

Revisiting the Higgs-mass calculation in the scale-invariant THDM

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We revisit the one-loop calculation of the Higgs-mass spectrum of the scale-invariant two-Higgs-doublet model, relying on a direct calculation of the relevant Feynman diagrams. We highlight a number of incorrect assumptions in earlier calculations that relied on the effective-potential approach. In contrast with the earlier findings, we show that the one-loop corrections can have an effect of $\mathcal{O}(10\%)$ on the predictions for the beyond the standard model (BSM) Higgs masses, and they can also induce non-negligible mixing between the standard model-like and BSM states in the neutral scalar sector.

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I. INTRODUCTION

The discovery of a Higgs boson with mass around 125 GeV [1,2] and properties compatible with the predictions of the standard model (SM) [3] goes a long way toward elucidating the mechanism of electroweak (EW) symmetry breaking, but does not by itself preclude the existence of additional, beyond the SM (BSM) Higgs bosons with masses around or even below the TeV scale, which could still be discovered in the current or future runs of the Large Hadron Collider (LHC).

The two-Higgs-doublet model (THDM) is one of the simplest and best-studied extensions of the SM (for reviews see, e.g., Refs. [4–6]). In the CP -conserving versions of the model, the Higgs sector includes five physical states: two CP -even scalars, h and H , one CP -odd scalar, A , and two charged scalars, $H^\pm$. In general realizations of the THDM, all of the scalar masses can be treated as free parameters, and the couplings of both h and H to fermions and gauge bosons deviate from the SM predictions. As discussed, e.g., in Ref. [7], the so-called “alignment” condition—in which one of the CP -even scalars has SM-like couplings to fermions and gauge bosons—can be realized through decoupling, when all of the other Higgs bosons are much heavier, or without decoupling, when a specific configuration of parameters in the Lagrangian suppresses the mixing between the SM-like scalar and the other CP -even scalar.

A version of the THDM that features (approximate) alignment without decoupling is the so-called “scale-invariant THDM” (SI-THDM) introduced in Ref. [8]. In this model there are no explicit mass parameters in the scalar potential, rendering the classical action of the theory scale invariant. However, quantum corrections to the scalar potential break both the scale invariance and the EW symmetry, inducing a vacuum expectation value (vev) v for the SM-like Higgs field, and thus masses proportional to combinations of couplings times v for all of the particles in the model. This mechanism of radiative symmetry breaking was first discussed by Coleman and Weinberg (CW) in Ref. [9], and later extended to models with additional scalar fields by Gildener and Weinberg (GW) in Ref. [10]. Owing to the reduced number of free parameters with respect to the usual THDM—namely, the three mass parameters for the Higgs doublets are set to zero—the Higgs sector of the SI-THDM is particularly constrained. Even after v is generated radiatively, one of the CP -even scalars, which we identify with h , remains massless at the tree level,¹ has SM-like couplings to fermions and gauge bosons, and does not mix with the other CP -even scalar. A mass for this SM-like Higgs boson, M_h , is generated at one loop, and a sum rule connects it with all of the other particle masses. Thus, if $M_h \approx 125$ GeV is treated as an input, the sum rule constrains one of the masses of the BSM Higgs bosons.

In view of the crucial role played by radiative corrections in the SI-THDM, several studies have been devoted over the years to the precise determination of its Higgs-mass spectrum. After the original one-loop calculation of Ref. [8], the one-loop corrections to the mass matrix for the CP -even scalars were revisited by different groups in

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¹With a common abuse of language, we refer to a “tree-level” mass spectrum even if v itself is induced at one loop.

Refs. [11–13]. Furthermore, the dominant two-loop corrections were computed: first along the h direction in Ref. [14], and later for the full mass matrix in Ref. [15]. All of these calculations relied on the effective-potential approach, which neglects the external-momentum effects, and the two-loop calculations also neglected the effects controlled by the EW gauge couplings. However, the one-loop calculations presented in Refs. [8,11–13] appear to be incorrect. First of all, they find that there are no one-loop corrections to the masses of A and $H^\pm$. As a result, the predictions for those masses, which do not vanish at the tree level, would be both scale and gauge dependent at the one-loop order. Furthermore, those calculations find that the only one-loop contributions to the mixing between h and H arise from diagrams involving the top quark, despite the presence in the Lagrangian of both $h\Phi\Phi$ and $H\Phi\Phi$ couplings (where $\Phi = H, A, H^\pm$ denotes the BSM Higgs bosons). A closer inspection of the effective-potential calculations of Refs. [8,11–13] reveals that both of these problems stem from an incorrect assumption for the field-dependent masses of BSM-Higgs and gauge bosons, which does not properly account for their dependence on the fields other than h . We remark that this problem also affects the two-loop calculation of Ref. [15], but it does not affect the two-loop calculation of Ref. [14], which was performed only along the h direction.²

Due to the presence in the Higgs sector of the SI-THDM of four neutral and four charged scalar fields that interact with each other—counting also the would-be-Goldstone bosons, G^0 and $G^\pm$ —the correct determination of the field-dependent masses required in the effective-potential calculation is not trivial (see Ref. [18] for a formal approach to this issue). In this paper, we bypass all complications related to the field-dependent masses by computing directly all of the Feynman diagrams that are relevant to the one-loop calculation of the Higgs-mass spectrum of the SI-THDM. Besides correcting the errors in the literature, this approach allows us to account for the external-momentum effects in the self-energies of the BSM Higgs bosons, eventually resulting in predictions for the masses that are both scale and gauge independent up to the perturbative order considered in our calculation.

The rest of the article is organized as follows: in Sec. II we first introduce the Higgs sector of the SI-THDM at the tree level, and then describe our calculation of the one-loop

corrections to the Higgs masses. In Sec. III we illustrate the numerical impact of the corrections to the Higgs masses in two representative scenarios, and Sec. IV contains our conclusions. Finally, Appendix A contains explicit formulas for all of the one-loop quantities relevant to the Higgs-mass calculation, and in Appendix B we discuss the role of the so-called Gildener-Weinberg scale introduced in Ref. [10].

II. THE HIGGS SECTOR OF THE SI-THDM

We start this section by describing the “tree-level” mass spectrum of the SI-THDM in the presence of a Higgs vev v generated radiatively through the CW/GW mechanism. We then describe the full one-loop calculation of the masses of all of the physical scalars in the model.

A. The scalar potential and the Higgs-mass spectrum at the tree level

We consider a version of the THDM where flavor-changing neutral-current interactions are forbidden at the tree level by a Z_2 symmetry. In the basis where this Z_2 symmetry applies, the scalar potential can be parametrized as

$$V_0 = \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \frac{\lambda_5}{2} [(\Phi_1^\dagger \Phi_2)^2 + \text{H.c.}], \quad (1)$$

where all of the quartic couplings are assumed to be real to ensure CP conservation. We do not need here to distinguish between different THDM “types” according to the form of their Higgs-fermion interactions, because in our calculation of the one-loop corrections to the Higgs masses we neglect all Yukawa couplings except the one of the top quark, which we assume to couple only to Φ_2 . We decompose the two $SU(2)$ doublets as

$$\Phi_k = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi_k^+ \\ v_k + \phi_k^0 + ia_k \end{pmatrix} \quad (k = 1, 2), \quad (2)$$

where the two (real) vevs are related by $v_1^2 + v_2^2 = v^2$, with $v \approx 246$ GeV, and we define $\tan\beta \equiv v_2/v_1$. In the absence of explicit mass parameters, the minimum conditions for the scalar potential when $v \neq 0$ reduce to relations between the quartic couplings and $\tan\beta$,

$$\lambda_1 = -\lambda_{345} \tan^2\beta, \quad \lambda_2 = -\lambda_{345} \cot^2\beta, \quad (3)$$

where we define $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$ and, for later convenience, $\lambda_{45} \equiv \lambda_4 + \lambda_5$. In this constrained version of the THDM, the mass matrices for the scalar, pseudoscalar, and charged components of the Higgs doublets in Eq. (2) are all diagonalized by the same rotation $R(\beta)$, defined as

