

Next-to-minimal freeze-in dark matter

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If the dark matter mass exceeds the highest temperature of the thermal bath, then dark matter production is Boltzmann suppressed. This opens new possibilities for dark matter model building. In particular, weakly interacting massive particle models that are experimentally excluded can be revived in this setting; conversely, freeze-in models, which would typically be beyond experimental reach, are potentially discoverable in the Boltzmann suppressed regime. In a recent letter, we highlighted these aspects for the case of electroweak doublet fermion dark matter assuming instantaneous inflationary reheating. Due to its elegance and simplicity, we coin this *minimal freeze-in* (MFI) dark matter. Here we consider next-to-minimal extensions of MFI dark matter. We present the implications for noninstantaneous reheating, including scenarios beyond the standard picture in which the Universe is initially matter dominated prior to reheating. Furthermore, we explore model variations within the electroweak dark matter scenario. Specifically, we consider fermion dark matter in higher representations of $SU(2)_L$, exploring the current limits and the near-future discovery potential.

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I. INTRODUCTION

There is an elegance in minimality, and models in which dark matter is connected to the Standard Model only via electroweak interactions epitomize this principle. Indeed, this is one of the original drivers of the popular weakly interacting massive particle (WIMP) scenario. In a companion paper, we laid out *minimal freeze-in* (MFI) dark matter based on this notion [1]. The particle content of MFI is a single pair of Weyl fermions that transform as doublets under $SU(2)_L$. The neutral component of these doublets is the dark matter. It was highlighted that electroweak doublet dark matter is not excluded by direct detection (e.g., LZ [2]) if the dark matter mass m is sufficiently large; $m \gtrsim 10^{10}$ GeV.¹

In addition to the requirements imposed by direct detection, to successfully realize MFI one must be in the Boltzmann suppressed freeze-in regime [1,4–14], with the dark matter mass exceeding the maximum temperature of the Universe; $m > T_{\text{max}}$. Notably, with this hierarchy the production rate typically never exceeds the Hubble rate and

thus the dark matter never thermalizes. In the case where inflationary reheating is efficient, then $T_{\text{max}} = T_{\text{rh}} \simeq \sqrt{\Gamma M_{\text{Pl}}}$, where Γ is the decay width of the inflaton. Notably, with this instantaneous reheating approximation MFI has only two free parameters, the dark matter mass m and T_{rh} , which makes MFI both highly predictive and potentially discoverable at near-future experiments such as DARWIN [1,15].

This paper explores extensions of the MFI scenario, thus *next-to-minimal freeze-in dark matter*. We extend beyond the assumption of instantaneous inflationary reheating and consider variations on the original MFI particle content. On the cosmological side we explore noninstantaneous inflationary reheating, including scenarios in which the equation of state of the Universe is different from matter dominated immediately prior to reheating. Standard freeze-in calculations [16,17] can be significantly impacted by departures from the instantaneous reheating approximation [18–27]. On the particle physics side we examine fermion dark matter in higher representations of $SU(2)_L$, drawing inspiration from the “minimal dark matter” scenario [28]. For higher representations, freeze-in is again mediated by the electroweak gauge bosons, but phenomenological differences arise. For dark matter with $SU(2)_L$ representation of dimension $n \leq 9$ indirect detection limits on dark matter annihilation exclude the possibility that the relic density is set by thermal freeze-out [29]. These dark matter annihilation constraints only weakly constrain models of Boltzmann suppressed freeze-in since the dark matter is very much heavier [1], thus reviving these low n scenarios.

¹If the Dirac fermion is split into distinct Majorana mass eigenstates by higher-dimensional operators, these limits are relaxed to 300 GeV, but this requires new high-scale physics [1,3].

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A major motivation of minimal dark matter is that certain representations are cosmologically stable, without the need for *ad hoc* stabilizing symmetries. Interestingly, as dark matter can be heavier in Boltzmann suppressed freeze-in, the dark matter lifetime can be quite different. Certain scenarios retain the nice feature of dark matter metastability, whereas other models are constrained or excluded by searches for decaying dark matter.

This paper is structured as follows. In Sec. II we discuss the general coupling structure of arbitrary dimension representations of $SU(2)_L$, and derive the relevant production cross sections. We specialize to a number of models of particular interest in Sec. III. Working in the instantaneous reheating approximation, we identify the viable parameter space for dark matter being triplet (3), quintuplet (5), or septuplet (7) under $SU(2)_L$. Section IV studies phenomenological aspects, including the dark matter relic abundance, cosmological bounds, direct and indirect detection searches. In Sec. V we explore the impact of deviating from instantaneous reheating, restricting our analysis to the doublet model. In Sec. VI, we note the irremovable contribution from gravitational production, but show that it is always subleading. Section VII provides some concluding remarks.

II. COUPLINGS AND CROSS SECTIONS

Two left-handed (LH) Weyl spinors that transform identically under the non-Abelian gauge groups, but with equal and opposite hypercharge, are referred to as a vectorlike pair. Such vectorlike pairs are interesting as they provide the minimal manner of adding nontrivial representations of the Standard Model gauge groups while avoiding gauge anomalies. Here we consider a vectorlike fermion pair (χ_1, χ_2) that transforms as n dimensional representations of $SU(2)_L$ and singlets under $SU(3)_c$, with hypercharge Y , thus

$$\chi_1 = (\mathbf{1}, \mathbf{n})_Y, \quad \chi_2 = (\mathbf{1}, \mathbf{n})_{-Y}. \quad (1)$$

The isospin of each is $T = (n-1)/2$ with their components labeled by $T_3 \in \{-T, -T+1, \dots, T\}$. The electric charge of the state with third component isospin T_3 is

$$Q = T_3 + Y. \quad (2)$$

After electroweak symmetry breaking, for each Q , one may form a Dirac fermion Ψ_Q composed of matching electromagnetic (EM) charge components from χ_1 and $\bar{\chi}_2$. Equivalently, for each isospin component $\mathcal{T}$ we identify the LH and RH components of the Dirac fermion as $\chi_1^{(\mathcal{T})}$ and $\bar{\chi}_2^{(-\mathcal{T})}$, respectively, so that $T_3^L = T_3^R := \mathcal{T}$. Accordingly, $Q[\chi_1^{(\mathcal{T})}] = Q[\bar{\chi}_2^{(-\mathcal{T})}] \equiv Q$. A neutral state occurs if and only if $T_3 = -Y$ for some allowed value of T_3 . Explicit examples are given in Sec. III.

The couplings of the new fermions follow from their electroweak representations and hypercharge. For a Dirac fermion Ψ with weak isospin T_3^L and T_3^R for its LH and RH components and electric charge Q , the vector and axial-vector couplings to the Z are

$$\begin{aligned} g_{V,Q}^{\Psi,Z} &= \frac{e}{2s_W c_W} (T_3^L + T_3^R - 2Qs_W^2), \\ g_{A,Q}^{\Psi,Z} &= \frac{e}{2s_W c_W} (T_3^L - T_3^R), \end{aligned} \quad (3)$$

where $s_W \equiv \sin \theta_W$, $c_W \equiv \cos \theta_W$, and $e = g_2 s_W$. The expressions we will derive shortly will be for a general vector mediator with the couplings corresponding to a Lagrangian of the form,

$$\mathcal{L} \supset Z_\mu \bar{\Psi} \gamma^\mu (g_{V,Q}^{\Psi,Z} - g_{A,Q}^{\Psi,Z} \gamma^5) \Psi. \quad (4)$$

Substituting into Eq. (3) gives, for the $\mathcal{T}$ th component,

$$\begin{aligned} g_{V,Q}^{\Psi,Z} &= \frac{e}{s_W c_W} (\mathcal{T}(1 - s_W^2) - Ys_W^2), \\ g_{A,Q}^{\Psi,Z} &= 0. \end{aligned} \quad (5)$$

Thus, for vectorlike pairs the Z coupling is purely vector,² whereas for the W the couplings are

$$\begin{aligned} g_V^{u_i d_j, W} &= g_A^{u_i d_j, W} = \frac{e}{2\sqrt{2}s_W} V_{ij}, \\ g_{V,Q}^{\Psi, W} &= g_{A,Q}^{\Psi, W} = \frac{e}{2\sqrt{2}s_W} \sqrt{(T+\mathcal{T})(T-\mathcal{T}+1)}, \end{aligned} \quad (6)$$

here, the Q subscript in the coupling denotes the higher-charge state in the coproduction of a $\Psi_Q \Psi_{Q-1}$ pair.

We define $\beta(s) \equiv \sqrt{1 - 4(m^2/s)}$, then the cross section for vector mediated production (as derived in Appendix A) is

$$\sigma(q\bar{q}' \rightarrow \Psi_Q \bar{\Psi}_{Q'}) \simeq \frac{\beta}{24\pi s} \mathcal{K}_{QQ'}^{q\bar{q}'}, \quad (7)$$

where Q, Q' label the EM charges of the states, $q, q' = u_i, d_i$, and $\mathcal{K}_{QQ'}^{q\bar{q}'}$ is a combination of couplings. We see in Eq. (5) that the axial Z coupling is zero at tree-level for the vectorlike pairs we consider, although the axial $W^\pm$ coupling is nonzero.

The specific form of $\mathcal{K}_{QQ'}^{q\bar{q}'}$ is given by

$$\mathcal{K}_{QQ'}^{q\bar{q}'} \equiv \delta_{qq'} (\mathcal{C}_{Q,\gamma}^{q\bar{q}} + \mathcal{C}_{Q,Z}^{q\bar{q}}) + (1 - \delta_{qq'}) \mathcal{C}_{QQ',W}^{q\bar{q}'}, \quad (8)$$

²Provided that the χ_i do not obtain an induced Majorana mass from higher-dimension operators, cf. Refs. [1,3].

TABLE I. $\mathcal{C}_Z$.

Process	Doublet	Triplet ($Y = 1$)	Triplet ($Y = 0$)	Quintuplet ($Y = 0$)	Septuplet ($Y = 0$)
$u_i \bar{u}_i \rightarrow \Psi^+ \Psi^-$	1.59×10^{-3}	1.17×10^{-3}	1.30×10^{-2}	1.30×10^{-2}	1.30×10^{-2}
$u_i \bar{u}_i \rightarrow \Psi^{++} \Psi^{--}$	...	6.34×10^{-3}	...	5.18×10^{-2}	5.18×10^{-2}
$u_i \bar{u}_i \rightarrow \Psi^{+++} \Psi^{---}$	...	...	...	...	1.17×10^{-1}
$d_i \bar{d}_i \rightarrow \Psi^+ \Psi^-$	2.04×10^{-3}	1.51×10^{-3}	1.67×10^{-2}	1.67×10^{-2}	1.67×10^{-2}
$d_i \bar{d}_i \rightarrow \Psi^{++} \Psi^{--}$	...	8.17×10^{-3}	...	6.68×10^{-2}	6.68×10^{-2}
$d_i \bar{d}_i \rightarrow \Psi^{+++} \Psi^{---}$	...	...	...	...	1.50×10^{-1}

TABLE II. $\mathcal{C}_\gamma$.

Process	Doublet	Triplet ($Y = 1$)	Triplet ($Y = 0$)	Quintuplet ($Y = 0$)	Septuplet ($Y = 0$)
$u_i \bar{u}_i \rightarrow \Psi^+ \Psi^-$	4.28×10^{-3}	4.28×10^{-3}	4.28×10^{-3}	4.28×10^{-3}	4.28×10^{-3}
$u_i \bar{u}_i \rightarrow \Psi^{++} \Psi^{--}$	...	1.71×10^{-2}	...	1.71×10^{-2}	1.71×10^{-2}
$u_i \bar{u}_i \rightarrow \Psi^{+++} \Psi^{---}$	...	...	...	...	3.86×10^{-2}
$d_i \bar{d}_i \rightarrow \Psi^+ \Psi^-$	1.07×10^{-3}	1.07×10^{-3}	1.07×10^{-3}	1.07×10^{-3}	1.07×10^{-3}
$d_i \bar{d}_i \rightarrow \Psi^{++} \Psi^{--}$	...	4.28×10^{-3}	...	4.28×10^{-3}	4.28×10^{-3}
$d_i \bar{d}_i \rightarrow \Psi^{+++} \Psi^{---}$	...	...	...	...	9.64×10^{-3}

with each term being a combination of couplings involving a different mediator $i = \gamma, Z, W$, defined by

$$\mathcal{C}_{Q,i}^{q\bar{q}'} \equiv \left(\left(g_V^{q\bar{q}',i} \right)^2 + \left(g_A^{q\bar{q}',i} \right)^2 \right) \left(\left(g_{V,Q}^{\Psi,i} \right)^2 + \beta^2 \left(g_{A,Q}^{\Psi,i} \right)^2 \right). \quad (9)$$

For the photon and Z channel $q = q'$ and $Q = Q'$. At leading order in β this is

$$\mathcal{C}_{Q,i}^{q\bar{q}'} \approx \left(\left(g_V^{q\bar{q}',i} \right)^2 + \left(g_A^{q\bar{q}',i} \right)^2 \right) \left(g_{V,Q}^{\Psi,i} \right)^2. \quad (10)$$

Note that $g_{A,Q}^{\Psi,i} = 0$ for $i = \gamma, Z$, whereas for W mediation the axial term in $\mathcal{C}_{Q,i}^{q\bar{q}'}$ involves a $\mathcal{O}(\beta^2)$ suppression. This

leading order limit is valid in the Boltzmann suppressed regime, since $m \gg T_{\text{th}}$. For a given model, these couplings are determined by the $\text{SU}(2)_L$ representation and the hypercharge of the vectorlike pair. We provide numerical computations of $\mathcal{C}_{Q,i}^{q\bar{q}'}$ [as given in Eq. (10)] in Tables I–III for a selection of models of interest. Further exploration of these specific models is presented in Sec. III (the list of models is not exhaustive).

Notably, Eq. (7) can be adapted to different initial and final states by taking the appropriate couplings, cf. Eq. (5). It is enlightening to consider two illustrative examples; the neutral current arising from the $\bar{u}u$ and the charged current arising from the $\bar{u}d$ initial state,

$$\begin{aligned} \sigma(u\bar{u} \rightarrow \Psi_Q \bar{\Psi}_Q) &\simeq \frac{\beta}{24\pi s} \left[\underbrace{(eQ_u)^2 (eQ)^2}_{\mathcal{C}_{Q,\gamma}^u} + \underbrace{\left[\left(\frac{e}{2s_W c_W} \left(\frac{1}{2} - \frac{4}{3} s_W^2 \right) \right)^2 + \left(\frac{e}{4s_W c_W} \right)^2 \right] \left(\frac{e}{s_W c_W} (T - Q s_W^2) \right)^2}_{\mathcal{C}_{Q,Z}^u} \right], \\ \sigma(u\bar{d} \rightarrow \Psi_Q \bar{\Psi}_{Q-1}) &= \frac{\beta}{24\pi s} \mathcal{C}_{Q,W}^{u\bar{d}} \simeq \frac{\beta}{24\pi s} \frac{e^4}{32s_W^4} (T + \mathcal{T})(T - \mathcal{T} + 1) |V_{ud}|^2. \end{aligned} \quad (11)$$

Other cross sections mediated by the W bosons can be obtained by using the appropriate Cabibbo-Kobayashi-Maskawa (CKM) element.

