

Angular coefficients of the Drell-Yan process across different rapidity and kinematical ranges

Valery E. Lyubovitskij^{1,2}, Alexey S. Zhevlakov^{3,4} and Iurii Anikin^{5,3}

¹*Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

²*Millennium Institute for Subatomic Physics at the High-Energy Frontier (SAPHIR) of ANID, Fernández Concha 700, Santiago, Chile*

³*Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia*

⁴*Matrosov Institute for System Dynamics and Control Theory, SB RAS, Lermontov strasse, 134, 664033, Irkutsk, Russia*

⁵*Department of Physics, Tomsk State University, 634050 Tomsk, Russia*



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We present a comprehensive analysis of the angular structure of the Drell-Yan process at different rapidity and kinematical ranges using predictions of theoretical approaches (collinear perturbative QCD and the geometrical method) at next-to-leading order and data of ATLAS, LHCb, and CMS Collaborations at the CERN LHC.

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I. INTRODUCTION

Discovery of composite structure of hadrons at SLAC in the 1960s supported by theoretical progress (quark model building, Feynman parton model, etc.) stimulated comprehensive studies of hadron physics. The Drell-Yan (DY) process is one of the most powerful and key methods for the study of perturbative and nonperturbative structures of hadrons. Study of DY processes gives a unique opportunity to shed light on strong interaction properties—perturbative (resummation of soft and collinear singularities) and non-perturbative [extraction of parton distribution functions (PDFs) of quarks and gluons in composite hadrons]. Different types of PDFs, being one of the fundamental properties of hadron constituents, describe the distribution of quarks and gluons in hadrons (nuclei) and encode information about longitudinal hadron momentum, transverse momenta of partons, and spin correlations of hadrons and partons. Theoretical study of the DY processes is motivated by currently running and planned precise experiments at worldwide facilities (see, e.g., Refs. [1–12]).

The LHC experiments (CMS [1], ATLAS [2], and LHCb [3]) can be considered as a main provider of DY data using pp collisions with the possibility of studying different ranges of invariant mass Q , transverse momentum Q_T , and

the rapidity of the lepton pair y . Analysis of the lepton angular structure of DY processes (in particular, angular coefficients) gives direct access to information about the inner structure of hadrons. The knowledge of angular coefficients also plays an important role in more precise determination of such parameters as the W boson mass m_W or Weinberg angle θ_W . For a recent overview of the current status of quantum chromodynamics (QCD) precision predictions for DY processes, see, e.g., Ref. [13].

Latest advanced calculations of the DY angular distributions and coefficients provide a crucial opportunity for high-precision tests of the electroweak sector of the Standard Model. Analytical studies of angular coefficients were done in next-to-leading order (NLO) in Refs. [14–16] and in next-to-next-to-leading order (NNLO) in [17]. A comprehensive comparison of the available experimental data for the Drell-Yan lepton angular coefficients λ and ν to calculations at NLO and NNLO of perturbative QCD (pQCD) has been presented in Ref. [18]. Notably, the precision of both inclusive and fully differential DY cross sections has been extended from NNLO [19–25] to N^3 LO [26–28]. A careful and consistent inclusion of electroweak corrections has been carried out in Refs. [29–33]. At present time we need to perform precise and fast theoretical calculations, which are provided by different existing Monte Carlo (MC) generators. In particular, DY processes were analyzed using NNLOjet [34], PYTHIA [35], Herwig [36], DYNLO [23], and FEWZ [24,37]. In general, the coefficients are not extracted directly at calculation in the MC event generators. However, they can be extracted from the angular distribution shapes using the method proposed in Ref. [38], due to the orthogonality of the polynomials.

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Using analytical results [15,16], we can obtain angular coefficients' behavior without using methods based on the orthogonality of the polynomials.

Utilizing the NNLOjet MC generator [34], the DY angular coefficients $A_{0,1,2,3,4}$ were calculated, compared with data provided by CMS and ATLAS Collaborations, and predictions were made for the LHCb experiment. These studies included N³LO computation of the angular coefficients using the method proposed in Ref. [38]. Besides, data were managed by PYTHIA [35], Herwig [36], DYNNLO [23], and FEWZ [24,37] in the experimental papers [1–3]. These analyses showed an importance of the NNLO and higher-order corrections to the DY angular distributions/coefficients. In particular, it is crucial for the A_1 and A_2 coefficients that provide an adjustment for sensitive agreement with data and violation of the Lam-Tung (LT) relation [34,39]. Besides, in Ref. [34] it was noted that the N³LO $O(\alpha_s^3)$ corrections are observed to have an important influence on the predicted shapes of the A_0 , A_1 , and A_2 coefficients' distributions. However, it is small for the A_3 and A_4 distributions and not sensitive for prediction at the existing experimental level of accuracy. Additionally, it should be noted that the DY angular coefficients are a good tool for extraction of PDFs [40], in particular, the one done by using the open-source fit platform xFitter [41] for Higgs production [42]. Besides, many collaborations [1–3] used ResBes [43,44] and DYTurbo [45–47] packages for analysis.

Other possibilities for predictions of angular coefficients were proposed in Refs. [48–50] based on the geometrical method. This method is based on the idea to use parametrization for Q_T dependence of the angular coefficients induced by the $q\bar{q}$ and qg subprocesses at NLO. In this way, the set of free parameters is fixed using data. Such a scheme provides a simple method for prediction of the Q_T behavior of the angular coefficients. This method raises two main questions: (1) how accurately does this method capture the behavior of the angular coefficients for DY processes? and (2) Can we determine angular coefficients from rotation of the hadronic plane relative to the quark plane using the $q\bar{q} \rightarrow l\bar{l}$ cross section? It will be tested using calculation in pQCD in the framework of collinear approximation [16] and analyzing existing data from LHC experiments. The calculation includes only the NLO contribution, as this is required for comparison with the geometrical method. In the future, we plan to expand our calculations on NNLO.

The paper is organized as follows. In Sec. II, we give a brief overview of the DY process and its basic

blocks—angular coefficients. Later, we also briefly discuss dependencies of angular coefficients in the geometrical method and results in the framework of collinear approximation pQCD in NLO calculation. In Sec. III, we present statistical analysis of both approaches (ours and geometrical) in comparison with available data from the LHC experiments for angular coefficients. In Sec. IV, we summarize our results. In Appendix we include statistical test results.

II. THEORY

A. Basics of the Drell-Yan reactions

We consider the process of lepton pair production in the inelastic DY reaction $p(p_1) + p(p_2) \rightarrow \gamma^*/Z + X \rightarrow l_1(k_1) + l_2(k_2) + X$ with intermediate photon γ^* and neutral weak Z boson. The cross section of this reaction can be presented as the contraction of leptonic $L_{\mu\nu}$ and hadronic $W_{\mu\nu}$ tensors,

$$\frac{d\sigma}{d\Omega d^4q} = \frac{\alpha^2}{2(2\pi)^4 Q^4 s^2} L_{\mu\nu} W^{\mu\nu}, \quad (1)$$

where $s = (P_1 + P_2)^2$ is the hadron-level total energy, $\alpha = 1/137.036$ is the electromagnetic fine-structure constant, and $Q^2 = q^2$ is the timelike vector boson momentum squared. Hadronic momenta P_1 and P_2 are chosen in the Collins-Soper (CS) frame and related to the parton momenta $p_i = \xi_i P_i$, where ξ_i is the partonic momentum fraction, and Q^+ , Q^- , and Q_T are the gauge boson longitudinal and transverse momentum components, respectively, with $Q^\pm = x_{1,2} \sqrt{s}/2 = e^{\pm y} \sqrt{(Q^2 + Q_T^2)/2}$. We introduce the following notations: $\rho = Q_T/Q$ is the ratio of the transverse component and magnitude $Q = \sqrt{Q^2}$ of the vector boson momentum, $x_{1,2} = 2P_{2,1}q/s$ are the momentum fractions of the light-cone components of the finale vector boson, and $y = (1/2) \log(x_1/x_2)$ is the rapidity. We also define the x_i^0 fraction factors at $Q_T^2 = 0$ as $x_{1,2}^0 = e^{\pm y} Q/\sqrt{s}$.