²For completeness, we mention here two additional recent studies of the SI-THDM, Refs. [16,17], which also feature one-loop calculations of the Higgs masses in the effective-potential approach. As will be discussed later, the treatment of the minimum conditions for the scalar potential in Ref. [16] imposes an arbitrary restriction on the parameter space of the model. Reference [17] has even broader issues, stemming in part from an incorrect identification of the SM-like state at the tree level. Neither of these studies specifies the definitions used for the field-dependent masses.

$$R(\beta) \equiv \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \quad (4)$$

where we introduce the shortcuts $c_\theta \equiv \cos \theta$ and $s_\theta \equiv \sin \theta$ for a generic angle θ . It is thus convenient to rotate the whole Higgs doublets from the Z_2 -symmetric basis to the so-called Higgs basis,

$$\begin{pmatrix} \Phi_{\text{SM}} \\ \Phi_{\text{BSM}} \end{pmatrix} = R(\beta) \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \quad (5)$$

in which one of the doublets develops the full SM-like vev v and the other has vanishing vev,³

$$\Phi_{\text{SM}} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix}, \quad \Phi_{\text{BSM}} = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}. \quad (6)$$

Using the minimum conditions in Eq. (3), we get the tree-level masses

$$\begin{aligned} m_h^2 = m_{G^0}^2 = m_{G^\pm}^2 = 0, \quad m_H^2 = -\lambda_{345} v^2, \\ m_A^2 = -\lambda_5 v^2, \quad m_{H^\pm}^2 = -\frac{\lambda_{45}}{2} v^2. \end{aligned} \quad (7)$$

In practice, h is the massless state—called “scalón” in Ref. [10]—associated with the flat direction in field space along which the vev v is generated by the quantum corrections to the scalar potential. Being aligned in field space with the vev, h has SM-like couplings to gauge bosons and to fermions. In contrast, the three BSM Higgs bosons acquire squared masses proportional to v^2 times combinations of the three quartic couplings λ_3 , λ_4 , and λ_5 . Their trilinear couplings to pairs of gauge bosons vanish, and their couplings to the top quark are all proportional to $\cot \beta$.

While the tree-level alignment of the Higgs sector of the SI-THDM is a welcome feature, the masslessness of the SM-like Higgs boson is obviously unrealistic. Luckily, as we will see in the next section, a suitable mass for h can be generated by the one-loop corrections, albeit at the price of some misalignment from the SM-like direction.

B. One-loop corrections to the Higgs-boson masses

Beyond the tree level, the minimum conditions for the effective potential become

³Note that in this paper the convention for H differs from the one in Refs. [19,20]: here we define $H = \phi_{\text{BSM}}^0$, whereas in our earlier papers on the aligned THDM we had $H \rightarrow -\phi_{\text{BSM}}^0$ in the alignment limit.

$$\lambda_1 = -\lambda_{345} \tan^2 \beta - \frac{2}{v^3 c_\beta^2} (T_h - T_H \tan \beta),$$

$$\lambda_2 = -\lambda_{345} \cot^2 \beta - \frac{2}{v^3 s_\beta^2} (T_h + T_H \cot \beta), \quad (8)$$

where $\tan \beta$ and the five couplings λ_i are now interpreted as $\overline{\text{MS}}$ -renormalized parameters at some scale Q . The quantities T_φ denote the finite parts of the one-loop tadpole diagrams⁴ for the fields $\varphi = (h, H)$. By making use of the minimum conditions in Eq. (8), we can write the loop-corrected mass matrices for the scalar (h, H), pseudoscalar (G^0, A), and charged ($G^\pm, H^\pm$) sectors as

$$\begin{aligned} \mathcal{M}_S^2(p^2) = & \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_{345} v^2 \end{pmatrix} + \begin{pmatrix} \Pi_{hh}(p^2) & \Pi_{hH}(p^2) \\ \Pi_{hH}(p^2) & \Pi_{HH}(p^2) \end{pmatrix} \\ & - \frac{3}{v} \begin{pmatrix} T_h & T_H \\ T_H & \tilde{T} \end{pmatrix}, \end{aligned} \quad (9)$$

$$\begin{aligned} \mathcal{M}_P^2(p^2) = & \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_5 v^2 \end{pmatrix} + \begin{pmatrix} \Pi_{G^0 G^0}(p^2) & \Pi_{G^0 A}(p^2) \\ \Pi_{G^0 A}(p^2) & \Pi_{AA}(p^2) \end{pmatrix} \\ & - \frac{1}{v} \begin{pmatrix} T_h & T_H \\ T_H & \tilde{T} \end{pmatrix}, \end{aligned} \quad (10)$$

$$\begin{aligned} \mathcal{M}_C^2(p^2) = & \begin{pmatrix} 0 & 0 \\ 0 & -\frac{\lambda_{45}}{2} v^2 \end{pmatrix} + \begin{pmatrix} \Pi_{G^\pm G^\mp}(p^2) & \Pi_{G^\pm H^\mp}(p^2) \\ \Pi_{G^\pm H^\mp}(p^2) & \Pi_{H^\pm H^\mp}(p^2) \end{pmatrix} \\ & - \frac{1}{v} \begin{pmatrix} T_h & T_H \\ T_H & \tilde{T} \end{pmatrix}, \end{aligned} \quad (11)$$

where $\Pi_{\varphi\varphi'}(p^2)$ are the finite parts of the one-loop self-energy matrices for the fields (φ, φ') , p^2 is the external momentum, and we defined $\tilde{T} = T_h + 2 \cot 2\beta T_H$. Again, the squared vev v^2 and the couplings λ_i entering the tree-level parts of the mass matrices are now interpreted as $\overline{\text{MS}}$ -renormalized parameters at some scale Q . We remark that the identities $\Pi_{G^0 G^0}(0) = \Pi_{G^\pm G^\mp}(0) = T_h/v$ and $\Pi_{G^0 A}(0) = \Pi_{G^\pm H^\mp}(0) = T_H/v$ ensure that the corrections to the masses and mixing of the would-be-Goldstone bosons vanish at $p^2 = 0$. For what concerns the physical states, we note that the mixing terms in the mass matrices of Eqs. (9)–(11) are purely of one-loop order, and thus affect the mass eigenvalues only starting from the two-loop order. Therefore, in a strict one-loop calculation, the “pole” mass for a given physical state can be identified with the real part of the corresponding diagonal element of the mass matrix, evaluated at an external momentum equal to the tree-level mass of the particle itself. For convenience, we also choose to replace the $\overline{\text{MS}}$ -renormalized Higgs vev v , which is both

⁴Decomposing the one-loop effective potential as $V_0 + \Delta V$, we also have $T_\varphi = d\Delta V/d\varphi|_{\min}$.

