

No room for minimal monopole dark matter

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The magnetic monopole of a dark sector has been advocated as an appealing dark matter candidate. We revisit the computation of the monopole abundance Ω_M , generated by a thermal phase transition in the minimal 't Hooft-Polyakov model. We explore the three regimes where the phase transition is second order, weakly first order, or supercooled, identifying the parameter space regions where Ω_M can match the observed dark matter abundance. However, the dark sector necessarily contains a stable electrically charged particle, namely a massive vector boson, with a calculable abundance $\Omega_{W'}$. We show that, under minimal assumptions, $\Omega_{W'}$ is always far larger than Ω_M : dark monopoles cannot constitute a sizeable fraction of dark matter.

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Introduction. A gauge interaction is said to belong to a dark sector if none of the Standard Model (SM) particles are charged under it. Dark-sector gauge symmetries may be broken by the Higgs mechanism, and may thus give rise to magnetic monopoles. These are extended topological defects, consisting of nontrivial gauge-field and Higgs-field configurations. At low energies, they effectively behave as massive classical particles.

Since monopoles are topologically stable, they will contribute to the dark matter (DM) of the Universe. Historically, this was regarded as a problem for grand-unified theories, as the breaking of the unified gauge group to the SM gauge group would abundantly produce super-heavy magnetic monopoles of ordinary electromagnetism, and thus overclose the Universe. If, however, the monopoles are part of a dark sector, the symmetry breaking scale can be much lower than the unification scale. Then the monopole abundance need not lead to overclosure. In fact, being stable massive particles, dark-sector monopoles might well account for all or part of the observed DM.

The production mechanisms for monopole DM are quite different from ordinary particle DM. This makes it an interesting object for study. The possibility that all of the DM consists of monopoles, produced via a thermal phase

transition in the early Universe, has been analyzed in [1–9]. Monopoles produced during the stage of preheating have been studied in [10].

The minimal dark-sector model featuring monopoles with calculable properties has an $SO(3)$ gauge group, broken to $SO(2)$ by a Higgs triplet vacuum expectation value [11], leading to 't Hooft-Polyakov monopoles [12,13]. In this model there exists another stable DM candidate, namely a massive W' gauge boson, which is the lightest state carrying electric charge under the unbroken symmetry. Indeed, in any model featuring a magnetic monopole, the lightest electrically charged state will be stable as well. It is an interesting question whether the DM abundance can be dominated by dark-sector monopoles, rather than by elementary particles with dark electric charge.

In this Letter, we will show that the answer to this question is negative for all of the parameter space of the minimal 't Hooft-Polyakov monopoles, produced during a thermal phase transition. With rather generic assumptions (essentially, we demand that the dark-sector couplings are in the perturbative regime, and the interactions with the SM play a subleading role), we find that the thermal W' abundance always exceeds the monopole abundance by far.

Minimal dark-sector monopoles. Let $G = SO(3)$ be a dark-sector gauge group and ϕ be a real scalar $SO(3)$ triplet. The most general renormalizable potential for ϕ reads as

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}(\phi^2)^2 + \frac{\lambda_{\phi H}}{2}\phi^2|H|^2, \quad (1)$$

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with H the SM Higgs doublet. For $\lambda > 0$ and $\mu^2 > 0$, and neglecting the Higgs portal term for the time being, the potential is minimized at $\langle \phi^2 \rangle = \mu^2/\lambda \equiv \eta^2$, and $\text{SO}(3)$ is broken to $\text{SO}(2)$. There is a massive scalar radial mode ρ with $m_\rho^2 = 2\lambda\eta^2$, and two massive gauge bosons $W'^\pm$ with $m_{W'}^2 = g^2\eta^2$, where g is the dark gauge coupling. A third vector boson γ' , associated to the unbroken $\text{SO}(2)$, remains massless.

The second homotopy group of the vacuum manifold is nontrivial, $\pi_2[\text{SO}(3)/\text{SO}(2)] = \mathbb{Z}$. The model therefore features stable monopole configurations, famously constructed in [12,13]. The monopole M with unit winding number has mass $m_M \approx 4\pi\eta/g$, magnetic charge $q_M = 4\pi/g$, no electric charge (for minimality, we assume a vanishing theta term for the dark gauge fields), and spin zero. The monopole core radius r_M scales as $r_M \approx (g\eta)^{-1}$. For perturbatively small values of g , r_M is much larger than the monopole Compton wavelength $\lambda_M \equiv 1/m_M \leq (g^2/4\pi)(g\eta)^{-1}$; therefore the monopole can be treated as a classical object.

