

Three-loop QCD corrections to the production of a Higgs boson and a jet

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We compute three-loop QCD corrections to the scattering amplitude of a Higgs boson and three partons interfered with its tree level counterpart. Specifically, we derive our results in the generalized leading color limit. Our results are represented in terms of so-called multiple polylogarithms and thus are ready for use in phenomenological predictions. We provide the three-loop amplitudes necessary to compute the production cross section of a Higgs boson and a hadronic jet at hadron colliders and the decay probability of a Higgs boson to three hadronic jets at lepton colliders.

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Introduction. The discovery of the Higgs boson by the ATLAS [1] and CMS [2] experiment at the Large Hadron Collider (LHC) ushered in a new era of exploration of fundamental forces. For the first time we gained experimental access to probe the interactions of all fundamental particles of the Standard Model and thus a brand new opportunity to test the foundations of our understanding of nature. Today, we explore the rich implications of the Higgs boson by performing high precision comparison of experimental data with theoretical predictions for the outcome of LHC scattering experiments. The large datasets already collected by the LHC and in particular, the advent of the upcoming high-luminosity phase [3], enable a precision phenomenology program testing the Standard Model and searching for new physics beyond this paradigm. The high experimental precision must be matched by equally precise theoretical predictions based on the cutting edge of our understanding of quantum field theory (QFT). In this Letter, we achieve a new milestone of precision calculations at the core of such predictions.

Precision computations for the LHC are performed exploiting the factorization of high energetic scattering processes of the QCD partons—the constituents of the two scattering protons—and the low-energy dynamics of the individual protons. Within this framework, we compute scattering cross sections based on the first principle Lagrangian of the Standard Model or other theories of

nature using perturbative QFT. Precision is achieved by computing cross sections to higher and higher power in an expansion in coupling constants. The largest of all coupling constants at LHC energies is the coupling of the strong force α_s . Achieving high order results in the expansion in powers of α_s is a formidable task challenging our technical and theoretical abilities of using QFT.

An overwhelming number of about ninety percent of all Higgs bosons at the LHC are produced via the gluon fusion mechanism. This mechanism describes the fusion of two initial state gluons via a virtual top-quark loop which in turn radiates the Higgs boson. Perturbative QCD corrections for this production mechanism are notoriously large, and consequently, very high order predictions are required to rival experimental precision. A powerful approach is the construction of an effective field theory [4–7] integrating out the degrees of freedom of the top quark, yielding the first term in an expansion in inverse powers of the large top quark mass. This approach reduces the top-quark loop at Born level to a pointlike interaction coupling the Higgs field H directly to the gluon field strength $G_a^{\mu\nu}$ via a Wilson coefficient [8–12] C^0 .

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM,5} - \frac{C^0}{4} H G_a^{\mu\nu} G_{a,\mu\nu}. \quad (1)$$

The application of this effective field theory has enabled frontier computations of the inclusive Higgs production cross section through next-to-next-to-next-to-leading order ($N^3\text{LO}$) in QCD perturbation theory [13–18], and predictions including the massive top quark were obtained already through next-to-next-to-leading order (NNLO) [19–23]. Higgs phenomenology is however much richer than only inclusive rates, and a vast range of observables differential in the decay products of the Higgs boson allow us to study this new particle in great detail. The advances in

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our understanding of the differential cross section of the past two decades enabled the computation of fully differential predictions for the production of a Higgs boson through $N^3\text{LO}$ in QCD perturbation theory [24–31]. A leap in our ability to perform differential computations with hadronic jets occurred about one decade ago [32–39], leading to fully differential predictions of the gluon fusion production of a Higgs boson in association with a jet at NNLO [40–43]. The additional final state jet is a requirement for any observable dependent on the Higgs boson having nonvanishing transverse momentum and enables us to further broaden the set of observables of Higgs boson physics. Extending predictions for the production of a Higgs boson in association with a jet to $N^3\text{LO}$ remains a formidable challenge. Motivated by the ambition to enable one-percent precision phenomenology at the LHC [44] we take here a decisive step toward this goal.