Further, we recall that if an EM neutral Dirac fermion exists (as appropriate to be dark matter), then $Q = 0$ requires $Y = -\mathcal{T}$. This implies that for Ψ^0 the corresponding Z vector coupling is

$$g_{V,Q=0}^{\Psi,Z} = -\frac{e}{s_W c_W} Y. \quad (12)$$

It can be seen from Eq. (12) that for integer isospin $T \in \mathbb{Z}$ (i.e., odd n) and $Y = 0$, the neutral state corresponds to $\mathcal{T} = 0$ which satisfies $g_{V,0}^{\Psi^0,Z} = g_{A,0}^{\Psi^0,Z} = 0$, thus there is

TABLE III. $\mathcal{C}_W$.

Process	Doublet	Triplet ($Y = 1$)	Triplet ($Y = 0$)	Quintuplet ($Y = 0$)	Septuplet ($Y = 0$)
$u\bar{d} \rightarrow \Psi^+\bar{\Psi}^0$	5.35×10^{-3}	1.07×10^{-2}	1.07×10^{-2}	3.21×10^{-2}	6.43×10^{-2}
$c\bar{s} \rightarrow \Psi^+\bar{\Psi}^0$	5.46×10^{-3}	1.09×10^{-2}	1.09×10^{-2}	3.28×10^{-2}	6.57×10^{-2}
$t\bar{b} \rightarrow \Psi^+\bar{\Psi}^0$	5.69×10^{-3}	1.14×10^{-2}	1.14×10^{-2}	3.41×10^{-2}	6.84×10^{-2}
$u\bar{d} \rightarrow \Psi^{++}\Psi^-$	$\dots$	1.07×10^{-2}	$\dots$	2.14×10^{-2}	5.35×10^{-2}
$c\bar{s} \rightarrow \Psi^{++}\Psi^-$	$\dots$	1.09×10^{-2}	$\dots$	2.18×10^{-2}	5.46×10^{-2}
$t\bar{b} \rightarrow \Psi^{++}\Psi^-$	$\dots$	1.14×10^{-2}	$\dots$	2.27×10^{-2}	5.69×10^{-2}
$u\bar{d} \rightarrow \Psi^{+++}\Psi^{--}$	$\dots$	$\dots$	$\dots$	$\dots$	3.21×10^{-2}
$c\bar{s} \rightarrow \Psi^{+++}\Psi^{--}$	$\dots$	$\dots$	$\dots$	$\dots$	3.28×10^{-2}
$t\bar{b} \rightarrow \Psi^{+++}\Psi^{--}$	$\dots$	$\dots$	$\dots$	$\dots$	3.41×10^{-2}

no diagonal tree-level coupling to the Z . The absence of tree-level Z -exchange significantly suppresses the direct detection cross section which will only be generated at loop level. In this case, the Z -mediated freeze-in cross section for Ψ^0 also vanishes at leading order; however, the W -mediated piece persists at leading order and charged Ψ states are also produced without suppression.

III. MODELS OF INTEREST

We next outline explicitly a number of models of interest and then study the phenomenological implications. Specifically, we study the doublet (2), the triplet (3), the quintuplet (5), and the septuplet (7). We restrict our attention to fermion dark matter. We note that for $SU(2)_L$ representations larger than dimension 7, tree-level unitarity violate becomes a concern [30].

A. Doublet representations

We start with the familiar case of the fundamental representation of $SU(2)_L$. For $n = 2$ one has isospin $T = \frac{1}{2}$, and thus the components take values $\mathcal{T} \in \{-\frac{1}{2}, +\frac{1}{2}\}$. For $Y = \pm\frac{1}{2}$ one of the components is EM neutral after electroweak symmetry breaking (EWSB). Taking $Y = \frac{1}{2}$ for definiteness, the electric charges are given by $Q_{\mathcal{T}} = \mathcal{T} + \frac{1}{2} \in \{0, +1\}$, with the EM neutral state corresponding to the $\mathcal{T} = -\frac{1}{2}$ component.

Labeling the two LH Weyl doublets by their hypercharge as $\chi_{\mathcal{T}}^{(+1/2)}$ and $\chi_{\mathcal{T}}^{(-1/2)}$. The associated Dirac fermions are then constructed as

$$\Psi^0 \equiv \begin{pmatrix} \chi_{-1/2}^{(+1/2)} \\ \bar{\chi}_{+1/2}^{(-1/2)} \end{pmatrix}, \quad \Psi^+ \equiv \begin{pmatrix} \chi_{+1/2}^{(+1/2)} \\ \bar{\chi}_{-1/2}^{(-1/2)} \end{pmatrix},$$

together with $\Psi^- \equiv (\Psi^+)^c$. The diagonal Z couplings follow from Eq. (5),

$$g_{V,Q}^{\Psi,Z} \Big|_{\substack{n=2 \\ Y=\frac{1}{2}}} = \frac{e}{s_W c_W} \left(\mathcal{T} - (\mathcal{T} + Y)s_W^2 \right). \quad (13)$$

Recall that in this case (and all cases studied below) the axial vector Z coupling vanishes $g_{A,Q}^{\Psi,Z} = 0$, due to the fact that we only consider vectorlike pairs; see Eq. (5).

For the neutral component ($\mathcal{T} = -\frac{1}{2}$, $Q = 0$) one has

$$g_{V,0}^{\Psi,Z} \Big|_{\substack{n=2 \\ Y=\frac{1}{2}}} = -\frac{e}{2s_W c_W}, \quad (14)$$

while for the charged component

$$g_{V,+1}^{\Psi,Z} \Big|_{\substack{n=2 \\ Y=\frac{1}{2}}} = \frac{e}{2s_W c_W} (1 - 2s_W^2). \quad (15)$$

Choosing instead $Y = -1/2$ simply relabels which $\mathcal{T}$ component is neutral and flips the overall sign of $g_{V,0}^{\Psi,Z}$; all production and scattering rates are unchanged since they depend only on $(g_{V,0}^{\Psi,Z})^2$.

The construction above is precisely the MFI model studied in the companion paper [1]. We present this as the point of comparison for the higher representations. Moreover, we will restrict our attention to this fermion doublet model in the nonminimal cosmology discussion presented in Secs. V and VI.

B. Triplet representations

The triplet 3 is the adjoint of $SU(2)_L$. With $n = 3$ one has isospin $T = 1$, and thus the components take values $\mathcal{T} \in \{-1, 0, +1\}$. We shall first consider the hypercharge $Y = 0$, followed by $Y = 1$. The case of $Y = 0$ is distinct from $Y \neq 0$ since the tree-level vector couplings vanish.

For $Y = 0$, the electric charge is $Q = \mathcal{T}$ and a neutral component exists ($\mathcal{T} = 0$). We label the two left-handed Weyl triplets $i = 1, 2$ by $\chi_{\mathcal{T}}^{(1)}$ and $\chi_{\mathcal{T}}^{(2)}$ (the hypercharges

match so we replace this label), then the associated Dirac fermions for $Y = 0$ are³

$$\Psi^0 \equiv \begin{pmatrix} \chi_0^{(1)} \\ \bar{\chi}_0^{(2)} \end{pmatrix}, \quad \Psi^+ \equiv \begin{pmatrix} \chi_{+1}^{(1)} \\ \bar{\chi}_{-1}^{(2)} \end{pmatrix}, \quad \Psi^- \equiv \begin{pmatrix} \chi_{-1}^{(1)} \\ \bar{\chi}_{+1}^{(2)} \end{pmatrix}.$$

The diagonal Z couplings come from Eq. (5),

$$g_{V,Q}^{\Psi,Z} \Big|_{\substack{n=3 \\ Y=0}} = \frac{e}{s_W c_W} (\mathcal{T} - \mathcal{T} s_W^2) = \frac{e c_W}{s_W} \mathcal{T}. \quad (16)$$

Thus, the tree-level vector coupling is $g_{V,Q}^{\Psi,Z(0)} = 0$ for the neutral component. For $\Psi^\pm$ the couplings are

$$g_{V,\pm 1}^{\Psi,Z} \Big|_{\substack{n=3 \\ Y=0}} = \pm \frac{e c_W}{s_W}. \quad (17)$$

In contrast, in the $Y = 1$ case, the EM charges are given by $Q = \mathcal{T} + 1 \in \{0, 1, 2\}$. Note that the neutral Dirac states Ψ^0 correspond to $\mathcal{T} = -1$. In this case, the set of Dirac fermions is formed as follows:

$$\Psi^0 \equiv \begin{pmatrix} \chi_{1,-1} \\ \bar{\chi}_{2,+1} \end{pmatrix}, \quad \Psi^+ \equiv \begin{pmatrix} \chi_{1,0} \\ \bar{\chi}_{2,0} \end{pmatrix}, \quad \Psi^{++} \equiv \begin{pmatrix} \chi_{1,+1} \\ \bar{\chi}_{2,-1} \end{pmatrix},$$

the charge-conjugate fields are identified with Ψ^- and Ψ^{--} . Observe the doubly charged states in the spectrum, in contrast to the $Y = 0$ case.

The diagonal Z couplings are

$$g_{V,Q}^{\Psi,Z} \Big|_{\substack{n=3 \\ Y=1}} = \frac{e}{s_W c_W} (\mathcal{T} - (\mathcal{T} + 1) s_W^2). \quad (18)$$

In particular, the neutral component ($\mathcal{T} = -1$, $Q = 0$) has a nonzero coupling to the Z given by

$$g_{V,0}^{\Psi,Z} \Big|_{\substack{n=3 \\ Y=1}} = -\frac{e}{s_W c_W}. \quad (19)$$

For charged states, the couplings are

$$g_{V,\pm 1}^{\Psi,Z} \Big|_{\substack{n=3 \\ Y=1}} = \pm \frac{e s_W}{c_W},$$

$$g_{V,\pm 2}^{\Psi,Z} \Big|_{\substack{n=3 \\ Y=1}} = \pm \frac{e}{s_W c_W} (1 - 2 s_W^2). \quad (20)$$

Similarly to the doublet, taking $Y = -1$ instead simply relabels which $\mathcal{T}$ component is electrically neutral.

³For odd n and $Y = 0$ the theory is anomaly free with only a single Weyl fermion, the EW spectrum remains unchanged (up to multiplicity), and a Majorana mass is permitted. We retain the pair of χ fields here to maintain uniformity between cases.

C. Quintuplet representation

Moving to higher representations, the quintuplet corresponds to the $n = 5$ representation, with isospin $T = 2$ thus $\mathcal{T} \in \{-2, -1, 0, +1, +2\}$. We first consider the case $Y = 0$ and thus $Q = \mathcal{T}$. Accordingly, the Dirac fermions are

$$\Psi^0 \equiv \begin{pmatrix} \chi_{1,0} \\ \bar{\chi}_{2,0} \end{pmatrix}, \quad \Psi^+ \equiv \begin{pmatrix} \chi_{1,+1} \\ \bar{\chi}_{2,-1} \end{pmatrix}, \quad \Psi^{++} \equiv \begin{pmatrix} \chi_{1,+2} \\ \bar{\chi}_{2,-2} \end{pmatrix},$$

with the remaining components forming Ψ^- and Ψ^{--} . For $Y = 0$, it follows from Eq. (5) that the diagonal Z couplings are identical to Eq. (16). Similarly, for the charged states,

$$g_{V,\pm 1}^{\Psi,Z} = \pm \frac{e c_W}{s_W}, \quad g_{V,\pm 2}^{\Psi,Z} = (\pm 2) \frac{e c_W}{s_W}. \quad (21)$$

The quintuplet is highlighted as a metastable representation in the original “minimal dark matter” [28]. However, since the decay rate of particles typically scales with the mass of the decaying state, the strong stability statements which apply for the orthodox TeV-scenario must be reexamined. As we show below the 5 representations are not found to be automatically stable for the entire parameter space for which they can account for the dark matter relic abundance.

For the quintuplet model ($n = 5$) with $Y = 0$, the Ψ^0 states that constitute the dark matter can decay via dimension-six operators of the form,

$$\mathcal{O}_{n=5}^{Y=0} \sim \frac{1}{\Lambda^2} \chi_1 (L H H \tilde{H}), \quad (22)$$

where $\tilde{H} = i\sigma_2 H^*$ and Λ is the scale at which these operators are generated. Observe that the two fields $\chi_{1,0}$ and $\bar{\chi}_{2,0}$, which comprise Ψ^0 , involve slightly different contractions with the Standard Model fields to form gauge invariant operators, but after EWSB these subtleties become irrelevant.⁴ After EWSB, the neutral-component couplings take the form,

$$\mathcal{L}_{\text{eff}} \supset \frac{v^2}{\Lambda^2} (\chi_1^0 \nu h + \bar{\chi}_2^0 \nu h) + \dots, \quad (23)$$

permitting dark matter decays via $\chi^0 \rightarrow h\nu$. There is also a mixing-induced coupling of χ to the electroweak gauge bosons (which can be seen by calculating the mass eigenstates of the $\chi - \nu$ system), which leads to (similarly suppressed) decay channels through νZ and lW .

⁴Mass shifts to χ_1^0 and χ_2^0 from these operators are negligible, since $\Delta \equiv m_1 - m_2$ comes only via mixing with the Standard Model leptons such that $\Delta \sim v^6/(\Lambda^4 m)$. For high scale Λ (and we anticipate $\Lambda \sim M_{\text{Pl}}$) this will *not* lead to the “pseudo-Dirac” direct detection suppression discussed in Footnote 1.

For $m \gg m_W, m_Z, m_h$, the total decay width is parametrically,

$$\Gamma_{\chi^0} \sim \frac{3}{8\pi} \frac{v^4}{\Lambda^4} m, \quad (24)$$

where we include a factor of 3 to account for the multiplicity of decay channels. Accordingly, the dark matter lifetime for $n = 5$ and $Y = 0$ is $\tau = 1/\Gamma_{\chi^0}$. The characteristic limit on dark matter decaying in the manner above is $\tau \gtrsim 10^{30}$ s [31,32], which implies

$$m_5^{(Y=0)} \lesssim 5 \times 10^{10} \text{ GeV} \left(\frac{\Lambda}{M_{\text{Pl}}} \right)^4. \quad (25)$$

We return to calculate this limit with care in Sec. IV.