Phase transitions. We will assume that, in the early Universe, the dark sector and the SM are thermalized via the Higgs portal coupling $\lambda_{\phi H}$, and that the reheating temperature is larger than the $\text{SO}(3)$ -symmetry breaking scale η . Thermal effects then correct the zero-temperature potential in Eq. (1) and lead to symmetry restoration. As the Universe cools down, a symmetry-breaking phase transition occurs at some critical temperature T_c . The Universe eventually settles in the symmetry-breaking vacuum, either smoothly in the case of a second-order phase transition (SOPT), or via bubble nucleation in the case of a first-order phase transition (FOPT). In either case, dark monopoles are produced.

We assume that the couplings g and λ are perturbatively small, and that $\lambda_{\phi H}$ is small enough not to significantly affect the dark-sector thermal effective potential. The nature of the phase transition depends on the ratio λ/g^2 , as sketched in Fig. 1.

- (1) For $\lambda \ll g^2$, finite-temperature perturbation theory can be used to establish that the phase transition is of the first order [14]. If, moreover, $\lambda \gg g^4$, then the one-loop thermal effective potential close to T_c ,

$$V_{\text{eff}}(\phi, T) \approx \frac{m^2(T)}{2}\phi^2 - \frac{\delta(T)}{3}|\phi|^3 + \frac{\lambda(T)}{4}(\phi^2)^2, \quad (2)$$

can be computed in a high-temperature expansion [14–17]. The phase transition in this region is found to be *weakly* first order, since bubbles start nucleating at temperatures immediately below T_c .

- (2) For $\lambda < g^4$, perturbation theory can still be used but the high-temperature expansion of the effective potential is unreliable. This includes, e.g., the Coleman-Weinberg scenario, where the second derivative

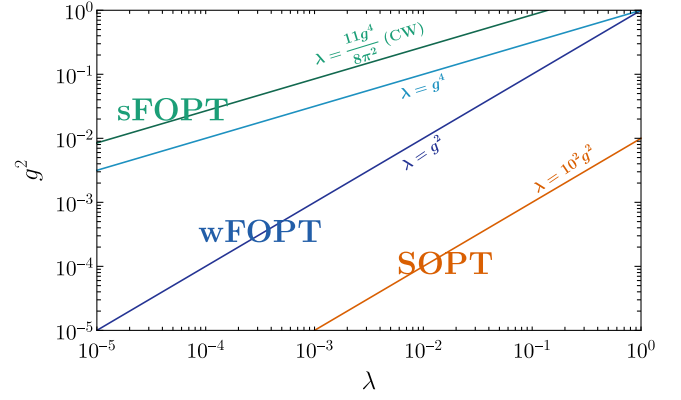


FIG. 1. The nature of the phase transition depends on the choice of the couplings λ and g^2 . Above the dark blue line, finite-temperature perturbation theory holds. Below the light blue line, high-temperature expansion holds. The green line indicates the Coleman-Weinberg scenario. The orange line indicates a benchmark for a SOPT.

of the $T = 0$ one-loop effective potential at $\phi = 0$ is renormalized to zero. This implies that symmetry breaking is radiatively induced, and that $\lambda = 11g^4/8\pi^2$, where λ is the renormalized coupling in the symmetry-breaking vacuum. The phase transition is *strongly* first order, as the tunneling rate to the true vacuum at $T \lesssim T_c$ is still highly suppressed. Bubbles of true vacuum will only form at a much lower nucleation temperature, $T_n \ll T_c$. Such phase transitions are dubbed *supercooled*.

- (3) For $g^2 < \lambda$, perturbation theory cannot be used to establish the nature of the phase transition. Lattice studies of the closely related electroweak (EW) phase transition in the SM [18–20] and of the Abelian Higgs model [21,22] indicate that it turns into a crossover at some critical ratio $(g^2/\lambda)_{\text{crit}} \sim \mathcal{O}(0.1)$ in the nonperturbative region. It is reasonable to expect that the model we are studying will behave similarly. In the limit $g \rightarrow 0$ of a global $\text{SO}(3)$ symmetry, the phase transition is of the second order, since by dimensional reduction [23] the theory can be mapped to the $\text{O}(3)$ model in three Euclidean dimensions [24]. By continuity, the phase transition should also be of the second order “for all practical purposes” for nonzero but sufficiently small g .