The DY lepton angular distribution $dN/d\Omega$ depends on the polar θ and azimuthal ϕ angles and can be conventionally expanded in terms of the nine helicity structure functions, corresponding to the specific polarization of the gauge boson. These structure functions play important role and encode the polar and azimuthal asymmetries occurring in the DY process. In the CS frame, the DY lepton angular distribution reads [14–17,39,51,52]

$$\begin{aligned} \frac{dN}{d\Omega} = \frac{d\sigma}{d\Omega d^4q} \left(\frac{d\sigma}{d^4q} \right)^{-1} = \frac{3}{8\pi(2W_T + W_L)} [g_T W_T + g_L W_L + g_\Delta W_\Delta + g_{\Delta\Delta} W_{\Delta\Delta} \\ + g_{T_p} W_{T_p} + g_{\nabla_p} W_{\nabla_p} + g_{\nabla} W_{\nabla} + g_{\Delta\Delta_p} W_{\Delta\Delta_p} + g_{\Delta_p} W_{\Delta_p}], \end{aligned} \quad (2)$$

where $g_i = g_i(\theta, \phi)$ are the angular coefficients,

$$\begin{aligned} g_T &= 1 + \cos^2\theta, & g_L &= 1 - \cos^2\theta, & g_{T_p} &= \cos\theta, \\ g_{\Delta\Delta} &= \sin^2\theta \cos 2\phi, & g_{\Delta} &= \sin 2\theta \cos \phi, & g_{\nabla_p} &= \sin\theta \cos \phi, \\ g_{\Delta\Delta_p} &= \sin^2\theta \sin 2\phi, & g_{\Delta_p} &= \sin 2\theta \sin \phi, & g_{\nabla} &= \sin\theta \sin \phi. \end{aligned} \quad (3)$$

There is equivalent parametrization of the $dN/d\Omega$ using the set of the angular coefficients A_i in Refs. [14,15,17,39,51],

$$\begin{aligned} \frac{dN}{d\Omega} &= \frac{3}{16\pi} \left(1 + \cos^2\theta + \frac{A_0}{2}(1 - 3\cos^2\theta) + A_1 \sin 2\theta \cos \phi + \frac{A_2}{2} \sin^2\theta \cos 2\phi \right. \\ &\quad \left. + A_3 \sin \theta \cos \phi + A_4 \cos \theta + A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi \right), \end{aligned} \quad (4)$$

where

$$\begin{aligned} A_0 &= \frac{2W_L}{2W_T + W_L}, & A_1 &= \frac{2W_{\Delta}}{2W_T + W_L}, & A_2 &= \frac{4W_{\Delta\Delta}}{2W_T + W_L}, \\ A_3 &= \frac{2W_{\nabla_p}}{2W_T + W_L}, & A_4 &= \frac{2W_{T_p}}{2W_T + W_L}, & A_5 &= \frac{2W_{\Delta\Delta_p}}{2W_T + W_L}, \\ A_6 &= \frac{2W_{\Delta_p}}{2W_T + W_L}, & A_7 &= \frac{2W_{\nabla}}{2W_T + W_L}. \end{aligned} \quad (5)$$

We also define the forward-backward (FB) asymmetry coefficient A_{FB} and the convexity (transverse-longitudinal hadronic structure) asymmetry A_{conv} [16],

$$A_{\text{FB}} = \frac{3}{8}A_4, \quad A_{\text{conv}} = \frac{3}{8}(2 - 3A_0). \quad (6)$$

Hadronic structure functions $W_j = W_j(x_1, x_2)$ characterizing the DY process with colliding hadrons $H_1(p_1)$ and $H_2(p_2)$ are related to the partonic-level structure functions $w_j^{ab}(x_1, x_2)$ by the QCD collinear factorization formula [52]

$$W_j(x_1, x_2) = \frac{1}{x_1 x_2} \sum_{a,b} \int_{x_1}^1 dz_1 \int_{x_2}^1 dz_2 w_j^{ab}(z_1, z_2) f_{a/H_1}\left(\frac{x_1}{z_1}\right) f_{b/H_2}\left(\frac{x_2}{z_2}\right), \quad (7)$$

where $f_{a/H}(\xi)$ with $\xi_i = x_i/z_i$ is the PDF describing the collinear ξ distribution of partons of type a in a hadron H .

As we know, the DY angular coefficients A_0 and A_2 at NLO are related by the LT identity $A_0 = A_2$. For the first time, this identity was discovered in the naive parton model [53] and then confirmed in pQCD at order $\mathcal{O}(\alpha_s)$ based on the collinear factorization (see Refs. [14,15,54]). Violation of the LT identity was experimentally observed and numerically simulated using different MC generators, and violation starts at computation of NNLO contribution to the DY cross section [2,34].

B. Geometrical method

In Refs. [48,49,55], the geometrical approach for the study of the angular coefficients' dependence on small Q_T and rapidity y was proposed and developed. In Fig. 1 we specify the kinematics of the DY process in the CS frame.

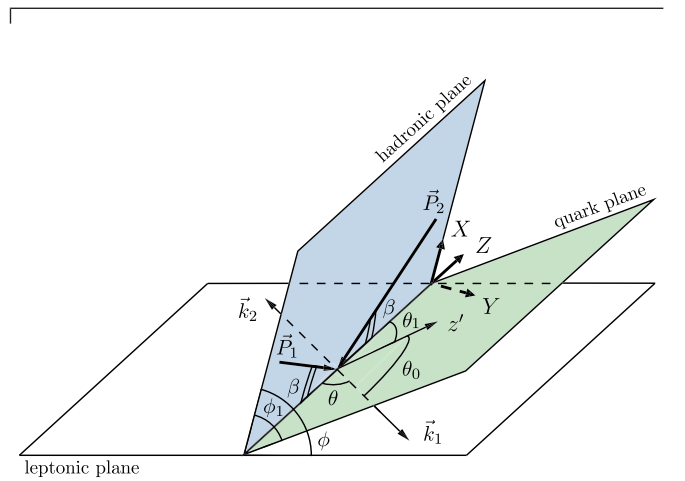


FIG. 1. DY process in the CS frame. The hadron, quark, and lepton planes are colored by blue, green, and white colors, respectively.

The CS frame is the rest frame of the virtual (γ^*/Z) boson or, equivalently, of the lepton pair with special definition of the hadronic plane formed by the spatial momenta of the beam ($\vec{P}_1$) and the target ($\vec{P}_2$) hadrons. In particular, in the CS frame the Z axis is fixed as pointing in the direction that bisects the angle β between the three-vectors $\vec{P}_1$ and $-\vec{P}_2$ with $\sin \beta = \rho/\sqrt{1+\rho^2}$ and $\cos \beta = 1/\sqrt{1+\rho^2}$. In Fig. 1, we define (1) three planes—hadronic, quark, and leptonic, which are specified by pairs of the corresponding three-momenta of colliding hadrons, quark and antiquark, lepton and antilepton, (2) the angle θ_0 , which is the angle between lepton ℓ^- and quark three-momenta, and (3) two pairs of polar and azimuthal angles: (θ, ϕ) defining relative orientation of hadronic and leptonic frames and (θ_1, ϕ_1) defining relative orientation of hadronic and quark frames.

The main idea of the geometrical method [48,49,55,56] is to use the “natural” quark-antiquark axis for definition of the DY angular lepton distribution. In particular, the latter is

azimuthally symmetric with respect to this axis and only depends on the polar angle θ_0 ,

$$\frac{dN}{d\Omega} = \frac{3}{16\pi} [1 + \cos^2 \theta_0 + a \cos \theta_0]. \quad (8)$$

Note, the third term on the r.h.s. of Eq. (8) was added in Refs. [49,50] for the description of the weak gauge boson production. Here, the coefficient a is the FB asymmetry originating from the parity-violating coupling of quarks and leptons with weak gauge boson. The angle θ_0 can be expressed through the set of angles ϕ , θ_1 , and ϕ_1 using identity [48,50,55]

$$\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos(\phi - \phi_1). \quad (9)$$

Using relation (9), one can present the DY angular distribution in the form [50,55]

$$\begin{aligned} \frac{dN}{d\Omega} = \frac{3}{16\pi} & \left[1 + \cos^2 \theta + \frac{\sin^2 \theta_1}{2} (1 - 3\cos^2 \theta) + \frac{1}{2} \sin 2\theta_1 \cos \phi_1 \sin 2\theta \cos \phi + \frac{1}{2} \sin^2 \theta_1 \cos 2\phi_1 \sin^2 \theta \cos 2\phi \right. \\ & + a \sin \theta_1 \cos \phi_1 \sin \theta \cos \phi + a \cos \theta_1 \cos \theta + \frac{1}{2} \sin^2 \theta_1 \sin 2\phi_1 \sin^2 \theta \sin 2\phi \\ & \left. + \frac{1}{2} \sin 2\theta_1 \sin \phi_1 \sin 2\theta \sin \phi + a \sin \theta_1 \sin \phi_1 \sin \theta \sin \phi \right]. \end{aligned} \quad (10)$$

Matching Eqs. (4) and (10) gives, for the angular coefficients [50,55],

$$\begin{aligned} A_0 &= \sin^2 \theta_1, & A_1 &= \frac{1}{2} \sin 2\theta_1 \cos \phi_1, & A_2 &= \sin^2 \theta_1 \cos 2\phi_1, & A_3 &= a \sin \theta_1 \cos \phi_1, \\ A_4 &= a \cos \theta_1, & A_5 &= \frac{1}{2} \sin^2 \theta_1 \sin 2\phi_1, & A_6 &= \frac{1}{2} \sin 2\theta_1 \sin \phi_1, & A_7 &= a \sin \theta_1 \sin \phi_1. \end{aligned} \quad (11)$$

In case of the quark-antiquark annihilation subprocess $q\bar{q} \rightarrow g\gamma(Z, W)$, the $\sin \theta_1$ is identified with $\sin \beta = \rho/\sqrt{1+\rho^2}$ [49,55], while in case of the quark-gluon Compton scattering $qg \rightarrow q\gamma(Z, W)$, the $\sin \theta_1$ is approximately equal to $\sqrt{5}\rho/\sqrt{1+5\rho^2}$ [49,55].