scale and gauge dependent, with the Fermi constant G_F , the two being related at the tree level by $v = (\sqrt{2}G_F)^{-1/2}$. We can thus write⁵

$$M_h^2 = \Pi_{hh}(0) - 3 \frac{T_h}{v}, \quad (12)$$

$$M_H^2 = -\frac{\lambda_{345}}{\sqrt{2}G_F} - m_H^2 \frac{\delta v^2}{v^2} + \text{Re} \Pi_{HH}(m_H^2) - 3 \frac{\tilde{T}}{v}, \quad (13)$$

$$M_A^2 = -\frac{\lambda_5}{\sqrt{2}G_F} - m_A^2 \frac{\delta v^2}{v^2} + \text{Re} \Pi_{AA}(m_A^2) - \frac{\tilde{T}}{v}, \quad (14)$$

$$M_{H^\pm}^2 = -\frac{\lambda_{45}}{2\sqrt{2}G_F} - m_{H^\pm}^2 \frac{\delta v^2}{v^2} + \text{Re} \Pi_{H^\pm H^\mp}(m_{H^\pm}^2) - \frac{\tilde{T}}{v}, \quad (15)$$

where we defined $(\sqrt{2}G_F)^{-1} = v^2 + \delta v^2$. We computed all of the one-loop tadpoles and self-energies entering Eqs. (9)–(15) in a generic R_ξ gauge with the help of FeynArts [21]. We also compared our results for the one-loop self-energies with those obtained by adapting the general results of Ref. [22] to the SI-THDM, and found agreement except for terms proportional to ξ^2 in $\Pi_{G^0 G^0}(p^2)$ and $\Pi_{G^\pm G^\mp}(p^2)$ (those are related to an issue in Ref. [22], already noticed in Ref. [23], concerning the couplings of the would-be-Goldstone bosons to ghost pairs). For δv^2 we combined the SM contributions from Refs. [24–26] with the BSM Higgs contribution to the self-energy of the W boson. As a nontrivial check of the correctness of our results, we find that all of the pole-mass predictions in Eqs. (12)–(15) are both gauge and scale independent at the one-loop order, once we take into account the scale dependence of the $\overline{\text{MS}}$ -renormalized quartic couplings entering the tree-level parts of the BSM-Higgs masses. Explicit formulas for all of the one-loop quantities relevant to the Higgs-mass calculation in the SI-THDM are provided in Appendix A.

Combining the different terms in Eqs. (14) and (15) we find that the one-loop corrections to the masses of A and $H^\pm$ do not vanish, contrary to what was found in Refs. [8,11,12]. The finding in Refs. [13,15] that one-loop contributions to M_H^2 arise only from diagrams involving the top quark is also incorrect. All of this remains the case even if we neglect the external-momentum effects and the connection of v with G_F , as was done in those papers. As mentioned in Sec. I, these discrepancies appear to stem from an unjustified assumption in Refs. [8,11–13,15] on the structure of the field-dependent masses of the BSM-Higgs and gauge bosons, which are all taken to be

proportional to $(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)$ or, equivalently, to $(\Phi_{\text{SM}}^\dagger \Phi_{\text{SM}} + \Phi_{\text{BSM}}^\dagger \Phi_{\text{BSM}})$. It is easy to see how, for example, this assumption would incorrectly imply that the BSM Higgs bosons do not contribute at all to T_H and $\Pi_{HH}(0)$, because it misses terms proportional to hH in their field-dependent masses.

For the mass of the SM-like Higgs boson, see Eq. (12), our result agrees with the earlier papers,

$$M_h^2 = \frac{1}{8\pi^2 v^2} (m_H^4 + m_A^4 + 2m_{H^\pm}^4 + 6m_W^4 + 3m_Z^4 - 4N_c m_t^4), \quad (16)$$

where $N_c = 3$ is a color factor. However, for the mixing term in the mass matrix for the scalar sector at vanishing external momentum we find

$$[\mathcal{M}_S^2(0)]_{12} = \frac{1}{8\pi^2 v^2} [m_H^2 (3m_H^2 + m_A^2 + 2m_{H^\pm}^2) \cot 2\beta - 4N_c m_t^4 \cot \beta], \quad (17)$$

to be contrasted with the results given in Refs. [13,15], where $[\mathcal{M}_S^2(0)]_{12}$ was found to be scale dependent and fully proportional to m_t^4 . As mentioned in Sec. I, in phenomenological analyses of the SI-THDM it is customary to treat $M_h \approx 125$ GeV as an input, and use the sum rule in Eq. (16) to constrain one of the masses of the BSM Higgs bosons. We note that, in principle, a further constraint on the BSM-Higgs masses could be imposed by requiring that $[\mathcal{M}_S^2(0)]_{12} = 0$, in analogy with the one-loop alignment condition discussed in Refs. [19,20]. However, Eq. (17) shows that a cancellation between the BSM-Higgs and top contributions to the mixing term is possible only for $\tan \beta < 1$.

While the effects of mixing on the scalar masses are formally of two-loop order and higher, thus beyond the accuracy of the present calculation, an assessment of their magnitude may still prove useful (e.g., to check whether their omission in the two-loop calculation of Ref. [14] was justified). To this purpose, we compute the eigenvalues $M_{h'}^2$ and $M_{H'}^2$ of the full scalar mass matrix in Eq. (9). More precisely, to properly account for the momentum dependence of the calculation and for the possibility of crossing between states, we define $M_{h'}$ as the mass of the most SM-like eigenstate of the mass matrix computed at $p^2 = 0$, and $M_{H'}$ as the mass of the least SM-like eigenstate of the mass matrix computed at $p^2 = m_H^2$. Note that the degree of “SM likeness” of each eigenstate is determined by the rotation angle that leads from the basis (h, H) to the basis (h', H') . We remark that $M_{h'}$ and $M_{H'}$ are gauge-dependent quantities, because both $[\mathcal{M}_S^2(m_H^2)]_{11}$ and $[\mathcal{M}_S^2(0)]_{22}$ are gauge dependent. This is just a reflection of the fact that $M_{h'}$ and $M_{H'}$ contain incomplete sets of two-loop effects.

⁵Here and thereafter, we use M_φ^2 to denote the squared pole mass of a scalar φ . We instead use m_φ^2 for the tree-level parts of Eqs. (13)–(15), as well as when the precise definition of the mass amounts to a higher-order effect.

To conclude this section, we comment on the choice of the renormalization scale Q entering the one-loop quantities in Eqs. (12)–(15). In earlier studies of the SI-THDM it was customary to use the so-called Gildener-Weinberg scale Q_{GW} , defined in Ref. [10] as the scale at which the tree-level potential has a flat direction along h and the one-loop SM-like tadpole T_h vanishes. As we discuss in more detail in Appendix B, this particular choice of scale is by no means compulsory, after all, the scale dependence of the individual terms in Eqs. (12)–(15) cancels out in their sum. We also note that Refs. [13,15] assume that, at the scale Q_{GW} , the minimum conditions for the scalar potential given in Eq. (3) remain valid even beyond the tree level. As is clear from Eq. (8), this assumption is not correct, because T_H generally does not vanish at the scale at which T_h vanishes. Similarly, the condition $T_H = 0$ imposed in Ref. [16] is an arbitrary constraint on the parameter space of the model.

III. NUMERICAL IMPACT OF THE CORRECTIONS TO THE HIGGS BOSON MASSES

To illustrate the numerical impact of the corrections to the Higgs-boson masses in the SI-THDM, we start by adopting a benchmark scenario similar to the one considered in Fig. 1 of Ref. [15]. Once we take into account the minimum conditions for the scalar potential, the free parameters of the model can be chosen as three quartic couplings of the Z_2 -symmetric basis, namely, λ_3 , λ_4 , and λ_5 , plus the ratio of vevs $\tan\beta$, all interpreted as $\overline{\text{MS}}$ -renormalized parameters at some scale Q . We treat the one-loop-induced mass of the SM-like Higgs boson as an input, $M_h = 125$ GeV, and we use Eq. (16) to constrain λ_3 as a function of λ_4 and λ_5 , identifying m_H^2 , m_A^2 , and $m_{H^\pm}^2$ with the tree-level parts of the expressions for the respective pole masses in Eqs. (13)–(15). We also assume the constraint $\lambda_4 = \lambda_5$, in which case $m_A = m_{H^\pm}$, see Eq. (7), and the one-loop BSM Higgs contributions to the ρ parameter vanish [27], limiting possible tensions with precision EW measurements. Finally, we fix $\tan\beta = 2$, so that the BSM Higgs contributions to the hH mixing term in Eq. (17) add up with the top-quark contribution.⁶ As a result of these choices, in our benchmark scenario the Higgs-mass spectrum of the SI-THDM depends on a single quartic coupling of the Z_2 -symmetric basis.

