

Diffractive production of heavy quarkonia at the Electron Ion Collider

Mohammad Yousuf Jamal¹, Ajaharul Islam¹, Aritra Bandyopadhyay^{2,3} and Santosh K. Das⁴

¹*Institute of Particle Physics and Key Laboratory of Quark and Lepton Physics (MOE),
Central China Normal University, Wuhan 430079, China*

²*Institute of Theoretical Physics, University of Wrocław, Plac Maksa Borna 9, PL-50204 Wrocław, Poland*

³*Department of Physics, West University of Timișoara, Bulevardul Vasile Pârvan 4,
Timișoara 300223, Romania*

⁴*School of Physical Sciences, Indian Institute of Technology Goa, Ponda 403401, Goa, India*



(Received 22 June 2026; accepted 23 July 2026; published 13 August 2026)

We study diffractive photo- and electroproduction of the S -wave heavy quarkonia J/ψ , $\psi(2S)$, and $\Upsilon(nS)$ at energies relevant for the Electron Ion Collider (EIC). The production amplitude is evaluated while retaining the full transverse-momentum (ℓ_t) dependence of the hard two-gluon kernel, that is, without expanding the impact-parameter Bessel kernel as is done in the small-size color-dipole limit. The quarkonia light-cone wave functions are built from Cornell-potential solutions of the Schrödinger equation, normalized to the measured leptonic widths, and combined with a modern collinear gluon distribution. After benchmarking the framework against the full set of HERA charmonium cross-section ratio data, we provide a consistent set of bottomonium cross-section ratio predictions in EIC kinematics. We find that the full ℓ_t -resolved treatment systematically improves the description of the radially excited states relative to the leading dipole limit, and we identify the kinematic windows where this difference is largest.

DOI: [10.1103/bplx-hsh5](https://doi.org/10.1103/bplx-hsh5)

Introduction. Diffractive photo- and electroproduction of heavy vector mesons has long been recognized as a clean probe of small- x gluon dynamics. The large quark mass provides a hard scale that justifies a perturbative treatment, while the exclusive final state selects coherent two-gluon exchange from the target. In the leading logarithmic approximation, the forward amplitude is proportional to the gluon momentum density $xG(x, \mu^2)$, evaluated at a momentum fraction x and a factorization scale μ^2 of order the heavy-quark mass squared. This proportionality yields the characteristic rise of the cross section with the photon-proton center-of-mass energy W , which was among the clearest signatures of gluon dominance at small Bjorken- x observed at HERA [1–4].

The forthcoming Electron Ion Collider (EIC), operating at $\sqrt{s} \simeq 29$ to 140 GeV with high luminosity and excellent forward instrumentation, will turn these measurements into precision imaging of gluons in protons and nuclei [5–8]. It will reach x as low as 10^{-4} to 10^{-5} and span Q^2 from quasireal photoproduction to several tens of GeV^2 , a regime where the gluon density remains only moderately constrained and exclusive quarkonium production offers complementary sensitivity to $xG(x, \mu^2)$ [9,10]. Extending

these studies to nuclear targets will further probe gluon shadowing, saturation, and nuclear geometry [11,12].

The standard theoretical tool is the color-dipole picture, in which the $q\bar{q}$ pair produced by the photon is assumed compact relative to the scale of the target gluon field [3,13,14]. This description works well for J/ψ production [15] and has been applied extensively to coherent and incoherent production off nuclei [9,16]. For radially excited states, however, it is more delicate, because the ratio $\sigma(\psi(2S))/\sigma(J/\psi)$ and its analogs are governed by node-induced cancellations in the $2S$ and $3S$ wave functions, which are sensitive to the full distribution of dipole sizes and gluon virtualities [17–19]. The transverse separation of the pair, b , and the gluon transverse momentum, ℓ_t , enter the hard kernel through the combination $\ell_t b$, and the dipole limit amounts to expanding the kernel for $\ell_t b \ll 1$, which discards precisely the structure to which the excited states are most sensitive. Dipole and saturation-based (BK) approaches give complementary predictions for these ratios [19,20]. Here, we quantify the effects of the full gluon transverse-momentum dependence beyond the dipole approximation.