In the geometrical approach, all angular T-even coefficients $A_{i:g}$ are given by linear combinations of $q\bar{q}$ and qg contributions using mixing parameter f [49],

$$\begin{aligned} A_{0:g} &= fA_{0:g}^{q\bar{q}} + (1-f)A_{0:g}^{qg} = f \frac{\rho^2}{1+\rho^2} + (1-f) \frac{5\rho^2}{1+5\rho^2}, \\ A_{1:g} &= fA_{1:g}^{q\bar{q}} + (1-f)A_{1:g}^{qg} = r_1 \left[f \frac{\rho}{1+\rho^2} + (1-f) \frac{\sqrt{5}\rho}{1+5\rho^2} \right], \\ A_{2:g} &= r_2 A_{0:g}, \\ A_{3:g} &= fA_{3:g}^{q\bar{q}} + (1-f)A_{3:g}^{qg} = r_3 \left[f \frac{\rho}{\sqrt{1+\rho^2}} + (1-f) \frac{\sqrt{5}\rho}{\sqrt{1+5\rho^2}} \right], \\ A_{4:g} &= fA_{4:g}^{q\bar{q}} + (1-f)A_{4:g}^{qg} = r_4 \left[f \frac{1}{\sqrt{1+\rho^2}} + (1-f) \frac{1}{\sqrt{1+5\rho^2}} \right], \end{aligned} \quad (12)$$

TABLE I. Parameters of the geometrical method fitted to data from CMS [1], ATLAS [2], and LHCb [3] Collaborations.

Data	CMS		ATLAS		LHCb	
	Rapidity $ y $		Rapidity $ y $		Rapidity $ y $	
Parameter	[0, 1]	[1, 2.1]	[0, 1]	[1, 2]	[2, 3.5]	[2, 4.5]
f	0.509		0.446		0.619	
r_1	0.024	0.112	0.028	0.130	...	0.267
r_3	0.015	0.034	0.000	0.032	0.093	0.099
r_4	0.018	0.073	0.021	0.063	0.136	0.143

where r_i with $i = 1, 2, 3, 4$ are the reduction factors, which play certain role in the angular coefficients' phenomenology. In particular, the r_1 is related to the rotation of the hadronic plane relative to the quark plane, r_2 parametrizes a violation of the LT identity $A_0 = A_2$ (we remind that the LT relation is violated starting from NNLO (α_s^2) [17]), while r_3 and r_4 are related to the forward-backward asymmetry. The parameters r_1 , r_3 , and r_4 are fitted accordingly using different experimental datasets at specific values of total energy $\sqrt{s}$ and rapidity y . After fitting free parameters, the geometrical method can predict a behavior of angular coefficients in some interval of Q_T . We test the geometrical method by comparison with the results for angular coefficients computed by us in the pQCD at NLO based on the collinear factorization [16]. We make statistical analysis using currently available data from CMS [1], ATLAS [2], and LHCb [3] Collaborations. Parameters of the geometrical method fitted to different sets of the LHC data are shown in Table I. Note, in the case of comparisons with LHCb data [3], the rapidity region is strictly forward.

C. DY partonic structure functions in pQCD

For analysis, we use our analytical results for the DY angular coefficients computed in Ref. [16] using collinear pQCD in the first order of α_s (NLO), i.e., at the same order of accuracy as was used in the geometrical approach and taking into account the partonic subprocesses of

quark-antiquark annihilation (see Fig. 2) and Compton quark (antiquark)-gluon scattering (see Fig. 3).

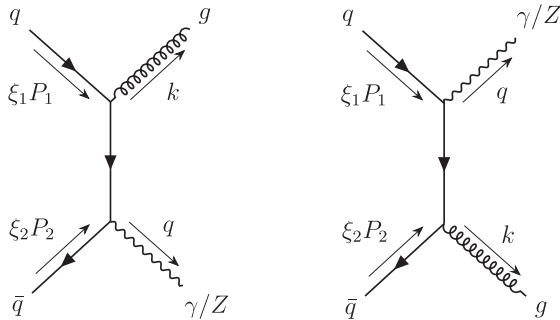
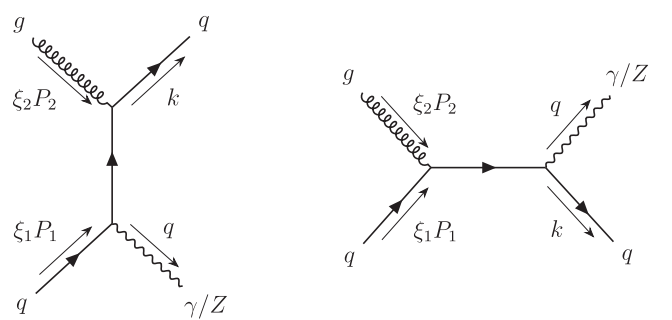
Angular coefficients are related to partonic-level structure functions with Eqs. (5) and (7). Next, we specify partonic-level structure functions occurring at NLO [14–16],

$$\begin{aligned}
 w_j^{ab}(z_1, z_2, \rho^2) &= \tilde{w}_j^{ab}(z_1, z_2, \rho^2) \delta((\hat{s} + \hat{t} + \hat{u} - Q^2)/\hat{s}) \\
 &= \tilde{w}_j^{ab}(z_1, z_2, \rho^2) \delta\left((1 - z_1)(1 - z_2) - \frac{\rho^2 z_1 z_2}{1 + \rho^2}\right), \quad (13)
 \end{aligned}$$

where we use the variables introduced before, $z_1 = x_1/\xi_1$ and $z_2 = x_2/\xi_2$.

Here, $\hat{s}$, $\hat{t}$, and $\hat{u}$ are parton-level Mandelstam variables and are expressed and obey the identities as

$$\begin{aligned}
 \hat{s} &= \frac{Q^2 + Q_T^2}{z_1 z_2} = \frac{Q_T^2}{(1 - z_1)(1 - z_2)}, \\
 \hat{t} &= Q^2 - \frac{Q^2 + Q_T^2}{z_1} = -\frac{Q_T^2}{1 - z_2}, \\
 \hat{u} &= Q^2 - \frac{Q^2 + Q_T^2}{z_2} = -\frac{Q_T^2}{1 - z_1}, \\
 \frac{\hat{t}}{\hat{s}} &= z_1 - 1, \quad \frac{\hat{u}}{\hat{s}} = z_2 - 1, \quad \frac{Q^2}{\hat{s}} = z_1 + z_2 - 1. \quad (14)
 \end{aligned}$$

FIG. 2. Partonic-level quark-antiquark annihilation diagrams that contribute to the DY cross section at order $\mathcal{O}(\alpha_s)$.FIG. 3. Partonic-level quark-gluon Compton scattering diagrams that contribute to the DY cross section at order $\mathcal{O}(\alpha_s)$.

The parton-level helicity structure functions from the $q\bar{q}$ annihilation subprocess have a form

$$\begin{aligned}\tilde{w}_T^{q\bar{q}} &= g_{q\bar{q};1} \frac{1}{\rho^2} \left(1 + \frac{\rho^2}{2} \right) \frac{z_1^2 + z_2^2}{z_1 z_2}, \\ \tilde{w}_L^{q\bar{q}} &= 2\omega_{\Delta\Delta}^{q\bar{q}} = g_{q\bar{q};1} \frac{z_1^2 + z_2^2}{z_1 z_2}, \\ \tilde{w}_\Delta^{q\bar{q}} &= g_{q\bar{q};1} \frac{1}{\rho} \frac{z_1^2 - z_2^2}{z_1 z_2}, \\ \tilde{w}_{T_p}^{q\bar{q}} &= g_{q\bar{q};2} \frac{\sqrt{1+\rho^2}}{\rho^2} \frac{z_1^2 + z_2^2}{z_1 z_2}, \\ \tilde{w}_{\nabla_p}^{q\bar{q}} &= g_{q\bar{q};2} \frac{\sqrt{1+\rho^2}}{\rho} \frac{z_1^2 - z_2^2}{z_1 z_2}.\end{aligned}\quad (15)$$

For the qg subprocess, we have

$$\begin{aligned}\tilde{w}_T^{qg} &= g_{qg;1} \frac{1}{\rho^2} \frac{1-z_2}{z_1 z_2} \left(z_2^2 + (1-z_1 z_2)^2 \right. \\ &\quad \left. + \rho^2 \left(1 - \frac{z_1^2}{2} - z_1 z_2 (z_1 + z_2) \right) \right), \\ \tilde{w}_L^{qg} &= 2\omega_{\Delta\Delta}^{qg} = g_{qg;1} \frac{1-z_2}{z_1 z_2} (z_2^2 + (z_1 + z_2)^2), \\ \tilde{w}_\Delta^{qg} &= g_{qg;1} \frac{1}{\rho} \frac{1-z_2}{z_1 z_2} (z_1^2 - 2z_2^2), \\ \tilde{w}_{T_p}^{qg} &= g_{qg;2} \frac{\sqrt{1+\rho^2}}{\rho^2} \frac{1-z_2}{z_1 z_2} (z_2^2 + (1-z_2)^2 - (1-z_1)^2), \\ \tilde{w}_{\nabla_p}^{qg} &= g_{qg;2} \frac{\sqrt{1+\rho^2}}{\rho} \frac{1-z_2}{z_1 z_2} \\ &\quad \times (1 - 2z_2^2 - (1-z_1)^2 + 2z_2(1-z_1)).\end{aligned}\quad (16)$$