In the above we took $n = 5$ and $Y = 0$, had we taken $Y = \pm 1, +2$ then this also leads to decay operators arising at mass dimension six. For $Y = 1$ the leading operator is $\chi L(H\tilde{H}\tilde{H})$ and for $Y = 2$ it is $\chi(L\tilde{H}\tilde{H}\tilde{H})$. However, for $Y = -2$ (the only remaining choice consistent with a dark matter candidate) the leading decay operator arises at mass dimension seven, namely,

$$\mathcal{O}_{n=5}^{Y=-2} \sim \frac{1}{\Lambda^3} \chi e^c (HHH\tilde{H}). \quad (26)$$

The corresponding lifetime is $\tau_5^{(Y=-2)} \propto (1/m)(\Lambda/v)^6$. Taking the characteristic limit $\tau \gtrsim 10^{30}$ s [31,32] this lifetime limit implies a mass bound,

$$m_5^{(Y=-2)} \lesssim 5 \times 10^{42} \text{ GeV} \left(\frac{\Lambda}{M_{\text{Pl}}} \right)^6. \quad (27)$$

For $\Lambda \sim M_{\text{Pl}}$ decaying dark matter limits do not lead to a meaningful mass bound in this case. We can rephrase the question to ask how low the cutoff can be before the dark matter mass is constrained. Reinterpreting Eq. (27), we note that even for the heaviest dark matter $m \sim M_{\text{Pl}}$ for $\Lambda \gtrsim 2 \times 10^{14}$ GeV the mass is unconstrained by searches for decaying dark matter for $n = 5$ and $Y = -2$.

D. Septuplet representation

The final example we consider is the septuplet representation, which corresponds to $n = 7$, thus $T = 3$ and $T \in \{-3, -2, -1, 0, +1, +2, +3\}$. We will consider the $Y = 0$ case for which $Q = T$. The corresponding Dirac fermions are $\Psi^0, \Psi^\pm, \Psi^{\pm\pm}, \Psi^{\pm\pm\pm}$. The diagonal Z couplings follow from Eq. (5),

$$g_{V,Q}^{\Psi,Z} = \frac{ec_W}{s_W} T, \quad g_{A,Q}^{\Psi,Z} = 0, \quad (28)$$

again, we have $g_{V,0}^{\Psi,Z} = 0$ since we consider the case $Y = 0$.

This representation can be sufficiently metastable to be a viable dark matter candidate without any additional stabilizing symmetry. If Ψ^0 decays are induced from an intermediate scale Λ this will generically lead to dimension eight operators of the form,

$$\mathcal{O}_{n=7}^{Y=0} \sim \frac{1}{\Lambda^4} \chi_1 L H H H \tilde{H} \tilde{H}, \quad (29)$$

and after EWSB this induces $1 \rightarrow 2$ decay operators for Ψ^0 similar to the 5 case, with $\tau \sim \frac{8\pi}{3} (\Lambda^8/v^8 m)$. This implies the following restriction on the pair m, Λ :

$$m \lesssim 10^{18} \text{ GeV} \left(\frac{\Lambda}{2 \times 10^{11} \text{ GeV}} \right)^8. \quad (30)$$

We see that dark matter is unconstrained by searches for decaying dark matter provided that $\Lambda \gtrsim 2 \times 10^{11}$ GeV. This intermediate mass scale is reminiscent of scales invoked with the PQ mechanism [33] and the neutrino seesaw scale [34], so it remains an interesting prospect that signals might arise in future indirect detection experiments from the septuplet model.

IV. PHENOMENOLOGY

Having established the relevant details for our models of interest, we proceed to study the phenomenological implications. In this section we calculate the dark matter relic abundance in the instantaneous reheating approximation, along with the associated limits from direct and indirect detection experiments.

A. Dark matter relic abundance

The masses of the Ψ states are split via radiative corrections which lead to the charged states picking up a small positive correction, the size of which is dependent on the dimension of the representation and the hypercharge. Specifically, the mass splitting between $\Psi^\pm$ and Ψ^0 is given by [28]

$$\Delta \equiv m_\pm - m_0 \simeq \begin{cases} 341 \text{ MeV} & n = 2, Y = 1/2, \\ 525 \text{ MeV} & n = 3, Y = 1, \\ 166 \text{ MeV} & n = 3, Y = 0, \\ 166 \text{ MeV} & n = 5, Y = 0, \\ 166 \text{ MeV} & n = 7, Y = 0. \end{cases}$$

The splitting is so small that elsewhere we do not make a distinction between the mass of the components and simply use $m \equiv m_0 \simeq m_Q$.

Importantly, the EM neutral state Ψ^0 is always the lightest of the Ψ states. The Ψ states carry a conserved quantum number, either by construction (i.e., via a $\mathbb{Z}_2$) or by accident. Hence, the heavier Ψ states will decay to the

neutral states. For the singly charged $\Psi^\pm$ this occurs via $\Psi^\pm \rightarrow \pi^\pm \Psi^0$, while for states with multiple electric charges, the decay to Ψ^0 is through a ladder of off shell $W^\pm$ cascades. Consequently, the relic abundance of Ψ^0 dark matter Y_∞^0 is the sum of the freeze-in abundances of all the Ψ states,

$$Y_\infty^0 = \sum_Q Y_Q|_{\text{FI}}. \quad (31)$$

In particular, freeze-in production can proceed in two distinct manners for each state:

- (i) Neutral channels $q\bar{q} \rightarrow (\gamma/Z) \rightarrow \Psi_Q \bar{\Psi}_Q$;
- (ii) ‘‘Coproduction’’ of adjacent isospin states via charged mediators e.g., $u\bar{d} \rightarrow W^+ \rightarrow \Psi_{Q+1} \bar{\Psi}_Q$.

It can be seen from Eq. (5) that the cross section for the Ψ production via Z mediation scales as (neglecting Y)

$$\sigma(q\bar{q} \rightarrow \Psi_Q \bar{\Psi}_Q) \propto (g_{V,Q}^{\Psi,Z})^2 \propto T^2. \quad (32)$$

Thus, higher isospin components are produced more readily (and will subsequently decay to Ψ^0). It follows that the production rate of the T^{th} pair with zero net-charge $\Psi_Q \bar{\Psi}_Q$ compared to the pair of singly charged states⁵ is parametrically,

$$\sigma_{q\bar{q} \rightarrow \Psi_Q \bar{\Psi}_Q} \simeq T^2 \times \sigma_{q\bar{q} \rightarrow \Psi^- \Psi^+}.$$

Similarly, the charged channel coproduction $\Psi_Q \bar{\Psi}_{Q-1}$ (with isospins T and $T-1$) scales as

$$\sigma_{q\bar{q}' \rightarrow \Psi_Q \bar{\Psi}_{Q-1}} \simeq (T - (T-1))(T+T) \sigma_{q\bar{q}' \rightarrow \Psi^+ \bar{\Psi}^0},$$

summing over all quarks and taking the appropriate sign final state to match the initial-state electric charge. To a good approximation, we can just include intrageneration W processes, as alternative charged channels are CKM suppressed. A more detailed treatment will lead to only an $\mathcal{O}(1)$ correction. In our numerical results, we include all tree-level cross sections.

To proceed, we should calculate the freeze-in abundances for a general Ψ_Q , and then sum these up to obtain the relic abundance, cf. Eq. (31). The relic abundance for Ψ_Q is obtained by evaluating the Boltzmann equation,

$$\dot{n}_Q + 3Hn_Q = \gamma_Q(T). \quad (33)$$

Neglecting quark masses, the reaction density for $q\bar{q}' \rightarrow \Psi_Q \bar{\Psi}_{Q'}$ is given by

⁵Choosing the reference cross section to correspond to Ψ^0 is dangerous since for $Y = 0$ the tree-level cross section vanishes.

$$\gamma_Q = \frac{T}{32\pi^4} \sum_{q,q'} \int_{4m^2}^{\infty} ds \left[(s-4m^2) \sqrt{s} \sigma_{q\bar{q}' \rightarrow \Psi_Q \bar{\Psi}_{Q'}} \right] K_1 \left(\frac{\sqrt{s}}{T} \right),$$

in terms of the modified Bessel function K_1 .

Since we are working in the Boltzmann suppressed freeze-in regime $T \ll m$, production is dominated near-threshold. Thus, we parametrize the center-of-mass energy as $s \simeq 4m^2 + \varepsilon$ taking $\varepsilon \ll 4m^2$ and hence

$$\beta \simeq \sqrt{\frac{\varepsilon}{4m^2}} \ll 1. \quad (34)$$

Moreover, $(s-4m^2)\sqrt{s}\sigma(s)$ as appears in the expression for γ_Q (in the square brackets) can be approximated by

$$(s-4m^2)\sqrt{s}\sigma(s) \simeq \left(\frac{\mathcal{K}_{QQ'}^{q\bar{q}'}}{96\pi m^2} \right) \varepsilon^{3/2}, \quad (35)$$

where we recall $\mathcal{K}_{QQ'}^{q\bar{q}'}$ collects all the various couplings together, cf. Eq. (9). Taking the Bessel function large value approximation $K_1(z) \simeq \sqrt{\frac{\pi}{2z}} e^{-z}$ for $z \gg 1$, and working to leading order in ε yields

$$K_1 \left(\frac{\sqrt{s}}{T} \right) \simeq \sqrt{\frac{T\pi}{4m}} e^{-\frac{2m}{T}} e^{-\frac{\varepsilon}{4mT}}. \quad (36)$$

It follows that the reaction density can be rewritten as an integral over ε in the following manner:

$$\gamma_Q \simeq \sum_{q,q'} \frac{T}{32\pi^4} \frac{\mathcal{K}_{QQ'}^{q\bar{q}'}}{96\pi m^2} \sqrt{\frac{\pi T}{4m}} e^{-\frac{2m}{T}} \int_0^\infty d\varepsilon \varepsilon^{3/2} e^{-\frac{\varepsilon}{4mT}}.$$

Evaluating the ε integral yields the reaction for $m \gg T$,

$$\gamma_Q \simeq \sum_{q,q'} \left(\frac{\mathcal{K}_{QQ'}^{q\bar{q}'}}{256\pi^4} \right) T^4 e^{-\frac{2m}{T}}, \quad (37)$$

where we must evaluate the combined coupling $\mathcal{K}_{QQ'}^{q\bar{q}'}$.

Returning to the Boltzmann equation, Eq. (33), we can use the form of $\gamma_Q(T)$ to calculate the freeze-in yields $Y \equiv n_Q/s$ for each component via

$$\frac{dY}{dT} = -\frac{\gamma_Q(T)}{sHT}, \quad (38)$$

where $H(T) = \sqrt{\frac{\pi^2 g_\star}{90}} \frac{T^2}{M_{\text{Pl}}}$ and $s(T) = \frac{2\pi^2}{45} g_\star T^3$ in terms of the effective entropy degrees of freedom $g_\star$, and the relativistic degrees of freedom $g_\star$, and the reduced Planck

TABLE IV. Values of m/T_{rh} required to fit the whole observed dark matter abundance (assuming instantaneous reheating), for the different $\text{SU}(2)_L$ representations.

	m/T_{rh}
Doublet	22.4
Triplet ($Y = 1$)	23.2
Triplet ($Y = 0$)	23.0
Quintuplet ($Y = 0$)	23.8
Septuplet ($Y = 0$)	24.3

mass $M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV. Substituting $\gamma_Q(T)$ from Eq. (37), it follows that

$$\frac{dY}{dT} = -\frac{135\sqrt{10}}{512\pi^7} \sum_{q,q'} \left(\frac{\mathcal{K}_{QQ'}^{q\bar{q}'}}{g_{\star s} \sqrt{g_{\star}}} \right) \frac{M_{\text{Pl}}}{T^2} e^{-\frac{2m}{T}}.$$

We assume that the initial abundances of Ψ states are negligible and that there is no appreciable Ψ production from direct inflaton decays. We work in the Boltzmann suppressed regime $m \gg T_{\text{rh}}$ and restrict our attention (in this section) to the case of *instantaneous* reheating after inflation to a reheat temperature T_{rh} . In this setting, the freeze-in yield of the Ψ state with charge Q is given by

$$Y_{\text{FI}}^{\text{inst}} = \frac{135\sqrt{10}}{512\pi^7} \sum_{q,q'} \frac{\mathcal{K}_{QQ'}^{q\bar{q}'}}{g_{\star s} \sqrt{g_{\star}}} M_{\text{Pl}} \int_0^{T_{\text{rh}}} dT \frac{e^{-\frac{2m}{T}}}{T^2}. \quad (39)$$

Integrating gives the yield (for the case of instantaneous reheating),

$$Y_{\text{FI}}^{\text{inst}} \simeq \frac{135\sqrt{10}}{1024\pi^7} \sum_{q,q'} \frac{\mathcal{K}_{QQ'}^{q\bar{q}'}}{g_{\star s} \sqrt{g_{\star}}} \frac{M_{\text{Pl}}}{m} e^{-\frac{2m}{T_{\text{rh}}}}. \quad (40)$$

The freeze-in yields scale as $Y_{\text{FI}} \propto \exp(-2m/T_{\text{rh}})$, as anticipated for Boltzmann suppressed freeze-in ($m \gg T_{\text{rh}}$). From Y_{FI} we can calculate the freeze-in yield of each Dirac fermion by taking the appropriate $\mathcal{K}$ in turn, as outlined in Eq. (8). We recall that the numerical values for $\mathcal{K}$ are provided in Tables I–III for selected models of interest. The yield can be subsequently compared with the observed relic abundance by noting that $\Omega_{\text{DM}} = mY_{\text{FI}}s_0/\rho_c$ where s_0 is the entropy density today and ρ_c is the critical density.

In the instantaneous reheating approximation, for a fixed representation, the model has only two degrees of freedom T_{rh} and m . As a result, this class of models is highly predictive. Note that the exponential in Eq. (40) strongly determines the value of $Y_{\text{FI}}^{\text{inst}}$, thus it is the ratio m/T_{rh} that is critical to reproducing the observed relic abundance. In Table IV we provide the approximate value which gives the correct value of Ω_{DM} in each of our models of interest.

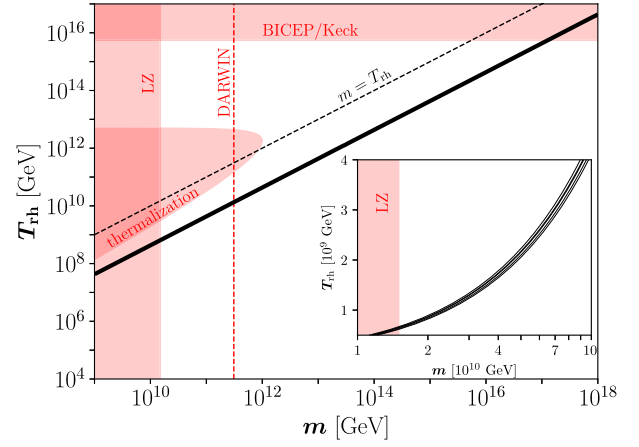


FIG. 1. The solid lines correspond to the reheat temperature T_{rh} which gives the observed dark matter relic density of mass m assuming instantaneous reheating, for the different $\text{SU}(2)_L$ representations. The inset plot shows a zoom of the different lines. The vertical red regions indicate the parameter space excluded by direct detection (LZ); the projected sensitivity of DARWIN is also shown. The black dashed line indicates $T_{\text{rh}} = m$, above which freeze-in is no longer Boltzmann suppressed. The red shaded “thermalization” region indicates parameter values for which Ψ^0 would enter equilibrium with the Standard Model.