A treatment that retains the full ℓ_t dependence avoids this expansion. The impact-parameter formalism of Refs. [2,3,21] provides a natural setting, in which the Bessel function $J_0(\ell_t b)$ is kept exact and the dipole limit is recovered only upon its small-argument expansion. A realistic quarkonium wave function with the correct radial-node structure is the second key ingredient. Early analyses relied on simple light-cone *Ansätze* or

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

potential-model inputs without a controlled nonrelativistic-to-light-cone mapping [22,23], whereas more systematic constructions start from a Cornell potential and perform the full transformation to light-cone variables [3,21]. We build on that approach here.

In this Letter, we combine Cornell-potential wave functions, normalized to the measured leptonic widths Γ_{ee} , with a direct numerical evaluation of the full ℓ_t -dependent amplitude. For the gluon distribution we use the HERAPDF2.0 NLO set (variation series) [24], accessed through the LHAPDF library [25], with an infrared matching scale $Q_0^2 = 1.001 \text{ GeV}^2$. We benchmark the framework against HERA data for $\sigma(\psi(2S))/\sigma(J/\psi)$ in Q^2 , W , and $|t|$, and then present bottomonium-ratio predictions in EIC kinematics. The chosen ranges $0 \leq Q^2 \leq 40 \text{ GeV}^2$, $40 \leq W \leq 200 \text{ GeV}$, and $0 \leq |t| \leq 1 \text{ GeV}^2$ are broadly compatible with the kinematic reach anticipated for the EIC. While the Q^2 and $|t|$ intervals lie well within the expected acceptance, the upper end of the W range approaches that of the highest-energy EIC configurations. These ranges therefore cover the phase space most relevant for exclusive heavy-quarkonium production studies [6,26]. Throughout, the full ℓ_t -resolved result is compared with the leading dipole limit obtained from the same framework and the same inputs, so that the comparison isolates the effect of the ℓ_t expansion alone. Predictions for Υ ratios in related approaches have appeared before [18,19,27], and our results provide an independent, ℓ_t -resolved determination obtained with potential-model wave functions calibrated to the HERA charmonium data.

Framework. We consider the process $\gamma^*(Q^2) + p \rightarrow V + p$, shown schematically in Fig. 1, at small Bjorken $x = (Q^2 + 4m_q^2)/W^2$ and for $V = J/\psi, \psi(2S), \Upsilon(1S, 2S, 3S)$. The z -dependent forward amplitude is written as .

$$\mathcal{A}(z; Q^2, W) = \int d^2\mathbf{b} \phi_\gamma(z, \mathbf{b}; Q^2) \mathcal{M}(x, \mathbf{b}) \psi_V(z, \mathbf{b}), \quad (1)$$

where z is the light-cone momentum fraction carried by the quark, $\mathbf{b}$ is the transverse separation of the $q\bar{q}$ pair, ϕ_γ and ψ_V are the photon and vector meson light-cone (LC) wave functions, and $\mathcal{M}$ is the convolution kernel that encodes the coupling to the gluon distribution through two-gluon exchange.

Quarkonium wave functions. We solve the S -wave radial Schrödinger equation with a Cornell potential,

$$\mathcal{V}_C(r) = -\frac{C_F \alpha_s}{r} + \sigma r, \quad (2)$$

with $C_F = 4/3$, $\alpha_s = 0.3$, and $\sigma = 0.18 \text{ GeV}^2$, obtaining radial wave functions $R_{nS}(r)$ for charmonium and