In Fig. 1 we plot the predictions for the neutral scalar masses in our benchmark scenario as a function of $-\lambda_4$ (or, equivalently, $-\lambda_5$) at different levels of accuracy. The quartic couplings on the x -axis are interpreted as $\overline{\text{MS}}$ -renormalized parameters at the scale Q_{GW} , for which an explicit expression is given in Appendix B. In the range

⁶This is equivalent to the choice $\tan\beta = 0.5$ in Ref. [15], because in that paper the top quark couples only to Φ_1 .

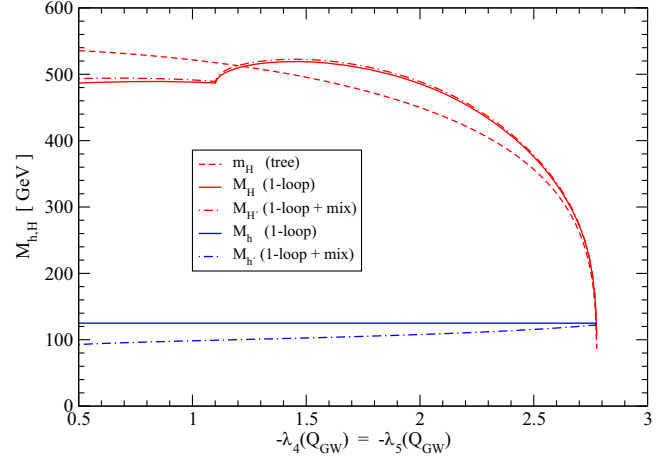


FIG. 1. Neutral-scalar masses as a function of the quartic couplings, in a scenario where $\lambda_4 = \lambda_5$, λ_3 is fixed by the value of M_h , and $\tan\beta = 2$. The meaning of the lines is explained in the text.

covered by the plot, the values of the quartic couplings correspond to tree-level masses m_A and $m_{H^\pm}$ that increase between approximately 174 and 410 GeV. The red dashed line in Fig. 1 represents the tree-level value of the BSM Higgs mass m_H , as extracted from Eq. (16) for a given value of m_A and $m_{H^\pm}$: note that it drops to zero when the sum rule for M_h is saturated by the pseudoscalar and charged-scalar masses alone. The red and blue solid lines correspond to the full one-loop predictions for M_H and M_h , respectively, as given in Eqs. (13) and (12). While the one-loop prediction for the SM-like Higgs mass is fixed by construction to $M_h = 125$ GeV, we see that the one-loop prediction for the mass of the BSM Higgs scalar can differ by almost 50 GeV from the tree-level mass, i.e., the one-loop corrections are of $\mathcal{O}(10\%)$. This is to be contrasted with the analogous plot in Fig. 1 of Ref. [15], where the tree-level and one-loop predictions for the H mass are essentially overlapping. We also note, in our Fig. 1, a kink in the prediction for M_H around $\lambda_4, \lambda_5 \approx -1.1$. This stems from the thresholds in $\text{Re } \Pi_{HH}(m_H^2)$ for $m_H = 2m_A$ and for $m_H = 2m_{H^\pm}$, and would obviously not be reproduced by a pure effective-potential calculation.

To assess the effects of mixing, which we recall are formally of two-loop order and higher, we plot the eigenvalues $M_{H'}$ and $M_{h'}$ of the full mass matrix as the red and blue dot-dashed lines, respectively. By comparing with the corresponding solid lines, we see that in this scenario the difference between $M_{H'}$ and M_H amounts to a few GeV, i.e., it is of $\mathcal{O}(1\%)$. In contrast, $M_{h'}$ can be lower than M_h by more than 30 GeV at the lowest considered values of $|\lambda_4|, |\lambda_5|$. The difference between M_h and $M_{h'}$ then decreases for increasing (absolute) values of the quartic couplings, due to the strong dependence of the hH mixing term in the mass matrix, see Eq. (17), on the BSM scalar mass m_H , which is larger on the left side of the plot. Again,

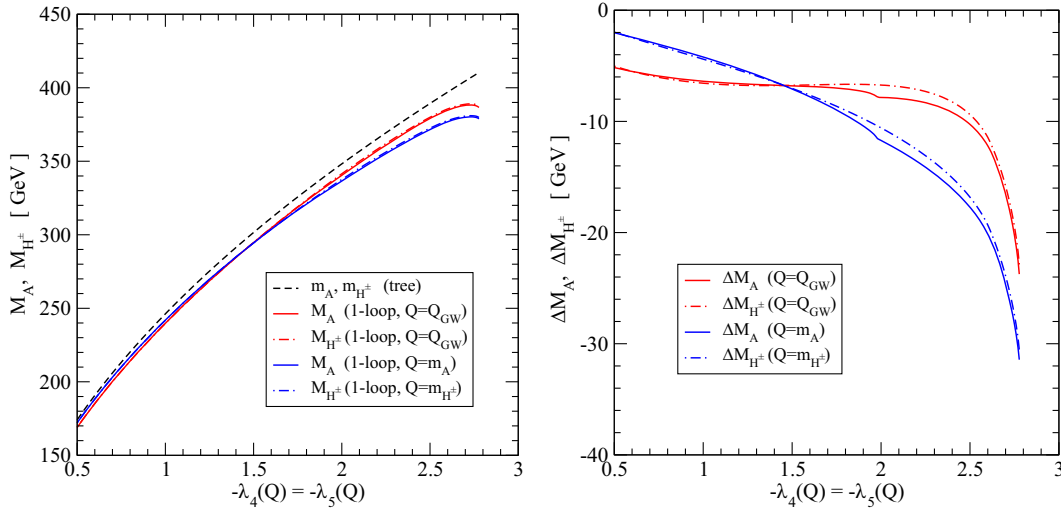


FIG. 2. Left: pseudoscalar and charged-scalar masses as a function of the quartic couplings, in the same scenario as in Fig. 1. Right: differences between loop-corrected and tree-level masses. The meaning of the lines is explained in the text.

this behavior should be contrasted with the one in Fig. 1 of Ref. [15], where the lines corresponding to M_h and $M_{h'}$ are essentially overlapping. We also recall that $M_{h'}$ and $M_{H'}$ are gauge-dependent quantities. The values shown in Fig. 1 are obtained with $\xi = 0$, while the values obtained with $\xi = 1$ can differ by a few tens of MeV (i.e., negligibly) in the case of $M_{H'}$, but up to 1.5 GeV in the case of $M_{h'}$. As mentioned in the previous section, this is merely a reflection of the fact that $M_{h'}$ and $M_{H'}$ contain incomplete sets of two-loop corrections.