Details on how these distributions are changing at changing types of partons are discussed in Ref. [16]. In Eqs. (15) and (16), we introduce couplings $g_{q\bar{q};i}$ and $g_{qg;i}$, defined as

$$g_{q\bar{q};i} = (8\pi^2 e_q^2 \alpha_s) C_{q\bar{q}} g_{EW;i}^{Z\gamma/W}, \quad (17)$$

$$g_{qg;i} = (8\pi^2 e_q^2 \alpha_s) C_{qg} g_{EW;i}^{Z\gamma/W}, \quad (18)$$

where $C_{q\bar{q}} = \frac{4}{9}$, $C_{qg} = \frac{1}{6}$, $C_F = \frac{4}{3}$, $C_A = 3$, and $T_F = \frac{1}{2}$, while e_q is the charge of quarks. For calculation of the T -even structure functions, we need the following combinations of weak couplings and Breit-Wigner propagator of the weak gauge boson:

$$\begin{aligned}g_{EW;1}^{Z\gamma} &= 1 + 2g_{Zq}^V g_{Zl}^V \text{Re}[D_Z(Q^2)] \\ &\quad + ((g_{Zq}^V)^2 + (g_{Zq}^A)^2)((g_{Zl}^V)^2 + (g_{Zl}^A)^2) |D_Z(Q^2)|^2, \\ g_{EW;2}^{Z\gamma} &= 4g_{Zq}^A [2g_{Zq}^V (g_{Zl}^A g_{Zl}^V) |D_Z(Q^2)|^2 + g_{Zl}^A \text{Re}[D_Z(Q^2)]],\end{aligned}\quad (19)$$

where

$$\begin{aligned}g_{Z\ell}^V &= -\frac{1 - 4\sin^2 \theta_W}{2\sin 2\theta_W}, & g_{Z\ell}^A &= -\frac{1}{2\sin 2\theta_W}, \\ g_{Zu}^V &= \frac{1 - 8/3 \sin^2 \theta_W}{2e_q \sin 2\theta_W}, & g_{Zd}^V &= -\frac{1 - 4/3 \sin^2 \theta_W}{2e_q \sin 2\theta_W}, \\ g_{Zu}^A &= \frac{1}{2e_q \sin 2\theta_W}, & g_{Zd}^A &= -\frac{1}{2e_q \sin 2\theta_W},\end{aligned}\quad (20)$$

and

$$\begin{aligned}\text{Re}[D_G(Q^2)] &= \frac{(M_G^2 - Q^2)Q^2}{(M_G^2 - Q^2)^2 + M_G^2 \Gamma_G^2}, \\ \text{Im}[D_G(Q^2)] &= \frac{M_G \Gamma_G Q^2}{(M_G^2 - Q^2)^2 + M_G^2 \Gamma_G^2}.\end{aligned}\quad (21)$$

Here, the Weinberg angle θ_W , masses M_G , and total widths Γ_G of the weak gauge bosons are taken from the Particle Data Group [57]: $\sin^2 \theta_W = 0.23129 \pm 0.00004$, $M_{W^\pm} = 80.3692 \pm 0.0133$ GeV, $M_Z = 91.1880 \pm 0.0020$ GeV, $\Gamma_{W^\pm} = 2.085 \pm 0.042$ GeV, and $\Gamma_Z = 2.4955 \pm 0.0023$ GeV. To perform numerical calculations, we utilize the LHAPDF library [58], specifically employing the CT18NLO parametrization for PDFs from CTEQ-TEA Collaboration [59]. Which order of PDFs needs to be used for computation of different orders of perturbative part calculation is discussed in [60].

III. COMPARISON OF THE PQCD AND GEOMETRIC APPROACH PREDICTIONS WITH DATA

In this section, we compare the predictions for the DY angular coefficients obtained in two theoretical approaches (pQCD approach at NLO [16] and the geometric approach [49,55]) with available data from the LHC experiments (CMS [1], ATLAS [2], and LHCb [3]).

Let us specify how we proceed:

- (1) In our analysis, we use unregularized LHC data for different ranges of rapidity and do not use regularized data from ATLAS Collaboration.
- (2) For every range of rapidity, we take into account results of the geometrical method specified by an appropriate fit of free parameters (see the list of fitted parameters in Table I).
- (3) The geometrical method is based on fitting of the set parameters f, r_i , which occur in the expressions for angular coefficients contributed by the sum $q\bar{q}$ and qg subprocesses.
- (4) In our analysis, we also consider the total sum of subprocesses $q\bar{q}$, $\bar{q}q$, and qg , gq , $\bar{q}g$, and $g\bar{q}$ as separate contributions of the $q\bar{q}$ and qg subprocesses.

- (5) Since experimental measurements were not performed at a fixed value of Q but rather within a Z boson invariant mass window ($Q \in [81, 101]$ GeV for CMS, $Q \in [80, 100]$ GeV for ATLAS, and $Q \in [75, 105]$ GeV for LHCb Collaboration), we simulate this kinematic region in both approaches to ensure a correct comparison with data. For the geometrical method, the central curve was fitted at $Q = 90$ GeV.
- (6) In our plots, we present the bands bounded by the two curves due to a variation of Q from its minimal to maximal experimental values.
- (7) In contrast, for our pQCD calculations we perform a numerical simulation by randomly sampling from a normal distribution within the same Q range as in the corresponding experiment. For each Q_T value, this process generates a band that accounts for both the simulation procedure and numerical integration uncertainties. For clarity, in all subsequent figures, we omit the central curve and retain only the band bounded by the aforementioned features.

A. Angular coefficient A_0

When we include the contribution of all perturbative subprocesses $q\bar{q}$, $\bar{q}q$, qg , gq , $\bar{q}g$, and $g\bar{q}$ to the angular coefficient A_0 , the latter is symmetric under interchange $z_1 \leftrightarrow z_2$ or $y \leftrightarrow -y$. In addition, A_0 has stable behavior at the change of the rapidity range. In Fig. 4, we perform a comparison of theoretical results (pQCD at NLO and the geometrical method) with available data from the LHC experiments (CMS data [1], ATLAS [2], and LHCb [3]). Moreover, Fig. 4 shows the results of the reduced χ^2 test for statistical consistency between theoretical predictions and experimental measurements applied to both theoretical approaches (pQCD at NLO and the geometrical method). One can see that the predictions of both theoretical frameworks demonstrate reasonable agreement with ATLAS and CMS data in the rapidity range $|y| < 2$, although the geometrical method curve displays overfitting behavior in most scenarios. Note, for the ATLAS data in the rapidity range $2 < |y| < 3.5$, agreement is slightly worse for both theoretical frameworks. Furthermore, both pQCD and geometric curves exhibit poor agreement with LHCb data points in the rapidity range $2 < y < 4.5$. It is worth noting that in the LHCb data case, apparent agreement is observed, but the statistical results remain poor. This discrepancy arises due to the limited number of data points and the small experimental uncertainties associated with measurements in the low- Q_T region. (The dependence on the transverse momentum of the hadron and parton, k_T , lies beyond the scope of the collinear approach considered here and is, therefore, not taken into account. Including such a non-perturbative contribution is important in the region of small Q_T and requires calculations within the transverse-momentum-dependent framework. In particular, recent

results presented in Refs. [61,62] demonstrate a good agreement with experimental data for angular coefficients of DY reactions in the low- Q_T region.)

We also show the partial contributions of different subprocesses to the Q_T dependence of the angular coefficient A_0 . In pQCD, the qg subprocess gives the main contribution for the LHC energies due to large-scale dominance of the gluon PDF. We should note that contributions of the partonic subprocesses in the geometrical method are in some inconsistency with the ones produced by the pQCD at NLO. Such a disagreement can be explained by specifics of the fitting procedure in the geometrical method. Besides, the geometrical method does not take into account the behavior of the PDFs. It is important for LHC data with dominant small x ranges, where gluon PDFs play an important role and $qg + \bar{q}g$ contribution is dominant. The geometrical method is not sensitive to parton distribution, and the fit constructed on existing data gives erroneous behavior of prediction. Such data dependence for fit coefficients (we will see it in the next sections) of the geometrical method shrinks the prediction force and, for the part of angular coefficients, gives a quality different behavior on Q_T in comparison with pQCD calculation.

In Fig. 5, for completeness, we also show the Q_T behavior of the convexity asymmetry A_{conv} (6) proposed in Ref. [16]. We compare our predictions with ATLAS [2] and CMS [1] data at $\sqrt{s} = 8$ TeV and in rapidity ranges $0 < |y| < 1$ and $1 < |y| < 2.1$. Here, one needs to note that A_0 and A_{conv} at NLO are stable to a change of rapidity range.

B. Angular coefficient A_1

Next, in Fig. 6 we analyze the Q_T dependence of the angular coefficient A_1 following our strategy in case of A_0 . This coefficient is connected with a single spin-flip hadronic structure function W_Δ . In the rapidity range $0 < |y| < 1$, the geometrical method has poor agreement with CMS data [1] and exhibits good agreement with ATLAS data [2], while pQCD compares well with both datasets, but better with ATLAS measurements. Such a discrepancy with the CMS experiment is due to the broader Q_T range, which results in a larger number of experimental points for comparison. The statistical test against CMS data in the rapidity region $1 < |y| < 2.1$ demonstrates good agreement of the CMS measurements with predictions of both theoretical approaches. In contrast, only the geometrical method's curve yields good agreement with ATLAS data in the rapidity region $1 < |y| < 2$. Finally, there is no agreement of both theoretical approaches with LHCb data for rapidity region $2.0 < y < 4.5$. For pQCD, it is shown that the behavior at small Q_T is corrected by inclusion of the NNLO correction (see Ref. [2]).

