

Production of heavy tetraquarks in rare exclusive decays of the Higgs boson

F. A. Martynenko^{✉,*}, A. V. Eskin[✉], and A. P. Martynenko[✉]
Samara University, Samara, Russia

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The relativistic quark model is used to study the processes of heavy tetraquark ($cc\bar{c}\bar{c}$) production in the Higgs boson decay. We consider quark-gluon, ZZ -boson, and quark-gluon loop decay mechanisms with the production of a fully charmed tetraquark. Relativistic interaction amplitudes are constructed and decay widths are calculated taking into account relativistic corrections in the decay amplitude connected with the relative motion of quarks and antiquarks.

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I. INTRODUCTION

The investigation of production processes of heavy quark bound states in various nuclear reactions has been and remains an important area of work. Such reactions allow one to study the properties of resonances being produced, the interaction constants of particles of the Standard Model, the bound state production and decay mechanisms, and to test the models used to describe bound states of quarks. Such processes are studied both theoretically and experimentally. It can be noted that, very often at the initial stage, theoretical work is ahead of experimental work. For example, long before the discovery of the Higgs boson, the reactions of its production and decay were studied theoretically. Such work served as a kind of guideline for experimenters in setting up experiments. But as soon as experimental data appeared, it was usually necessary to refine the theoretical calculations and implement them with greater accuracy.

In the last decade, exotic bound states of heavy quarks that do not fit the standard quark model, most notably tetraquarks and pentaquarks, have come under intense study. The discovery of the charmoniumlike state $X(3872)$ by the Belle Collaboration in 2003 brought the study of exotic states to the forefront of hadron physics. Although many tetraquark candidates containing heavy (c,b) quarks are now known [1–11], many questions remain about them, including which of them are truly exotic hadrons and what their internal structure is.

For the production of exotic bound states, one can use well-developed methods that have been successfully applied

to the production of ordinary heavy mesons and baryons. In this paper, we study the processes of heavy tetraquark production in the Higgs boson decay on the basis of the relativistic approach previously used to describe the decay of the Higgs boson with the pair production of bound states of quarks and leptons [12–14]. We further consider the production of a vector tetraquark consisting of two c quarks and two $\bar{c}$ antiquarks in the decay $H \rightarrow T_{cc\bar{c}\bar{c}} + \gamma$. The LHCb Collaboration detected the $X(6900)$ resonance, which was confirmed in the ATLAS and CMS experiments and is a candidate for the tetraquark ($cc\bar{c}\bar{c}$) along with numerous other resonances discovered in recent years [1–11]. An analysis of the invariant mass distribution in the proton-proton interaction [3] revealed three resonance structures: two new states, $X(6600)$ and $X(7100)$, and confirmation of the previously observed $X(6900)$. The first determination of quantum numbers for fully charmed tetraquarks was performed. In connection with the appearance of experimental data on the production of exotic bound states (tetraquarks) of quarks and gluons, a significant number of papers have appeared in which the mass spectrum and cross sections of their production in proton-proton interactions, photoproduction, electron-positron annihilation, and B-meson decays were calculated [11,15–30] (see other references in review papers [7–9]).

The production of tetraquarks in the decay of the Higgs boson $H \rightarrow T_{cc\bar{c}\bar{c}} + \gamma$ is another channel for their study. Despite the fact that such processes are rare, their study is of interest in connection with the construction of various Higgs boson factories. The goal of this paper is to obtain estimates for corresponding Higgs boson decay widths. In doing so, we use the relativistic approach to study the reactions of quark bound state production, which allows one to consistently take into account relativistic effects connected with the relative motion of quarks when constructing amplitudes and decay widths [12–14]. In the case of pair production of charmonium in the decay of the H boson, which we studied earlier in [12], two bound states ($c\bar{c}$) arise in the final state.

*Contact author: f.a.martynenko@gmail.com

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In this regard, it was interesting to study the production of a bound state of four particles ($cc\bar{c}\bar{c}$) (tetraquark $T_{cc\bar{c}\bar{c}}$) also in the decay of the Higgs boson in the relativistic quark model. We concentrate on the study of the production of a tetraquark built from quarks and antiquarks of the same kind, using a configuration of two pairs of particles $[cc][\bar{c}\bar{c}]$ for its construction, where square brackets denote the color antitriplet state. This model is structurally similar to the diquark-antiquark model, but we consider the tetraquark as a four-particle state. Since the quantum problem with a number of particles greater than or equal to 3 has no exact solution, various models are used to calculate the observed characteristics of multiparticle states in QCD. For example, the variational method is widely used, allowing for high-precision calculations of the wave functions and energy levels of tetraquarks. The numerical results obtained for calculating the parameters of bound states within this method are in good agreement with existing experimental data. Within the relativistic quark model, it is possible to consistently calculate relativistic effects due to relative motion of quarks forming a bound state. In the production of bound states of c quarks, relativistic corrections are essential for achieving high accuracy of calculation.

