

Double Higgs boson production via vector boson fusion in SMEFT

Athanasios Dedes^{1,*}, Janusz Rosiek^{2,†} and Michał Ryczkowski^{3,‡}

¹*Department of Physics, University of Ioannina, GR 45110, Ioannina, Greece*

²*Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*

³*Dipartimento di Fisica e Astronomia “Galileo Galilei,” Università di Padova, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Padova, Padova, I-35131, Italy*



(Received 3 July 2025; accepted 28 August 2025; published 29 September 2025)

While gluon fusion dominates Higgs pair production at the LHC, vector boson fusion (VBF) offers a unique window into beyond the Standard Model (BSM) physics through its distinctive kinematic features and direct sensitivity to Higgs-vector boson interactions. We perform a comprehensive analysis of double Higgs production via VBF in the Standard Model effective field theory (SMEFT), systematically investigating how dimension-6 and dimension-8 bosonic operators—particularly those involving field derivatives—can enhance the production rate. We identify the most relevant Wilson coefficients (WCs) affecting the trilinear Higgs coupling (hhh) and Higgs-vector boson interactions (hVV , $hhVV$). Using constraints from global fits and interpolating fit results for unconstrained WCs with naive dimensional analysis, we assess their effects on the $VV \rightarrow hh$ ($V = W, Z$) scattering amplitudes and cross sections. Our analysis includes a study of EFT convergence and validity in models with scalar extensions of the SM. Numerical simulations for the planned High-Luminosity LHC experiment (HL-LHC) in general reveal only a modest and challenging to detect enhancement of the VBF di-Higgs production rate over the SM prediction. However, we show that in optimistic scenarios, such a process could be observed at the HL-LHC. In certain cases, when the enhancement is dominated by the dimension-6 or dimension-8 operators containing field derivatives (and thus leading to stronger energy-dependent effects), this channel becomes competitive with di-Higgs production via gluon fusion. This work highlights the role of VBF di-Higgs production as a complementary channel for probing anomalous Higgs couplings and their impact on BSM physics.

DOI: [10.1103/bywd-2dy1](https://doi.org/10.1103/bywd-2dy1)

I. INTRODUCTION

The discovery of the Higgs boson at the Large Hadron Collider (LHC) in 2012 [1,2] marked a pivotal moment in particle physics, confirming the mechanism of electroweak symmetry breaking (EWSB) as described by the Standard Model (SM) [3–5]. However, there are couplings present in the SM have not yet been measured experimentally. Among them, the trilinear Higgs self-coupling is perhaps one of the most important, as it is vital for determining the exact shape of the Higgs potential and for assessing the stability of the Universe. This coupling can be probed at the LHC through nonresonant Higgs boson pair production (commonly termed as di-Higgs production), which is one of the most

promising directions in the search for the physics beyond the Standard Model (BSM). Existing observed and expected 95% CL experimental upper limits for inclusive gluon fusion (ggF) and vector boson fusion (VBF) double Higgs production modes read (with significant progress expected with the High-Luminosity LHC upgrade) [6,7]

$$\begin{aligned}\sigma(pp \rightarrow hh)_{\text{Obs(Exp)}}^{\text{ggF+VBF}} &= 2.9(2.4) \times \sigma^{\text{SM}}(pp \rightarrow hh), \\ &\text{ATLAS,} \\ \sigma(pp \rightarrow hh)_{\text{Obs(Exp)}}^{\text{ggF+VBF}} &= 3.5(2.5) \times \sigma^{\text{SM}}(pp \rightarrow hh), \\ &\text{CMS.}\end{aligned}\tag{1.1}$$

These numbers indicate significant potential for new physics to affect this process by enhancing its cross section relative to the SM prediction, leading lately to growing interest in studying the process of double Higgs boson production.

The two main production channels for Higgs boson pair production in the SM are gluon fusion and vector boson fusion processes [8–11]. While gluon fusion ($gg \rightarrow hh$) [12–45] dominates in the SM, vector boson fusion ($qq \rightarrow qqVV \rightarrow qqhh$) [46–58] constitutes a complementary

*Contact author: adedes@uoi.gr

†Contact author: Janusz.Rosiek@fuw.edu.pl

‡Contact author: michaljakub.ryczkowski@unipd.it

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

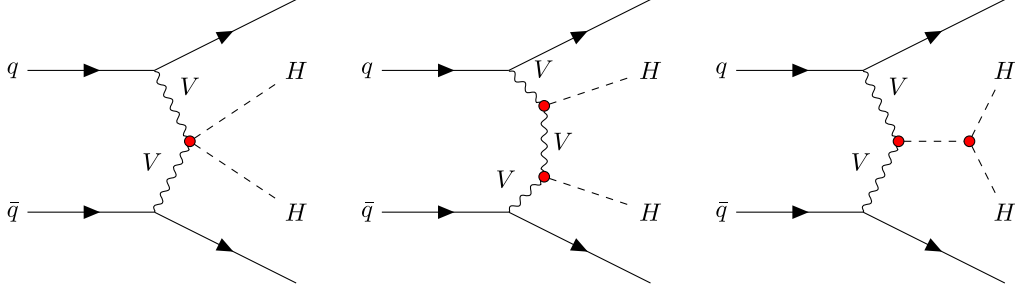


FIG. 1. Diagrams contributing to the double Higgs production via vector boson fusion at tree-level and in unitary gauge (the u -channel is not shown).

channel with distinct experimental signatures, including presence of forward jets and low hadronic activity in the central region. Moreover, although VBF di-Higgs production is suppressed by an order of magnitude relative to ggF channel, it is highly sensitive to anomalous Higgs couplings—namely, the trilinear coupling hhh and Higgs-gauge boson couplings hVV and $hhVV$ (where “ V ” stands for either a W -boson or a Z -boson). Such couplings commonly arise in various BSM theories. For reference, the leading-order tree-level diagrams contributing to di-Higgs VBF production at LHC are shown in Fig. 1, with anomalous couplings marked by red blobs.

The Standard Model effective field theory (SMEFT) [59–63] provides a model-independent framework to parametrize BSM effects through higher-dimensional operators, suppressed by a cutoff scale Λ . These operators modify Higgs and gauge boson interactions, leading to deviations in cross sections and kinematic distributions. In the context of double Higgs VBF production, dimension-6 and dimension-8 operators can, in principle, significantly alter the scattering amplitudes, offering a window into new physics entering the trilinear Higgs coupling that may be inaccessible in single-Higgs processes.

Over the years, numerous studies have investigated double Higgs production in the context of various BSM scenarios [64–71]. A number of recent analyses have employed EFT approach, including both SMEFT and Higgs effective field theory (HEFT) [72–76], to study single, double, and multi Higgs production. Some focused on the ggF channel [77–85], while others examined modifications to the trilinear Higgs coupling [86–92], or concentrated on the VBF channel [93–100].

In this work, we investigate double Higgs production via VBF in the SMEFT framework, examining the impact of higher-dimensional operators on the production rate and differential distributions, including terms $\propto \frac{1}{\Lambda^2}$ (LO in the EFT expansion) and $\propto \frac{1}{\Lambda^4}$ (NLO in the EFT expansion). We identify the most sensitive operators and assess the prospects for detecting BSM effects at current (LHC) and future (HL-LHC) collider experiments. Our analysis highlights the importance of VBF di-Higgs production as a unique probe of Higgs interactions and the Higgs sector, and

underlines the potential of SMEFT as a tool for uncovering indirect signs of new physics. Finally, utilizing SMEFT as a tool should be treated with care, as we also illustrate in this study.

The paper is organized as follows. Section II identifies the dimension-6 and dimension-8 SMEFT operators relevant for this analysis, and outlines our assumptions leading to bounds on maximal values of their Wilson coefficients used further in numerical analysis. In Sec. III we discuss the EFT convergence and accuracy, illustrating it with examples of the three BSM models involving heavy scalar singlets and triplets, by comparing the amplitudes calculated in renormalizable UV-theories with the corresponding ones obtained in the decoupling limit. Analytical formulas for $VV \rightarrow hh$ helicity amplitudes and cross sections are presented in Sec. IV and Appendices A and B. Finally, Sec. V contains the results of our numerical simulations, directly addressing the question of maximal potential enhancement of VBF di-Higgs production in SMEFT for LHC and HL-LHC experiments. Section VI concludes the paper. In the Appendices, we present the helicity amplitudes for the $WW \rightarrow HH$ and $ZZ \rightarrow HH$ processes, including SMEFT contributions from the dimension-6 and dimension-8 operators collected in Table II. For each amplitude, we display only the leading order terms in the high energy limit ($M_V^2/s \rightarrow 0$), including interference terms between different Wilson coefficients. We start with the $WW \rightarrow HH$ process in Appendix A and present the Standard Model contribution in Eq. (A1), the dimension-6 and dimension-6² contributions in Eq. (A2), and the dimension-8 contributions in Eqs. (A3) and (A4). Similarly, in Appendix B, for the $ZZ \rightarrow HH$ process, we present the SM contribution in Eq. (B1), the dimension-6 and dimension-6² contributions in Eq. (B2), and the dimension-8 contributions in Eqs. (B3) and (B4).

II. CONTRIBUTING OPERATORS AND BOUNDS ON WILSON COEFFICIENTS

A. Relevant operators

To affect the process of double Higgs production via VBF, the SMEFT operators can be inserted into three types of vertices: hhh , hVV , and $hhVV$ (red dots on diagrams in

TABLE I. Classes of SMEFT dimension-6 and dimension-8 operators affecting vertices relevant for double Higgs boson production via VBF.

Vertex	Operator class	
	Dimension-6	Dimension-8
hhh	$\varphi^6, \varphi^4 D^2$	$\varphi^8, \varphi^6 D^2$
hVV	$\varphi^4 D^2, X^2 \varphi^2$	$\varphi^6 D^2, X^2 \varphi^4, X \varphi^4 D^2$
$hhVV$	$\varphi^4 D^2, X^2 \varphi^2$	$\varphi^4 D^4, \varphi^6 D^2, X^2 \varphi^4, X^2 \varphi^2 D^2$

Fig. 1, h indicates the Higgs and V indicates the gauge boson). The relevant classes of the SMEFT operators that can impact this process are all summarized in Table I.

Table II presents the specific operators that may affect one or both of the two VBF processes of interest: $WW \rightarrow hh$ and $ZZ \rightarrow hh$. All these operators have been included in the corresponding matrix elements presented in Sec. IV and analytical results are given in Appendices A and B.

Some important comments are in place here. As we are interested in the most general features of the VBF part of the process, we neglect the impact of the SMEFT operators on the external quark lines and on further decays of the Higgs boson pair into the final states. At the dimension-6 level, the primary potential contributions may originate from the dipole operators $\mathcal{Q}_{uW}^{11}, \mathcal{Q}_{dW}^{11}$ that are chirality-flipping, as well as the chiral operators $\mathcal{Q}_{qud}^{11}$ and $[\mathcal{Q}_{\varphi q}^{(3)}]^{11}$. Especially the latter could amplify the longitudinal W -boson scattering, and as it was shown in ref. [101] for $WW \rightarrow WW$ scattering, these operators result in distinct kinematic distributions, possibly leading to the same effects in $WW \rightarrow hh$. However, they are strongly constrained by the Z -pole precision observables, e.g., for $[\mathcal{Q}_{\varphi q}^{(3)}]^{11}$, a recent electroweak analysis [102] shows a tight bound on its corresponding WC, of less than $0.02 \frac{1}{\text{TeV}^2}$. Despite that, the concern about such fermionic operators affecting studied process remains, especially at the dimension-8 level. Effects of a subset of such operators have been recently studied in Ref. [85] in the case of single Higgs production. Additionally, we neglect all CP -violating operators. We also do not consider the effects of the renormalization group equation (RGE) running of the WCs. Although it has been shown in the literature (see, e.g., [80, 103–106]) that RGE effects are often non-negligible, we have verified that, in our case, they can change the values of the dimension-6 WCs by about $\mathcal{O}(20\%)$. This may slightly modify our results, but with no significant impact on the final conclusions.

B. Bounds on Wilson coefficients

To estimate the possible impact of SMEFT operators on double Higgs production via VBF, one must assign numerical values to the Wilson coefficients. For dimension-6 WCs, one can assume the maximum values that

TABLE II. Chosen bosonic SMEFT dimension-6 and dimension-8 operators affecting the $VV \rightarrow hh$ process, and consequently, double Higgs boson production via VBF. Operators have been categorized based on their field structure: X denotes the field strength tensor, D the covariant derivative and φ the Higgs doublet.

Dimension-6		Dimension-8	
	φ^6		φ^8
$\mathcal{Q}_\varphi$	$(\varphi^\dagger \varphi)^3$	$\mathcal{Q}_{\varphi^8}$	$(\varphi^\dagger \varphi)^4$
	$\varphi^4 D^2$		$\varphi^4 D^4$
$\mathcal{Q}_{\varphi \square}$	$(\varphi^\dagger \varphi) \square (\varphi^\dagger \varphi)$	$\mathcal{Q}_{\varphi^4 D^4}^{(1)}$	$(D_\mu \varphi^\dagger D_\nu \varphi) (D^\nu \varphi^\dagger D^\mu \varphi)$
$\mathcal{Q}_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$	$\mathcal{Q}_{\varphi^4 D^4}^{(2)}$	$(D_\mu \varphi^\dagger D_\nu \varphi) (D^\nu \varphi^\dagger D^\mu \varphi)$
		$\mathcal{Q}_{\varphi^4 D^4}^{(3)}$	$(D_\mu \varphi^\dagger D^\mu \varphi) (D_\nu \varphi^\dagger D^\nu \varphi)$
			$\varphi^6 D^2$
		$\mathcal{Q}_{\varphi^6 D^2}$	$(\varphi^\dagger \varphi) (\varphi^\dagger D_\mu \varphi)^* (\varphi^\dagger D^\mu \varphi)$
		$\mathcal{Q}_{\varphi^6 \square}$	$(\varphi^\dagger \varphi)^2 \square (\varphi^\dagger \varphi)$
	$X^2 \varphi^2$		$X^2 \varphi^4$
$\mathcal{Q}_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{Q}_{W^2 \varphi^4}^{(1)}$	$(\varphi^\dagger \varphi)^2 W_{\mu\nu}^I W^{I\mu\nu}$
$\mathcal{Q}_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	$\mathcal{Q}_{B^2 \varphi^4}^{(1)}$	$(\varphi^\dagger \varphi)^2 B_{\mu\nu} B^{\mu\nu}$
$\mathcal{Q}_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$		$X^2 \varphi^2 D^2$
		$\mathcal{Q}_{W^2 \varphi^2 D^2}^{(1)}$	$(D^\mu \varphi^\dagger D^\nu \varphi) W_{\mu\rho}^I W_\nu^{I\rho}$
		$\mathcal{Q}_{W^2 \varphi^2 D^2}^{(2)}$	$(D^\mu \varphi^\dagger \tau^I D^\nu \varphi) W_{\mu\rho}^I W_\nu^{I\rho}$
		$\mathcal{Q}_{B^2 \varphi^2 D^2}^{(1)}$	$(D^\mu \varphi^\dagger D^\nu \varphi) B_{\mu\rho} B_\nu^\rho$
		$\mathcal{Q}_{B^2 \varphi^2 D^2}^{(2)}$	$(D^\mu \varphi^\dagger D_\mu \varphi) B_{\nu\rho} B^{\nu\rho}$
			$X \varphi^4 D^2$
		$\mathcal{Q}_{W \varphi^4 D^2}^{(1)}$	$(\varphi^\dagger \varphi) (D^\mu \varphi^\dagger \tau^I D^\nu \varphi) W_{\mu\nu}^I$
		$\mathcal{Q}_{B \varphi^4 D^2}^{(1)}$	$(\varphi^\dagger \varphi) (D^\mu \varphi^\dagger D^\nu \varphi) B_{\mu\nu}$

current fits to experimental data allow, as detailed in Refs. [107–109]. These values are displayed in Table III.

The maximum allowed values for C_φ (not shown in Table III) can be obtained from the current allowed 95% confidence level intervals for the trilinear Higgs coupling modifier $\kappa_{\lambda_3} = \lambda_3 / \lambda_3^{\text{SM}}$. Bounds on κ_{λ_3} derived from combined single- and double Higgs production searches read (with other couplings set to their SM predictions) [110, 111]¹

$$\begin{aligned} \kappa_{\lambda_3} &\in [-0.4, 6.3], \text{ ATLAS,} \\ \kappa_{\lambda_3} &\in [-1.2, 7.5], \text{ CMS.} \end{aligned} \quad (2.1)$$

¹There is also new CMS measurement constraining κ_{λ_3} to similar range, $\kappa_{\lambda_3} \in [-1.39, 7.02]$, based on di-Higgs production only [7].

TABLE III. The 95% CL bounds on the EFT coefficients from linear, $\mathcal{O}(\Lambda^{-2})$, and quadratic, $\mathcal{O}(\Lambda^{-4})$ EFT fits taken from [109]. $\Lambda = 1$ TeV is assumed. Results from individual and marginalized fits are presented separately.

WC	95% CL bounds, $\mathcal{O}(\Lambda^{-2})$		95% CI bounds, $\mathcal{O}(\Lambda^{-4})$	
	Individual	Marginalized	Individual	Marginalized
$C_{\varphi B}$	$[-0.005, 0.062]$	$[-0.310, 0.573]$	$[-0.005, 0.002] \cup [0.085, 0.092]$	$[-0.103, 0.152]$
$C_{\varphi W}$	$[-0.018, 0.006]$	$[-0.273, 0.707]$	$[-0.017, 0.006] \cup [0.282, 0.305]$	$[-0.067, 0.338]$
$C_{\varphi WB}$	$[0.007, 0.003]$	$[-0.525, 0.504]$	$[-0.007, 0.003]$	$[-0.190, 0.263]$
$C_{\varphi \square}$	$[0.416, 1.193]$	$[-1.715, 1.879]$	$[-0.429, 1.141]$	$[-1.856, 1.199]$
$C_{\varphi D}$	$[-0.027, -0.003]$	$[-1.063, 1.149]$	$[-0.027, -0.003]$	$[-0.513, 0.483]$

However, within linear field redefinitions in SMEFT, the trilinear Higgs coupling hhh is a momentum-dependent quantity through the WCs $C_{\varphi D}$ and $C_{\varphi \square}$ and related WCs associated with dimension-8 operators. Under the assumption that all other WCs are negligibly small compared to C_{φ} and C_{φ^8} , this limit can be translated into a bound on the following combination in the $\{G_F, m_W, m_Z\}$ -input parameter scheme [84,112],

$$\kappa_{\lambda_3} = 1 - \frac{C_{\varphi}}{(G_F m_h^2)(G_F \Lambda^2)} - \frac{\sqrt{2}C_{\varphi^8}}{(G_F m_h^2)(G_F \Lambda^2)^2}, \quad (2.2)$$

where $\lambda_3^{\text{SM}} = \sqrt{2}G_F m_h^2$. Assuming $C_{\varphi^8} = 0$ and $\Lambda = 1$ TeV, one can translate the bound on κ_{λ_3} from Eq. (2.1) into the direct bound on C_{φ} . Using the less stringent constraints from CMS one gets,

$$-\frac{13.8}{\text{TeV}^2} \lesssim C_{\varphi} \lesssim \frac{4.7}{\text{TeV}^2}. \quad (2.3)$$

While our study adopts conservative bounds on C_{φ} , we note that recent trilinear coupling analysis [113,114] suggests that somewhat stronger constraints may apply.