Observe that for low n electroweak representations one typically requires $m/T_{\text{rh}} \simeq 23 \pm 1$.

In Fig. 1 we show the reheat temperature T_{rh} required to reproduce the observed dark matter relic abundance as a function of the dark matter mass m . In calculating the relic abundance, we sum over all the quark initial states (neglecting other channels) and all the Ψ final states and then account for charged Ψ decays to the (meta)stable neutral state Ψ^0 . Observe in Fig. 1 that all of the low n models studied here have comparable relic density curves, indeed they are only distinguishable in the “zoomed” subpanel. In Sec. V we study the impact of deviating from instantaneous reheating.

B. Cosmological bounds

Having demonstrated that the relic abundance $\Omega_{\text{DM}} h^2 \simeq 0.12$ can be reproduced, we turn to the limits on these models. We first examine the observational and consistency constraints that arise from cosmology. The reheat temperature is constrained at the higher and lower ranges. The cosmic abundances of nuclei agree well with the predictions of big bang nucleosynthesis (BBN). Consistency with BBN requires that the Standard Model thermal bath was in thermal equilibrium at temperatures around a few MeV. This implies a lower bound on the reheat temperature [35–37],

$$T_{\text{rh}} \gtrsim 5 \text{ MeV}. \quad (41)$$

The upper temperature limit of T_{rh} is constrained by cosmological observations by BICEP/Keck [38]. Specifically, BICEP/Keck searches for primordial B-modes constrain the tensor-to-scalar ratio r . This can be related to the Hubble scale during inflation H_I [39],

$$H_I = \pi M_{\text{Pl}} \sqrt{r A_s / 2}, \quad (42)$$

where $A_s \simeq 2.1 \times 10^{-9}$ is the measured scalar amplitude. Applying the BICEP/Keck bound $r < 0.036$ [38] one obtains an upper bound on H_I [39],

$$H_I \lesssim 6 \times 10^{13} \text{ GeV}. \quad (43)$$

This in turn constrains radiation energy density and hence the maximum temperature of the thermal bath via

$$\rho_{\text{rh}} = \frac{g_*(T_{\text{rh}})\pi^2}{30} T_{\text{rh}}^4 = 3M_{\text{Pl}}^2 H_{\text{rh}}^2 \leq 3M_{\text{Pl}}^2 H_I^2. \quad (44)$$

Assuming instantaneous reheating, it follows that

$$T_{\text{rh}} \lesssim 6 \times 10^{15} \text{ GeV} \left(\frac{106.75}{g_*} \right)^{1/4} \left(\frac{H_I}{6 \times 10^{13} \text{ GeV}} \right)^{1/2},$$

corresponding to the upper red band in Fig. 1.

Finally, we should identify the parameter range for which the dark matter enters equilibrium with the thermal bath of Standard Model particles. Equilibration is avoided, provided that the production rate is smaller than the expansion rate H , this restriction can be expressed as

$$\gamma(T_{\text{rh}}) < n_{\text{eq}}(T_{\text{rh}})H(T_{\text{rh}}), \quad (45)$$

where n_{eq} is the equilibrium number density of dark matter. The region in which thermalization occurs is marked in Fig. 1. Note that the line $m = T_{\text{rh}}$ separates the relativistic and nonrelativistic regimes. The inflection in the bounding curve of the thermalization region corresponds to the transition from relativistic to nonrelativistic regimes. Furthermore, we highlight that these cosmological limits do not meaningfully constrain the parameter space of our models of interest.

C. Indirect detection

Experimental observations of extragalactic γ -rays and cosmic rays constrain dark matter decays and annihilation. In models with $Y = 0$ TeV scale masses are allowed without conflict with direct detection, in this case searches for dark matter annihilation place lower limits on the mass. The analysis of indirect detection signals for annihilating dark matter arising from electroweak dark matter must take into account Sommerfeld enhancement

of the annihilation cross section [40,41]. An updated analysis is beyond the scope of this work, so we draw on existing analyses, [29,42,43]

$$m \gtrsim \begin{cases} 3.5 \text{ TeV} & n = 3, \\ 20 \text{ TeV} & n = 5, \\ 20 \text{ TeV} & n = 7. \end{cases} \quad (46)$$

We highlight that these limits derived in Ref. [42] are based on older HESS data [44], while recent analyzes have found improvements using Fermi data [29,45] (although restricted to the $\sim$ TeV mass range). Moreover, the exact numbers are dependent on the assumed dark matter halo profile (the above assume the Einasto profile [46]).

Furthermore, while the **5** and **7** representations can be naturally metastable without an additional (e.g., $\mathbb{Z}_2$) stabilizing symmetry, in the high mass limit, decays of Ψ^0 descending from **5** are constrained by decaying dark matter bounds for $Y = 0, \pm 1, +2$ (unless an additional stabilizing symmetry is assumed). Recall from Sec. III, that the Ψ^0 decays dominantly to $h\nu$. In this case Auger places a lower bound on the dark matter lifetime of order 10^{30} s. We provided analytic estimates of the lifetime constraints in Eqs. (25), (27), and (30), in Fig. 2 we present the detailed bounds, drawing on the channel specific analysis in [32]. These limits are comparable to the bounds on other decay channels presented in Ref. [31].

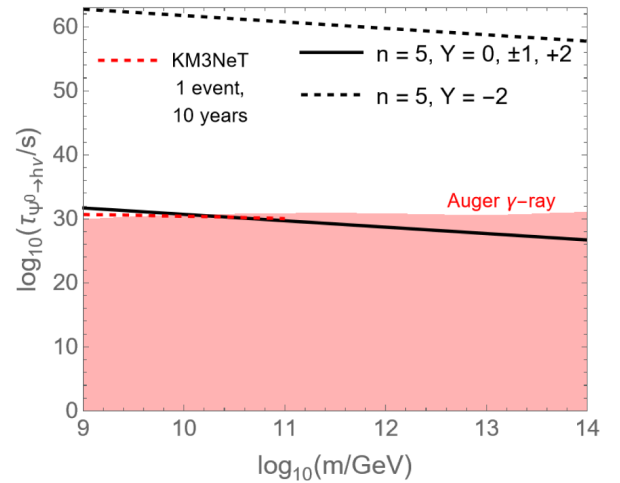


FIG. 2. We show the lifetimes of the quintuplet (5) representations due to Planck induced higher dimension operators for the two distinct cases $Y = 0, \pm 1, +2$, and $Y = -2$. This is plotted as a function of mass m . We overlay the indirect detection limits for decaying dark matter coming from Auger's observation of galactic γ -rays in red, specialized for the decay channel $\Psi^0 \rightarrow h\nu$ [32]. For $Y = 0, \pm 1, +2$ Auger places an upper bound on the dark matter mass. We also show the anticipated reach of KM3NeT (based on the assumption of a single event over 10 years) [47] as the red dotted curve.

For $Y = 0, \pm 1, 2$, decays of Ψ^0 induced by Planck-suppressed operators [saturating $\Lambda \sim M_{\text{Pl}}$ in Eq. (25)] are constrained by Pierre Auger observations of galactic γ -rays, yielding an upper mass bound,

$$m \lesssim 1.2 \times 10^{10} \text{ GeV}. \quad (47)$$

This is in agreement with our earlier analytic estimates. We note that for $Y = -2$ the dark matter Ψ^0 is cosmologically stable (and thus unbounded) unless the decay operators are induced well-below the Planck scale (i.e., $\Lambda \ll M_{\text{Pl}}$), as described by Eq. (26).

Finally, in Fig. 2 we also indicate the expected reach of next-generation indirect detection experiments to constrain the dark matter lifetime, specifically KM3NeT (1 event, 10 years) [47], although this seemingly offers limited improvement. Moreover, it is anticipated that the Cherenkov Telescope Array Observatory (CTAO) will strengthen the annihilation bounds on dark matter coming from 5 (7) representation, with a reach of around 30 TeV (75 TeV) [42] (see also Ref. [48]). Thus, one expects an improvement in the lower mass bounds on these models in the near future.

D. Direct detection

Next, we examine the constraints that arise from direct detection, most prominently from the LZ experiment [2]. Comparable constraints have also been derived by XENONnT [49] and PandaX [50]; we also highlight the LZ dedicated high mass analysis [51]. To proceed we should calculate the direct detection cross sections for the dark matter models arising in Sec. III. The cases with $Y \neq 0$ lead to tree-level Z couplings for the dark matter Ψ^0 in which case the cross section for Ψ^0 to scatter on an individual nucleon $N = p, n$ is given by [52]

$$\sigma_{\text{SI}}^{(N)} = \frac{\mu_N^2}{\pi} \left(\frac{g_{V,0}^{\Psi,Z} g_V^{N,Z}}{m_Z^2} \right)^2. \quad (48)$$

The coupling $g_{V,0}^{\Psi,Z}$ is given in Eq. (5) for a vectorlike pair of general $\text{SU}(2)_L$ representation. Provided that $g_{V,Q=0}^{\Psi,Z} \neq 0$, to compute $\sigma_{\text{SI}}^{(N)}$ all that we require is the effective nucleon couplings $g_V^{N,Z}$. To find $g_V^{N,Z}$ we first ascertain the couplings of the Standard Model quarks which arise from Eq. (3) (with $T_3^R = 0$), namely for $q = u, d$

$$\begin{aligned} g_V^{u,Z} &= \frac{e}{2s_W c_W} \left(\frac{1}{2} - \frac{4}{3} s_W^2 \right), \\ g_V^{d,Z} &= \frac{e}{2s_W c_W} \left(-\frac{1}{2} + \frac{2}{3} s_W^2 \right), \\ g_A^{u,Z} &= -g_A^{d,Z} = \frac{e}{4s_W c_W}. \end{aligned} \quad (49)$$

We will take $\sin^2 \theta_W|_{\mu=m_Z} \approx 0.231$ which is the Z pole value, however, this does not change significantly via running to intermediate scales [1]. The coupling to protons (p) and neutrons (n) can then be obtained via

$$\begin{aligned} g_V^{p,Z} &= 2g_V^{u,Z} + g_V^{d,Z}, \\ g_V^{n,Z} &= g_V^{u,Z} + 2g_V^{d,Z}. \end{aligned} \quad (50)$$

From this we can write the direct detection cross section for all cases of interest with $Y \neq 0$,

$$\sigma_{\text{SI}} \simeq \begin{cases} 4.7 \times 10^{-40} \text{ cm}^2 & n = 2, Y = \pm 1/2, \\ 1.9 \times 10^{-39} \text{ cm}^2 & n = 3, Y = \pm 1, \\ 1.9 \times 10^{-39} \text{ cm}^2 & n = 5, Y = \pm 1, \\ 7.6 \times 10^{-39} \text{ cm}^2 & n = 5, Y = \pm 2. \end{cases}$$

If $Y = 0$ the scattering cross section is significantly reduced since the tree-level Z coupling vanishes, and the elastic SI cross section arises at loop level [28,53,54]. In the following, we report the cross sections computed in Ref. [54] for the $Y = 0$ models,⁶

$$\sigma_{\text{SI}} \simeq \begin{cases} [3.4, 13] \times 10^{-48} \text{ cm}^2 & n = 3, Y = 0, \\ [3.1, 12] \times 10^{-47} \text{ cm}^2 & n = 5, Y = 0, \\ [1.2, 4.8] \times 10^{-46} \text{ cm}^2 & n = 7, Y = 0. \end{cases}$$

In particular, for $Y = 0$ the scattering cross sections are reduced by a factor of 10^8 relative to their $Y = 1$ counterparts. We note that $n = 5, Y = 1$ is identical to the triplet $n = 3, Y = 1$ since their couplings $g_{V,0}^{\Psi,Z}$ are identical, cf. Eq. (12).

Notably, the current constraint from LZ constrains the dark matter scattering cross section to be [2]

$$\sigma_{\text{SI}}^{(n,p)} \Big|_{m \gg m_Z} < 3 \times 10^{-47} \text{ cm}^2 \left(\frac{m}{\text{TeV}} \right). \quad (51)$$

We overlay this limit on Fig. 1. Moreover, in Fig. 3 we show the constraints on the various models of interest in the familiar direct detection style plot, also combining the constraints from indirect detection and cosmology. In particular, smaller masses are constrained by dark matter annihilation in the cases of zero hypercharge for triplets, quintuplets and septuplets; cf. Eq. (46). Additionally, an upper bound on the mass comes from dark matter decays in the case of the quintuplet (cf. Fig. 2). The combination of direct detection and decaying dark matter bounds completely exclude the $n = 5$ case with $Y = \pm 1, 2$. Since the 5 with $Y = -2$ is metastable for M_{Pl} induced operators

⁶We thank the authors of Ref. [54] for providing numerical values.

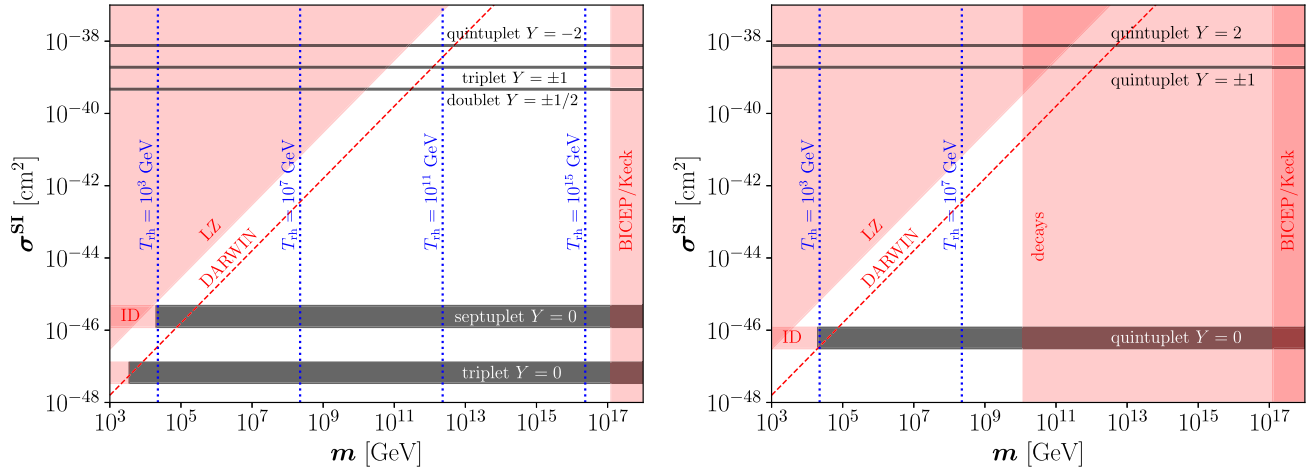


FIG. 3. Constraints from spin-independent limits LZ, for the different representations. We also show the projections of the proposed DARWIN experiment. For instantaneous reheating the dark matter mass uniquely determines the required T_{rh} and we plot some characteristic contours. We also collect together the cosmological limits from BICEP/Keck which constrain T_{rh} and the bounds from dark matter annihilations [cf. Eq. (46)], marked “ID.” We separate the selection of models which are constrained by dark matter decays limits, which we show on the right-hand panel, marked “decays” (cf. Fig. 2).