The non-negligible impact of the mixing effects on $M_{h'}$ suggests that a two-loop calculation along the h direction alone, as was performed in Ref. [14], is not sufficient for a precise prediction of the SM-like Higgs mass in the SI-THDM. On the other hand, by computing the angle that rotates (h, H) into (h', H') , we find that, in this scenario, the couplings of h' to gauge bosons and to the top quark are modified with respect to the SM predictions by $\kappa_V \approx 0.99$ and $\kappa_t \approx 1.06$, respectively. This is well within the 2σ bounds from the Run 2 of the LHC [3]. The impact of the mixing effects on the couplings of h' to down-type fermions depends on the form of the Higgs-fermion interactions: in a “type-I” THDM, where all fermions couple only to Φ_2 in the Z_2 -symmetric basis, we get $\kappa_{b,\tau,\mu} = \kappa_t$, also within the 2σ bounds. However, in a “type-II” THDM, where the down-type fermions couple only to Φ_1 , we get $\kappa_{b,\tau,\mu} \lesssim 0.74$ in this scenario, in some tension with the 2σ bounds at least for what concerns κ_τ .⁷

We now move on to discussing the impact of the one-loop corrections to the masses of the pseudoscalar and of the charged scalar. In the left plot of Fig. 2 we show the predictions for those masses as a function of $-\lambda_4$

(or, equivalently, $-\lambda_5$), in the same scenario as in Fig. 1. The black dashed line represents the common tree-level masses $m_A = m_{H^\pm}$, while the solid and dot-dashed lines correspond to the full one-loop predictions for M_A and $M_{H^\pm}$, respectively, as given in Eqs. (14) and (15). We show two sets of solid and dot-dashed lines: the red lines are the predictions obtained assuming that the quartic couplings on the x-axis are evaluated at the scale Q_{GW} defined in Appendix B, while the blue lines assume that the couplings are evaluated at a scale equal to the tree-level masses m_A and $m_{H^\pm}$. We stress that the red and blue lines in Fig. 2 map two different regions of the SI-THDM parameter space, thus their spread should not be interpreted as a theory uncertainty of the one-loop calculation. To allow for a better assessment of the size and behavior of the one-loop corrections, in the right plot of Fig. 2 we show the differences between the loop-corrected pseudoscalar and charged-scalar masses and the corresponding tree-level masses. The meaning of the color and the style of the different lines is the same as in the left plot.

Figure 2 shows that, contrary to the finding in Refs. [8,11,12], the pseudoscalar and charged-scalar masses do receive one-loop corrections of the order of a few percent. Comparing the solid and dot-dashed lines in each of the blue and red sets, we also see a small split between M_A and $M_{H^\pm}$, which is driven by the corrections controlled by the EW gauge couplings or by the top Yukawa coupling. In particular, the small kinks in the solid lines around $\lambda_5 \approx -2$ stem from the threshold in $\text{Re} \Pi_{AA}(m_A^2)$ for $m_A = 2m_t$, while $\text{Re} \Pi_{H^\pm H^\mp}(m_{H^\pm}^2)$ does not have a threshold there because, in that case, the fermion loop involves both top and bottom quarks. In contrast, the corrections controlled by the quartic scalar couplings are the same for the pseudoscalar and the charged scalar, due to an unbroken $\text{SO}(3)$ symmetry of the scalar potential of the

⁷Note that the identification of h' with the observed Higgs boson is anyway precluded by the predictions for $M_{h'}$.

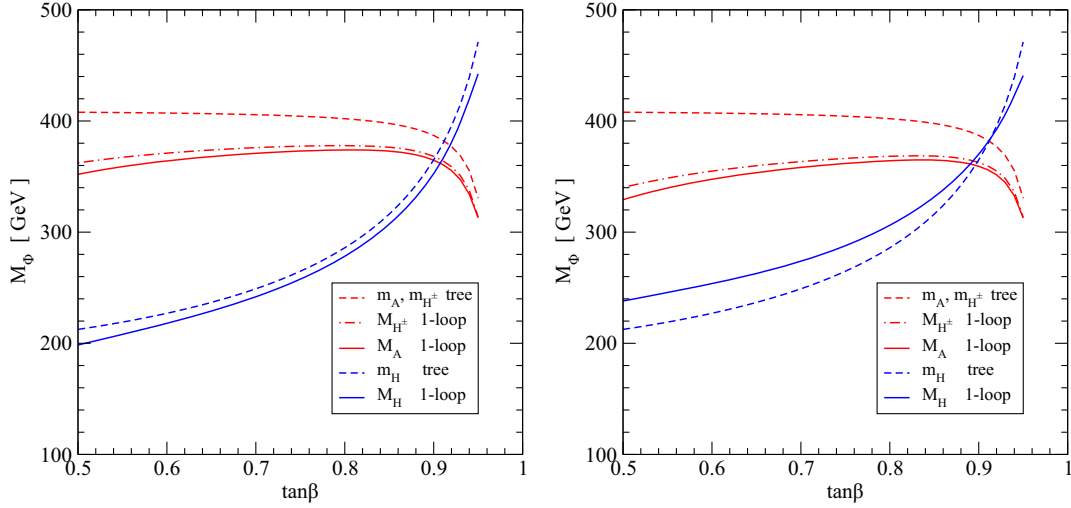


FIG. 3. Left: masses of the BSM Higgs bosons as a function of $\tan\beta$, in an aligned scenario in which $m_A = m_{H^\pm}$, $M_h = 125$ GeV, and $[\mathcal{M}_S^2(0)]_{12} = 0$, with $Q = Q_{\text{GW}}$. Right: same as the left plot, but with $Q = m_A, m_{H^\pm}$. The meaning of the lines is explained in the text.

SI-THDM that is realized when $\lambda_4 = \lambda_5$ [28]. Finally, comparing the red and blue sets of lines we see that the one-loop corrections to the pseudoscalar and charged-scalar masses are of the same order of magnitude independently of whether the $\overline{\text{MS}}$ -renormalized quartic couplings entering the tree-level masses are taken as input at $Q = Q_{\text{GW}}$ or at $Q = m_A, m_{H^\pm}$, although the former choice yields a somewhat flatter dependence of the corrections on the couplings before the rapid fall around $\lambda_4, \lambda_5 \approx -2.5$. As discussed in Appendix B, any choice of renormalization scale for the input parameters that is in the same ballpark as v is bound to lead to a reasonable perturbative behavior of the loop corrections.

Deviations from the SM predictions of a few percent in the couplings of the mass eigenstate h' to fermions and gauge bosons, such as the ones we found in the scenario of Fig. 1, are currently within the LHC bounds, but can in principle be tested at the HL-LHC [29]. However, Eq. (17) shows that it is even possible to devise an aligned scenario in which the effective mixing angle between the SM-like and BSM scalar states—as defined from the one-loop mass matrix computed at zero external momentum—vanishes altogether, so that h' coincides with h and its couplings to fermions and gauge bosons are fully SM-like. Together with the sum rule for M_h in Eq. (16) and with the condition $m_A = m_{H^\pm}$, which we retain to improve compatibility with the precision EW measurements, the condition $[\mathcal{M}_S^2(0)]_{12} = 0$ fixes all three of the BSM-Higgs masses m_H , m_A , and $m_{H^\pm}$. Even in this case we need to specify a renormalization scale Q for those masses, because in the calculation of the one-loop corrections we identify them with the tree-level parts of Eqs. (13)–(15), which involve the $\overline{\text{MS}}$ -renormalized quartic couplings. The remaining free parameter is then $\tan\beta$, and Eq. (17) shows that a cancellation between the BSM-Higgs and top contributions to the mixing is possible only when $\tan\beta < 1$.