In the case of dimension-8 Wilson coefficients, there have been some attempts to impose constraints based on available experimental data and theoretical considerations, such as unitarity bounds [71,115]. However, these constraints are significantly weaker than the values adopted in this work following assumptions, which we list below. Consequently, we do not include them in our analysis. Instead, one can roughly estimate their maximal values using the approach of [116], based on the study of the validity of the EFT expansion, which leads to a particular power-counting of WCs, also known as naive dimensional analysis (NDA),

$$\frac{C_i}{\Lambda^{D-4}} = \frac{g_*^{n_i-2} \times c_i}{\Lambda^{D-4}}, \quad (2.4)$$

where D is the dimension of SMEFT operator, n_i is the number of fields involved, $g_* \leq 4\pi$ is a generic UV coupling, and c_i is an additional suppression factor related, e.g., to additional symmetries such as shift symmetry or

custodial symmetry. As a result, we get,

$$\begin{aligned} C_{\varphi^8} &= g_*^6 c_{\varphi^8}, & C_{\varphi^6 \square} &= g_*^4 c_{\varphi^6 \square}, \\ C_{\varphi^6 D^2} &= g_*^4 c_{\varphi^6 D^2}, & C_{\varphi^4 D^4}^{(i)} &= g_*^2 c_{\varphi^4 D^4}^{(i)}, \\ C_{V^2 \varphi^4}^{(i)} &= g_*^4 c_{V^2 \varphi^4}^{(1)}, & C_{V^2 \varphi^2 D^2}^{(i)} &= g_*^2 c_{V^2 \varphi^2 D^2}^{(1)}, \\ C_{V \varphi^4 D^2}^{(i)} &= g_*^3 c_{V \varphi^4 D^2}^{(1)}, \end{aligned} \quad (2.5)$$

with $V = W$ or B .

Further assumption of the maximal possible value of $g_* = 4\pi$ and no additional suppression factors ($c_i = 1$) is in principle allowed in strongly coupled UV scenarios, but may lead to problems with EFT expansion convergence. Let us consider as an example the Higgs boson mass in SMEFT. At the tree level, it receives contributions from all higher order operators of the form $\mathcal{Q}_{\varphi^n}(\varphi^\dagger \varphi)^{n+3}$. After EWSB, assuming the NDA form of (2.4) for C_{φ^n} , one has [117],

$$m_h^2 = \frac{1}{Z_h^2} \left[\lambda v^2 - g_*^2 v^2 \sum_{n=0}^{\infty} (n+2)(n+3) c_{\varphi^n} \left(\frac{g_*^2 v^2}{2\Lambda^2} \right)^{n+1} \right], \quad (2.6)$$

where the constant Z_h is required to ensure canonical normalization of the Higgs field in the kinetic terms, and is dependent on WCs of the form $\varphi^{4+2n} D^2$, $n = 0, 1, 2, \dots$ in Table II. Assuming that all c_{φ^n} are of order $\mathcal{O}(1)$ (or at least not growing faster than n to some fixed power), such a power series converges if,

$$\frac{g_*^2 v^2}{2\Lambda^2} < 1. \quad (2.7)$$

Incidentally, Eq. (2.6) displays a certain fine-tuning for keeping $m_h^2 \ll \Lambda^2$, when the upper bound of (2.7) is nearly saturated. Similar constraints on WCs containing Higgs field derivatives follow from the requirements of consistency of the SMEFT Lagrangian. E.g., Z_h factor at dimension-6 level has the form [118],

$$Z_h = 1 + \frac{1}{4} C_{\varphi D} \frac{v^2}{M^2} - C_{\varphi \square} \frac{v^2}{M^2}. \quad (2.8)$$

This has to be greater than zero (otherwise a Higgs tachyon in SMEFT appears), leading to the condition,

$$\frac{|4C_{\varphi\Box} - C_{\varphi D}|v^2}{\Lambda^2} < \sqrt{2}, \quad (2.9)$$

or equivalently, using NDA for WCs through Eq. (2.5), to a bound,

$$\frac{g_*^2 v^2}{\Lambda^2} \lesssim \mathcal{O}(1). \quad (2.10)$$

Consequently, in the context of NDA estimates for the upper bounds on the magnitudes of Wilson coefficient, the assumption $g_* \sim 4\pi$ is generally invalid when the scale Λ is below approximately 3.1 TeV, where to ensure the convergence of the EFT series, the effective coupling g_* should satisfy a tighter constraint $g_* \lesssim \frac{\Lambda}{v}$.

Additionally, assuming the NDA scaling of Eq. (2.4), the bounds on dimension-6 Wilson coefficients obtained from fits to the experimental dataset a natural limit on UV coupling g_* . The most stringent limit arises from constraints on C_φ in Eq. (2.3), leading to,

$$g_* \lesssim \min \left(|C_\varphi^{\max}|^{1/4} \sqrt{\frac{\Lambda}{1 \text{ TeV}}}, 4\pi \right), \quad (2.11)$$

where, to be conservative, we use a maximal allowed value of $C_\varphi^{\max} = 13.8$ derived from Eq. (2.3). Likewise, applying the NDA estimation to the Wilson coefficient $C_{\varphi\Box}$ yields the following bound,

$$g_* \lesssim \min \left(\sqrt{|C_{\varphi\Box}^{\max}|} \frac{\Lambda}{1 \text{ TeV}}, 4\pi \right), \quad (2.12)$$

with $|C_{\varphi\Box}^{\max}| \approx 2$ following from Table III. For our estimates of possible enhancements to double Higgs production via the VBF channel in the SMEFT, we use the g_* limits displayed above as the maximal NDA value for WCs that currently have no experimental constraints. As operators with and without derivatives may arise from distinct underlying physics processes, we have chosen to employ different bounding schemes for each category, using the constraint (2.11) for C_{φ^8} and (2.12) for the remaining WCs not yet included in fits to experimental data.

Indeed, the above-mentioned “educated-NDA” procedure works quite well for keeping the EFT convergence under control. To illustrate this point, if we take the scaling of C_{φ^8} from Eq. (2.5) and impose the bound from Eq. (2.11), we arrive at (Λ expressed in TeV)

$$\frac{C_{\varphi^8}}{\Lambda^4} \sim \frac{g_*^6}{\Lambda^4} \sim \frac{10^{3/2}}{\Lambda}. \quad (2.13)$$

This demonstrates that $\frac{C_{\varphi^8}}{\Lambda^4}$ decreases as Λ grows and remains bounded. Critically, assuming a large coupling $g_* = 4\pi$ even at a relatively high scale like $\Lambda = 10 \text{ TeV}$ would lead to a drastically different result, $\frac{C_{\varphi^8}}{\Lambda^4} = 125\pi$, which is orders of magnitude larger and likely inconsistent with Eq. (2.2) and the convergence of EFT expansion.

In summary, to check the role of VBF processes in di-Higgs production, we employ the following bounds. For WCs corresponding to a nonderivative operator $\mathcal{Q}_{\varphi^8}$, we use the experimental CMS bound [111] on κ_{λ_3} and the resulting limits on C_φ in Eq. (2.3) together with the NDA scaling of (2.4) for $c_{\varphi^8} = 1$. We then obtain,

$$-\min \left(1.8 \sqrt{\frac{\Lambda}{1 \text{ TeV}}}, 4\pi \right) \lesssim g'_* \lesssim \min \left(1.4 \sqrt{\frac{\Lambda}{1 \text{ TeV}}}, 4\pi \right),$$

$$-\frac{31.6}{\text{TeV}^4} \lesssim \frac{C_{\varphi^8}}{\Lambda^4} \lesssim \frac{7.4}{\text{TeV}^4}. \quad (2.14)$$

However, as can be seen from the analytical expressions of the helicity amplitudes $VV \rightarrow hh$ given in Appendices A and B, the dominant contribution to di-Higgs production both in WW or ZZ scattering is proportional to the same combination of C_φ and C_{φ^8} as the one entering κ_{λ_3} in Eq. (2.2). For this reason, individual estimates of both WCs do not significantly affect numerical results and remain bounded primarily by already known experimental limits on κ_{λ_3} , as we discuss further in Sec. V.

For derivative operators, again being conservative, we assume maximal values slightly larger than those derived from global fits for *marginalized* and *linear* case presented in Table III,

$$|C_{\varphi\Box}| \lesssim \frac{2}{\text{TeV}^2}, \quad |C_{\varphi D}| \lesssim \frac{1.5}{\text{TeV}^2}, \quad |C_{\varphi(W,B,WB)}| \lesssim \frac{1}{\text{TeV}^2}, \quad (2.15)$$

which, after making use of (2.12) result in bounds,

$$g_* \lesssim \max \left(\sqrt{2} \frac{\Lambda}{1 \text{ TeV}}, 4\pi \right),$$

$$\frac{|C_{\varphi^6 D^2}|}{\Lambda^4} = \frac{|C_{V^2 \varphi^4}|}{\Lambda^4} = \frac{g_*^4}{\Lambda^4} \lesssim \frac{4}{(1 \text{ TeV})^4},$$

$$\frac{|C_{\varphi^4 D^4}|}{\Lambda^4} = \frac{|C_{V^2 \varphi^2 D^2}|}{\Lambda^4} = \frac{g_*^2}{\Lambda^4} \lesssim \frac{2}{(1 \text{ TeV})^2 \Lambda^2},$$

$$\frac{|C_{W\varphi^4 D^2}|}{\Lambda^4} = \frac{|C_{V\varphi^4 D^2}|}{\Lambda^4} = \frac{g_*^3}{\Lambda^4} \lesssim \frac{2\sqrt{2}}{(1 \text{ TeV})^3 \Lambda}. \quad (2.16)$$

The bounds from Eqs. (2.14) and (2.16) will serve as the primary input values for the subsequent analysis.

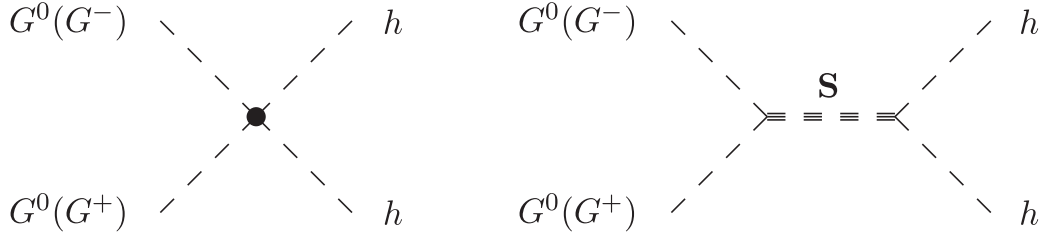


FIG. 2. Dominant diagrams contributing to the di-Higgs production in the real scalar SM extension. The black dot indicates the quartic Higgs coupling λ . The fields $G^0, G^\pm$ are the Goldstone bosons in the R_ξ -gauge we are using and h is the Higgs field.

III. WILSON COEFFICIENTS AND EFT CONVERGENCE IN SCALAR EXTENSIONS OF THE SM

Several questions arise when discussing the maximal enhancement of the double Higgs boson production within SMEFT. First, are the NDA estimates realistic, i.e., can one explicitly construct UV models which after decoupling of heavy particles lead to large values of WCs? Second, if WCs are large, can they affect the convergence of the EFT expansion in inverse powers of heavy mass scale? Third, how reliable is the SMEFT approximation for the growing energy of the process?

We address these questions by considering examples of a few simple extensions of the SM. These UV-completions are models with a heavy real scalar singlet field S or two types of heavy scalar triplets T_1, T_2 . They provide us with a useful theoretical laboratory for studying and testing the VBF di-Higgs production in the SMEFT.

A. Real scalar model

The matching onto the SMEFT of a heavy scalar singlet model has been worked out in several studies, with dimension-6 operators at tree-level [119,120] and one-loop [121,122] as well as dimension-8 operators [123]. By incorporating all renormalizable interactions between the heavy singlet S and the SM fields, we have

$$\mathcal{L}_S = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - \alpha M S \varphi^\dagger \varphi - \frac{1}{2}\kappa|\varphi|^2 S^2 - \frac{1}{3!}\beta M S^3 - \frac{1}{4!}\lambda_S S^4, \quad (3.1)$$

where $\mathcal{L}_{\text{SM}}$ is the SM-Lagrangian in the notation of Refs. [63,118] and φ is the SM Higgs doublet. Besides the heavy singlet mass M , all other parameters are dimensionless. By promoting the parameters of the theory into auxiliary fields, a background Z_2 -symmetry under which $\varphi \rightarrow \varphi$ and $S \rightarrow -S$, assigns charges (-1) to α and β and $(+1)$ to all other couplings. The Wilson coefficients in front of the Z_2 -invariant EFT expansions in terms of v^2/M^2 (or s/M^2 when calculating momentum-dependent terms in scattering amplitudes) should also have an Z_2 -invariant polynomial form,

$$C_i = f(\alpha^{n_1} \beta^{n_2}, \dots, \kappa, \lambda_S), \quad n_1 + n_2 = \text{even}, \quad (3.2)$$

Alternatively to the standard matching procedure, one can try to identify SMEFT WCs by comparing the corresponding terms of the amplitudes in both UV and effective theory in the energy region $v^2 \ll s \ll M^2$. Relevant to the di-Higgs production processes, we can use longitudinally polarized vector bosons $W_L W_L \rightarrow hh$ and $Z_L Z_L \rightarrow hh$ scattering, although the method works as well for other choices (e.g., $W_L W_L \rightarrow W_L W_L$ and $W_L W_L \rightarrow Z_L Z_L$). Let us compare the leading (not suppressed by additional v^2/M^2 powers) contributions to coefficients of s powers in the amplitude. In the high $s \gg v^2$ regime, the Goldstone boson equivalence theorem [124–126] simplifies the calculation since at high energy in SMEFT we only have to calculate $G^+ G^- \rightarrow hh$ and $G^0 G^0 \rightarrow hh$ and match to the same amplitudes in full theory at $s \ll M^2$. The tree level longitudinal amplitudes in the UV model for both processes are given by the same expression, including contributions from the SM contact 4-scalar interaction and the single s -channel exchange of the heavy scalar (see Fig. 2),

$$\begin{aligned} \mathcal{M}_{00}^{(G^+ G^- \rightarrow hh)}(s) &= \mathcal{M}_{00}^{(G^0 G^0 \rightarrow hh)}(s) \\ &= -\lambda + \frac{(-i\alpha M)^2}{s - M^2} \stackrel{s \ll M^2}{=} -\lambda \\ &\quad + \alpha^2 \left[1 + \frac{s}{M^2} + \frac{s^2}{M^4} \cdots + \right]. \end{aligned} \quad (3.3)$$

where λ is the quartic Higgs coupling in $\mathcal{L}_{\text{SM}}$: $-\frac{\lambda}{2}(\varphi^\dagger \varphi)^2$.

It is crucial to note the same prefactor α^2 for coefficients of $(s/M^2)^n$, $n = 0, 1, 2, \dots$ powers. On the other hand, the SMEFT amplitudes at large- s , but always below the singlet mass squared, give (in dimension-6 basis of [63] and dimension-8 basis of [127]; the explicit expressions for the amplitudes are presented in Appendices A and B)

$$\begin{aligned} \mathcal{M}_{00}^{(G^+ G^- \rightarrow hh)}(v^2 \ll s \ll M^2) &= -\lambda_{\text{eff}} - \frac{1}{2}(4C^{\varphi\Box} - C^{\varphi D}) \frac{s}{M^2} \\ &\quad + \frac{1}{8} \left[(C_{\varphi^4 D^4}^{(1)} + C_{\varphi^4 D^4}^{(2)})(1 + \cos^2 \theta) + 4C_{\varphi^4 D^4}^{(3)} \right] \frac{s^2}{M^4}, \end{aligned} \quad (3.4)$$

$$\begin{aligned}
\mathcal{M}_{00}^{(G^0 G^0 \rightarrow hh)}(v^2 \ll s \ll M^2) \\
= -\lambda_{\text{eff}} - (2C_{\varphi\Box} + C_{\varphi D}) \frac{s}{M^2} \\
+ \frac{1}{4} \left[2(C_{\varphi^4 D^4}^{(1)} + C_{\varphi^4 D^4}^{(3)}) - C_{\varphi^4 D^4}^{(2)} (1 - \cos^2 \theta) \right] \frac{s^2}{M^4}, \quad (3.5)
\end{aligned}$$

where θ is the scattering angle. By comparing the s^0 , s^1 , s^2 pieces of Eqs. (3.3), (3.4), and (3.5) we get

$$\lambda_{\text{eff}} = \lambda - \alpha^2, \quad (3.6)$$

$$C_{\varphi\Box} = -\frac{\alpha^2}{2}, \quad C_{\varphi D} = 0, \quad (3.7)$$

$$C_{\varphi^4 D^4}^{(3)} = 2\alpha^2, \quad C_{\varphi^4 D^4}^{(1)} = C_{\varphi^4 D^4}^{(2)} = 0, \quad (3.8)$$

where λ_{eff} corresponds to low-energy quartic SM Higgs coupling.

These results are in agreement with the literature [119–123]. Note also that the dimension-8 WCs are positive semidefinite according to the corresponding positivity bound [128]. Furthermore, C_φ can be found by comparing the corresponding amplitudes for φ^6 processes in full and effective theory. It is [122]

$$C_\varphi = -\frac{\alpha^2}{2} \left(\kappa - \frac{\alpha\beta}{3} \right). \quad (3.9)$$

The above procedure suggests the matching conditions to be taken at $v^2 \ll s = \Lambda_0^2 \ll M^2$, with Λ_0 could naturally be chosen at the scale defined by the unitarity violation of SMEFT. As usual, WCs at low scale can be then calculated using the RGEs evolution.

From the UV model's perspective, α is a free parameter, eventually constrained by perturbativity of couplings. In such case, the UV loop-expansion parameter should be limited to

$$\frac{\alpha^2}{(4\pi)^2} \lesssim 1, \quad (3.10)$$

so that $\alpha \lesssim 4\pi$ and maximal value of $C_{\varphi\Box}$ can saturate the NDA limit. Therefore, the SM extension with a real scalar provides a simple, explicit example of a model that can really produce large values for WCs in the low-energy limit.²

²However, one should note that some WCs may be constrained by additional conditions, such as the properties of the Higgs potential. Higher order operators contribute to it, ultimately affecting its stability and positions of minima. We do not study these issues in detail here, as our goal is simply to provide an example of consistent UV models which may produce large WCs.

Actually, it is possible to get even larger WCs from perturbative UV models. If instead of one heavy real scalar we add to the SM n copies of such fields with couplings $\alpha_1, \alpha_2, \dots$ and masses $M_1 \leq M_2 \leq \dots$ (in addition we assume for simplicity that heavy scalars interact feebly with each other and mainly couple to the SM Higgs-doublet), the formula for $C_{\varphi\Box}$ takes the form,

$$C_{\varphi\Box} = -\frac{1}{2M_1^2} \sum_{i=1}^n \alpha_i^2 M_i^2. \quad (3.11)$$

Obviously by increasing the number of heavy fields with similar masses one can make $C_{\varphi\Box}$ considerable, while keeping all the UV couplings α_i being perturbative.

To address the question of EFT convergence, let us note that comparison of Eq. (3.3) with Eqs. (3.4) and (3.5) shows that WCs of *all* higher order operators contributing to the dominant terms in the amplitude are proportional to α^2 and not to the higher powers of α . Consequently, they do not grow uncontrollably with increasing mass dimension and do not endanger the EFT series convergence. This remark is also in agreement with NDA as on dimensional ground all such couplings should be proportional to g_*^2 . From Eq. (3.3), and, at tree level in $\hbar$ -expansion, it is clear that EFT series always converges if we are below the heavy mass threshold, $s < M^2$, independently of WC values. Other WCs which may contribute to subdominant terms in the amplitude are proportional to higher powers of UV parameters and thus can be larger, but they are always multiplied by powers of (v/M) and therefore should not affect EFT convergence as long as condition (2.7) is fulfilled.

B. Triplet scalar models

As can be seen from Eq. (3.3), the dominant contribution to $W_L W_L \rightarrow hh$ and $Z_L Z_L \rightarrow hh$ in real scalar model has always opposite sign to SM term $-\lambda_{\text{eff}}$ (λ_{eff} should be positive for Higgs potential to be stable and to have proper minima). Thus, in such a model both processes are suppressed compared to the SM, unless the α -parameter is huge and BSM terms dominate the SM amplitude. This may not be the case for scalar triplet extensions.

Let us consider models containing heavy scalar triplets, first neutral $T_1(1, 3, 0)$ and then hypercharged $T_2(1, 3, 1)$ (numbers in parenthesis denote as usual $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers). In the first case, the interesting part of the Lagrangian is

$$\mathcal{L}_{T_1} = \mathcal{L}_{\text{SM}} + (D_\mu T_1)^\dagger (D^\mu T_1) - \alpha_{T_1} M \varphi^\dagger \sigma^a T_1^a \varphi + \dots \quad (3.12)$$

where σ^a are the Pauli matrices. By exploiting the same Z_2 -symmetry arguments as for the real singlet scalar, we deduce that WCs will only depend on $\alpha_{T_1}^2$.

