

Large black-hole scalar charges induced by cosmology in Horndeski theories

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The regularity of black-hole solutions, embedded in an expanding universe, is studied in a subclass of Horndeski theories, namely the sum of the simplest quadratic, cubic, and quintic actions. We find that in the presence of a time derivative of the scalar field, driven by the cosmological expansion, this regularity generically imposes large scalar charges for black holes, even when assuming strictly no direct coupling of matter to the scalar field. Such charges cause a significant accretion of the scalar field by the black holes, driving its local time derivative to a small value. This phenomenon, together with the Vainshtein screening typical of these theories, strongly suppresses observable scalar effects. We show that this full class of models is consistent with LIGO/Virgo detections of gravitational waves, but that the LISA mission should be able to constrain the coefficient of the quintic term at the 10^{-30} level in a self-acceleration scenario, an improvement by 16 orders of magnitude with respect to what is imposed by the speed of gravitational waves.

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I. INTRODUCTION

The uniqueness of black holes is one of the key predictions of general relativity (GR). The detection of gravitational waves (GW) from the final stages of the inspiral of binary compact objects has finally opened up a direct avenue for verifying this property.

Scalar-tensor theories provide concrete alternative models allowing for the exploration of consistent compact-object phenomenology differing from the GR setup. They allow for the existence of hair attached to scalar charges carried by compact objects, providing an opportunity for testing gravity. Static hairy solutions for black holes were first considered in the context of conformally coupled

theories of gravity, where they are singular on the horizon [1,2]. Neutron stars, on the other hand, admit regular hairy solutions with a mass-dependent sensitivity [3], allowing for the production of dipolar emission from binaries. Observations of such binary pulsars over a long time span still provide the best constraints for such nonminimal couplings [4,5].

More general theories, all part of the Horndeski class [6] giving standard second-order equations of motion, allow for alternative (derivative) interaction terms with gravity. In the presence of potentials for the scalar field, these tend to be suppressed and the phenomenology is similar to the conformal case. When the scalar-field action is shift symmetric, the new operators can be important, but alternative no-hair theorems exist [7]. The emission of dipolar radiation does not take place, and therefore these models are not strongly constrained [8].

However, the above conclusions are predicated on *both* a static configuration for the scalar field and asymptotically trivial boundary conditions at infinity. Already in Ref. [9] it was pointed out that an evolving scalar field at the boundary leads to an induced scalar charge for a black hole even in conformally coupled theories. This effect is of utmost relevance for black holes embedded in cosmology,

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although for conformally coupled theories the charge remains hard to observe [10]. Second, the existence of a cosmological horizon requires that a static profile for the scalar field sourced by a charge be augmented by a time-dependent term [11], the lack of which can even lead to singularities on horizons in some models [12]. The consistent setup for a black hole in cosmology therefore has *both* an induced scalar charge and a time-dependent profile. Whether this is a matter of principle or actually relevant to phenomenology is the central question of this paper.

Indeed, the fast-rolling scalar field is exploited in cosmology to provide a mechanism for late-time acceleration different from the cosmological constant or scalar-field potential energy [13,14]. Such models can modify the speed of GWs in cosmology and therefore have been constrained by its measurement [15–18]; nonetheless, a wide range of parameters are still allowed. The typical construction is one where the canonical kinetic term for the scalar is chosen to have the wrong sign, and therefore fluctuations around the trivial scalar background are ghosts. The cosmological dynamics then chooses a nontrivial Lorentz-violating background which nonetheless asymptotically looks similar to de Sitter spacetime. This introduces another barrier to the existence of static black hole (BH) solutions, since one would then have to connect the spacelike gradients of the scalar field near the BH to the timelike gradients at large distances without probing the unstable backgrounds. A configuration of the scalar gradient that is timelike everywhere but stationary owing to the shift symmetry is a natural resolution of this tension.

The fact that the cosmological evolution of ϕ can induce an effective coupling to a local source has been already discussed in the literature in different contexts. In particular, it was noticed in Ref. [19] that the matter-scalar coupling strength is modified due to the cosmological evolution of ϕ , in the case of the cubic Galileon model. This effect can be attributed to the kinetic scalar-graviton mixing in the presence of material sources, such as stars. In the case of black holes in the background of a time-dependent scalar field, a nontrivial configuration of the scalar field also arises, as it has been shown for asymptotic Minkowski spacetime [20,21]. However, this is owing to a different physical reason in this case; namely, this is a consequence of the requirement of nondivergence of observable quantities at the BH horizon. Therefore a nontrivial scalar configuration arises around a black hole even in the case of minimal coupling. In fact, the appearance of a nontrivial scalar configuration around a black hole for nonzero time derivative $\dot{\phi}$ can be viewed as a process of accretion, which is well understood for perfect fluids (see, e.g., [22,23]).

A construction of smooth everywhere nonsingular solutions, interpolating between the black hole and cosmological horizons, may be problematic due to various issues. In Ref. [12] it was demonstrated that a particular model involving the linear scalar-Gauss-Bonnet (sGB) term leads

to singular behavior at a horizon. In this paper, we focus on theories which do not present this issue (see Appendix A); i.e., a solution can be constructed such that observables are regular at both the cosmological and the black-hole horizons. Nonetheless, we demonstrate that another problem generically arises, related to a smooth transition from one branch of the solution to another as the distance from the black hole increases. Making a parallel with accretion of a perfect fluid, one has to build an analog of a transonic solution, which implies jumping from one branch to another. As we show in the present paper, it turns out to be challenging to have smooth branch transitions, when taking into account that normally two such points arise—one in the vicinity of the black-hole horizon, one at cosmological distances.

The paper is organized as follows. Section II defines the subclass of Horndeski theories we consider, derives their test scalar-field solution, and discusses two generic phenomena which significantly reduce observable scalar-field deviations from general relativity: Scalar-field accretion by BHs makes their scalar charge decrease with time, and the Vainshtein mechanism screens scalar exchanges between two BHs as well as the binary’s energy loss via scalar waves. Section III focuses on the quadratic plus cubic Galileon model. It shows that the discriminant of a quadratic equation needs to have double roots at some precise locations, explains our technique to compute them, and derives the BH’s scalar charge and the local time derivative of the scalar field needed to get a linear time-dependent solution in the whole universe. Section IV considers another subclass of models, involving the simplest quintic Horndeski action in addition to the standard quadratic kinetic term for the scalar field. It illustrates the strong accretion of the scalar field which occurs when too large scalar charges are predicted for BHs. Section V is devoted to the phenomenologically richer case of the quadratic, cubic, and quintic kinetic terms together. It is particularly interesting when cosmology is dominated by the quadratic and cubic Galileon terms, while the local physics of BHs is dominated by the quintic term. Section VI discusses the observational consequences of these three models in binary black-hole coalescences. While all of them are consistent with the present LIGO/Virgo data, we show that the coefficient of the quintic term will be tightly constrained by LISA. Our conclusions are given in Sec. VII.

II. ACTION AND TEST SCALAR-FIELD SOLUTION

A. Quadratic, cubic, and quintic Horndeski

In the present paper, we focus on the quadratic, cubic, and quintic Horndeski actions, together with the standard Einstein-Hilbert term and a possible cosmological constant. The corresponding action may be written as

$$\begin{aligned}
S = & M_{\text{Pl}}^2 \int \sqrt{-g} d^4x \left\{ \frac{R}{2} - \Lambda_{\text{bare}} \right. \\
& + G_2(X) + G_3(X) \square \varphi + G_5(X) G^{\mu\nu} \varphi_{\mu\nu} \\
& \left. - \frac{1}{3} G'_5(X) [(\square \varphi)^3 - 3 \square \varphi \varphi_{\mu\nu} \varphi^{\mu\nu} + 2 \varphi_{\mu\nu} \varphi^{\nu\rho} \varphi_{\rho}{}^{\mu}] \right\} \\
& + S_{\text{matter}}[\psi, e^{2\alpha\varphi} g_{\mu\nu}], \quad (1)
\end{aligned}$$

where $M_{\text{Pl}} \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass (in units such that $\hbar = c = 1$), R is the curvature scalar of the metric $g_{\mu\nu}$ with the sign conventions of Ref. [24] (notably the mostly plus signature), φ is a dimensionless scalar field whose first derivative is denoted as $\varphi_\mu \equiv \partial_\mu \varphi$, similarly $\varphi_{\mu\nu} \equiv \nabla_\nu \nabla_\mu \varphi$ for its second covariant derivative, and $X \equiv -\varphi_\mu^2 = -g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$.

We have also included the action specifying how matter fields, globally denoted as ψ , are universally coupled to a physical metric which may differ from $g_{\mu\nu}$. The exponential factor we wrote here means that matter is assumed to be linearly coupled to the scalar field with a bare coupling constant α (which is dimensionless). However, its actual coupling generically differs from α because of the nonlinearities of these theories [19,25–28]. Moreover, we will mainly focus on black holes in the present work, i.e., vacuum solutions for which this bare α does not play any role. We will actually derive that a black-hole effective coupling constant to the scalar field, say α_{BH} , depends on the parameters entering the functions $G_2(X)$, $G_3(X)$, and $G_5(X)$, on the Hubble constant H , and on the Schwarzschild radius r_s of the black hole.

In the absence of matter sources, action (1) only depends on the derivatives of the scalar field; therefore, it is “shift-symmetric,” i.e., invariant when one adds a constant to the scalar field. Noether’s theorem implies that there exists a conserved current related to this symmetry. This is simply

$$J^\mu \equiv -\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta \partial_\mu \varphi}, \quad (2)$$

and its covariant conservation $\nabla_\mu J^\mu = 0$ is the scalar-field equation of motion.

We further restrict action (1) by imposing that the three functions $G_2(X)$, $G_3(X)$, and $G_5(X)$ are linear in X :

$$G_2(X) = k_2 X, \quad (3a)$$

$$G_3(X) = \frac{k_3}{M^2} X, \quad (3b)$$

$$G_5(X) = \frac{k_5}{M^4} X, \quad (3c)$$

where k_2 , k_3 , and k_5 are dimensionless constants and M is a constant mass parameter. Note that, while these choices of $G_2(X)$ and $G_3(X)$ correspond to the respective covariant

Galileons, our $G_5(X)$ does not.¹ Nevertheless, we will loosely refer to our models as Galileons. Some factors could easily be reabsorbed in the definition of φ and these constants, but it is useful to keep track of the origin of the various terms in our results below. Moreover, a positive value of k_2 corresponds to a standard positive-energy degree of freedom on a background with vanishing scalar derivatives. Instead, we will take a negative value of k_2 —this implies that the standard configuration with vanishing scalar derivatives is unstable. However, there now exists a well-behaved attractor in which both the background and fluctuations of the scalar have positive energy, which generates an accelerated expansion of the universe as the end point of expansion, even when $\Lambda_{\text{bare}} = 0$ [14,19,29].

In this work we will be interested in classical nonlinear regimes in these theories, where the higher-dimensional (HD) operators $G_3(X)$ and/or $G_5(X)$ either balance or dominate over the lower dimensional $G_2(X)$. This is the case in cosmology and in the Vainshtein regime, respectively. From the effective field theory (EFT) point of view, the coexistence of such a regime with the absence of strong coupling is possible, as the nonlinearity energy scale is generically much smaller than the strong-coupling scale, at which quantum effects become important [30]. This is no different from GR. Here, this is in part due to the non-renormalization properties enjoyed by Horndeski theories [31]. These two scales are background dependent and their comparison should be done in a case by case basis. Ultimately, as the strong-coupling scale is the naive cutoff of the EFT (it may be lower), it should be kept sufficiently large as to allow one to describe the physics at the scales of interest.

B. Cosmological background

Let us first consider an isotropic and homogeneous universe described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = -d\tau^2 + a(\tau)^2(d\rho^2 + \rho^2 d\Omega^2). \quad (4)$$

The field equations for the metric tensor give the value of $H \equiv \dot{a}/a$ in terms of its sources.² There may exist several such sources, notably a bare cosmological constant Λ_{bare} as in action (1), our Galileon field φ , and possibly other fields. It will not be necessary to write explicitly these equations for our study below.

¹The quintic Galileon would correspond instead to $G_5(X) \propto X^2$.

²Note that this H is defined in the “Horndeski frame,” i.e., with respect to the metric $g_{\mu\nu}$ of Eq. (1). The Jordan-frame (observable) Hubble expansion rate reads $e^{-\alpha\varphi}(H + \alpha\dot{\varphi})$, where α is the matter-scalar coupling constant entering action (1). For $|\alpha| \lesssim 1$, such a change of frame does not modify our order-of-magnitude estimates of Sec. VI, and the two frames strictly coincide when assuming $\alpha = 0$, i.e., no direct matter-scalar coupling.

In the homogeneous universe we consider, the scalar-field equation of motion (2) reduces to $\partial_t(a^3 J^0) = 0$, assuming no direct coupling to other fields; therefore, the shift-charge density $J^0 = \text{const}/a^3$ tends toward 0 during cosmological expansion. The asymptotic cosmological solution for the time derivative of the scalar field, say $\dot{\varphi}_c$, is thus given by $J^0 = 0$ with

$$\frac{J^0}{M_{\text{Pl}}^2} = -2k_2\dot{\varphi}_c + \frac{6H}{M^2} \left[k_3 - \left(\frac{H}{M} \right)^2 k_5 \right] \dot{\varphi}_c^2. \quad (5)$$

Aside from the trivial solution $\dot{\varphi}_c = 0$, for which the scalar field does not contribute at all in the cosmological expansion, and which is unstable owing to our choice of $k_2 < 0$, there is an additional non-vanishing solution for $\dot{\varphi}_c$ when $k_2 \neq 0$ and k_3 or k_5 do not vanish either. The value of $\dot{\varphi}_c$ varies as an inverse power of H as the universe evolves, with H given by the Friedmann equation. The energy density carried by the homogeneous scalar field in cosmology is in general given by

$$\rho_\varphi = -J^0\dot{\varphi}_c - 2k_2\dot{\varphi}_c^2 + \frac{2k_5}{M^4} H^3 \dot{\varphi}_c^3, \quad (6)$$

where the J^0 term disappears quickly as the shift charges dilute. The value $\dot{\varphi}_c$ given by $J^0 = 0$ provides a nontrivial contribution to the energy density, giving the asymptotic value of H , which is then only a function of the parameters k_i , M , and Λ_{bare} .

With $k_2 \sim -1$ and one of $k_{3,5} \sim 1$ and $\Lambda_{\text{bare}} = 0$, we have $H \sim M$. We will refer to this particular choice as *full self-acceleration*, where the scalar is alone responsible for driving the expansion of the universe.³ This result exhibits the utility of using our parametrization for the Horndeski functions (3), for which the dependence on M_{Pl} factors out; in geometrical units the equations would carry a small parameter. Retaining a positive nonzero $\Lambda_{\text{bare}} \gg M^2$ implies that the bare cosmological constant drives the acceleration. We stress here the inversion that occurs with respect to the usual situation: In our models a smaller M implies a smaller effect for the background, as opposed to the usual discussion where large mass scales in operators suppress their effects. In this context, the nonlinearity scale is M , while the naive strong-coupling scale is $\Lambda_3 \sim (M_P M^2)^{1/3}$, i.e., the scale that suppresses the HD operators when expressed in terms of canonically normalized fields, and clearly $M \ll \Lambda_3$. Therefore this type of solution is well-described within the EFT.

We underline here the peculiarity resulting from the choice of $k_2 < 0$, necessary for self-acceleration: there is a minimum value of the scalar-field gradient. As it is

approached, the normalization of the acoustic (effective) metric for fluctuations goes to zero, implying strong coupling. Strong coupling is also present for a class of black-hole solutions in scalar-tensor theories [32–34]. It should be stressed that the solutions we consider below do not belong to this “stealth” class; therefore, we do not expect strong coupling in our scenario. In kinetic gravity braiding, it was shown that there is a pressure singularity at this point [35] on the other side of which the scalar degree of freedom is a ghost [36]. A similar mechanism should be present for generic Galileon theories and is in fact a sign that the effective field theory description is no longer valid. It is thus not possible to consistently connect the timelike cosmological scalar-field gradient to a static spacelike gradient or even just a vacuum configuration within the same theory.

We also note that the presence of k_5 modifies the speed of propagation of gravitational waves on the cosmological background [29]

$$\alpha_T \equiv c_T^2 - 1 = \frac{2k_5\dot{\varphi}_c^3 H}{M^4 - 2k_5\dot{\varphi}_c^3 H}. \quad (7)$$

This expression should also contain $\ddot{\varphi}_c$, which vanishes for the future asymptotic background solution, and in any case $\ddot{\varphi} \sim H\dot{\varphi}$ and we will neglect it. The existence of such an operator is very strongly constrained [15–18] by the measurement of the speed of gravitational waves by LIGO/Virgo [37]

$$|\alpha_T^0| < 10^{-15}, \quad (8)$$

where we have added the superscript 0 to signify that this constraint arises at low redshift and therefore for our models, it implies that the deviation of GW speed from luminal must have been even smaller in the past, as a result of the inverse relationship between $\dot{\varphi}_c$ and H implied by Eq. (5).

It is worth observing that, when the scale $M \sim H$ and the scalar is fully responsible for the acceleration, the corresponding strong-coupling scale Λ_3 is quite low and, in terms of frequency, it lies just within the LIGO band. This makes the EFT prediction of a nonluminal speed of gravitational waves not robust for LIGO gravitational waves [38]. It may well be that the speed of tensors instead approaches the speed of light around the LIGO band and therefore the above constraint would not be nearly as strong, if it were at all.

C. Test scalar-field generated by a black hole

Let us now consider a static black hole of Schwarzschild radius r_s embedded within such an expanding universe. Although the scalar field may be responsible for this expansion, i.e., the value of H , we assume that the *local* scalar field generated by the black hole is small enough not

³In Secs. V and VID, we shall consider a model containing the three functions (3) but with k_5 small. The case $|k_2| \sim |k_3| \sim M/H \sim 1$ will in any case correspond to full self-acceleration.

to backreact⁴ on the Schwarzschild–de Sitter metric in static coordinates⁵

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad (9)$$

where

$$f(r) = 1 - \frac{r_S}{r} - (Hr)^2. \quad (10)$$

We look for a stationary solution of our test scalar field in the form

$$\varphi = \dot{\varphi}_{\text{BH}} t + \phi(r), \quad (11)$$

where $\dot{\varphi}_{\text{BH}}$ is assumed to be constant [19–21,39]. Since the radial derivative $\varphi' = \phi'$, we shall actually no longer use the notation ϕ in the following. The scalar-field equation $\nabla_\mu J^\mu = 0$ reads in such a case $\partial_r(r^2 J^r) = 0$; therefore, $r^2 J^r$ is given by an integration constant. In the case of material bodies [19], writing this equation within matter and integrating it over r would imply that this integration constant reads αr_S at lowest post-Newtonian order, where α is the dimensionless matter-scalar coupling constant entering the physical metric in action (1). In the present case of a black hole, i.e., of a vacuum solution, the integration constant is not fixed by any matter source, but rather by the regularity of the solution. Let us denote it as $\alpha_{\text{BH}} r_S$ by analogy with the matter case. The scalar-field equation reads then

$$\frac{J^r}{M_{\text{Pl}}^2} = \frac{\alpha_{\text{BH}} r_S}{r^2}, \quad (12)$$

where

$$\frac{J^r}{M_{\text{Pl}}^2} = A\varphi'^2 + B\varphi' + C, \quad (13a)$$

$$A = \frac{f}{M^2} \left[\left(\frac{4f}{r} + f' \right) k_3 + \frac{3f-1}{(Mr)^2} f' k_5 \right], \quad (13b)$$

$$B = 2fk_2, \quad (13c)$$

$$C = - \left[k_3 + \frac{f-1}{(Mr)^2} k_5 \right] \frac{f' \dot{\varphi}_{\text{BH}}^2}{fM^2}. \quad (13d)$$

⁴In a physically relevant scenario, the scalar field either plays the role of the cosmological constant or is a spectator field. Therefore, barring extra contributions from the spatial derivatives—which are suppressed due to the Vainshtein mechanism when it operates—one expects that the backreaction is at most of order of that of the cosmological constant, i.e., can be safely neglected at least locally. We shall come back to this question below, notably when estimating the scalar accretion rate in Sec. II D.