II. GENERAL FORMALISM

There are various mechanisms of tetraquark production in the decay $H \rightarrow T_{cc\bar{c}\bar{c}} + \gamma$: quark-gluon [Fig. 1(a)], ZZ-boson [Fig. 1(b)], quark-gluon (photon) loop [Figs. 2(a) and 2(b)], W-boson loop, and others. The

differences between them are connected with physical processes that occur at the microscopic level and ultimately lead to the formation of a tetraquark in the final state. The amplitudes of such production mechanisms differ in the degrees of interaction constants α (fine structure constant) and α_s (strong interaction constant), as well as in characteristic ratios of particle masses.

In the case of the ZZ-boson mechanism, there are two vertices of particle interaction. One vertex of the $H \rightarrow ZZ$ interaction is determined by the factor [31]

$$V^{\alpha\beta} = \frac{2e}{\sin 2\theta_W} M_Z g^{\alpha\beta}. \quad (1)$$

The second vertex of interaction of each Z boson with c quarks has the form [31]

$$\Gamma^\alpha = \frac{e}{\sin 2\theta_W} \gamma^\alpha \left[\frac{1}{2}(1 - \gamma_5) - a_z \right], \quad a_z = 2Q_c \sin^2 \theta_W. \quad (2)$$

For the quark-gluon mechanism, the interaction vertex $H \rightarrow c\bar{c}$ includes the factor

$$V(H \rightarrow c\bar{c}) = -\frac{e}{\sin 2\theta_W} \frac{m}{M_Z}, \quad (3)$$

where m is the mass of the c quark, and M_Z is the mass of the Z boson.

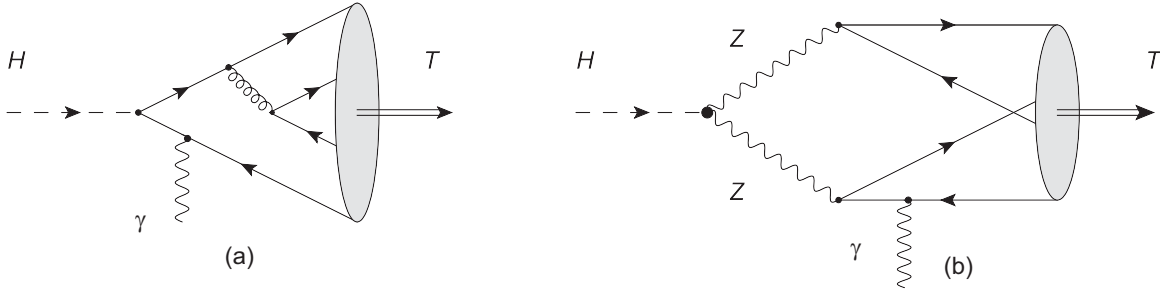


FIG. 1. Quark-gluon (a) and ZZ-boson (b) mechanisms of heavy tetraquark production in the Higgs boson decays $H \rightarrow T + \gamma$. The gluon is indicated by a helical line.

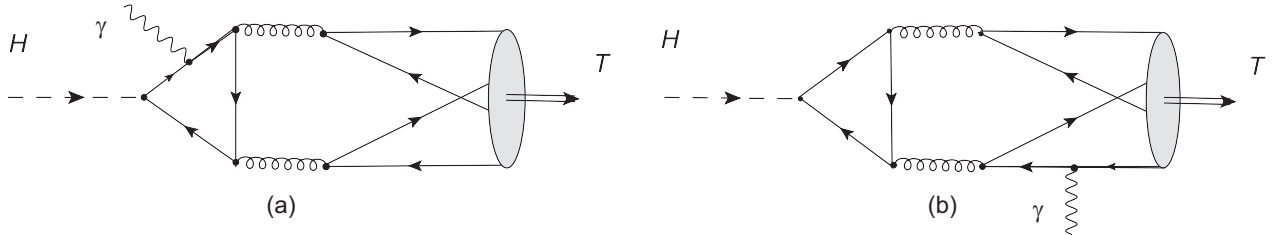


FIG. 2. Quark-gluon loop mechanism of heavy tetraquark production in the Higgs boson decay $H \rightarrow T + \gamma$. The gluon is indicated by a helical line. In (a) a photon is emitted by quarks in a loop. In (b) a photon is emitted by quarks and antiquarks in the final state.

A. ZZ production mechanism

Let us consider the ZZ-boson mechanism of tetraquark production shown in Fig. 1(b) in the reaction $H \rightarrow T_{cc\bar{c}\bar{c}} + \gamma$. Our goal is to obtain an estimate of the decay width of the Higgs boson with the production of the tetraquark $T_{cc\bar{c}\bar{c}}$ taking into account relativistic corrections, determined by the momenta of relative motion of heavy quarks during the formation of a tetraquark.