Regarding the partial contribution of the partonic subprocesses to the total Q_T dependence of the coefficient A_1

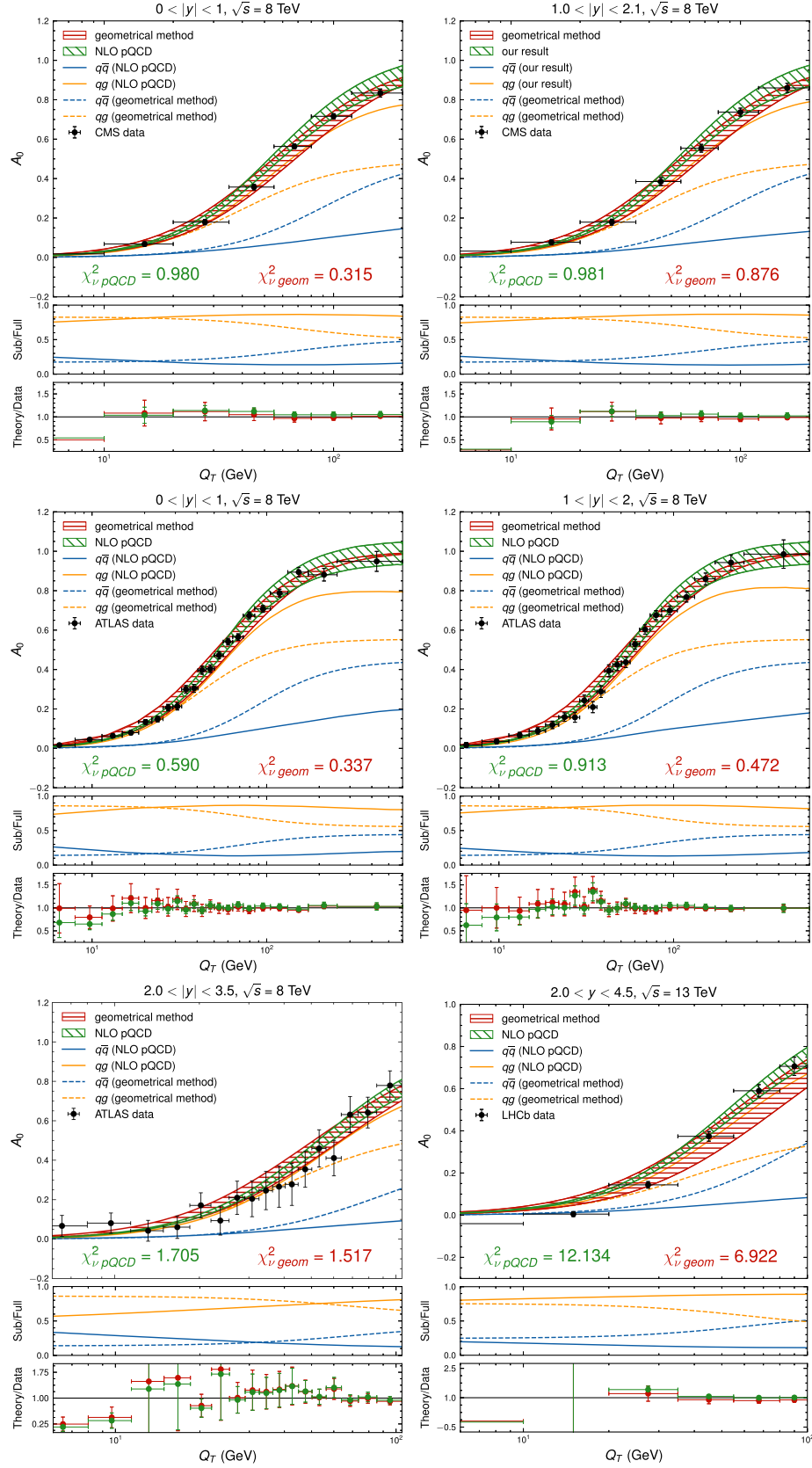


FIG. 4. Angular coefficient A_0 : comparison of theoretical results (pQCD and the geometrical method) with CMS [1], ATLAS [2], and LHCb [3] data at $\sqrt{s} = 8$ TeV in five rapidity regions, $0 < |y| < 1$ and $1 < |y| < 2.1$, $0 < |y| < 1$, $1 < |y| < 2$, and $2 < |y| < 3.5$, and at $\sqrt{s} = 13$ TeV in rapidity region $2 < y < 4.5$.

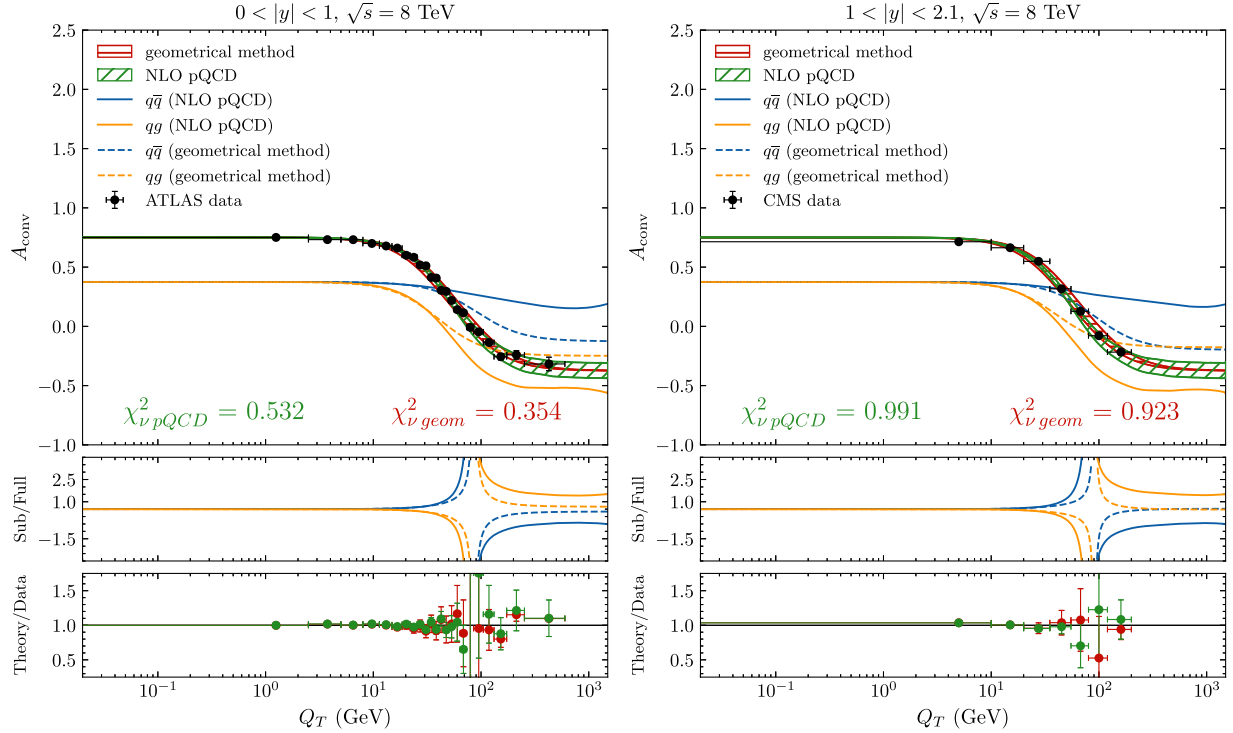


FIG. 5. Convexity asymmetry A_{conv} : comparison of theoretical results (pQCD and the geometrical method) with ATLAS [2] and CMS [1] data at $\sqrt{s} = 8$ TeV in two rapidity regions, $0 < |y| < 1$ and $1 < |y| < 2.1$.

within the pQCD framework, the qg contribution dominates in the whole Q_T range for most of the rapidity regions. However, in the rapidity range $0 < |y| < 1$, the contributions of the $q\bar{q}$ and qg subprocesses become comparable for $Q_T > 200$ GeV. In the case of the geometrical method, the picture changes. As Q_T increases, the ratio between the contributions changes and the contribution of the $q\bar{q}$ subprocess becomes dominant, rather than the qg one.

C. Angular coefficient A_3

Analysis of the angular coefficient A_3 is shown in Fig. 7. Except LHCb, pQCD NLO calculations demonstrate excellent agreement with ATLAS and CMS measurements for all available rapidity ranges. Agreement of the geometric approach with ATLAS and CMS data is slightly worse. In case of A_3 , the qg contribution dominates in both the geometrical method and most of the Q_T range within the pQCD framework. The only regions within the pQCD framework, where the $q\bar{q}$ subprocess exceeds the qg contribution, are those where the value of A_3 approaches zero. We also note that the geometrical method cannot yield negative contributions from individual subprocesses. In contrast, in the rapidity range $|y| < 2.1$, the $q\bar{q}$ subprocess calculated within the pQCD framework contributes negatively to the full A_3 coefficient.