In the plots of Fig. 3 we show the predictions for the masses of the BSM Higgs bosons as a function of $\tan\beta$ in this aligned scenario. The dashed lines represent the common tree-level masses $m_A = m_{H^\pm}$ (red) and the tree-level mass m_H (blue), as obtained from the combined conditions $M_h = 125$ GeV and $[\mathcal{M}_S^2(0)]_{12} = 0$ for a given value of $\tan\beta$. We start from $\tan\beta = 0.5$ to limit the size of the top Yukawa coupling—recalling that $y_t = \sqrt{2}m_t/(vs_\beta)$ —and we find solutions up to $\tan\beta \approx 0.95$. The solid and dot-dashed red lines represent the one-loop predictions for M_A and $M_{H^\pm}$, respectively, as given in Eqs. (14) and (15), while the solid blue line represents the one-loop prediction for M_H , as given in Eq. (13). The left plot is obtained with $Q = Q_{\text{GW}}$, and the right plot is obtained with $Q = m_A, m_{H^\pm}$. We stress again that the two choices of scale correspond to two different slices of the SI-THDM parameter space, thus the differences between the two plots should not be interpreted as a theory uncertainty of the one-loop calculation.

Figure 3 shows that in this aligned scenario all of the BSM-Higgs masses are relatively light, and that the charged-scalar and pseudoscalar masses are constrained in a band between approximately 310 and 370 GeV (in contrast, the mass of the BSM neutral scalar ranges between approximately 200 and 440 GeV). In this scenario the effects of the radiative corrections on the BSM-Higgs masses are of $\mathcal{O}(10\%)$, and the comparison between the left and right plots shows that the choice $Q = Q_{\text{GW}}$ leads to somewhat smaller corrections than the choice $Q = m_A, m_{H^\pm}$.

We conclude this section with a comment on the use of the conditions $M_h = 125$ GeV and $[\mathcal{M}_S^2(0)]_{12} = 0$ to constrain some of the BSM-Higgs masses. While this procedure proves convenient for defining benchmark scenarios and for comparing with earlier analyses in the literature, the discussion of Figs. 2 and 3 above shows

that it suffers from intrinsic ambiguities. Indeed, since both M_h and $[\mathcal{M}_S^2(0)]_{12}$ are purely one-loop-induced quantities, see Eqs. (16) and (17), the precise definitions of the BSM-Higgs masses entering their expressions amount to higher-order effects. Thus, we are free to identify those masses with the tree-level parts of Eqs. (13)–(15), but then, as shown in Figs. 2 and 3, different choices of scale for the $\overline{\text{MS}}$ -renormalized quartic couplings entering the tree-level masses correspond to different regions of the parameter space. Note that we could even choose to identify the BSM-Higgs masses entering Eqs. (16) and (17) directly with the pole masses, which would correspond to yet another region of the parameter space. These ambiguities could in principle be resolved by computing M_h and $[\mathcal{M}_S^2(0)]_{12}$ at the two-loop level, although in that case the extraction of the relevant relations between the masses would be nontrivial and likely to require numerical methods. This said, we stress that there is no ambiguity in the one-loop determination of the BSM-Higgs masses via Eqs. (13)–(15) when all three of the $\overline{\text{MS}}$ -renormalized couplings λ_3 , λ_4 , and λ_5 are directly treated as input at a given renormalization scale.

IV. CONCLUSIONS

The most attractive features of the SI-THDM are a constrained mass spectrum for the BSM Higgs bosons, which must all be relatively light, and tree-level alignment, so that one of the neutral scalars has SM-like couplings to gauge bosons and fermions. However, most of the earlier calculations of the Higgs-mass spectrum of the SI-THDM, which relied on the effective-potential approach, were plagued by an incorrect assumption for the field-dependent masses [8,11–13,15] and/or by an incorrect treatment of the minimum conditions for the scalar potential [13,15,16]. In this paper we revisited the one-loop calculation of the Higgs-mass spectrum of the SI-THDM, but we relied on a direct calculation of the relevant Feynman diagrams. This allowed us to bypass the complications related to the definition of the field-dependent masses, and also to account for the external-momentum effects, which are necessary to ensure the scale and gauge independence of the pole masses of the BSM Higgs bosons. Explicit formulas for all of the one-loop quantities relevant to our calculation are provided in Appendix A.

Exploring the numerical implications of our calculation, we showed that, in contrast with the findings of the earlier papers, the one-loop corrections to the BSM-Higgs masses can be of $\mathcal{O}(10\%)$. Furthermore, we found that the

one-loop corrections can induce a non-negligible mixing between the SM-like and BSM states in the neutral scalar sector. This induces a shift in the mass of the SM-like state that is formally of two-loop order, but was not taken into account in the two-loop calculations of Refs. [14,15]. We then considered an aligned scenario in which the condition of vanishing mixing induces a further constraint on the BSM-Higgs masses. Finally, we discussed how the definitions of benchmark scenarios for the SI-THDM can depend on the choice of renormalization scale, and elucidated the role of the Gildener-Weinberg scale introduced in Ref. [10] (see also Appendix B).

The work presented in this paper could of course be extended in several directions. On the calculation side, our full one-loop results for the neutral scalar mass matrix could be combined with the two-loop results for M_h in Ref. [14], so that the mixing effects are properly taken into account in the latter. The two-loop calculation could in turn be extended to the full mass matrix for the neutral scalars, thus correcting the results of Ref. [15], and even to the pseudoscalar and charged scalar masses. On the phenomenology side, the constraints on the parameter space of the SI-THDM that arise from the measured properties of the 125 GeV Higgs boson and from the searches for BSM Higgs bosons at the LHC could be reassessed in view of the correct calculation of masses and mixing. We leave all of this for future work, hoping that the results presented in this paper will help the collective effort to use the Higgs sector as a probe of what lies beyond the SM.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: ONE-LOOP TADPOLES AND SELF-ENERGIES IN THE SI-THDM

In this appendix we list explicit formulas for the one-loop quantities that are relevant to our calculation, including also the counterterm to the relation between v and G_F . The tadpoles and self-energies for the physical scalar fields ($h, H, A, H^\pm$) in a generic R_ξ gauge are

$$16\pi^2 v T_h = m_H^2 A_0(m_H^2) + m_A^2 A_0(m_A^2) + 2m_{H^\pm}^2 A_0(m_{H^\pm}^2) - 4N_c m_t^2 A_0(m_t^2) + m_Z^2 [3A_0(m_Z^2) + 2m_Z^2] + 2m_W^2 [3A_0(m_W^2) + 2m_W^2], \quad (\text{A1})$$

$$16\pi^2 v T_H = m_H^2 \cot 2\beta [3A_0(m_H^2) + A_0(m_A^2) + 2A_0(m_{H^\pm}^2)] - 4N_c m_t^2 \cot \beta A_0(m_t^2), \quad (\text{A2})$$

$$16\pi^2 v^2 \Pi_{hh}(p^2) = m_H^2 F(p^2, m_H^2) + m_A^2 F(p^2, m_A^2) + 2m_{H^\pm}^2 F(p^2, m_{H^\pm}^2) - 2N_c m_t^2 [2F(p^2, m_t^2) + p^2 B_0(p^2, m_t^2, m_t^2)] \\ + f_1(p^2, m_Z^2) + 2f_1(p^2, m_W^2), \quad (\text{A3})$$

$$16\pi^2 v^2 \Pi_{hH}(p^2) = m_H^2 \cot 2\beta [3F(p^2, m_H^2) + F(p^2, m_A^2) + 2F(p^2, m_{H^\pm}^2)] \\ - 2N_c m_t^2 \cot \beta [2F(p^2, m_t^2) + p^2 B_0(p^2, m_t^2, m_t^2)], \quad (\text{A4})$$