In this case, the amplitudes $\mathcal{M}_{00}^{G^0 G^0 \rightarrow hh}$ and $\mathcal{M}_{00}^{G^+ G^- \rightarrow hh}$ are different, at high energy the first one comes from the s – channel only with neutral component of the triplet mediator T_1^0 plus the contact SM-like graph in Fig. 2, while the second one also contains contributions from the t and u channels with charged mediators exchange (not shown in Fig. 2). The amplitudes are

$$\begin{aligned} \mathcal{M}_{00}^{G^0 G^0 \rightarrow hh}(s) &= -\lambda + \frac{(i\alpha_{T_1} M)^2}{s - M^2} \\ &= -\lambda + \alpha_{T_1}^2 \left[1 + \frac{s}{M^2} + \frac{s^2}{M^4} + \dots \right], \end{aligned} \quad (3.13)$$

and

$$\begin{aligned} \mathcal{M}_{00}^{G^+ G^- \rightarrow hh}(s) &= -\lambda + \alpha_{T_1}^2 M^2 \left[\frac{1}{s - M^2} - \frac{1}{t - M^2} - \frac{1}{u - M^2} \right] \\ &= -\lambda + \alpha_{T_1}^2 - 2\alpha_{T_1}^2 \frac{s}{M^2} \\ &\quad - \frac{\alpha_{T_1}^2 s^2}{2 M^4} (1 - \cos^2 \theta) + \dots \end{aligned} \quad (3.14)$$

By performing the UV and EFT matching procedure we described earlier for the amplitudes in the real scalar scenario, and using the leading terms of amplitudes given in Appendices A and B, we arrive at,

$$\lambda_{\text{eff}} = \lambda - \alpha_{T_1}^2, \quad (3.15)$$

$$C_{\varphi\Box} = \frac{1}{2} \alpha_{T_1}^2, \quad C_{\varphi D} = -2\alpha_{T_1}^2, \quad (3.16)$$

$$C_{\varphi^4 D^4}^{(1)} = 4\alpha_{T_1}^2, \quad C_{\varphi^4 D^4}^{(2)} = 0, \quad C_{\varphi^4 D^4}^{(3)} = -2\alpha_{T_1}^2. \quad (3.17)$$

These results are consistent with Ref. [123] derived with functional matching. In such a model, $C_{\varphi\Box}$ is positive but nonvanishing negative $C_{\varphi D}$ is generated, so that effectively $Z_L Z_L \rightarrow hh$ cross section is still suppressed compared to SM but $W_L W_L \rightarrow hh$ becomes enhanced. Nevertheless, both effects cannot be large due to constraints on the $C_{\varphi D}$ which enters alone in the electroweak ρ -parameter as [118]

$$\rho - 1 = \frac{1}{2} \frac{v^2}{\Lambda^2} C_{\varphi D}. \quad (3.18)$$

The ρ -parameter deviation from unity is constrained by the electroweak precision observables to be at one per mille level. Hence, $|\alpha_{T_1}| \lesssim 0.01$ and the leading neutral scalar triplet contribution affecting di-Higgs production at high energies is strongly constrained.

The situation is different in charged heavy triplet model $T_2(1, 3, 1)$, with the Lagrangian

$$\begin{aligned} \mathcal{L}_{T_2} &= \mathcal{L}_{\text{SM}} + (D_\mu T_2)^\dagger (D^\mu T_2) \\ &\quad - \alpha_{T_2} M \tilde{\varphi}^\dagger \sigma^a T_2^a \varphi + \text{H.c.} + \dots \end{aligned} \quad (3.19)$$

In this case, high energy scattering amplitudes are given by

$$\begin{aligned} \mathcal{M}_{00}^{G^0 G^0 \rightarrow hh}(s) &= -\lambda + 2\alpha_{T_2}^2 M^2 \left[\frac{1}{s - M^2} - \frac{1}{t - M^2} - \frac{1}{u - M^2} \right] \\ &= -\lambda + 2\alpha_{T_2}^2 - 4\alpha_{T_2}^2 \frac{s}{M^2} \\ &\quad - \alpha_{T_2}^2 (1 - \cos^2 \theta) \frac{s^2}{M^4} + \dots, \end{aligned} \quad (3.20)$$

$$\begin{aligned} \mathcal{M}_{00}^{G^+ G^- \rightarrow hh}(s) &= -\lambda - \alpha_{T_2}^2 M^2 \left[\frac{1}{t - M^2} + \frac{1}{u - M^2} \right] \\ &= -\lambda + 2\alpha_{T_2}^2 - \alpha_{T_2}^2 \frac{s}{M^2} \\ &\quad + \frac{\alpha_{T_2}^2}{2} (1 + \cos^2 \theta) \frac{s^2}{M^4} + \dots \end{aligned} \quad (3.21)$$

and matching to SMEFT as before provides

$$\lambda_{\text{eff}} = \lambda - 2\alpha_{T_2}^2, \quad (3.22)$$

$$C_{\varphi\Box} = \alpha_{T_2}^2, \quad C_{\varphi D} = 2\alpha_{T_2}^2, \quad (3.23)$$

$$C_{\varphi^4 D^4}^{(1)} = 0, \quad C_{\varphi^4 D^4}^{(2)} = 4\alpha_{T_2}^2, \quad C_{\varphi^4 D^4}^{(3)} = 0. \quad (3.24)$$

Comparing to the singlet and the neutral triplet scenarios, both $C_{\varphi\Box}$ and $C_{\varphi D}$ are now positive and both $Z_L Z_L \rightarrow hh$ and $W_L W_L \rightarrow hh$ channels of di-Higgs production are enhanced in this case. In addition, adding both types of triplets (with identical masses) to the SM, one has

$$\begin{aligned} \lambda_{\text{eff}} &= \lambda - \alpha_{T_1}^2 - 2\alpha_{T_2}^2, \\ C_{\varphi\Box} &= \frac{1}{2} \alpha_{T_2}^2 + \alpha_{T_2}^2, \end{aligned} \quad (3.25)$$

$$\begin{aligned} C_{\varphi D} &= -2(\alpha_{T_1}^2 - \alpha_{T_2}^2), \\ C_{\varphi^4 D^4}^{(1)} &= 4\alpha_{T_1}^2, \quad C_{\varphi^4 D^4}^{(2)} = 4\alpha_{T_2}^2, \quad C_{\varphi^4 D^4}^{(3)} = -2\alpha_{T_1}^2. \end{aligned} \quad (3.26)$$

and in case of approximately (or exactly, due to some underlying symmetry) equal couplings, $\alpha_{T_1} \approx \alpha_{T_2}$, it is possible to get enhancement of the double Higgs production avoiding the constraints from the ρ -parameter. One should note that RGEs evolution can mix various WCs after decoupling and in principle lead, at lower energies, to problems with ρ -parameter constraints even if they were initially fulfilled. However, we have checked that in SMEFT for considered low Λ scales such effect is small and does not affect our conclusions.

In summary, we demonstrated with explicit examples that it is possible to find renormalizable UV-models which

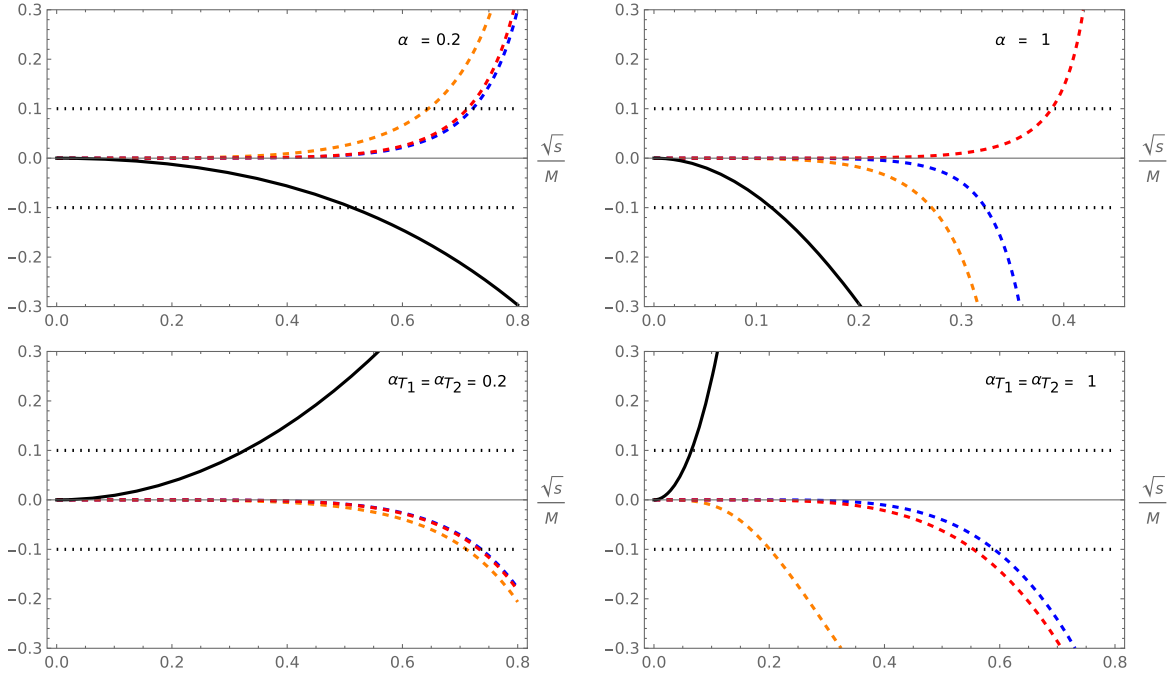


FIG. 3. The plots compare the relative accuracy of the $W_L W_L \rightarrow hh$ high energy scattering cross section as predicted by the SMEFT and underlying UV models. The dashed lines represent Δ_{EFT} from Eq. (3.27) as a function of $\sqrt{s}/M$. The orange curve shows the EFT result with dimension-6 terms in the amplitude alone, with higher-order terms truncated in the cross section. The blue and red curves include dimension-8 terms in the amplitude, either truncated or fully kept in its square, respectively. The dotted lines mark the $\pm 10\%$ range of relative accuracy. The solid black line shows the deviation of the cross section ratios from unity in the scalar models relative to the SM. The upper panels present the findings for a real scalar model, and the lower panels show the findings for a model extended with both neutral and charged triplets.

produce at low energies large WCs in SMEFT, that may increase or decrease the di-Higgs production rates through VBF and are consistent with EFT expansion convergence.

C. EFT accuracy for large $\sqrt{s}$

The issue related to the EFT convergence is how well the EFT approximation reproduces the full UV-model, depending on the value of $\sqrt{s}$. For a rough estimate, in Fig. 3 we plotted the relative accuracy of EFT approximation defined as

$$\Delta_{EFT} = \left| \frac{\mathcal{M}_{00(EFT)}^{(G^+ G^- \rightarrow hh)}}{\mathcal{M}_{00(UV)}^{(G^+ G^- \rightarrow hh)}} \right|^2 - 1, \quad (3.27)$$

with $\mathcal{M}_{00(EFT)}^{(G^+ G^- \rightarrow hh)}$ and $\mathcal{M}_{00(UV)}^{(G^+ G^- \rightarrow hh)}$ given by expressions for the real scalar or triplet models given above. Figure 3 presents results for two scenarios: the upper panels correspond to the real scalar model, and the lower panels to a Standard Model extension incorporating both neutral and charged triplets with degenerate masses and identical couplings, denoted by $\alpha_{T_1} = \alpha_{T_2}$. We also assumed that λ_{eff} is predominantly determined by the Higgs mass value, so that $\lambda_{\text{eff}} \approx 0.26$.

Figure 3 indicates that for small couplings $\alpha(\alpha_{T_i}) = 0.2$, the EFT remains a reliable tool for all considered UV-models up to at least $\sqrt{s}/M \sim 0.7$, provided dimension-8 terms are accounted for. With large couplings, around $\mathcal{O}(1)$, the EFT approximation maintains good agreement with UV-models in scenarios with enhanced cross sections. However, its validity is reduced to $\sqrt{s}/M \sim 0.4$ when SM-BSM interference is negative, potentially leading to a complete cancelation of the amplitude. This effect is exacerbated by the smallness of UV-amplitude in the denominator of Eq. (3.27), increasing the discrepancy. Considering the limited experimental interest in suppressed cross sections, the EFT offers a safe approach for studying enhanced di-Higgs VBF production at the LHC, even provided that the new physics scale is not much larger than the highest investigated invariant mass of the Higgs pair. Based on the preceding analysis, we adopt the bound on the ratio of the maximal center-of-mass energy to the new physics (NP) scale to ensure EFT convergence

$$\frac{\sqrt{s_{\text{max}}}}{\Lambda} \leq x \approx 1. \quad (3.28)$$

Although the value $x \approx 0.7$ is theoretically preferable for inequality (3.28), we adopt the more relaxed value $x = 1$.

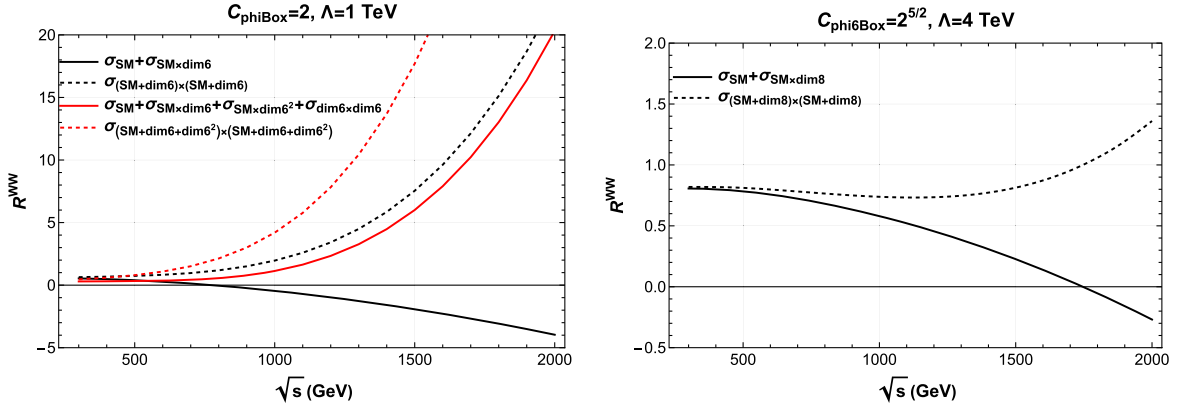


FIG. 4. Comparison of truncation schemes using the ratio R^{WW} between angular-integrated cross sections in the SMEFT and the SM [cf., Eq. (4.7)] for $C_{\phi\Box}$ dimension-6 WC and $C_{\phi^6\Box}$ dimension-8 WC (with numerical values displayed in the corresponding plots). Solid lines indicate a cross section truncated to terms of the same maximal dimension as included in amplitude, dashed lines indicate a cross section calculated by squaring amplitudes without any truncation.

This choice is justified by our numerical analysis in Sec. V, which shows that even $x = 1$ severely limits potential process enhancements. Consequently, the stricter bound $x \approx 0.7$ would yield even less favorable predictions.

It is also important to note that retaining only linear terms in dimension-6 Wilson coefficients yields considerably less accurate results. Furthermore, Fig. 3 demonstrates that when the discrepancy between EFT and UV predictions becomes significant for stronger couplings. In some cases, EFT more accurately reproduces the full result if the amplitude is truncated at dimension-8 terms, even when formally higher-order terms (up to dimension-12) arising from squaring the amplitude are retained in the cross section. More details are provided in the following subsection.

Finally, constraints on the maximum $\sqrt{s}$ for consistent EFT application can also arise from perturbative unitarity requirements (see, e.g., Refs. [101,115,129–134]). However, for center-of-mass energies achievable at the LHC, unitarity violation typically does not impose stringent bounds. For instance, for WCs of operators involving Higgs doublet derivatives, it leads to a condition of the form $C_{\Lambda^2} \lesssim \mathcal{O}(4\pi)$. This condition is generally satisfied if the relevant dimension-6 WCs remain within the bounds derived from fits to experimental data, as summarized in Eqs. (2.15) and (2.16). Similarly, our checks indicate that unitarity constraints from $2 \rightarrow 3$ and $2 \rightarrow 4$ processes, as discussed in Ref. [134], also do not provide meaningful constraints for $\sqrt{s} \lesssim 2$ TeV. Therefore, we do not consider such constraints in our analysis.³

³We also note that the existing literature on unitarity constraints in SMEFT generally does not consistently account for dimension-6² contributions and thus cannot be directly applied to our analysis, which includes such terms.

D. A note on truncation: Amplitude vs cross section

The general structure of the EFT amplitude includes terms at the leading order $\propto \frac{1}{\Lambda^2}$ (LO) and next-to-leading order $\propto \frac{1}{\Lambda^4}$ (NLO) in the EFT expansion,

$$\mathcal{M} = \mathcal{M}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{M}_{\text{dim } 6} + \frac{1}{\Lambda^4} \mathcal{M}_{\text{dim } 6^2} + \frac{1}{\Lambda^4} \mathcal{M}_{\text{dim } 8}, \quad (3.29)$$

where $\mathcal{M}_{\text{dim } 6}$ contains single insertions of dimension-6 WCs, $\mathcal{M}_{\text{dim } 6^2}$ their double insertions, and $\mathcal{M}_{\text{dim } 8}$ single insertions of dimension-8 WCs. At the level of cross section, this leads to corrections of order as high as $\propto \frac{1}{\Lambda^8}$,

$$\begin{aligned} \sigma \propto |\mathcal{M}|^2 &\equiv \sigma_{(\text{SM} + \text{dim } 6 + \text{dim } 6^2 + \text{dim } 8) \times (\text{SM} + \text{dim } 6 + \text{dim } 6^2 + \text{dim } 8)} \\ &\supset \frac{1}{\Lambda^8} |\mathcal{M}_{\text{dim } 6^2} + \mathcal{M}_{\text{dim } 8}|^2. \end{aligned} \quad (3.30)$$

In this work, we truncate only the amplitude in the EFT expansion. This means we calculate the amplitude up to a specific order but retain all higher-order terms, as those in Eq. (3.30), that arise when squaring it to obtain the cross section. Truncation at the cross section level would omit terms like those presented in Eq. (3.30), which are formally of dimension-12. While Fig. 3 shows that amplitude truncation does not always guarantee better accuracy than cross section truncation at a given order, a significant drawback of cross section truncation is its potential to yield negative cross section values. This behavior is illustrated in Fig. 4.

As a concrete example, let us discuss the issue of truncation within the real scalar singlet model of this section. At a very high energy, the amplitude (3.3) governs the dynamics of the longitudinal vector bosons scattering $V_L V_L \rightarrow hh$ with $V = Z, W^\pm$,

$$\mathcal{M}_{00}^{V_L V_L \rightarrow hh}(s) = -\lambda_{\text{eff}} + \alpha^2 \left(\frac{s}{M^2} + \frac{s^2}{M^4} + \dots \right), \quad (3.31)$$

where $\lambda_{\text{eff}} = \lambda - \alpha^2$. Now we square (3.31), to find

$$|\mathcal{M}_{00}^{V_L V_L \rightarrow hh}(s)|^2 = \underbrace{\lambda_{\text{eff}}^2 - 2\lambda_{\text{eff}}\alpha^2 \frac{s}{M^2} + \alpha^4 \frac{s^2}{M^4}}_{\sigma_{1/\Lambda^2}} + \underbrace{-2\lambda_{\text{eff}}\alpha^2 \frac{s^2}{M^4} + 2\alpha^2 \frac{s^3}{M^6} + \alpha^4 \frac{s^4}{M^8}}_{\sigma_{1/\Lambda^4}}. \quad (3.32)$$

Clearly, truncating the cross section at order $O(\sigma_{1/\Lambda^2})$ or $O(\sigma_{1/\Lambda^4})$ can lead to negative values. In the strong coupling regime ($\alpha \gtrsim 1$), this readily occurs for energies above $\sqrt{s} > \frac{M}{\alpha} \sqrt{\frac{\lambda_{\text{eff}}}{2}}$. Conversely, truncating the amplitude in such cases allows the noninterference term to dominate, preventing negative cross sections. Therefore, unless the coupling α is weak, comparable to λ_{eff} , truncating the full amplitude is the correct approach. Issues related to truncation and EFT validity have been discussed in Refs. [83,135,136], where similar conclusions were reached.