⁵Related to the Friedmann coordinates of Eq. (4) by the transformation (38).

This is thus a mere quadratic equation for φ' , generalizing to $k_5 \neq 0$ the one derived for the cubic Galileon case in [19] (see also [40,41]). Denoting its discriminant as

$$\Delta \equiv B^2 - 4A \left(C - \frac{\alpha_{\text{BH}} r_S}{r^2} \right), \quad (14)$$

we have thus the very simple solution

$$\varphi' = \frac{-B \pm \sqrt{\Delta}}{2A}. \quad (15)$$

Note that this closed form is a consequence of our assumption of linear Horndeski functions (3). It will allow us to analyze in detail its behavior at various locations close to the black hole and at cosmologically large distances. Let us immediately underline a crucial point: One must have $\Delta \geq 0$ for φ' to be real. This means that when Δ reaches 0 at a given radius r , it must have a *double* root to remain positive on both sides. We shall see below that this actually provides the relationship between α_{BH} and $\dot{\varphi}_{\text{BH}}$. In the full domain, two such radii exist, and this then actually fixes the values of these two quantities. It is worth noting here that a modification of $\dot{\varphi}_{\text{BH}}$ away from its cosmological value $\dot{\varphi}_c$ does not prevent the recovery of the homogeneous cosmological background at large distances [11] (see Appendix B).

Note also that the time derivative of the scalar field, $\dot{\varphi}_{\text{BH}}$, enters in Eq. (13d) as a second source term for its radial derivative φ' , in addition to the right-hand side of Eq. (12). This consequence of the nonlinearity of Horndeski theories implies that the cosmological expansion has a direct effect on local solutions. This had already been underlined for material bodies in [19,25–28], as well as for black holes in other contexts [21]. As we will compute later, the nontrivial background $\dot{\varphi}_{\text{BH}}$ induces a scalar charge α_{BH} for the black holes with $\alpha_{\text{BH}} \propto (\dot{\varphi}_{\text{BH}})^2$. Since in our self-accelerating setup there exists a minimum value of $\dot{\varphi}_{\text{BH}}$ for which the scalar-field fluctuations are nonghosts, this background cannot be removed completely and a stationary black hole must always carry such a charge. We expect a similar situation also for more general functions $G_i(X)$, as a source term C_0 analogous to (13d), that is, a φ' -independent term, is always present for generic $G_3(X)$ and $G_5(X)$ functions, namely

$$C_0 = - \left[G'_3(X) + \frac{f-1}{r^2} G'_5(X) \right] \frac{f' \dot{\varphi}_{\text{BH}}^2}{f}, \quad (16)$$

with both $G'_3(X)$ and $G'_5(X)$ evaluated at $X = \dot{\varphi}_{\text{BH}}^2/f$. Interestingly, $G_2(X)$ and $G_4(X)$ never contribute to it. We shall see below that it has important observational consequences for black holes in the present models.

D. Accretion of scalar field

The ansätze we have assumed in Eqs. (9) and (11) may not be entirely consistent with one another. While the shift symmetry guarantees that the linear time dependence of the scalar field does not appear in the equations explicitly, the solutions to the combined field equations may still show a nontrivial time evolution. This is due to a nonvanishing energy flux through the black-hole horizon, as given by the off-diagonal components of the stress-energy tensor,

$$T^r_t = -\dot{\phi}_{\text{BH}} J^r|_{r=r_S}, \quad (17)$$

where we take the horizon to approximately be at $r = r_S$. This form is general and a consequence of diffeomorphism invariance [42], and it implies that the simultaneous presence of a time derivative of the scalar field $\dot{\phi}_{\text{BH}}$ and a nonzero shift-charge flux J^r gives rise to an energy flux. In Eq. (12) we see that the shift-charge flux into the black hole is in turn proportional to the scalar charge α_{BH} . Note that there are subclasses of Horndeski theory which allow for solutions with a time-dependent scalar (11) and zero energy flux (see Refs. [39,43]). For the solution we consider here, however, there is always a nonzero accretion.

To trust our stationary ansätze, we must demand that accretion of the scalar field into the black hole is a sufficiently slow process. This condition may be expressed in terms of the accretion rate associated with the above energy flux (for details, see, e.g., [44]),

$$F_{\text{acc}} \sim r_S^2 |T^r_t| = M_{\text{Pl}}^2 r_S |\dot{\phi}_{\text{BH}} \alpha_{\text{BH}}|, \quad (18)$$

where we used Eq. (12) in the second equality. Then using

$$\frac{dm}{dt} = \frac{M_{\text{Pl}}^2}{2} \frac{dr_S}{dt} = F_{\text{acc}}, \quad (19)$$

we find the characteristic time of the black-hole mass change, Γ_{acc}^{-1} ,

$$\Gamma_{\text{acc}} \sim |\dot{\phi}_{\text{BH}} \alpha_{\text{BH}}|, \quad (20)$$

with $\alpha_{\text{BH}} \propto (\dot{\phi}_{\text{BH}})^2$ in the models we consider here, as we will demonstrate in the following sections and as may already be anticipated from Eq. (13d).

Upon the formation of the black hole in the presence of the cosmological background of the scalar field $\dot{\phi}_c$, a stationary solution is not possible without a charge appearing. This is a result of the singularity in Eq. (15) which appears unless we have a double root when $\Delta = 0$ in the vicinity of r_S . The black hole must then evolve on time-scales of order Γ_{acc}^{-1} by accreting shift charge, until this accretion is quenched, conservatively when the configuration is such that

$$\Gamma_{\text{acc}}^{-1} \sim H^{-1}, \quad (21)$$

the lifetime of the universe.⁶ In the following, we will estimate the potential for observability of such black holes assuming this conservative state. In principle, the black holes could be seen before they reach this final quasistationary state and therefore with a larger charge and more hair.

The black hole then is in a quasistationary state. This happens as a result of the two possible scenarios:

- (I) *Small accretion rate:* The cosmological $\dot{\phi}_c$ is such that the accretion rate is already slow enough. The evolution of the black hole is effectively already frozen and

$$\dot{\phi}_{\text{BH}} \approx \dot{\phi}_c. \quad (22)$$

We will demonstrate that this is the scenario for the cubic Galileon model.

- (II) *Quenched accretion:* Alternatively, if the charge induced by the cosmological $\dot{\phi}_c$ makes the accretion rate large, the black hole instead begins to consume the energy stored in the scalar field's cosmological background configuration, reducing the value of $\dot{\phi}$ in its vicinity. The accretion rate falls, but on the timescale of the lifetime of the universe, cannot decrease below the inverse lifetime of the universe. So, again conservatively, the depletion would effectively freeze when

$$\dot{\phi}_{\text{BH}} \ll \dot{\phi}_c, \quad (23)$$

reaching a value low enough so that Eq. (21) is satisfied. The charge of such a black hole would then be disconnected from that implied by the cosmological background. This is the scenario for sufficiently small black holes when a quintic Horndeski operator is present.

We stress that the alternative, maybe more usual, end point with no scalar background or just a static spatial gradient is *not* a consistent solution that can be described by our action. Scalar-field fluctuations on such backgrounds are ghosts and the approach to the boundary between nonghosts and ghosts necessarily leads through a strong-coupling regime where calculations are outside the validity of the effective field theory.

We should also remind the reader that H is a function of time, with $\dot{\phi}_c$ given by Eq. (5) and therefore smaller in the past. Our requirements on the accretion rate should be interpreted as related to the time when the black hole exists and, e.g., is emitting radiation. This means that higher accretion rates would be considered slow in the past, while charges would typically be smaller. We will show that the

⁶Accretion can be considered quenched when its characteristic time is longer than the lifetime of the object. To be conservative, we assume that this time is the lifetime of the universe.

total effect of this time dependence is quite subtle and model dependent, as a result of a dependence of any observables also on the screening.

E. Vainshtein screening

As we will show in concrete models, black-hole charges induced by cosmology in our class of theories are (very) large. This should immediately appear as a potentially dangerous modification from the standard situation, since it could spoil the experimental tests of general relativity.

But the crucial difference with respect to the standard scalar-tensor theories [45] [i.e., with a canonical kinetic term (3) alone] is that there exists a Vainshtein screening in the nonlinear Galileon theories, which reduces the effect of the charge (see, e.g., Ref. [46] for a review on the Vainshtein mechanism).

A good way to understand this is by noting that small-amplitude high-frequency scalar perturbations effectively propagate in the (inverse) acoustic metric $Z^{\mu\nu}$ and not the usual spacetime (see, e.g., Ref. [47] for a pedagogical explanation of this phenomenon). When exchanging scalar perturbations, the interaction between two bodies A and B is proportional to the product of their scalar charges $\alpha_A \alpha_B$, while the scalar propagator is built from the inverse of the kinetic term. The interaction strength, compared to that prevalent at cosmological scales away from the body, is thus reduced by a multiplicative factor proportional to

$$z_\lambda \equiv \frac{Z_\lambda''}{Z_c''}, \quad (24)$$

where $Z_c'' \sim |k_2|$ is the acoustic metric normalization at cosmological distances and Z_λ'' is the normalization of the acoustic metric valid at the relevant scales, e.g., the wavelength of the gravitational radiation. It is worth noting that the acoustic metric is not always well-defined, since the signature of the metric may be wrong, and in this case either gradient instability or a ghost may appear. In particular, some black-hole solutions with the configuration of the scalar field $X = \text{const}$ indeed suffer from this pathology [48,49]. We stress, however, that we consider here solutions that do not fall into this class, since $X \neq \text{const}$. Moreover, in Ref. [19] it was demonstrated that close to the material source, the perturbations in the $G_2 + G_3$ model are stable, which suggests that this may also be valid when a black hole is considered in the same model. A full stability analysis is postponed to future work.

We will now present the method for estimating the relevant Z_λ'' . In the presence of derivative interactions (i.e., noncanonical kinetic terms) the acoustic metric depends on the background configuration of the scalar field. The particular feature of the present Galileon models is that the dependence on the scalar gradients is strong and the acoustic metric changes significantly between that on the homogeneous cosmological configuration and that in the vicinity

of the black hole. For the cubic Galileon, the metric is given in full generality in Eq. (16) of Ref. [19] (or Eq. (2.15) of Ref. [14]), where the relevant terms involve φ' and φ'' .

A complication arises for Horndeski operators beyond $G_2(X)$, namely the kinetic mixing between scalar and spin-2 degrees of freedom. This is not problematic specifically for cubic Horndeski, as for any $G_3(X)$ it is always possible to decouple them generically for arbitrary backgrounds [14,19,50]. However, for quintic Horndeski this is not possible and the procedure instead becomes considerably more difficult. Since we are only interested here in order-of-magnitude estimates, in what follows we may proceed without carrying out the diagonalization procedure, and instead estimate separately the pure scalar part and the mixing term, as a means of identifying the relevant overall scaling of the effective metric for scalar perturbations.

In any case, the contributions involve the scalar-field gradient as sourced by the black hole. They will thus crucially depend on the scalar charge α_{BH} induced by the cosmology, giving a highly nonlinear behavior.

For radii much larger than the Schwarzschild radius r_S but much smaller than the cosmological horizon $1/H$, the background scalar-field solution around the black hole (12) and (13) takes one of three forms, depending on which term dominates the equation of motion. In G_5 dominance,

$$\varphi'^2 = \frac{\alpha_{\text{BH}} M^4 r^2}{2k_5} \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) + \mathcal{O}(H^2 r^2) \right]; \quad (25)$$

in regions where G_3 is most important,

$$\varphi'^2 = \frac{\alpha_{\text{BH}} M^2 r_S}{4k_3 r} \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) + \mathcal{O}(H^2 r^2) \right]; \quad (26)$$

and for the G_2 kinetic term,

$$\varphi' = \frac{\alpha_{\text{BH}} r_S}{2k_2 r^2} \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) + \mathcal{O}(H^2 r^2) \right]. \quad (27)$$

As we will demonstrate in the subsequent analysis, the value of α_{BH} depends on the particular model and its parameters, but whatever it is, it creates the profiles Eqs. (25)–(27). Depending on the choices of the model parameters k_i and M , and the mass of the black hole, various terms will dominate in different regions. Generically, G_5 dominates the solution at small radii, G_3 at intermediate radii, and G_2 at large radii. However, parameters can be chosen where either G_3 or G_5 is never the relevant solution (not in the least, when those operators are absent from the model).

Comparing the order of magnitude of the terms (25)–(27) allows us to define three different Vainshtein radii. The first, typically the smallest one, occurs when Eq. (25) is of the order of (26), and gives

$$r_{V35}^3 = \frac{|k_5| r_S}{2|k_3| M^2}, \quad (28)$$

where the subscript “35” recalls that we are comparing the G_3 - and G_5 -dominated expressions. An intermediate Vainshtein radius can be defined when Eq. (25) is of the order of the square of (27),

$$r_{V25}^3 = \frac{\sqrt{|k_5 \alpha_{\text{BH}}|} r_S}{\sqrt{2}|k_2| M^2}. \quad (29)$$

Finally, a third typically largest Vainshtein radius occurs when Eq. (26) is of the order of the square of (27),

$$r_{V23}^3 = \frac{|k_3 \alpha_{\text{BH}}| r_S}{k_2^2 M^2}. \quad (30)$$

Note that the dependence of the Vainshtein radii on M^2 also enters through α_{BH} , which we will show is model dependent.

The schematic behavior of the scalar field is illustrated in the log-log plot of Fig. 1, with values calculated for an example of the model presented in Sec. V. Of course, the actual solution has a smoother shape, and the cosmological corrections $\mathcal{O}(H^2 r^2)$ we neglected start to have an influence at large distances.

Given the solutions φ' and the Vainshtein radii, we can now calculate the acoustic metric suppression factors. The G_2 contribution to the metric is just $Z_2'' \sim |k_2|$. The form of the G_3 contribution to $Z^{\mu\nu}$ implies that [19]

$$Z_3''(r) \sim |k_2| \left(\frac{r_{V23}}{r} \right)^{3/2}. \quad (31)$$

For the quintic Horndeski Lagrangian, at quadratic order in scalar fluctuations, we find that the dominant contribution to the purely scalar effective metric reads

$$Z_5^{\mu\nu} \sim \frac{k_5}{M^4} R^{\mu\alpha\nu\beta} \nabla_\alpha \nabla_\beta \varphi, \quad (32)$$

where φ is the background scalar field. Evaluating the tt component, by inspection it is possible to see that there is at least one derivative acting on the metric function f (from the curvature factor), providing a factor of r_S in the intermediate region far from r_S and H^{-1} . Moreover, the whole expression itself is linear in φ , with at least one derivative. The rest is dimensional analysis. Therefore, we can estimate the leading contribution in this region to behave as

$$Z_5'' \sim \frac{k_5}{M^4} \frac{r_S}{r^4} \varphi'. \quad (33)$$

Using the expression for φ' in the nonlinear regime (25), we find that the acoustic metric behaves as

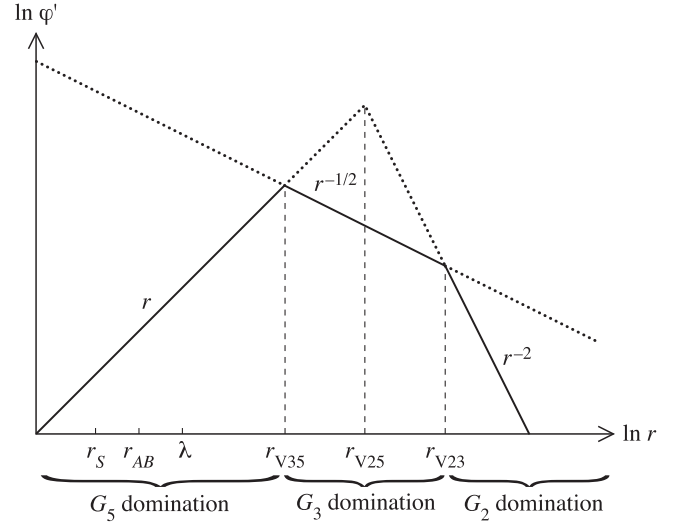


FIG. 1. The various regimes of the radial derivative φ' in the quadratic + cubic + small quintic Galileon model for a choice of black hole which is G_5 -dominated. The gravitational wavelength is denoted as λ , and r_{AB} is the interbody distance.

$$Z_5'' \sim |k_2| \left(\frac{r_{V25}}{r} \right)^3, \quad (34)$$

and therefore the Vainshtein screening is much stronger than for G_3 . The above estimate was obtained without accounting for the kinetic mixing of the scalar with the gravitational fluctuations. However, this is reliable as the mixing is negligible in all the cases of interest that will be discussed in the later sections (see Appendix C).

Whenever a particular G_i Lagrangian term dominates the background, it also dominates the acoustic metric. We will thus not need to distinguish in the remainder of this paper between these two types of scales. Note that r_{V35} , Eq. (28), is independent of the black-hole charge α_{BH} , depending on the model parameters k_i , M , and the mass of the black hole only. A G_5 -dominated region surrounds a particular black hole only if $r_{V35} > r_S$. Otherwise, the black hole itself is G_3 -dominated, and it is the G_3 term which can be seen to set the black-hole charge.

If G_5 dominates for a particular black hole, the G_3 region is transitory and does not affect the Z_5 term of the acoustic metric beyond its intermediate effect on φ' . Note that only r_{V25} enters Eq. (34), and not r_{V35} , even though the location $r = r_{V25}$ does not correspond to anything particular for the background solution plotted in Fig. 1.

Even if the G_5 -dominated region exists, a separate question is the scale at which the acoustic metric is probed, λ . For the emission of gravitational radiation in inspirals, this is the wavelength of the GWs, i.e., $\lambda \sim 300 r_S$ at the moment the black-hole binaries enter the LIGO/Virgo band. This is a somewhat larger scale than r_S , and therefore, there exist choices of parameters for which the black hole itself is in the G_5 -dominated region but where the radiation

production is determined by the G_3 term in the action. We will discuss this in detail in Sec. VI.

Concerning the validity of the EFT to describe this very nonlinear regime, it is interesting that the effect of the Vainshtein screening in suppressing the interactions of the scalar field, Eq. (24), tends to *increase* the strong-coupling scale well beyond the $\Lambda_3 \sim (M_P M^2)^{1/3}$ that we have identified in the cosmological context. On the other hand, we can identify a nonlinearity scale as r_V^{-1} , for the largest of the Vainshtein radii. These make the two scales vastly different, making any such solutions well-described within the EFT.

Having described the general features of the Galileon models, we now turn to crux of this paper—the computation of the black-hole charges induced by cosmology, in the construction with self-acceleration, $k_2 < 0$. We do this for two simpler subcases, where the solutions and approximation are clear, and then generalize to the full model involving both the cubic and quintic Horndeski terms.

III. CUBIC GALILEON: SMALL ACCRETION

In the present section, we consider the particular case $k_5 = 0$ in Eqs. (3), i.e., when only the two functions $G_2(X)$ and $G_3(X)$ define the dynamics of the scalar field.

The cosmological attractor Eq. (5) in this model implies

$$\dot{\varphi}_c = \frac{k_2 M^2}{3k_3 H}, \quad (35)$$

which in turn gives the energy density of the scalar field

$$\rho_\varphi = -k_2 \dot{\varphi}_c^2 = \frac{|k_2|^3 M^4}{9k_3^2 H^2}. \quad (36)$$

This energy grows in time, and therefore this kind of dark energy is a phantom [14]. Since $k_5 = 0$, the propagation of gravitational waves is not modified in this model and $\alpha_T = 0$.