The color structure of the tetraquark state ($cc\bar{c}\bar{c}$) can be either of the hadronic molecule type, $3_c \otimes 3_c \otimes \bar{3}_{\bar{c}} \otimes \bar{3}_{\bar{c}} \rightarrow 1_{[\bar{c}c]_1[\bar{c}c]_1}$, or of the diquark-antidiquark type, $3_c \otimes 3_c \otimes \bar{3}_{\bar{c}} \otimes \bar{3}_{\bar{c}} \rightarrow 1_{[cc]_3[\bar{c}\bar{c}]_3} \oplus 1_{[cc]_6[\bar{c}\bar{c}]_6}$. Pairs of quarks in the color antitriplet state are attracted to each other, and in the color sextet state they repel each other. Similarly, in a quark-antiquark pair, the attractive force arises in the color singlet state, and in the color octet state, the repulsive force arises. Thus, various configurations of four quarks are possible, which lead to the formation of a tetraquark [23,25,32–35]. We consider the case of production, when a tetraquark is formed from two pairs, quark-quark and antiquark-antiquark, each of which has a spin 1 and is in a color-antisymmetric state. When a tetraquark consists of identical quarks and antiquarks, the Pauli principle leads to additional restrictions on the quantum numbers of a pair of identical quarks (diquark). Indeed, permutation of quark indices should change the sign of the total wave function of a pair of quarks. The color part of the wave function of a pair of quarks is antisymmetric. The coordinate wave function is symmetric, since the quarks are in the S-wave state, therefore the spin part of wave function should also be symmetric. Therefore, total spin of the S-wave state of the quark pair must be equal to 1. In terms of the configuration we are considering (diquark-antidiquark), total angular momentum $\mathbf{J}$ of a fully charmed tetraquark ($cc\bar{c}\bar{c}$) is obtained by adding total angular momentum $\mathbf{J}_{cc}$ of doubly charmed state (cc), total angular momentum $\mathbf{J}_{\bar{c}\bar{c}}$ of doubly charmed state ($\bar{c}\bar{c}$), and relative orbital angular momentum $\mathbf{L}_\sigma$ (relative coordinates ρ, λ, σ are defined in [36]) between the state (cc) and the state ($\bar{c}\bar{c}$).

Introducing the total P , Q and relative p , q four-momenta in pairs (cc), ($\bar{c}\bar{c}$), we can express the momenta of individual quarks and antiquarks as follows:

$$p_1 = \frac{1}{2}P + p, \quad p_2 = \frac{1}{2}P - p, \quad q_1 = \frac{1}{2}Q + q, \quad q_2 = \frac{1}{2}Q - q. \quad (4)$$

Let us further introduce total T and relative t four-momenta for these pairs,

$$P = \frac{1}{2}T + t, \quad Q = \frac{1}{2}T - t. \quad (5)$$

Denoting the photon momentum k and the Higgs boson momentum r , we obtain from the law of momentum conservation,

$$r = k + T, \quad kT = \frac{1}{2}(M_H^2 - M_T^2), \quad rT = \frac{1}{2}(M_H^2 + M_T^2), \quad (6)$$

where M_T is the tetraquark mass, and M_H is the Higgs boson mass.

Total angular momentum of a tetraquark, obtained by adding two spins 1 of pairs of quarks (cc) and antiquarks ($\bar{c}\bar{c}$) with zero orbital angular momentum of quarks, can take three values $J = 0, 1, 2$, so that the vectors of corresponding states can be represented as follows:

$$|0^{++} \rangle_T = |S_{cc} = 1, \quad S_{\bar{c}\bar{c}} = 1, \quad S_T = 0, \quad L_T = 0 \rangle_{J_T=0}, \quad (7)$$

$$|1^{+-} \rangle_T = |S_{cc} = 1, \quad S_{\bar{c}\bar{c}} = 1, \quad S_T = 1, \quad L_T = 0 \rangle_{J_T=1}, \quad (8)$$

$$|2^{++} \rangle_T = |S_{cc} = 1, \quad S_{\bar{c}\bar{c}} = 1, \quad S_T = 2, \quad L_T = 0 \rangle_{J_T=0}, \quad (9)$$

where the spatial and charge parities of the tetraquark are determined by the formulas

$$P_T = (-1)^{L_T}, \quad C_T = (-1)^{S_T+L_T}. \quad (10)$$

The photon is emitted by one of the quarks or antiquarks, so for the ZZ-boson production mechanism we have four amplitudes in Fig. 1(b).

