathbf{p}') = f_i^\dagger(\mathbf{p}) \frac{\lambda}{I - \lambda \sum_k I_k(E)} f_j(\mathbf{p}'), \quad (5)$$

where $f_i(\mathbf{p}) = (f_{1i}(\mathbf{p}'), f_{2i}(\mathbf{p}'), \dots)$ and $\lambda = \text{diag}(1/(E - E_1), 1/(E - E_2), \dots)$. The term $I_k(E)$ encapsulates the contribution from the k -th hadron channels to the pole positions of physical states and is given by

$$I_k(E) = \int d^3q \frac{f_k(\mathbf{q}) f_k^\dagger(\mathbf{q})}{E - m_k^{\text{th}} - q^2/(2\mu_k)}. \quad (6)$$

We focus on the energy region just above the $D\bar{D}$ threshold. The bare states $\psi(1D)$ and $\psi(3S)$ are included, with masses fixed at 3.82 GeV and 4.10 GeV, respectively, as obtained from a potential model calculation [11]. Although the $\psi(2S)$ lies slightly below the $D\bar{D}$ threshold, it is also included with a mass of 3.688 GeV.

Among the various hadronic channels, the open-charm channels $D^{(*)}\bar{D}^{(*)}$ play the dominant role, accounting for nearly the entire decay width of $c\bar{c}$ states above the open-charm threshold. The transition function $f_{i,n}(\mathbf{p})$ is calculated

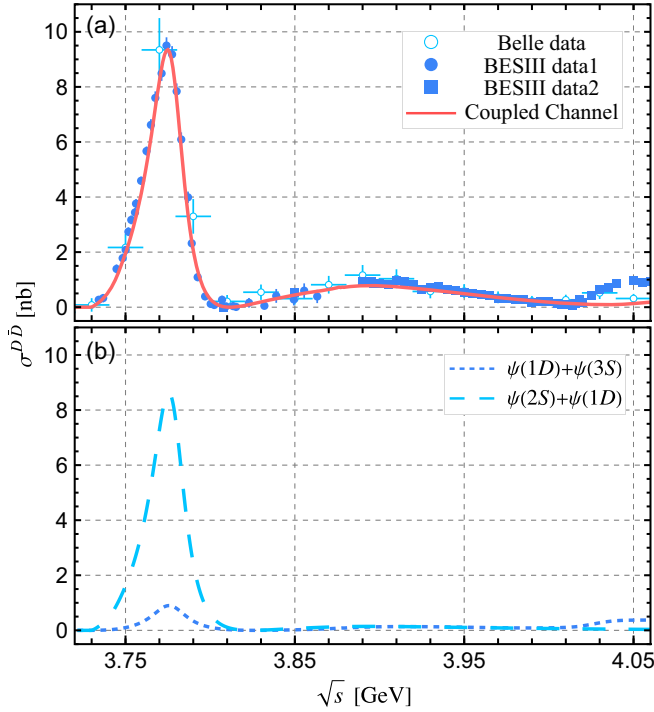


FIG. 1. (a) Coupled-channel description of the $e^+e^- \rightarrow D\bar{D}$ cross section. The red solid line represents the full model including $\psi(2S)$, $\psi(1D)$, and $\psi(3S)$. Experimental data are from Belle [19] and BESIII [16,24]. (b) Cross section when either $\psi(2S)$ or $\psi(3S)$ is removed.

using the quark pair creation (QPC) model [29,30], which describes the coupling between a $c\bar{c}$ state and a $D^{(*)}\bar{D}^{(*)}$ pair via creation of a light-quark pair with the quantum numbers $J^{PC} = 0^{++}$. The transition function takes the form

$$f_{i,n}(\mathbf{p}) = \gamma_n \mathcal{M}_{M_n \rightarrow \{M_i\}}^{\text{QPC}}(\mathbf{p}) e^{-\frac{\mathbf{p}^2}{2\Lambda^2}}, \quad (7)$$

where an exponential form factor is introduced to suppress high-momentum contributions inherent in the original QPC model [31–35]. The parameter γ_n quantifies the strength of quark pair creation. For $\psi(2S)$, we include its coupling to $D\bar{D}$, though its mass lies below the $D\bar{D}$ threshold; Couplings to other open-charm channels are neglected due to their higher threshold. For $\psi(1D)$ and $\psi(3S)$, coupling to $D\bar{D}^{(*)}$ and $D^{(*)}\bar{D}^{(*)}$, respectively, are included. The details of the QPC amplitudes can be found in Supplemental Material (SM) [36].