IV. HELICITY AMPLITUDES AND CROSS SECTIONS

Double Higgs boson production via VBF is a complicated multiparticle process in the SM, which becomes even more involved in SMEFT. For this reason, before performing the exact numerical calculations of the cross section for the $pp \rightarrow hhjj$ process, it is instructive to study the anatomy of individual on-shell subamplitudes contributing to the full amplitude. More specifically, we study the on-shell scattering of opposite-sign $W^\pm$ bosons or a pair of neutral Z -bosons into a pair of Higgs bosons h , namely

$$V(p_1, \lambda_1) + V(p_2, \lambda_2) \rightarrow h(p_3) + h(p_4), \quad (4.1)$$

where $V = W^\pm$ or Z , p_i are the four-momenta of the initial and final states, and λ_i represent the polarizations of the initial heavy vector bosons (two transverse— $\pm$ and longitudinal—0). The helicity amplitude for the $VV \rightarrow hh$ process can be expressed as

$$\mathcal{M}_{\lambda_1 \lambda_2}^{VV} \equiv \mathcal{M}_{\lambda_1 \lambda_2}^{VVHH}(\theta, s, G_F, M_W, M_Z, M_H, C_i), \quad (4.2)$$

where $\theta \in [0, \pi]$ is the scattering angle between the incoming and outgoing particles in the center-of-mass frame.

Not all the 9 possible helicity configurations are independent. As we neglect CP -violating WCs, the following relations hold:

$$\mathcal{M}_{++}^{VV} = \mathcal{M}_{--}^{VV}, \quad \mathcal{M}_{+-}^{VV} = \mathcal{M}_{-+}^{VV}, \quad \mathcal{M}_{0\pm}^{VV} = \mathcal{M}_{\pm 0}^{VV}. \quad (4.3)$$

leaving a total of four independent helicity structures. The differential cross section for a given helicity is given by,

$$\left(\frac{d\sigma^{VV}}{d\Omega} \right)_{\lambda_1 \lambda_2} = \frac{1}{2!} \frac{1}{9} \frac{1}{64\pi^2 s} \times |\mathcal{M}_{\lambda_1 \lambda_2}^{VV}(\theta, s, G_F, M_W, M_Z, M_H, C_i)|^2, \quad (4.4)$$

and the helicity cross section,

$$\sigma(s)_{\lambda_1 \lambda_2}^{VV} = \int_0^{2\pi} d\varphi \int_{\theta_{\text{cut}}}^{\pi - \theta_{\text{cut}}} d\theta \sin \theta \left(\frac{d\sigma^{VV}}{d\Omega} \right)_{\lambda_1 \lambda_2}. \quad (4.5)$$

By summing over all relevant helicities, the total cross section is

$$\sigma(s)^{VV} = \sigma(s)_{00}^{VV} + 2\sigma(s)_{++}^{VV} + 2\sigma(s)_{+-}^{VV} + 4\sigma(s)_{0+}^{VV}. \quad (4.6)$$

It is important to note that, as discussed in Sec. III, although we consistently truncate all amplitudes to dimension-8 level, we *do not* truncate expressions for cross sections in the same way, keeping instead all terms arising from squaring the amplitudes. This means that we include in cross section calculations also the subset of terms up to dimension-12 (such subset is gauge-invariant⁴). For large values of WCs it can lead to numerically significant differences comparing to cross sections truncated to dimension-8 terms only.

Explicit formulas for the leading order helicity amplitudes are provided in Appendix A for the $WW \rightarrow HH$ process and in Appendix B for the $ZZ \rightarrow HH$ process. All amplitudes and corresponding numerical results in this section were generated using the following chain of programs [112,137,138]:

⁴We have analytically and explicitly checked (with the help of `SmeftFR`-code [112]) that the amplitude $VV \rightarrow hh$ is gauge invariant, i.e., gauge-fixing ξ -parameter independent, at tree level for dim6 and dim6² contributions alone. We expect the same holds for the dim8 amplitude terms, although we have not verified the latter cases.

SmeftFR $\rightarrow$ FeynArts
 $\rightarrow$ FeynCalc

The new feature of this analysis is the *consistent* inclusion of $O(1/\Lambda^4)$ dimension-6² terms. Neglecting such terms, e.g., in normalization factors $Z_h, Z_W, \dots$ between weak eigenbasis and physical fields (see Refs. [112,118]), leads to error by a factor of $\mathcal{O}(10)$ in the amplitudes when the dimension-6² terms are relevant. Even more important is the fact that, failure of a consistent truncation in Z -normalization factors here would lead to gauge-fixing parameter dependent results and therefore meaningless S -matrix elements. This omission may turn out to be important in setting bounds to Wilson coefficients or predictions for physical observables in SMEFT.

Inspecting the formulas for helicity amplitudes displayed in Appendices A and B one can see that contributions from some dimension-6 operators ($Q_{\phi\Box}, Q_{\phi D}, Q_{\phi B}, Q_{\phi W}, Q_{\phi WB}$) may grow linearly with s . Including dimension-6² terms does not change this conclusion, the new contributions have the form $C^2 s / (G_F \Lambda^4)$. However, corrections to amplitudes due to some dimension-8 operators ($Q_{\phi^4 D^4}^{(1)}, Q_{\phi^4 D^4}^{(2)}, Q_{\phi^4 D^4}^{(3)}, Q_{B^2 \phi^2 D^2}^{(1)}, Q_{W^2 \phi^2 D^2}^{(1)}, Q_{W^2 \phi^2 D^2}^{(2)}$) grow faster with energy, like s^2 .

In Fig. 5 we illustrate possible enhancement of the $ZZ \rightarrow HH$ cross sections in SMEFT due to effects growing with energy. We present results in terms of the ratio to the SM cross section, defined as:

$$R^{VV} = \frac{\sigma_{\text{SMEFT}}(VV \rightarrow HH)}{\sigma_{\text{SM}}(VV \rightarrow HH)}, \quad V = (W^\pm, Z). \quad (4.7)$$

We show only the results for the $W^+W^- \rightarrow HH$ process, as for all operators which contribute to ZZ scattering their s -dependence is qualitatively similar to the W^+W^- case. For all plots and numerical results, we assume one nonvanishing WC at a time and its sign is chosen to maximize the cross section.

Plots in Fig. 5 are chosen to illustrate several important points. First, one can see that for dimension-6 WCs values close to the upper bounds dictated by fits to experimental data (see Table III), dimension-6² terms become dominant for $\sqrt{s} \gtrsim 600$ GeV (excluding C_ϕ , for which there are no double insertions at the level of the amplitude, hence the LO and NLO results are identical). Moreover, one can expect significant enhancement of the $VV \rightarrow HH$ cross section from all the classes of dimension-6 WCs, with specific examples presented in Fig. 5: ϕ^6 : C_ϕ , $\phi^4 D^2$: $C_{\phi\Box}$ and $X^2 \phi^2$: $C_{\phi W}$. For dimension-8 WCs, the most promising classes are ϕ^8 : C_{ϕ^8} , $\phi^6 D^2$: $C_{\phi^6 \Box}$ and $X^2 \phi^4$: $C_{W^2 \phi^4}^{(1)}$. The relative magnitude of enhancement from various WCs is primarily dependent on their maximal allowed values given

by numerical fits or NDA. The s -dependence can, for some operators, further significantly increase the R^{VV} ratio, but one should remember that in the high- s range, the number of produced events is lower and statistical errors of measurements larger, so this tail effect may be more difficult to be observed. However, even if not relevant for the HL-LHC, it might become more of significance for future planned experiments such as FCC-hh.

V. MAXIMAL ENHANCEMENT OF THE DOUBLE HIGGS PRODUCTION VIA VBF IN SMEFT

We now turn our attention toward finally answering the question: *What is the maximum enhancement in SMEFT of the double Higgs boson production rate through vector boson fusion at the HL-LHC?*

The answer requires an analysis of the full process $pp \rightarrow hh + \text{jets}$, rather than just the $VV \rightarrow hh$ subprocess which we discussed in detail in the previous sections, to analytically understand the most important effects of the higher-dimensional SMEFT operators. To this end, we performed a numerical Monte Carlo analysis of this process for the LHC and its upgrade HL-LHC. The results were obtained using MADGRAPH 5—v3_4_1 [139] event generator utilizing UFO [140] model files generated with SmeftFR v3.02 [112]. The process of interest is

$$pp > hhjj$$

where p represents the initial protons, h the Higgs bosons, and j the final-state jets. We utilized the VBF-specific cuts on kinematic variables that are based on the ATLAS recommendations (following [93,101,130]) on M_{jj} —invariant mass of a pair of jets, $\Delta\eta_{jj}$ —the rapidity separation of jets, η_j —the rapidity of individual jets, Δp_{Tj} —the transverse momentum of jets, and M_{hh} —the invariant mass of the pair of Higgs bosons

$$\begin{aligned} M_{jj} &\geq 700 \text{ GeV}, & \Delta\eta_{jj} &\geq 5, & |\eta_j| &\leq 4.5, \\ p_{Tj} &\geq 25 \text{ GeV}, & M_{hh} &\geq 400 \text{ GeV}. \end{aligned} \quad (5.1)$$

It should be noted that a more detailed study including Higgs decays to final states, may require more refined and final-state-specific cuts on kinematic variables.

Moreover, we assumed all fermion masses—apart from the top quark—to be equal to zero, and the flavor mixing matrices to be equal to identity. The numerical values of input parameters are in line with the default SmeftFR v3 “ G_F ” default input scheme and read

$$\begin{aligned} G_F &= 1.1638 \times 10^{-5} \text{ GeV}^{-2}, & M_Z &= 91.1876 \text{ GeV}, \\ M_W &= 80.379 \text{ GeV}, & M_H &= 125.35 \text{ GeV}, \\ M_t &= 172.76 \text{ GeV}. \end{aligned} \quad (5.2)$$

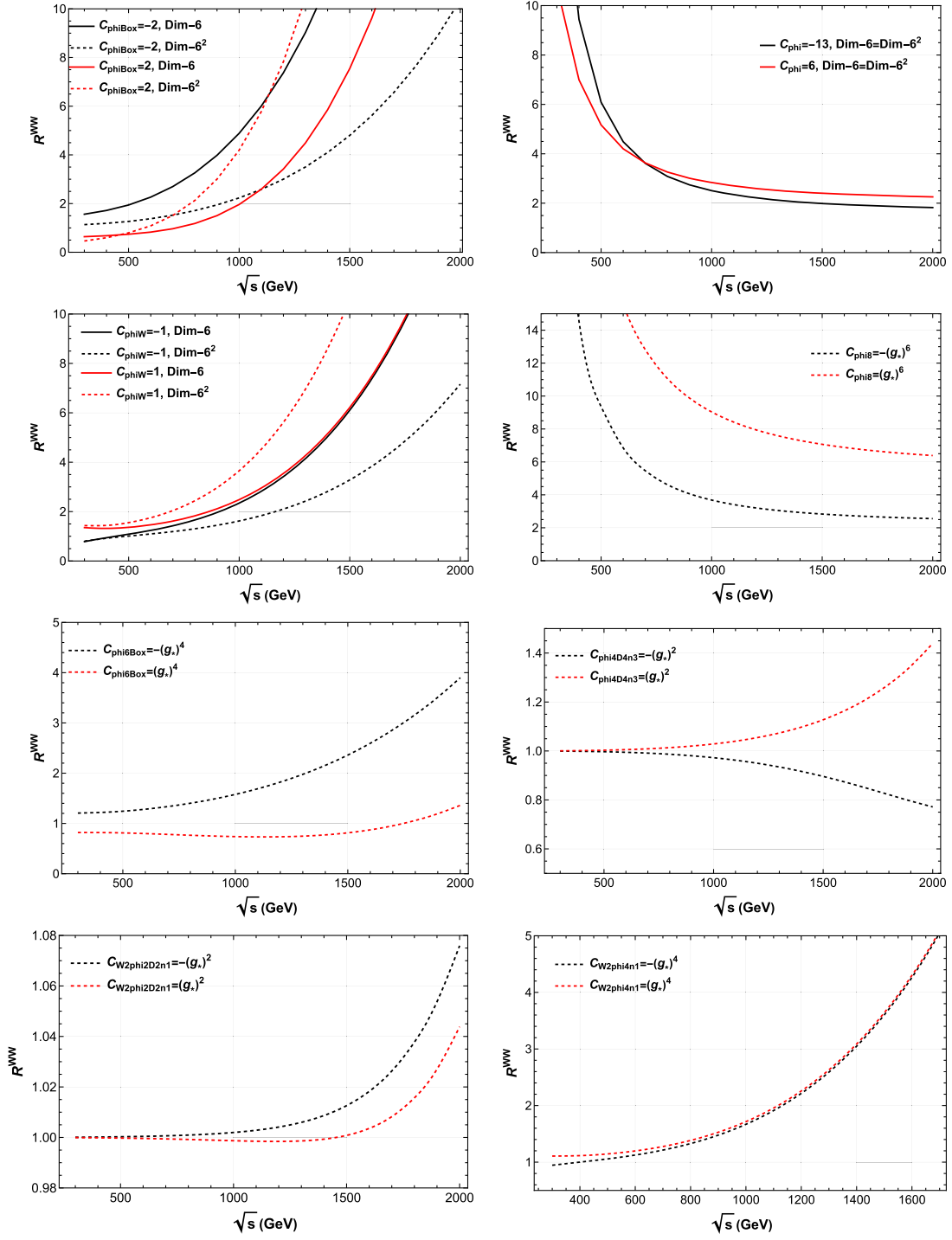


FIG. 5. Ratio R^{WW} between angular-integrated cross sections in the SMEFT and the SM for chosen dimension-6 WCs $C_{\varphi\Box}$, C_{φ} , $C_{\varphi W}$, including dimension-6 (solid lines) and dimension-6² (dashed lines) terms, and for dimension-8 WCs C_{φ^8} , $C_{\varphi^6\Box}$, $C_{\varphi^4 D^4}^{(3)}$, $C_{W^2\varphi^2 D^2}^{(1)}$, $C_{W^2\varphi^4}^{(1)}$. Values of dimension-6 WCs are given on corresponding plots and $\Lambda = 1$ TeV is assumed. For dimension-8 WCs, NDA scaling is assumed with numerical values of $g_* = \sqrt{2} \frac{\Lambda}{1 \text{ TeV}}$ and $\Lambda = 4$ TeV.

We run the simulations for the LHC and HL-LHC experiments, in the rest of the paper assuming the energy $\sqrt{s} = 13$ TeV and integrated luminosity of $\mathcal{L} = 30 \text{ fb}^{-1}$ for the LHC and $\sqrt{s} = 14$ TeV, $\mathcal{L} = 3000 \text{ fb}^{-1}$ for the HL-LHC.

As the maximal values of Wilson coefficients allowed by current experimental data (eventually extrapolated using the NDA assumption) we adopt their numerical values listed in Sec. II B. As some of them depend on the new

TABLE IV. Maximal enhancement of the di-Higgs production via VBF for C_φ modifying κ_{λ_3} .

	LHC		
	WC	$\Delta\sigma$	ΔN
$\frac{C_\varphi}{1 \text{ TeV}^2}$	4.7	6.0	81
	-13.8	22.2	336

	HL-LHC		
	WC	$\Delta\sigma$	ΔN
$\frac{C_\varphi}{1 \text{ TeV}^2}$	4.7	5.9	9550
	-13.8	21.5	39784

physics scale Λ , we employ the EFT validity criterion of Eq. (3.28) to determine its minimal value, limited by the condition that less than 1% of all generated Higgs pairs have invariant mass higher than Λ (see Sec. VB for more details).

We start with a study assuming one nonzero WC at a time (with an exception to C_φ and C_{φ^8} affecting κ_{λ_3} , see discussion in Sec. VA). For each run, we calculate the cross section and the number-of-event enhancements understood as

$$\Delta\sigma = \frac{\sigma^{\text{SMEFT}}}{\sigma^{\text{SM}}}, \quad \Delta N = N^{\text{SMEFT}} - N^{\text{SM}}. \quad (5.3)$$

Whenever necessary, we further distinguish the contributions from various subclasses of terms, e.g., $\Delta\sigma_{D6}$, $\Delta\sigma_{D6^2}$, $\Delta\sigma_{D8}$ denote cross section enhancement considering, respectively, dimension-6 terms only, dimension-6 and dimension-6² terms or dimension-8 terms only (and similarly for event excesses ΔN_{D6} , ΔN_{D6^2} , ΔN_{D8}). We use such a self-explanatory notation in all following tables and figures.

We display the results for both positive and negative ends of the allowed WC ranges. Moreover, we present kinematic distributions of Higgs pair invariant mass M_{hh} to illustrate where the BSM effects can be easiest to identify.

A. WCs affecting Higgs self-couplings

We start by analysing the enhancement associated with the triple Higgs coupling modifier κ_{λ_3} . As discussed in Sec. II B, experimental constraints on κ_{λ_3} can be translated into bounds on two Wilson coefficients that primarily affect this coupling: C_φ and C_{φ^8} . However, as can be seen from the formulas in Appendices A and B, the dominant contributions from both WCs affect mainly longitudinal scattering amplitudes $\mathcal{M}_{00}^{WW(ZZ)}$ and enter into relevant expression in the same linear combination as into κ_{λ_3} (note also that there are no C_φ^2 contributions, distinguishing LO and NLO orders of EFT expansion). Therefore, bounds on di-Higgs production are in this case directly given by current limits on κ_{λ_3} [see Eq. (2.2)], independently of individual values of C_φ and C_{φ^8} . For illustration, we present them assuming $C_\varphi \neq 0$ and $C_{\varphi^8} = 0$ using limits of Eq. (2.3). The numerical results of the impact of these Wilson coefficients on di-Higgs production are shown in Table IV, while the invariant mass distributions are displayed in Fig. 6. As can be seen from Table IV, the enhancement can be substantial, especially in the negative sign case. The distributions in Fig. 6 show, consistent with Fig. 5, that the most significant excess of events is expected at low energies, with $M_{hh} = \sqrt{s} \lesssim 1000$ GeV.

It should be noted that while in this instance the enhancement can reach factor $\mathcal{O}(20)$, gluon fusion would also cause a similar excess in the dominant production channel, resulting in a much earlier measurement. For this reason, we consider other Wilson coefficients, endemic for

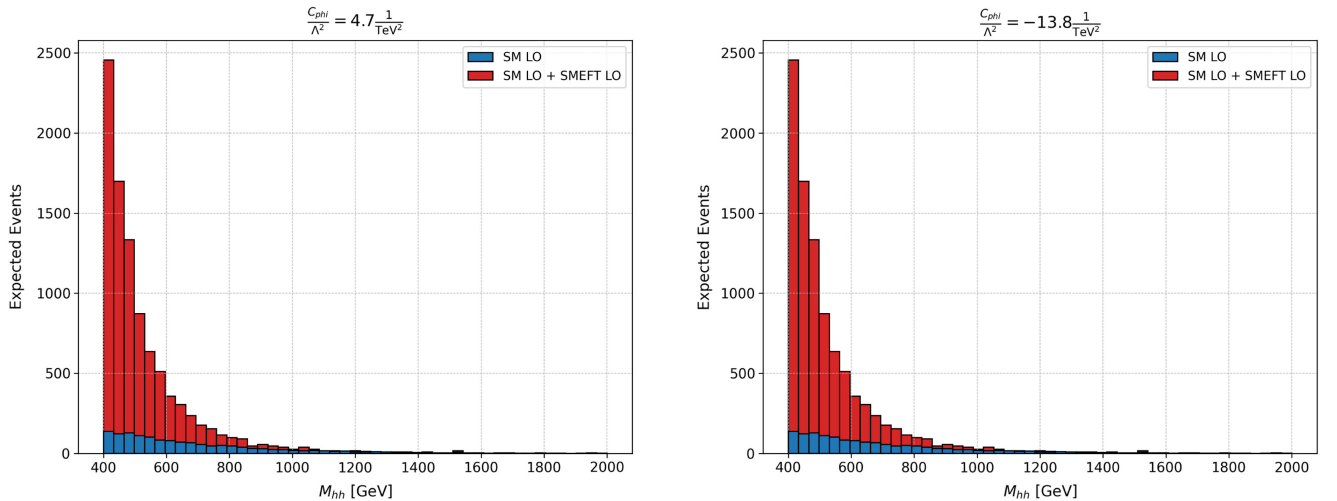
FIG. 6. Higgs pair invariant mass M_{hh} distribution for C_φ modifying κ_{λ_3} .