The original expression for relativistic decay amplitude is a convolution of the production amplitude of two quarks c , two antiquarks $\bar{c}$, and a photon with the Bethe-Salpeter amplitude describing the bound state of four particles. This convolution has the same general structure as in the case of two particles [37,38]. After transforming this convolution to a three-dimensional quasipotential form, we obtain a convolution of the interaction amplitude $\mathcal{T}(\mathbf{p}, \mathbf{q}, \mathbf{t}, T, k)$ projected onto positive-frequency states and the tetraquark quasipotential wave function $\Psi_T(\mathbf{p}, \mathbf{q}, \mathbf{t})$,

$$\mathcal{M}(H \rightarrow T + \gamma) = \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{d\mathbf{t}}{(2\pi)^3} \bar{\Psi}_T^{\lambda_1 \lambda_2 \lambda_3 \lambda_4}(\mathbf{p}, \mathbf{q}, \mathbf{t}) \mathcal{T}^{\lambda_1 \lambda_2 \lambda_3 \lambda_4}(\mathbf{p}, \mathbf{q}, \mathbf{t}, T, k). \quad (11)$$

The law of transformation of a tetraquark wave function during a transition from a reference frame with momentum T to a reference frame at rest is similar in structure to the case of a two-particle bound state [39,40],

$$\bar{\Psi}_T^{\mu\nu\lambda\sigma}(\mathbf{p}, \mathbf{q}, \mathbf{t}) = \bar{\Psi}_0^{\alpha\beta\rho\omega}(\mathbf{p}, \mathbf{q}, \mathbf{t}) D_1^{+1/2\alpha\mu}(R_{L_T}^W) D_2^{+1/2\beta\nu}(R_{L_T}^W) D_3^{+1/2\rho\lambda}(R_{L_T}^W) D_4^{+1/2\omega\sigma}(R_{L_T}^W), \quad (12)$$

where R^W is the Wigner rotation, L_T is the Lorentz boost from the tetraquark rest frame to the moving reference frame, and $D^{1/2}(R)$ is the rotation matrix, which is defined as follows:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} 1D_{1,2}^{1/2}(R_{L_T}^W) = S^{-1}(\mathbf{p}_{1,2})S(\mathbf{T})S(\mathbf{p}), \quad 1\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} 1D_{3,4}^{1/2}(R_{L_T}^W) = S^{-1}(\mathbf{q}_{1,2})S(\mathbf{T})S(\mathbf{q}), \quad (13)$$

where the matrix of the Lorentz transformation of the Dirac bispinor has the form

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p) + m}{2m}} \left(1 + \frac{(\boldsymbol{\alpha}\mathbf{p})}{\epsilon(p) + m} \right). \quad (14)$$

To further transform the amplitude (11), we use the following formulas for transforming the Dirac wave functions [40]:

$$S_{\alpha\beta}(\Lambda)u_{\beta}^{\lambda}(p) = \sum_{\sigma=\pm 1/2} u_{\alpha}^{\sigma}(\Lambda p) D_{\sigma\lambda}^{1/2}(R_{\Lambda p}^W), \quad (15)$$

$$\bar{u}_{\beta}^{\lambda}(p)S_{\beta\alpha}^{-1}(\Lambda) = \sum_{\sigma=\pm 1/2} D_{\lambda\sigma}^{+1/2}(R_{\Lambda p}^W)\bar{u}_{\alpha}^{\sigma}(\Lambda p). \quad (16)$$

As a result of transformations using (12)–(16), the expressions for four decay amplitudes in the case of the ZZ-boson mechanism take the form

$$\begin{aligned} \mathcal{M}_{T+\gamma}^{(1)} &= \frac{e^4}{4 \sin 2\theta M_Z m^2} \int d\mathbf{p} \int d\mathbf{q} \int d\mathbf{t} \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D_Z^{\alpha\lambda}(k_1) D_Z^{\alpha\sigma}(k_2) \frac{N(\mathbf{p}, \mathbf{t})N(\mathbf{q}, \mathbf{t})}{\left[\frac{1}{4} \left(M_H^2 - \frac{3}{4} M_T^2 \right) - m^2 \right]} \\ &\quad \times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \hat{\epsilon}_{\gamma}(\hat{p}_1 + \hat{k} + m) \Gamma^{\lambda} \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \Gamma^{\sigma} \right\}, \end{aligned} \quad (17)$$

$$\begin{aligned} \mathcal{M}_{T+\gamma}^{(2)} &= \frac{e^4}{4 \sin 2\theta M_Z m^2} \int d\mathbf{p} \int d\mathbf{q} \int d\mathbf{t} \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D_Z^{\alpha\lambda}(k_1) D_Z^{\alpha\sigma}(k_2) \frac{N(\mathbf{p}, \mathbf{t})N(\mathbf{q}, \mathbf{t})}{\left[\frac{1}{4} \left(M_H^2 - \frac{3}{4} M_T^2 \right) - m^2 \right]} \\ &\quad \times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \Gamma^{\lambda} \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \Gamma^{\sigma} (\hat{p}_1 + \hat{k} + m) \hat{\epsilon}_{\gamma} \right\}, \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{M}_{T+\gamma}^{(3)} &= \frac{e^4}{4 \sin 2\theta M_Z m^2} \int d\mathbf{p} \int d\mathbf{q} \int d\mathbf{t} \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D_Z^{\alpha\lambda}(k_1) D_Z^{\alpha\sigma}(k_2) \frac{N(\mathbf{p}, \mathbf{t})N(\mathbf{q}, \mathbf{t})}{\left[\frac{1}{4} \left(M_H^2 - \frac{3}{4} M_T^2 \right) - m^2 \right]} \\ &\quad \times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \Gamma^{\lambda} (-\hat{q}_1 - \hat{k} + m) \hat{\epsilon}_{\gamma} \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \Gamma^{\sigma} \right\}, \end{aligned} \quad (19)$$