A. The structure of the solution

As is clear from Eq. (15), there are two possible branches of the solution, which correspond to the plus and minus signs, respectively. The choice of the sign is determined by the physical requirements on the scalar-field profile; i.e., one needs to choose the physically relevant solution. To this end, let us first consider the scalar-field profile at large distances from a black hole. One expects to recover the homogeneous scalar profile, i.e., a solution of the scalar in a homogeneous FLRW universe, with small corrections due to the presence of the black hole. Setting $r_S = 0$, Eq. (15) must therefore reduce to the homogeneous solution, which in cosmological coordinates reads

$$\varphi = \dot{\varphi}_c \tau. \quad (37)$$

The static and cosmological coordinates without the black hole are related through

$$t = \tau - \frac{1}{2H} \log [1 - (H e^{H\tau} \rho)^2], \quad (38a)$$

$$r = e^{H\tau} \rho, \quad (38b)$$

so that [19]

$$\varphi' = -\dot{\varphi}_c \frac{Hr}{1 - (Hr)^2}. \quad (39)$$

Note that the homogeneous solution at large distances is recovered from φ' , and *not* from $\dot{\varphi}$ as may have been naively expected. If we choose $\dot{\varphi} \neq \dot{\varphi}_c$ in static coordinates, we would still recover the cosmological solution, admittedly at the price of some inhomogeneity decaying past the cosmological horizon (see Appendix B for details).

Still for $r_S = 0$, the discriminant (14) becomes a pure square, $\Delta = [2k_2(1 - 3H^2 r^2)/3]^2$, so that solution (15) reads

$$\varphi' = -\dot{\varphi}_c Hr \frac{3(1 - H^2 r^2) \pm |1 - 3H^2 r^2|}{2(1 - H^2 r^2)(2 - 3H^2 r^2)}. \quad (40)$$

To recover Eq. (39), one must choose $\pm = \text{sign}(1 - 3H^2 r^2)$, i.e., plus for $Hr < 1/\sqrt{3}$ and minus for $Hr > 1/\sqrt{3}$. The wrong sign would give Eq. (39) divided by $(2 - 3H^2 r^2)$, corresponding to an inhomogeneous scalar background in FLRW coordinates. The point $Hr = 1/\sqrt{3}$, where the discriminant is zero, is thus a branching point, where the sign entering solution (15) must change from + to − as r increases.

Moreover, the choice of the − branch at very large radii, $Hr > 1/\sqrt{3}$, ensures that solution (15) is regular at the point where $A = 0$ [i.e., $4f + rf' = 0$ for the model of the cubic Galileon, as one can read off Eq. (13b)], which corresponds to $Hr = \sqrt{2/3}$ for the homogeneous solution. On the contrary, the choice of the + branch would lead to a singularity at $Hr = \sqrt{2/3}$. Indeed, close to this radius, Eq. (15) can be expanded as

$$\varphi' = \frac{|B| \pm |B|}{2A} \mp \frac{C}{|B|} + \mathcal{O}(A), \quad (41)$$

showing that the upper sign is singular while the lower sign gives a regular expression.

For a nonzero r_S , we expect a similar behavior of the solution at large distances, with small corrections to the scalar profile and the position of the cosmological branching point.

However, the presence of the black hole naturally introduces another branching point in the vicinity of the

TABLE I. Special points of the test scalar-field solution in the quadratic plus cubic Galileon model. We only display the first two terms of the expansions in powers of the small quantity Hr_S (note that the corrections are proportional to Hr_S^2 for some terms but to the much smaller $H^2r_S^3$ for others). In the first line, $Z^{\mu\nu}$ is the inverse effective metric in which scalar perturbations propagate. The last column displays the sign to be imposed in solution (15) in order for it not to deviate too much from the cosmological background $\varphi = \dot{\varphi}_c\tau$.

Description	Equation	r	$\pm\sqrt{\Delta}$
Sound horizon	$Z^{rr} = 0$	$\frac{3}{4}r_S - \frac{9}{32}\sqrt{3}Hr_S^2$	—
Pole	$4f + rf' = 0$	$\frac{3}{4}r_S + \frac{81}{128}H^2r_S^3$	—
Branching point	$\Delta = 0$	$\frac{3}{4}r_S + \frac{9}{128}\sqrt{3}Hr_S^2$	$\mp$
Metric horizon	$f = 0$	$r_S + H^2r_S^3$	+
Branching point	$\Delta = 0$	$1/(\sqrt{3}H) + \frac{15}{2}r_S$	$\pm$
Pole	$4f + rf' = 0$	$\sqrt{2}/(\sqrt{3}H) - \frac{3}{8}r_S$	—
Metric horizon	$f = 0$	$1/H - \frac{1}{2}r_S$	—

black hole horizon. Indeed, the + branch of the solution that we chose above to match the cosmological background at smaller radii, $Hr < 1/\sqrt{3}$, diverges inside the Schwarzschild horizon at a radius corresponding to $A = 0$, namely at $r \simeq \frac{3}{4}r_S$ when neglecting the corrections due to the nonzero H . To avoid this singularity at $A = 0$ for small r , one needs to change again to the — branch at $\Delta = 0$, similar to our description of the behavior of the solution at cosmologically large distances.⁷ As we will confirm later, the branching point $\Delta = 0$ is located at a slightly larger radius than the one for which $A = 0$, which allows the solution to avoid the singularity at $A = 0$.

The structure of the solution that we desire to construct is summarized in Table I, while the details of calculations will be given in the next subsections.

It is worth comparing the case of a black hole in cosmology with the somewhat similar study of an asymptotically flat spacetime as in Ref. [21]. Galileon accretion in asymptotically flat spacetime requires the existence of a single branching point, which is located in the vicinity of the BH horizon. At this point, a branch of the solution which is well-behaved at spatial infinity is matched to the branch well-behaved near the horizon. In the case of accretion of a perfect fluid, such a transition happens at the so-called transonic point. For a black hole in a FLRW universe, as we have seen above, there is an extra branching point at a cosmologically large distance. This makes the

⁷One may think that such a singularity, if present, may be disregarded, since it is inside the Schwarzschild horizon. However, scalar perturbations in fact pass through this horizon, as it generically happens for superluminal perturbations [20]. Therefore, a physically relevant solution must be nonsingular up to the horizon for scalar perturbations, which lies inside the Schwarzschild radius.

construction of an everywhere smooth and regular solution rather challenging, as each branching point brings extra conditions on the profile of the solution. We discuss this point in detail below.

B. Branching points

We underlined in Sec. II C that the discriminant Δ , Eq. (14), needs to remain positive or null at all radii for our test scalar-field solution (15) to make sense. Since the locations of the roots of Δ depend on α_{BH} and $\dot{\varphi}_{\text{BH}}$, we must define a procedure to fix these parameters so that these roots are always *double* roots. A single branching point actually imposes a *relation* between α_{BH} and $\dot{\varphi}_{\text{BH}}$, and one thus needs two branching points to fix both of them. This allows us to construct a quasistationary solution valid in the whole spacetime.

Let us describe how α_{BH} can be determined in terms of $\dot{\varphi}_{\text{BH}}$ by enforcing a double root at one specific radius. To do so, we write $\Delta = N(r)/D(r)$ as a ratio of polynomials depending on r . Its numerator $N(r)$ is of the ninth degree. Without yet knowing the radius $r = r_{\text{root}}$ where $N(r)$ vanishes, we impose that its radial derivative must also vanish at the same location. We have thus a set of two polynomial equations, $N(r_{\text{root}}) = 0$ and $N'(r_{\text{root}}) = 0$. One of them may be used to write α_{BH} in terms of the (still unknown) r_{root} , and be replaced in the other equation. This gives now a polynomial of the 14th degree. Since its exact real roots cannot be written in a closed form, one then looks for them in a perturbative way, by increasing progressively the relative power of the very small quantity Hr_S up to which the solution is correct.⁸ More specifically, we start by a trial value r_{trial} and look for a next approximation in the form $r_{\text{trial}} + \delta r$ that we insert within the 14th-degree polynomial which must vanish. Solving for this small δr (at first order, or at second order for the first step) gives the next trial value, and we iterate this procedure. Once r_{root} has been determined with enough precision, say up to relative order $\mathcal{O}(H^n r_S^n)$, one can check that both $N(r_{\text{root}})$ and $N'(r_{\text{root}})$ vanish, but that the second derivative $N''(r_{\text{root}})$ does *not* (otherwise, this would be a third root and the discriminant Δ could become negative). More specifically, if $N(r) \propto (r - r_{\text{root}})^2$, then $N'(r) \propto (r - r_{\text{root}})$; therefore, $N(r_{\text{root}})$ should be of order $\mathcal{O}(H^{2n} r_S^{2n})$, $N'(r_{\text{root}})$ of order $\mathcal{O}(H^n r_S^n)$, while $N''(r_{\text{root}})$ should not have any such factor. Finally, when r_{root} has been obtained from this procedure, one can replace it in the expression of α_{BH} , and this fixes in a unique way the integration constant $\alpha_{\text{BH}} r_S$ entering Eq. (12), in terms of $\dot{\varphi}_{\text{BH}}$.

Repeating the same procedure at the second branching point generates another relation, and their combination allows us to determine both α_{BH} and $\dot{\varphi}_{\text{BH}}$.

⁸ $Hr_S \sim 10^{-22}$ for the typical black holes observed in the LIGO/Virgo interferometers, and $\sim 10^{-18}$ to 10^{-16} for the heavy ones expected to be detected with the LISA mission.

C. Branching point close to the black hole

When looking for such a double root of Δ at small radii of the order of the Schwarzschild radius r_S , and assuming that $\dot{\varphi}_{\text{BH}}$ is of the order of magnitude of the cosmological background value (35), one finds

$$r_{\text{root}} = \frac{3}{4} r_S \left[1 + \frac{\sqrt{3} k_2 M^2}{32 k_3 \dot{\varphi}_{\text{BH}}} r_S + \mathcal{O}(H^2 r_S^2) \right], \quad (42a)$$

$$\alpha_{\text{BH}} = 3k_3 \left(\frac{\dot{\varphi}_{\text{BH}}}{M} \right)^2 + \frac{3\sqrt{3}}{4} k_2 \dot{\varphi}_{\text{BH}} r_S + \mathcal{O}(H^2 r_S^2). \quad (42b)$$

To simplify, we only write the first two terms in the expansions, but we did compute several orders more to check the behaviors of $N(r_{\text{root}})$, $N'(r_{\text{root}})$, and $N''(r_{\text{root}})$ as explained above.

With the assumption that $\dot{\varphi}_{\text{BH}} = \dot{\varphi}_c$ strictly, this gives

$$r_{\text{root}}^{(c)} = \frac{3}{4} r_S \left[1 + \frac{3\sqrt{3}}{32} H r_S + \mathcal{O}(H^2 r_S^2) \right], \quad (43a)$$

$$\alpha_{\text{BH}}^{(c)} = \frac{1}{3k_3} \left(\frac{k_2 M}{H} \right)^2 \left[1 + \frac{3\sqrt{3}}{4} H r_S + \mathcal{O}(H^2 r_S^2) \right], \quad (43b)$$

where the upper index (c) recalls this assumption. When the Galileon field φ is responsible alone for cosmological expansion, $\alpha_{\text{BH}}^{(c)}$ is maximal. In that case, Ref. [19] showed that $H^2 = (|k_2|/3)^{3/2} M^2/|k_3|$, and we get

$$\frac{\alpha_{\text{BH}}^{(c)}}{\text{sign}(k_3) \sqrt{-k_2}} = \sqrt{3} + \frac{9}{4} H r_S + \mathcal{O}(H^2 r_S^2). \quad (44)$$

Let us recall that it is always possible to choose $k_2 = -1$ (or $-\frac{1}{2}$, depending on the reader's preferences) by reabsorbing it in the definition of φ in action (1). The important point to already note here is that this dimensionless scalar charge is of order 1, independently of any direct matter-scalar coupling constant α which may have been assumed in action (1), and even if $\alpha = 0$ strictly. In other words, even if one assumes that the scalar field is not directly coupled at all to any matter, the regularity of the solution implies that black holes must be significantly coupled to it. This surprising effect is a result of needing to avoid a potential singularity in the solution close to the black hole $r = r_{\text{root}}^{(c)}$, Eq. (43a), in the presence of $\dot{\varphi}_{\text{BH}}$. Despite being driven by the cosmological background, it is a local effect which forces the relation (42b).

In asymptotic Minkowski spacetime with $\dot{\varphi}_c = 0$, pure Schwarzschild black holes without any scalar hair ($\varphi = \text{const}$ everywhere) are solutions of the field equations. Here, this is because Eq. (13d) behaves as a source for φ' that we find such a hair.

D. Small accretion rate

Let us check that this configuration is the (approximate) end point of the evolution of the black hole, as we discussed in Sec. II D. The accretion rate for the charge (43b) can be estimated as

$$\Gamma_{\text{acc}} \approx \frac{|k_2|^3 M^4}{9|k_3|^2 H^3} = \frac{\rho_\varphi}{H} \lesssim H, \quad (45)$$

where we have also used Eq. (36) for the energy density of the scalar ρ_φ . Thus, for this solution, scalar-field accretion into the black hole is negligible, and the charges are long lived. We are within scenario I of Sec. II D. Note that the accretion rate in this model is even smaller in the past, $\rho_\varphi \propto H^{-2}$.

We may also estimate the contribution of the energy-momentum tensor of the scalar field close to the black hole, to check that our assumed background Schwarzschild–de Sitter metric (9) is not significantly affected, and, therefore, that our computation of the scalar charge α_{BH} can be trusted. It is dominated by the G_3 term entering action (1), of order $k_3 X \square \varphi / M^2$. Its integral within a sphere of radius a few times r_S gives a result of order $\mathcal{O}(H r_S^2)$ and, therefore, is extremely small with respect to the assumed r_S in metric (9). This is another way to confirm that the scalar's backreaction is negligible.

E. Branching point at large distance

As we have already demonstrated in Sec. III A, there exists a second branching point at a cosmologically large distance $r \approx 1/(\sqrt{3}H)$ from the black hole. We show in Appendix D that when assuming $\dot{\varphi}_{\text{BH}} = \dot{\varphi}_c$ strictly, it is strongly inconsistent with the local branching point studied in Sec. III C above. It would, indeed, need a small $\mathcal{O}(H r_S)$ scalar charge, instead of the large $\mathcal{O}(1)$ value derived in Eqs. (43b) or (44). To make these two branching points consistent with each other, we must now tune our second free parameter, $\dot{\varphi}_{\text{BH}}$, which may slightly differ from $\dot{\varphi}_c$.

This discussion is actually valid for any scalar charge, even the bare one α assumed for matter in action (1). Let us thus drop for a while the subscript “BH” and assume

$$\dot{\varphi}_{\text{local}} = \dot{\varphi}_c \times [1 + \kappa H r_S + \mathcal{O}(H^2 r_S^2)], \quad (46)$$

where κ is an $\mathcal{O}(1)$ dimensionless parameter we wish to determine. Using again the perturbative technique described in Sec. III B, we now assume that α is fixed (either by the action for matter bodies, or from the regularity of the local solution for black holes), and we look for $\dot{\varphi}_{\text{local}}$ such that the discriminant Δ admits a double root near $r \approx 1/(\sqrt{3}H)$. One finds

$$\dot{\varphi}_{\text{local}} = \dot{\varphi}_c \times \left[1 + \frac{9\sqrt{3}k_3 H^2}{2k_2^2 M^2} \alpha H r_S + \mathcal{O}(H^2 r_S^2) \right], \quad (47a)$$

$$r_{\text{root}} = \frac{1}{\sqrt{3}H} + \left(\frac{3}{2} + \frac{18k_3\alpha H^2}{k_2^2 M^2} \right) r_S + \mathcal{O}(Hr_S^2). \quad (47b)$$

Note that the corrections to $\dot{\phi}_c$ and $1/(\sqrt{3}H)$ are extremely small, even for a scalar charge α of order 1, since $Hr_S \sim 10^{-22}$. In conclusion, our assumption of a linear time-dependent test scalar field of the form (11) is now consistent at all radii, even for the case of matter bodies considered in Ref. [19]. The only price to pay is a tiny modification of the local time derivative of the scalar field with respect to its asymptotic cosmological value. In the case of black holes we are considering in the present paper, the conclusion is that such a tiny modification of $\dot{\phi}_{\text{BH}}$ suffices to make both branching points near $r \approx \frac{3}{4}r_S$ and $r \approx 1/(\sqrt{3}H)$ consistent with each other. Of course, this change of $\dot{\phi}_{\text{BH}}$ also implies very small modifications with respect to Eqs. (43). Our final results read

$$\dot{\phi}_{\text{BH}} = \dot{\phi}_c \times \left[1 + \frac{3}{2} \sqrt{3}Hr_S + \mathcal{O}(H^2r_S^2) \right], \quad (48a)$$

$$r_{\text{root}}^{\text{close}} = \frac{3}{4}r_S \left[1 + \frac{3\sqrt{3}}{32}Hr_S + \mathcal{O}(H^2r_S^2) \right], \quad (48b)$$

$$r_{\text{root}}^{\text{far}} = \frac{1}{\sqrt{3}H} + \frac{15r_S}{2} + \mathcal{O}(Hr_S^2), \quad (48c)$$

$$\alpha_{\text{BH}} = \frac{1}{3k_3} \left(\frac{k_2 M}{H} \right)^2 \left[1 + \frac{15}{4} \sqrt{3}Hr_S + \mathcal{O}(H^2r_S^2) \right]. \quad (48d)$$

Here again, we have actually computed several orders more in these expansions, but we only display the first two terms in each equation. Note that at this order, the position of the close root, Eq. (48b), did not change with respect to (43a). On the other hand, the second terms of Eqs. (48d) and (48c) are 5 times larger than what we obtain while assuming $\dot{\phi}_{\text{BH}} = \dot{\phi}_c$ strictly in Eq. (43b) above and in Eq. (D1a) of Appendix D. Let us also quote the value of the scalar charge when the Galileon field is responsible alone for the cosmological expansion, i.e., the corrected version of Eq. (44):

$$\frac{\alpha_{\text{BH}}}{\text{sign}(k_3)\sqrt{-k_2}} = \sqrt{3} + \frac{45}{4}Hr_S + \mathcal{O}(H^2r_S^2). \quad (49)$$

We should underline that the above modification of $\dot{\phi}_{\text{BH}}$ with respect to $\dot{\phi}_c$ is consistent with our assumption (11) of a linear time dependence of the scalar field everywhere in spacetime, but that other solutions are also possible. From a physical viewpoint, the local fields must react quickly to avoid any singularity, but the inconsistency we find in Appendix D at the cosmologically large distance $r \approx 1/(\sqrt{3}H)$ needs much more time to backreact on $\dot{\phi}_{\text{BH}}$. We can thus argue that in the realistic setup of the formation of a black hole, for instance from the collapse of matter not

directly coupled to the Galileon [i.e., $\alpha = 0$ action (1)], the actual scalar field should have a more complex time dependence than (11). Once it is formed in a background with a time derivative of the scalar field equal to $\dot{\phi}_c$, it quickly stabilizes to avoid local singularities; therefore, it adjusts its scalar charge to Eq. (43b) so that the discriminant Δ has a double root near $r \approx \frac{3}{4}r_S$. Its scalar hair then propagates at a finite velocity toward large radii, and when it reaches $r \approx 1/(\sqrt{3}H)$, the field equation “realizes” that $\dot{\phi}_{\text{BH}}$ needs to be adjusted for a double root of Δ to also exist there. It then sends back this information, again at a finite velocity, toward the location of the black hole. Therefore, one may argue that the imprecise scalar charge (43b) has probably more physical meaning than the correct one (48d) needed for an exact linear time dependence everywhere. Moreover, as soon as there exist several black holes of different masses in the universe, as well as the possibly coupled matter bodies [$\alpha \neq 0$ in action (1)], then a uniform $\dot{\phi}$ is obviously no longer possible everywhere. But since all local $\dot{\phi}_{\text{BH}}$ and the background $\dot{\phi}_c$ only differ by a tiny amount of relative order $\mathcal{O}(Hr_S) \sim 10^{-22}$, this does not change anything to the observational consequences discussed in Sec. VI.