(cf. Fig. 2) it remains a viable dark matter candidate provided $m \gtrsim 2 \times 10^{11}$ GeV.

We also mark the reach of DARWIN [15] on Figs. 1 and 3. Notably, Darwin improves the bounds on dark matter by two orders of magnitude at all mass scales. The parameter space below DARWIN’s reach is in the neutrino fog (cf. Ref. [55]) and novel experimental advances will be required for further progress.

V. NONINSTANTANEOUS REHEATING

In our companion paper and the considerations above we have worked under the assumption that inflationary reheating is instantaneous. Notably, in this case the maximum temperature of the Universe is exactly the reheat temperature,

$$T_{\text{max}}|_{\text{instant}} = T_{\text{rh}} \simeq \sqrt{\Gamma_{\phi} M_{\text{Pl}}}, \quad (52)$$

where Γ_{ϕ} is the decay rate of the inflaton. However, beyond the instantaneous inflaton decay approximation, the maximum temperature of the thermal bath can exceed the reheat temperature [4], i.e., $T_{\text{max}} > T_{\text{rh}}$. Moreover, in the regime $T_{\text{max}} > T > T_{\text{rh}}$ the freeze-in dynamics is sensitive to the equation-of-state parameter of the Universe. Indeed, details on how the energy density of the Universe is transmitted from the inflaton to the Standard Model plasma can radically alter expectations for freeze-in [18–27].

Suppose that during cosmic reheating, the dominant component of the Universe in terms of energy density has an equation-of-state parameter ω , and that the Standard Model temperature scales as a power law. Let T_{rh} and a_{rh} denote the Standard Model temperature and scale factor at

the onset of the radiation-dominated era, respectively. Under these assumptions, the evolution of the Hubble expansion rate H as a function of the scale factor a is given by [56]

$$H(a) \simeq H_{\text{rh}} \times \begin{cases} \left(\frac{a_{\text{rh}}}{a}\right)^{\frac{3(1+\omega)}{2}} & \text{for } a \leq a_{\text{rh}}, \\ \left(\frac{a_{\text{rh}}}{a}\right)^2 & \text{for } a_{\text{rh}} \leq a, \end{cases} \quad (53)$$

while the Standard Model bath temperature evolves as

$$T(a) \simeq T_{\text{rh}} \times \begin{cases} \left(\frac{a_{\text{rh}}}{a}\right)^{\alpha} & \text{for } a \leq a_{\text{rh}}, \\ \left(\frac{a_{\text{rh}}}{a}\right) & \text{for } a_{\text{rh}} \leq a. \end{cases} \quad (54)$$

In conventional cosmology $\alpha > 0$ so that the Standard Model temperature always decreases with time, as is typical (but in certain scenarios $\alpha \leq 0$ is possible [57]).

Taking into account that at the end of reheating $\rho_R(a_{\text{rh}}) = \rho_{\phi}(a_{\text{rh}})$, the maximum energy density reached by the Standard Model thermal bath at the beginning of reheating (that is, at $a = a_I$) can be estimated and corresponds to a temperature $T_I \geq T_{\text{rh}}$ given by [56,58]

$$T_I \simeq T_{\text{rh}} \left[\frac{90}{\pi^2 g_{\star}} \frac{H_I^2 M_{\text{Pl}}^2}{T_{\text{rh}}^4} \right]^{\frac{\alpha}{3(1+\omega)}}. \quad (55)$$

We assume existence of a thermalized bath during the entirety of the reheating process. In some models of inflationary reheating, thermalization may be delayed which can lead to a lower maximum temperature [59,60].

In a general cosmic background where the Standard Model entropy is not conserved, instead of the yield Y , it is convenient to track the evolution of $N_Q \equiv n_Q a^3$. Therefore, for each component, the Boltzmann equation, Eq. (33) can be expressed as

$$\frac{dN_Q}{da} = \frac{a^2 \gamma_Q}{H}. \quad (56)$$

The dark matter abundance produced during reheating (between a_I and a_{rh}), in the case where $\alpha \neq 0$, can be written as

$$Y_{\text{FI}}^{\text{rh}} \simeq \frac{405}{256\pi^7} \frac{\mathcal{K}_{QQ'}}{g_{\star s} \alpha} \sqrt{\frac{10 M_{\text{Pl}}}{g_{\star} T_{\text{rh}}}} \left(\frac{T_{\text{rh}}}{2m}\right)^{\eta} \times \left[\Gamma\left(\eta, \frac{2m}{T_I}\right) - \Gamma\left(\eta, \frac{2m}{T_{\text{rh}}}\right) \right], \quad (57)$$

where $\Gamma(\eta, x)$ is the incomplete gamma function and

$$\eta \equiv \frac{9 + 3\omega - 8\alpha}{2\alpha} \quad (58)$$

collects the cosmological parameters ω and α .

In particular, when $\alpha > 0$ (corresponding to the thermal bath cooling with time, as typically occurs), then Eq. (57) can be approximated by

$$Y_{\text{FI}}^{\text{rh}} \simeq Y_{\text{FI}}^{\text{inst}} \times \begin{cases} \frac{1}{\alpha} \left(\frac{T_{\text{rh}}}{T_I}\right)^{\eta-1} e^{\frac{2m}{T_{\text{rh}}} - \frac{2m}{T_I}} & \text{for } T_{\text{rh}} < T_I \ll m, \\ \frac{\Gamma(\eta)}{\alpha} \left(\frac{T_{\text{rh}}}{2m}\right)^{\eta-1} e^{\frac{2m}{T_{\text{rh}}}} & \text{for } T_{\text{rh}} < m \ll T_I, \end{cases} \quad (59)$$

where $Y_{\text{FI}}^{\text{inst}}$ is the dark matter yield produced after reheating, given by Eq. (40). The two limits of Eq. (59) correspond respectively to the cases in which the largest temperature reached during reheating is still below the dark matter mass, $T_I \ll m$, and in which the bath initially reaches temperatures above the dark matter mass, $m \ll T_I$. The exponential factors in Eq. (59) should therefore not be interpreted as a universal enhancement of Boltzmann-suppressed freeze-in. Rather, they compensate the Boltzmann factor appearing in the postreheating result $Y_{\text{FI}}^{\text{inst}}$, because production during reheating can occur at temperatures higher than T_{rh} . In the regime $T_{\text{rh}} < T_I \ll m$, the production is Boltzmann suppressed at all times, as indicated by the factor e^{-2m/T_I} . By contrast, in the regime $T_{\text{rh}} < m \ll T_I$, the bath is initially hot enough to produce dark matter without Boltzmann suppression, and the yield is therefore controlled by the highest-temperature reached in the reheating history and carries only the power-law suppression $(T_{\text{rh}}/2m)^{\eta-1}$. In this latter regime the expression should be understood as the freeze-in yield provided

the dark matter interaction rate remains below the Hubble rate throughout reheating; if the rate becomes larger than H , the dark matter would thermalize and the freeze-in treatment would no longer apply.

The opposite behavior to above occurs for the (atypical) case of $\alpha < 0$,

$$Y_{\text{FI}}^{\text{rh}} \simeq Y_{\text{FI}}^{\text{inst}} \times \begin{cases} \frac{1}{|\alpha|} & \text{for } T_I < T_{\text{rh}} \ll m, \\ \frac{1}{\alpha \eta} \frac{2m}{T_{\text{rh}}} e^{\frac{2m}{T_{\text{rh}}}} & \text{for } T_I < m \ll T_{\text{rh}}, \end{cases} \quad (60)$$

which corresponds to a subdominant dark matter production during reheating. Finally, in the (unusual) case that the temperature is constant during reheating, then $\alpha = 0$ [6,25,57,61] (in which case $T_I = T_{\text{max}} = T_{\text{rh}}$) and the yield from reheating is

$$Y_{\text{FI}}^{\text{rh}} \simeq \frac{135}{128\pi^7} \frac{\mathcal{K}_{QQ'}}{g_{\star s} (3 + \omega)} \sqrt{\frac{10 M_{\text{Pl}}}{g_{\star} T_{\text{rh}}}} e^{-\frac{2m}{T_{\text{rh}}}} \times \left[1 - \left(\frac{a_I}{a_{\text{rh}}}\right)^{\frac{3(3+\omega)}{2}} \right]. \quad (61)$$

Thus, parametrically

$$Y_{\text{FI}}^{\text{rh}} \simeq \frac{4}{3(3 + \omega)} \frac{m}{T_{\text{rh}}} Y_{\text{FI}}^{\text{inst}}. \quad (62)$$

This corresponds to a modest amplification with respect to the production after reheating. We emphasize that the *total* dark matter yield corresponds to the production during and after reheating, and therefore the two contributions have to be added,

$$Y_{\text{FI}} = Y_{\text{FI}}^{\text{rh}} + Y_{\text{FI}}^{\text{inst}}. \quad (63)$$

In Fig. 4 we indicate how nonstandard cosmology impacts the expectations for freeze-in of the doublet model outlined in Sec. III A. We consider different choices of ω and α ; the top-left panel corresponds to an early matter dominated era ($\omega = 0$, $\alpha = 3/8$), the top right to kination ($\omega = \alpha = 1$), and the lower panels to two versions of a radiation-dominated era ($\omega = 1/3$ and $\alpha = 1/4$ or $\alpha = 3/4$). The gray regions correspond to the parameter space where the whole dark matter abundance can be fitted, by choosing T_{max} appropriately, as illustrated in Fig. 5 (for the case $\omega = 0$ with $\alpha = 3/8$). Thus, this gray area brackets the uncertainty on the duration of the reheating era.

The upper boundary of the gray areas correspond to instantaneous reheating (that is, $T_I = T_{\text{rh}}$). The lower bounds correspond to the maximum obtainable value of T_{max} . In particular, the bound on the inflationary scale H_I in Eq. (43) implies that the duration of reheating has to

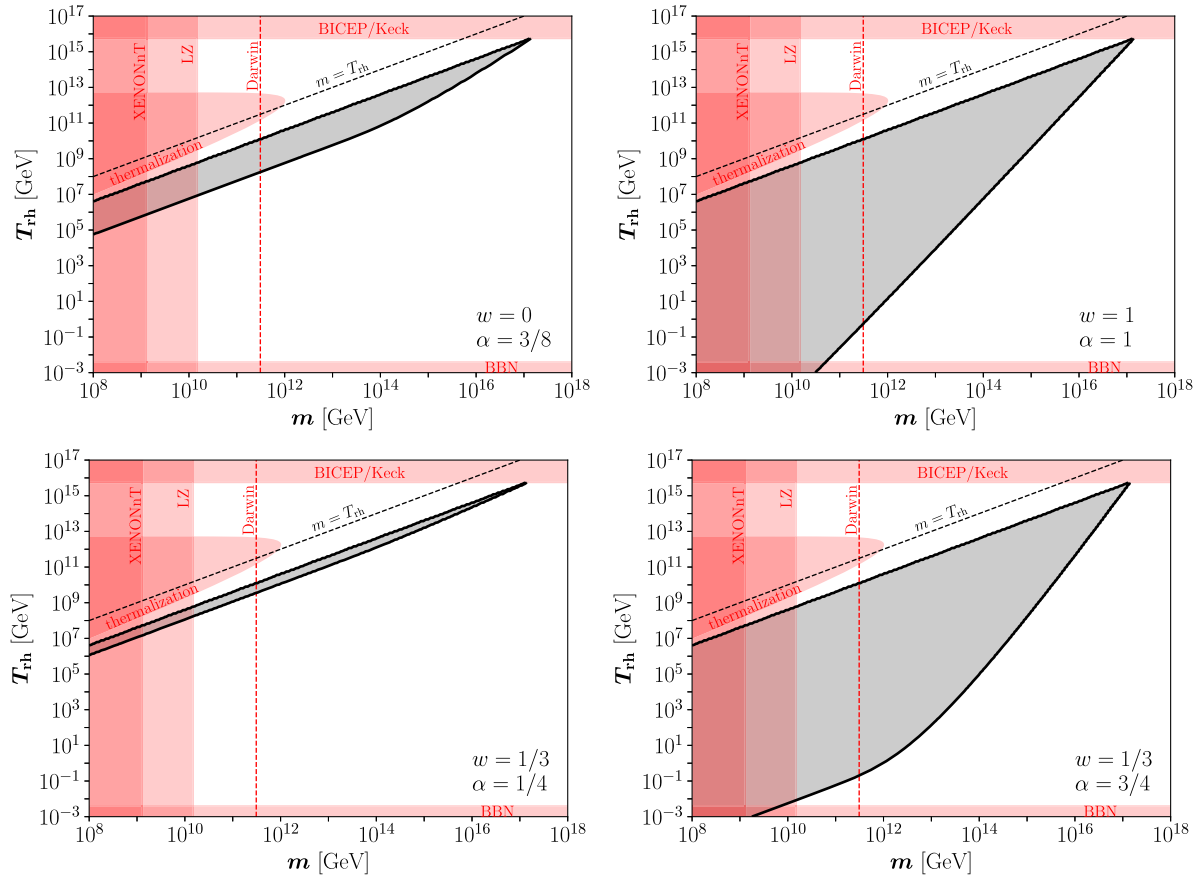


FIG. 4. Noninstantaneous reheating. The shaded region indicates viable parameters. The upper boundary is set by instantaneous reheating ($T_I = T_{\text{rh}}$); the lower boundary corresponds to the maximal duration of reheating, specifically $H(T_I) < 6 \times 10^{13} \text{ GeV}$ consistent with BICEP/Keck [38]. The “thermalization” region indicates parameters for which the dark matter enters equilibrium with the thermal bath, in which case the observed relic abundance is not reproduced.

decrease if T_{rh} grows, and therefore the bands shrink and collapse to a point when $T_{\text{rh}} \simeq 6 \times 10^{15} \text{ GeV}$. The two slopes of the lower bounds reflect the cases in which m is higher and lower than the maximal temperature T_I reached

by the thermal bath. The thickness of the gray regions is related to the exponent $\eta - 1$ in Eq. (59), with wider bands corresponding to smaller values of $\eta - 1$.

Figure 4 is overlaid with the region in which dark matter thermalizes with the Standard Model, and the bounds from direct detection for the doublet representation.

VI. GRAVITATIONAL PRODUCTION

Beyond the electroweak interactions discussed previously, dark matter and other heavier states are also inevitably produced via the annihilation of Standard Model particle pairs through s -channel graviton exchange. The squared amplitudes for fermionic dark matter are presented in Appendix B (cf. Refs. [62–64]).