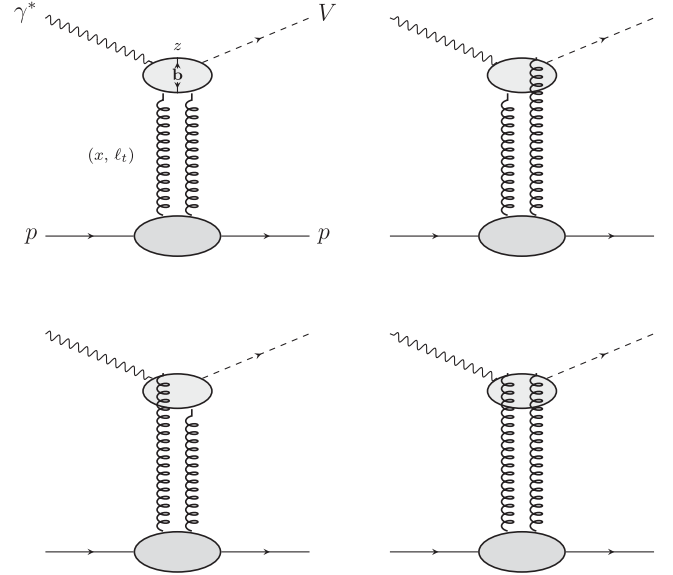


FIG. 1. Schematic of exclusive diffractive vector-meson production $\gamma^*(Q^2) + p \rightarrow V + p$ via color-singlet two-gluon exchange. The four diagrams correspond to the possible attachments of the exchanged gluons to the quark–antiquark dipole generated by the virtual photon. Here z denotes the quark light-cone momentum fraction, b the transverse separation of the $q\bar{q}$ dipole, and ℓ_t the transverse momentum of the exchanged gluons.

bottomonium. The short-distance normalization is fixed by the measured leptonic width through $\Gamma_{ee}(V \rightarrow e^+e^-) = 4\alpha_{\text{em}}^2 e_Q^2 |R_V(0)|^2 / M_V^2$, M_V being the vector meson mass. The nonrelativistic (NR) momentum-space wave functions are then mapped to LC momentum space using

$$\psi_V(z, k_t) = \left[\frac{k_t^2 + m_q^2}{4[z(1-z)]^3} \right]^{1/4} \psi_{\text{NR}} \left(\sqrt{\frac{k_t^2 + (2z-1)^2 m_q^2}{4z(1-z)}} \right), \quad (3)$$

and then transformed to impact-parameter space by a Hankel transform. This mapping is the standard kinematical Terentev prescription and does not include the Melosh spin rotation or the D -wave admixture generated by a massive photonlike $V \rightarrow Q\bar{Q}$ vertex, which are known to affect the $\psi(2S)/J/\psi$ ratio, in particular its W dependence at low Q^2 [28–30]. The sensitivity of excited to ground-state ratios to the wave-function model was also noted early on for Υ production [31]. The Cornell input mitigates this model dependence, since it contains both the Coulomb short-distance and the linear confining behavior, its normalization is fixed by the measured leptonic widths, and realistic potentials give similar ratios [18]. Since the full and dipole results are computed with identical wave functions, the difference between them, which is the focus of this Letter, is insensitive to this choice. The photon

LC wave function is $\phi_\gamma(z, b; Q^2) = K_0(bQ_{\text{eff}})$, with $Q_{\text{eff}}^2(z) = z(1-z)Q^2 + m_q^2$. This transverse virtuality is the only hard scale in the process, so it also sets the upper limit of the gluon evolution below.

Full amplitude and dipole limit. The central ℓ_t -dependent overlap is

$$A(\ell_t, z; Q^2) = \frac{1}{\ell_t^2} \int_0^\infty db b [1 - J_0(\ell_t b)] \phi_\gamma(z, b; Q^2) \psi_V(z, b), \quad (4)$$

where $[1 - J_0(\ell_t b)]/\ell_t^2 \rightarrow b^2/4$ as $\ell_t \rightarrow 0$. The full z -dependent amplitude then reads

$$\mathcal{A}_{\text{full}}(z; Q^2, W) = 4\pi\alpha_s \left\{ xG(x, Q_0^2) A_0(z; Q^2) + \int_{Q_0^2}^{Q_{\text{eff}}^2(z)} d\mu^2 \frac{\partial xG(x, \mu^2)}{\partial \mu^2} A\left(\sqrt{\mu^2}, z; Q^2\right) \right\}, \quad (5)$$

where $A_0(z; Q^2) = \frac{1}{4} \int_0^\infty db b^3 \phi_\gamma \psi_V$ is the dipole-limit overlap and Q_0^2 is an infrared separation scale below which $A(\ell_t, z; Q^2) \approx A_0(z; Q^2)$. The boundary term proportional to $xG(x, Q_0^2)$ resums the infrared region $\ell_t^2 < Q_0^2$, while the integral builds up the ℓ_t -resolved contribution through the scale derivative $\partial xG/\partial \mu^2$. The leading dipole approximation follows by replacing $A(\ell_t, z; Q^2) \rightarrow A_0(z; Q^2)$ throughout Eq. (5), which collapses the amplitude to

$$\mathcal{A}_{\text{dipole}}(z; Q^2, W) = 4\pi\alpha_s xG(x, Q_{\text{eff}}^2) A_0(z; Q^2). \quad (6)$$