An essential ingredient of perturbative corrections is the availability of scattering amplitudes to the required order in perturbation theory. In this Letter, we focus on three-loop corrections to amplitudes describing the scattering of a Higgs boson and three partons. Results at one and two loops for these scattering amplitudes were obtained in Refs. [45–50]. The past computations of these scattering amplitudes have provided not only key ingredients to today’s LHC phenomenology program but also led to deeper understanding of the functions comprising these amplitudes (see for example Refs. [51,52]), to improved computational techniques [53], and to a flurry of activity of understanding formal gauge theory [54–56,56–58]. The technology enabling the computation of these particular scattering amplitudes is then applicable to a range of scattering processes, most notably the scattering amplitudes relating three partons and a vector boson [59–62].

Figure 1 shows an example Feynman diagram contributing to the three-loop corrections of the scattering amplitude relating a Higgs boson and three gluons. To tame the immense complexity of these amplitudes we restrict our computation to the generalized leading color contributions, treating the number of colors N_c of QCD and number of light quark flavors n_f as large and neglect corrections that are suppressed beyond the leading term in these parameters by inverse powers of the number of colors. Due to the direct coupling of the Higgs boson to gluons in the effective theory of Eq. (1) and the color-singlet nature of the Higgs, the generalized leading color limit of the scattering

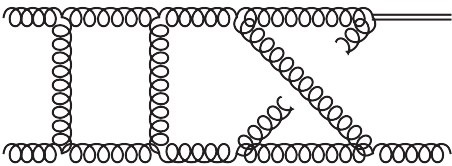


FIG. 1. Example diagram for the leading-color, three-loop scattering amplitude of three gluons and a Higgs boson.

amplitude is not only given by so-called planar diagrams [63]. Nonplanar diagrams, such as the one displayed in Fig. 1, drastically increase the complexity of perturbative computations and put our results at the forefront of the field.

Our results will find application beyond the prediction of the production cross section for a Higgs boson and a jet. First, our scattering amplitudes are a natural ingredient to the inclusive production probability of a Higgs boson at the next order, $N^4\text{LO}$. Furthermore, the advent of future lepton collider facilities promises to study hadronic decays of the Higgs in astounding detail. A range of planned facilities like the FCC-ee [64,65], CEPC [66], ILC [67], C^3 [68], or future muon colliders would provide the opportunity to study jets as decay products of the Higgs boson. While inclusive hadronic decay rates are known to be of astounding fourth order in QCD [69–72], differential two and three jets rates are currently available through $N^3\text{LO}$ [73] and NNLO [74,75], respectively. Advancing the latter to $N^3\text{LO}$ requires the computation of the scattering amplitudes computed here.

Setup. We compute three-loop scattering amplitudes relating a Higgs boson H with momentum q and three partons with momenta k_i , $i \in \{1, 2, 3\}$. We start by considering the *decay region*, which describes the process of the Higgs boson decaying to three gluons or to a gluon and a quark antiquark pair.

$$\begin{aligned} H(q) &\rightarrow g(k_1) + g(k_2) + g(k_3), \\ H(q) &\rightarrow g(k_1) + q(k_2) + \bar{q}(k_3). \end{aligned} \quad (2)$$

The scattering amplitude $\mathcal{A}_X$ for the process X (e.g., $X = H \rightarrow ggg$) can then be expanded in powers of the bare strong coupling constant $\alpha_S = \frac{g_S^2}{4\pi}$, as

$$\mathcal{A}_X(\alpha_S, C^0) = g_S C^0 \sum_{n=0}^{\infty} a_S^n \mathcal{A}_X^{(n)}, \quad a_S = \frac{\alpha_S}{\pi} N_e^{-1}, \quad (3)$$