TABLE V. Maximal enhancement of VBF di-Higgs production process for dimension-6 WCs.

LHC					
WC	$\frac{C}{1 \text{ TeV}^2}$	$\Delta\sigma_{D6}$	ΔN_{D6}	$\Delta\sigma_{D6^2}$	ΔN_{D6^2}
$C_{\phi\Box}$	2	1.04	1	1.53	9
	-2	2.41	23	1.41	7
$C_{\phi D}$	1.5	1.08	1	1.07	1
	-1.5	1.00	0	1.02	0
$C_{\phi W}$	1	1.74	12	2.49	24
	-1	2.20	19	1.62	10
$C_{\phi WB}$	1	1.27	4	1.14	2
	-1	1.23	4	1.41	7
$C_{\phi B}$	1	1.39	6	1.69	11
	-1	1.38	6	1.17	3

HL-LHC					
WC	$\frac{C}{1 \text{ TeV}^2}$	$\Delta\sigma_{D6}$	ΔN_{D6}	$\Delta\sigma_{D6^2}$	ΔN_{D6^2}
$C_{\phi\Box}$	2	1.07	134	1.59	1144
	-2	2.42	2770	1.41	800
$C_{\phi D}$	1.5	1.07	140	1.07	131
	-1.5	1.01	23	1.02	38
$C_{\phi W}$	1	1.85	1645	2.72	3340
	-1	2.31	1504	1.67	1303
$C_{\phi WB}$	1	1.28	540	1.14	275
	-1	1.25	481	1.44	848
$C_{\phi B}$	1	1.40	773	1.72	1396
	-1	1.39	756	1.17	336

VBF double Higgs production, to be more interesting, as discussed in the following sections.

B. WCs affecting Higgs-vector boson interactions

1. Dimension-6 operators

In the case of dimension-6 Wilson coefficients, we assume numerical values based on the results of global fits cited in Table III. We use wider, “marginalized” bounds, as we first aim to examine their individual effects, keeping in mind that, in realistic scenarios, they never appear in isolation. We illustrate the magnitude of difference in results when only single insertions of dimension-6 WCs in the amplitudes are allowed (i.e., linear order), with the case in which we allow double insertions (quadratic order). Moreover, as argued in Sec. III D, we do not truncate higher order EFT terms at the level of the cross section as the consistent and correct choice.

Our results are presented in Table V and illustrated in Fig. 7 for the $C_{\phi\Box}$ and $C_{\phi W}$ WCs, which correspond to the strongest enhancements. These results indicate that the strongest enhancement of di-Higgs VBF production due to dimension-6 Wilson coefficients at the HL-LHC reaches at most 2.7 for $C_{\phi W}$, and is significantly lower for all

remaining WCs. Furthermore, they highlight the importance of NLO (dimension-6²) contributions in comparison to LO (dimension-6) ones. For example, in the case of $C_{\phi\Box} = -2$, the LO contribution is much larger than the NLO one, consistent with the behavior shown in the plots in Fig. 5. This is further illustrated by the Higgs invariant mass histograms in Fig. 7. The reason for this difference is the negative interference between the SM, LO, and NLO contributions in the negative-sign case, which can be examined in Eq. (A2). The histograms in Fig. 7 also demonstrate the energy growth of the cross section, as can be seen in Fig. 5. Finally, from these results, one can deduce the EFT validity scale in line with the requirement of Eq. (3.28): $\Lambda \gtrsim 3 \text{ TeV}$ for $C_{\phi\Box}$ and $\Lambda \gtrsim 4 \text{ TeV}$ for $C_{\phi W}$.

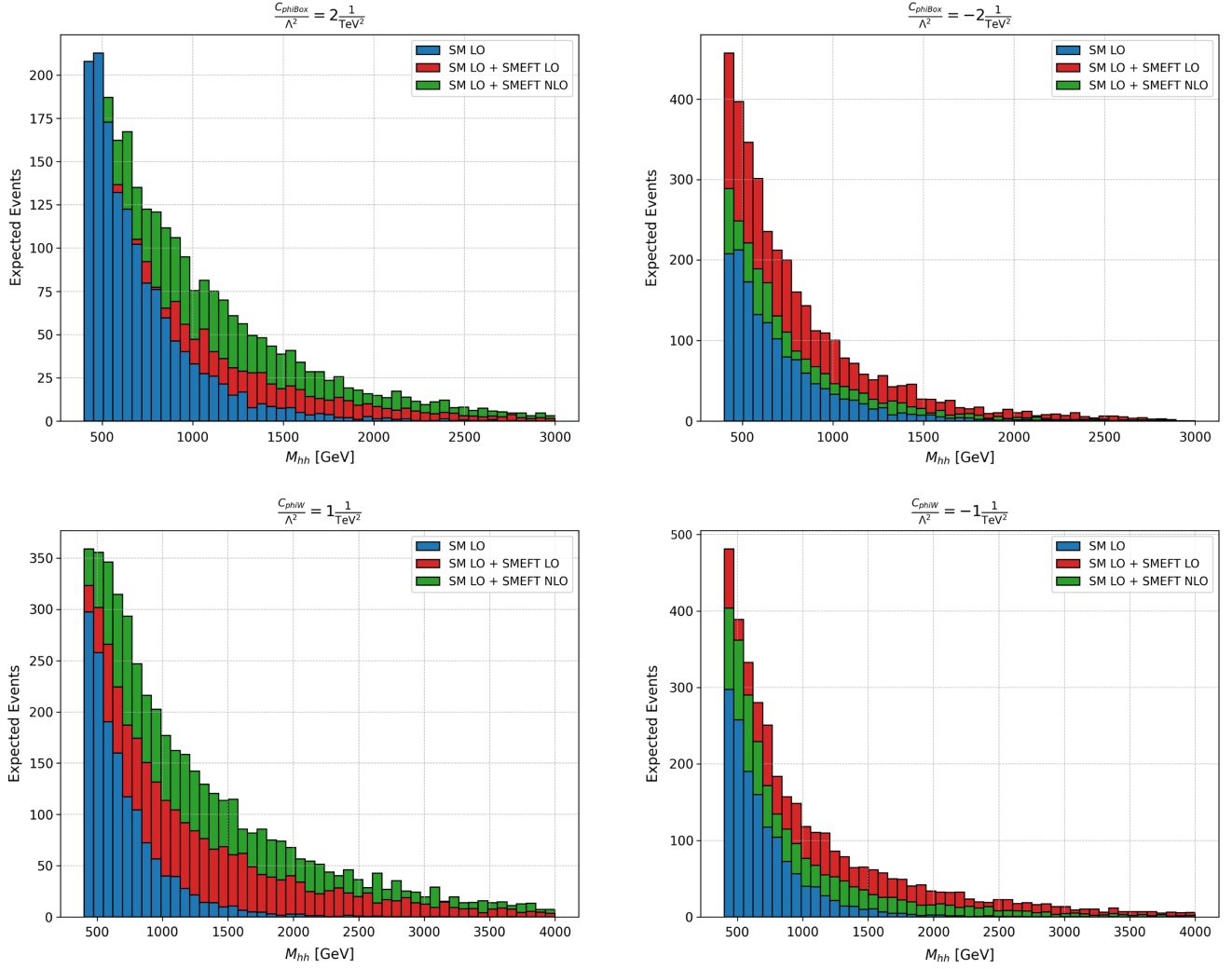
2. Dimension-8 operators

In the case of the dimension-8 Wilson coefficients considered in this paper, we assume their numerical values in line with Eq. (2.16), leading to coefficients $C_{\phi^4 D^4}$, $C_{V^2 \phi^2 D^2}$, and $C_{V \phi^4 D^2}$ scaling with the new physics scale Λ . To estimate their maximal effect on di-Higgs production, we must first find a minimal scale Λ consistent with the validity criterion of Eq. (3.28).

We do that by generating several event samples for $\Lambda = 1, 2, 3, 4, 5 \text{ TeV}$, while assigning corresponding numerical values to the Wilson coefficients, assuming their scaling in Eq. (2.16). For each sample, we impose a cut on the Higgs pair invariant mass, assuming $M_{hh} < \sqrt{s_{\text{max}}} = \Lambda$, and define the “minimal” value of Λ as the one for which the number of events excluded by this cut is of the order of $\mathcal{O}(1\%)$. We emphasize once more that a more stringent scrutiny of our assumptions would only result in a further reduction of the event excess. This procedure yields maximal absolute values for dimension-8 WCs ranging from $0.13/\text{TeV}^4$ to $4/\text{TeV}^4$, with EFT validity scale Λ generally exceeding 2–4 TeV. Then, at NLO, the most significant enhancements of di-Higgs VBF production at the HL-LHC, compared to Standard Model predictions, are at most a factor of 1.6 for $C_{W^2 \phi^4}^{(1)}$ and 1.3 for $C_{\phi^6 \Box}$. Enhancements for other WCs are considerably smaller. These increases are even less pronounced than those from dimension-6 WCs, making them unlikely to produce any observable effects given the expected HL-LHC sensitivity.

C. Interference effects of multiple WCs

If one assumes that several WCs do not vanish at the same time, at the NLO level, quadratic contributions from dimension-6² WCs may produce interesting interference effects, rising the possibility of additional di-Higgs production enhancement.

FIG. 7. Higgs pair invariant mass M_{hh} distribution for $C_{\phi\Box}$ and $C_{\phi W}$.

To assess the impact of several selected nonzero Wilson coefficients on VBF double Higgs production, we collect the results for chosen benchmark points defined in Table VI. The relations between WCs in points BP1 and BP2 follow the structure generated in the scalar singlet model discussed in Sec. III (see also, e.g., [123]). In such model, the leading contributions arise from the C_ϕ , C_{ϕ^8} , $C_{\phi\Box}$ and $C_{\phi^6\Box}$ WCs. BP1 includes only dimension-6 WCs, whereas BP2 also considers dimension-8 operators.⁵ Points BP3 and BP4 include $C_{\phi\Box}$, $C_{\phi^6\Box}$, $C_{\phi W}$, and $C_{W^2\phi^4}^{(1)}$ WCs (all potentially loop-generated here) and are chosen to illustrate

⁵At the amplitude level, dimension-8 WCs add linearly to other terms. However, as we do not truncate higher-order terms in cross sections, also dimension-8 terms may lead to nontrivial interference with other WCs; thus we include them in our benchmark points.

the possible enhancement with $\kappa_{\lambda_3} \simeq 1$, since C_ϕ and C_{ϕ^8} are set to zero.

The results of Monte Carlo simulations are summarized in Table VII and in the histograms of Fig. 8. One can draw the following conclusions:

- (i) BP1—The enhancement is primarily driven by modifications of κ_{λ_3} , resulting in scenario C in maximal increase of $\mathcal{O}(30)$ at linear order and up to $\mathcal{O}(65)$ at quadratic order, for both the LHC and HL-LHC. With luck, such a significant effect could be observed even before the HL-LHC era. Quadratic effects are large and have to be taken into account for viable cross section estimates, especially that at the NLO order they can lead to both significant enhancement or suppression of the final result, depending on relative WC signs.
- (ii) BP2—The conclusions mirror those for BP1, with the main difference arising from the inclusion of C_{ϕ^8}

TABLE VI. Four benchmark scenarios used to evaluate possible enhancement of the VBF di-Higgs production process. New physics scale $\Lambda = 4$ TeV is assumed. Letters A, B, C, and D label subscenarios that take into account different signs configuration.

BP1					
WC		C_φ	$C_{\varphi\Box}$		
$\frac{C}{1\text{ TeV}^2}$	A	4.7	2		
	B	4.7	-2		
	C	-13.8	2		
	D	-13.8	-2		
BP2					
WC		C_φ	C_{φ^8}	$C_{\varphi\Box}$	$C_{\varphi^6\Box}$
$\frac{C}{1\text{ TeV}^{(2,4)}}$	A	3.8	7.4	2	4
	B	3.8	7.4	-2	-4
	C	-10	-13.6	2	4
	D	-10	-13.6	-2	-4
BP3					
WC		$C_{\varphi\Box}$		$C_{\varphi W}$	
$\frac{C}{1\text{ TeV}^2}$	A	2		1	
	B	2		-1	
	C	-2		1	
	D	-2		-1	
BP4					
WC		$C_{\varphi\Box}$	$C_{\varphi^6\Box}$	$C_{\varphi W}$	$C_{W^2\varphi^4}^{(1)}$
$\frac{C}{1\text{ TeV}^{(2,4)}}$	A	2	4	1	4
	B	2	4	-1	-4
	C	-2	-4	1	4
	D	-2	-4	-1	-4

in the κ_{λ_3} modification, along with additional contributions from $C_{\varphi^6\Box}$.

- (iii) BP3—In the absence of contributions to κ_{λ_3} , the derivative WCs $C_{\varphi\Box}$ and $C_{\varphi W}$ can only produce smaller enhancement, reaching $\mathcal{O}(5)$ in scenario A. Again, quadratic effects are large and can lead to both constructive or destructive interference between WCs.
- (iv) BP4—The conclusions are consistent with those for BP3, with the enhancement of the cross section reaching $\mathcal{O}(9)$ after including also WCs of dimension-8 operators $C_{\varphi^6\Box}$ and $C_{W^2\varphi^4}^{(1)}$.

These results highlight the importance of including quadratic contributions in the analysis, both when calculating total cross sections and in finding the shape of invariant mass distribution (as illustrated in Fig. 8). Although operators related to κ_{λ_3} dominate the enhancement, interference between C_φ , C_{φ^8} and other WCs can

lead to large effects, potentially comparable to those in the gluon fusion channel. The important role is played here by the relative signs of various WCs. Even without tree-level triple Higgs boson coupling modification, i.e., when $\kappa_{\lambda_3} = 1$, enhancement from derivative WCs can reach up to $\mathcal{O}(10)$. This conclusion should be interpreted with caution, as we do not include the full set of dimension-8 operators; the result may therefore depend on the choice of operator basis.

We should also comment that our analysis is preliminary and thus does not specify more refined experimental cuts. In case of momentum-dependent Higgs-vector boson couplings, which are particularly interesting as they are weakly or not at all probed by the ggF channel, even a simple cut on invariant mass of the Higgs pair may further improve a ratio of signal from momentum dependent operators from the SM background. In Table VIII we show the results obtained assuming $M_{hh} \geq 1$ TeV. As one can see, comparing to numbers in Tables V and VII, such a cut can increase the signal to SM prediction ratio by additional factors of up to almost 5, while still leaving the total number of events large enough to be observable and statistically significant.

The presented results should be put into a context with expected experimental accuracy. Following the recent ATLAS report [141] on the projected HL-LHC sensitivity to double Higgs boson production, we quote it for the VBF-induced double Higgs production as

$$\mu_{\text{VBF-}hh} \equiv \Delta\sigma \gtrsim 10. \quad (5.4)$$

This value is significantly above the enhancements predicted in the previous sections, where only a single Wilson coefficient was considered at a time, leading to at most $\Delta\sigma \lesssim 3$ and limited prospects for observing VBF di-Higgs boson production at the HL-LHC within the regime of EFT validity.

However, as demonstrated in this section, in more elaborated multiple WC scenarios, or using cuts on invariant mass of the Higgs pair, the enhancement can become large enough to make the VBF channel detectable in future, in parallel to the ggF channel. Measuring (or putting upper limits) on both of them is valuable, as the sets of SMEFT WCs which they probe overlap only partially. The results shown highlight also the importance of considering the quadratic terms in the EFT expansion and interference-driven effects between multiple Wilson coefficients. Such effects can produce sizeable cross section enhancement in the VBF channel, even in cases where the ggF channel remains unchanged.

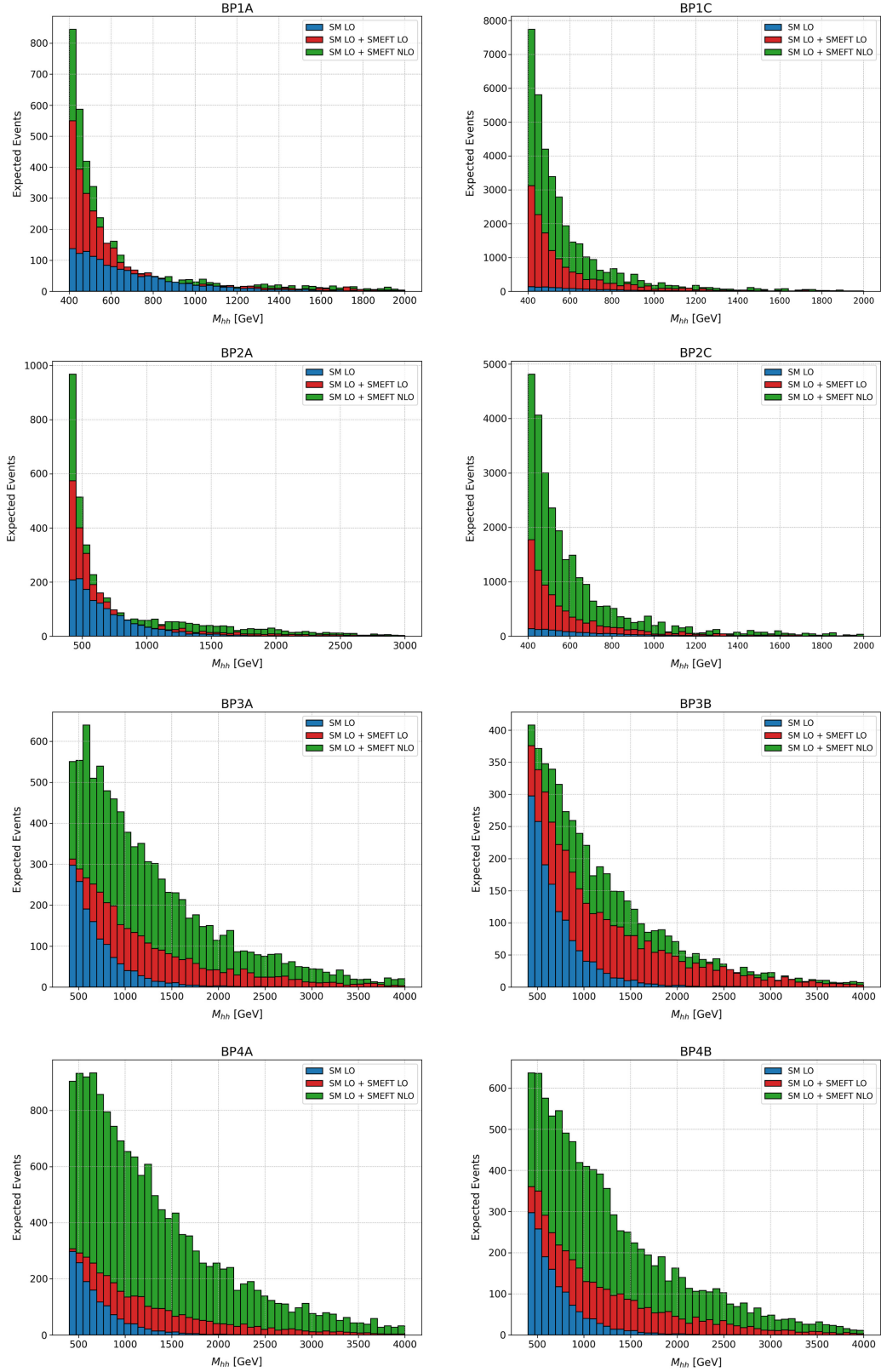


FIG. 8. Higgs pair invariant mass M_{hh} distribution for chosen BPs from Table VI for HL-LHC.

TABLE VII. Enhancement of VBF di-Higgs production for benchmark scenarios BP1–BP4. $\Delta\sigma_{D6}$ and $\Delta\sigma_{D6^2+D8}$ represent cross section enhancements at linear and full quadratic order (i.e., including in the amplitude dimension-6 terms only or also dimension-6² and dimension-8 contributions), respectively. ΔN_{D6} and ΔN_{D6^2+D8} indicate the respective event yield enhancements.