The convergence of the small- ℓ_t expansion underlying Eq. (6) is examined quantitatively in Supplemental Material [32], where we show that it fails for the radially excited states.

From the amplitude to the cross section. Finally, performing the z integration requires classifying the longitudinal and transverse projections of the amplitude,

$$I_L = \int_0^1 dz z(1-z) \mathcal{A}(z), \quad I_T = \int_0^1 dz \frac{\mathcal{A}(z)}{z(1-z)}, \quad (7)$$

which reflect the LC structure of the $\gamma^* \rightarrow q\bar{q}$ vertex, with longitudinal photons weighting the amplitude near $z \simeq 1/2$ and transverse photons receiving enhanced contributions from the endpoints $z \rightarrow 0, 1$. The forward differential cross section at $t = 0$ for polarization $\lambda = L, T$ is then

$$\left. \frac{d\sigma_\lambda}{dt} \right|_{t=0} = \frac{\mathcal{N}^2}{16\pi} (1 + \rho^2) I_\lambda^2(Q^2, W), \quad (8)$$

where ρ is the ratio of the real to imaginary parts of the amplitude, estimated from the standard logarithmic derivative, and $\mathcal{N}^2 \propto \alpha_s^2 e_Q^2 / M_V$ collects the overall normalization, including the skewness correction [9]. Assuming the standard exponential t dependence of diffractive vector-meson production at small $|t|$,

$$\frac{d\sigma}{dt} = \left. \frac{d\sigma}{dt} \right|_{t=0} e^{-B_V(W)|t|}, \quad (9)$$

integration over t gives

$$\sigma_\lambda(W, Q^2) = B_V^{-1}(W) \left. \frac{d\sigma_\lambda}{dt} \right|_{t=0}, \quad (10)$$

and the measured electroproduction cross section is $\sigma = \sigma_T + \epsilon \sigma_L$, with ϵ the virtual-photon polarization parameter. The diffractive slope follows the Regge parametrization

$$B_V(W) = B_0 + 4\alpha' \ln(W/W_0), \quad (11)$$

with $W_0 = 90$ GeV and soft-Pomeron slope $\alpha' = 0.135$ GeV⁻² from the H1 elastic J/ψ analysis [33]. The intercepts B_0 are taken from the corresponding HERA measurements, where available, and extrapolated by transverse-size scaling for the higher bottomonium states, with the values used and a test of their impact given in Supplemental Material [32]. In the cross-section ratios studied below, the normalization $\mathcal{N}$, the skewness factor, and the slope B_V largely cancel, leaving the observables sensitive mainly to the interplay of the wave function and the gluon density.

Gluon input. A central feature of this work is the use of a modern collinear gluon PDF. The gluon momentum density $xG(x, \mu^2)$ is read directly from the HERAPDF2.0 NLO variation set [24] through the LHAPDF6 interface [25], without any external reparametrization. Because xG enters both the boundary term and the evolution integral in Eq. (5), the scale derivative $\partial xG/\partial \mu^2$ controls how the different ℓ_t modes contribute to the amplitude, and its shape directly affects the predicted Q^2 dependence of the ratios. The numerical procedure, the choice of Q_0^2 , and the stability of the results against its variation are detailed in Supplemental Material [32].