D. Angular coefficient A_4

Finally, we show results for the A_4 coefficient in Fig. 8. One can see that agreement between both theoretical frameworks and LHC data is excellent except some experimental points with big statistical errors. Comparing the individual contributions of the partonic subprocesses, we see that in the low- Q_T region, pQCD predicts a dominance of the $q\bar{q}$ subprocess, while the qg contribution approaches zero. As Q_T increases, the contribution of the qg subprocess dominates over the $q\bar{q}$ one. A similar behavior of the individual contributions of the partonic subprocesses occurs in the case of the geometrical method. However, key differences from pQCD calculations emerge in the details of this behavior. Referring to Fig. 8, the $q\bar{q}$ and qg subprocesses for geometrical method prediction by fitting ATLAS and LHCb data obtain reverse roles relative to pQCD predictions. Additionally, both subprocesses contribute with comparable weight to the A_4 value. However, comparing theoretical approaches with CMS data, we see that the hierarchy of the subprocess contributions in the geometric approach aligns with predictions of pQCD. A key observation is that the geometric curve, when fitted to the CMS and ATLAS data within the same rapidity range $0 < |y| < 1$, shows significantly different ratios of the $q\bar{q}$ and qg subprocess contributions. This inconsistency highlights a notable limitation of the

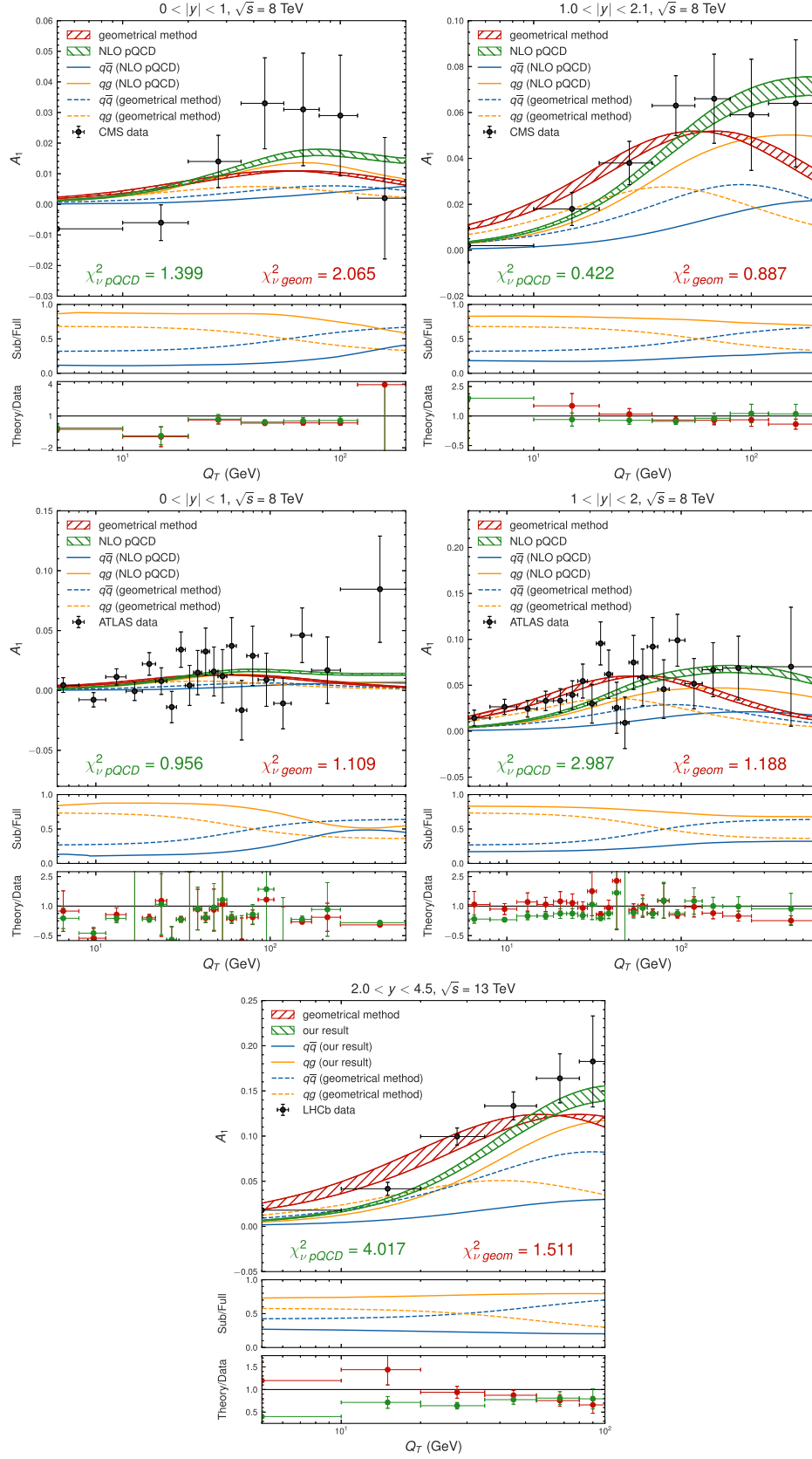


FIG. 6. Angular coefficient A_1 : comparison of theoretical results (pQCD and the geometrical method) with CMS [1], ATLAS [2], and LHCb [3] data at $\sqrt{s} = 8$ TeV in three rapidity regions, $0 < |y| < 1$, $1 < |y| < 2$, and $1 < |y| < 2.1$, and at $\sqrt{s} = 13$ TeV in rapidity region $2 < y < 4.5$.

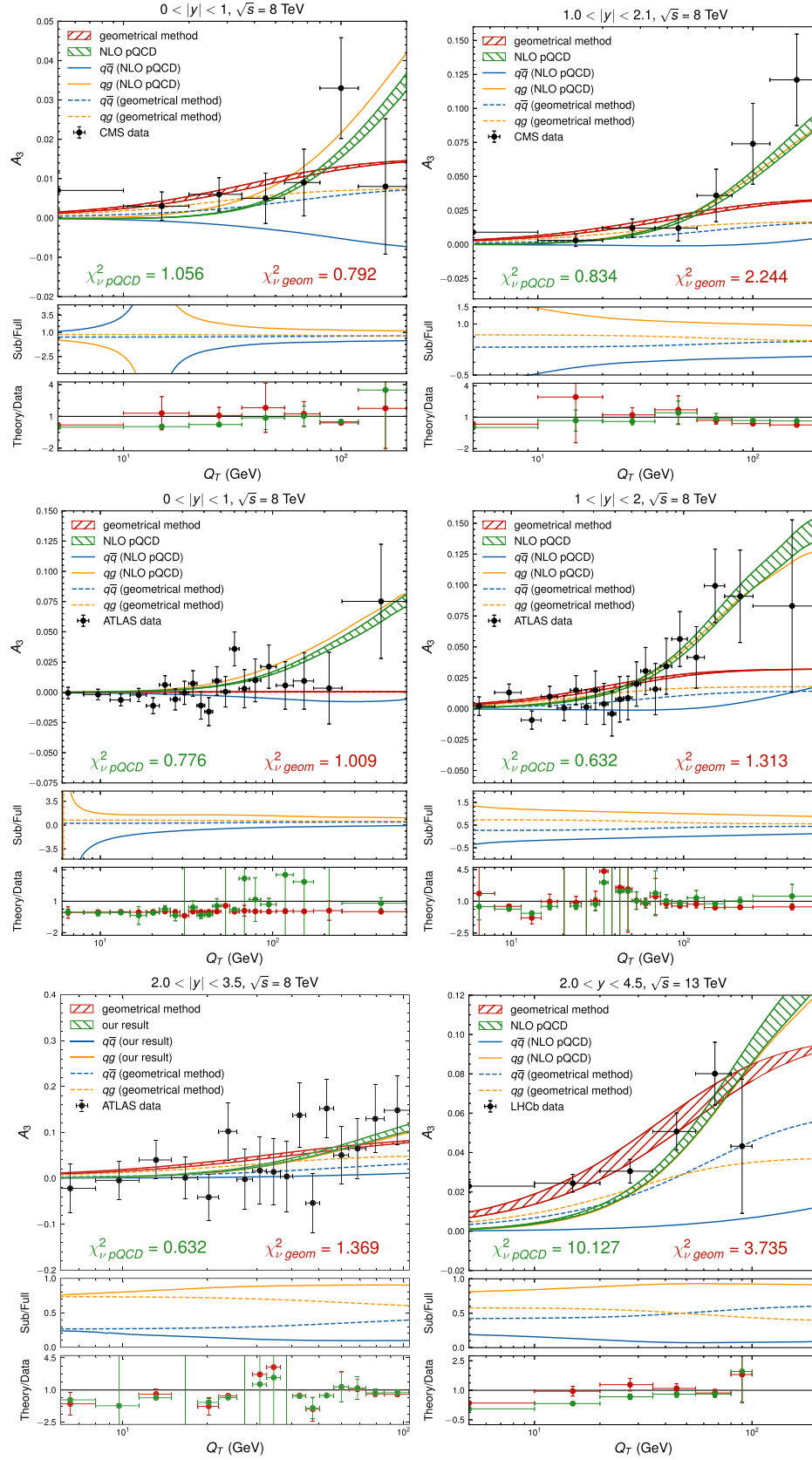


FIG. 7. Angular coefficient A_3 : comparison of theoretical results (pQCD and the geometrical method) with CMS [1], ATLAS [2], and LHCb [3] data at $\sqrt{s} = 8$ TeV in four rapidity regions, $0 < |y| < 1$, $1 < |y| < 2$, $1 < |y| < 2.1$, and $2 < |y| < 3.5$, and at $\sqrt{s} = 13$ TeV in rapidity region $2 < |y| < 4.5$.