		LHC			
Scenario		$\Delta\sigma_{D6}$	ΔN_{D6}	$\Delta\sigma_{D6^2+D8}$	ΔN_{D6^2+D8}
BP1	A	3.95	48	6.36	87
	B	9.63	140	3.73	44
	C	28.17	440	65.64	1046
	D	16.97	259	3.98	48
BP2	A	2.90	31	5.17	68
	B	7.77	110	5.19	68
	C	15.30	231	46.48	736
	D	7.63	107	3.16	35
BP3	A	1.96	15	4.73	60
	B	2.06	17	2.76	29
	C	2.99	32	1.89	14
	D	3.77	45	1.64	10
BP4	A	1.96	15	8.00	113
	B	2.06	17	5.10	66
	C	2.99	32	3.80	45
	D	3.77	45	2.90	31

		HL-LHC			
Scenario		$\Delta\sigma_{D6}$	ΔN_{D6}	$\Delta\sigma_{D6^2+D8}$	ΔN_{D6^2+D8}
BP1	A	3.87	5581	6.23	10177
	B	9.48	16498	3.69	5230
	C	27.69	51934	65.02	124564
	D	16.57	30304	3.91	5656
BP2	A	2.87	3634	5.13	8035
	B	7.66	12955	5.13	8029
	C	15.13	27490	46.08	87724
	D	7.53	12700	3.12	4132
BP3	A	2.10	2134	5.21	8197
	B	2.21	2356	2.98	3856
	C	3.12	4129	1.98	1912
	D	3.91	5653	1.66	1282
BP4	A	2.10	2134	8.87	15316
	B	2.21	2356	5.62	8983
	C	3.12	4129	4.13	6097
	D	3.91	5653	3.02	3934

VI. CONCLUSIONS

Our study explores the influence of higher-dimensional operators of the Standard Model effective field theory on the double Higgs boson production process via vector boson fusion. Although the VBF channel is subdominant compared to gluon fusion, it is characterized by a unique experimental signature and provides sensitivity to a different spectrum of BSM interactions.

TABLE VIII. Maximal enhancement of VBF di-Higgs production for dimension-6 Wilson coefficients (WCs) obtained after applying $M_{hh} \geq 1$ TeV cut.

		HL-LHC			
WC	$\frac{C}{1 \text{ TeV}^{2,\pi}}$	$\Delta\sigma_{D6}$	ΔN_{D6}	$\Delta\sigma_{D6^2+D8}$	ΔN_{D6^2+D8}
$C_{\varphi\Box}$	2	2.62	332	4.96	810
$C_{\varphi W}$	1	6.35	1095	11.12	2070
$C_{\varphi^6\Box}$	−4	...	...	1.43	88
$C_{W^2\varphi^4}^{(1)}$	−4	...	...	3.80	573
BP3 scenario A		7.92	1415	23.09	4517
BP4 scenario A		7.92	1415	41.86	8357

We incorporated all bosonic CP -conserving dimension-6 operators and a chosen representative subset of dimension-8 operators, arranged in Table II. These operators may significantly impact the tree-level Higgs bosons self-interactions, and its interactions with W and Z gauge bosons.⁶ For the dimension-8 operators, we believe that our selection is adequate for capturing the overarching conclusions regarding the maximum possible enhancement of the VBF double Higgs production process within SMEFT. Our choice encompasses all main classes of dimension-8 operators corresponding to possible configurations of energy growth in the amplitudes. The complete set of such operators is very large and would obscure, rather than clarify, the conclusions and discussion presented here. Moreover, since the specific set of operators generated depends on the underlying UV model, a broader selection would not necessarily be more valid or useful. Our aim in this work is to illustrate representative patterns of discussed behavior, which can then be straightforwardly extrapolated to more realistic scenarios.

We determined realistic numerical values for the Wilson coefficients associated with the chosen SMEFT operators in Sec. II B. This involved either leveraging existing bounds (as detailed in Table III), that are derived from numerical fits to experimental data, or extrapolating such bounds to currently unconstrained WCs by applying naive dimension analysis assumptions. To further illustrate these concepts, we employed examples of singlet and triplet scalar extensions of the SM in Sec. III. This enabled us to demonstrate when and how such Wilson coefficient values could be realized after the decoupling of complete UV-models. We utilized a natural matching technique, deployed directly in the SMEFT mass basis, and leveraged the Goldstone boson equivalence theorem. This allowed us to directly map

⁶In SMEFT there are also $VV'h$ and $VV'hh$ vertices with $V, V' = \gamma, Z$ that are driven by $C_{\varphi W}, C_{\varphi B}, C_{\varphi WB}$ WCs. These are, of course, included in our numerical results but not shown analytically in the Appendices.

UV-model parameters to the relevant SMEFT Wilson coefficients, as evident from the high energy analytical forms of specific amplitudes (provided in the Appendices A and B). Concurrently, we also examined the convergence criteria for the effective field theory expansion using these same SM scalar extensions.

Subsequently, in Secs. IV and V, we conducted an in-depth analysis of the $VV \rightarrow hh$ and the full $pp \rightarrow hhjj$ cross sections, respectively. Our analysis included the full suite of next-to-leading order bosonic contributions (in the EFT expansion), specifically dimension-6, dimension-6², and dimension-8 terms, which contribute to the tree-level amplitudes. Through this analysis, we pinpointed which WCs exert the most significant impact and how these contributions vary with the invariant mass of the Higgs boson pair in the final state. The key findings and implications of our work, arranged in Tables IV–VIII and Figs. 6–8 are summarized as follows:

- (1) *Nonderivative operators* (C_ϕ, C_{ϕ^8}): these WCs contribute to the trilinear Higgs coupling and are by far the most significant in potential enhancement of the VBF di-Higgs cross section, by a factor of at most $\mathcal{O}(20)$ (see Table IV). Nevertheless, we also note that this kind of modification would also drastically change the ggF channel, which could result in its much earlier detection.
- (2) *Derivative operators* (e.g., $C_{\phi\Box}, C_{\phi W}$): the VBF di-Higgs production cross section exhibits only modest enhancements, typically at most a factor of $\mathcal{O}(2-3)$ for dimension-6 WCs, as presented in Table V. Even smaller improvement can be seen in dimension-8 WCs, where the increase is no more than of order $\mathcal{O}(1.5)$. Consequently, these effects are difficult to detect at the LHC and continue to be marginal with the High-Luminosity LHC (HL-LHC) upgrade. Nevertheless, we noticed that the inclusion of dimension-6² terms in the amplitude, may alter the predicted enhancement by $\approx 50\%$ barring cancellations with the full set of dimension-8 operators.
- (3) *In scenarios where multiple WCs contribute constructively*, particularly those involving derivative operators (e.g., $C_{\phi\Box}, C_{\phi W}$), the cross section can be enhanced by up to an order of magnitude. Such enhancements are most pronounced in kinematic regions with high invariant mass of the Higgs pair ($M_{hh} \gtrsim 1$ TeV), where the energy-dependent terms dominate—see Tables VI and VIII. This underscores the significance of incorporating contributions from

dimension-6² and dimension-8 operators, as their interference can substantially alter predictions.

In summary, while the VBF di-Higgs channel alone may not provide definitive evidence of BSM physics at the HL-LHC, it serves as an important complementary probe, especially for derivative interactions and more realistic, multi-WC scenarios. Our results highlight the importance of including higher-order EFT contributions, and emphasize the need for a careful assessment of EFT validity, motivating further experimental and theoretical efforts to explore this process in greater detail.

ACKNOWLEDGMENTS

The authors acknowledge support from the COMETA COST Action CA22130. The work of J.R. was supported by the Polish National Science Center under the research Grant No. DEC2019/35/B/ST2/02008. M.R. acknowledges support from the University of Padua under the 2023 STARS Grants@Unipd programme (Acronym and title of the project: HiggsPairs—Precise Theoretical Predictions for Higgs pair production at the LHC). We would like to thank Stefan Pokorski for numerous fruitful discussions.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: $W^+W^- \rightarrow HH$ HELICITY AMPLITUDE

1. SM

$$\begin{aligned}
 \mathcal{M}_{00}^{WW,SM} &= \sqrt{2}G_F M_H^2 \left[1 + 2 \left(1 - \frac{4}{\sin^2\theta} \right) \frac{M_W^2}{M_H^2} \right], \\
 \mathcal{M}_{\pm\pm}^{WW,SM} &= \mathcal{O}\left(\frac{1}{s}\right), \\
 \mathcal{M}_{\pm\mp}^{WW,SM} &= 2\sqrt{2}G_F M_W^2, \\
 \mathcal{M}_{0\pm}^{WW,SM} &= \mathcal{M}_{\pm 0}^{WW,SM} = \mathcal{O}\left(\frac{1}{\sqrt{s}}\right).
 \end{aligned} \tag{A1}$$

2. Dimension-6 SMEFT

$$\begin{aligned}
\mathcal{M}_{00}^{WW,D6} &= -3\sqrt{2}C_\varphi \left(\frac{1}{G_F\Lambda^2} \right) - 20C_{\varphi W} \left(\frac{M_W^2}{\Lambda^2} \right) + \frac{1}{2}(4C_{\varphi\Box} - C_{\varphi D}) \left(\frac{s}{\Lambda^2} \right) & \text{dim -6} \\
&+ \frac{1}{2\sqrt{2}}(4C_{\varphi\Box} - C_{\varphi D})^2 \left(\frac{1}{G_F\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right) - 3C_\varphi(4C_{\varphi\Box} - C_{\varphi D}) \left(\frac{1}{G_F\Lambda^2} \right)^2, & (\text{dim -6})^2 \\
\mathcal{M}_{\pm\pm}^{WW,D6} &= -(4C_{\varphi\Box} - C_{\varphi D}) \left(\frac{M_W^2}{\Lambda^2} \right) + 2C_{\varphi W} \left(\frac{s}{\Lambda^2} \right) \\
&+ \sqrt{2}C_{\varphi W}(4C_{\varphi\Box} - C_{\varphi D} + 6C_{\varphi W}) \left(\frac{1}{G_F\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right) - 6C_\varphi C_{\varphi W} \left(\frac{1}{G_F\Lambda^2} \right)^2, \\
\mathcal{M}_{\pm\mp}^{WW,D6} &= +(4C_{\varphi\Box} - C_{\varphi D}) \left(\frac{M_W^2}{\Lambda^2} \right) + 2\sqrt{2}C_{\varphi W}^2 \left(\frac{1}{G_F\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right), \\
\mathcal{M}_{0\pm}^{WW,D6} &= -4\sqrt{2}\cot\theta C_{\varphi W} \left(\frac{\sqrt{s}M_W}{\Lambda^2} \right) - 2C_{\varphi W}(4C_{\varphi W} + 4C_{\varphi\Box} - C_{\varphi D})\cot\theta \left(\frac{1}{G_F\Lambda^2} \right) \left(\frac{\sqrt{s}M_W}{\Lambda^2} \right). & (A2)
\end{aligned}$$

3. Dimension-8 SMEFT

For clarity, we divide the expressions for the dimension-8 terms into two classes.

a. $\varphi^8, \varphi^6 D^2, \varphi^4 D^4$ operator classes

$$\begin{aligned}
\mathcal{M}_{00}^{WW,D8} &= -6C_{\varphi^8} \left(\frac{1}{G_F^2\Lambda^4} \right) + \frac{\sqrt{2}}{4}(8C_{\varphi^6\Box} - C_{\varphi^6 D^2}) \frac{s}{\Lambda^2} \frac{1}{G_F\Lambda^2} \\
&- \frac{1}{8}[(C_{\varphi^4 D^4}^{(1)} + C_{\varphi^4 D^4}^{(2)} + 4C_{\varphi^4 D^4}^{(3)}) + (C_{\varphi^4 D^4}^{(1)} + C_{\varphi^4 D^4}^{(2)})\cos^2\theta] \left(\frac{s}{\Lambda^2} \right)^2, \\
\mathcal{M}_{\pm\pm}^{WW,D8} &= -\frac{1}{\sqrt{2}}(8C_{\varphi^6\Box} - C_{\varphi^6 D^2}) \left(\frac{M_W^2}{\Lambda^2} \right) \left(\frac{1}{G_F\Lambda^2} \right) \\
&+ \frac{1}{4}[4C_{\varphi^4 D^4}^{(3)} + (C_{\varphi^4 D^4}^{(1)} + C_{\varphi^4 D^4}^{(2)})\sin^2\theta] \left(\frac{M_W^2}{\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right), \\
\mathcal{M}_{\pm\mp}^{WW,D8} &= +\frac{\sqrt{2}}{4}(8C_{\varphi^6\Box} - C_{\varphi^6 D^2}) \frac{M_Z^2}{\Lambda^2} \frac{1}{G_F\Lambda^2} + \frac{1}{4}(C_{\varphi^4 D^4}^{(1)} + C_{\varphi^4 D^4}^{(2)})\sin^2\theta \left(\frac{M_W^2}{\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right), \\
\mathcal{M}_{0\pm}^{WW,D8} &= +\frac{1}{8\sqrt{2}}(C_{\varphi^4 D^4}^{(1)} + C_{\varphi^4 D^4}^{(2)}) \sin 2\theta \left(\frac{s^{3/2}M_W}{\Lambda^4} \right). & (A3)
\end{aligned}$$

b. $X^2\varphi^4, X^2\varphi^2 D^2, X\varphi^4 D^2$ operator classes

$$\begin{aligned}
\mathcal{M}_{00}^{WW,D8} &= +\left[\frac{1}{2}C_{W^2\varphi^2 D^2}^{(1)}(\sin^2\theta + 1) + 2C_{W^2\varphi^2 D^2}^{(2)} \right] \left(\frac{M_W^2}{\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right) \\
&- 14\sqrt{2}C_{W^2\varphi^4}^{(1)} \left(\frac{M_W^2}{\Lambda^2} \right) \left(\frac{1}{G_F\Lambda^2} \right) + \frac{5i}{2^{5/4}}C_{W\varphi^4 D^2}^{(1)} \left(\frac{M_W}{\sqrt{G_F}\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right), \\
\mathcal{M}_{\pm\pm}^{WW,D8} &= -\left(\frac{1}{4}C_{W^2\varphi^2 D^2}^{(1)} + C_{W^2\varphi^2 D^2}^{(2)} \right) \left(\frac{s^2}{\Lambda^4} \right) + 3\sqrt{2}C_{W^2\varphi^4}^{(1)} \left(\frac{1}{G_F\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right) - \frac{3i}{2^{5/4}}C_{W\varphi^4 D^2}^{(1)} \left(\frac{M_W}{\sqrt{G_F}\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right),
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{\pm\mp}^{WW,D8} &= -\frac{1}{8}\sin^2\theta C_{W^2\varphi^2D^2}^{(1)}\left(\frac{s^2}{\Lambda^4}\right) + 2^{3/4}C_{W\varphi^4D^2}^{(1)}\left(\frac{M_W}{\sqrt{G_F}\Lambda^2}\right)\left(\frac{M_W^2}{\Lambda^2}\right), \\
\mathcal{M}_{0\pm}^{WW,D8} &= +2^{1/4}iC_{W\varphi^4D^2}^{(1)}\cot\theta\left(\frac{M_W}{\sqrt{G_F}\Lambda^2}\right)\left(\frac{M_W\sqrt{s}}{\Lambda^2}\right) \\
&\quad - 4C_{W^2\varphi^4}^{(1)}\cot\theta\left(\frac{s^{1/2}M_W}{\Lambda^2}\right)\left(\frac{1}{G_F\Lambda^2}\right) - \frac{1}{8\sqrt{2}}C_{W^2\varphi^2D^2}^{(1)}\sin 2\theta\left(\frac{s^{3/2}M_W}{\Lambda^4}\right).
\end{aligned} \tag{A4}$$

APPENDIX B: $ZZ \rightarrow HH$ HELICITY AMPLITUDE

1. SM

$$\begin{aligned}
\mathcal{M}_{00}^{ZZ,SM} &= \sqrt{2}G_F M_H^2 \left[1 + 2\left(1 - \frac{4}{\sin^2\theta}\right)\frac{M_Z^2}{M_H^2}\right], \quad \mathcal{M}_{\pm\pm}^{ZZ,SM} = \mathcal{O}\left(\frac{1}{s}\right), \quad \mathcal{M}_{\pm\mp}^{ZZ,SM} = 2\sqrt{2}G_F M_Z^2, \\
\mathcal{M}_{0\pm}^{ZZ,SM} &= \mathcal{M}_{\pm 0}^{ZZ,SM} = \mathcal{O}\left(\frac{1}{\sqrt{s}}\right).
\end{aligned} \tag{B1}$$