To connect with experimental cross sections, we also introduce the coupling f_{n,e^+e^-} between bare states and the e^+e^- channel. The value of f_{n,e^+e^-} is related to the dilepton width of the bare state within the potential model via

$$\Gamma_n^{e^+e^-} = 8\pi^2 \frac{|\mathbf{p}_n^{cm}|}{m_n} E_e^2 \frac{1}{2J_n + 1} \sum_{\text{spin}} |f_{n,e^+e^-}|^2, \quad (8)$$

where E_e is the electron energy. Clearly, $I_{e^+e^-}(E)$ is negligibly small compared to contribution from open-charm channels. The on-shell T -matrix element relevant to $e^+e^- \rightarrow D\bar{D}$ is

$$T_{D\bar{D},e^+e^-}(E + i\epsilon; \mathbf{p}_D^{cm}) = \frac{f_{D\bar{D}}(\mathbf{p}_D^{cm}) \lambda f_{e^+e^-}}{I - \lambda(I_{D\bar{D}} + I_{D\bar{D}^*} + I_{D^*\bar{D}^*}) + i\epsilon}, \quad (9)$$

and the cross section in our nonrelativistic convention is

$$\frac{d\sigma^{D\bar{D}}}{d\Omega} = (2\pi)^4 \frac{\sqrt{s} |\mathbf{p}_D^{cm}|}{8} \frac{1}{4} \sum_{\text{spin}} |T_{D\bar{D},e^+e^-}|^2. \quad (10)$$

Figure 1 shows the comparison of the coupled-channel calculation with experimental data; the corresponding parameters are listed in Table II. The dilepton widths for the bare states are

$$\begin{aligned} \Gamma_{2S}^{e^+e^-} &= 5.50 \text{ keV}, & \Gamma_{1D}^{e^+e^-} &= 0.02 \text{ keV}, \\ \Gamma_{3S}^{e^+e^-} &= 4.91 \text{ keV}. \end{aligned}$$

These values are comparable to prediction from the potential model [11]. The asymmetric line shape of the $\psi(3770)$ is naturally reproduced within this framework. Specifically, the $\psi(2S) - D\bar{D} - \psi(1D)$ coupling leads to a relatively large dilepton width for $\psi(3770)$. Meanwhile, the $\psi(1D) - D\bar{D}^{(*)} - \psi(3S)$ coupling induces an interference that produces a visible bump near 3.9 GeV in the $D\bar{D}$ channel. Removing either $\psi(2S)$ or $\psi(3S)$ eliminates this structure, indicating a nontrivial interference pattern.

The poles corresponding to physical resonances are obtained via analytic continuation from the physical region to the relevant Riemann sheet. On the sheet connected to the physical region, we find three poles near the real axis (see Table III), corresponding to the $\psi(3686)$, $\psi(3770)$, and $\psi(4040)$. The inclusion of $D\bar{D}$ coupled-channel dynamics shifts the mass of the bare $\psi(1D)$ state from 3.82 GeV to that of physical $\psi(3770)$. No additional pole is found near 3.9 GeV in the vicinity of the physical axis.

The $R(3760)$, $R(3780)$ and $R(3810)$ in the nonopen-charm hadron channel. We now extend the coupled-channel framework to include nonopen-charm hadron channels.

TABLE II. The parameters in the coupled-channel model. The coupling f_{n,e^+e^-} are given in units of $\text{GeV}^{-\frac{1}{2}}$.

γ_{2S}	γ_{1D}	γ_{3S}	$f_{\psi(2S),e^+e^-}$	$f_{\psi(1D),e^+e^-}$	$f_{\psi(3S),e^+e^-}$	Λ (GeV)
3.54	6.51	6.15	2.48×10^{-4}	0.15×10^{-4}	2.11×10^{-4}	0.65

TABLE III. Poles in the coupled-channel model. The Riemann sheets are labeled according to the sheets used for the $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$ channels. For example, (I, II, I) denotes the first Riemann sheet for the $D\bar{D}$ channel, the second sheet for the $D\bar{D}^*$ channel, and the first sheet for the $D^*\bar{D}^*$ channel.