where $N_e = e^{\epsilon\gamma_E} (4\pi)^{-\epsilon}$. Throughout this Letter we work in the framework of dimensional regularization with $d = 4 - 2\epsilon$ spacetime dimensions in the modified minimal subtraction scheme $\overline{\text{MS}}$ to regulate infrared and ultraviolet singularities of our scattering amplitudes. The definition of a_S in the above equation anticipates renormalization in the $\overline{\text{MS}}$ by absorbing universal factors of (4π) and the Euler-Mascheroni constant γ_E . To compute the ingredient necessary for physical scattering cross sections we multiply our scattering amplitudes with their tree level counterparts and sum over spin and color degrees of freedom of the external partons. We denote the resulting interfered matrix elements by

$$\mathcal{M}_X^{(n)} = \sum_{\text{spin,color}} \mathcal{A}_X^{(n)} \mathcal{A}_X^{\dagger,(0)}. \quad (4)$$

These matrix elements are scalar functions of the Lorentz invariant scalar products of the external momenta, and we choose in particular the parametrization

$$s = \frac{(k_1 + k_2)^2}{q^2}, \quad t = \frac{(k_2 + k_3)^2}{q^2}, \quad u = \frac{(k_1 + k_3)^2}{q^2}. \quad (5)$$

Note that $k_i^2 = 0$ and $q^2 = m_h^2$ for an on-shell Higgs boson. The three variables are constrained by the identity

$$1 = s + t + u, \quad (6)$$

and we choose t and u as our independent variables alongside q^2 , which carries the energy dimension of the amplitude. In addition, we introduce the variable $Q^2 = -q^2 - i0$, which has an infinitesimal imaginary part such that the matrix elements in decay kinematics expressed in terms of Q^2 , t , and u do not have any explicit imaginary parts. In decay kinematics we find

$$s > 0, \quad t > 0, \quad u > 0. \quad (7)$$

There are three distinct matrix elements required for the production of a Higgs boson and a parton in the collision of two partons. We refer to this configuration as *scattering region*.

$$\begin{aligned} g(-k_1) + g(-k_2) &\rightarrow H(q) + g(k_3), \\ g(-k_1) + q(-k_2) &\rightarrow H(q) + q(k_3), \\ q(-k_1) + \bar{q}(-k_2) &\rightarrow H(q) + g(k_3). \end{aligned} \quad (8)$$

We obtain the associated matrix elements by crossing the decay matrix elements and permuting of external momenta. Exchanging quarks and antiquarks leaves the interfered matrix elements invariant, and all other configurations can simply be obtained by relabeling of the momenta. In the scattering region, we find that

$$s > 0, \quad t < 0, \quad u < 0. \quad (9)$$

For convenience, we introduce the variables $\hat{t} = -t$ and $\hat{u} = -u$ with which we express our scattering matrix elements in the scattering region.

The main results of this Letter are the scattering matrix elements $\mathcal{M}_X^{(n)}$ through three loops, $n \in \{0, 1, 2, 3\}$. Through two loops we perform our computation for QCD with n_f massless quark flavors and general color gauge group of $SU(N_c)$. In the case of the three-loop amplitudes, we limit our computation to the generalized leading color limit. In this limit we treat the number of colors N_c and flavors n_f as large and neglect contributions suppressed by

powers of $1/N_c^2$. This generalized leading color limit has shown to be a highly effective approximation for similar scattering amplitudes [61,76,77], as can be easily understood from the fact that it parametrically approximates matrix elements up to $\mathcal{O}(10\%)$ corrections. Specifically, this implies that we include the coefficients of the terms proportional to any one of the factors $\{N_c^6, N_c^5 n_f, N_c^4 n_f^2, N_c^3 n_f^3\}$ when all three external partons are gluons and $\{N_c^5, N_c^4 n_f, N_c^3 n_f^2, N_c^2 n_f^3\}$ otherwise. Our results are expressed in terms of a Laurent expansion in the dimensional regulator ϵ including all terms required for a N³LO computation of the discussed scattering and decay processes. The coefficients of these expansions are expressed in terms of ratios of polynomials and generalized polylogarithms [78] of our kinematic variables.