The scale structure of Eq. (5) parallels the Next to Leading Order collinear treatment of Refs. [10,36], where a Sudakov factor suppresses gluon emissions with transverse momenta above a hard cutoff scale. At leading order their amplitude is proportional to the gluon density at a single effective scale, which corresponds to the dipole limit of Eq. (5), while the Sudakov factor restores sensitivity to the gluon transverse-momentum distribution, in analogy with the explicit $\partial xG/\partial \mu^2$ integral retained here. The two

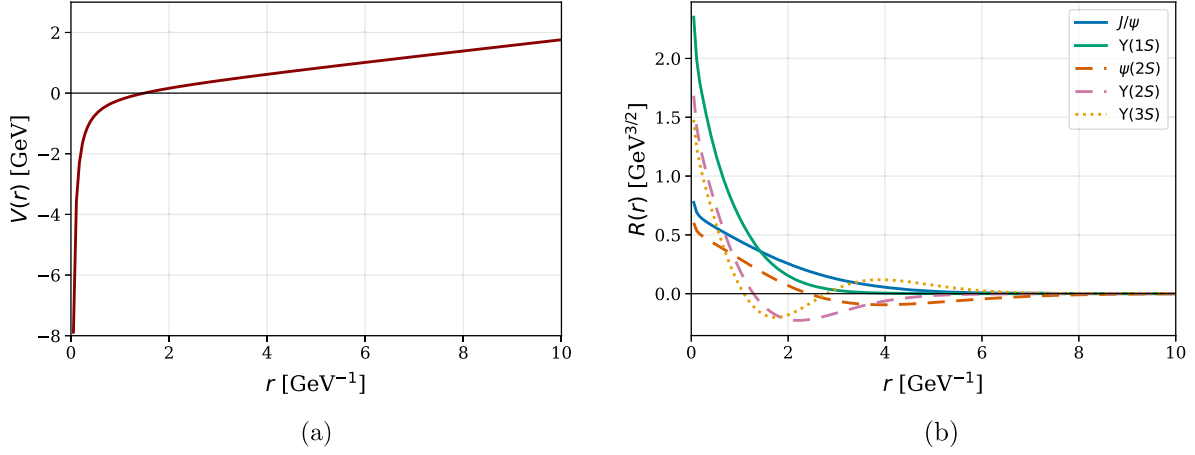


FIG. 2. (a) Cornell potential of Eq. (2) for $\alpha_s = 0.30$ and $\sigma = 0.18 \text{ GeV}^2$. (b) Radial wave functions $R_{nS}(r)$ for the charmonium states J/ψ and $\psi(2S)$, and the bottomonium states $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$, obtained by solving the nonrelativistic Schrödinger equation with the Cornell potential.

treatments are therefore complementary probes of the same scale physics.

Results. Figure 2 shows the Cornell potential and the resulting radial wave functions. The ground states are nodeless and spatially compact, with bottomonium more localized than charmonium because of the heavier quark mass. The excited states display the expected radial nodes and larger transverse sizes; a quantitative convergence test of the dipole expansion, and three-dimensional visualizations of the key intermediate quantities are provided in Supplemental Material [32].

Figure 3 shows the ratio $R = \sigma(\psi(2S))/\sigma(J/\psi)$ as a function of Q^2 , W , and $|t|$. The full calculation (solid curves) reproduces the mild Q^2 dependence and the overall normalization of the HERA data from H1 and ZEUS [15,37–40], whereas the dipole limit (dashed curves) lies systematically lower and exhibits a different Q^2 shape. Since many systematic effects cancel in the ratio, this difference reflects the physics of the ℓ_i expansion rather

than a normalization choice. The same pattern holds for the W dependence at $Q^2 = 0$ in Fig. 3(b), where the full result tracks the data across the HERA energy range, and for the $|t|$ dependence in Fig. 3(c), where the dipole result shows a systematically harder slope.

The physical origin of this difference is the radial node of the $\psi(2S)$ wave function, which sits near the typical transverse separation of the pair and produces a partial cancellation between the inner and outer lobes of the overlap with the photon. The size of that cancellation depends on the distribution of gluon virtualities probed, which the dipole limit fixes to a single effective scale while the full calculation retains in its entirety. This retained spread of scales is precisely what the ratio is sensitive to, and it accounts for the cleaner description of the data in the full treatment.