As the phase transition takes place, the order parameter changes from $\langle \phi \rangle = 0$ to a nonzero value. While the absolute value of $\langle \phi \rangle$ is unequivocally determined by energy minimization, its orientation in field space is not: the field takes random orientations in the vacuum manifold on scales larger than the field correlation length ξ . Therefore, the Universe is fragmented into domains of typical length ξ , each characterized by a different orientation of $\langle \phi \rangle$. At the intersection of these domains, a monopole may form with a probability p which depends on the topology of the vacuum manifold. Therefore, the

monopole number density can be estimated as $n_M \approx p\xi^{-3}$. In our case, the vacuum manifold is S^2 and $p = 1/8$ [25]. As we review below, the computation of the correlation length ξ strongly depends on the details of the phase transition.

Monopoles from second-order phase transitions. During a SOPT, monopoles are formed by the Kibble-Zurek (KZ) mechanism [25,26]; see, e.g., [27,28] for reviews. As the temperature approaches the critical temperature T_c , both the correlation length and the relaxation time diverge with critical exponents ν and μ respectively. Since our model is in the Heisenberg universality class of the $O(3)$ model in three dimensions, we take $\nu \approx 0.7$ [24], and since its dispersion relation is relativistic, we have $\mu \approx \nu$ [1,27]. Close to T_c , the system can no longer equilibrate due to the divergent relaxation time, and fluctuations freeze at a spatial scale:

$$\xi = H(T_c)^{-1} [H(T_c)\xi_0]^{\frac{1}{1+\nu}} \equiv \xi_{KZ}, \quad (3)$$

where $\xi_0^2 \approx 1/m_\rho^2$ and H is the Hubble parameter, $H^2 = \gamma_* T^4/M_{\text{Pl}}^2$, with $M_{\text{Pl}} \approx 2.4 \times 10^{18}$ GeV, $\gamma_* \equiv \pi^2 g_*/90$, and g_* the number of relativistic degrees of freedom.

We define the comoving monopole number density by $Y_M \equiv n_M/s$, with the entropy density $s = 4\gamma_* T^3$. Using $n_M \approx \xi^{-3}/8$ and $T_c^2 \approx 12\eta^2/5$, we obtain from Eq. (3)

$$Y_M \approx \frac{1}{32} \left(\frac{5}{6} \lambda \right)^{\frac{3}{2+2\nu}} \gamma_*^{\frac{\nu-2}{2+2\nu}} \left(\frac{T_c}{M_{\text{Pl}}} \right)^{\frac{3\nu}{1+\nu}}. \quad (4)$$

This scaling with T_c matches the generic prediction [1] for monopoles from a SOPT, while the prefactor is specific to the 't Hooft-Polyakov model studied here. In Fig. 2, the green line labeled “KZ” shows the monopole abundance obtained from Eq. (4).

The above discussion is valid as long as the monopoles are effectively pointlike. However, in the global limit $g \rightarrow 0$, the monopole mass and radius diverge. For $r_M > \xi_{KZ}$, the KZ estimate of the number density must clearly break down. Analytical studies and numerical simulations have shown that global monopoles enter a scaling regime with an $\mathcal{O}(1)$ number ζ of monopoles per Hubble volume, $n_M = \zeta H^3$ [29–32]. In radiation domination, [31] finds $\zeta = 3.44 \pm 0.56$. This global limit is a good approximation as long as $r_M > 1/H$. To roughly estimate the present-day monopole number density, we assume that the scaling regime ends once $r_M \lesssim 1/H$, and that afterward the monopoles redshift as matter. This leads to a comoving number density,

$$Y_M \approx \frac{\zeta}{4} \gamma_*^{-1/4} (r_M M_{\text{Pl}})^{-3/2}; \quad (5)$$

see the green line labeled “global” in Fig. 2. In the intermediate case $\xi_{KZ} < r_M < H(T_c)^{-1}$, the field configuration after the phase transition still consists of many overlapping monopoles which will efficiently annihilate. Assuming that