F. Vainshtein screening

For the cubic Galileon model, the relevant Vainshtein radius is given by Eqs. (30) and (48d):

$$r_{V23}^3 = \frac{k_3\alpha_{\text{BH}}r_S}{k_2^2M^2} \approx \frac{r_S}{3H^2}. \quad (50)$$

It is interesting to note that it only depends on the physical quantities r_S and H , but no longer on any parameter entering action (1), i.e., k_2 , k_3 , nor M . In particular, it keeps strictly the same value even if the Galileon field is not responsible alone for the accelerated expansion of the universe, and even if $M \ll H$. This independence of the Vainshtein radius from the theory parameters comes from the fact that we are considering only two linear Galileon kinetic terms (3). It generically does depend on M in other models, as Sec. V C will illustrate.

Substituting Eq. (50) into Eq. (31), we obtain

$$Z_3'' \sim \frac{|k_2|}{\sqrt{3}Hr_S} \left(\frac{r_S}{r} \right)^{3/2}. \quad (51)$$

We discuss the observational consequences of the combination of the large scalar charge (48d) and the acoustic metric (51) in Sec. VI B.

IV. SIMPLEST QUINTIC HORNDESKI TERM

A. Scalar-field solution

Let us now consider the particular case $k_3 = 0$ in Eqs. (3), i.e., when only the two functions $G_2(X)$ and

$G_5(X)$ define the dynamics of the scalar field. In such a case, Eq. (5) imposes

$$\dot{\varphi}_c = -\frac{k_2 M^4}{3k_5 H^3}, \quad (52)$$

instead of Eq. (35). This theory is very simple and the energy density of the scalar (6) can be directly related to the speed of gravitational waves (7),

$$\frac{\rho_\varphi}{12H^2} = \alpha_T = \frac{2}{27} \frac{|k_2|^3}{k_5^2} \left(\frac{M}{H}\right)^8. \quad (53)$$

The constraint (8) then implies that such a model cannot drive the acceleration of the universe. If we choose $|k_2| \sim |k_5| \sim 1$ which can be reabsorbed in the definitions of φ and M [cf. Eqs. (3)], this translates as a limit on $M/H \lesssim 2 \times 10^{-2}$, still a mild hierarchy of scales. Note also the H^{-8} dependence in expression (53): However small α_T is allowed to be today, in this model it would have been much smaller in the recent past.

We follow the same procedure as described in Sec. III B to determine the precise location of a double root of the discriminant Δ close to the black hole, and the corresponding scalar charge α_{BH} . A difference is that when writing the discriminant $\Delta = N(r)/D(r)$ as a ratio of polynomials, its numerator $N(r)$ is now of 14th degree. And when replacing the expression of α_{BH} imposed by one of the two equations $N(r_{\text{root}}) = 0$ or $N'(r_{\text{root}}) = 0$ into the other, we now get a polynomial of 23rd degree. The perturbative search for such a double root is, however, similar, and we now find that it must be close to $r_{\text{root}} \approx \frac{3}{2} r_S$ (as compared to $\frac{3}{4} r_S$ in Sec. III C).

We then find that the analogs of Eqs. (43) read in the present quadratic plus quintic model

$$r_{\text{root}}^{(c)} = \frac{3}{2} r_S \left[1 + \frac{27}{8} H^2 r_S^2 + \mathcal{O}(H^3 r_S^3) \right], \quad (54a)$$

$$\alpha_{\text{BH}}^{(c)} = \frac{2}{k_5} \left(\frac{2k_2 M^2}{9H^3 r_S} \right)^2 \left[1 - \frac{27}{2} H^2 r_S^2 + \mathcal{O}(H^3 r_S^3) \right], \quad (54b)$$

where the upper index (c) recalls our assumption $\dot{\varphi}_{\text{BH}} = \dot{\varphi}_c$. The crucial difference with Sec. III is that the scalar charge is now proportional to $1/(Hr_S)^2$, as compared to $\mathcal{O}(1)$ in Eq. (43b) or (44). In other words, in this model, the requirement of stationarity of the solution in the presence of a nonvanishing $\dot{\varphi}_c$ implies extremely large scalar charges for black holes. This comes from the coefficient $1/(Mr)^2$ entering Eq. (13d). Moreover, the dimensionless charges now depend on the Schwarzschild radius r_S ; therefore, black holes of different masses have different scalar charges, so that dipolar radiation becomes possible. As an illustration, taking $Hr_S \sim 10^{-22}$ (for LIGO black holes), we could thus

expect scalar charges as high as order 10^{42} in the fully self-accelerated case $|k_2| \sim |k_5| \sim M/H \sim 1$.

Attempting to correct this solution by taking account of the root at cosmological distances as in the cubic Galileon case does not produce a good solution. A second branching point near $r \approx 1/(\sqrt{3}H)$ should exist. We can thus let the local $\dot{\varphi}_{\text{BH}}$ differ from the background $\dot{\varphi}_c$ and look as before for the value which would allow for a double root of the discriminant Δ near this radius. We find that this is actually impossible, because the existence of such a double root would need a *negative* value of the square $\dot{\varphi}_{\text{BH}}^2$. Our assumption (11) of a linear time dependence everywhere is inconsistent. As we will demonstrate, the accretion rate implied by the cosmological background in this model is typically large and does not support the type of stationary solution described in Sec. III E valid everywhere in spacetime (the solution would have evolved on the time-scale required to adjust to the root at cosmological distances). As argued previously, fixing the behavior at small distances is sufficient for our discussion of the local physics of radiation.

B. Accretion scenarios

Given the large scalar charge for black holes in this model, one may expect that the accretion rate can be rather large. Let us make a connection with possible observations by assuming that we have a given fixed black-hole mass represented by its r_S . For the accretion rate (20) onto such a black hole to be slow, $\Gamma_{\text{acc}}^{(c)} \lesssim H$, the expressions for $\dot{\varphi}_c$ in Eq. (52) and α_{BH} in Eq. (54b) imply that we require

$$\frac{M}{H} \lesssim \left(\frac{3^5 k_5^2}{2^3 |k_2|^3} H^2 r_S^2 \right)^{1/8}. \quad (55)$$

In other words,

$$\Gamma_{\text{acc}}^{(c)} = \frac{2^3 |k_2|^3 M^8}{3^5 k_5^2 H^9 r_S^2} \approx \left(\frac{2}{3Hr_S} \right)^2 \alpha_T H, \quad (56)$$

where we have used Eq. (53) and α_T scales as H^{-8} . The requirement for small accretion in this model then implies that $\alpha_T < (3Hr_S/2)^2 \sim 10^{-43}$ for LIGO/Virgo black holes with masses $\sim 10m_\odot$, and $\alpha_T < 10^{-35}$ for LISA supermassive black holes with masses $\sim 10^5 m_\odot$. For M/H larger than implied by these limits, black holes of sizes that we will observe would not be in the small accretion scenario.

However, since $\Gamma_{\text{acc}}^{(c)}/H \propto H^{-10}$, the range of black-hole masses which are accreting slowly on the cosmological background is larger at high redshift in this model—for a source at redshift $z = 2$ these above conditions are relaxed by a factor of $[H(z=2)/H(z=0)]^{10} \sim 10^5$, and $\sim 10^9$ for a source at redshift $z = 5$.

Indeed, for any value of the parameters of the action, sufficiently small black holes, $(Hr_S)^2 < \alpha_T$, are in the

quenched accretion scenario of Sec. II D. When such black holes form on the cosmological background $\dot{\varphi}_c$, the accretion rate is initially large, and they absorb the scalar-field background reducing the local $\dot{\varphi}$ and therefore their charge. The asymptotic future configuration is decoupled from the cosmological background. Once the timescale of accretion reduces to $\Gamma_{\text{acc}} \ll r_S^{-1}$, we can think of the quasistationary configuration, determined by requiring the presence of the double root at $r \approx \frac{3}{2}r_S$ given some $\dot{\varphi}_{\text{local}}$, as being the approximate solution which evolves adiabatically. In such a case we have

$$\alpha_{\text{BH}} \approx 2k_5 \left(\frac{2\dot{\varphi}_{\text{local}}}{3M^2 r_S} \right)^2. \quad (57)$$

The accretion rate will continue to fall as the scalar background is absorbed, but it cannot go below that implied by the lifetime of the universe, $\Gamma_{\text{acc}} \gtrsim H$. We thus have a *lower* bound for the local value of the scalar derivative and the charge in this model, both dependent on r_S ,

$$|\dot{\varphi}_{\text{local}}| \gtrsim \left(\frac{9HM^4 r_S^2}{8|k_5|} \right)^{1/3}, \quad (58a)$$

$$|\alpha_{\text{BH}}| \gtrsim 2 \left(\frac{|k_5| H^2}{9M^4 r_S^2} \right)^{1/3}. \quad (58b)$$

Depending at which point in their evolution these black holes are observed, the charge will be somewhere between the initial value implied by the cosmological background (54b) and the lower bound (58b). This is a generic feature of such a quintic operator, and in the subsequent we will assume that the charge is at this lower bound for all black holes which have undergone the quenched accretion scenario. This is a very conservative estimate.

When they are in this quenched state, one may also estimate the contribution of the energy-momentum tensor of the scalar field close to the black hole. It is dominated by the G_5 term entering action (1), of order $k_5 X \varphi'' f'' / M^4$. Its integral within a sphere of radius a few times r_S gives again a result of order $\mathcal{O}(H r_S^2)$, therefore negligible with respect to the assumed r_S in metric (9).

C. Vainshtein screening

As we have already previewed in Sec. II E, in the intermediate range $r_S \ll r \ll 1/H$, the scalar-field solution in (12) and (13) is dominated by its G_5 kinetic term at small distances and the G_2 term at large distances, giving the approximate solutions (25) and (27), respectively. Assuming that our black hole was formed large enough to accrete slowly, the Vainshtein radius determined by its charge is universal for all such black holes and given by Eq. (29),

$$r_{\text{V25}}^3 = \frac{\sqrt{k_5 \alpha_{\text{BH}}} r_S}{\sqrt{2}|k_2| M^2} = \frac{2}{9H^3}, \quad (59)$$

where α_{BH} is given by Eq. (54b), i.e., a very large radius of a size comparable to the observable universe.⁹ Obviously, our assumption $r \ll 1/H$ is not satisfied at such a large radius, but this anyway shows that coalescences of such black holes happen deep within their Vainshtein region.

On the other hand, when the black hole initially accretes quickly, the quenched charge at the time of observation is much reduced. This leads to a smaller Vainshtein radius, still given by the same Eq. (29), but with the charge instead determined through the lower bound of the accretion condition, Eq. (58b),

$$r_{\text{V25,local}}^9 \gtrsim \frac{k_5^2 H r_S^2}{3|k_2|^3 M^8} = \left(\frac{3H r_S}{2} \right)^2 \frac{r_{\text{V25}}^9}{\alpha_T}. \quad (60)$$

The condition determining one or the other scenario of accretion—hence the Vainshtein radius—can be read off Eq. (56). For $\alpha_T / (H r_S)^2 \lesssim 1$ the Vainshtein radius is given by (59), while otherwise we have (60). In a given theory, i.e., a fixed value of M , these two Vainshtein radii coincide for a BH mass such that $\Gamma_{\text{acc}}^{(c)} \sim H$. In principle, if a black hole is observed before being fully quenched, its effective Vainshtein radius could lie between r_{V25} and the lower bound of $r_{\text{V25,local}}$.

Substituting Eq. (59) into Eq. (34) we obtain for the acoustic metric in the small-accretion scenario

$$Z_{5(c)}'' \sim \frac{2|k_2|}{9(H r_S)^3} \left(\frac{r_S}{r} \right)^3. \quad (61)$$

On the other hand, when accretion is initially large and undergoes quenching, the acoustic metric is reduced together with the Vainshtein radius

$$Z_{5,\text{local}}'' \sim \left(\frac{(3H r_S)^2}{4\alpha_T} \right)^{1/3} Z_{5(c)}''. \quad (62)$$

Although Z_5'' is proportional to the small quantity $(r_S/r)^3$, as compared to $(r_S/r)^{3/2}$ for Z_3'' in the cubic Galileon model, Eq. (51), it is enhanced by a factor $(H r_S)^{-2}$, and therefore, the G_5 screening is much stronger than G_3 . We calculate the observational consequences of these results in Sec. VI C.

V. CUBIC GALILEON WITH A SMALL QUINTIC TERM

A. Scalar-field solution

We now consider the full case of Eqs. (3), where the three functions $G_2(X)$, $G_3(X)$, and $G_5(X)$ define the dynamics of the scalar field. As we will see, this model

⁹Note that in the present model again, this Vainshtein radius does not depend on the theory parameters, but only on the physical quantity H (and not even r_S , here).

is flexible enough to allow us to separate the questions of dark energy and of black-hole charges. Given the results of the previous section, we will assume that k_5 is much smaller than k_2 and k_3 (themselves possibly of order 1). This will not only avoid too large scalar charges generated by the $1/(Mr)^2$ term entering Eq. (13d), but also allow the model to pass the known constraint (8) on GW speed. More precisely, we shall assume here that

$$(Hr_S)^2 \ll \left| \frac{k_5}{k_3} \right| \left(\frac{H}{M} \right)^2 \ll Hr_S, \quad (63)$$

which numerically means $10^{-44} \ll |k_5/k_3|(H/M)^2 \ll 10^{-22}$ for the LIGO/Virgo experiments, and $10^{-36} \ll |k_5/k_3|(H/M)^2 \ll 10^{-18}$ for the LISA mission. Although this may seem a fine-tuned choice, since our theory is part of Horndeski gravity it enjoys a weakly broken Galileon symmetry [31] which protects the k_i coefficients from large quantum corrections,¹⁰ which are instead suppressed by the small ratio $(M/M_{\text{Pl}})^{2/3} \lesssim 10^{-40}$. Comparing with the general expression for r_{V35} , Eq. (28), we see that the left inequality is equivalent to $r_S \ll r_{V35}$, i.e., the requirement that this black hole be surrounded by a G_5 -dominated region. Otherwise, the G_5 part of the solution is never relevant for this black hole.

With the above assumptions, Eq. (5) imposes

$$\dot{\varphi}_c = \frac{k_2 M^2}{3k_3 H} \left[1 + \mathcal{O}\left(\frac{k_5 H^2}{k_3 M^2}\right) \right], \quad (64)$$

which is thus almost equal to Eq. (35), up to a fully negligible relative correction, much smaller than $Hr_S \sim 10^{-18}$ for LISA black holes. Thus the energy density of the background under these assumptions is essentially the same as in the G_2 - G_3 model, Eq. (36),

$$\rho_\varphi = \frac{|k_2|^3 M^4}{9k_3^2 H^2} \left[1 + \mathcal{O}\left(\frac{k_5 H^2}{k_3 M^2}\right) \right]. \quad (65)$$

On the other hand, the speed of GWs on the cosmological background is corrected by

$$\alpha_T \approx 2 \left(\frac{k_2}{3k_3} \right)^3 k_5 \left(\frac{M}{H} \right)^2. \quad (66)$$

Note that α_T scales as H^{-2} here, compared to H^{-8} for the G_2 - G_5 model, while the energy density scales as H^{-2} (vs H^{-6})—the ratio α_T/ρ_φ is constant in the present model. With these expressions we can also rewrite our condition for the validity of our perturbative expansion (63) as

¹⁰A caveat is that the results of Ref. [31] were derived around flat space, which by assumption is not stable in the theories we consider here.

$$(Hr_S)^2 \ll \frac{3H^2}{2\rho_\varphi} |\alpha_T| \ll Hr_S, \quad (67)$$

which confirms that whenever our expansion is valid, $|\alpha_T| \ll 10^{-18}$ for LISA black holes, well within the constraint (8) even for full self-acceleration. The small quintic contribution will only have local consequences, at distances of order of the Schwarzschild radius r_S . We shall see that it actually imposes the scalar charge α_{BH} .

Since the perturbative method of Sec. III B relies on expansions in powers of the small dimensionless quantity Hr_S , we must, however, be careful when assuming that k_5 is small, as ratios of small parameters might be of any size. A convenient technique is to define a new parameter

$$\bar{k}_5 \equiv \frac{k_5}{(Hr_S)^2}, \quad (68)$$

of “reasonable” size. Since this can be visually useful to understand the order of magnitude of the various terms we will write below, let us copy our assumptions (63) in terms of this new notation,

$$1 \ll \left| \frac{\bar{k}_5}{k_3} \right| \left(\frac{H}{M} \right)^2 \ll \frac{1}{Hr_S} \sim 10^{22} \quad (69)$$

(or $\sim 10^{18}$ for the LISA mission). When $|k_3| \sim M/H \sim 1$, the parameter $|\bar{k}_5|$ is thus assumed to be negligible with respect to $1/(Hr_S)$, so that one can perform safely our expansions in powers of Hr_S . And when both k_3 and $\bar{k}_5$ occur at the same order, we may neglect the former, because of the first inequality in (69). We did check that these assumptions are consistent with our results below at each step of our calculations.

Because of them, the large-distance behavior of the solution is extremely close to that of Sec. III, and we notably recover Eqs. (47) for a double root of the discriminant Δ to exist near $r \approx 1/(\sqrt{3}H)$. On the other hand, the quintic term plays a dominant role at small distances, and we find that the existence of a local double root of Δ imposes

$$\dot{\varphi}_{\text{BH}} = \frac{k_2 M^2}{3k_3 H} \left[1 + \left(\frac{4\bar{k}_5 H^2}{3\sqrt{3}k_3 M^2} + 6\sqrt{3} - \frac{3^6 \sqrt{3} k_3 M^2}{2^4 \bar{k}_5 H^2} \right) Hr_S + \mathcal{O}(H^2 r_S^2) \right], \quad (70a)$$

$$r_{\text{root}}^{\text{close}} = \frac{3}{2} r_S \left[1 + \frac{3^4 \sqrt{3} |k_3| M^2 r_S}{2^5 \sqrt{5} \bar{k}_5 H} + \mathcal{O}(H^2 r_S^2) \right], \quad (70b)$$

$$r_{\text{root}}^{\text{far}} = \frac{1}{\sqrt{3}H} + \left(\frac{2^4 \bar{k}_5 H^2}{3^2 k_3 M^2} + \mathcal{O}(1) \right) r_S + \mathcal{O}(Hr_S^2), \quad (70c)$$

$$\alpha_{\text{BH}} = 2^3 \left(\frac{k_2}{9k_3} \right)^2 \bar{k}_5 \left[1 + \frac{3^3 k_3 M^2}{2 \bar{k}_5 H^2} + \mathcal{O} \left(\frac{k_3^2}{\bar{k}_5^2} \right) \right. \\ \left. + \frac{(2^6 \bar{k}_5^2 H^4 + 2^5 3^3 k_3 \bar{k}_5 H^2 M^2 - 3^8 k_3^2 M^4)^2}{2^9 3 \sqrt{3} k_3 \bar{k}_5^3 H^6 M^2} H r_s \right. \\ \left. + \mathcal{O}(H^2 r_s^2) \right]. \quad (70d)$$

As before, we only display the first two terms of these expansions, although we did compute higher orders. We also display the large-distance double root $r_{\text{root}}^{\text{far}}$ in Eq. (70c), obtained from the replacement of (70d) into (47b), to parallel the presentation of Eqs. (48) for the cubic Galileon of Sec. III. These intricate expressions show that such a case of G_2 , G_3 , and G_5 together is highly nontrivial. In particular, the existence of simultaneous ratios $\bar{k}_5/k_3$ and $k_3/\bar{k}_5$ underlines that fully neglecting one of these two parameters would be inconsistent.¹¹ But the 1 starting all square brackets indeed dominate over the next terms, under our assumptions (69), and the global factors of such square brackets thus give the lowest-order values. We can therefore conclude that in the present model, it is possible to assume a linear time dependence (11) of the scalar field everywhere. The two double roots of the discriminant Δ are indeed consistent with each other if the time derivative (70a) only slightly differs from its cosmological background (64). This confirms that the present model behaves as the (quadratic plus) cubic one of Sec. III at large distances. On the other hand, Eq. (70b) shows that the location of the double root of Δ at small distances almost equals the one we found in (54a) for the quintic Galileon of Sec. IV. Therefore, the G_5 term dominates at small distances, as expected from its $1/(Mr)^2$ contribution to Eq. (13d). The predicted scalar charge (70d) reads at lowest order

$$\alpha_{\text{BH}} \approx 8 \left(\frac{k_2}{9k_3 H r_s} \right)^2 k_5, \quad (71)$$

where we have now replaced the intermediate notation (68) by its actual expression in terms of the theory parameter k_5 . We can express this in terms of the physical parameters of the cosmological background of this model as

$$\alpha_{\text{BH}} \approx \left(\frac{2}{3 H r_s} \right)^2 \sqrt{\frac{-k_2}{3}} \frac{\alpha_T}{\sqrt{\rho_\phi/3H^2}}. \quad (72)$$

¹¹It is interesting to note that when the left-hand side of our hypothesis (63) or (69) is not satisfied, then the first correction in Eq. (70d) behaves as $(2k_2 M/H)^2/(3k_3)$, i.e., very similar to what we found in Eq. (48d) for the pure quadratic plus cubic Galileon model, up to a factor 4. This confirms that when $|k_5/k_3|$ is even smaller than $(H r_s)^2$, the present model progressively behaves as this cubic Galileon of Sec. III. The next correction in (70d), proportional to $1/\bar{k}_5$, has a meaning only when our assumption (69) is satisfied, on the other hand.