2. Dimension-6 SMEFT

$$\begin{aligned}
\mathcal{M}_{00}^{ZZ,D6} &= -3\sqrt{2}C_\varphi\left(\frac{1}{G_F\Lambda^2}\right) + (2C_{\varphi\Box} + C_{\varphi D})\left(\frac{s}{\Lambda^2}\right) \quad \mathbf{dim -6} \\
&\quad - 20C_{\varphi W}\left(\frac{M_W^2}{\Lambda^2}\right) + 20C_{\varphi B}\left(\frac{M_W^2 - M_Z^2}{\Lambda^2}\right) - 20C_{\varphi WB}\left(\frac{M_W\sqrt{M_Z^2 - M_W^2}}{\Lambda^2}\right) \\
&\quad + \sqrt{2}\left(4C_{\varphi\Box}^2 - \frac{3}{4}C_{\varphi D}^2\right)\left(\frac{1}{G_F\Lambda^2}\right)\left(\frac{s}{\Lambda^2}\right) - \frac{3}{2}C_\varphi(8C_{\varphi\Box} - C_{\varphi D})\left(\frac{1}{G_F\Lambda^2}\right)^2, \quad (\mathbf{dim -6})^2 \\
\mathcal{M}_{\pm\pm}^{ZZ,D6} &= -2(2C_{\varphi\Box} + C_{\varphi D})\left(\frac{M_Z^2}{\Lambda^2}\right) + 2C_{\varphi W}\left(\frac{M_W^2}{M_Z^2}\right)\left(\frac{s}{\Lambda^2}\right) + 2C_{\varphi B}\left(1 - \frac{M_W^2}{M_Z^2}\right)\left(\frac{s}{\Lambda^2}\right) \\
&\quad + 2C_{\varphi WB}\frac{M_W}{M_Z}\sqrt{1 - \frac{M_W^2}{M_Z^2}}\left(\frac{s}{\Lambda^2}\right) + 6\sqrt{2}C_{\varphi W}^2\left(\frac{M_W^2}{M_Z^2}\right)\left(\frac{1}{G_F\Lambda^2}\right)\left(\frac{s}{\Lambda^2}\right) \\
&\quad + 6\sqrt{2}C_{\varphi B}^2\left(1 - \frac{M_W^2}{M_Z^2}\right)\left(\frac{1}{G_F\Lambda^2}\right)\left(\frac{s}{\Lambda^2}\right) + 2\sqrt{2}C_{\varphi WB}^2\left(1 - \frac{M_W^2}{2M_Z^2}\right)\left(\frac{1}{G_F\Lambda^2}\right)\left(\frac{s}{\Lambda^2}\right) \\
&\quad - 6C_\varphi\left[C_{\varphi W}\left(\frac{M_W^2}{M_Z^2}\right) + C_{\varphi B}\left(1 - \frac{M_W^2}{M_Z^2}\right) + C_{\varphi WB}\left(\frac{M_W}{M_Z}\right)\sqrt{1 - \frac{M_W^2}{M_Z^2}}\right]\left(\frac{1}{\Lambda^2 G_F}\right)^2 \\
&\quad + \left[-\frac{1}{\sqrt{2}}C_{\varphi W}C_{\varphi D}\left(\frac{M_W^2}{M_Z^2}\right) + 4\sqrt{2}C_{\varphi W}C_{\varphi\Box}\left(\frac{M_W^2}{M_Z^2}\right)\right. \\
&\quad \left.- \sqrt{2}C_{\varphi B}C_{\varphi D}\left(1 - \frac{M_W^2}{2M_Z^2}\right) + 4\sqrt{2}C_{\varphi B}C_{\varphi\Box}\left(1 - \frac{M_W^2}{M_Z^2}\right)\right. \\
&\quad \left.- \frac{3\sqrt{2}}{4}C_{\varphi WB}C_{\varphi D}\frac{M_W}{\sqrt{M_Z^2 - M_W^2}}\left(1 - \frac{2M_W^2}{3M_Z^2}\right) + 4\sqrt{2}C_{\varphi WB}C_{\varphi\Box}\frac{M_W}{M_Z}\sqrt{1 - \frac{M_W^2}{M_Z^2}}\right. \\
&\quad \left.+ 5\sqrt{2}C_{\varphi B}C_{\varphi WB}\left(\frac{M_W}{M_Z}\right)\sqrt{1 - \frac{M_W^2}{M_Z^2}} + 7\sqrt{2}C_{\varphi B}C_{\varphi WB}\left(\frac{M_W}{M_Z}\right)\sqrt{1 - \frac{M_W^2}{M_Z^2}}\right]\left(\frac{1}{G_F\Lambda^2}\right)\left(\frac{s}{\Lambda^2}\right),
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{\pm\mp}^{ZZ,D6} &= +(4C_{\varphi\Box} + C_{\varphi D}) \left(\frac{M_Z^2}{\Lambda^2} \right) \\
&+ \left[4\sqrt{2}C_{\varphi W}^2 \left(\frac{M_W^2}{M_Z^2} \right) + 4\sqrt{2}C_{\varphi B}^2 \left(\frac{M_W^2}{M_Z^2} \right) \left(1 - \frac{M_W^2}{M_Z^2} \right) + \sqrt{2}C_{\varphi WB}^2 \right. \\
&+ 2\sqrt{2}C_{\varphi WB}(C_{\varphi W} + C_{\varphi B}) \left(\frac{M_W}{M_Z} \right) \sqrt{1 - \frac{M_W^2}{M_Z^2}} \left. \left(\frac{1}{G_F\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right), \mathcal{M}_{0\pm}^{ZZ,D6} \right. \\
&= -4\sqrt{2}\cot\theta C_{\varphi W} \left(\frac{M_W^2}{M_Z^2} \right) \left(\frac{\sqrt{s}M_Z}{\Lambda^2} \right) - 4\sqrt{2}\cot\theta C_{\varphi B} \left(1 - \frac{M_W^2}{M_Z^2} \right) \left(\frac{\sqrt{s}M_Z}{\Lambda^2} \right) \\
&- 4\sqrt{2}\cot\theta C_{\varphi WB} \sqrt{1 - \frac{M_W^2}{M_Z^2}} \left(\frac{\sqrt{s}M_W}{\Lambda^2} \right) \\
&+ \left[-16\cot\theta C_{\varphi W}^2 \left(\frac{M_W^2}{M_Z^2} \right) - 16\cot\theta C_{\varphi B}^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) \left(\frac{1}{G_F\Lambda^2} \right) \left(\frac{\sqrt{s}M_Z}{\Lambda^2} \right) - 8\cot\theta C_{\varphi WB}^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) \right. \\
&- 2\cot\theta C_{\varphi W}(4C_{\varphi\Box} + C_{\varphi D}) \left(\frac{M_W^2}{M_Z^2} \right) - 2\cot\theta C_{\varphi B} \left(4C_{\varphi\Box} \left(1 - \frac{M_W^2}{M_Z^2} \right) + C_{\varphi D} \left(\frac{M_W^2}{M_Z^2} \right) \right) \\
&- 8\cot\theta C_{\varphi WB} C_{\varphi\Box} \left(\frac{M_W}{M_Z} \right) \sqrt{1 - \frac{M_W^2}{M_Z^2}} - \cot\theta C_{\varphi WB} C_{\varphi D} \left(\frac{M_W}{M_Z} \right) \left(1 - 2\frac{M_W^2}{M_Z^2} \right) \left(1 - \frac{M_W^2}{M_Z^2} \right)^{-1/2} \\
&- 4C_{\varphi WB}(3C_{\varphi W} + C_{\varphi B}) \left(\frac{M_W}{M_Z} \right) \sqrt{1 - \frac{M_W^2}{M_Z^2}} \left. \left(\frac{1}{G_F\Lambda^2} \right) \left(\frac{\sqrt{s}M_Z}{\Lambda^2} \right). \right. \tag{B2}
\end{aligned}$$

3. Dimension-8 SMEFT

a. φ^8 , $\varphi^6 D^2$, $\varphi^4 D^4$ operator classes

$$\begin{aligned}
\mathcal{M}_{00}^{ZZ,D8} &= -6C_{\varphi^8} \left(\frac{1}{G_F^2\Lambda^4} \right) + \sqrt{2}(2C_{\varphi^6\Box} + C_{\varphi^6 D^2}) \frac{s}{\Lambda^2} \frac{1}{G_F\Lambda^2} - \frac{1}{2} \left(C_{\varphi^4 D^4}^{(1)} - \frac{1}{2}C_{\varphi^4 D^4}^{(2)}\sin^2\theta + C_{\varphi^4 D^4}^{(3)} \right) \left(\frac{s}{\Lambda^2} \right)^2, \\
\mathcal{M}_{\pm\pm}^{ZZ,D8} &= -2\sqrt{2}(2C_{\varphi^6\Box} + C_{\varphi^6 D^2}) \frac{M_Z^2}{\Lambda^2} \frac{1}{G_F\Lambda^2} + \left[\left(C_{\varphi^4 D^4}^{(1)} - \frac{1}{2}C_{\varphi^4 D^4}^{(2)} + C_{\varphi^4 D^4}^{(3)} \right) - \frac{1}{2}C_{\varphi^4 D^4}^{(2)}\cos^2\theta \right] \left(\frac{M_Z^2}{\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right), \\
\mathcal{M}_{\pm\mp}^{ZZ,D8} &= +\sqrt{2} \left(2C_{\varphi^6\Box} + \frac{3}{4}C_{\varphi^6 D^2} \right) \frac{M_Z^2}{\Lambda^2} \frac{1}{G_F\Lambda^2} + \frac{1}{2}C_{\varphi^4 D^4}^{(2)}\sin^2\theta \left(\frac{M_Z^2}{\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right), \\
\mathcal{M}_{0\pm}^{ZZ,D8} &= +\frac{1}{4\sqrt{2}}C_{\varphi^4 D^4}^{(2)}\sin 2\theta \left(\frac{s^{3/2}M_Z}{\Lambda^4} \right). \tag{B3}
\end{aligned}$$

b. $X^2\varphi^4$, $X^2\varphi^2 D^2$, $X\varphi^4 D^2$ operator classes

$$\begin{aligned}
\mathcal{M}_{00}^{ZZ,D8} &= +14\sqrt{2}C_{B^2\varphi^4}^{(1)} \left(\frac{M_W^2 - M_Z^2}{\Lambda^2} \right) \left(\frac{1}{G_F\Lambda^2} \right) + 14\sqrt{2}C_{W^2\varphi^4}^{(1)} \left(\frac{M_W^2}{\Lambda^2} \right) \left(\frac{1}{G_F\Lambda^2} \right) \\
&+ \left[\frac{1}{2}C_{B^2\varphi^2 D^2}^{(1)}(\sin^2\theta + 1) + 2C_{B^2\varphi^2 D^2}^{(2)} \right] \left(1 - \frac{M_W^2}{M_Z^2} \right) \left(\frac{M_Z^2}{\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right)
\end{aligned}$$

$$\begin{aligned}
& + \left[\frac{1}{2} C_{W^2 \varphi^2 D^2}^{(1)} (\cos^2 \theta + 1) + 2 C_{W^2 \varphi^2 D^2}^{(2)} \right] \frac{M_W^2}{M_Z^2} \left(\frac{M_Z^2}{\Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right) \\
& - \frac{5i}{2^{5/4}} C_{B \varphi^4 D^2}^{(1)} \left(\frac{\sqrt{M_Z^2 - M_W^2}}{\sqrt{G_F} \Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right) + \frac{5i}{2^{5/4}} C_{W \varphi^4 D^2}^{(1)} \left(\frac{M_W}{\sqrt{G_F} \Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right), \\
\mathcal{M}_{\pm\pm}^{ZZ,D8} = & + 3\sqrt{2} C_{B^2 \varphi^4}^{(1)} \left(1 - \frac{M_W^2}{M_Z^2} \right) \left(\frac{1}{G_F \Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right) + 3\sqrt{2} C_{W^2 \varphi^4}^{(1)} \frac{M_W^2}{M_Z^2} \left(\frac{1}{G_F \Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right) \\
& - \left[\frac{1}{4} C_{B^2 \varphi^2 D^2}^{(1)} + C_{B^2 \varphi^2 D^2}^{(2)} \right] \left(1 - \frac{M_W^2}{M_Z^2} \right) \left(\frac{s^2}{\Lambda^4} \right) - \left[\frac{1}{4} C_{W^2 \varphi^2 D^2}^{(1)} + C_{W^2 \varphi^2 D^2}^{(2)} \right] \frac{M_W^2}{M_Z^2} \left(\frac{s^2}{\Lambda^4} \right) \\
& - \frac{3i}{2^{5/4}} C_{B \varphi^4 D^2}^{(1)} \left(\frac{\sqrt{M_Z^2 - M_W^2}}{\sqrt{G_F} \Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right) - \frac{3i}{2^{5/4}} C_{W \varphi^4 D^2}^{(1)} \left(\frac{M_W}{\sqrt{G_F} \Lambda^2} \right) \left(\frac{s}{\Lambda^2} \right), \\
\mathcal{M}_{\pm\mp}^{ZZ,D8} = & - \frac{1}{8} C_{B^2 \varphi^2 D^2}^{(1)} \left(1 - \frac{M_W^2}{M_Z^2} \right) \sin^2 \theta \left(\frac{s^2}{\Lambda^4} \right) - \frac{1}{8} C_{W^2 \varphi^2 D^2}^{(1)} \frac{M_W^2}{M_Z^2} \sin^2 \theta \left(\frac{s^2}{\Lambda^4} \right) \\
& + 2^{3/4} i C_{B \varphi^4 D^2}^{(1)} \left(\frac{\sqrt{M_Z^2 - M_W^2}}{\sqrt{G_F} \Lambda^2} \right) \left(\frac{M_Z^2}{\Lambda^2} \right) + 2^{3/4} i C_{W \varphi^4 D^2}^{(1)} \left(\frac{M_W}{\sqrt{G_F} \Lambda^2} \right) \left(\frac{M_Z^2}{\Lambda^2} \right), \\
\mathcal{M}_{0\pm}^{ZZ,D8} = & - 4 C_{B^2 \varphi^4}^{(1)} \left(1 - \frac{M_W^2}{M_Z^2} \right) \cot \theta \left(\frac{s^{1/2} M_Z}{\Lambda^2} \right) \left(\frac{1}{G_F \Lambda^2} \right) - 4 C_{W^2 \varphi^4}^{(1)} \frac{M_W^2}{M_Z^2} \cot \theta \left(\frac{s^{1/2} M_Z}{\Lambda^2} \right) \left(\frac{1}{G_F \Lambda^2} \right) \\
& - \frac{1}{8\sqrt{2}} C_{B^2 \varphi^2 D^2}^{(1)} \left(1 - \frac{M_W^2}{M_Z^2} \right) \sin 2\theta \left(\frac{s^{3/2} M_Z}{\Lambda^4} \right) - \frac{1}{8\sqrt{2}} C_{W^2 \varphi^2 D^2}^{(1)} \frac{M_W^2}{M_Z^2} \sin 2\theta \left(\frac{s^{3/2} M_Z}{\Lambda^4} \right) \\
& + 2^{1/4} i C_{B \varphi^4 D^2}^{(1)} \cot \theta \left(\frac{\sqrt{M_Z^2 - M_W^2}}{\sqrt{G_F} \Lambda^2} \right) \left(\frac{M_Z \sqrt{s}}{\Lambda^2} \right) + 2^{1/4} i C_{W \varphi^4 D^2}^{(1)} \cot \theta \left(\frac{M_W}{\sqrt{G_F} \Lambda^2} \right) \left(\frac{M_Z \sqrt{s}}{\Lambda^2} \right). \quad (\text{B4})
\end{aligned}$$

-
- [1] ATLAS Collaboration, Observation of a new particle in the search for the standard model Higgs boson with the ATLAS detector at the LHC, *Phys. Lett. B* **716**, 1 (2012).
- [2] CMS Collaboration, Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, *Phys. Lett. B* **716**, 30 (2012).
- [3] S. Glashow, Partial symmetries of weak interactions, *Nucl. Phys.* **22**, 579 (1961).
- [4] S. Weinberg, A model of leptons, *Phys. Rev. Lett.* **19**, 1264 (1967).
- [5] A. Salam, in *Proceedings of the Eighth Nobel Symposium*, edited by N. Svartholm (Wiley, New York, 1968) (1969), p. 367.
- [6] ATLAS Collaboration, Combination of searches for Higgs boson pair production in pp collisions at $s = 13$ TeV with the ATLAS detector, *Phys. Rev. Lett.* **133**, 101801 (2024).
- [7] CMS Collaboration, Combination of searches for nonresonant Higgs boson pair production in proton-proton collisions at $\sqrt{s} = 13$ TeV, CMS-PAS-HIG-20-011, 2024.
- [8] J. Baglio, A. Djouadi, R. Gröber, M. M. Mühlleitner, J. Quevillon, and M. Spira, The measurement of the Higgs self-coupling at the LHC: Theoretical status, *J. High Energy Phys.* **04** (2013) 151.
- [9] R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, P. Torrielli, E. Vryonidou, and M. Zaro, Higgs pair production at the LHC with NLO and parton-shower effects, *Phys. Lett. B* **732**, 142 (2014).
- [10] M. Cepeda *et al.*, Report from working group 2: Higgs physics at the HL-LHC and HE-LHC, *CERN Yellow Rep. Monogr.* **7**, 221 (2019).
- [11] J. Alison, M. Gouzevitch, J. Mazzitelli, and C. Vernieri, Higgs boson potential at colliders: Status and perspectives, *Rev. Phys.* **5**, 100045 (2020).
- [12] E. W. N. Glover and J. J. van der Bij, Higgs boson pair production via gluon fusion, *Nucl. Phys.* **B309**, 282 (1988).
- [13] T. Plehn, M. Spira, and P. M. Zerwas, Pair production of neutral Higgs particles in gluon-gluon collisions, *Nucl. Phys.* **B479**, 46 (1996).
- [14] D. de Florian and J. Mazzitelli, Higgs boson pair production at next-to-next-to-leading order in QCD, *Phys. Rev. Lett.* **111**, 201801 (2013).
- [15] J. Grigo, J. Hof, and M. Steinhauser, Higgs boson pair production: Top quark mass effects at NLO and NNLO, *Nucl. Phys.* **B900**, 412 (2015).
- [16] D. de Florian and J. Mazzitelli, Higgs pair production at next-to-next-to-leading logarithmic accuracy at the LHC, *J. High Energy Phys.* **09** (2015) 053.

- [17] S. Borowka, N. Greiner, G. Heinrich, S. P. Jones, M. Kerner, J. Schlenk, U. Schubert, and T. Zirke, Higgs boson pair production in gluon fusion at next-to-leading order with full top-quark mass dependence, *Phys. Rev. Lett.* **117**, 012001 (2016).
- [18] S. Borowka, N. Greiner, G. Heinrich, S. P. Jones, M. Kerner, J. Schlenk, and T. Zirke, Full top quark mass dependence in Higgs boson pair production at NLO, *J. High Energy Phys.* **10** (2016) 107.
- [19] M. Grazzini, G. Heinrich, S. Jones, S. Kallweit, M. Kerner, J. M. Lindert, and J. Mazzeitelli, Higgs boson pair production at NNLO with top quark mass effects, *J. High Energy Phys.* **05** (2018) 059.
- [20] R. Bonciani, G. Degrossi, P. P. Giardino, and R. Gröber, Analytical method for next-to-leading-order QCD corrections to double-Higgs production, *Phys. Rev. Lett.* **121**, 162003 (2018).
- [21] R. Gröber, A. Maier, and T. Rauh, Reconstruction of top-quark mass effects in Higgs pair production and other gluon-fusion processes, *J. High Energy Phys.* **03** (2018) 020.
- [22] J. Davies, G. Mishima, M. Steinhauser, and D. Wellmann, Double-Higgs boson production in the high-energy limit: Planar master integrals, *J. High Energy Phys.* **03** (2018) 048.
- [23] X. Xu and L. L. Yang, Towards a new approximation for pair-production and associated-production of the Higgs boson, *J. High Energy Phys.* **01** (2019) 211.
- [24] J. Baglio, F. Campanario, S. Glaus, M. Mühlleitner, M. Spira, and J. Streicher, Gluon fusion into Higgs pairs at NLO QCD and the top mass scheme, *Eur. Phys. J. C* **79**, 459 (2019).
- [25] J. Davies, G. Mishima, M. Steinhauser, and D. Wellmann, Double Higgs boson production at NLO in the high-energy limit: Complete analytic results, *J. High Energy Phys.* **01** (2019) 176.
- [26] J. Davies, G. Heinrich, S. P. Jones, M. Kerner, G. Mishima, M. Steinhauser, and D. Wellmann, Double Higgs boson production at NLO: Combining the exact numerical result and high-energy expansion, *J. High Energy Phys.* **11** (2019) 024.
- [27] L.-B. Chen, H. T. Li, H.-S. Shao, and J. Wang, Higgs boson pair production via gluon fusion at N³LO in QCD, *Phys. Lett. B* **803**, 135292 (2020).
- [28] L.-B. Chen, H. T. Li, H.-S. Shao, and J. Wang, The gluon-fusion production of Higgs boson pair: N³LO QCD corrections and top-quark mass effects, *J. High Energy Phys.* **03** (2020) 072.
- [29] J. Baglio, F. Campanario, S. Glaus, M. Mühlleitner, J. Ronca, M. Spira, and J. Streicher, Higgs-pair production via gluon fusion at hadron colliders: NLO QCD corrections, *J. High Energy Phys.* **04** (2020) 181.
- [30] G. Wang, Y. Wang, X. Xu, Y. Xu, and L. L. Yang, Efficient computation of two-loop amplitudes for Higgs boson pair production, *Phys. Rev. D* **104**, L051901 (2021).
- [31] J. Baglio, F. Campanario, S. Glaus, M. Mühlleitner, J. Ronca, and M. Spira, $gg \rightarrow HH$: Combined uncertainties, *Phys. Rev. D* **103**, 056002 (2021).
- [32] J. Davies, G. Mishima, K. Schönwald, M. Steinhauser, and H. Zhang, Higgs boson contribution to the leading two-loop Yukawa corrections to $gg \rightarrow HH$, *J. High Energy Phys.* **08** (2022) 259.
- [33] L. Bellafronte, G. Degrossi, P. P. Giardino, R. Gröber, and M. Vitti, Gluon fusion production at NLO: Merging the transverse momentum and the high-energy expansions, *J. High Energy Phys.* **07** (2022) 069.
- [34] A. H. Ajjath and H.-S. Shao, N³LO + N³LL QCD improved Higgs pair cross sections, *J. High Energy Phys.* **02** (2023) 067.
- [35] J. Davies, K. Schönwald, M. Steinhauser, and H. Zhang, Next-to-leading order electroweak corrections to $gg \rightarrow HH$ and $gg \rightarrow gH$ in the large- m_t limit, *J. High Energy Phys.* **10** (2023) 033.
- [36] E. Bagnaschi, G. Degrossi, and R. Gröber, Higgs boson pair production at NLO in the POWHEG approach and the top quark mass uncertainties, *Eur. Phys. J. C* **83**, 1054 (2023).
- [37] J. Davies, K. Schönwald, M. Steinhauser, and M. Vitti, Three-loop corrections to Higgs boson pair production: Reducible contribution, *J. High Energy Phys.* **08** (2024) 096.
- [38] G. Heinrich, S. Jones, M. Kerner, T. Stone, and A. Vestner, Electroweak corrections to Higgs boson pair production: The top-Yukawa and self-coupling contributions, *J. High Energy Phys.* **11** (2024) 040.
- [39] H.-Y. Bi, L.-H. Huang, R.-J. Huang, Y.-Q. Ma, and H.-M. Yu, Electroweak corrections to double Higgs production at the LHC, *Phys. Rev. Lett.* **132**, 231802 (2024).
- [40] S. Jaskiewicz, S. Jones, R. Szafron, and Y. Ulrich, The structure of quark mass corrections in the $gg \rightarrow HH$ amplitude at high-energy, *J. High Energy Phys.* **09** (2025) 015.
- [41] J. M. Campbell, G. De Laurentis, and R. K. Ellis, Analytic amplitudes for a pair of Higgs bosons in association with three partons, *J. High Energy Phys.* **10** (2024) 230.
- [42] J. Davies, K. Schönwald, M. Steinhauser, and H. Zhang, Analytic next-to-leading order Yukawa and Higgs boson self-coupling corrections to $gg \rightarrow HH$ at high energies, *J. High Energy Phys.* **04** (2025) 193.
- [43] J. Davies, K. Schönwald, and M. Steinhauser, Three-loop large- N_c virtual corrections to $gg \rightarrow HH$ in the forward limit, *J. High Energy Phys.* **08** (2025) 192.
- [44] Z. Hu, T. Liu, and J. M. Yang, $gg \rightarrow HH$ amplitude induced by bottom quarks at two-loop level: planar master integrals, *arXiv:2503.10051*.
- [45] M. Bonetti, P. Rendler, and W. J. Torres Bobadilla, Two-loop light-quark Electroweak corrections to Higgs boson pair production in gluon fusion, *J. High Energy Phys.* **07** (2025) 024.
- [46] M. Ciccolini, A. Denner, and S. Dittmaier, Electroweak and QCD corrections to Higgs production via vector-boson fusion at the LHC, *Phys. Rev. D* **77**, 013002 (2008).
- [47] T. Figy, Next-to-leading order QCD corrections to light Higgs pair production via vector boson fusion, *Mod. Phys. Lett. A* **23**, 1961 (2008).
- [48] P. Bolzoni, F. Maltoni, S.-O. Moch, and M. Zaro, Higgs production via vector-boson fusion at NNLO in QCD, *Phys. Rev. Lett.* **105**, 011801 (2010).
- [49] M. J. Dolan, C. Englert, N. Greiner, and M. Spannowsky, Further on up the road: $hhjj$ production at the LHC, *Phys. Rev. Lett.* **112**, 101802 (2014).