$$\begin{aligned} \mathcal{M}_{T+\gamma}^{(4)} &= \frac{e^4}{4 \sin 2\theta M_Z m^2} \int d\mathbf{p} \int d\mathbf{q} \int d\mathbf{t} \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D_Z^{\alpha\lambda}(k_1) D_Z^{\alpha\sigma}(k_2) \frac{N(\mathbf{p}, \mathbf{t})N(\mathbf{q}, \mathbf{t})}{\left[\frac{1}{4} \left(M_H^2 - \frac{3}{4} M_T^2 \right) - m^2 \right]} \\ &\quad \times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \Gamma^{\lambda} \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \hat{\epsilon}_{\gamma} (-\hat{q}_2 - \hat{k} + m) \Gamma^{\sigma} \right\}, \end{aligned} \quad (20)$$

where the relativistic normalization factor has the form

$$N(\mathbf{p}, \mathbf{t}) = \left[\frac{\epsilon(\mathbf{p} + \mathbf{t}/2)}{m} \frac{(\epsilon(\mathbf{p} + \mathbf{t}/2) + m)}{2m} \frac{\epsilon(\mathbf{p} - \mathbf{t}/2)}{m} \frac{(\epsilon(\mathbf{p} - \mathbf{t}/2) + m)}{2m} \right]^{-1/2}. \quad (21)$$

The momenta in the Z -boson propagators are

$$k_1 = \frac{1}{2}T + p + q + t, \quad k_2 = \frac{1}{2}T - p - q. \quad (22)$$

The amplitudes (17)–(20) contain relativistic projection operators onto the state of a pair of quarks or a pair of antiquarks with spin 1,

$$\hat{\Pi}_i^V(\mathbf{p}, \mathbf{t}) = \left[\frac{\hat{v} - 1}{2} - \hat{v} \frac{(\epsilon(\mathbf{p} + \frac{\mathbf{t}}{2}) - m)}{2m} - \frac{(\hat{p} + \frac{\hat{t}}{2})}{2m} \right] \hat{\epsilon}_i (1 + \hat{v}) \left[\frac{\hat{v} + 1}{2} - \hat{v} \frac{(\epsilon(\mathbf{p} - \frac{\mathbf{t}}{2}) - m)}{2m} + \frac{(\hat{p} - \frac{\hat{t}}{2})}{2m} \right], \quad (23)$$

where ϵ_i ($i = 1, 2$) are polarization vectors describing pairs of quarks and antiquarks with spin 1. The color part of amplitudes (17)–(20) is allocated as a product of color factors describing pairs of quarks and antiquarks in an antisymmetric state: $\frac{1}{\sqrt{2}}\epsilon_{ijk}\frac{1}{\sqrt{2}}\epsilon_{ijl} = \delta_{kl}$.

Leaving in the numerator of amplitudes the quadratic terms in relative momenta $\omega_p = \mathbf{p}^2/m^2$, $\omega_q = \mathbf{q}^2/m^2$, $\omega_t = \mathbf{t}^2/m^2$, we obtain the following expression for the numerator of total amplitude of the production of a vector tetraquark and a photon:

$$\mathcal{N}_{T+\gamma} = N_1(v\epsilon_\gamma)(v_\gamma\epsilon_T) + N_2(\epsilon_\gamma\epsilon_T) + N_3\epsilon_{\mu\nu\sigma\lambda}v^\mu v_\gamma^\nu\epsilon_T^\sigma\epsilon_\gamma^\lambda, \quad (24)$$

$$N_1 = -4(1 - 2a_z) \left(1 - \frac{7}{48}r_2^2 + \frac{7}{48}r_3^2 - \frac{1}{192}r_2^2r_3^2 \right) \omega_t, \quad (25)$$

$$N_2 = -4(1 - 2a_z)\omega_t \left[r_3^{-1} \left(-\frac{1}{2}r_2^2 - \frac{1}{96}r_3^4 \right) + \frac{1}{4} - \frac{1}{24}r_2^2 + \frac{1}{128}r_2^4 + \frac{1}{32}r_3^2 - \frac{1}{192}r_3^2r_2^2 \right], \quad (26)$$