$$16\pi^2 v^2 \Pi_{HH}(p^2) = 2m_H^2 \cot^2 2\beta [3A_0(m_H^2) + A_0(m_A^2) + 2A_0(m_{H^\pm}^2)] - 4m_H^4 B_0(p^2, m_H^2, 0) \\ - 2m_H^4 \cot^2 2\beta [9B_0(p^2, m_H^2, m_H^2) + B_0(p^2, m_A^2, m_A^2) + 2B_0(p^2, m_{H^\pm}^2, m_{H^\pm}^2)] \\ - 2N_c m_t^2 \cot^2 \beta [2F(p^2, m_t^2) + p^2 B_0(p^2, m_t^2, m_t^2)] \\ + f_2(p^2, m_Z^2, m_A^2, m_H^2) + 2f_2(p^2, m_W^2, m_{H^\pm}^2, m_H^2), \quad (\text{A5})$$

$$16\pi^2 v^2 \Pi_{AA}(p^2) = 2m_H^2 \cot^2 2\beta [A_0(m_H^2) + 3A_0(m_A^2) + 2A_0(m_{H^\pm}^2)] - 4m_A^4 B_0(p^2, m_A^2, 0) - 4m_H^4 \cot^2 2\beta B_0(p^2, m_A^2, m_H^2) \\ - 2N_c m_t^2 \cot^2 \beta [2A_0(m_t^2) + p^2 B_0(p^2, m_t^2, m_t^2)] + f_2(p^2, m_Z^2, m_H^2, m_A^2) + 2f_2(p^2, m_W^2, m_{H^\pm}^2, m_A^2), \quad (\text{A6})$$

$$16\pi^2 v^2 \Pi_{H^+H^-}(p^2) = 2m_H^2 \cot^2 2\beta [A_0(m_H^2) + A_0(m_A^2) + 4A_0(m_{H^\pm}^2)] - 4m_{H^\pm}^4 B_0(p^2, m_{H^\pm}^2, 0) \\ - 4m_H^4 \cot^2 2\beta B_0(p^2, m_{H^\pm}^2, m_H^2) - 2N_c m_t^2 \cot^2 \beta [A_0(m_t^2) + (p^2 - m_t^2) B_0(p^2, m_t^2, 0)] \\ + f_2(p^2, m_W^2, m_H^2, m_{H^\pm}^2) + f_2(p^2, m_W^2, m_A^2, m_{H^\pm}^2) + c_{2\theta_W}^2 f_2(p^2, m_Z^2, m_{H^\pm}^2, m_{H^\pm}^2) \\ + s_{2\theta_W}^2 m_Z^2 \left[2(\xi - 1)p^2 + (3 - 2\xi)A_0(m_{H^\pm}^2) + \frac{m_{H^\pm}^2}{m_Z^2} A_0(\xi m_Z^2) + (3 - \xi)(p^2 + m_{H^\pm}^2) B_0(p^2, m_{H^\pm}^2, 0) \right], \quad (\text{A7})$$

where θ_W is the weak mixing angle defined by $c_{\theta_W} = m_W/m_Z$, and $A_0(m^2)$ and $B_0(p^2, m_1^2, m_2^2)$ are Passarino-Veltman functions. In particular, we can write

$$A_0(m^2) = m^2 \left(\ln \frac{m^2}{Q^2} - 1 \right), \quad B_0(0, m_1^2, m_2^2) = -\frac{A_0(m_1^2) - A_0(m_2^2)}{m_1^2 - m_2^2}, \quad (\text{A8})$$

while we use the code `LoopTools` [30] to account for the full p^2 dependence of B_0 . In Eqs. (A2)–(A7) we also introduced the auxiliary functions

$$F(p^2, m^2) = A_0(m^2) - 2m^2 B_0(p^2, m^2, m^2), \quad (\text{A9})$$

$$f_1(p^2, m_V^2) = 3m_V^2 [A_0(m_V^2) + 2m_V^2] - p^2 [A_0(m_V^2) - A_0(\xi m_V^2)] \\ - \frac{1}{2} (p^4 - 4p^2 m_V^2 + 12m_V^4) B_0(p^2, m_V^2, m_V^2) + \frac{p^4}{2} B_0(p^2, \xi m_V^2, \xi m_V^2), \quad (\text{A10})$$

$$f_2(p^2, m_V^2, m_1^2, m_2^2) = 2m_V^4 + (2m_V^2 + m_1^2) A_0(m_V^2) + m_V^2 A_0(m_1^2) - p^2 [A_0(m_V^2) - A_0(\xi m_V^2)] \\ - [m_1^4 + (p^2 - m_V^2)^2 - 2m_1^2(p^2 + m_V^2)] B_0(p^2, m_1^2, m_V^2) \\ + (p^2 - m_2^2)(p^2 + m_2^2 - 2m_1^2) B_0(p^2, m_1^2, \xi m_V^2). \quad (\text{A11})$$

The $\overline{\text{MS}}$ -renormalized vev v is connected with the Fermi constant G_F by the relation $(\sqrt{2}G_F)^{-1} = v^2 + \delta v^2$, with

$$\delta v^2 = v^2 \left[\frac{\Pi_{WW}(0)}{m_W^2} - \delta_{\text{VB}} \right], \quad (\text{A12})$$

where $\Pi_{WW}(0)$ is the finite part of the one-loop self-energy of the W boson at vanishing momentum, and δ_{VB} contains the one-loop contributions to muon decay from vertex and box diagrams. The SM contributions to $\Pi_{WW}(0)$ and δ_{VB}

were long ago computed in the Feynman gauge ($\xi = 1$) in Refs. [24,25], respectively. Supplementing $\Pi_{WW}(0)$ with the BSM contributions,⁸ and setting the tree-level mass of h to zero in the SM part, we get

$$\begin{aligned} \frac{16\pi^2}{g^2} \Pi_{WW}(0)|_{\xi=1} = m_W^2 & \left[\frac{27 - 34s_{\theta_W}^2}{8c_{\theta_W}^2} + \left(\frac{17}{s_{\theta_W}^2} + 8s_{\theta_W}^2 - 29 \right) \frac{\ln c_{\theta_W}^2}{4c_{\theta_W}^2} + \left(\frac{s_{\theta_W}^2}{c_{\theta_W}^2} - 1 \right) \ln \frac{m_W^2}{Q^2} \right] \\ & - \frac{N_c}{2} m_t^2 \left(\ln \frac{m_t^2}{Q^2} - \frac{1}{2} \right) + \tilde{B}_{22}(0, m_H^2, m_{H^\pm}^2) + \tilde{B}_{22}(0, m_A^2, m_{H^\pm}^2), \end{aligned} \quad (\text{A13})$$

$$\frac{16\pi^2}{g^2} \delta_{\text{VB}}|_{\xi=1} = 6 + \left(\frac{7}{2} - 6s_{\theta_W}^2 \right) \frac{\ln c_{\theta_W}^2}{s_{\theta_W}^2} - 4 \ln \frac{m_Z^2}{Q^2}, \quad (\text{A14})$$

with

$$\tilde{B}_{22}(0, m_1^2, m_2^2) = \frac{1}{2} \left(\frac{m_1^2 + m_2^2}{4} - \frac{m_1^2 m_2^2}{2(m_1^2 - m_2^2)} \ln \frac{m_1^2}{m_2^2} \right). \quad (\text{A15})$$

The ξ -dependent parts of $\Pi_{WW}(0)$ and δ_{VB} were computed in Ref. [26]. For the combination that is relevant to our calculation, we get⁹

$$\begin{aligned} \delta v^2 = \delta v^2|_{\xi=1} + \frac{1}{16\pi^2} (1 - \xi) & \left[m_Z^2 B_0(0, m_Z^2, \xi m_Z^2) \right. \\ & \left. + 2m_W^2 B_0(0, m_W^2, \xi m_W^2) \right]. \end{aligned} \quad (\text{A16})$$

Combining the ξ -dependent formulas for self-energies and δv , we can check that the pole masses in Eqs. (12)–(15) are indeed gauge independent. In the case of the charged scalar mass, this requires also the identity $B_0(m^2, m^2, 0) = 2 - \ln(m^2/Q^2)$.