Riemann sheet	(I, I, I)	(II, I, I)	(II, II, II)
Pole (GeV)	3.686	$3.778 - 0.012i$	$4.031 - 0.024i$
Physical state	$\psi(3686)$	$\psi(3770)$	$\psi(4040)$

The $R(3780)$ structure, with mass and width similar to those of the conventional $\psi(3770)$, can be identified with $\psi(3770)$. In contrast, the $R(3760)$ structure appear near the $D\bar{D}$ threshold in the nonopen-charm hadron cross section (See Fig. 2) and lacks a clear $c\bar{c}$ counterpart. A natural question is whether the coupling between bare $c\bar{c}$ states and the $D\bar{D}$ channel could produce a $D\bar{D}$ threshold cusp in the nonopen-charm hadron channel when $c\bar{c}$ -nOCH coupling are included. We conjecture that *a direct coupling between bare $c\bar{c}$ states and nonopen-charm hadron channel cannot yield any significant structure near the $D\bar{D}$ threshold.*

To clarify this point, consider the energy region very close to the $D\bar{D}$ threshold. When including $c\bar{c}$ -nonopen-charm hadron coupling, the scattering amplitude can be approximated as

$$T_{\text{nOCH}, e^+e^-}^0(E + i\epsilon) \approx \frac{f_{\text{nOCH}}(m_{D\bar{D}}^{\text{th}}) \lambda f_{e^+e^-}}{I - \lambda(I_{D\bar{D}}(E + i\epsilon) - \sum_{k \neq D\bar{D}} I_k(m_{D\bar{D}}^{\text{th}}))}.$$

Near the $D\bar{D}$ threshold, the only significant energy dependence comes from $I_{D\bar{D}}(E)$. The contribution from $I_{D^{(*)}\bar{D}^*}(E)$ and $I_{\text{nOCH}}(E)$ are smooth in this region due to their higher thresholds. The loop function $I_{D\bar{D}}$ behaves as

$$I_{D\bar{D}}(E + i\epsilon) \approx -2\mu_{D\bar{D}} \int dq (f_{D\bar{D}}^\dagger f_{D\bar{D}})(q) - i\pi\mu_{D\bar{D}} k_{D\bar{D}} (f_{D\bar{D}}^\dagger f_{D\bar{D}})(k_{D\bar{D}}) \quad (11)$$

with $k_{D\bar{D}} = \sqrt{2\mu_{D\bar{D}}E_{D\bar{D}}}$. Although the square root function could, in principle, produce a cusp, the coupling $f_{D\bar{D}}$ is in P -wave, leading to the behavior $f_{D\bar{D}} \approx ak_{D\bar{D}}$ as $E_{D\bar{D}} \rightarrow 0$. Consequently, the imaginary part is suppressed by $f_{D\bar{D}}^\dagger f_{D\bar{D}} \propto (2\mu_{D\bar{D}}E_{D\bar{D}})$, rendering the cusp effect negligible. This contrasts sharply with the case of an S -wave coupling, where $f_i^\dagger f_i \approx \text{constant}$ and a pronounced cusp appears.

Since direct $c\bar{c}$ -nonopen-charm hadron coupling cannot produce a noticeable threshold structure, we instead introduce a coupling between the $D\bar{D}$ and nonopen-charm hadron channels, corresponding to a rescattering process $D\bar{D} \rightarrow \text{nOCH}$. This is modeled by

$$V_{\text{nOCH}, D\bar{D}}(\mathbf{q}) = g_{\text{nOCH}} q e^{-q/\Lambda_{\text{nOCH}}} Y_1^{1*}(\Omega_{\mathbf{q}}), \quad (12)$$

where q is the momentum of the $D\bar{D}$ system and Y_1^1 is the spherical harmonic function. The momentum dependence of nonopen-charm hadron channels can be neglected due to their lower thresholds. This form captures two essential features: (1) the P -wave nature of the coupling to the vector charmonium state, leading to $V_{\text{nOCH}, D\bar{D}} \propto q Y_1^{1*}(\Omega_{\mathbf{q}})$ as $q \rightarrow 0$; and (2) the characteristic momentum scale Λ_{nOCH} of the rescattering. The T -matrix for nonopen-charm hadron production is then