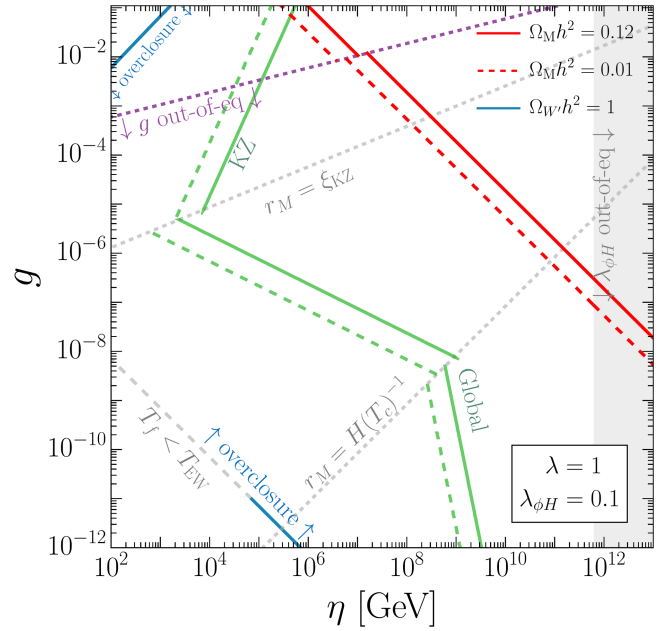


FIG. 2. Relic density of the vectors W' (blue) and of the monopoles M , just after a SOPT (green) and today (red). In the gray region the whole dark sector is not thermalized with the SM; below the purple line, the transverse dark gauge bosons are not thermalized with the rest of the dark sector. The W' overclosure abundance is determined by (non) relativistic freeze-out in the (top left) bottom left corner; we neglected EW symmetry breaking, which would further enhance $\Omega_{W'}$ for $T_f < T_{EW}$. For $r_M < \xi_{KZ}$, the monopoles are produced by the KZ mechanism, for $r_M > 1/H(T_c)$ they are produced as global monopoles, while for the intermediate region we took $\xi \approx r_M$; in all three cases, later monopole annihilations control the final abundance.

this takes place on a short timescale, the effective correlation length is given by $\xi \approx r_M$, and the comoving number density becomes

$$Y_M \approx \frac{1}{4} \gamma_*^{-1} (r_M T_c)^{-3}, \quad (6)$$

which interpolates between the KZ and global limits; see Fig. 2. In all cases the abundance is further reduced by monopole annihilations; see the section on Monopole annihilation.

Monopoles from first-order phase transitions. The hallmark of a FOPT is the presence of two degenerate minima of the thermal effective potential at the critical temperature, separated by a barrier. The scalar field remains trapped in the symmetry-preserving false vacuum as the Universe cools down. At the nucleation temperature, T_n , quantum tunneling or thermal fluctuations are efficient enough for the phase transition to proceed, via nucleation of bubbles of true vacuum in a background of metastable phase.

These bubbles expand and eventually percolate. The correlation length ξ at the end of the phase transition is

given by the average bubble radius at percolation R_p , such that the comoving monopole number density is

$$Y_M \approx \frac{1}{32} (\gamma_*^{\text{reh}})^{-1} (R_p T_{\text{reh}})^{-3}. \quad (7)$$

Here T_{reh} is the temperature to which the Universe is reheated by bubble collisions. The value of R_p strongly depends on the phase transition parameters, which control bubble nucleation and expansion dynamics. In the Supplemental Material [33] we review these subjects in more detail and show how to compute R_p in the various regimes of interest; below we summarize the main results.

A phase transition is weakly first-order (wFOPT) if it occurs when the Universe is still radiation dominated, so that $T_{\text{reh}} \approx T_p$. In this case the mean bubble radius at percolation is given by

$$R_p^{(w)} = \left(\frac{8\pi v_b^3}{0.29} \right)^{1/3} \beta^{-1}, \quad (8)$$

where $v_b \ll 1$ is the terminal bubble wall velocity. The rate of completion of the phase transition β is defined in terms of the bubble nucleation rate per unit volume, $\Gamma(T) \approx T^4 e^{-S_3/T}$, where S_3 is the Euclidean action of the bubble, according to

$$\beta \equiv \left. \frac{d \log \Gamma}{dt} \right|_{T_p} \simeq H(T_p) T_p \frac{d}{dT} \left(\frac{S_3}{T} \right) \Big|_{T_p}. \quad (9)$$

By numerically evaluating S_3 and T_p , one can use Eq. (7) to obtain the monopole abundance, illustrated by the red lines in Fig. 3 for two different values of v_b .