Under the assumption of Maxwell-Boltzmann statistics, the corresponding reaction density reads

$$\gamma_Q^h(T) = \frac{T^4}{3840\pi^5 M_{\text{Pl}}^4} (525m^4 + 1536m^3T + 2440m^2T^2 + 2508mT^3 + 1254T^4) e^{-\frac{2m}{T}}. \quad (64)$$

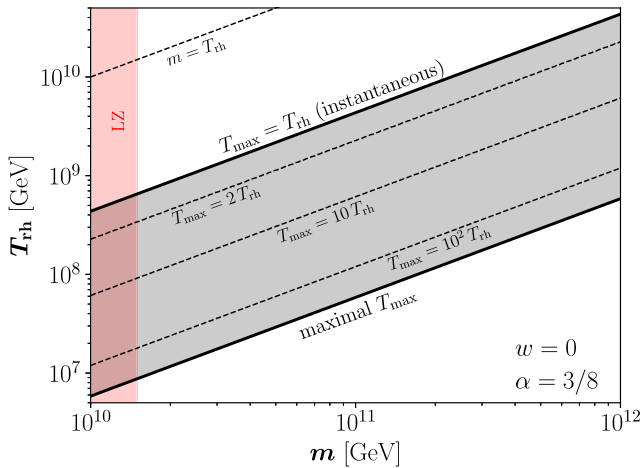


FIG. 5. Zoomed-in view of the top left panel of Fig. 4, overlaying contours corresponding to different values of T_{max} .

In the ultrarelativistic and nonrelativistic limits, γ_Q is

$$\gamma_Q^h(T) \simeq \begin{cases} \frac{209T^8}{640\pi^5 M_{\text{Pl}}^4} & \text{for } m \ll T, \\ \frac{35m^4 T^4}{256\pi^5 M_{\text{Pl}}^4} e^{-\frac{2m}{T}} & \text{for } T \ll m. \end{cases} \quad (65)$$

The ultrarelativistic limit was reported [62]; the results are compatible, with a difference of approximately 0.5%.

In the instantaneous reheating approximation, the solution of Eq. (38) yields

$$Y_{\text{FI}} = \frac{9}{1024\pi^8 g_{\star s}} \sqrt{\frac{10}{g_{\star}}} \frac{f(m, T_{\text{rh}})}{M_{\text{Pl}}^3} e^{-\frac{2m}{T_{\text{rh}}}}, \quad (66)$$

where

$$f(m, T_{\text{rh}}) \equiv 525m^3 + 1536m^2 T_{\text{rh}} + 1672m T_{\text{rh}}^2 + 836T_{\text{rh}}^3.$$

Taking the ultrarelativistic and nonrelativistic limits,

$$Y_{\text{FI}} \simeq \begin{cases} \frac{1881}{256\pi^8 g_{\star s}} \sqrt{\frac{10}{g_{\star}}} \left(\frac{T_{\text{rh}}}{M_{\text{Pl}}}\right)^3 & \text{for } m \ll T_{\text{rh}}, \\ \frac{4725}{1024\pi^8 g_{\star s}} \sqrt{\frac{10}{g_{\star}}} \left(\frac{m}{M_{\text{Pl}}}\right)^3 e^{-\frac{2m}{T_{\text{rh}}}} & \text{for } T_{\text{rh}} \ll m, \end{cases}$$

the general case beyond the instantaneous reheating approximation can be computed straightforwardly; however, the resulting expressions are rather lengthy and provide limited additional physical insight. We therefore report them only in Appendix B, see Eq. (B8).

Figure 6 is analogous to Fig. 4, with the gravitational production channel overlaid in blue. The blue regions indicate the parameter space where the observed dark matter abundance can be generated solely through gravitational production. Again, the thicknesses of the regions bracket the uncertainty on the duration of the reheating era. In addition, owing to the strong Planck-mass suppression of this mechanism, sizable dark matter masses and large reheating temperatures are required to achieve the correct relic density. Since this necessitates temperatures higher than those relevant for the electroweak channel, gravitational production never dominates electroweak production in the present scenario.

The dark matter yield from gravitational production is insensitive to the representation. A mild dependence arises, from heavier states that subsequently decay into dark matter. This additional contribution modifies the yield only by an $\mathcal{O}(1)$ factor, which is not discernible over the wide parameter range shown in Fig. 6.

Before closing this section, we note as an aside, that gravitational production is particularly relevant for the

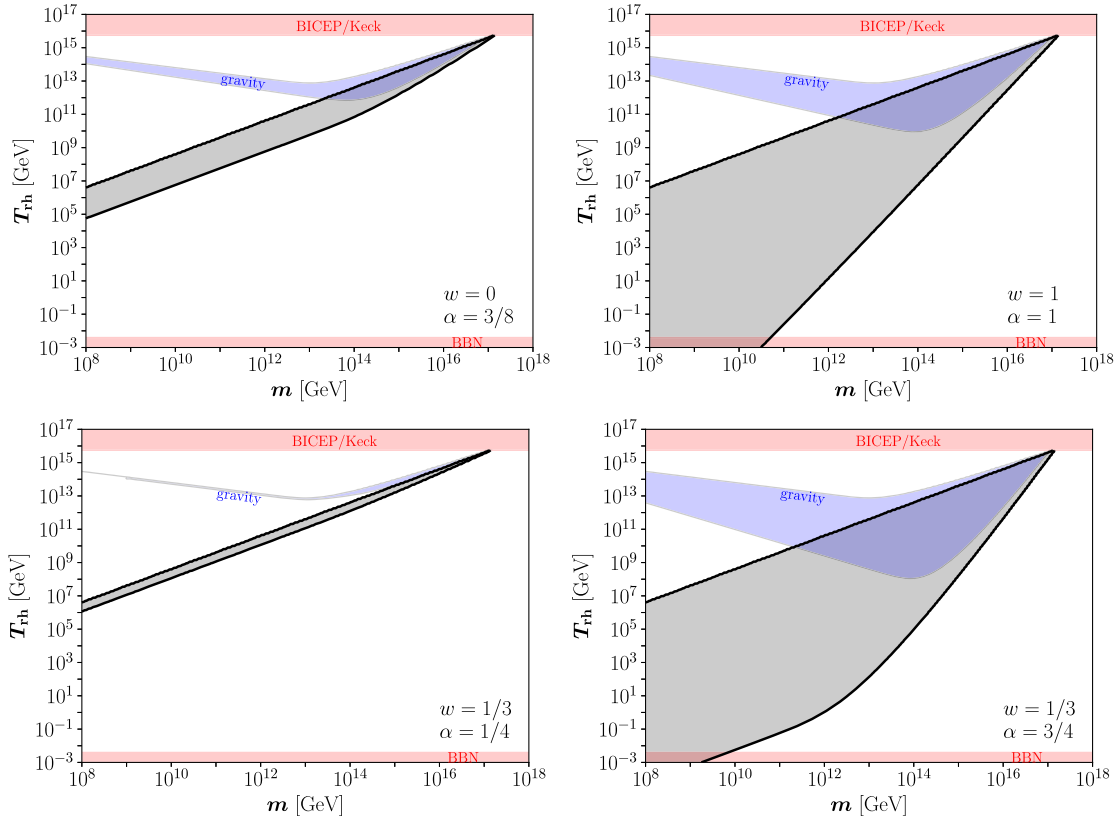


FIG. 6. Same as Fig. 4, overlaying in blue the gravitational production.

singlet fermion representation, as it constitutes the only available production channel, since singlets do not have tree-level couplings to the electroweak gauge bosons.

VII. SUMMARY AND CONCLUSIONS

Freeze-in in the regime $m > T_{\text{max}}$ offers the prospect of interesting models of dark matter which are highly distinct from *both* orthodox freeze-in, and the classic freeze-out WIMP picture, and with the benefit of being potentially discoverable at near-future direct and indirect experiments. Such Boltzmann suppressed freeze-in models,⁷ first outlined in Ref. [4], have recently been attracting renewed attention [1,5–14,65–69].

A. Summary

In a recent letter [1], we outlined the prospect of the neutral component of a pair of electroweak fermion doublets being dark matter with their relic abundance set via Boltzmann suppressed freeze-in. Due to its extreme minimality we argued that this scenario was worthy of the title; *minimal freeze-in dark matter*. In this companion paper, we have presented the next-to-minimal variants, both in terms of field content and cosmology.

Specifically, we have studied dark matter that arises from different representations of $SU(2)_L$ focusing on the triplet, quintuplet, and septuplet. While the relic density curves are largely unchanged between the small n representations (see Fig. 1), large differences can arise in expectations for direct and indirect detection (see Fig. 3). We note, from a field content perspective, the **3** with hypercharge $Y = 0$ (or any odd n with $Y = 0$) could arguably be considered more minimal than the doublet model of [1], as this provides an anomaly-free extension with just a single Weyl fermion (cf. Footnote 3). As a counterpoint, there is a compelling simplicity in models with matter only in the fundamental representation.

On the cosmology side, we have explored the implications of deviating from the instantaneous reheating approximation. Moreover, we have highlighted that for noninstantaneous reheating the details of the equation of state of the Universe prior to reheating can have a significant impact on the freeze-in dynamics. Inspection of Fig. 4 indicates the wider parameter freedom that nonminimal cosmological assumptions permit.

B. Scalar variants

We have endeavored to explore the cleanest and best motivated examples of Boltzmann suppressed freeze-in electroweak dark matter. In particular, we have restricted our considerations to fermion dark matter. The reasons we

favor fermions over scalars are two-fold; (i) A second scalar implies a second hierarchy problem, (ii) Naturalness suggests that generically the operators,

$$\lambda_3 |\phi|^2 |H|^2, \quad \lambda_4 |H^\dagger \phi|^2, \quad \lambda_5 (H^\dagger \phi)^2,$$

will appear with $\lambda_i \sim \mathcal{O}(1)$, as in the case of the inert doublet model [70]. The latter point is not so much a problem, rather it makes the analysis less clean as it introduces another free parameter.

Putting aside naturalness considerations, one might consider dark matter, which is a complex scalar ϕ transforming as a nontrivial representation under $SU(2)_L$. Let us briefly discuss the case in which ϕ is a scalar doublet. This is reminiscent of a second Higgs without Yukawa couplings. Similarly to the fermion case, for an $SU(2)_L$ doublet complex scalar ϕ with hypercharge Y , for one of the components to be neutral, the hypercharge must be $Y = \pm 1/2$.

The Lagrangian contribution corresponding to the scalar electroweak doublet is given by

$$\begin{aligned} \Delta\mathcal{L} = & (D_\mu \phi)^\dagger (D^\mu \phi) - m_\phi^2 |\phi|^2 + \lambda_3 |\phi|^2 |H|^2 \\ & + \lambda_4 |H^\dagger \phi|^2 + \frac{\lambda_5}{2} [(H^\dagger \phi)^2 + \text{H.c.}], \end{aligned} \quad (67)$$

where the above assumes a $\mathbb{Z}_2$ symmetry $\phi \rightarrow -\phi$. To make the model predictive, one must assume $\lambda_i \ll 1$ (although see Refs. [5,71] for studies of the Higgs portal). If the Higgs portal is negligible (which is an assumption which goes against technical naturalness), the leading interactions are via the electroweak gauge bosons, which interestingly leads to derivative couplings. Thus, the scalar dark matter variant is a nontrivial extension of the models considered here, as such we leave it for future studies.

C. Motivations for higher representations

Electroweak dark matter is independently motivated from a UV perspective. Notably, in the context of supersymmetry [72], Higgsinos and Winos provide quintessential examples of doublet and triplet dark matter (stabilized by R-parity). It is natural for the Higgsino to be the lightest supersymmetric particle (LSP) in scenarios involving the Giudice-Masiero mechanism [73], whereas gauginos such as the Wino are naturally the LSP in R-symmetric models, e.g., [74–80]. Moreover, it has been suggested that in supersymmetric models the inclusion of 5 can resolve the “little hierarchy problem” if the Standard Model superpartners are pushed to ~ 10 TeV [81]. The LSP dark matter in this model is the neutral fermion component of the 5, and thus is similar to the scenario considered here.

Aside from supersymmetry, alternative motivations have been found, especially for the 5 representation. For instance, the quintuplet has been proposed to have a possible origin in $SU(5)$ GUTs [82], as well as potentially arising in

⁷Also called “freeze-in at stronger coupling” [5], we prefer Boltzmann suppressed freeze-in, as it better includes UV freeze-in [65].

extra dimensional settings, in the context of 5D gauge-Higgs unification [83].

It should also be emphasised that the automatic metastability of dark matter arising from the 5 and 7 representations is a highly desirable feature. Not only does metastability remove the need for an *ad hoc* stabilizing symmetry, it also avoids the question of UV completion. We note that if a global symmetry [e.g., $\mathbb{Z}_2$ or $U(1)$] stabilizes the dark matter, then Planck induced operators are generically expected to violate this symmetry [84]. Accordingly, dimension five Planck-suppressed operators will commonly introduce dark matter decays unless these operators are forbidden by UV completing the global symmetry into a local symmetry or if the dark matter is accidentally metastable. This issue can be evaded if dark matter arises from a **5** or **7** of $SU(2)_L$, since dark matter can be accidentally metastable by construction.

D. Closing remarks

Forthcoming and proposed experiments offer the potential to discover this class of dark matter particles. Notably, direct and indirect detection already constrain the parameter space of these models, and near-future observations will continue to advance the search for these states. In particular, DARWIN [15] and CTAO [42,48] (for direct and indirect detection, respectively) improve on the reach of current experiments by orders-of-magnitude [85].

Arguably, the most attractive variants are those that are automatically metastable. It is notable that in contrast to the regular TeV scale freeze-out minimal dark matter, heavy dark matter coming from the quintuplet representation can lead to decay signals that may be detectable over the lifetime of KM3NeT [47,86]. We also highlight the tentative claim that a PeV neutrino event may be indicative of decaying dark matter [47]. As seen in Sec. III, decaying dark matter is consistent with Ψ^0 arising from the (viable) **5** model with $Y = 0$.

Although the collider bounds do not currently provide competitive constraints [43,87], future colliders could constrain (or discover) $\sim \text{TeV}$ $SU(2)_L$ multiplets. Studies have been made of minimal dark matter searches using proposed colliders based on linear e^+e^- [88], 100 TeV pp [89], and muon colliders [90]. In particular, this next-to-minimal freeze-in scenario appears to be an ideal benchmark for any future wakefield collider; see, e.g., [91,92].

The models outlined above are extremely predictive since the dark matter phenomenology is entirely determined by the dark matter mass for a given representation (since the couplings are fixed to be electroweak). Unlike conventional freeze-in which is coupled extremely weakly to the Standard Model, the models presented here offer the

potential for discovery in forthcoming experiments and provide excellent benchmarks for future searches.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: PRODUCTION CROSS SECTION

Below we derive the production cross sections,

$$q\bar{q}' \rightarrow \Psi^+\Psi^-, \Psi^0\bar{\Psi}^0, \Psi^\pm\bar{\Psi}^0, \Psi^-\Psi^{++}, \Psi^{--}\Psi^{++}, \dots,$$

This section largely follows the Appendix of [1] with a few generalizations. The process $q(p_1)\bar{q}'(p_2) \rightarrow \Psi(k_1)\Psi'(k_2)$ leading to the production of a pair of fermions with mass m and momenta k_1, k_2 from two quarks with momenta p_1, p_2 via a vector boson arises from the Lagrangian,

$$\mathcal{L}_{\text{int}} = V_\mu \left[\bar{q}\gamma^\mu \left(g_V^{q\bar{q}'} - g_A^{q\bar{q}'}\gamma^5 \right) q' + \bar{\Psi}\gamma^\mu \left(g_V^\Psi - g_A^\Psi\gamma^5 \right) \Psi' \right],$$

with generic vector couplings $g_V^{q\bar{q}'}, g_V^\Psi$ and axial-vector couplings $g_A^{q\bar{q}'}, g_A^\Psi$, as given in Eq. (3).