$$\begin{aligned} T_{\text{nOCH}, e^+e^-}(E + i\epsilon) &= V_{\text{nOCH}, D\bar{D}} G^{D\bar{D}}(E + i\epsilon) T_{D\bar{D}, e^+e^-}(E + i\epsilon) \\ &= \int d^3q \frac{V_{\text{nOCH}, D\bar{D}}(\mathbf{q}) T_{D\bar{D}, e^+e^-}(E + i\epsilon; \mathbf{q})}{E - m_{D\bar{D}}^{\text{th}} - q^2/(2\mu_{D\bar{D}}) + i\epsilon}, \end{aligned}$$

where $T_{D\bar{D}, e^+e^-}$ is approximated by the amplitude in Eq. (9).¹ To model the collective effect of multiple non-open-charm hadron channels with thresholds well below $D\bar{D}$, we introduce an effective two-body channel with

$$\mu_{\text{nOCH}} = 0.7 \text{ GeV}, \quad m_{\text{nOCH}}^{\text{th}} = 3.0 \text{ GeV}. \quad (13)$$

In addition to the $R(3760)$ near the $D\bar{D}$ threshold, a small peaklike structure $R(3810)$ is observed around 3.81 GeV in the nonopen-charm hadron channel, just above the $h_c\pi\pi$ threshold and with no counterpart in $D\bar{D}$. This suggests a contribution from $e^+e^- \rightarrow h_c\pi\pi$. We therefore include an $h_c\pi\pi$ channel in the model. Lacking theoretical constraints on its coupling to $c\bar{c}$ states, and noting that the bare $\psi(1D)$ mass (3.82 GeV) is close to the $h_c\pi\pi$ threshold, we introduce a $\psi(1D) - h_c\pi\pi$ coupling. Assuming the two pions are in relative P -wave ($L_{\pi\pi} = 1$), the transition matrix element as

$$\begin{aligned} f_{\psi(1D), h_c\pi\pi}(\mathbf{q}, \mathbf{p}) &= \frac{g_{h_c\pi\pi}}{\sqrt{4\pi}} p e^{-(q^2+p^2)/\Lambda_c^2} \frac{1}{\sqrt{2}} \left(Y_1^{0*}(\Omega_p) - Y_1^{1*}(\Omega_p) \right), \end{aligned}$$

where $\mathbf{q}$ is the h_c momentum in the c.m. frame, and $\mathbf{p}$ is the relative momentum between the pions. The contribution to the T -matrix is

$$\begin{aligned} I_{h_c\pi\pi}(E) &= \int d^3q d^3p \frac{f_{h_c\pi\pi}(\mathbf{q}, \mathbf{p}) f_{h_c\pi\pi}^\dagger(\mathbf{q}, \mathbf{p})}{E - m_{h_c\pi\pi}^{\text{th}} - q^2/(2\mu_{h_c(\pi\pi)}) - p^2/(2\mu_{\pi\pi})}, \end{aligned}$$

¹Strictly speaking, the $T_{D\bar{D}, e^+e^-}$ should be obtained by solving the full LS equation, including the $V_{\text{nOCH}, D\bar{D}}$ interaction. However, this requires knowledge of the explicit momentum dependence of nOCH channel $V_{\text{nOCH}, D\bar{D}}(\mathbf{p}_{\text{nOCH}}, \mathbf{p}_{D\bar{D}})$, which introduces additional parameters. As a first-order approximation, we can approximate $T_{D\bar{D}, e^+e^-}$ as the T matrix calculated in the absence of $V_{\text{nOCH}, D\bar{D}}$. Additional discussion of the full LS equation is provided in SM [36].