A phase transition is strongly first-order (sFOPT) if the Universe becomes vacuum dominated before bubbles start nucleating, so that there is substantial supercooling. If supercooling ends by the potential barrier disappearing at some temperature $T > 0$, the phase transition can be a fast sFOPT. An example is given by a deformed Coleman-Weinberg scenario with a small negative $V''_{\text{eff}}(0) \equiv -m_0^2$ at zero temperature. In this case, the critical bubble radius R_c of bubbles at nucleation may be sizeable and should be added to the bubble radius gained from expansion, in order to estimate the final radius at percolation, $R_p = R_c + R_{\text{exp}}$. Here R_{exp} is still given by the expression in Eq. (8), now with $v_b \approx 1$. In the thick-wall approximation, in terms of the effective potential parameters of Eq. (2), one has

$$R_c^2 = \frac{1}{m^2(T_n)} \frac{3\kappa(T_n)}{3\kappa(T_n) - 1 + \sqrt{1 - 4\kappa(T_n)}}, \quad (10)$$

with $\kappa(T_n) \equiv \lambda(T_n) m^2(T_n) / \delta^2(T_n)$. The red lines in Fig. 4 show the resulting monopole abundance created by a sFOPT, for two different values of m_0 . The radius at percolation is dominated by R_{exp} for small values of η (rising red curves) and by R_c for large values (falling red curves).

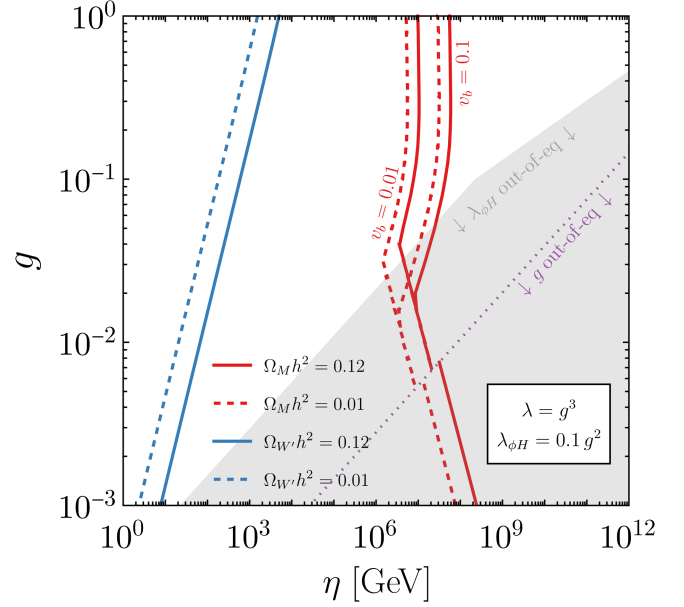


FIG. 3. The monopole abundance (red) after a wFOPT, taking into account later annihilations which become relevant for $g \lesssim 2 \times 10^{-2}$. The corresponding W' abundance (blue) is determined by standard freeze-out, driven by annihilations into dark photons. Gray region and purple line are defined as in Fig. 2.

Monopole annihilation. The monopole number density produced by the phase transition may be reduced by monopole-antimonopole annihilation [44,45] (see [46] for a review). Monopoles can dissipate their energy by moving through a plasma of relativistic charged particles, in our case the W' gauge bosons. This allows the formation of monopole-antimonopole bound states, which eventually annihilate. The diffusive capture process takes effect as long as the mean free path of the monopole in the plasma is smaller than the capture radius. It stops once the monopole abundance is sufficiently diluted, or when the W' bosons become non-relativistic, whichever happens first. It may also happen that the monopole density at production was never large enough to allow for efficient annihilation in the first place. When annihilation is relevant, the final yield is [45]

$$Y_M = \frac{\mathcal{B} g^2}{16\pi} \gamma_*^{-1/2} \frac{T_a}{M_{\text{Pl}}}, \quad (11)$$

with T_a the temperature where annihilation ceases to be effective, and $\mathcal{B} = g_{W'} \zeta(3) / \pi^2$ for a plasma of W' bosons.