Neglecting the mediator mass, the propagator is $-i/s$ the tree-level amplitude is

$$\mathcal{M} = \frac{-i}{s} \left[\bar{v}(p_2)\gamma^\mu \left(g_V^{q\bar{q}'} - g_A^{q\bar{q}'}\gamma^5 \right) u(p_1) \right] \times \left[\bar{u}(k_1)\gamma_\mu \left(g_V^\Psi - g_A^\Psi\gamma^5 \right) v(k_2) \right]. \quad (\text{A1})$$

In the following, we neglect quark masses and take the limit $m_Z^2 \ll m^2, s$. We square the matrix element and sum over the final-state colors and spins, averaging over initial-state colors and spins, it follows that

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \frac{1}{12s^2} \text{Tr} \left[\not{p}_2 \gamma^\mu \left(g_V^{q\bar{q}'} - g_A^{q\bar{q}'}\gamma^5 \right) \not{p}_1 \gamma^\nu \left(g_V^{q\bar{q}'} - g_A^{q\bar{q}'}\gamma^5 \right) \right] \\ &\times \text{Tr} \left[(\not{k}_1 + m) \gamma_\mu \left(g_V^\Psi - g_A^\Psi\gamma^5 \right) \right. \\ &\left. \cdot (\not{k}_2 - m) \gamma_\nu \left(g_V^\Psi - g_A^\Psi\gamma^5 \right) \right]. \quad (\text{A2}) \end{aligned}$$

We can reexpress this in terms of the Mandelstam invariants,

$$\begin{aligned} s &= (p_1 + p_2)^2, \\ t &= (p_1 - k_1)^2, \\ u &= (p_1 - k_2)^2, \end{aligned} \quad (\text{A3})$$

with $s + t + u = 2m^2$ for massless initial-state quarks. Accordingly, one has

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \frac{2}{3s^2} \left[\left((g_V^q)^2 + (g_A^q)^2 \right) \left((g_V^\Psi)^2 (t^2 + u^2 + 2m^2 s) \right. \right. \\ &\quad \left. \left. + (g_A^\Psi)^2 (t^2 + u^2 - 2m^2 s) \right) \right. \\ &\quad \left. + 4g_V^{q\bar{q}'} g_A^{q\bar{q}'} g_V^\Psi g_A^\Psi s (t - u) \right]. \end{aligned} \quad (\text{A4})$$

$$\frac{d\sigma}{d\cos\vartheta} = \frac{\beta}{32\pi s} \frac{1}{3} \left\{ \left((g_V^{q\bar{q}'})^2 + (g_A^{q\bar{q}'})^2 \right) \left[(g_V^\Psi)^2 (1 + \cos^2\vartheta + (1 - \beta^2) \sin^2\vartheta) + (g_A^\Psi)^2 \beta^2 (1 + \cos^2\vartheta) \right] + 4g_V^{q\bar{q}'} g_A^{q\bar{q}'} g_V^\Psi g_A^\Psi \beta \cos\vartheta \right\},$$

where we used $m^2 = \frac{s}{4}(1 - \beta^2)$ and have integrated the azimuthal angle over 2π . Integrating the differential cross section over $\cos\vartheta \in [-1, 1]$ leads to the cross section for $q\bar{q} \rightarrow \Psi\Psi$ given by

$$\begin{aligned} \sigma &= \frac{\beta}{24\pi s} \left[(g_V^{q\bar{q}'})^2 + (g_A^{q\bar{q}'})^2 \right] \\ &\quad \times \left((g_V^\Psi)^2 - \frac{\beta^2}{3} \left((g_V^\Psi)^2 - 2(g_A^\Psi)^2 \right) \right). \end{aligned}$$

Note that when integrating over $\cos\vartheta$ the term proportional to $g_V^{q\bar{q}'} g_A^{q\bar{q}'} g_V^\Psi g_A^\Psi$ vanishes. We reexpress this in the following manner:

$$\sigma_{q\bar{q}' \rightarrow \Psi_Q \bar{\Psi}_{Q'}}(\beta) \simeq \frac{\beta}{24\pi s} \mathcal{K}_{Q,Q'}^{q\bar{q}'}, \quad (\text{A7})$$

with $\mathcal{K}_{Q,Q'}^{q\bar{q}'}$ defined [as in Eq. (8)] to be

$$\mathcal{K}_{Q,Q'}^{q\bar{q}'} \equiv \delta_{qq'} \left(C_{Q,\gamma}^{q\bar{q}} + C_{Q,Z}^{q\bar{q}} \right) + (1 - \delta_{qq'}) C_{Q,Q'}^{q\bar{q}'}. \quad (\text{A8})$$

Recall, at leading order [cf. Eq. (10)] for $i = \gamma, Z$,

$$C_{Q,i}^{q\bar{q}'} \approx \left((g_V^{q\bar{q}',i})^2 + (g_A^{q\bar{q}',i})^2 \right) (g_{V,Q}^{\Psi,i})^2. \quad (\text{A9})$$

For the photon channel ($q = q'$) the quark axial coupling vanishes $g_A^{q,\gamma} = 0$, the vector coupling is $g_V^{q,\gamma} = eQ_q$, and the $\mathcal{T}^{\text{th}}$ component coupling is $g_{V,Q}^{\Psi,\gamma} = eQ$, hence

Defining $\beta \equiv \sqrt{1 - \frac{4m^2}{s}}$, and ϑ as the angle between $\vec{p}_1$ and $\vec{k}_1$, one has

$$\begin{aligned} t &= m^2 - \frac{s}{2}(1 - \beta \cos\vartheta), \\ u &= m^2 - \frac{s}{2}(1 + \beta \cos\vartheta). \end{aligned} \quad (\text{A5})$$

The differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{k}|}{|\vec{p}|} |\overline{\mathcal{M}}|^2 = \frac{\beta}{64\pi^2 s} |\overline{\mathcal{M}}|^2. \quad (\text{A6})$$

Substituting Eq. (A4) and integrating the azimuthal angle to get a factor 2π gives

$$C_{Q,\gamma}^q = e^4 Q_q^2 Q^2. \quad (\text{A10})$$

For the Z channel ($q = q'$) the quark couplings are the Standard Model ones [cf. Eq. (49)], and recall

$$g_{V,Q}^{\Psi,Z} = \frac{e}{s_W c_W} (\mathcal{T} - Q s_W^2), \quad (\text{A11})$$

hence for the isospin component $\mathcal{T}$ with charge Q one has

$$C_{Q,Z}^q = \left((g_V^{q,Z})^2 + (g_A^{q,Z})^2 \right) \left[\frac{e}{s_W c_W} (\mathcal{T} - Q s_W^2) \right]^2. \quad (\text{A12})$$

$W^\pm$ induced “coproduction” processes, such as

$$\begin{aligned} u(p_1) \bar{d}(p_2) &\rightarrow W^+ \rightarrow \Psi^+(k_1) \bar{\Psi}^0(k_2), \\ d(p_1) \bar{u}(p_2) &\rightarrow W^- \rightarrow \Psi^-(k_1) \Psi^0(k_2), \end{aligned} \quad (\text{A13})$$

the cross section in the limit $\beta \ll 1$ is of the form,

$$\sigma_{q\bar{q}' \rightarrow \Psi_Q \bar{\Psi}_{Q-1}}^{(W)} \Big|_{\beta \ll 1} \simeq \frac{\beta}{24\pi s} C_{Q,W}^{q\bar{q}'}. \quad (\text{A14})$$

Equation (A7) can be specialized to this case by using couplings of the form (for $q \neq q'$),

$$\begin{aligned} g_{V,W}^{q\bar{q}'} &= g_{A,W}^{q\bar{q}'} = g_{V,W}^{q'\bar{q}} = g_{A,W}^{q'\bar{q}} = \frac{e}{2\sqrt{2}s_W} V_{qq'}, \\ g_{V,Q}^{\Psi,W} &= g_{A,Q}^{\Psi,W} = \frac{e}{2\sqrt{2}s_W} \sqrt{(T + \mathcal{T})(T - \mathcal{T} + 1)}, \end{aligned} \quad (\text{A15})$$

yielding

$$C_{Q,W}^{u,d_j} = \frac{1}{32} \frac{e^4}{s_W^4} |V_{ij}|^2 (T + \mathcal{T})(T - \mathcal{T} + 1). \quad (\text{A16})$$

In the above, we have worked in the (γ, Z) basis and the Ψ_Q fields, since direct detection limits require freeze-in to occur at temperatures $T \gg m_Z$ it may be more intuitive to work in the unbroken basis [e.g., in terms of χ fields and basis (B, W_3)]. Since at a late time we wish to identify the relic abundance of the dark matter state Ψ^0 , it seems simpler to always work in the basis of the broken phase.

APPENDIX B: GRAVITON EXCHANGE

The total amplitude squared for graviton exchange can be expressed as the sum of three contributions, weighted by the Standard Model content in fields,

$$|\mathcal{M}|^2 = 4|\mathcal{M}_0|^2 + 45|\mathcal{M}_{\frac{1}{2}}|^2 + 12|\mathcal{M}_1|^2. \quad (\text{B1})$$

Here we give the squared amplitudes for fermionic dark matter (where we indicate the spin of the initial-state particle in the subscript) [62–64],

$$\begin{aligned} |\mathcal{M}_0|^2 &= \frac{1}{32M_{\text{Pl}}^4 s^2} (m^2 - t)(s + t - m^2)(s + 2t - 2m^2)^2, \\ |\mathcal{M}_{\frac{1}{2}}|^2 &= \frac{1}{128M_{\text{Pl}}^4 s^2} \left[32m^8 - 32m^6(s + 4t) + 2m^4(5s^2 + 64st + 96t^2) \right. \\ &\quad \left. - 4m^2(s^3 + 13s^2t + 40st^2 + 32t^3) + s^4 + 10s^3t + 42s^2t^2 + 64st^3 + 32t^4 \right], \\ |\mathcal{M}_1|^2 &= \frac{-1}{8M_{\text{Pl}}^4 s^2} \left[m^4 - 2m^2t + t(s + t) \right] \left[2(m^4 - 2m^2t + t(s + t)) + s^2 \right]. \end{aligned} \quad (\text{B2})$$

Then the dark matter production rate density γ_Q^h is [93]

$$\gamma_Q^h(T) = \frac{1}{4(2\pi)^4} \int_m^\infty dE_1 dE_2 e^{-\frac{E_1}{T}} e^{-\frac{E_2}{T}} \int_{4m^2}^{4E_1 E_2} ds \frac{s}{\sqrt{\lambda(s, m^2, m^2)}} \int_{t_-}^{t_+} dt \frac{|\mathcal{M}|^2}{8\pi s}, \quad (\text{B3})$$

where

$$\lambda(s, m^2, m^2) = s(s - 4m^2), \quad (\text{B4})$$

$$t_- = -\frac{1}{4s} \left(s + \sqrt{s(s - 4m^2)} \right)^2, \quad (\text{B5})$$

$$t_+ = m^2 + \frac{1}{2} \left(-s + \sqrt{s(s - 4m^2)} \right), \quad (\text{B6})$$

one gets

$$\begin{aligned} \gamma_Q^h(T) &= \frac{T^4}{3840\pi^5 M_{\text{Pl}}^4} \left(525m^4 + 1536m^3T \right. \\ &\quad \left. + 2440m^2T^2 + 2508mT^3 + 1254T^4 \right) e^{-\frac{2m}{T}}, \end{aligned} \quad (\text{B7})$$

which corresponds to Eq. (64).

Before closing, we report the dark matter yield produced during reheating, which corresponds to the result of Eq. (56) with γ_Q^h . One gets

$$\begin{aligned} Y_{\text{FI}}^{\text{rh}} &= \frac{9}{2^{10}\pi^8 g_{\star s}} \sqrt{\frac{10}{g_{\star}}} \left(\frac{m}{M_{\text{Pl}}} \right)^3 \left(\frac{T_{\text{rh}}}{2m} \right)^{q-2} \\ &\quad \times [525\tilde{\Gamma}(q-1) + 3072\tilde{\Gamma}(q-2) + 9760\tilde{\Gamma}(q-3) \\ &\quad + 20064(\tilde{\Gamma}(q-4) + \tilde{\Gamma}(q-5))], \end{aligned} \quad (\text{B8})$$

with

$$\begin{aligned} \tilde{\Gamma}(x) &\equiv \Gamma\left(x, \frac{2m}{T_{\text{rh}}}\right) - \Gamma\left(x, \frac{2m}{T_I}\right), \\ q &\equiv \frac{3(1+\omega)}{2\alpha}. \end{aligned} \quad (\text{B9})$$