Calculation. We generate the Feynman diagrams for our scattering amplitudes with QGRAF [79]. We compute gauge invariant scattering amplitudes in conventional dimensional regularization using tensor decomposition (see, for example, Refs. [49,53,59,80,81]) through two loops and interferences of tree and three-loop scattering amplitudes. We use our in-house *Mathematica* and C++ packages to compute our amplitudes, including contracting the Lorentz and color indices. Then, we express all the squared amplitudes as linear combinations of scalar integrals and classify them in terms of integral families. We utilize the package CalcLoop [82] to fully exploit external symmetry among Feynman integrals. Once we identify a minimal set of Feynman integrals that can no longer be reduced by symmetry relations of the external and loop momenta, we employ the package Blade to perform integration-by-part (IBP) [83–85,85] reductions to reduce our integrals to a set of so-called master integrals. This step represents one of the most challenging portions of our computation due to the high algebraic complexity of rank-six integrals in non-planar topologies.

Specifically, we adopt an improved seeding strategy [86,87]¹ and divide the reduction into spanning sectors [86]. This reduces the memory usage of the numerical IBP reduction to $\mathcal{O}(100)$ GB and allows the calculation to be carried out on typical high-performance computing infrastructure, rather than requiring custom computing

¹In Blade, improved seeding is implemented through the option `FilterLevel`. Given an initial rank r_0 and assuming that the top sector contains t_0 propagators, one specifies a vector

$$\vec{c} = \{c_0, \dots, c_k\}$$

such that an integral with t propagators and rank $\leq r_0 + c_{t_0-t}$ is chosen as a seed, with $c_i = c_k$ for $i > k$. In practice, we determine a suitable choice of $\vec{c}$ empirically by testing lower-rank reductions, for instance $r = 3, 4, 5$, and then apply the same choice to the rank-six reduction. Choices such as $\{0, -1, -2, -3\}$, $\{1, 0, -1, -2\}$, and $\{1, 1, 0, -1, -2\}$ are frequently used.

hardware. The improved seeding and spanning-sector strategies also improve numerical efficiency.

We use a relatively small number of numerical IBP runs, $\mathcal{O}(10^2\text{--}10^3)$, to construct the block-triangular form [88,89].² For the most complicated spanning sectors, the resulting block-triangular form typically contains $\mathcal{O}(10^4)$ equations, which is significantly smaller than the number of IBP equations, $\mathcal{O}(10^6)$. We then solve this much more efficient block-triangular system to sample a large number of points, $\mathcal{O}(10^4\text{--}10^5)$, for functional reconstruction. The memory consumption for solving the block-triangular form is negligible, which allows us to make use of all available CPUs. The finite-field computations are performed using FiniteFlow [90].

We also make practical modifications to Blade to reduce I/O overhead³ and to enable parallelization across prime fields. Finally, we exploit external-momentum symmetries to reduce the master integrals in each family to a minimal set containing $\mathcal{O}(1500)$ elements.

To compute the residual set of master integrals we make use of the method of differential equations [91–95] and relate them to a basis of so-called canonical master integrals [95]. In particular, all genuinely three-loop canonical integrals were identified in Refs. [52,96–99], and we relate our master integrals to the canonical results in the literature. We checked the existing results for canonical differential equations. For the remaining one-loop and two-loop integrals, we find a canonical set of master integrals using algorithmic methods [100]. We compute boundary conditions for the differential equations using the methods of for example Refs. [101–103] and confirm existing results in the decay region. We solve the canonical master integrals in terms of Chen iterated integrals [104] and insert them into our newly computed scattering amplitudes in the decay region. Next, we rewrite the remaining iterated integrals in terms of multiple polylogarithms [78] using methods outlined for example in Refs. [105–110]. The functions appearing in these scattering amplitudes were studied in some detail in Ref. [52]. The multiple polylogarithms can readily be evaluated using public software, see for example Refs. [111,112]. To obtain results for amplitudes in the scattering region we cross the external particles accordingly and analytically continue our functions.