Vector boson abundance. The W' vector bosons, as the lightest (and only) particles carrying dark electric charge, are also stable DM candidates. The abundance of transversely polarized W' bosons and dark photons γ' may be parametrically small in the limit of small g . The relevant process is $W' - \phi$ scattering before the dark-sector phase transition, with

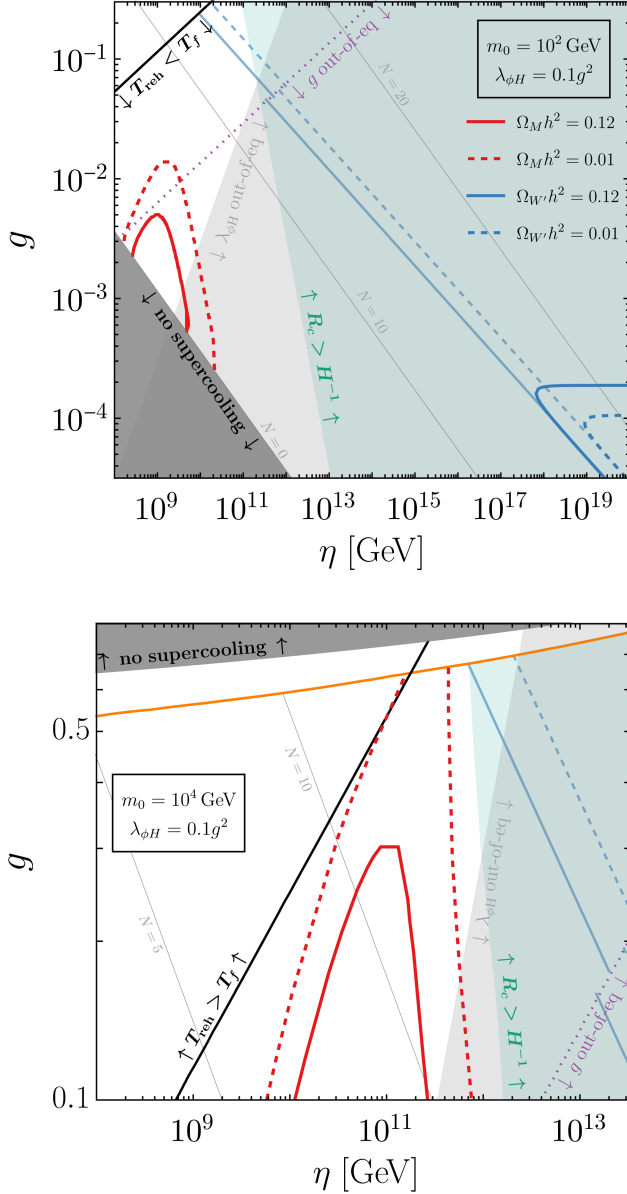


FIG. 4. Relic abundance of monopoles (red) and dark gauge bosons (thick blue), for a supercooled sFOPT, followed by instantaneous reheating. Thin blue lines show the W' abundance at the end of supercooling, neglecting the subthermal population generated after reheating. Gray lines show isocontours of the number N of e -folds of thermal inflation, which dilute the W' s. Above the orange line, nucleation occurs independently of m_0 at much larger temperatures so that N sharply decreases. Above the black line, the Universe is reheated to a temperature large enough to restore W' thermal equilibrium. Inside the dark gray region the phase transition completes before the Universe enters vacuum domination. The green shading indicates the region where the effect of gravity on the tunneling rate should be taken into account.

$$\langle\sigma v\rangle_{W'W'\rightarrow\phi\phi} = \frac{41g^4}{4608\pi T^2}. \quad (12)$$

The transverse polarizations of the three dark gauge bosons are not thermalized at the phase transition if $n_{W'}^{\text{eq}}\langle\sigma v\rangle < H$,

which requires $T_c \gtrsim g^4(1.2 \times 10^{15} \text{ GeV})$. However, by assumption ϕ is thermalized with the SM via the Higgs portal, and therefore there is always a thermal population of dark Higgs bosons and of longitudinally polarized W' s, which are Nambu-Goldstone bosons in the global limit. From the thermally averaged cross section for (massless) ϕ —Higgs boson scattering,

$$\langle\sigma v\rangle_{\phi\phi\rightarrow HH} = \frac{\lambda_{\phi H}^2}{128\pi T^2}, \quad (13)$$

one deduces that the two sectors are thermalized at the time of the phase transition, $n_{\phi}^{\text{eq}}\langle\sigma v\rangle > H$, provided that $T_c \lesssim \lambda_{\phi H}^2(10^{14} \text{ GeV})$.