$$\begin{aligned} N_3 = & 32 - 2r_2^2 - 4r_3^2 + \frac{1}{2}r_2^2r_3^2 - a_z(1 - a_z) - (\omega_p + \omega_q) \left[-\frac{112}{3} + \frac{7}{3}r_2^2 + \frac{14}{3}r_3^2 - \frac{7}{12}r_2^2r_3^2 \right. \\ & + a_z(1 - a_z) \left(\frac{224}{3} + \frac{2}{3}r_2^2 + \frac{4}{3}r_3^2 - \frac{1}{6}r_2^2r_3^2 \right) \left. \right] - \omega_t \left[-\frac{61}{3} + \frac{59}{48}r_2^2 + \frac{67}{24}r_3^2 - \frac{65}{192}r_2^2r_3^2 \right. \\ & + a_z(1 - a_z) \left(\frac{122}{3} - \frac{1}{2}r_2^2 - r_3^2 + \frac{1}{8}r_2^2r_3^2 \right) \left. \right], \end{aligned} \quad (27)$$

where auxiliary four-vectors $v = \frac{T}{M_T}$, $v_\gamma = \frac{k}{M_T}$ are introduced for convenience. Parameters r_1 , r_2 , r_3 are determined by the ratios of particle masses in the form

$$r_1 = \frac{m}{M_T}, \quad r_2 = \frac{M_H}{M_Z}, \quad r_3 = \frac{M_T}{M_Z}. \quad (28)$$

The corresponding decay width, taking into account the leading-order relativistic corrections, is

$$\Gamma(H \rightarrow ZZ \rightarrow \gamma + T) = \frac{256\pi^3 \alpha^4 r_3^2 (r_2^2 - r_3^2) |\Psi(0, 0, 0)|^2}{\sin^2 2\theta_W M_Z^8 r_2^3 (4 - r_3^2)^2 (2r_2^2 - r_3^2 - 4)^2 \left[\frac{1}{4} (r_2^2 - \frac{3}{4} r_3^2) - r_1^2 r_3^2 \right]^2} \times \left[3N_2^2 + 2N_3^2 (vv_\gamma)^2 - N_1^2 (vv_\gamma)^2 \right], \quad (vv_\gamma) = \frac{(r_2^2 - r_3^2)}{2r_3^2}. \quad (29)$$

One of parameters that enters (29) is the value of the tetraquark wave function at zero coordinate values $\Psi(0, 0, 0)$. This parameter can be found by solving the eigenvalue problem for the Hamiltonian of the four-particle system. As discussed above, the total tetraquark wave function can be represented as the product of spin wave function χ_s , color wave function χ_c , and coordinate wave function Ψ in the general form

$$\Phi = \mathcal{A}(\chi_s \otimes \chi_c \otimes \Psi), \quad (30)$$

where $\mathcal{A}$ is the antisymmetrization operator of two identical quarks and antiquarks. In the rest frame of the tetraquark, its coordinate wave function is obtained within the variational method as an expansion in the Gaussian trial functions. The quark interaction potential in nonrelativistic approximation can be represented as the sum of pair interaction potentials in the Jacobi coordinates in the form

$$\Delta V(\rho, \lambda, \sigma) = \Delta V^C(\rho, \lambda, \sigma) + \Delta V^{\text{conf}}(\rho, \lambda, \sigma), \quad (31)$$

$$\Delta V^C(\rho, \lambda, \sigma) = \left[-\frac{2\alpha_{s12}}{3\rho} - \frac{2\alpha_{s34}}{3\lambda} - \frac{4\alpha_{s13}}{3} \frac{1}{|\sigma + \frac{m_4}{m_{34}}\lambda - \frac{m_2}{m_{12}}\rho|} - \frac{4\alpha_{s14}}{3} \frac{1}{|\sigma - \frac{m_3}{m_{34}}\lambda - \frac{m_2}{m_{12}}\rho|} - \frac{4\alpha_{s23}}{3} \frac{1}{|\sigma + \frac{m_4}{m_{34}}\lambda + \frac{m_1}{m_{12}}\rho|} - \frac{4\alpha_{s24}}{3} \frac{1}{|\sigma - \frac{m_3}{m_{34}}\lambda + \frac{m_1}{m_{12}}\rho|} \right], \quad (32)$$

$$\Delta V^{\text{conf}}(\rho, \lambda, \sigma) = \left[\frac{1}{2} A|\rho| + \frac{1}{2} A|\lambda| + A|\sigma + \frac{m_4}{m_{34}}\lambda - \frac{m_2}{m_{12}}\rho| + A|\sigma - \frac{m_3}{m_{34}}\lambda - \frac{m_2}{m_{12}}\rho| + A|\sigma + \frac{m_4}{m_{34}}\lambda + \frac{m_1}{m_{12}}\rho| + A|\sigma - \frac{m_3}{m_{34}}\lambda + \frac{m_1}{m_{12}}\rho| + B \right], \quad (33)$$