Figure 4 presents the bottomonium ratios $\sigma(\Upsilon(nS))/\sigma(\Upsilon(1S))$ for $n = 2, 3$, for which we are not aware of any existing experimental data. The heavier b mass reduces typical dipole sizes, so the node-induced cancellations are milder than for charmonium, yet the full and dipole results

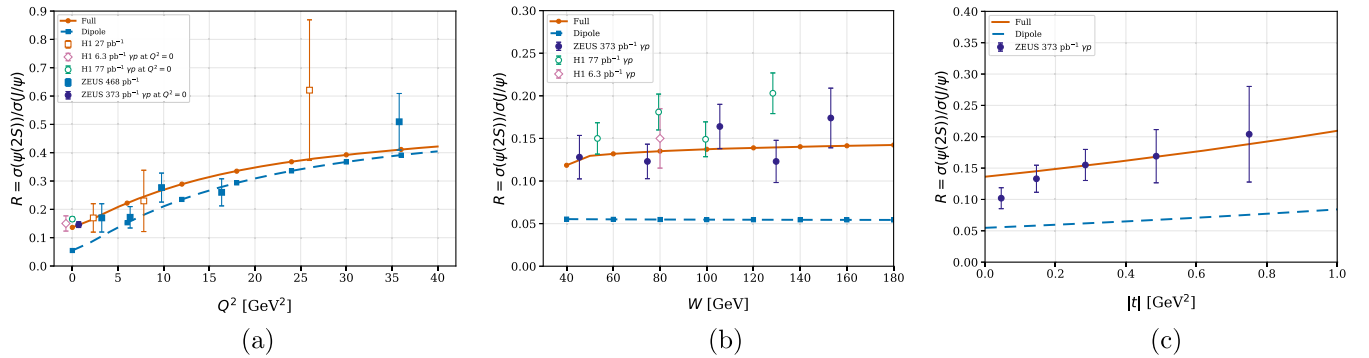


FIG. 3. Ratio $R = \sigma(\psi(2S))/\sigma(J/\psi)$ in exclusive photo- and electroproduction: (a) versus Q^2 at $W = 90 \text{ GeV}$, (b) versus W at $Q^2 = 0$, and (c) versus $|t|$ at $W = 90 \text{ GeV}$, $Q^2 = 0$. Solid (dashed) curves are the full (dipole) results within the present framework. Data are from ZEUS [37,38] and H1 [15,39,40].

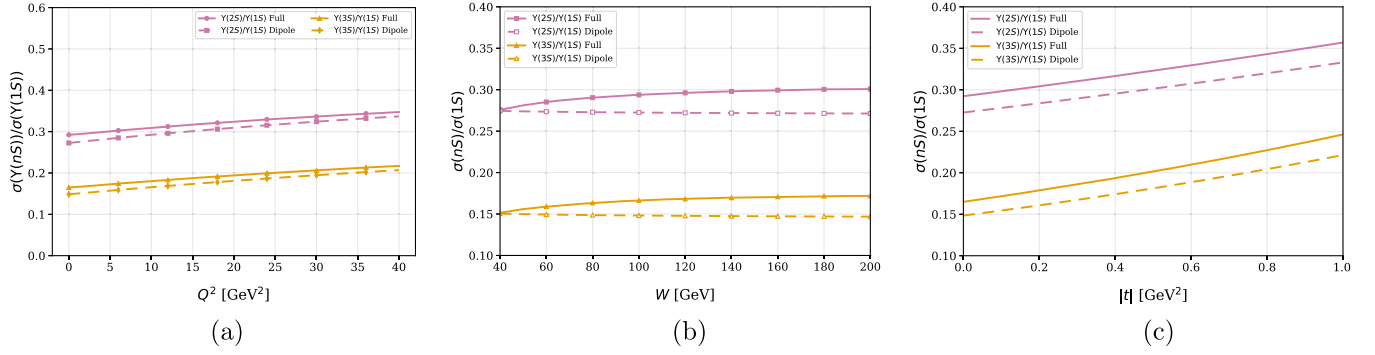


FIG. 4. EIC predictions for $\sigma(Y(nS))/\sigma(Y(1S))$ ($n = 2, 3$): (a) versus Q^2 at $W = 90$ GeV, (b) versus W at $Q^2 = 0$, and (c) versus $|t|$ at $W = 90$ GeV, $Q^2 = 0$. Solid (dashed) curves are the full (dipole) results.