where $f_{h_c\pi\pi} = (0, f_{1D, h_c\pi\pi}(\mathbf{q}, \mathbf{p}), 0)$, since couplings of $\psi(2S/3S)$ to $h_c\pi\pi$ are neglected. The full T -matrix is approximated as

$$T_{ij} = f_i \frac{\lambda}{I - \lambda(I_{D\bar{D}} + I_{D\bar{D}^*} + I_{D^*\bar{D}^*} + I_{h_c\pi\pi} + I_{2S})} f_j, \quad (14)$$

where a small constant term $I_{2S} = i\delta_{2S}\text{diag}(1, 0, 0)$ is introduced to give $\psi(2S)$ a small width of ~ 300 keV [37]. Its value is determined by the relation

$$\delta_{2S} = \Gamma_{2S}/2 \approx 150 \text{ keV}.$$

The $D^0\bar{D}^0$ and D^+D^- channels are treated separately. The nonopen-charm hadron cross section is then

$$\sigma^{\text{nOCH}} = \sigma^{D\bar{D} \rightarrow \text{nOCH}} + \sigma^{h_c\pi\pi}, \quad (15)$$

and the inclusive hadronic cross section is

$$\sigma^{\text{hadron}} = \sigma^{D\bar{D}} + \sigma^{D\bar{D} \rightarrow \text{nOCH}} + \sigma^{h_c\pi\pi}. \quad (16)$$

The final results for the nonopen-charm hadron cross section, including both $V_{\text{nOCH}, D\bar{D}}$ and $f_{\psi(1D), h_c\pi\pi}$ couplings, are shown in Fig. 2. The additional parameters are

$$\begin{aligned} g_{\text{nOCH}} &= 11.5 \text{ GeV}^{-1/2}, & \Lambda_{\text{nOCH}} &= 90 \text{ MeV}, \\ g_{h_c\pi\pi} &= 8 \times 10^3 \text{ GeV}^{-1}, & \Lambda_{h_c} &= 40 \text{ MeV}. \end{aligned} \quad (17)$$

The uncertainty of parameters Λ_{nOCH} and Λ_{h_c} is discussed in SM [36]. The structures just above the $D\bar{D}$ threshold arise from $D\bar{D} \rightarrow \text{nOCH}$ rescattering. The scale $\Lambda_{\text{nOCH}} \approx 0.09$ GeV restricts the process to small relative momenta ($\lesssim 90$ MeV), naturally explaining the significant nonopen-charm hadron decay width of the $\psi(3770)$. No direct $\psi(1D) \rightarrow \text{nOCH}$ coupling is required, consistent with the small non- $D\bar{D}$ width of the $\psi(3686)$. The large nonopen-charm hadron branching fraction of the $\psi(3770)$ is due to its proximity to the $D\bar{D}$ threshold, enabling decay via rescattering $D\bar{D} \rightarrow \text{nOCH}$. Similar structures may appear in $e^+e^- \rightarrow J/\psi X$ cross section [38].

Analogous rescattering effects near other thresholds (e.g., $D\bar{D}^*$ and $D^*\bar{D}^*$) are expected due to similar interactions in the heavy-quark limit. The $\psi(4040)$, located close to the $D^*\bar{D}^*$ threshold ($\Delta m \approx 20$ MeV) and having a large $D^*\bar{D}^*$ branching fraction, may also exhibit enhanced nonopen-charm hadron decays. Furthermore, similar threshold structures could appear in the hadronic decays of charmoniumlike structures such as $Z_c(3900)$ [39,40] and $Z_c(4020)$ [41], where cusp effects may mimic resonant behavior [42–45].

Inclusion of the $h_c\pi\pi$ channel yields a resonancelike structure $R(3810)$ at $\sqrt{s} = 3.81$ GeV in the $e^+e^- \rightarrow h_c\pi\pi$ cross section, visible in nonopen-charm hadron but not open-charm channels, with minimal impact on the $\psi(3770)$