where α_{sij} is the strong interaction constant of the pair of particles i and j . The parameters in the confinement potential (33) and in the Coulomb potential are chosen in exactly the same form as in the case of the interaction of quarks and antiquarks in mesons and baryons [36]. In the Jacobi coordinates, the coordinate wave function has the form

$$\Psi(\rho, \lambda, \sigma) = \sum_{I=1}^K C_I e^{-\frac{1}{2} [A_{11}(I)\rho^2 + 2A_{12}(I)\rho\lambda + A_{22}(I)\lambda^2 + 2A_{13}(I)\rho\sigma + 2A_{23}(I)\lambda\sigma + A_{33}(I)\sigma^2]}, \quad (34)$$

where $A_{ij}(I)$ is the matrix of nonlinear variational parameters, and C_I are linear variational parameters.

The normalization coefficient in the wave function of the system in coordinate representation is expressed analytically in terms of nonlinear parameters,

$$\mathcal{N} = \sum_{I=1}^K \sum_{J=1}^K C_I C_J \frac{(8\pi^3)^{3/2}}{B_{33}^{3/2}(I, J) (\tilde{B}_{11}(I, J) \tilde{B}_{22}(I, J) - \tilde{B}_{12}^2(I, J))^{3/2}}, \quad (35)$$

$$\tilde{B}_{11} = B_{11} - \frac{B_{13}^2}{B_{33}}, \quad \tilde{B}_{22} = B_{22} - \frac{B_{23}^2}{B_{33}}, \quad \tilde{B}_{12} = B_{12} - \frac{B_{13}B_{23}}{B_{33}}, \quad B_{kl} = A_{kl}(I) + A_{kl}(J). \quad (36)$$

In momentum representation, the wave function of a four-particle system can be obtained by performing the Fourier transform (34) in the form

$$\Psi(\mathbf{p}, \mathbf{q}, \mathbf{t}) = \sum_{I=1}^K \frac{C_I}{\sqrt{\mathcal{N}}(\det A)^{3/2}(2\pi)^9} e^{-\frac{1}{2\det A}[\mathbf{p}^2(A_{22}A_{33}-A_{23}^2)+\mathbf{q}^2(A_{11}A_{33}-A_{13}^2)+\mathbf{t}^2(A_{11}A_{22}-A_{12}^2)]} \\ \times e^{-\frac{1}{2\det A}[2\mathbf{p}\mathbf{t}(A_{12}A_{23}-A_{13}A_{22})+2\mathbf{q}\mathbf{t}(A_{12}A_{13}-A_{11}A_{23})+2\mathbf{p}\mathbf{q}(A_{13}A_{23}-A_{12}A_{33})]}, \quad (37)$$

where $\mathbf{p}$, $\mathbf{q}$, $\mathbf{t}$ are relative momenta for particles 12, 34, and the pair of particles 12 and 34, and all matrix elements $A_{kl} = A_{kl}(I)$.

The normalization factor of wave function (37), which is obtained after calculating the integrals over relative momenta, is equal to 1,

$$\langle \Psi | \Psi \rangle = \int |\Psi(\mathbf{p}, \mathbf{q}, \mathbf{t})|^2 \frac{d\mathbf{p}d\mathbf{q}d\mathbf{t}}{(2\pi)^9} = \sum_{I=1}^K \sum_{J=1}^K \frac{C_I C_J 16\sqrt{2}\pi^{\frac{9}{2}}}{\mathcal{N}} \frac{1}{\det Q(I, J)^{\frac{3}{2}}} \frac{1}{\det A(I)^{\frac{3}{2}}} \frac{1}{\det A(J)^{\frac{3}{2}}}. \quad (38)$$

The elements of the matrix Q in (38) are expressed in terms of nonlinear parameters A_{ij} as follows:

$$Q_{11} = \frac{(A_{22}(I)A_{33}(I) - A_{23}^2(I))}{\det A(I)} + \frac{(A_{22}(J)A_{33}(J) - A_{23}^2(J))}{\det A(J)}, \quad (39)$$

$$Q_{22} = \frac{(A_{11}(I)A_{33}(I) - A_{13}^2(I))}{\det A(I)} + \frac{(A_{11}(J)A_{33}(J) - A_{13}^2(J))}{\det A(J)}, \quad (40)$$

$$Q_{33} = \frac{(A_{11}(I)A_{22}(I) - A_{12}^2(I))}{\det A(I)} + \frac{(A_{11}(J)A_{22}(J) - A_{12}^2(J))}{\det A(J)}, \quad (41)$$

$$Q_{13} = \frac{(A_{12}(I)A_{23}(I) - A_{13}(I)A_{22}(I))}{\det A(I)} + \frac{(A_{12}(J)A_{23}(J) - A_{13}(J)A_{22}(J))}{\det A(J)}, \quad (42)$$

$$Q_{23} = \frac{(A_{12}(I)A_{13}(I) - A_{11}(I)A_{23}(I))}{\det A(I)} + \frac{(A_{12}(J)A_{13}(J) - A_{11}(J)A_{23}(J))}{\det A(J)}, \quad (43)$$

$$Q_{12} = \frac{(A_{13}(I)A_{23}(I) - A_{12}(I)A_{33}(I))}{\det A(I)} + \frac{(A_{13}(J)A_{23}(J) - A_{12}(J)A_{33}(J))}{\det A(J)}. \quad (44)$$