still differ appreciably over a broad Q^2 range, as seen in Fig. 4(a). The full result is smoother and more monotonic, while the dipole limit shows a stronger Q^2 variation and a shifted normalization. The W dependence in Fig. 4(b) is nearly flat at $Q^2 = 0$, with the $Y(3S)/Y(1S)$ ratio lying below $Y(2S)/Y(1S)$ as a consequence of the stronger node suppression of the $3S$ state, and the $|t|$ dependence in Fig. 4(c) shows a mild rise in the full calculation against the flatter dipole prediction. A precision EIC measurement of these ratios would therefore provide a clean and direct test of the ℓ_t -resolved framework.

Conclusions. We have computed diffractive heavy-quarkonium production amplitudes while retaining the full ℓ_t dependence of the hard kernel, using Cornell-potential LC wave functions normalized to leptonic widths, together with a modern collinear gluon distribution. Three conclusions emerge. First, the full ℓ_t -resolved amplitude markedly improves the description of $R_{\psi(2S)}$ across Q^2 , W , and $|t|$ relative to the leading dipole limit evaluated with identical inputs, where the node-induced cancellations in the $\psi(2S)$ wave function are essential. Second, even for bottomonium, where the dipole picture is expected to be more reliable, nontrivial differences between the full and dipole treatments persist and are accessible at the EIC. Third, the framework naturally interpolates between the dipole and k_T -factorization descriptions and can be extended to nuclear targets, offering a unified ℓ_t -resolved description of exclusive quarkonium production in the EIC era.

The $\gamma^{(*)} + p \rightarrow V + p$ amplitudes computed here are also the direct theoretical input for exclusive quarkonium production in ultraperipheral collisions at the LHC, where the $\psi(2S)/J/\psi$ ratio has recently been measured as a function of rapidity by LHCb [41] and ALICE [42]. An extension of the present framework to ultraperipheral kinematics is, therefore, a natural next step.

Acknowledgments. The authors are deeply grateful to Jan-e Alam and Bedangadas Mohanty for several fruitful discussions and valuable suggestions. M. Y. J. and A. I. gratefully acknowledge Central China Normal University (CCNU), Wuhan, China, for its hospitality and financial support through postdoctoral fellowships. A. B. acknowledges support from the ULAM fellowship program of the Polish National Agency for Academic Exchange (NAWA), Program No. BNI/ULM/2024/1/00193 and EU's NextGenerationEU instrument through the National Recovery and Resilience Plan of Romania-Pillar III-C9-I8, managed by the Ministry of Research, Innovation and Digitization, within the project entitled “Facets of Rotating Quark-Gluon Plasma” (FORQ), Contract No. 760079/23.05.2023 code CF 103/15.11.2022. S. K. D. acknowledges the support from Anusandhan National Research Foundation (ANRF), India, under Grant No. ANRF/ARG/2025/002424/PS.

Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] M. G. Ryskin, *Z. Phys. C* **57**, 89 (1993).
- [2] M. G. Ryskin, R. G. Roberts, A. D. Martin, and E. M. Levin, *Z. Phys. C* **76**, 231 (1997).
- [3] L. Frankfurt, W. Koepf, and M. Strikman, *Phys. Rev. D* **57**, 512 (1998).

- [4] S. Chekanov *et al.* (ZEUS Collaboration), *Eur. Phys. J. C* **24**, 345 (2002).
- [5] R. Abir *et al.*, *SciPost Phys. Lect. Notes* **92** (2025), 10.21468/SciPostPhysLectNotes.92.
- [6] R. Abdul Khalek *et al.*, *Nucl. Phys. A* **1026**, 122447 (2022).