In the case of a SOPT or a wFOPT, the present-day W' abundance is determined by ordinary freeze-out, with different possible annihilation channels. Detailed expressions are provided in the Supplemental Material [33]. The resulting W' abundance is shown by the blue curves in Figs. 2 and 3.

In contrast, in the case of a sFOPT with supercooling, the thermal population of massless W' s is diluted, before the phase transition, by a phase of thermal inflation. After the transition however, if the reheating temperature T_{reh} is much larger (not much smaller) than the W' freeze-out temperature T_f , then the W' will (partially) thermalize again; see Supplemental Material [33] for explicit expressions. The resulting W' abundance is shown in Fig. 4 by thick (thin) blue lines, assuming instantaneous reheating (suppressing the reheating contribution).

Conclusions and perspectives. We charted the magnetic monopole relic abundance across the parameter space of a minimal 't Hooft-Polyakov dark sector, which is fully characterized by three parameters: the symmetry-breaking scale η , the (electric) gauge coupling g , and the scalar self-coupling λ (or, equivalently, a scalar mass parameter m_0). Our analysis shows that $\Omega_M \ll \Omega_{W'}$ for all possible (perturbative) ranges for g , λ and η . In particular, $\Omega_{W'}$ can be parametrically small in the limit $m_{W'} \rightarrow 0$; however in this limit Ω_M is even more suppressed (see Fig. 2). In addition, while a period of thermal inflation during a supercooled FOPT may significantly reduce the W' relic density, W' s may then be recreated after reheating. Even if their resulting abundance is subthermal, they still overclose the Universe by far in the parameter space region where $\Omega_M \sim 0.1$; see Fig. 4. To conclude, the DM relic density is always dominated by the lightest electrically charged particle, and never by the lightest magnetically charged state.

Comparing with the previous literature, we agree with the prediction of Ω_M for a SOPT found by [1]. However, this work did not study monopoles embedded in a complete, realistic model, such as the 't Hooft-Polyakov model including W' bosons. We agree with [3] that, in the specific region with $\Omega_{W'} \sim 0.1$ and a SOPT, Ω_M is negligible.

Finally, we disagree with [4] that monopoles can account for an $\mathcal{O}(1)$ fraction of the observed DM relic density.¹

The negative conclusion for monopole DM may change if one relaxes some of our assumptions:

- (1) There are potentially large corrections when the dark-sector couplings become nonperturbative, or when gravity becomes significant for the phase-transition tunneling process.
- (2) If the Higgs portal $\lambda_{\phi H}$ (or any alternative interaction between the dark sector and the SM) is too small to maintain thermalization, the dark-sector temperature T' can be smaller than T and the ratio $\Omega_M/\Omega_{W'}$ may vary with T'/T . On the other hand, if $\lambda_{\phi H}$ is comparable to or larger than both λ and g^2 , then the dark phase transition can be significantly deformed, and interfere with the electroweak phase transition. In particular, the role of m_0^2 in triggering a sFOPT could

be taken by $\lambda_{\phi H} v^2$ with v the Higgs vacuum expectation value.

- (3) The dark sector could be extended: (i) light charged particles could allow W' to decay; (ii) charged scalars other than ϕ could determine different spontaneous symmetry-breaking patterns, with other kinds of monopoles and additional topological defects; (iii) $\text{SO}(3)$ can be replaced by a larger gauge group.

In a separate paper [47], we will show that some of these extensions allow for monopoles to naturally dominate the DM abundance, with a specific associated phenomenology.

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Data availability. The data that support the findings of this article are openly available [48].

¹Most significantly, we find that the monopole abundance after annihilation resulting from Eq. (4) scales differently with g than Eq. (3.39) of the arXiv version 3 of [4].

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