This is to be compared to Eq. (54b) that we found for the quintic Galileon of Sec. IV—up to a factor of $\sqrt{8}$, it can be shown to be the same expression when translated into the physical parameters, despite the different proportionality with respect to k_5 . For this general model, we are free to adjust ρ_ϕ independently of k_5 (or α_T), and therefore, the charge can be small even in the case of full self-acceleration. In other words, even if we assume $|k_2| \sim |k_3| \sim M/H \sim 1$, the scalar charges of black holes are no longer required to be huge, contrary to Sec. IV.

Let us also underline that contrary to Eqs. (48d) and (54b) of the previous sections, α_{BH} is independent of the theory parameter M in the present model. This means that when the Galileon is not fully self-accelerating (i.e., M/H is small if we choose $|k_2| \sim |k_3| \sim 1$), the scalar charge (71) does not change. It is quite surprising that the consistency of the present solution imposes such a large scalar charge even when the scalar field has actually a negligible influence in cosmology.

B. Accretion

The effect of accretion may already be estimated from Eqs. (70a) and (70d). Its rate (20) here evaluates to

$$\Gamma_{\text{acc}}^{(c)} = |\dot{\phi}_c \alpha_{\text{BH}}| \approx \frac{2^3 |k_2|^3}{3^5 |k_3|^3 H^3 r_s^2} |k_5| M^2 \\ \approx \left(\frac{2}{3 H r_s} \right)^2 |\alpha_T| H, \quad (73)$$

which is the same expression as for the quintic Galileon Eq. (56): It is strongly enhanced by the $1/r_s^2$ dependence of the scalar charge α_{BH} . However, here we can vary α_T independently of the Galileon energy density, by reducing k_5 , requiring¹²

$$|k_5| \left(\frac{M}{H} \right)^2 \lesssim (H r_s)^2, \quad (74)$$

when setting $|k_2| \sim |k_3| \sim 1$ by redefining the variables entering functions (3).

In fact, we can rewrite our condition (63) as

$$1 \ll \frac{\Gamma_{\text{acc}}^{(c)}/H}{\rho_\phi/(3H^2)} \ll (H r_s)^{-1}. \quad (75)$$

As we have already mentioned after Eq. (63), when the left inequality is not satisfied, $r_s > r_{V35}$ and there is no G_5 -dominated region around this black hole. For large enough black holes, we would instead recover the cubic Galileon behavior already studied in Sec. III, in which accretion is always small. If there is a G_5 -dominated region

¹²When combining the limit of Eq. (74), $|k_5|(M/H)^2 \sim (H r_s)^2$, with our assumptions (63), this implies $(H r_s)^2 \ll |k_5| \ll (H r_s)^{3/2}$ with $1 \gtrsim M/H \gtrsim (H r_s)^{1/4}$.

at all, then expression (75) means that a black hole can only slowly accrete on the cosmological background, $\Gamma_{\text{acc}}^{(c)} < H$, when the Galileon is not driving the acceleration, $\rho_\phi < 3H^2$. In the full self-acceleration case, all black holes slowly accreting on the cosmological background are in G_3 domination.

It is also worth reiterating that the middle term in Eq. (75) is constant [see Eqs. (65) and (66)], so if the black hole is in the G_5 -dominated accretion regime now, it was so in the past. This directly results from the constancy of r_{V35} , Eq. (28). In any case, LIGO/Virgo black holes are slowly accreting in this model only if $|\alpha_T| \lesssim 10^{-43}$, which would pass the constraint (8) by 28 orders of magnitude and be safely quasistationary whenever conditions (75) are satisfied.

Just as in the case of the simple quintic Galileon, there are always small enough black holes, $(3Hr_S/2)^2 < \alpha_T$, which have a large accretion rate on the cosmological background $\dot{\phi}_c$. These black holes undergo the quenched accretion scenario and absorb the energy density stored in the scalar, decreasing their charge. The expressions for the configuration once the accretion is quenched are just as in the quintic model of Sec. IV; i.e., we have

$$\alpha_{\text{BH}} \approx 2k_5 \left(\frac{2\dot{\phi}_{\text{local}}}{3M^2 r_S} \right)^2 \quad (76)$$

[cf. Eq. (57)], and requiring $\Gamma_{\text{acc}} = |\alpha_{\text{BH}} \dot{\phi}_{\text{local}}| \gtrsim H$ gives again

$$|\dot{\phi}_{\text{local}}| \gtrsim \left(\frac{9HM^4 r_S^2}{8|k_5|} \right)^{1/3}, \quad (77a)$$

$$|\alpha_{\text{BH}}| \gtrsim 2 \left(\frac{|k_5| H^2}{9M^4 r_S^2} \right)^{1/3} \quad (77b)$$

[cf. Eqs. (58a) and (58b)]. Again, this is a conservative lower bound for the charge in this alternative scenario. The above bounds are identical for both the models since these black holes decouple from the cosmological background produced by the G_3 term. The black-hole charge is determined by the fact that the object is in G_5 domination and it is the G_5 term, shared by both the present model and the simple quintic Galileon, that sets all the relevant properties of the quenched quasistationary configuration. The contribution of the energy-momentum tensor of the scalar field close to the black hole is thus the same as in Sec. IV B, i.e., negligible with respect to the assumed r_S in metric (9), whose form can thus be trusted.¹³

¹³Curiously enough, in the present model, this scalar's energy is *also* negligible during the process of fast scalar accretion, even when the scalar charge takes its largest value (71). This is due to our assumption of a small parameter $|k_5|$ in Eq. (63). One indeed finds that the scalar's energy contributes at order $\sim (M/H)^2 |k_5|/H \ll (M/H)^4 r_S$, therefore negligible with respect to r_S .

Screening does not change any of this discussion, since the unscreened charge α_{BH} is responsible for the energy flux (17) into the horizon. Nevertheless, screening is still important for the observable effects, as it does suppress the scalar-wave emission. We turn to this question now.

C. Vainshtein screening

In this general model, we have the possibility of the full set of Vainshtein screened regimes, as illustrated in Fig. 1.

In the setup with small accretion, we obtain the Vainshtein radii from the general expressions Eqs. (28)–(30) using Eq. (71) for the black-hole charge. The smallest one is independent of the black-hole charge and of time,

$$r_{V35}^3 = \frac{|k_5| r_S}{2|k_3| M^2} = \frac{3|\alpha_T|}{4\rho_\phi} r_S, \quad (78)$$

where we have again expressed the model parameters in terms of the combination of the physical properties of the cosmological background. We repeat here that the lower limit of condition (63) is equivalent to $r_S \ll r_{V35}$, i.e., G_5 -domination on the scales of r_S . The intermediate Vainshtein radius is given by

$$r_{V25}^3 \approx \frac{2|k_5|}{9|k_3| HM^2} = \frac{4 r_{V35}^3}{9 H r_S}. \quad (79)$$

It corresponds to what we found in Eq. (59) of Sec. IV, but the difference is that α_{BH} now takes the value (71) instead of (54b). Finally, a third and the largest Vainshtein radius is given by

$$r_{V23}^3 \approx \frac{8}{(9HM)^2} \frac{|k_5|}{|k_3| r_S} = \frac{4 r_{V25}^3}{9 H r_S}. \quad (80)$$

The hierarchy of the Vainshtein radii is determined by the black-hole size Hr_S . Contrary to Eqs. (50) and (59) of the previous sections, the three Vainshtein radii (78)–(80) here depend on the theory parameter M , and more precisely, they are all proportional to $1/M^2$. This is due to the fact that the scalar charge (71) is now independent of M . When self-acceleration is not full, these three transition radii become larger, and the Vainshtein screening is more efficient.

In the quenched-accretion scenario, we need to instead use the bound on the quenched charge (77b) to obtain

$$r_{V25,\text{local}}^9 \gtrsim \frac{k_5^2 H r_S^2}{3|k_2|^3 M^8} = \left(\frac{3Hr_S}{2} \right)^2 \frac{r_{V25}^9}{|\alpha_T|}, \quad (81)$$

recovering expression Eq. (60) for the simple quintic model, while the largest Vainshtein radius becomes

$$r_{V23,\text{local}}^9 \gtrsim \frac{8|k_3^3 k_5|}{9k_2^6 M^{10}} H^2 r_S = \left(\frac{3Hr_S}{2} \right)^4 \frac{r_{V23}^9}{\alpha_T^2}, \quad (82)$$

where we have used the expression for α_T on the cosmological background of this model, Eq. (66), to relate the Vainshtein radii of the local quenched solutions to the radii (79) and (80). Equation (73) implies that large accretion occurs whenever $(Hr_S)^2/|\alpha_T| < 1$. Thus, the local Vainshtein radii are always smaller than the respective cosmological ones, as Eqs. (29) and (30) also show because the scalar charge $|\alpha_{\text{BH}}|$ is reduced. Using the upper limit of our assumptions (63), one can also prove that the order $r_{V35} < r_{V25,\text{local}} < r_{V23,\text{local}}$ is maintained.

The important criterion to note is the relative size of r_{V35} versus the wavelength of the emitted gravitational waves, $\lambda \sim 300r_S$. If $r_{V35} \gg \lambda$, then the emission of gravitational waves will be governed by the G_5 term in the acoustic metric $Z^{\mu\nu}$. Otherwise, $r_{V35} < \lambda$ and the emission is governed by the G_3 term, even if the black hole itself is in G_5 domination, $r_{V35} > r_S$.

The effect of the screening in the end boils down to the normalization of the effective metric at the scale λ . Taking the expression (34) and reexpressing r_{V25} using Eq. (79) gives

$$Z_{5(c)}'' \sim \frac{1}{Hr_S} \left(\frac{r_{V35}}{\lambda} \right)^3 |k_2|. \quad (83)$$

For the quenched accretion scenario, we can use Eq. (81) to obtain

$$Z_{5,\text{local}}'' \sim \left[\frac{(3Hr_S)^2}{4|\alpha_T|} \right]^{1/3} Z_{5(c)}'', \quad (84)$$

which is reduced compared to $Z_{5(c)}''$.

Finally when the scale λ is in a G_3 -domination region, $r_{V35} < \lambda$, but the black hole itself is in G_5 domination, using Eqs. (80) and (82) in Eq. (31), we obtain

$$Z_{3(c)}'' \sim \frac{1}{Hr_S} \left(\frac{r_{V35}}{\lambda} \right)^{3/2} |k_2|, \quad (85)$$

$$Z_{3,\text{local}}'' \sim \left[\frac{(3Hr_S)^2}{4|\alpha_T|} \right]^{1/3} Z_{3(c)}'', \quad (86)$$

and we recover that the G_3 and G_5 contributions to the total $Z^{\mu\nu}$ are of the same magnitude at $r = r_{V35}$ and they have a common time dependence.

Again, the overall radiation flux will depend on the combination of the metric normalization Z'' and the black-hole charges. We discuss the potential for observability for this model in Sec. VID.

VI. OBSERVATIONAL CONSEQUENCES

A. Radiation from binary inspirals

The most direct observational constraint on these black-hole charges results from an additional channel for radiation in black-hole mergers, allowing them to proceed more quickly than in general relativity.

In a binary system of two bodies A and B at equilibrium, with masses of the same order of magnitude, $m_A \sim m_B \sim r_S/(2G)$, and a negligible eccentricity, the energy flux carried away by gravitational radiation is dominated in general relativity by the quadrupole term

$$F_{\text{GR}} \approx \frac{2}{5G} \left(\frac{r_S}{r_{AB}} \right)^5, \quad (87)$$

where r_{AB} denotes the interbody distance. This is the simplest writing, but it is useful to reexpress it in terms of the orbital angular frequency $\Omega_p \equiv 2\pi/P$, where P is the orbital period. This is achieved thanks to Kepler's third law (at its lowest, Newtonian, order), $\Omega_p^2 r_{AB}^3 = G(m_A + m_B) \approx r_S$, which implies

$$\frac{r_S}{r_{AB}} \approx (\Omega_p r_S)^{2/3}. \quad (88)$$

Therefore, Eq. (87) is proportional to $(\Omega_p r_S)^{10/3}$. Twice the orbital frequency is the wave frequency, which is directly observable, while the chirp mass is of order r_S .

In scalar-tensor theories, the binary also emits scalar waves, whose dominant contributions to the energy flux read [45]

$$F_{\text{scalar}} = \frac{F_{\text{scalar}}^{\text{dipole}}}{z_{\lambda,1}} + \frac{F_{\text{scalar}}^{\text{quadrupole}}}{z_{\lambda,2}}, \quad (89a)$$

$$F_{\text{scalar}}^{\text{dipole}} \approx \frac{1}{48G|k_2|} \left(\frac{r_S}{r_{AB}} \right)^4 (\alpha_A^{\text{eff}} - \alpha_B^{\text{eff}})^2, \quad (89b)$$

$$F_{\text{scalar}}^{\text{quadrupole}} \approx \frac{1}{15G|k_2|} \left(\frac{r_S}{r_{AB}} \right)^5 \alpha_A^{\text{eff}} \alpha_B^{\text{eff}}. \quad (89c)$$

The coefficients $z_{\lambda,\ell}$ are the Vainshtein screening factors discussed in Sec. IIE that we shall further describe below. The quadrupolar term (89c) is of the same order of magnitude as the general relativistic prediction (87), but multiplied by the square of the dimensionless scalar charge $\alpha_{\text{BH}} \sim \alpha_A^{\text{eff}} \sim \alpha_B^{\text{eff}}$, which is of order 1 in the cubic Galileon model of Sec. III but may be large in the presence of the quintic term, as seen in Secs. IV and V. On the other hand, the dipolar term (89b) is generically larger than the GR quadrupole (87), because it involves a smaller power of $r_S/r_{AB} = 2Gm/(r_{AB}c^2)$; i.e., it is of a *lower* post-Newtonian order. However, note that it needs the two scalar charges α_A^{eff} and α_B^{eff} to differ. Although we assume

that the two black holes have similar masses $m_A \sim m_B$, this means that they must not be strictly equal for this dipolar term (89b) to be significant. Moreover, even when $m_A \neq m_B$, the cubic Galileon model of Sec. III predicted that α_{BH} , Eq. (48d), does not depend on the black-hole mass (up to negligible relative corrections of order Mr_S). Therefore, although the dipole (89b) is generically the dominant scalar contribution to the energy flux, it happens to be negligible in the case of the cubic Galileon model.

In Eqs. (89), the superscripts “eff” of α_A^{eff} and α_B^{eff} come from the fact that our test scalar-field solution (15) depends on the discriminant Δ , Eq. (14), which involves the combination $(C - \alpha_{\text{BH}} r_S / r^2)$ and not only the scalar charge $\alpha_{\text{BH}} r_S$. This means that the actual black-hole–scalar coupling strength is not merely α_{BH} , but rather the coefficient of the $-r_S / r^2$ main term of this combination, namely

$$C - \frac{\alpha_{\text{BH}} r_S}{r^2} = -\left(\frac{k_3 \dot{\phi}_{\text{BH}}^2}{M^2} + \alpha_{\text{BH}}\right) \frac{r_S}{r^2} + \mathcal{O}\left(\frac{r_S^2}{r^3}\right). \quad (90)$$

The coupling constants entering Eqs. (89), for each body A and B, are thus given by

$$\alpha_{\text{BH}}^{\text{eff}} \equiv \alpha_{\text{BH}} + \frac{k_3 \dot{\phi}_{\text{BH}}^2}{M^2}. \quad (91)$$

This is the black-hole analog of the effective matter–scalar coupling strength α_{eff} defined in Eq. (10) of Ref. [19]. In the cubic Galileon model of Sec. III, Eqs. (35), (48a), and (48d) imply that this effective scalar charge reads

$$\alpha_{\text{BH}}^{\text{eff}} = \frac{4}{9k_3} \left(\frac{k_2 M}{H}\right)^2 + \mathcal{O}(Mr_S) = \frac{4}{3} \alpha_{\text{BH}} + \mathcal{O}(Mr_S), \quad (92)$$

which is of the same order of magnitude as α_{BH} . In the quintic Galileon model of Sec. IV, the quantity C, Eq. (13d), does not involve any correction proportional to $1/r^2$; therefore, $\alpha_{\text{BH}}^{\text{eff}} = \alpha_{\text{BH}}$. Finally, in the full (quadratic plus cubic plus quintic) model of Sec. V, the $k_3 \dot{\phi}_{\text{BH}}^2 / M^2$ correction entering (91) does not vanish but is negligibly small because of our assumptions (69),

$$\left| \frac{k_3 \dot{\phi}_{\text{BH}}^2}{M^2 \alpha_{\text{BH}}} \right| \approx \left(\frac{3M}{2H} \right)^2 \left| \frac{k_3}{2\bar{k}_5} \right| \ll 1; \quad (93)$$

therefore, $\alpha_{\text{BH}}^{\text{eff}} \approx \alpha_{\text{BH}}$.

The crucial difference with standard scalar-tensor theories [45] [i.e., with a standard kinetic term (3a) alone] is that there exists a Vainshtein screening in the present nonlinear Galileon theories, which grossly changes the normalization of the scalar fluctuations, as described in Sec. II E: Scalar perturbations and interactions via scalar exchange behave as if each of the scalar charges were renormalized by a small factor $z_\lambda^{-1/2}$. This ensures that the effect of the scalar wave emission on the binary dynamics remains

perturbative with respect to GR and can be described by Eqs. (89) evaluated on the unmodified post-Newtonian trajectories. For estimating the scalar force between the bodies, the relevant scale λ is the interbody distance r_{AB} . However, the emission of gravitational waves (including helicity-0 ones due to the scalar field) is a collective phenomenon which builds up at the scale of the gravitational wavelength; therefore, the scale λ to insert in the reduction factor is rather this wavelength. This is what was carefully derived in Eq. (A.42) of Ref. [51] for the case of the cubic Galileon in the usual static, asymptotically flat configuration, rather than the cosmological boundary being considered here. We will adapt these results for our case, under the assumption that, deep in the Vainshtein region, the effect of the cosmological boundary is limited to inducing the scalar charges [as confirmed below Eq. (34) of Ref. [19] for $r \gg r_S$, where the same Vainshtein screening factor was found for both of these asymptotic cases]. This is not strictly correct, since the presence of $\dot{\phi}_c$ gives rise to mixed tr terms in the acoustic metric and, therefore, a tilting of the sound cone with respect to the light cone. We are only looking for order-of-magnitude estimates, so we will not study this detail in the present work. Reference [51] proved that the Vainshtein reduction factor reads

$$z_{\lambda, \ell}^{\text{cubic}} \approx \frac{1}{4} (\Omega_p r_{AB})^{3-\ell} (\Omega_p r_{V23})^{3/2}. \quad (94)$$

The large factor $(\Omega_p r_{V23})^{3/2}$ is the expected one from evaluating the Vainshtein screening at the distance of a few (π) wavelengths. The extra factor $(\Omega_p r_{AB})^{3-\ell}$ is more subtle, as it depends on the ratio of orthoradial to radial velocities of scalar perturbations. For $\ell = 2$ (quadrupole), it is of order 0.2 for LIGO/Virgo binaries, and 0.1 for LISA. We shall take it into account below when the binaries are in the G_3 -dominated region, but they do not change significantly our order-of-magnitude estimates.