- [7] D. Boer, C. Pisano, and P. Tael, *Phys. Rev. D* **103**, 074012 (2021).
- [8] C. Fanelli *et al.*, *Nucl. Instrum. Methods Phys. Res., Sect. A* **1047**, 167748 (2023).
- [9] A. D. Martin, C. Nockles, M. G. Ryskin, and T. Teubner, *Phys. Lett. B* **662**, 252 (2008).
- [10] S. P. Jones, A. D. Martin, M. G. Ryskin, and T. Teubner, *J. High Energy Phys.* **11** (2013) 085.
- [11] T. Lappi and H. Mantysaari, *Phys. Rev. C* **87**, 032201 (2013).
- [12] Z. Tu (STAR Collaboration), *Proc. Sci. HardProbes2020* (2021) 100.
- [13] J. Nemchik, N. N. Nikolaev, E. Predazzi, B. G. Zakharov, and V. R. Zoller, *J. Exp. Theor. Phys.* **86**, 1054 (1998).
- [14] M. Wusthoff and A. D. Martin, *J. Phys. G* **25**, R309 (1999).
- [15] C. Adloff *et al.* (H1 Collaboration), *Phys. Lett. B* **541**, 251 (2002).
- [16] L. L. Frankfurt, M. F. McDermott, and M. Strikman, *J. High Energy Phys.* **02** (1999) 002.
- [17] P. Hoyer and S. Peigne, *Phys. Rev. D* **61**, 031501 (2000).
- [18] J. Cepila, J. Nemchik, M. Krelina, and R. Pasechnik, *Eur. Phys. J. C* **79**, 495 (2019).
- [19] M. A. Peredo and M. Hentschinski, *Phys. Rev. D* **109**, 014032 (2024).
- [20] H. Mäntysaari and J. Penttala, *Phys. Lett. B* **823**, 136723 (2021).
- [21] K. Suzuki, A. Hayashigaki, K. Itakura, J. Alam, and T. Hatsuda, *Phys. Rev. D* **62**, 031501 (2000).
- [22] W. Buchmuller and S. H. H. Tye, *Phys. Rev. D* **24**, 132 (1981).
- [23] C. Quigg and J. L. Rosner, *Phys. Lett.* **71B**, 153 (1977).
- [24] H. Abramowicz *et al.* (H1 and ZEUS Collaborations), *Eur. Phys. J. C* **75**, 580 (2015).
- [25] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht, M. Schönherr, and G. Watt, *Eur. Phys. J. C* **75**, 132 (2015).
- [26] A. Accardi *et al.*, *Eur. Phys. J. A* **52**, 268 (2016).
- [27] M. B. Gay Ducati, F. Kopp, M. V. T. Machado, and S. Martins, *Phys. Rev. D* **94**, 094023 (2016).
- [28] M. Krelina, V. P. Goncalves, and J. Cepila, *Nucl. Phys.* **A989**, 187 (2019).
- [29] M. Krelina, J. Nemchik, and R. Pasechnik, *Eur. Phys. J. C* **80**, 92 (2020).
- [30] C. Henkels, E. G. de Oliveira, R. Pasechnik, and H. Trebien, *Phys. Rev. D* **102**, 014024 (2020).
- [31] A. Rybarska, W. Schafer, and A. Szczurek, *Phys. Lett. B* **668**, 126 (2008).
- [32] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/bplx-hsh5> for parameters, numerical details, the convergence analysis of the small- $\ell_t b$ expansion, and three-dimensional visualizations of the intermediate quantities, which includes Refs. [3,15,21,24,25,33–35].
- [33] C. Alexa *et al.* (H1 Collaboration), *Eur. Phys. J. C* **73**, 2466 (2013).
- [34] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [35] T.-J. Hou *et al.*, *Phys. Rev. D* **103**, 014013 (2021).
- [36] S. P. Jones, A. D. Martin, M. G. Ryskin, and T. Teubner, *J. Phys. G* **41**, 055009 (2014).
- [37] I. Abt *et al.* (ZEUS Collaboration), *J. High Energy Phys.* **12** (2022) 164.
- [38] S. Chekanov *et al.* (ZEUS Collaboration), *Nucl. Phys.* **B695**, 3 (2004).
- [39] A. Aktas *et al.* (H1 Collaboration), *Eur. Phys. J. C* **46**, 585 (2006).
- [40] C. Adloff *et al.* (H1 Collaboration), *Eur. Phys. J. C* **3**, 247 (1998).
- [41] R. Aaij *et al.* (LHCb Collaboration), *SciPost Phys.* **18**, 071 (2025).
- [42] D. A. H. Abdallah *et al.* (ALICE Collaboration), *arXiv*: 2605.13569.