- [50] M. Cacciari, F. A. Dreyer, A. Karlberg, G. P. Salam, and G. Zanderighi, Fully differential vector-boson-fusion Higgs production at next-to-next-to-leading order, *Phys. Rev. Lett.* **115**, 082002 (2015).
- [51] M. J. Dolan, C. Englert, N. Greiner, K. Nordstrom, and M. Spannowsky, $hhjj$ production at the LHC, *Eur. Phys. J. C* **75**, 387 (2015).
- [52] F. A. Dreyer and A. Karlberg, Vector-boson fusion Higgs production at three loops in QCD, *Phys. Rev. Lett.* **117**, 072001 (2016).
- [53] J. Cruz-Martinez, T. Gehrmann, E. W. N. Glover, and A. Huss, Second-order QCD effects in Higgs boson production through vector boson fusion, *Phys. Lett. B* **781**, 672 (2018).
- [54] F. A. Dreyer and A. Karlberg, Vector-boson fusion Higgs pair production at N^3 LO, *Phys. Rev. D* **98**, 114016 (2018).
- [55] F. A. Dreyer and A. Karlberg, Fully differential vector-boson fusion Higgs pair production at next-to-next-to-leading order, *Phys. Rev. D* **99**, 074028 (2019).
- [56] F. A. Dreyer, A. Karlberg, and L. Tancredi, On the impact of non-factorisable corrections in VBF single and double Higgs production, *J. High Energy Phys.* **10** (2020) 131.
- [57] F. A. Dreyer, A. Karlberg, J.-N. Lang, and M. Pellen, Precise predictions for double-Higgs production via vector-boson fusion, *Eur. Phys. J. C* **80**, 1037 (2020).
- [58] B. Jäger, A. Karlberg, and S. Reinhardt, Precision tools for the simulation of double-Higgs production via vector-boson fusion, *J. High Energy Phys.* **06** (2025) 022.
- [59] S. Weinberg, Effective gauge theories, *Phys. Lett.* **91B**, 51 (1980).
- [60] S. R. Coleman, J. Wess, and B. Zumino, Structure of phenomenological Lagrangians. 1, *Phys. Rev.* **177**, 2239 (1969).
- [61] C. G. Callan, Jr., S. R. Coleman, J. Wess, and B. Zumino, Structure of phenomenological Lagrangians. 2, *Phys. Rev.* **177**, 2247 (1969).
- [62] W. Buchmüller and D. Wyler, Effective Lagrangian analysis of new interactions and flavor conservation, *Nucl. Phys.* **B268**, 621 (1986).
- [63] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, Dimension-six terms in the standard model Lagrangian, *J. High Energy Phys.* **10** (2010) 085.
- [64] J. Cao, Z. Heng, L. Shang, P. Wan, and J. M. Yang, Pair production of a 125 GeV Higgs boson in MSSM and NMSSM at the LHC, *J. High Energy Phys.* **04** (2013) 134.
- [65] U. Ellwanger, Higgs pair production in the NMSSM at the LHC, *J. High Energy Phys.* **08** (2013) 077.
- [66] B. Hespel, D. Lopez-Val, and E. Vryonidou, Higgs pair production via gluon fusion in the two-Higgs-doublet model, *J. High Energy Phys.* **09** (2014) 124.
- [67] C.-Y. Chen, S. Dawson, and I. M. Lewis, Exploring resonant di-Higgs boson production in the Higgs singlet model, *Phys. Rev. D* **91**, 035015 (2015).
- [68] G. Cacciapaglia, H. Cai, A. Carvalho, A. Deandrea, T. Flacke, B. Fuks, D. Majumder, and H.-S. Shao, Probing vector-like quark models with Higgs-boson pair production, *J. High Energy Phys.* **07** (2017) 005.
- [69] R. Grober, M. Muhlleitner, and M. Spira, Higgs pair production at NLO QCD for CP -violating Higgs sectors, *Nucl. Phys.* **B925**, 1 (2017).
- [70] K. Cheung, A. Jueid, C.-T. Lu, J. Song, and Y. W. Yoon, Disentangling new physics effects on nonresonant Higgs boson pair production from gluon fusion, *Phys. Rev. D* **103**, 015019 (2021).
- [71] A. Cappati, R. Covarelli, P. Torrielli, and M. Zaro, Sensitivity to new physics in final states with multiple gauge and Higgs bosons, *J. High Energy Phys.* **09** (2022) 038.
- [72] F. Feruglio, The chiral approach to the electroweak interactions, *Int. J. Mod. Phys. A* **08**, 4937 (1993).
- [73] G. Buchalla and O. Cata, Effective theory of a dynamically broken electroweak standard model at NLO, *J. High Energy Phys.* **07** (2012) 101.
- [74] R. Alonso, M. B. Gavela, L. Merlo, S. Rigolin, and J. Yepes, The effective chiral Lagrangian for a light dynamical “Higgs particle”, *Phys. Lett. B* **722**, 330 (2013).
- [75] I. Brivio, T. Corbett, O. J. P. Éboli, M. B. Gavela, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia, L. Merlo, and S. Rigolin, Disentangling a dynamical Higgs, *J. High Energy Phys.* **03** (2014) 024.
- [76] I. Brivio, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia, and L. Merlo, The complete HEFT Lagrangian after the LHC Run I, *Eur. Phys. J. C* **76**, 416 (2016).
- [77] F. Goertz, A. Papaefstathiou, L. L. Yang, and J. Zurita, Higgs boson pair production in the $D = 6$ extension of the SM, *J. High Energy Phys.* **04** (2015) 167.
- [78] A. Azatov, R. Contino, G. Panico, and M. Son, Effective field theory analysis of double Higgs boson production via gluon fusion, *Phys. Rev. D* **92**, 035001 (2015).
- [79] R. Grober, M. Muhlleitner, M. Spira, and J. Streicher, NLO QCD corrections to Higgs pair production including dimension-6 operators, *J. High Energy Phys.* **09** (2015) 092.
- [80] F. Maltoni, E. Vryonidou, and C. Zhang, Higgs production in association with a top-antitop pair in the standard model effective field theory at NLO in QCD, *J. High Energy Phys.* **10** (2016) 123.
- [81] N. Deutschmann, C. Duhr, F. Maltoni, and E. Vryonidou, Gluon-fusion Higgs production in the standard model effective field theory, *J. High Energy Phys.* **12** (2017) 063.
- [82] G. Buchalla, M. Capozzi, A. Celis, G. Heinrich, and L. Scyboz, Higgs boson pair production in non-linear effective field theory with full m_t -dependence at NLO QCD, *J. High Energy Phys.* **09** (2018) 057.
- [83] G. Heinrich, J. Lang, and L. Scyboz, SMEFT predictions for $gg \rightarrow HH$ at full NLO QCD and truncation uncertainties, *J. High Energy Phys.* **08** (2022) 079.
- [84] L. Alasfar *et al.*, Effective field theory descriptions of Higgs boson pair production, *SciPost Phys. Community Rep.* **2024**, 2 (2024).
- [85] B. Assi and A. Martin, Energy-enhanced dimension eight SMEFT effects in VBF Higgs production, *J. High Energy Phys.* **02** (2025) 029.
- [86] S. Di Vita, C. Grojean, G. Panico, M. Riembau, and T. Vantalón, A global view on the Higgs self-coupling, *J. High Energy Phys.* **09** (2017) 069.
- [87] L. Di Luzio, R. Gröber, and M. Spannowsky, Maxi-sizing the trilinear Higgs self-coupling: How large could it be?, *Eur. Phys. J. C* **77**, 788 (2017).

- [88] A. Falkowski and R. Rattazzi, Which EFT, *J. High Energy Phys.* **10** (2019) 255.
- [89] S. Chang and M. A. Luty, The Higgs trilinear coupling and the scale of new physics, *J. High Energy Phys.* **03** (2020) 140.
- [90] P. N. Bhattiprolu and J. D. Wells, A sensitivity target for an impactful Higgs boson self coupling measurement, *Phys. Lett. B* **861**, 139263 (2025).
- [91] H. T. Li, Z.-G. Si, J. Wang, X. Zhang, and D. Zhao, Improved constraints on Higgs boson self-couplings with quartic and cubic power dependencies of the cross section*, *Chin. Phys. C* **49**, 023107 (2025).
- [92] J. t. Hoeve, L. Mantani, J. Rojo, A. N. Rossia, and E. Vryonidou, The Higgs trilinear coupling in the SMEFT at the HL-LHC and the FCC-ee, [arXiv:2504.05974](https://arxiv.org/abs/2504.05974).
- [93] F. Bishara, R. Contino, and J. Rojo, Higgs pair production in vector-boson fusion at the LHC and beyond, *Eur. Phys. J. C* **77**, 481 (2017).
- [94] R. Gómez-Ambrosio, F. J. Llanes-Estrada, A. Salas-Bernárdez, and J. J. Sanz-Cillero, Distinguishing electroweak EFTs with $WLWL \rightarrow n \times h$, *Phys. Rev. D* **106**, 053004 (2022).
- [95] D. Domenech, M. J. Herrero, R. A. Morales, and M. Ramos, Double Higgs boson production at TeV e^+e^- colliders with effective field theories: Sensitivity to BSM Higgs couplings, *Phys. Rev. D* **106**, 115027 (2022).
- [96] Anisha, D. Domenech, C. Englert, M. J. Herrero, and R. A. Morales, Bosonic multi-Higgs correlations beyond leading order, *Phys. Rev. D* **110**, 095016 (2024).
- [97] S. Mahmud and K. Tobioka, Energy growth in $V_L V_L \rightarrow V_L V_L, V_L V_L h$ scattering to probe Higgs cubic and HEFT interactions, *J. High Energy Phys.* **09** (2024) 073.
- [98] R. L. Delgado, R. Gómez-Ambrosio, J. Martínez-Martín, A. Salas-Bernárdez, and J. J. Sanz-Cillero, Production of two, three, and four Higgs bosons: Where SMEFT and HEFT depart, *J. High Energy Phys.* **03** (2024) 037.
- [99] Anisha, D. Domenech, C. Englert, M. J. Herrero, and R. A. Morales, HEFT's appraisal of triple (versus double) Higgs weak boson fusion, *Phys. Rev. D* **111**, 055004 (2025).
- [100] J. Braun, P. Bredt, G. Heinrich, and M. Höfer, Double Higgs production in vector boson fusion at NLO QCD in HEFT, *J. High Energy Phys.* **07** (2025) 209.
- [101] A. Dedes, P. Kozów, and M. Szeleper, Standard model EFT effects in vector-boson scattering at the LHC, *Phys. Rev. D* **104**, 013003 (2021).
- [102] L. Bellafronte, S. Dawson, and P. P. Giardino, The importance of flavor in SMEFT electroweak precision fits, *J. High Energy Phys.* **05** (2023) 208.
- [103] R. Aoude, F. Maltoni, O. Mattelaer, C. Severi, and E. Vryonidou, Renormalisation group effects on SMEFT interpretations of LHC data, *J. High Energy Phys.* **09** (2023) 191.
- [104] S. Di Noi and R. Gröber, Renormalisation group running effects in $pp \rightarrow t\bar{t}h$ in the standard model effective field theory, *Eur. Phys. J. C* **84**, 403 (2024).
- [105] F. Maltoni, G. Ventura, and E. Vryonidou, Impact of SMEFT renormalisation group running on Higgs production at the LHC, *J. High Energy Phys.* **12** (2024) 183.
- [106] S. Di Noi, R. Gröber, and M. K. Mandal, Two-loop running effects in Higgs physics in standard model effective field theory, *J. High Energy Phys.* **12** (2025) 220.
- [107] J. Ellis, M. Madigan, K. Mimasu, V. Sanz, and T. You, Top, Higgs, Diboson and electroweak fit to the standard model effective field theory, *J. High Energy Phys.* **04** (2021) 279.
- [108] SMEFT Collaboration, Combined SMEFT interpretation of Higgs, Diboson, and top quark data from the LHC, *J. High Energy Phys.* **11** (2021) 089.
- [109] E. Celada, T. Giani, J. ter Hoeve, L. Mantani, J. Rojo, A. N. Rossia, M. O. A. Thomas, and E. Vryonidou, Mapping the SMEFT at high-energy colliders: From LEP and the (HL-) LHC to the FCC-ee, *J. High Energy Phys.* **09** (2024) 091.
- [110] ATLAS Collaboration, Constraints on the Higgs boson self-coupling from single- and double-Higgs production with the ATLAS detector using pp collisions at $s = 13$ TeV, *Phys. Lett. B* **843**, 137745 (2023).
- [111] CMS Collaboration, Constraints on the Higgs boson self-coupling from the combination of single and double Higgs boson production in proton-proton collisions at $s = 13$ TeV, *Phys. Lett. B* **861**, 139210 (2025).
- [112] A. Dedes, J. Rosiek, M. Ryczkowski, K. Suxho, and L. Trifyllis, SmeftFR v3—Feynman rules generator for the standard model effective field theory, *Comput. Phys. Commun.* **294**, 108943 (2024).
- [113] V. Maura, B. A. Stefanek, and T. You, The Higgs self-coupling at FCC-ee, [arXiv:2503.13719](https://arxiv.org/abs/2503.13719).
- [114] J. ter Hoeve, L. Mantani, J. Rojo, A. N. Rossia, and E. Vryonidou, The Higgs trilinear coupling in the SMEFT at the HL-LHC and the FCC-ee, *Phys. Rev. D* **112**, 013008 (2025).
- [115] L. C. Bresciani, G. Levati, and P. Paradisi, Amplitudes and partial wave unitarity bounds, [arXiv:2504.12855](https://arxiv.org/abs/2504.12855).
- [116] R. Contino, A. Falkowski, F. Goertz, C. Grojean, and F. Riva, On the validity of the effective field theory approach to SM precision tests, *J. High Energy Phys.* **07** (2016) 144.
- [117] L. Trifyllis, Theoretical and phenomenological aspects of the standard model effective field theory, Ph.D. thesis, University of Ioannina, 2022, [10.12681/eadd/52779](https://arxiv.org/abs/10.12681/eadd/52779).
- [118] A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek, and K. Suxho, Feynman rules for the standard model effective field theory in R_ξ -gauges, *J. High Energy Phys.* **06** (2017) 143.
- [119] D. Egana-Ugrinovic and S. Thomas, Effective theory of Higgs sector vacuum states, [arXiv:1512.00144](https://arxiv.org/abs/1512.00144).
- [120] S. Dawson, P. P. Giardino, and S. Homiller, Uncovering the high scale Higgs singlet model, *Phys. Rev. D* **103**, 075016 (2021).
- [121] U. Haisch, M. Ruhdorfer, E. Salvioni, E. Venturini, and A. Weiler, Singlet night in Feynman-ville: One-loop matching of a real scalar, *J. High Energy Phys.* **04** (2020) 164.
- [122] T. Cohen, X. Lu, and Z. Zhang, Functional prescription for EFT matching, *J. High Energy Phys.* **02** (2021) 228.
- [123] J. Ellis, K. Mimasu, and F. Zampedri, Dimension-8 SMEFT analysis of minimal scalar field extensions of the standard model, *J. High Energy Phys.* **10** (2023) 051.
- [124] J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Uniqueness of spontaneously broken gauge theories, *Phys. Rev. Lett.* **30**, 1268 (1973).

- [125] C. E. Vayonakis, Born helicity amplitudes and cross-sections in nonabelian gauge theories, *Lett. Nuovo Cimento* **17**, 383 (1976).
- [126] M. S. Chanowitz and M. K. Gaillard, The TeV physics of strongly interacting W's and Z's, *Nucl. Phys.* **B261**, 379 (1985).
- [127] C. W. Murphy, Dimension-8 operators in the standard model effective field theory, *J. High Energy Phys.* **10** (2020) 174.
- [128] G. N. Remmen and N. L. Rodd, Consistency of the standard model effective field theory, *J. High Energy Phys.* **12** (2019) 032.
- [129] T. Corbett, O. J. P. Éboli, and M. C. Gonzalez-Garcia, Unitarity constraints on dimension-six operators, *Phys. Rev. D* **91**, 035014 (2015).
- [130] J. Kalinowski, P. Kozów, S. Pokorski, J. Rosiek, M. Szleper, and S. Tkaczyk, Same-sign WW scattering at the LHC: Can we discover BSM effects before discovering new states?, *Eur. Phys. J. C* **78**, 403 (2018).
- [131] W. Kilian, S. Sun, Q.-S. Yan, X. Zhao, and Z. Zhao, Multi-Higgs boson production and unitarity in vector-boson fusion at future hadron colliders, *Phys. Rev. D* **101**, 076012 (2020).
- [132] E. d. S. Almeida, O. J. P. Éboli, and M. C. Gonzalez-Garcia, Unitarity constraints on anomalous quartic couplings, *Phys. Rev. D* **101**, 113003 (2020).
- [133] H. E. Logan, Lectures on perturbative unitarity and decoupling in Higgs physics, [arXiv:2207.01064](https://arxiv.org/abs/2207.01064).
- [134] S. Mahmud and K. Tobioka, High energy vector boson scattering in four-body final states to probe Higgs cubic, quartic, and HEFT interactions, *J. High Energy Phys.* **06** (2025) 153.
- [135] I. Brivio *et al.*, Truncation, validity, uncertainties, [arXiv:2201.04974](https://arxiv.org/abs/2201.04974).
- [136] L. Allwicher, D. A. Faroughy, M. Martines, O. Sumensari, and F. Wilsch, On the EFT validity for Drell-Yan tails at the LHC, *Eur. Phys. J. C* **85**, 463 (2025).
- [137] T. Hahn, Generating Feynman diagrams and amplitudes with FeynArts 3, *Comput. Phys. Commun.* **140**, 418 (2001).
- [138] V. Shtabovenko, R. Mertig, and F. Orellana, FeynCalc 9.3: New features and improvements, *Comput. Phys. Commun.* **256**, 107478 (2020).
- [139] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer, and T. Stelzer, MADGRAPH5: Going beyond, *J. High Energy Phys.* **06** (2011) 128.
- [140] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer, and T. Reiter, UFO—The universal FeynRules output, *Comput. Phys. Commun.* **183**, 1201 (2012).
- [141] ATLAS Collaboration, Projected sensitivity of measurements of Higgs boson pair production with the ATLAS experiment at the HL-LHC.