The same careful analysis has not been performed for the quintic Galileon, as there is no known covariant way to diagonalize the kinetic terms of the spin-2 and spin-0 degrees of freedom. But aside from some possible factors of $(\Omega_p r_{AB})$ which do not change much the orders of magnitude (and which would increase the predicted scalar effects; therefore, it is conservative not to include them), the above results confirm that the Vainshtein screening factor should be evaluated at a few wavelengths. From our discussion of Sec. II E, we may thus estimate that in a G_5 -dominated region, one should use

$$z_\lambda^{\text{quintic}} \sim (\Omega_p r_{V25})^3. \quad (95)$$

Note two crucial differences with respect to Eq. (94): The exponent is now 3 instead of $\frac{3}{2}$, and the Vainshtein radius entering this expression is r_{V25} instead of r_{V23} .

Before computing the scalar effects for our three models of Secs. III, IV, and V, let us quote the numerical values we shall use. The bandwidth of LIGO and Virgo interferometers is between 30 Hz and 10^3 Hz, but this is for the lowest frequencies that they accumulate the largest number of cycles of their inspiral phase, and can therefore significantly constrain deviations from general relativity. We shall thus take $\Omega_p = \pi\nu$, with $\nu \approx 30$ Hz (note the factor π instead of 2π because the GW frequency is twice that of the orbit). In the case of LISA, we shall similarly use the lowest frequency of its bandwidth, namely $\nu \approx 10^{-4}$ Hz. This gives

$$\Omega_p \approx 10^2 \text{ rad s}^{-1} \quad \text{for LIGO/Virgo,} \quad (96a)$$

$$\Omega_p \approx 3 \times 10^{-4} \text{ rad s}^{-1} \quad \text{for LISA.} \quad (96b)$$

The largest number of observed cycles correspond to rather light black holes, i.e., of about $10m_\odot$ in the case of LIGO/Virgo, and $10^5m_\odot$ for LISA. This corresponds to

$$\Omega_p r_S \approx 10^{-2} \quad \text{for LIGO/Virgo,} \quad (97a)$$

$$\Omega_p r_S \approx 3 \times 10^{-4} \quad \text{for LISA,} \quad (97b)$$

and

$$Hr_S \approx 2 \times 10^{-22} \quad \text{for LIGO/Virgo,} \quad (98a)$$

$$Hr_S \approx 2 \times 10^{-18} \quad \text{for LISA.} \quad (98b)$$

In LIGO/Virgo tests of general relativity, the parameter denoted as φ_{-2} in Ref. [52] quantifies the allowed correction to dipolar radiation. This reference's Fig. 6 gives the constraint $\varphi_{-2} < 10^{-3}$. The ratio of the dipolar term in Eq. (89a) to the GR prediction (87) is thus constrained by

$$\frac{F_{\text{scalar}}^{\text{dipole}}/z_{\lambda,1}}{F_{\text{GR}}} < 10^{-3} \quad \text{with LIGO/Virgo,} \quad (99)$$

with $z_{\lambda,1}$ given by Eq. (94) if the wavelength lies in the G_3 -dominated region, or by Eq. (95) if it lies in the G_5 -dominated region. In the cubic Galileon model of Sec. III, all black holes have the same α_{BH} (up to negligible corrections); therefore, the dipole vanishes, and we may use the constraints on the parameter φ_0 of this same Ref. [52], which quantifies the allowed correction to quadrupolar radiation. Its Fig. 6 gives $\varphi_0 < 5 \times 10^{-2}$, implying

$$\frac{F_{\text{scalar}}^{\text{quadrupole}}/z_{\lambda,2}}{F_{\text{GR}}} < 5 \times 10^{-2} \quad \text{with LIGO/Virgo.} \quad (100)$$

With the future LISA mission, one can expect to observe about 30000 cycles for black-hole masses of order $10^5m_\odot$ [53], with a large signal-to-noise ratio of a few hundreds. The

deviations from GR should thus be tested at least at the level

$$\frac{F_{\text{scalar}}}{F_{\text{GR}}} \lesssim \frac{1}{100 \times 30000} \approx 3 \times 10^{-7} \quad \text{with LISA,} \quad (101)$$

if the detected GWs are consistent with the general relativistic templates. Significantly tighter bounds are actually predicted in Ref. [54], for various populations of BHs and combined experiments (see notably its Fig. 11), and therefore, the above bound (101) is conservative.

B. Cubic Galileon

In the cubic Galileon model of Sec. III (i.e., with $k_5 = 0$), the Vainshtein radius (50) gives

$$\Omega_p r_{V23} \approx 2 \times 10^{12} \quad \text{for LIGO/Virgo,} \quad (102a)$$

$$\Omega_p r_{V23} \approx 10^8 \quad \text{for LISA.} \quad (102b)$$

These large values mean that the binaries are deep within the screened region where G_3 dominates.

We underlined above that scalar dipolar radiation is vanishingly small in the present cubic Galileon model, because all black holes have the same dimensionless scalar charge α_{BH} [see (48d)] up to negligible corrections of order $\mathcal{O}(Hr_S)$. The scalar energy flux (89) is thus given by $F_{\text{scalar}}^{\text{quadrupole}}/z_{\lambda,2}$, and Eqs. (48d), (87), (88), (91), and (94) allow us to write its ratio to the general relativistic prediction F_{GR} as

$$\begin{aligned} \frac{F_{\text{scalar}}}{F_{\text{GR}}} &\approx \frac{2^5 \alpha_{\text{BH}}^2}{3^3 |k_2| (\Omega_p r_{AB}) (\Omega_p r_{V23})^{3/2}} \\ &\approx \frac{2^5 |k_2|^3}{3^4 \sqrt{3} k_3^2} \frac{M^4}{H^3 \Omega_p} \frac{1}{(\Omega_p r_S)^{5/6}}. \end{aligned} \quad (103)$$

In the case of full self-acceleration, $M^4 = 3^3 k_3^2 H^4 / |k_2|^3$ takes its largest possible value, and therefore also the scalar charge (49), which reaches an $\mathcal{O}(1)$ value. This gives then the largest possible scalar effects

$$\frac{F_{\text{scalar}}}{F_{\text{GR}}} \approx \frac{2^5}{3\sqrt{3}} \frac{H}{\Omega_p} \frac{1}{(\Omega_p r_S)^{5/6}}. \quad (104)$$

The presence of the very small factor H , of cosmological origin, shows that the Vainshtein screening is very efficient, and even with $\mathcal{O}(1)$ scalar charges, experimental bounds are easily passed. The numerical values (97) and (98) indeed give

$$\frac{F_{\text{scalar}}}{F_{\text{GR}}} \approx 6 \times 10^{-18} \quad \text{for LIGO/Virgo,} \quad (105a)$$

$$\frac{F_{\text{scalar}}}{F_{\text{GR}}} \approx 4 \times 10^{-11} \quad \text{for LISA.} \quad (105b)$$

These predicted scalar effects are thus much smaller than the present constraint (100) provided by LIGO/Virgo data, and even the expected accuracy (101) which should be reached with LISA. They become even smaller if the Galileon field is not responsible alone for the accelerated expansion of the universe, i.e., that the theory parameter M is smaller than its maximum value of order H , as illustrated by Eq. (103). This can also be understood by noting that the black-hole scalar charge (48d) is proportional to M^2 while the Vainshtein radius (50) remains strictly the same.

In conclusion, although the time derivative of the scalar field imposed by cosmology generates $\mathcal{O}(1)$ scalar charges for black holes, the Vainshtein screening is so efficient that no deviation from GR can be observed in gravitational-wave experiments, in the quadratic plus cubic Galileon model of Sec. III. In other words, this full class of models passes experimental tests. We will see below that the situation changes drastically when considering the quintic Horndeski term.

C. Simplest quintic Horndeski term

In the quintic model of Sec. IV (i.e., with $k_3 = 0$), we saw in Eq. (55) that the scalar-field accretion is negligible when M/H is small enough. In such a case, the relevant Vainshtein radius (59) is of the order of the size of the observable universe, and therefore, binary black holes are always deep within the screened region where G_5 dominates. Since the dimensionless scalar charge $\alpha_{\text{BH}}^{(c)}$, Eq. (54b), is body dependent, dipolar radiation dominates the scalar energy flux (89), and Eqs. (87), (88), (95) give

$$\frac{F_{\text{scalar}}}{F_{\text{GR}}} \sim \frac{5\alpha_{\text{BH}}^2}{96|k_2|(\Omega_{\text{p}}r_{\text{S}})^{2/3}(\Omega_{\text{p}}r_{\text{V25}})^3} \quad (106a)$$

$$\sim \frac{5|k_2|^3}{3^7k_5^2} \left(\frac{M}{H}\right)^8 \frac{1}{(Hr_{\text{S}})(\Omega_{\text{p}}r_{\text{S}})^{11/3}} \quad (106b)$$

$$\lesssim \frac{5H}{72\Omega_{\text{p}}} \frac{1}{(\Omega_{\text{p}}r_{\text{S}})^{8/3}}, \quad (106c)$$

where the last inequality uses the small-accretion bound (55). Because of it, the tiny ratio H/Ω_{p} enters again the predicted effect, and the numerical values (97) and (98) give

$$\frac{F_{\text{scalar}}}{F_{\text{GR}}} \sim 4 \times 10^{-16} \quad \text{for LIGO/Virgo,} \quad (107a)$$

$$\frac{F_{\text{scalar}}}{F_{\text{GR}}} \sim 10^{-6} \quad \text{for LISA.} \quad (107b)$$

The first value is again much smaller than the present constraint (100) provided by LIGO/Virgo data, but the

second is larger than our conservative LISA accuracy (101). Moreover, in the present quintic model, we neglected the probable amplification factors similar to $(\Omega_{\text{p}}r_{\text{AB}})^{\ell-3}$ derived in Ref. [51] for the cubic case. Therefore, scalar effects should probably be larger than our order-of-magnitude estimate (107b), especially given $\ell = 1$ for dipolar radiation. Another amplification may also come from the larger value of H to insert in Eq. (106c), if the detected binary BH is at a significant redshift. It is interesting to note that this observable prediction corresponds to a model with a very small value of M/H [cf. Eq. (55) of Sec. IV B]. For $10^5 m_{\odot}$ black holes, this means $M/H < 6 \times 10^{-5}$, and one could thus naively think that the Galileon field has negligible influence on any physical prediction, in the same way it can be fully forgotten in the cosmological Friedmann equations. But because of the $1/(Mr)^2$ dependence of the C term in Eq. (13d), the nonvanishing time derivative $\dot{\varphi}_c$ generates large scalar charges for black holes, which do yield effects which will be observable with the LISA interferometer.

Denoting as δ the expected bound (101), LISA's consistency with GR would imply the constraint

$$\frac{M}{H} < \left(\frac{3^7k_5^2}{5|k_2|^3} Hr_{\text{S}}(\Omega_{\text{p}}r_{\text{S}})^{11/3}\delta \right)^{1/8} \sim 5 \times 10^{-5}. \quad (108)$$

In terms of the parameter α_T quantifying the speed deviation of GWs with respect to light, Eq. (53), this means that, in the context of this model, LISA should be able to constrain

$$|\alpha_T| < 3 \times 10^{-36}, \quad (109)$$

an improvement by 21 orders of magnitude with respect to the present experimental limit (8).

In Sec. IV B, we showed that for larger values of M/H than Eq. (55), but still consistent with a small-enough α_T (i.e., $M/H \lesssim 2 \times 10^{-2}$ for $|k_2| \sim |k_5| \sim 1$), the model predicts large initial scalar-field accretion rates, which are eventually quenched. The relevant Vainshtein radius is then given by Eq. (60), and numerically

$$\Omega_{\text{p}}r_{\text{V25,local}} \approx 6 \times 10^{14} \left(\frac{H}{M}\right)^{8/9} \quad \text{for LIGO/Virgo,} \quad (110a)$$

$$\Omega_{\text{p}}r_{\text{V25,local}} \approx 2 \times 10^{10} \left(\frac{H}{M}\right)^{8/9} \quad \text{for LISA.} \quad (110b)$$

These large numbers mean again that the binaries are deep within the screened region where G_5 dominates. The dimensionless quenched scalar charge (58b) is still body dependent; therefore, the dipolar energy flux entering Eq. (89) dominates, and we predict again expression (106a). The difference is that the scalar charge (58b) and

the Vainshtein radius (60) take other values, but they finally combine to give the same expression (106c), namely

$$\frac{F_{\text{scalar}}}{F_{\text{GR}}} \gtrsim \frac{5H}{72\Omega_p} \frac{1}{(\Omega_p r_s)^{8/3}}, \quad (111)$$

and numerically Eqs. (107). Note that this is now a lower bound, i.e., it corresponds to the final state where the scalar accretion rate Γ_{acc} is of order $\mathcal{O}(H)$. But if it happens that we observe a binary not too long after the formation of the BHs, i.e., that scalar accretion has not yet driven the scalar charges to the rather small limit (58b), then we may observe larger effects than Eqs. (107).

In conclusion, although the present quadratic plus quintic Galileon model generically predicts very large scalar charges for black holes, this also causes a large scalar-field accretion, which makes the local value of $\dot{\varphi}_{\text{local}}$ decrease. After such an accretion, the predicted scalar effects in binary black holes are too small to be of observational relevance for LIGO/Virgo, but should be easily detectable with the LISA mission. If its observations are consistent with the GR wave templates, this means that this class of models will be ruled out for $M/H \gtrsim 5 \times 10^{-5}$ [cf. Eq. (108)], thereby constraining α_T by 21 orders of magnitude tighter than the present experimental limit [cf. Eq. (109)].

D. Cubic Galileon with a small quintic term

Various scenarios are possible in the model of Sec. V, depending on whether scalar accretion is negligible or significant, and whether the BH binaries are within a G_5 - or G_3 -dominated region. The clearest way to discuss them is to plot a two-variable diagram. It is indeed always possible to set $|k_2| = |k_3| = 1$ by redefining φ and M in action (1) and functions (3), and there remains only two dimensionless parameters defining the model: k_5 and M/H .

Figure 2 illustrates the case of $10m_\odot$ black holes. The region of the plane consistent with our assumptions (63) is the central white strip going from the bottom-left to the top-right. In the gray triangle at the top, one has $|k_5/k_3|(H/M)^2 < (Hr_s)^2$, and therefore, the quintic Galileon term is no longer dominating at small distances $r \sim r_s$. When considering even lower values of $|k_5/k_3|(H/M)^2$, the model ultimately tends to the (quadratic plus) cubic Galileon discussed in Sec. VI B above. In the lower gray triangle, one has $|k_5/k_3|(H/M)^2 > Hr_s$; therefore, the neglected terms in our expansions (70) become significant, and the model ultimately tends to the (quadratic plus) quintic Galileon discussed in Sec. VI C.

The horizontal dashed line corresponds to the value of M/H giving full self-acceleration. The hatched upper half-plane is thus unphysical. The upper red triangle is forbidden by constraint (8) on the speed of GWs.

The lower blue triangle corresponds to the small values of M/H such that the scalar-field accretion rate (20) by the

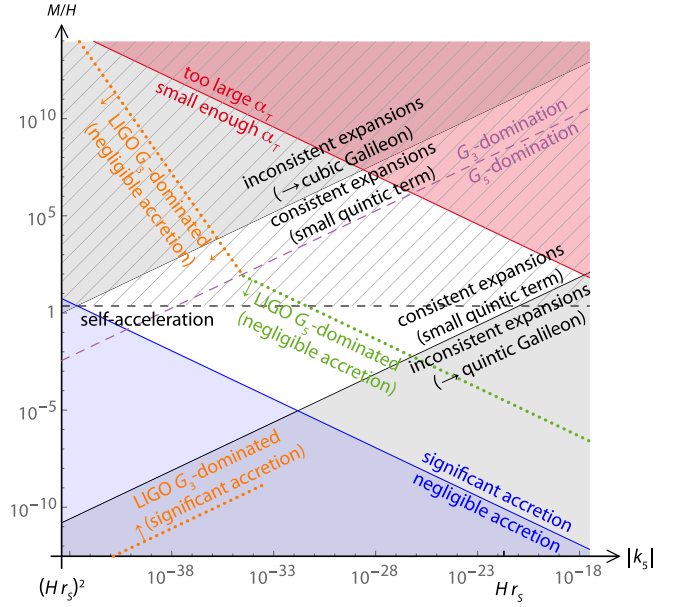


FIG. 2. Predictions for LIGO/Virgo in the parameter space $(|k_5|, M/H)$, for $|k_2| = |k_3| = 1$. In the central strip where our perturbative calculations are justified, all models are consistent with present experimental bounds.

BHs is small enough with respect to H . The models are also allowed in the white region above it, but the analysis of their predictions changes, because one must take into account the depletion of $\dot{\varphi}_{\text{local}}$ by scalar-field accretion, as explained in Sec. V B.

The diagonal violet dashed line separates the regions where the binary lies within a G_3 - or G_5 -dominated region, implying a different Vainshtein reduction factor (94) or (95).

Since the dimensionless scalar charge (71) is body dependent in the present model, the relevant LIGO/Virgo observational bound is the one on dipolar radiation, Eq. (99). More explicitly, the relative scalar contribution to the energy flux reads

$$\frac{F_{\text{scalar}}}{F_{\text{GR}}} \sim \frac{5}{32\sqrt{|k_3|}} \frac{M}{\Omega_p} \frac{|\alpha_{\text{BH}}|^{3/2}}{(\Omega_p r_s)^{11/6}}, \quad (112)$$

when G_3 dominates the orbital physics, and

$$\frac{F_{\text{scalar}}}{F_{\text{GR}}} \sim \frac{5}{48\sqrt{2|k_5|}} \left(\frac{M}{\Omega_p}\right)^2 \frac{|\alpha_{\text{BH}}|^{3/2}}{(\Omega_p r_s)^{5/3}}, \quad (113)$$

when G_5 dominates. Note in passing that these two predictions have the same $|\alpha_{\text{BH}}|^{3/2}$ dependence, in spite of the different Vainshtein screening factors (94) and (95), which are, respectively, proportional to $r_{V23}^{3/2}$ and r_{V25}^3 .

This comes from the fact that $r_{V23} \propto |\alpha_{\text{BH}}|^{1/3}$, Eq. (30), whereas $r_{V25} \propto |\alpha_{\text{BH}}|^{1/6}$, Eq. (29).

The long dotted orange and green lines display the bound (99) for the models which are within the lower blue region, i.e., with M/H small enough to predict negligible scalar-field accretion. In such a case, the scalar charge is given by Eq. (71). The allowed models lie below these lines, and since the blue region is already below them, this means that all of them pass the LIGO/Virgo experimental tests.

On the other hand, the dotted orange line near the bottom of the plane corresponds to the LIGO/Virgo bounds for the models lying within the white region (where scalar accretion was initially significant and is quenched), when G_3 dominates the orbital physics and the emission of scalar waves. The corresponding prediction is thus given by Eq. (112), but now with the scalar charge (77b). The allowed models are above this dotted line. But the corresponding models lie in the tiny white triangle at the left of the figure, above the violet dashed line, and are thus already above this dotted orange line. Therefore, once again, they all are consistent with the LIGO/Virgo bounds.

Finally, there also exists a large white region (significant scalar accretion followed by quenching) between the bottom blue triangle and the violet dashed line, where G_5 dominates the orbital physics. In such a case, the scalar energy flux reads

$$\frac{F^{\text{scalar}}}{F^{\text{GR}}} \sim \frac{5H}{72\Omega_p} \frac{1}{(\Omega_p r_s)^{8/3}} \gtrsim 4 \times 10^{-16}, \quad (114)$$

which does not depend on $|k_5|$ nor M/H ; therefore, no corresponding line is plotted on Fig. 2. But this value is 12 orders of magnitude smaller than the experimental bound (99), which means that all these models are also consistent with observation. Note that Eq. (114) coincides with (111) and (107a) we had found in Sec. VIC for the case of the quadratic plus quintic model.