- [1] N. Bernal, S. Mukherjee, and J. Unwin, companion paper, Minimal freeze-in dark matter: Reviving electroweak doublet dark matter with Boltzmann suppressed freeze-in, *Phys. Rev. D* **113**, 115065 (2026)..
- [2] LZ Collaboration, Dark matter search results from 4.2 tonne-years of exposure of the LUX-ZEPLIN (LZ) experiment, *Phys. Rev. Lett.* **135**, 011802 (2025).
- [3] D. Tucker-Smith and N. Weiner, Inelastic dark matter, *Phys. Rev. D* **64**, 043502 (2001).
- [4] G. F. Giudice, E. W. Kolb, and A. Riotto, Largest temperature of the radiation era and its cosmological implications, *Phys. Rev. D* **64**, 023508 (2001).
- [5] C. Cosme, F. Costa, and O. Lebedev, Freeze-in at stronger coupling, *Phys. Rev. D* **109**, 075038 (2024).
- [6] C. Cosme, F. Costa, and O. Lebedev, Temperature evolution in the early universe and freeze-in at stronger coupling, *J. Cosmol. Astropart. Phys.* **06** (2024) 031.
- [7] N. Okada and O. Seto, Superheavy WIMP dark matter from incomplete thermalization, *Phys. Lett. B* **820**, 136528 (2021).
- [8] N. Koivunen, O. Lebedev, and M. Raidal, Probing sterile neutrino freeze-in at stronger coupling, *Eur. Phys. J. C* **84**, 1234 (2024).
- [9] G. Arcadi, F. Costa, A. Goudelis, and O. Lebedev, Higgs portal dark matter freeze-in at stronger coupling: Observational benchmarks, *J. High Energy Phys.* **07** (2024) 044.
- [10] K. K. Boddy, K. Freese, G. Montefalcone, and B. Shams Es Haghi, Minimal dark matter freeze-in with low reheating temperatures and implications for direct detection, *Phys. Rev. D* **111**, 063537 (2025).
- [11] G. Arcadi, D. Cabo-Almeida, and O. Lebedev, Z' -mediated dark matter freeze-in at stronger coupling, *Phys. Lett. B* **861**, 139268 (2025).
- [12] N. Bernal, C. S. Fong, and Ó. Zapata, Probing low-reheating scenarios with minimal freeze-in dark matter, *J. High Energy Phys.* **02** (2025) 161.
- [13] N. Bernal, G. Cottin, B. Díaz Sáez, and M. López, Testing frozen-in pNGB dark matter with a long-lived dark Higgs, *J. High Energy Phys.* **01** (2026) 081.
- [14] D. Feiteira, O. Lebedev, and V. Oliveira, Warm dark matter from freeze-in at stronger coupling, *Phys. Lett. B* **877**, 140486 (2026).
- [15] DARWIN Collaboration, DARWIN: Towards the ultimate dark matter detector, *J. Cosmol. Astropart. Phys.* **11** (2016) 017.
- [16] L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, Freeze-in production of FIMP dark matter, *J. High Energy Phys.* **03** (2010) 080.
- [17] F. Elahi, C. Kolda, and J. Unwin, UltraViolet freeze-in, *J. High Energy Phys.* **03** (2015) 048.
- [18] M. A. G. García, Y. Mambrini, K. A. Olive, and M. Peloso, Enhancement of the dark matter abundance before reheating: Applications to gravitino dark matter, *Phys. Rev. D* **96**, 103510 (2017).
- [19] N. Bernal, F. Elahi, C. Maldonado, and J. Unwin, Ultra-violet freeze-in and non-standard cosmologies, *J. Cosmol. Astropart. Phys.* **11** (2019) 026.
- [20] M. A. G. García, K. Kaneta, Y. Mambrini, and K. A. Olive, Reheating and post-inflationary production of dark matter, *Phys. Rev. D* **101**, 123507 (2020).
- [21] N. Bernal, J. Rubio, and H. Veermäe, Boosting ultraviolet freeze-in in NO models, *J. Cosmol. Astropart. Phys.* **06** (2020) 047.
- [22] N. Bernal, J. Rubio, and H. Veermäe, UV freeze-in in Starobinsky inflation, *J. Cosmol. Astropart. Phys.* **10** (2020) 021.
- [23] R. Allahverdi *et al.*, The first three seconds: A review of possible expansion histories of the early Universe, *Open J. Astrophys.* **4**, 5 (2021).
- [24] M. A. G. García, K. Kaneta, Y. Mambrini, K. A. Olive, and S. Verner, Freeze-in from preheating, *J. Cosmol. Astropart. Phys.* **03** (2022) 016.
- [25] B. Barman, N. Bernal, Y. Xu, and Ó. Zapata, Ultraviolet freeze-in with a time-dependent inflaton decay, *J. Cosmol. Astropart. Phys.* **07** (2022) 019.
- [26] B. Batell *et al.*, Conversations and deliberations: Non-standard cosmological epochs and expansion histories, *Int. J. Mod. Phys. A* **40**, 2530004 (2025).
- [27] N. Bernal, K. Deka, and M. Losada, Dark matter ultraviolet freeze-in in general reheating scenarios, *Phys. Rev. D* **111**, 055034 (2025).
- [28] M. Cirelli, N. Fornengo, and A. Strumia, Minimal dark matter, *Nucl. Phys. B* **753**, 178 (2006).
- [29] B. R. Safdi and W. L. Xu, Wino and real minimal dark matter excluded by Fermi gamma-ray observations, *arXiv*: 2507.15934.
- [30] A. Milagre and L. Lavoura, Unitarity constraints on large multiplets of arbitrary gauge groups, *Nucl. Phys. B* **1004**, 116542 (2024).
- [31] S. Das, K. Murase, and T. Fujii, Revisiting ultrahigh-energy constraints on decaying superheavy dark matter, *Phys. Rev. D* **107**, 103013 (2023).
- [32] O. Deligny, Constraints on superheavy dark matter decaying into $h\nu$, $Z\nu$ and $W\ell$, *Eur. Phys. J. C* **85**, 985 (2025).
- [33] R. D. Peccei and H. R. Quinn, CP conservation in the presence of instantons, *Phys. Rev. Lett.* **38**, 1440 (1977).
- [34] T. Yanagida, Horizontal gauge symmetry and masses of neutrinos, *Conf. Proc. C* **7902131**, 95 (1979).
- [35] S. Sarkar, Big bang nucleosynthesis and physics beyond the standard model, *Rep. Prog. Phys.* **59**, 1493 (1996).
- [36] T. Hasegawa, N. Hiroshima, K. Kohri, R. S. L. Hansen, T. Tram, and S. Hannestad, MeV-scale reheating temperature and thermalization of oscillating neutrinos by radiative and hadronic decays of massive particles, *J. Cosmol. Astropart. Phys.* **12** (2019) 012.
- [37] M. Kawasaki, K. Kohri, and N. Sugiyama, Cosmological constraints on late time entropy production, *Phys. Rev. Lett.* **82**, 4168 (1999).
- [38] BICEP and Keck Collaborations, Improved constraints on primordial gravitational waves using Planck, WMAP, and BICEP/Keck observations through the 2018 observing season, *Phys. Rev. Lett.* **127**, 151301 (2021).
- [39] D. Baumann, Inflation, in *Theoretical Advanced Study Institute in Elementary Particle Physics: Physics of the Large and the Small* (World Scientific, Singapore, 2011), pp. 523–686, [10.1142/9789814327183_0010](https://arxiv.org/abs/1011.2519).
- [40] J. Hisano, S. Matsumoto, M. M. Nojiri, and O. Saito, Non-perturbative effect on dark matter annihilation and gamma ray signature from galactic center, *Phys. Rev. D* **71**, 063528 (2005).

- [41] J. L. Feng, M. Kaplinghat, and H.-B. Yu, Sommerfeld enhancements for thermal relic dark matter, *Phys. Rev. D* **82**, 083525 (2010).
- [42] C. García-Cely, A. Ibarra, A. S. Lamperstorfer, and M. H. G. Tytgat, Gamma-rays from heavy minimal dark matter, *J. Cosmol. Astropart. Phys.* **10** (2015) 058.
- [43] P. Panci, Electroweak multiplets as dark matter candidates: A brief review, *Proc. Sci. CORFU2023* (2024) 033 [arXiv:2405.05087].
- [44] H.E.S.S. Collaboration, Search for photon-line-like signatures from dark matter annihilations with H.E.S.S., *Phys. Rev. Lett.* **110**, 041301 (2013).
- [45] M. Aghaie, A. Dondarini, G. Marino, and P. Panci, Minimal dark matter in the sky: Updated indirect detection probes, arXiv:2507.17607.
- [46] J. Einasto, On the construction of a composite model for the galaxy and on the determination of the system of galactic parameters, *Tr. Astrofiz. Inst. Alma-Ata* **5**, 87 (1965).
- [47] K. Kohri, P. K. Paul, and N. Sahu, Super heavy dark matter origin of the PeV neutrino event: KM3-230213A, *Phys. Rev. D* **112**, L031703 (2025).
- [48] S. Abe *et al.*, Discovering the Higgsino at CTAO-north within the decade, arXiv:2506.08084.
- [49] XENON Collaboration, WIMP dark matter search using a 3.1 tonne-year exposure of the XENONnT experiment, *Phys. Rev. Lett.* **135**, 221003 (2025).
- [50] PandaX Collaboration, Dark matter search results from 1.54 tonneyear exposure of PandaX-4T, *Phys. Rev. Lett.* **134**, 011805 (2025).
- [51] LZ Collaboration, New constraints on ultraheavy dark matter from the LZ experiment, *Phys. Rev. D* **109**, 112010 (2024).
- [52] R. Essig, Direct detection of non-chiral dark matter, *Phys. Rev. D* **78**, 015004 (2008).
- [53] J. Hisano, S. Matsumoto, M. M. Nojiri, and O. Saito, Direct detection of the Wino and Higgsino-like neutralino dark matters at one-loop level, *Phys. Rev. D* **71**, 015007 (2005).
- [54] Q. Chen, G.-J. Ding, and R. J. Hill, General heavy WIMP nucleon elastic scattering, *Phys. Rev. D* **108**, 116023 (2023).
- [55] C. A. J. O’Hare, Can we overcome the neutrino floor at high masses?, *Phys. Rev. D* **102**, 063024 (2020).
- [56] N. Bernal, K. Deka, and M. Losada, Thermal dark matter with low-temperature reheating, *J. Cosmol. Astropart. Phys.* **09** (2024) 024.
- [57] R. T. Co, E. González, and K. Harigaya, Increasing temperature toward the completion of reheating, *J. Cosmol. Astropart. Phys.* **11** (2020) 038.
- [58] B. Barman and N. Bernal, Gravitational SIMPs, *J. Cosmol. Astropart. Phys.* **06** (2021) 011.
- [59] K. Harigaya and K. Mukaida, Thermalization after/during reheating, *J. High Energy Phys.* **05** (2014) 006.
- [60] K. Mukaida and M. Yamada, Thermalization process after inflation and effective potential of scalar field, *J. Cosmol. Astropart. Phys.* **02** (2016) 003.
- [61] D. Chowdhury and A. Hait, Thermalization in the presence of a time-dependent dissipation and its impact on dark matter production, *J. High Energy Phys.* **09** (2023) 085.
- [62] N. Bernal, M. Dutra, Y. Mambrini, K. Olive, M. Peloso, and M. Pierre, Spin-2 portal dark matter, *Phys. Rev. D* **97**, 115020 (2018).
- [63] M. Dutra, Origins for dark matter particles: From the WIMP miracle to the “FIMP wonder”, Ph.D. thesis, Orsay, LPT, 2019.
- [64] S. Cléry, Y. Mambrini, K. A. Olive, and S. Verner, Gravitational portals in the early Universe, *Phys. Rev. D* **105**, 075005 (2022).
- [65] N. Bernal, E. Cervantes, K. Deka, and A. Hryczuk, Freezing-in cannibals with low-reheating temperature, *J. High Energy Phys.* **09** (2025) 083.
- [66] H. M. Lee, M. Park, and V. Sanz, Gravity-mediated dark matter at a low reheating temperature, *J. High Energy Phys.* **05** (2025) 126.
- [67] G. Bélanger, N. Bernal, and A. Pukhov, Z' -mediated dark matter with low-temperature reheating, *J. High Energy Phys.* **03** (2025) 079.
- [68] S. Khan, J. Kim, and H. M. Lee, Higgs portal vector dark matter at a low reheating temperature, *J. Cosmol. Astropart. Phys.* **06** (2025) 040.
- [69] N. Bernal, J. P. Neto, J. Silva-Malpartida, and F. S. Queiroz, Enabling thermal dark matter within the vanilla $L_\mu - L_\tau$ model, *Phys. Rev. D* **112**, 075042 (2025).
- [70] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva., Theory and phenomenology of two-Higgs-doublet models, *Phys. Rept.* **516**, 1 (2012).
- [71] N. Bernal, S. Mukherjee, and J. Unwin, Boltzmann suppressed ultraviolet freeze-in, *J. Cosmol. Astropart. Phys.* **02** (2026) 010.
- [72] S. P. Martin, A supersymmetry primer, *Adv. Ser. Dir. High Energy Phys.* **18**, 1 (1998).
- [73] G. F. Giudice and A. Masiero, A natural solution to the μ problem in supergravity theories, *Phys. Lett. B* **206**, 480 (1988).
- [74] P. Fayet, Spontaneously broken supersymmetric theories of weak, electromagnetic and strong interactions, *Phys. Lett.* **69B**, 489 (1977).
- [75] G. R. Farrar and S. Weinberg, Supersymmetry at ordinary energies. II. R invariance, Goldstone bosons, and gauge fermion masses, *Phys. Rev. D* **27**, 2732 (1983).
- [76] L. J. Hall and L. Randall, U(1)-R symmetric supersymmetry, *Nucl. Phys.* **B352**, 289 (1991).
- [77] P. J. Fox, A. E. Nelson, and N. Weiner, Dirac gaugino masses and supersoft supersymmetry breaking, *J. High Energy Phys.* **08** (2002) 035.
- [78] G. D. Kribs, E. Poppitz, and N. Weiner, Flavor in supersymmetry with an extended R-symmetry, *Phys. Rev. D* **78**, 055010 (2008).
- [79] J. Unwin and T. Yildirim, A QCD R-axion, *Phys. Rev. D* **111**, 015048 (2025).
- [80] J. Unwin, R-symmetric high scale supersymmetry, *Phys. Rev. D* **86**, 095002 (2012).
- [81] M. Fabbrichesi and A. Urbano, Natural minimal dark matter, *Phys. Rev. D* **93**, 055017 (2016).
- [82] T. Toma, Minimal dark matter in SU(5) grand unification, *Phys. Rev. D* **111**, L051701 (2025).
- [83] N. Maru, N. Okada, and S. Okada, Fermionic minimal dark matter in 5D gauge-Higgs unification, *Phys. Rev. D* **96**, 115023 (2017).

- [84] T. Banks and N. Seiberg, Symmetries and strings in field theory and gravity, *Phys. Rev. D* **83**, 084019 (2011).
- [85] S. Das, J. A. Carpio, and K. Murase, Probing superheavy dark matter through lunar radio observations of ultrahigh-energy neutrinos and the impacts of neutrino cascades, *Phys. Rev. D* **111**, 083007 (2025).
- [86] K. C. Y. Ng *et al.*, Sensitivities of KM3NeT on decaying dark matter, [arXiv:2007.03692](#).
- [87] B. Ostdiek, Constraining the minimal dark matter fiveplet with LHC searches, *Phys. Rev. D* **92**, 055008 (2015).
- [88] N. Kumar and V. Sahdev, Alternative signatures of the quintuplet fermions at the LHC and future linear colliders, *Phys. Rev. D* **105**, 115016 (2022).
- [89] Y.-P. Zeng, C. Cai, D.-Y. Liu, Z.-H. Yu, and H.-H. Zhang, Probing quadruplet scalar dark matter at current and future pp colliders, *Phys. Rev. D* **101**, 115033 (2020).
- [90] S. Bottaro, A. Strumia, and N. Vignaroli, Minimal dark matter bound states at future colliders, *J. High Energy Phys.* **06** (2021) 143.
- [91] S. Gessner *et al.*, Design initiative for a 10 TeV pCM wakefield collider, [arXiv:2503.20214](#).
- [92] S. Chigusa *et al.*, Searches for electroweak states at future plasma wakefield colliders, [arXiv:2512.09995](#).
- [93] P. Gondolo and G. Gelmini, Cosmic abundances of stable particles: Improved analysis, *Nucl. Phys.* **B360**, 145 (1991).