Finite amplitudes—UV and IR subtraction. The infrared and ultraviolet singularities of our scattering amplitudes are

²The search for a block-triangular form is essentially based on making an ansatz for linear relations among Feynman integrals and solving this ansatz using numerical IBP solutions. We distinguish the dependence on ϵ from that on $\{t, u\}$ and make use of the mass-dimension properties of $\{t, u\}$ in constructing the ansatz. This reduces the number of unknowns and accelerates the search.

³We divide the reduction tables into smaller chunks and merge the spanning-sector results chunk by chunk.

manifested in terms of poles in the dimensional regulator ϵ . The poles of an amplitude at n th order are described by the following universal formula in terms of its lower order counterparts such that we can define a finite remainder $\mathcal{A}_X^F$:

$$\begin{aligned}\mathcal{A}_X^F &= \mathbf{Z}(\alpha_S(\mu^2), \{p_i\}) \mathcal{A}_X(\alpha_S, C^0) \\ &= \mathbf{Z}(\alpha_S(\mu^2), \{p_i\}) \mathcal{A}_X(\mathbf{Z}_{\alpha_S} N_\epsilon \alpha_S^R, \mathbf{Z}_{C^0} C^{0,R}).\end{aligned}\quad (10)$$

Above, α_S^R and $C^{0,R}$ are the renormalized strong coupling and Wilson coefficient, respectively; the operator $\mathbf{Z}_{\alpha_S}$ and $\mathbf{Z}_{C^0}$ implement the usual $\overline{\text{MS}}$ ultraviolet renormalization of the strong coupling constant and the effective operator of Eq. (1) using the QCD β -function [113–118]; and the infrared singularities are subtracted by the universal factor $\mathbf{Z}(\alpha_S(\mu^2), \{p_i\})$ [119–126].

$$\mathbf{Z}(\alpha_S(\mu^2), \{p_i\}, \epsilon) = \mathcal{P} e^{-\frac{1}{4} \int_0^{\mu^2} \frac{d\mu'^2}{\mu'^2} \Gamma(\alpha_S(\mu'^2), \{p_i\}, \epsilon)}, \quad (11)$$

with

$$\begin{aligned}\Gamma(\alpha_S(\mu^2), \{p_i\}, \epsilon) &= \sum_{i \neq j} \mathbf{T}_i^a \mathbf{T}_j^a \Gamma_{\text{cusp}}(\alpha_S(\mu^2)) \log \frac{-s_{ij}}{\mu^2} \\ &\quad + \frac{1}{2} \sum_i \mathbb{1} \gamma_c^{R_i} + \Delta(\alpha_S(\mu^2), \{p_i\}).\end{aligned}\quad (12)$$

Above, Γ_{cusp} refers to the cusp anomalous dimension [124], which is currently known exactly through four-loop order [127,128] and approximately at five loops [129]. Furthermore, γ_c^R is the collinear anomalous dimension, obtained through four-loop order in Refs. [128,130]. The above formula was derived and calculated through three-loop order in Ref. [119] and verified in $\mathcal{N} = 4$ super Yang-Mills (SYM) theory [131] and QCD [132–134]. In Ref. [126], its general structure was determined to four-loop order. The term $\Delta(\alpha_S(\mu^2), \{p_i\})$ is known as the correction of the dipole formula and starts at the three-loop order. We refer, for example, to Section V of Ref. [135] for further details.

We use the above definition of finite amplitudes to create a finite interference with the tree level amplitude.

$$\mathcal{M}_X^{F,(n)} = \sum_{\text{spin,color}} \mathcal{A}_X^{F,(n)} \mathcal{A}_X^{\dagger F,(0)}. \quad (13)$$

Finite interfered amplitudes like in the above equation are the ingredients directly feeding into the computation of scattering cross sections. We provide the finite interfered amplitudes for all external particle configurations in the scattering and in the decay region in Supplemental Material [136].