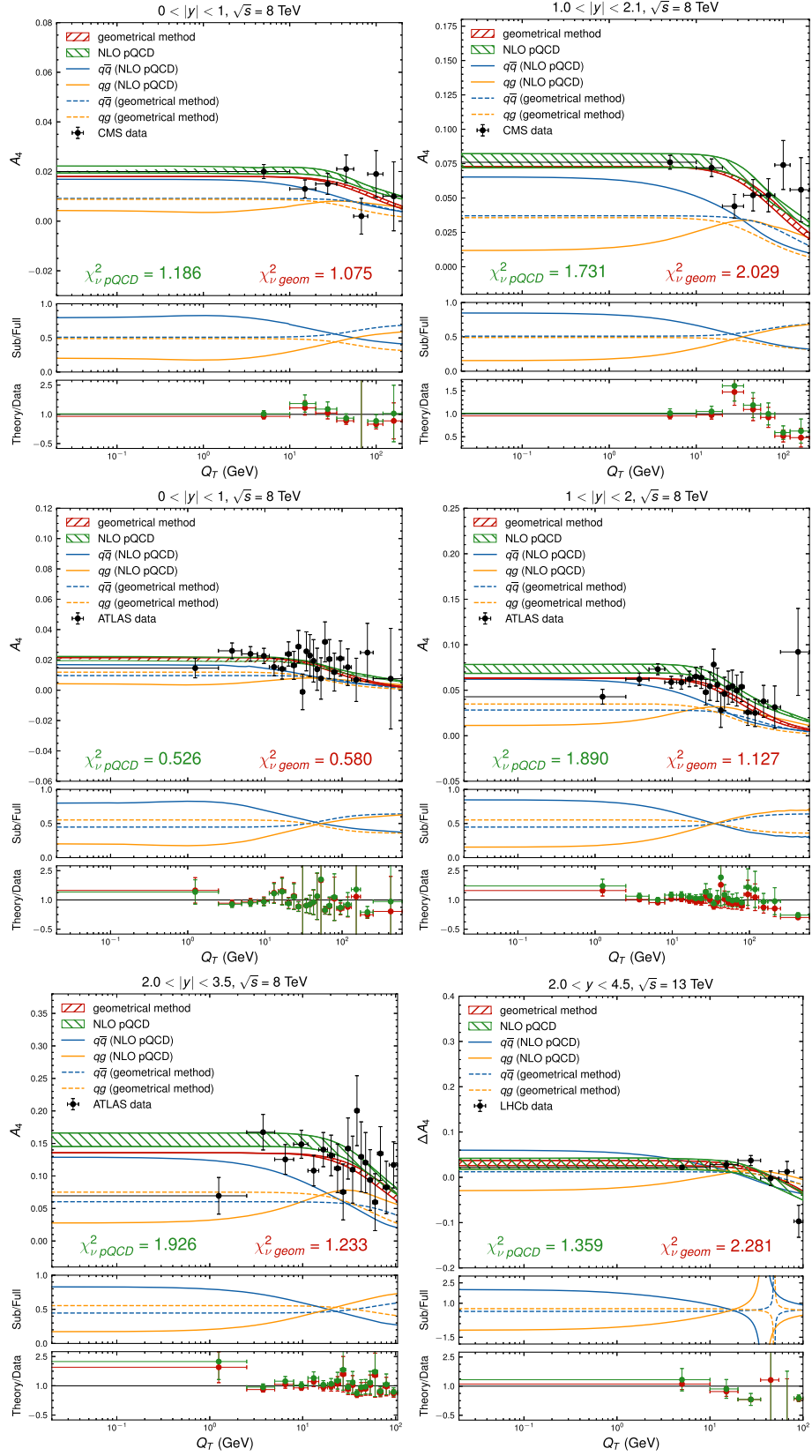


FIG. 8. Angular coefficient A_4 : comparison of theoretical results (pQCD and the geometrical method) with CMS [1], ATLAS [2], and LHCb [3] data at $\sqrt{s} = 8$ TeV in four rapidity regions, $0 < |y| < 1$, $1 < |y| < 2$, $1 < |y| < 2.1$, and $2 < |y| < 3.5$, and at $\sqrt{s} = 13$ TeV in rapidity region $2 < y < 4.5$.

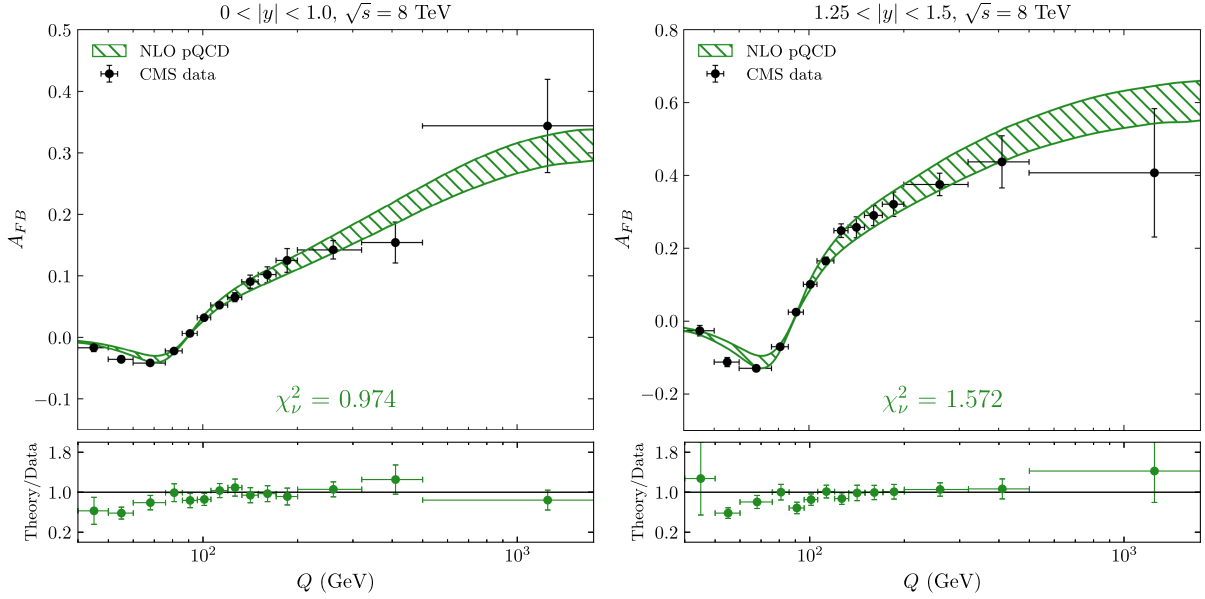


FIG. 9. Angular coefficient A_{FB} : comparison of pQCD at NLO with CMS data [63] at $\sqrt{s} = 8$ in two rapidity regions, $0 < |y| < 1$ and $1.25 < |y| < 1.5$.

geometrical method. The LHCb data are presented by analysis of ΔA_4 , which is defined as A_4 minus its mean value. Notably, ΔA_4 is the only dataset from LHCb Collaboration where both pQCD and the geometrical method predictions demonstrate at least moderate agreement with LHC experiments. These data include a wide diapason of rapidity. Both approaches show agreement, but the geometrical method is sensitive to data fitting.

In Fig. 9, we also show the Q behavior of the A_{FB} asymmetry parameter (6) and compare our NLO predictions with statistical analysis with CMS data [63] at $\sqrt{s} = 8$ in two rapidity regions, $0 < |y| < 1$ and $1.25 < |y| < 1.5$.

IV. SUMMARY

In this paper, we analyzed and compared the predictions for the DY angular coefficients A_0 , A_1 , A_3 , and A_4 in two theoretical approaches—pQCD at NLO based on the collinear factorization [14–16] and the geometrical method [48–50]—with LHC data collected by the CMS [1], ATLAS [2], and LHCb [3] experiments across various rapidity and kinematical regions and employing the reduced χ^2 statistical test. We found that the χ^2 statistical test shows an approximate consistency of both theoretical methods. The Q_T behavior of the A_0 and A_4 coefficients is similar in both approaches. However, these methods disagree in the predictions for the angular coefficients A_1 and A_3 at high- Q_T regions, even though their statistical outcomes remain comparable. These angular coefficients are related to single spin-flip hadronic structure functions. Notably, significant limitations

of the geometrical method were identified, stemming from its reliance on a fitting procedure central to the approach. It cannot predict the angular coefficients' behavior in a specific rapidity range without experimental data to guide the fit. Moreover, the geometric approach can yield noticeably different ratios of quark-antiquark and quark-gluon contributions to the total angular coefficient when parameters are fitted to different experimental datasets within the same rapidity range.

Poor agreement for the angular coefficients involving single spin-flip hadronic structure functions can be explained by the fact that the simple rotation done in the geometrical method is not sensitive to rapidity and does not reproduce behavior for different contributions. Not taking into account the parton distribution gives incorrect dependencies for parton contributions and spoils the predictive power for the geometrical method. In the near future, we plan to study the full rapidity dependence of the angular coefficients occurring in the DY process and extend our analysis to the α_s^2 order in strong coupling expansion.

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DATA AVAILABILITY

Our numerical analysis data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: SUMMARY OF REDUCED χ^2 TEST RESULTS

Here, in Tables II–V, we present the complete set of reduced χ^2 statistical test results, including additional pQCD calculations performed using LO PDFs.

TABLE II. Statistical tests of geometrical and pQCD predictions for the A_0 angular coefficient against experimental data from CMS [1], ATLAS [2], and LHCb [3] Collaborations.

A_0	Data	Rapidity $ y $	χ^2_ν		
			LO PDF	NLO PDF	Geometric
CMS		[0, 1]	0.774	0.980	0.315
		[1, 2.1]	0.949	0.981	0.876
ATLAS		[0, 1]	0.509	0.590	0.337
		[1, 2]	0.877	0.913	0.472
		[2, 3.5]	1.749	1.705	1.517
LHCb		[2, 4.5]	12.689	12.134	6.922

TABLE III. Statistical tests of geometrical and pQCD predictions for the A_1 angular coefficient against experimental data from CMS [1], ATLAS [2], and LHCb [3] Collaborations.