In conclusion, in spite of the large scalar charges for BHs predicted by this class of models, the Vainshtein screening and the scalar depletion by accretion are so efficient that no signature of scalar waves can be detected in LIGO/Virgo. Let us recall that the scalar charge (71) is $\propto k_5/(Hr_s)^2$ when accretion is negligible (blue region of Fig. 2), and that its lower bound (77b) after significant accretion (white region) is still $\propto [|k_5|H^2/(M^4 r_s^2)]^{1/3}$. Because of our assumptions (63), which imply $|k_5| \gg |k_3|(Mr_s)^2$, both these charges are thus generically much larger than 1. The first one, Eq. (71), may become smaller than 1 when $|k_5|$ is as small as our hypotheses allow and simultaneously $M/H \ll 1$ (bottom-left of the blue region in Fig. 2). On the other hand, the end of accretion limit (77b) is always large within our working interval (63) (white region of Fig. 2). This underlines how surprising is our prediction of no observable signature of such BH charges in LIGO/Virgo.

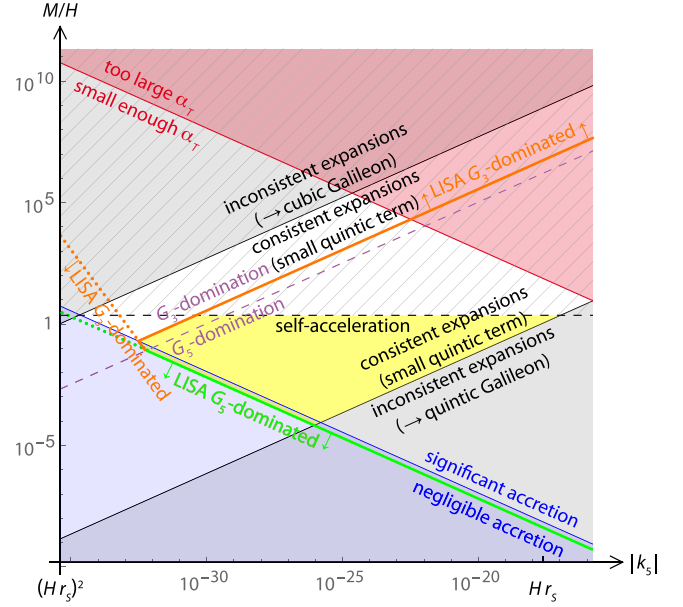


FIG. 3. Predictions for LISA in the parameter space $(|k_5|, M/H)$, for $|k_2| = |k_3| = 1$. The central yellow region will be probed, i.e., ruled out if observations are consistent with the general relativistic GW templates.

Figure 3 illustrates the case of $10^5 m_\odot$ black holes. The gray, hatched, red, and blue regions have the same meaning as above, as well as the diagonal violet dashed line separating G_3 - and G_5 -dominated cases. The novelty is that LISA will be sensitive to the effect of scalar-wave radiation in the central yellow region.

The orange and green lines indeed correspond to the expected LISA constraint (101) if its detections are consistent with the general-relativistic wave templates. In the blue region where scalar accretion is negligible, the models should lie below the dotted orange line when the orbital physics of the binary is dominated by G_3 , Eq. (112), and below the dotted or plain green lines when it is dominated by G_5 , Eq. (113). Since the corresponding blue regions are already below the dotted lines, these models will be consistent with observed data. On the other hand, the tiny strip between the plain green and blue lines will be probed. Note that the green line will go down when considering a less conservative accuracy than Eq. (101); i.e., a wider strip within this blue triangle (negligible accretion) should be probed.

The white region where scalar accretion is significant is again divided by the dashed violet line distinguishing G_3 - and G_5 -dominated cases. Above this dashed violet line, i.e., within a white triangle at the left of the figure, the models below the plain orange line will be probed by LISA (they are colored in yellow), whereas those which remain white (above the plain orange line) will pass the tests. The remaining of the central yellow region is dominated by G_5 , which gives again the literal expression (114) for the scalar

energy flux. Since it does not depend on $|k_5|$ nor M/H , no line is represented on Fig. 3. But its numerical value coincides with Eq. (107b) we found for the quadratic plus quintic case of Sec. VIC, explicitly

$$\frac{F^{\text{scalar}}}{F^{\text{GR}}} \sim \frac{5H}{72\Omega_p} \frac{1}{(\Omega_p r_s)^{8/3}} \gtrsim 10^{-6}. \quad (115)$$

Since it is larger than the LISA expected accuracy (101), the full central yellow region should be probed. If LISA observations are consistent with GR, this means that all the models within this yellow region should be ruled out.

Note that the models along the dashed horizontal line correspond to full self-acceleration. In such a case, Friedmann equations give $M/H_0 = (3/|k_2|)^{3/4} \sqrt{|k_3|}$, with H_0 the present value of the Hubble constant, and denoting as δ the expected LISA bound (101), we get

$$\begin{aligned} |k_5| &\lesssim \frac{2^7 \times 3^3 \sqrt{3}}{5^2 |k_2|^{3/2}} \left(\frac{H_0}{H}\right)^2 (\Omega_p r_s)^{17/3} (k_3 \delta)^2 \\ &\lesssim 4 \times 10^{-31} \left(\frac{H_0}{H}\right)^2 \frac{k_3^2}{|k_2|^{3/2}}. \end{aligned} \quad (116)$$

[It can easily be checked that $k_5 |k_2|^{3/2} / k_3^2$ is indeed the observable notation-independent ratio entering action (1)–(3).] Note that if the detected binary is at a significant redshift, the corresponding value of H is larger than the present H_0 ; therefore, the above constraint is even stronger.

In conclusion, not only LISA should probe a significant region of the theory plane, Fig. 3, but it should even be able to constrain the parameter k_5 at the 10^{-30} level for self-acceleration models. Such a tight constraint may be compared to what Eq. (8) imposes on the same self-accelerating class of models, namely $|k_5| < 3 \times 10^{-15} k_3^2 / |k_2|^{3/2}$. LISA will thus provide an improvement by at least 16 orders of magnitude on the coefficient of the quintic term. The generically large scalar charges predicted by the present class of Horndeski theories are responsible for such a tight bound, in spite of the Vainshtein screening and the phenomenon of scalar accretion, which very significantly reduce observable effects on GWs. Let us, for instance, mention that for the self-accelerating model saturating inequality (116), we still find a rather large value of

$$|\alpha_{\text{BH}}| \gtrsim 2|k_2| \left(\frac{|k_5| H^2}{3^5 k_3^2 H_0^4 r_s^2} \right)^{1/3} \approx 15|k_2|^{1/2}. \quad (117)$$

This means that the coupling strength of the BH to the scalar field is still 15 times larger than to gravitons. It is obviously even (much) larger for the other self-acceleration models at the top of the yellow region of Fig. 3, which correspond to values of $|k_5|$ larger by several orders of magnitude.

VII. CONCLUSIONS

In general relativity, there exists a so-called “effacement principle” [55], such that local and large-distance physics are almost decoupled, up to tidal effects which start manifesting on the motion of compact bodies only at the fifth post-Newtonian order ($1/c^{10}$). Scalar-tensor theories generically do not share this property, because the local value of a scalar field cannot be reabsorbed in a change of coordinates. This is the reason why the cosmological expansion of the universe can have an influence on local solutions, including on black holes.

In the present paper, we have shown that the regularity of black-hole solutions imposes that they must have a scalar hair as soon as the time derivative of the scalar field does not strictly vanish, and even if no matter-scalar coupling is assumed in the action. While the existence of the non-vanishing time derivatives is driven by cosmology, this is a local effect, arising from the requirement that a solution adopt a particular form near the Schwarzschild radius of the black hole. The regularity of the solution imposes a different requirement for material bodies; nonetheless, at least the coupling to $\dot{\varphi}^2$ remains and provides a contribution to the effective scalar charge.

We have shown that these charges are very large compared to expectations in typical modified-gravity setups, despite the extremely small scalar-field backgrounds with curvatures of the order of the Hubble rate today. For example, the quadratic plus cubic Galileon model of Sec. III predicts scalar charges of order 1, meaning that black holes couple to the scalar field with a strength similar to their coupling to gravity. This charge causes accretion of the background scalar field onto the black holes, but at a sufficiently small rate that a quasistationary solution can be found for the whole expanding spacetime, confirming this is a long-lived charge. Typically such a charge would imply a change to GR predictions excluded by many orders of magnitude. However, the Vainshtein screening of scalar effects innate to this type of models implies that the new emission channels are much too small to cause observable effects in gravitational-wave experiments detecting coalescences of binary black holes.

An even more astounding deviation from the usual setups appears in the presence of the quintic Galileon operator. The local consistency of the scalar-field profile requires a dimensionless black-hole charge of order $(r_s)^{-2}$. Given fixed model parameters, large enough black holes accrete slowly enough to allow them to be considered as quasistatic solutions. On the other hand, the charges for small black holes are large enough to make accretion fast. Studying the evolution for such configurations can only be done numerically and is beyond the scope of this paper. We argue that a depletion of the charge by the absorption of the local scalar-field configuration is the only possibility to achieve a stationary solution for a black hole in this class of models. This charge quenching cannot remove the charge

TABLE II. Summary of the most important results for the three classes of models we consider.

Scalar Lagrangian	Scalar accretion	Scalar charge α_{BH}	Equation number	LISA constraints
G_2, G_3	Negligible	$\mathcal{O}(1)$	(48d)	No
G_2, G_5	Formation	$\propto (Hr_S)^{-2}$	(54b)	Eq. (108)
	Quenched	$\propto (Hr_S)^{-2/3}$	(58b)	
G_2, G_3, G_5	Formation	$\propto k_5 (Hr_S)^{-2}$	(71)	Fig. 3 and Eq. (116)
	Quenched	$\propto \left(\frac{ k_5 }{H^2 r_S^2}\right)^{1/3}$	(77b)	

completely, but can only reduce it to the extent that the characteristic accretion timescale is reduced to no more than the lifetime of the universe. With this assumption, we show that if LISA does not see these effects, the predicted deviation of the speed of gravitational waves from luminal in these models will be constrained by many additional orders of magnitude, compared to the current bounds. We derive this phenomenology for two models—a pure quadratic-quintic Galileon and a full model containing linear G_2 , G_3 , and G_5 terms which we show interpolates between the two simpler models. In the quadratic-quintic case, for which the scalar field cannot contribute significantly to the accelerated expansion of the universe, LISA should be able to improve the constraint on the GW speed parameter α_T by 21 orders of magnitude. In a self-acceleration scenario of the full model, it should improve it by 16 orders of magnitude. Table II summarizes our most important results, and points to the corresponding equations and figure.

As we have pointed out, the models studied here do not exhibit a *two-horizon* problem like the one affecting the *vanilla* shift-symmetric sGB gravity [12], i.e., without any other operators. Nevertheless, it is interesting that the two situations bear some resemblance: Both the scalar charge and the local time derivative must be adjusted uniquely for a “good” solution to exist, not anymore due to regularity requirements but to those of reality and smoothness. It is not this distinction which dooms one model and saves the others, but the fact that the resulting value for $\dot{\varphi}_{\text{BH}}$ is unacceptably far from $\dot{\varphi}_c$ in the sGB case, clashing with the assumed cosmology. An open question is when the conditions on a solution may become too many to be accommodated simultaneously by fixing only two parameters, in which case the whole assumption of stationarity breaks down, necessitating a different approach.

One may wonder to what extent our results may apply to more general Horndeski theories of the same family, namely a combination of $G_2(X)$, $G_3(X)$, and $G_5(X)$. For general functions of X , the resulting field equation will generically be a higher-order polynomial in φ' , leading to many more branches of solutions and making an analytic study of the kind we did here much more complicated, if even possible at all. Nevertheless, there are a number of

shared properties with the models that we have considered here. First, for analytic choices of functions $G_3(X)$ and $G_5(X)$, there is always a cosmologically induced independent term [Eq. (16)] in said equation, given purely as a function of $\dot{\varphi}$, acting as a source term for φ' . For this reason we expect that black holes will generically have hair. Second, not all the possible roots of the field equation are actually relevant. Indeed, for our case of interest there must be at least one root of the $J^0 = 0$ equation in FLRW that delivers a self-accelerating cosmological solution, and which is given by the balance of the G_2 - and $G_{3,5}$ -type operators (if it were possible with only G_2 , then the model would become effectively a much simpler k -essence). Moreover, these self-accelerating solutions are in a part of phase space for $\dot{\varphi}_c$ which is disconnected from the Minkowski vacuum, since otherwise Minkowski would have been the asymptotic future state. This implies that in fully self-consistent collapse from perturbed cosmology, the solution would not be able to transition from the timelike gradients in cosmology to spacelike gradients of a would-be static solution through a vacuum configurations ($X = 0$) without some kind of pathology, just as in the case presented here. This forces the local root of the equation of motion to belong to the same vacuum as the cosmological one. For these reasons, one may imagine that there exists a rather large class of theories exhibiting a similar behavior to the simpler models we have considered, where the $G_i(X)$ functions end up acting effectively as mildly r -dependent k_i coefficients.

It is worth comparing the results of the present paper with respect to known black-hole solutions with a time-dependent scalar field (11) in scalar-tensor theories. A particularly interesting example is the class of stealth and self-tuning black holes [39,43,56,57]. These are exact solutions in Horndeski theories and their generalizations with a scalar field of the form (11). For these solutions, the scalar charge appearing in the r.h.s. of Eq. (12) must vanish, as a consequence of the linear time dependence of the scalar and the staticity of the metric [42]. One should note that such configurations are found in theories that do not coincide with those we considered here; in particular, for stealth/self-tuning solutions, the G_3 and G_5 Galileon terms should be effectively switched off [43]. For the cubic Galileon theory, however, nonstealth asymptotically de Sitter black-hole solutions were found by numerical methods in [58]. The assumptions of staticity of the metric and the linear time dependence of the scalar field were imposed, implying a vanishing scalar charge of the black hole. On the contrary, in our approach here, we assume negligible backreaction of the scalar field in the quasistationary state; thus, the scalar charge need not be strictly zero, and this is consistent if accretion is small. Note, however, that in [58], it was observed that numerical integration could not be continued below some radius inside the horizon, which might have been the consequence of missing the branching

point near the black hole, as discussed in Sec. III C above. This question can be resolved by studying the dynamical formation and evolution of a black hole, which goes beyond the scope of our paper.

Indeed, there remain two large open questions for the setup we consider, which require a separate study: (i) What are the details of the evolution of the black hole and its charge in the presence of large accretion, and what is the actual end point of this phase? Without a full numerical study, we have not been able, for example, to estimate the duration of any such transition to a quenched configuration, nor can we say what is the size of the local configuration of the scalar decoupled from cosmology. We are also working in the test-field approximation, while it may well turn out that the end point of the quenching is a black hole with a spacetime metric that significantly deviates from Schwarzschild at small radii. However, we reiterate that if the end point is quasistationary, it must have a charge, even if the underlying solution differs from the original black hole. (ii) Is the effect of non-staticity of the acoustic metric significant? For our estimates, we relied on the only computation of emission of scalar radiation in Vainshtein-screened regions so far attempted (see Ref. [51]). Not only is that setup purely for the cubic Galileon, as opposed to the quintic Galileon operator of most interest here, but it is based on the assumption of a static acoustic metric with a standard asymptotically flat spacetime at infinity. As we have mentioned, the presence of $\dot{\phi}$ would tilt the sound cone with respect to the light cone, e.g., creating a nonsymmetric setup with outgoing and incoming modes propagating at different sound speeds. The difference in the radial and orthoradial sound speeds was important in the final result of Ref. [51], and we would presume that this will change the details of the final answer. It is possible that the Vainshtein screening is weakened even further than for the static computation of Ref. [51]. Moreover, the kinetic mixing between the graviton and the scalar in the presence of the G_5 term may end up complicating this static-Vainshtein picture even further. Our computation is sufficient at the precision of order of magnitude, but for a detailed study these open questions must be addressed.

Understanding the answers to the above will also open another avenue for testing these models with gravitational-wave observations: their imprint on the ringdown phase of the merger. The nontrivial scalar background in the vicinity of the final charged black hole causes a significant amount of kinetic mixing between the scalar and gravitational degrees of freedom, which is expected to scramble the quasi-normal-mode (QNM) spectrum. The prospect of probing dark energy in this way was considered in the case where the shift-symmetric sGB coupling is present, acting as the source of the scalar hair, with the other standard operators of the Horndeski class providing the cosmology and a screening mechanism [59]. Here, the main difference is that the

sGB term is actually not needed to generate the hair, instead being sourced by the standard Horndeski operators themselves in the presence of the time-dependent background. We leave the detailed study of the QNMs associated with the hairy black-hole solutions found here for future work.

Finally, in this paper we have used a conservative method of estimating future constraints by using information from a single event. A population study of the type presented in Ref. [54] should result in a stronger constraint for the Galileon models, even without the additional leverage of multiband observations. However, the results of the existing analysis cannot be applied directly, since we predict significantly different phenomenology from the standard dipolar-radiation case: The black-hole charges and therefore the expected radiation depend at least on the Hubble parameter at emission as well as the black-hole mass. A reanalysis of the constraining power of future surveys is a natural follow-up to our work.

In this paper we have presented a rather unexpected result: The cosmological boundary affects local solutions very significantly, to the extent that we expect LISA to be able to probe modifications of gravity relevant for cosmic acceleration. Such complementary tests of modifications of gravity might prove key to understanding the mechanism behind dark energy, given the recent results showing growing inconsistency of the late-time data with the Λ CDM model [60].

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: NO TWO-HORIZON PROBLEM

A question that we should address is whether the values of α_{BH} and $\dot{\phi}_{\text{BH}}$ may be conditioned by the requirement of regularity of the scalar solutions at both the black-hole and cosmological horizons. This has been shown to be the case in the *vanilla* shift-symmetric scalar-Gauss-Bonnet model, i.e., in the absence of extra higher-dimensional operators¹⁴ [12]. In that model, the regularity conditions prevent the construction of stationary solutions with a physically reasonable approach to homogeneity, as $\dot{\phi}_{\text{BH}}$ differs too

¹⁴Such that the scalar-field equation is linear.

strongly from $\dot{\varphi}_c$. This was dubbed the two-horizon problem. Ultimately, this is rooted in a particularity of that theory: the r_s^{-1} behavior of the sGB source term. As also stressed in Ref. [12], however, the mere presence of a higher-dimensional operator is enough to resolve this tension at the price of introducing *strong* nonlinearities of the scalar field.

Let us examine these conditions in our present case of interest. Given the shift symmetry, we focus on the scalar quantity X to assess the regularity of the solution at each of the two gravitational horizons, i.e., $f \rightarrow 0$. For the ansätze (9) and (11), it takes the form $X = \dot{\varphi}_{\text{BH}}^2/f - f\varphi'^2$, and therefore, a regular X implies that near a horizon $r \simeq r_h$,

$$\varphi'(r) \simeq \pm \frac{\dot{\varphi}_{\text{BH}}}{f} + \text{finite}, \quad (\text{A1})$$

where the finite parts may depend on which horizon we are looking at. We place this form of $\varphi'(r)$ into the scalar-field equation (12), collect inverse powers of f , and demand it is satisfied order by order. At the leading order f^{-1} , which is the only possibly divergent one, the equation is automatically satisfied by the above form of $\varphi'(r)$. In other words, there are no further regularity conditions than the one of X itself. At subleading order, the constants α_{BH} and $\dot{\varphi}_{\text{BH}}$ appear and are balanced by the finite value $X(r_h)$ at each horizon. Eventually higher derivatives of $\varphi'(r)$ at the horizon also show up at higher orders. With only two horizons, these conditions are not sufficient to fully determine α_{BH} and $\dot{\varphi}_{\text{BH}}$, unlike the *vanilla* sGB case studied in Ref. [12].

We conclude then that there is no two-horizon problem in the theory we are considering here. The gravitational horizons play no role in fixing α_{BH} and $\dot{\varphi}_{\text{BH}}$, which may or may not be required to satisfy other conditions elsewhere.