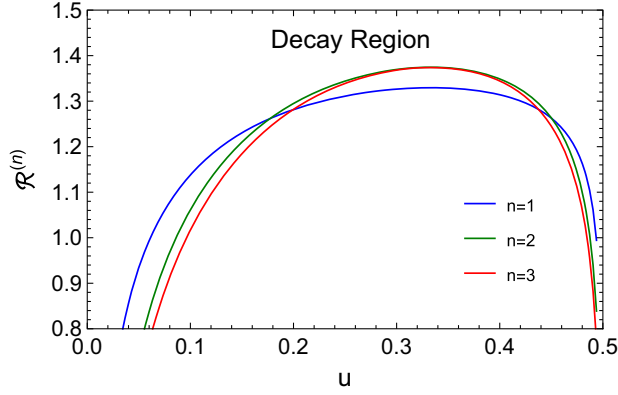


FIG. 2. The ratio $\mathcal{R}^{(n)}$ as defined in Eq. (14) for $n = 1$ (blue), $n = 2$ (green), and $n = 3$ (red) in the decay region for $a_S = 0.118/\pi$.

Figure 2 shows the ratio $\mathcal{R}^{(n)}$,

$$\mathcal{R}^{(n)} = \sum_{i=0}^n a_S^i \text{Re} \left[\frac{\mathcal{M}_{H \rightarrow ggg}^{F,(i)}}{\mathcal{M}_{H \rightarrow ggg}^{F,(0)}} \right], \quad (14)$$

for different orders in perturbation theory in the decay region for the line $q^2 = 1$ and $t = u$ as a function of u .⁴ The central region of the plot is away from any infrared logarithmic enhancement and displays excellent perturbative stability for three-loop corrections.

Validation. We performed a range of validations to ensure that our computation is correct.

- (1) Equation (10) describes the poles in the dimensional regulator of an amplitude based on lower loop amplitudes. The fact that our newly computed three-loop amplitudes satisfy this structure is a very stringent test.
- (2) The two-particle collinear limit of scattering amplitudes is described by universal splitting amplitudes, which were recently computed in Ref. [137] to third loop order, and we validate our results in this limit.
- (3) The limit of scattering amplitudes of one gluon having almost vanishing energy is described by a

⁴We use the *Mathematica* package NumPolyLog [112] to evaluate the multiple polylogarithms with 16-digit precision. On a laptop equipped with an Intel Core i7-8565U CPU at 1.8 GHz, the average evaluation time for a single phase-space point is roughly half a minute.

universal soft current, which was recently computed in Refs. [135,138] to third loop order, and we validate our results in this limit.

- (4) We find that the leading transcendental part of the $\mathcal{M}_{H \rightarrow ggg}^{(3)}$ amplitude corresponds [139] to the so-called $\text{tr}(\phi^2)$ form factor in $\mathcal{N} = 4$ super Yang-Mills theory [54–57]. We reproduce this form factor based on an integrand from Ref. [57] through three-loop order and confirm this remarkable correspondence between QCD and $\mathcal{N} = 4$ SYM theory for the first time at this loop order.
- (5) We perform numerical evaluations of our scattering amplitudes using the auxiliary mass flow method [140–142] implemented in AMFlow [143] in the decay region and find agreement with our analytic computation in terms of multiple polylogarithms.

Conclusions. In this Letter, we computed for the first time the scattering amplitudes for the decay of a Higgs boson to three partons as well as for the production of a Higgs boson alongside another parton in the scattering of two partons through third loop order in the generalized leading color approximation. These amplitudes are essential ingredients to key collider observables studying the Higgs boson at the forefront of precision phenomenology. Our result is the first computation of a four-particle scattering amplitude at third loop order in QCD involving a massive external particle and nonplanar diagrams. We attach our results in electronically readable files together with the submission of this Letter. Based on the performance of the reduction strategy used in the present work, we expect that current IBP techniques remain promising for extending the calculation beyond the generalized leading-color approximation.

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Data availability. The data that support the findings of this article are openly available [136].

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