APPENDIX B: ON THE GILDENER-WEINBERG SCALE

To elucidate the role of the so-called Gildener-Weinberg scale introduced in Ref. [10], it is convenient to rotate the scalar potential of Eq. (1) from the Z_2 -symmetric basis (Φ_1, Φ_2) to the Higgs basis $(\Phi_{\text{SM}}, \Phi_{\text{BSM}})$,

$$\begin{aligned} V_0 = \frac{\Lambda_1}{2} (\Phi_{\text{SM}}^\dagger \Phi_{\text{SM}})^2 + \frac{\Lambda_2}{2} (\Phi_{\text{BSM}}^\dagger \Phi_{\text{BSM}})^2 + \Lambda_3 (\Phi_{\text{SM}}^\dagger \Phi_{\text{SM}}) (\Phi_{\text{BSM}}^\dagger \Phi_{\text{BSM}}) + \Lambda_4 (\Phi_{\text{SM}}^\dagger \Phi_{\text{BSM}}) (\Phi_{\text{BSM}}^\dagger \Phi_{\text{SM}}) \\ + \left[\frac{\Lambda_5}{2} (\Phi_{\text{SM}}^\dagger \Phi_{\text{BSM}})^2 + (\Lambda_6 \Phi_{\text{SM}}^\dagger \Phi_{\text{SM}} + \Lambda_7 \Phi_{\text{BSM}}^\dagger \Phi_{\text{BSM}}) \Phi_{\text{SM}}^\dagger \Phi_{\text{BSM}} + \text{H.c.} \right]. \end{aligned} \quad (\text{B1})$$

In this basis the minimum conditions for the effective potential take on a simple form,

$$\Lambda_1 = -\frac{2}{v^3} T_h, \quad \Lambda_6 = -\frac{2}{v^3} T_H, \quad (\text{B2})$$

where we made use of the fact that $T_\varphi = d\Delta V/d\varphi|_{\text{min}}$. Equation (B2) above is equivalent to Eq. (8), in view of the relations

$$\begin{aligned} \Lambda_1 &= \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \frac{1}{2} \lambda_{345} s_{2\beta}^2, \\ \Lambda_6 &= -\frac{1}{2} s_{2\beta} (\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta}). \end{aligned} \quad (\text{B3})$$

For a choice of scale Q_{GW} such that $\Lambda_1(Q_{\text{GW}}) = 0$, so that the tree-level potential has a flat direction along h , the first minimum condition in Eq. (B2) implies $T_h = 0$. We stress again that this choice does not have any special implication for the second condition in Eq. (B2), contrary to what was assumed in Refs. [13,15]. Similarly, the requirement in

⁸There are no BSM contributions to δ_{VB} under our approximations for the Yukawa couplings.

⁹We thank G. Degrandi for clarifications on this point.

Ref. [16] that $\Lambda_6(Q_{\text{GW}}) = 0$, which implies $T_H = 0$ via the second minimum condition in Eq. (B2), should be interpreted as an arbitrary constraint on the parameter space of the model.

To work out the implications of the condition $T_h = 0$, we follow the approach of Ref. [10] and write the one-loop part of the effective potential along the h direction in the form

$$\Delta V = (v + h)^4 \left[A + B \ln \frac{(v + h)^2}{Q^2} \right]. \quad (\text{B4})$$

This is possible because, in scale-invariant models where the scalar potential does not include explicit mass parameters, all of the h -dependent squared masses entering the effective potential have the form $\tilde{\Lambda}(v + h)^2$, where by $\tilde{\Lambda}$ we denote generic combinations of quartic scalar couplings Λ_i , squared gauge couplings g^2 and g'^2 , or the squared top coupling y_t^2 . Working in the Landau gauge ($\xi = 0$) and in the $\overline{\text{MS}}$ renormalization scheme, the dimensionless parameters A and B read

$$A = \sum_{\varphi} \frac{\alpha_{\varphi} m_{\varphi}^4}{64\pi^2 v^4} \left(\ln \frac{m_{\varphi}^2}{v^2} - k_{\varphi} \right), \quad B = \sum_{\varphi} \frac{\alpha_{\varphi} m_{\varphi}^4}{64\pi^2 v^4}, \quad (\text{B5})$$

where $\alpha_{\varphi} = (1, 1, 2, 6, 3, -4N_c)$ for $\varphi = (H, A, H^{\pm}, W, Z, t)$, m_{φ}^2 are the squared masses at the minimum of the potential, and k_{φ} is equal to $5/6$ for the gauge bosons and to $3/2$ for the BSM Higgs bosons and the top quark. Note that both A and B are of $\mathcal{O}(\tilde{\Lambda}^2/16\pi^2)$, and that M_h^2 , as given in Eq. (16), is equal to $8v^2 B$. For $Q = Q_{\text{GW}}$, the condition $T_h = d\Delta V/dh|_{\min} = 0$ corresponds to

$$\ln \frac{v^2}{Q_{\text{GW}}^2} = -\frac{1}{2} - \frac{A}{B}. \quad (\text{B6})$$

Since A and B are of the same order in the couplings $\tilde{\Lambda}$ and in the loops, the logarithm in Eq. (B6) is generically of $\mathcal{O}(1)$, thus the perturbative expansion should be safe.

Moreover, Eq. (B6) can be interpreted as a Higgs vev of $\mathcal{O}(Q_{\text{GW}})$ being generated via “dimensional transmutation,”

$$v = e^{-\left(\frac{1}{4} + \frac{A}{2B}\right)} Q_{\text{GW}}. \quad (\text{B7})$$

We now discuss the implications of choosing a different scale Q for which $\Lambda_1 \neq 0$. In that case the minimum condition for the effective potential along the h direction corresponds to

$$\ln \frac{v^2}{Q^2} = -\frac{1}{2} - \frac{A}{B} - \frac{\Lambda_1}{8B}. \quad (\text{B8})$$

If we considered Λ_1 to be of $\mathcal{O}(\tilde{\Lambda})$ like the other couplings in the model, the third term on the rhs of Eq. (B8) would be of $\mathcal{O}(16\pi^2/\tilde{\Lambda})$, leading to a large logarithm and a breakdown of the perturbative expansion.¹⁰ However, this cannot happen as long as the scales Q and Q_{GW} are not very far from each other, so that $\ln(Q/Q_{\text{GW}})$ remains of $\mathcal{O}(1)$. In that case, since Q_{GW} is defined by the condition $\Lambda_1(Q_{\text{GW}}) = 0$, the renormalization-group evolution to Q can only induce a value of $\mathcal{O}(\tilde{\Lambda}^2/16\pi^2)$ for Λ_1 . Thus, even the third term on the rhs of Eq. (B8) is in fact of $\mathcal{O}(1)$, and the perturbative expansion remains safe.

In summary, we stress again that in the calculation of the Higgs-mass spectrum of the SI-THDM the choice $Q = Q_{\text{GW}}$ is by no means compulsory. Any scale choice in the same ballpark as the masses of the particles involved in the loops—or, equivalently, in the same ballpark as v —will lead to a reasonable perturbative expansion, and anyway the predictions for physical quantities must be independent of the scale, as we indeed find in Eqs. (12)–(15).

¹⁰Note that in Ref. [10] the role of our $\tilde{\Lambda}$ is played by e^2 . Curiously, the discussion after Eq. (3.24) of that paper seems indeed to suggest that the perturbative expansion would break down if we chose a scale different from Q_{GW} .

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