APPENDIX B: HOMOGENEITY FOR $\dot{\varphi}_{\text{BH}} \neq \dot{\varphi}_c$

The local time derivative $\dot{\varphi}_{\text{BH}}$ of the scalar field may differ from the cosmological background value $\dot{\varphi}_c$, as studied in the context of Brans-Dicke theory in Ref. [11]. Indeed, to recover a homogeneous cosmological solution of the type (37) sufficiently far away, the only requirement is that $\varphi'(r)$ agrees with Eq. (39) in the large distance limit in static coordinates ($r \rightarrow \infty$), explicitly

$$\varphi' \simeq \frac{\dot{\varphi}_c}{Hr}. \quad (\text{B1})$$

This is always satisfied by the solution to the field equation (12) in this limit. Instead, the term linear in the “local” time t does not directly contribute to the linear dependence in the cosmological time τ far away. This is an effect of the change of coordinates of Eq. (38). Indeed, beyond the cosmological horizon,

$$t = \tau - \frac{1}{2H} \log[-1 + (He^{H\tau}\rho)^2] \simeq -\frac{1}{H} \log(H\rho). \quad (\text{B2})$$

Therefore, the full φ , Eq. (11), expressed in cosmological coordinates in this limit ($\rho \rightarrow \infty$) goes like

$$\varphi \simeq \dot{\varphi}_c \tau + \frac{(\dot{\varphi}_c - \dot{\varphi}_{\text{BH}})}{H} \log(H\rho). \quad (\text{B3})$$

One may be concerned about the fact that $\varphi \rightarrow \infty$ when $\rho \rightarrow \infty$ due to the logarithmic term. However, this is a shift-symmetric theory, and therefore, the value of φ is of no physical consequence. We should instead inspect observable quantities such as X , in Friedmann coordinates

$$X \simeq \dot{\varphi}_c^2 - e^{-2H\tau} \frac{(\dot{\varphi}_c - \dot{\varphi}_{\text{BH}})^2}{H^2 \rho^2} \xrightarrow{\rho \rightarrow \infty} \dot{\varphi}_c^2, \quad (\text{B4})$$

which indeed approaches its expected homogeneous cosmological limit at a (static) distance of order,

$$r_{\text{homog}} \sim H^{-1} \left| 1 - \frac{\dot{\varphi}_{\text{BH}}}{\dot{\varphi}_c} \right|, \quad (\text{B5})$$

which should be kept not larger than H^{-1} . We may then allow $\dot{\varphi}_{\text{BH}} \lesssim \dot{\varphi}_c$ locally to satisfy regularity and existence conditions of the solution, without breaking basic assumptions about cosmology nor disagreeing with observations when the scalar drives the acceleration.

APPENDIX C: ESTIMATE OF THE KINETIC MIXING INDUCED BY G_5

To assess the importance of the kinetic mixing between the scalar and gravitational fluctuations around the backgrounds of interest, it will suffice to use the schematic form of this term as it can be read from the quadratic action corresponding to our G_5 term (3c), namely

$$Z_{5,\text{mix}} \sim \frac{k_5}{M^4} X \mathcal{R}, \quad (\text{C1})$$

where $\mathcal{R}$ here represents a generic curvature quantity. In this language, the purely scalar part discussed in Sec. II E, Eq. (32), may be represented instead as $Z_5'' \sim k_5 \mathcal{R} \nabla \nabla \varphi / M^4$. Under the same assumptions that lead to Eqs. (33) and (34), the ratio of these quantities may be expressed as

$$\frac{Z_{5,\text{mix}}}{Z_5''} \sim r \varphi' \sim r^2 M^2 \left| \frac{\alpha_{\text{BH}}}{k_5} \right|^{1/2}, \quad (\text{C2})$$

where in the second step we are considering the G_5 -dominated region. For the binary merger, the relevant distance at which to evaluate this ratio is the wavelength $r \sim \Omega_p^{-1}$. Let us define $\epsilon_{\text{mix}} \equiv (\Omega_p r_s)^2 Z_{\text{mix}} / Z_5''$ and evaluate it for the cases of interest.

For the $G_2 + G_5$ model of Sec. IV, we obtain

$$\varepsilon_{\text{mix}} \sim \frac{M^4}{H^2} \lesssim Hr_S, \quad \text{not quenched}, \quad (\text{C3})$$

$$\varepsilon_{\text{mix}} \sim \left(\frac{HM^4 r_S^5}{|k_5|} \right)^{1/3} \lesssim (Hr_S)^{5/3}, \quad \text{quenched}, \quad (\text{C4})$$

where we used $M \lesssim H$ to find the upper bounds, and here $k_5 \sim \mathcal{O}(1)$ is assumed.

For the $G_2 + G_3 + G_5$ model of Sec. V, we find

$$\varepsilon_{\text{mix}} \sim \frac{M^2 r_S}{H} \lesssim Hr_S, \quad \text{not quenched}, \quad (\text{C5})$$

$$\varepsilon_{\text{mix}} \sim \left(\frac{M^4 r_S^3}{|\bar{k}_5| H^2} \right)^{1/3} \ll Hr_S, \quad \text{quenched}, \quad (\text{C6})$$

where here we have replaced k_5 in favor of $\bar{k}_5$ as defined in Eq. (68), and assumed in this case the hierarchy in Eq. (69).

In conclusion, at a distance of a wavelength we may safely neglect the kinetic mixing of the quintic operator in the cases we have studied and estimate the Vainshtein screening solely from Z_5'' as given in Eq. (34).

APPENDIX D: CHARGE IMPLIED BY THE COSMOLOGICAL BRANCHING POINT IN THE CUBIC GALILEON MODEL

We saw in Sec. III E that the tiny modification (48a) of the local time derivative $\dot{\varphi}_{\text{BH}}$ of the scalar field, with respect to its cosmologically imposed value $\dot{\varphi}_c$, suffices to make both branching points near $r \approx \frac{3}{4}r_S$ and $r \approx 1/(\sqrt{3}H)$ consistent with each other. It is worth mentioning what happens when one tries to enforce $\dot{\varphi}_{\text{BH}} = \dot{\varphi}_c$ strictly: One actually finds a strong inconsistency between these two branching points.

If one assumes that the scalar charge α_{BH} takes its value (43b), imposed by the inner branching point near $r \approx \frac{3}{4}r_S$, one finds that the discriminant Δ does *not* vanish close to $r \approx 1/(\sqrt{3}H)$. It becomes very small, in the sense that $\Delta/B^2 = \mathcal{O}(Hr_S)$ for such a radius, with the notation of Eq. (14), but it does not admit any real root there. However, for solution (15) to be close to the cosmological background $\varphi = \dot{\varphi}_c \tau$, we saw in Sec. III A that the sign of $\pm\sqrt{\Delta}$ must be positive at radii smaller than $1/(\sqrt{3}H)$, and

negative at larger radii. Such a change of sign would here be discontinuous, which is impossible in the absence of a singular spherical shell source at this large distance. On the other hand, if solution (15) is continuous, then it cannot be close to the cosmological background either at smaller or at larger radii than $1/(\sqrt{3}H)$.

Another way to underline this inconsistency is to use again the technique described in Sec. III B, to tune α_{BH} so that there indeed exist a double root of Δ close to $r \approx 1/(\sqrt{3}H)$ while imposing $\dot{\varphi}_{\text{BH}} = \dot{\varphi}_c$ strictly. One then finds

$$r_{\text{root}}^{\text{inconsistent}} = \frac{1}{\sqrt{3}H} + \frac{3}{2}r_S + \mathcal{O}(Hr_S^2), \quad (\text{D1a})$$

$$\alpha_{\text{BH}}^{\text{inconsistent}} = \frac{\sqrt{3}k_2^2 M^2}{4k_3 H} r_S - \frac{9k_2^2 M^2}{2k_3} r_S^2 + \mathcal{O}(H^3 r_S^3). \quad (\text{D1b})$$

Let us also quote this last expression when the Galileon field φ is assumed to be responsible alone for the cosmological expansion:

$$\frac{\alpha_{\text{BH}}^{\text{inconsistent}}}{\text{sign}(k_3)\sqrt{-k_2}} = \frac{9}{4}Hr_S - \frac{27\sqrt{3}}{2}(Hr_S)^2 + \mathcal{O}(H^3 r_S^3). \quad (\text{D2})$$

We thus find that a double root of Δ would be possible at this large distance, but for a scalar charge which is $\mathcal{O}(Hr_S)$ smaller than the one we needed at small distances, Eq. (43b). Let us recall that $Hr_S \sim 10^{-22}$ (for a BH mass of $10m_\odot$); therefore, this is a very strong inconsistency between the two branching points.

It is thus quite surprising that the tiny modification (48a) of $\dot{\varphi}_{\text{BH}}$ sufficed to make them consistent with each other, while keeping a linear time dependence of the scalar field (11) in the whole universe. Independently of this surprise, our result (D1b) also illustrates the different orders of magnitude of the scalar charges generated by branching points close to the BH or at large distances. Other Horndeski theories may, for instance, hide the inner branching point inside the metric and sound horizons, and one should *a priori* only care about the large-distance branching points. In such a case, one may expect to predict much smaller scalar charges than in the present paper, and thereby negligible scalar effects in GW detections.

- [1] N. M. Bocharova, K. A. Bronnikov, and V. N. Melnikov, On an exact solution of the Einstein equations with a massless scalar field, *Vestn. Mosk. Univ., Fiz. Astron.* **6**, 706 (1970).
- [2] J. D. Bekenstein, Exact solutions of Einstein conformal scalar equations, *Ann. Phys. (N.Y.)* **82**, 535 (1974).
- [3] T. Damour and G. Esposito-Farèse, Nonperturbative strong field effects in tensor—scalar theories of gravitation, *Phys. Rev. Lett.* **70**, 2220 (1993).
- [4] P. C. C. Freire, N. Wex, G. Esposito-Farèse, J. P. W. Verbiest, M. Bailes, B. A. Jacoby, M. Kramer, I. H. Stairs, J. Antoniadis, and G. H. Janssen, The relativistic pulsar-white dwarf binary PSR J1738 + 0333 II. The most stringent test of scalar-tensor gravity, *Mon. Not. R. Astron. Soc.* **423**, 3328 (2012).
- [5] M. Kramer *et al.*, Strong-field gravity tests with the double pulsar, *Phys. Rev. X* **11**, 041050 (2021).
- [6] G. W. Horndeski, Second-order scalar-tensor field equations in a four-dimensional space, *Int. J. Theor. Phys.* **10**, 363 (1974).
- [7] L. Hui and A. Nicolis, No-hair theorem for the Galileon, *Phys. Rev. Lett.* **110**, 241104 (2013).
- [8] E. Barausse and K. Yagi, Gravitation-wave emission in shift-symmetric Horndeski theories, *Phys. Rev. Lett.* **115**, 211105 (2015).
- [9] T. Jacobson, Primordial black hole evolution in tensor scalar cosmology, *Phys. Rev. Lett.* **83**, 2699 (1999).
- [10] M. W. Horbatsch and C. P. Burgess, Cosmic black-hole hair growth and quasar OJ287, *J. Cosmol. Astropart. Phys.* **05** (2012) 010.
- [11] D. Glavan, S. P. Miao, T. Prokopec, and R. P. Woodard, Large logarithms from quantum gravitational corrections to a massless, minimally coupled scalar on de Sitter, *J. High Energy Phys.* **03** (2022) 088.
- [12] E. Babichev, I. Sawicki, and L. G. Trombetta, Black-hole hair in scalar-Gauss-Bonnet gravity is altered by cosmology, *Phys. Rev. D* **111**, L041502 (2025).
- [13] C. Armendariz-Picon, V. F. Mukhanov, and P. J. Steinhardt, A dynamical solution to the problem of a small cosmological constant and late-time cosmic acceleration, *Phys. Rev. Lett.* **85**, 4438 (2000).
- [14] C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, Imperfect dark energy from kinetic gravity braiding, *J. Cosmol. Astropart. Phys.* **10** (2010) 026.
- [15] T. Baker, E. Bellini, P. G. Ferreira, M. Lagos, J. Noller, and I. Sawicki, Strong constraints on cosmological gravity from GW170817 and GRB 170817A, *Phys. Rev. Lett.* **119**, 251301 (2017).
- [16] P. Creminelli and F. Vernizzi, Dark energy after GW170817 and GRB170817A, *Phys. Rev. Lett.* **119**, 251302 (2017).
- [17] J. M. Ezquiaga and M. Zumalacárregui, Dark energy after GW170817: Dead ends and the road ahead, *Phys. Rev. Lett.* **119**, 251304 (2017).
- [18] J. Sakstein and B. Jain, Implications of the neutron star merger GW170817 for cosmological scalar-tensor theories, *Phys. Rev. Lett.* **119**, 251303 (2017).
- [19] E. Babichev and G. Esposito-Farèse, Time-dependent spherically symmetric covariant Galileons, *Phys. Rev. D* **87**, 044032 (2013).
- [20] E. Babichev, V. F. Mukhanov, and A. Vikman, Escaping from the black hole?, *J. High Energy Phys.* **09** (2006) 061.
- [21] E. Babichev, Galileon accretion, *Phys. Rev. D* **83**, 024008 (2011).
- [22] E. Babichev, V. Dokuchaev, and Y. Eroshenko, Black hole mass decreasing due to phantom energy accretion, *Phys. Rev. Lett.* **93**, 021102 (2004).
- [23] E. O. Babichev, V. I. Dokuchaev, and Y. N. Eroshenko, Black holes in the presence of dark energy, *Phys. Usp.* **56**, 1155 (2013).
- [24] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, New York, 1973).
- [25] T. Anson and E. Babichev, Vainshtein screening for slowly rotating stars, *Phys. Rev. D* **102**, 044046 (2020).
- [26] J. Sakstein, Disformal theories of gravity: From the solar system to cosmology, *J. Cosmol. Astropart. Phys.* **12** (2014) 012.
- [27] T. Kobayashi, Y. Watanabe, and D. Yamauchi, Breaking of Vainshtein screening in scalar-tensor theories beyond Horndeski, *Phys. Rev. D* **91**, 064013 (2015).
- [28] E. Babichev, K. Koyama, D. Langlois, R. Saito, and J. Sakstein, Relativistic stars in beyond Horndeski theories, *Classical Quantum Gravity* **33**, 235014 (2016).
- [29] E. Bellini and I. Sawicki, Maximal freedom at minimum cost: Linear large-scale structure in general modifications of gravity, *J. Cosmol. Astropart. Phys.* **07** (2014) 050.
- [30] L. Heisenberg, J. Noller, and J. Zosso, Horndeski under the quantum loupe, *J. Cosmol. Astropart. Phys.* **10** (2020) 010.
- [31] D. Pirtskhalava, L. Santoni, E. Trincherini, and F. Vernizzi, Weakly broken Galileon symmetry, *J. Cosmol. Astropart. Phys.* **09** (2015) 007.
- [32] S. Mukohyama, Black holes in the ghost condensate, *Phys. Rev. D* **71**, 104019 (2005).
- [33] E. Babichev, C. Charmousis, G. Esposito-Farèse, and A. Lehébel, Hamiltonian unboundedness vs stability with an application to Horndeski theory, *Phys. Rev. D* **98**, 104050 (2018).
- [34] C. de Rham and J. Zhang, Perturbations of stealth black holes in degenerate higher-order scalar-tensor theories, *Phys. Rev. D* **100**, 124023 (2019).
- [35] O. Pujolas, I. Sawicki, and A. Vikman, The imperfect fluid behind kinetic gravity braiding, *J. High Energy Phys.* **11** (2011) 156.
- [36] E. Babichev, Emergence of ghosts in Horndeski theory, *J. High Energy Phys.* **07** (2020) 038.
- [37] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), GW170817: Observation of gravitational waves from a binary neutron star inspiral, *Phys. Rev. Lett.* **119**, 161101 (2017).
- [38] C. de Rham and S. Melville, Gravitational rainbows: LIGO and dark energy at its cutoff, *Phys. Rev. Lett.* **121**, 221101 (2018).
- [39] E. Babichev and C. Charmousis, Dressing a black hole with a time-dependent Galileon, *J. High Energy Phys.* **08** (2014) 106.
- [40] E. Babichev, C. Charmousis, and A. Lehébel, Asymptotically flat black holes in Horndeski theory and beyond, *J. Cosmol. Astropart. Phys.* **04** (2017) 027.
- [41] A. Lehébel, Compact astrophysical objects in modified gravity, Ph.D. thesis, Orsay, 2018.

- [42] E. Babichev, C. Charmousis, and M. Hassaine, Charged Galileon black holes, *J. Cosmol. Astropart. Phys.* **05** (2015) 031.
- [43] E. Babichev and G. Esposito-Farèse, Cosmological self-tuning and local solutions in generalized Horndeski theories, *Phys. Rev. D* **95**, 024020 (2017).
- [44] E. Babichev, V. Dokuchaev, and Y. Eroshenko, Backreaction of accreting matter onto a black hole in the Eddington-Finkelstein coordinates, *Classical Quantum Gravity* **29**, 115002 (2012).
- [45] T. Damour and G. Esposito-Farèse, Tensor multiscalar theories of gravitation, *Classical Quantum Gravity* **9**, 2093 (1992).
- [46] E. Babichev and C. Deffayet, An introduction to the Vainshtein mechanism, *Classical Quantum Gravity* **30**, 184001 (2013).
- [47] I. Sawicki, G. Trenkler, and A. Vikman, Causality and stability from acoustic geometry, *arXiv:2412.21169*.
- [48] J. Khoury, M. Trodden, and S. S. C. Wong, Existence and instability of hairy black holes in shift-symmetric Horndeski theories, *J. Cosmol. Astropart. Phys.* **11** (2020) 044.
- [49] K. Takahashi and H. Motohashi, Black hole perturbations in DHOST theories: Master variables, gradient instability, and strong coupling, *J. Cosmol. Astropart. Phys.* **08** (2021) 013.
- [50] J. M. Ezquiaga and M. Zumalacárregui, Gravitational wave lensing beyond general relativity: Birefringence, echoes and shadows, *Phys. Rev. D* **102**, 124048 (2020).
- [51] F. Dar, C. De Rham, J. T. Deskins, J. T. Giblin, and A. J. Tolley, Scalar gravitational radiation from binaries: Vainshtein mechanism in time-dependent systems, *Classical Quantum Gravity* **36**, 025008 (2019).
- [52] R. Abbott *et al.* (LIGO Scientific, Virgo, and KAGRA Collaborations), Tests of general relativity with GWTC-3, *arXiv:2112.06861*.
- [53] L. Blanchet, G. Faye, Q. Henry, F. Larrouturou, and D. Trestini, Gravitational-wave phasing of quasicircular compact binary systems to the fourth-and-a-half post-Newtonian order, *Phys. Rev. Lett.* **131**, 121402 (2023).
- [54] S. E. Perkins, N. Yunes, and E. Berti, Probing fundamental physics with gravitational waves: The next generation, *Phys. Rev. D* **103**, 044024 (2021).
- [55] T. Damour, The problem of motion in Newtonian and Einsteinian gravity, in *300 Years of Gravity: A Conference to Mark the 300th Anniversary of the Publication of Newton's Principia*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1987), p. 128.
- [56] T. Kobayashi and N. Tanahashi, Exact black hole solutions in shift symmetric scalar-tensor theories, *Prog. Theor. Exp. Phys.* **2014**, 073E02 (2014).
- [57] H. Motohashi and M. Minamitsuji, Exact black hole solutions in shift-symmetric quadratic degenerate higher-order scalar-tensor theories, *Phys. Rev. D* **99**, 064040 (2019).
- [58] E. Babichev, C. Charmousis, A. Lehébel, and T. Moskalaets, Black holes in a cubic Galileon universe, *J. Cosmol. Astropart. Phys.* **09** (2016) 011.
- [59] J. Noller, L. Santoni, E. Trinchini, and L. G. Trombetta, Black hole ringdown as a probe for dark energy, *Phys. Rev. D* **101**, 084049 (2020).
- [60] M. Abdul Karim *et al.* (DESI Collaboration), DESI DR2 Results II: Measurements of baryon acoustic oscillations and cosmological constraints, *arXiv:2503.14738*.