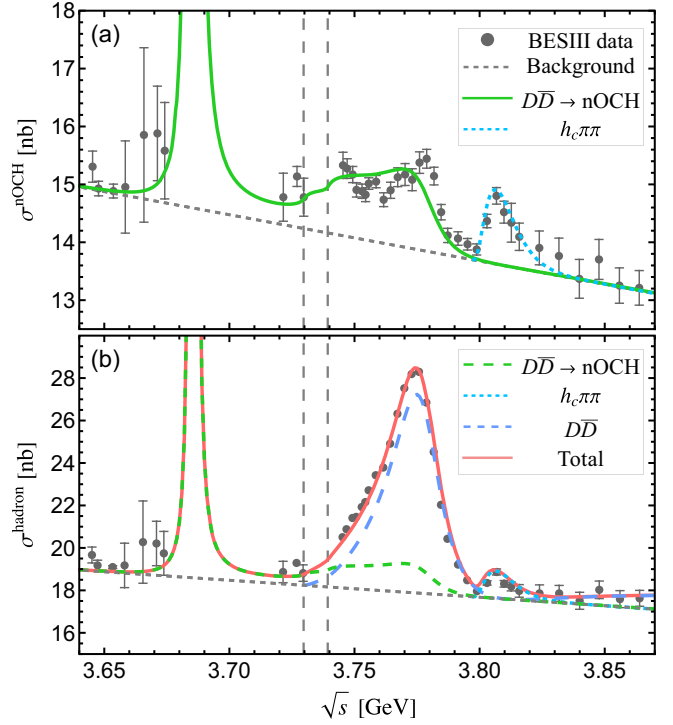


FIG. 2. Coupled-channel descriptions of (a) the $e^+e^- \rightarrow$ non-open-charm (nOCH) hadron cross section and (b) the inclusive hadronic cross sections. Vertical dashed lines indicate the $D^0\bar{D}^0$ and $D^+\bar{D}^-$ thresholds. Nonopen-charm hadron data are from Ref. [28], inclusive hadronic data are from Ref. [27].

pole. The small fitted value $\Lambda_{h_c} = 0.04$ GeV implies the coupling $f_{\psi(1D), h_c\pi\pi}(\mathbf{q}, \mathbf{p})$ is significant only at low momenta ($q, p \lesssim 0.04$ GeV), consistent with soft-gluon emission in QCD multipole expansion [46]. Further experimental study of $e^+e^- \rightarrow h_c\pi\pi$ is warranted.

Summary. The discovery of numerous charmonium and charmoniumlike states near open-charm thresholds has posed significant challenges to our understanding of non-perturbative quantum chromodynamics. In particular, the energy region around the $D\bar{D}$ threshold exhibits a proliferation of structures— $\psi(3770)$, $G(3900)$, $R(3760)$, $R(3780)$, and $R(3810)$ —that defy conventional quark model classifications and suggest the presence of complex coupled-channel effects. The $\psi(3770)$, nominally assigned as the 1^3D_1 charmonium state, shows several anomalies including a mass shift from potential model predictions, a large nonopen-charm decay fraction, and a distorted line shape. Recent high-precision measurements of $e^+e^- \rightarrow$ hadrons cross sections further reveal multiple near-threshold enhancements, indicating that a unified description beyond conventional hadron or exotic state interpretations is required.

In this paper, we develop a coupled-channel framework incorporating both open-charm and nonopen-charm hadron channels to simultaneously describe all five near-threshold

structures. The model includes bare $c\bar{c}$ states— $\psi(2S)$, $\psi(1D)$, and $\psi(3S)$ —with masses and transition matrix elements constrained by phenomenological and QPC model inputs. By considering couplings to $D^{(*)}\bar{D}^{(*)}$ channels and introducing rescattering via an effective s -channel potential, we reproduce the asymmetric line shape of the $\psi(3770)$ and the prominent $G(3900)$ peak in the $D\bar{D}$ cross section. The $R(3780)$ is unambiguously identified with the $\psi(3770)$ resonance. The threshold enhancement $R(3760)$ is explained as a consequence of $D\bar{D} \rightarrow n\text{OCH}$ rescattering, without requiring a direct $c\bar{c}$ coupling. The $R(3810)$ structure is attributed to the coupling between the $\psi(1D)$ state and the $h_c\pi\pi$ channel.

Our analysis provides a natural explanation for the large nonopen-charm hadron branching fraction of the $\psi(3770)$ through its proximity to the $D\bar{D}$ threshold, which enhances rescattering decay amplitudes. The model successfully describes both open-charm, nonopen-charm and inclusive hadronic cross sections with a minimal set of parameters. The results underscore the necessity of coupled-channel effects in interpreting near-threshold phenomena and mark

a transition from a static, quenched quark model to a dynamic, unquenched spectroscopy. The framework presented here is general and can be extended to bottomonium systems, offering a predictive approach for analyzing future high-precision data from facilities such as Belle II.

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Data availability. The data that support the findings of this article are openly available [16,19,24,27,28].

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