A_1	Data	Rapidity $ y $	χ^2_ν		
			LO PDF	NLO PDF	Geometric
CMS		[0, 1]	1.551	1.399	2.065
		[1, 2.1]	1.372	0.422	0.887
ATLAS		[0, 1]	1.002	0.956	1.109
		[1, 2]	4.408	2.987	1.188
LHCb		[2, 4.5]	5.903	4.017	1.511

TABLE IV. Statistical tests of geometrical and pQCD predictions for the A_3 angular coefficient against experimental data from CMS [1], ATLAS [2], and LHCb [3] Collaborations.

A_3	Data	Rapidity $ y $	χ^2_ν		
			LO PDF	NLO PDF	Geometric
CMS		[0, 1]	1.134	1.056	0.792
		[1, 2.1]	0.927	0.834	2.244
ATLAS		[0, 1]	0.733	0.776	1.009
		[1, 2]	0.622	0.632	1.313
		[2, 3.5]	0.645	0.632	1.369
LHCb		[2, 4.5]	10.346	10.127	3.735

TABLE V. Statistical tests of geometrical and pQCD predictions for the A_4 angular coefficient against experimental data from CMS [1], ATLAS [2], and LHCb [3] Collaborations.

A_4	Data	Rapidity $ y $	χ^2_ν		
			LO PDF	NLO PDF	Geometric
CMS		[0, 1]	1.189	1.186	1.075
		[1, 2.1]	1.713	1.731	2.029
ATLAS		[0, 1]	0.523	0.526	0.580
		[1, 2]	1.953	1.890	1.127
		[2, 3.5]	1.800	1.926	1.233
LHCb		[2, 4.5]	1.347	1.359	2.281

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- [1] V. Khachatryan *et al.* (CMS Collaboration), *Phys. Lett. B* **750**, 154 (2015).
[2] G. Aad *et al.* (ATLAS Collaboration), *J. High Energy Phys.* **08** (2016) 159.
[3] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **129**, 091801 (2022).
[4] M. Aghasyan *et al.* (COMPASS Collaboration), *Phys. Rev. Lett.* **119**, 112002 (2017).
[5] G. D. Alexeev *et al.* (COMPASS Collaboration), *Phys. Rev. Lett.* **133**, 071902 (2024).
[6] B. Adams *et al.*, arXiv:1808.00848.
[7] L. Y. Zhu *et al.* (NuSea Collaboration), *Phys. Rev. Lett.* **99**, 082301 (2007).
[8] T. Aaltonen *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **106**, 241801 (2011).
[9] L. Adamczyk *et al.* (STAR Collaboration), *Phys. Rev. Lett.* **116**, 132301 (2016).
[10] C. Aidala *et al.* (PHENIX Collaboration), *Phys. Rev. D* **99**, 072003 (2019).
[11] A. Chen *et al.* (SeaQuest Collaboration), *Proc. Sci. SPIN2018* (2019) 164.
[12] A. Accardi *et al.*, *Eur. Phys. J. A* **52**, 268 (2016).
[13] S. Alekhin *et al.*, *Eur. Phys. J. C* **85**, 406 (2025).
[14] D. Boer and W. Vogelsang, *Phys. Rev. D* **74**, 014004 (2006).
[15] E. L. Berger, J.-W. Qiu, and R. A. Rodriguez-Pedraza, *Phys. Rev. D* **76**, 074006 (2007).
[16] V. E. Lyubovitskij, A. S. Zhevlakov, and I. A. Anikin, *Phys. Rev. D* **110**, 074028 (2024).
[17] E. Mirkes, *Nucl. Phys.* **B387**, 3 (1992).

- [18] M. Lambertsens and W. Vogelsang, *Phys. Rev. D* **93**, 114013 (2016).
- [19] R. Hamberg, W. L. van Neerven, and T. Matsuura, *Nucl. Phys. B* **359**, 343 (1991); **B644**, 403(E) (2002).
- [20] W. L. van Neerven and E. B. Zijlstra, *Nucl. Phys. B* **382**, 11 (1992); **B680**, 513(E) (2004).
- [21] C. Anastasiou, L. J. Dixon, K. Melnikov, and F. Petriello, *Phys. Rev. Lett.* **91**, 182002 (2003).
- [22] K. Melnikov and F. Petriello, *Phys. Rev. Lett.* **96**, 231803 (2006).
- [23] S. Catani, L. Cieri, G. Ferrera, D. de Florian, and M. Grazzini, *Phys. Rev. Lett.* **103**, 082001 (2009).
- [24] R. Gavin, Y. Li, F. Petriello, and S. Quackenbush, *Comput. Phys. Commun.* **182**, 2388 (2011).
- [25] R. Gavin, Y. Li, F. Petriello, and S. Quackenbush, *Comput. Phys. Commun.* **184**, 209 (2013).
- [26] C. Duhr, F. Dulat, and B. Mistlberger, *Phys. Rev. Lett.* **125**, 172001 (2020).
- [27] J. Baglio, C. Duhr, B. Mistlberger, and R. Szafron, *J. High Energy Phys.* **12** (2022) 066.
- [28] S. Camarda, L. Cieri, and G. Ferrera, *Eur. Phys. J. C* **82**, 575 (2022).
- [29] U. Baur, O. Brein, W. Hollik, C. Schappacher, and D. Wackerroth, *Phys. Rev. D* **65**, 033007 (2002).
- [30] S. Dittmaier and M. Krämer, *Phys. Rev. D* **65**, 073007 (2002).
- [31] A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, G. Nanava, and R. Sadykov, *Eur. Phys. J. C* **46**, 407 (2006); **50**, 505(E) (2007).
- [32] A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, G. Nanava, and R. Sadykov, *Eur. Phys. J. C* **54**, 451 (2008).
- [33] C. M. Carloni Calame, G. Montagna, O. Nicrosini, and A. Vicini, *J. High Energy Phys.* **10** (2007) 109.
- [34] R. Gauld, A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, and A. Huss, *J. High Energy Phys.* **11** (2017) 003.
- [35] A. D. Martin, R. G. Roberts, W. J. Stirling, and R. S. Thorne, *Eur. Phys. J. C* **39**, 155 (2005).
- [36] J. Bellm *et al.*, *Eur. Phys. J. C* **76**, 196 (2016).
- [37] Y. Li and F. Petriello, *Phys. Rev. D* **86**, 094034 (2012).
- [38] E. Mirkes and J. Ohnemus, *Phys. Rev. D* **50**, 5692 (1994).
- [39] C. S. Lam and W.-K. Tung, *Phys. Rev. D* **18**, 2447 (1978).
- [40] E. Accomando *et al.*, *J. High Energy Phys.* **10** (2019) 176.
- [41] S. Alekhin *et al.*, *Eur. Phys. J. C* **75**, 304 (2015).
- [42] S. Amoroso, J. Fiaschi, F. Giuliani, A. Glazov, F. Hautmann, and O. Zenaiev, *Phys. Lett. B* **821**, 136613 (2021).
- [43] C. Balazs and C. P. Yuan, *Phys. Rev. D* **56**, 5558 (1997).
- [44] G. A. Ladinsky and C. P. Yuan, *Phys. Rev. D* **50**, R4239 (1994).
- [45] S. Camarda *et al.*, *Eur. Phys. J. C* **80**, 251 (2020); **80**, 440(E) (2020).
- [46] S. Camarda, L. Cieri, and G. Ferrera, *Phys. Rev. D* **104**, L111503 (2021).
- [47] S. Camarda, L. Cieri, and G. Ferrera, *Phys. Lett. B* **845**, 138125 (2023).
- [48] E. N. Argyres and C. S. Lam, *Phys. Rev. D* **26**, 114 (1982).
- [49] W.-C. Chang, R. E. McClellan, J.-C. Peng, and O. Teryaev, *Phys. Rev. D* **96**, 054020 (2017).
- [50] Y. Lyu, W.-C. Chang, R. E. McClellan, J.-C. Peng, and O. Teryaev, *Phys. Rev. D* **103**, 034011 (2021).
- [51] J. C. Collins and D. E. Soper, *Phys. Rev. D* **16**, 2219 (1977).
- [52] V. E. Lyubovitskij, W. Vogelsang, F. Wunder, and A. S. Zhevlakov, *Phys. Rev. D* **109**, 114023 (2024).
- [53] C. S. Lam and W.-K. Tung, *Phys. Lett. B* **80**, 228 (1979).
- [54] J. C. Collins, D. E. Soper, and G. F. Sterman, *Nucl. Phys. B* **250**, 199 (1985).
- [55] J.-C. Peng, W.-C. Chang, R. E. McClellan, and O. Teryaev, *Phys. Lett. B* **758**, 384 (2016).
- [56] P. Faccioli, C. Lourenco, J. Seixas, and H. K. Wohri, *Phys. Rev. D* **83**, 056008 (2011).
- [57] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [58] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht, M. Schönherr, and G. Watt, *Eur. Phys. J. C* **75**, 132 (2015).
- [59] M. Yan, T.-J. Hou, P. Nadolsky, and C. P. Yuan, *Phys. Rev. D* **107**, 116001 (2023).
- [60] A. Sherstnev and R. S. Thorne, *Eur. Phys. J. C* **55**, 553 (2008).
- [61] S. Piloneta and A. Vladimirov, *J. High Energy Phys.* **12** (2024) 059.
- [62] A. Arroyo-Castro, I. Scimemi, and A. Vladimirov, *J. High Energy Phys.* **06** (2025) 202.
- [63] V. Khachatryan *et al.* (CMS Collaboration), *Eur. Phys. J. C* **76**, 325 (2016).