Then the magnitude of coordinate wave function at zero relative coordinates, included in (29), has the form

$$\Psi(0, 0, 0) = \frac{1}{\sqrt{\langle \Psi | \Psi \rangle}} \sum_{I=1}^K C_I \frac{1}{16\sqrt{2}\pi^{\frac{9}{2}}} \frac{1}{\det \tilde{Q}(I)^{\frac{3}{2}}} \frac{1}{\det A(I)^{\frac{3}{2}}}, \quad (45)$$

where matrix elements $\tilde{Q}$ are determined by the first half of terms in matrix elements Q .

An increase in calculational accuracy of decay rates can be achieved by taking into account various corrections to nonrelativistic approximation, including relativistic corrections determined by relative momenta in the four-particle system. Relativistic corrections in the production of bound states of four particles are determined by momentum integrals of the following form:

$$\left\langle \frac{\mathbf{p}^2}{m^2} \right\rangle = \int \Psi(\mathbf{p}, \mathbf{q}, \mathbf{k}) \frac{\mathbf{p}^2}{m^2} \frac{d\mathbf{p}d\mathbf{q}d\mathbf{k}}{(2\pi)^9} = \sum_{I=1}^K \frac{3C_I}{\sqrt{\mathcal{N}}} \frac{(\tilde{Q}_{22}\tilde{Q}_{33} - \tilde{Q}_{23}^2)}{\det A(I)^{\frac{3}{2}} \det \tilde{Q}(I)^{\frac{5}{2}} m^2} \\ \times \left[\text{Erf} \left(\sqrt{\frac{m^2 \det \tilde{Q}}{2(\tilde{Q}_{22}\tilde{Q}_{33} - \tilde{Q}_{23}^2)}} \right) - \frac{\sqrt{2}}{3\sqrt{\pi}} \frac{m^3 (\det \tilde{Q})^{3/2}}{(\tilde{Q}_{22}\tilde{Q}_{33} - \tilde{Q}_{23}^2)^{3/2}} \left(1 + \frac{3(\tilde{Q}_{22}\tilde{Q}_{33} - \tilde{Q}_{23}^2)}{m^2 \det \tilde{Q}} \right) e^{-\frac{m^2 \det \tilde{Q}}{2(\tilde{Q}_{22}\tilde{Q}_{33} - \tilde{Q}_{23}^2)}} \right], \quad (46)$$

TABLE I. Numerical results for relative decay widths in nonrelativistic approximation and taking into account relativistic corrections.

Decay	Nonrelativistic decay branching Br_{nr}	Relativistic decay branching Br_{rel}
$H \rightarrow ZZ \rightarrow T(1^{+-}) + \gamma$	0.38×10^{-14}	0.61×10^{-14}
$H \rightarrow Q\bar{Q} \rightarrow T(1^{+-}) + \gamma$	0.15×10^{-8}	0.21×10^{-8}
$H \rightarrow \text{loop} Q \rightarrow T(1^{+-}) + \gamma$	0.32×10^{-11}	0.50×10^{-11}
$H \rightarrow T(1^{--}) + \gamma$	0.58×10^{-10}	0.88×10^{-10}

$$\left\langle \frac{\mathbf{q}^2}{m^2} \right\rangle = \int \Psi(\mathbf{p}, \mathbf{q}, \mathbf{k}) \frac{\mathbf{q}^2}{m^2} \frac{d\mathbf{p}d\mathbf{q}d\mathbf{k}}{(2\pi)^9} = \sum_{I=1}^K \frac{3C_I}{\sqrt{N}} \frac{(\tilde{Q}_{11}\tilde{Q}_{33} - \tilde{Q}_{13}^2)}{\det A(I)^{\frac{3}{2}} \det \tilde{Q}(I)^{\frac{5}{2}} m^2} \times \left[\text{Erf} \left(\sqrt{\frac{m^2 \det \tilde{Q}}{2(\tilde{Q}_{11}\tilde{Q}_{33} - \tilde{Q}_{13}^2)}} \right) - \frac{\sqrt{2}}{3\sqrt{\pi}} \frac{m^3 (\det \tilde{Q})^{3/2}}{(\tilde{Q}_{11}\tilde{Q}_{33} - \tilde{Q}_{13}^2)^{3/2}} \left(1 + \frac{3(\tilde{Q}_{11}\tilde{Q}_{33} - \tilde{Q}_{13}^2)}{m^2 \det \tilde{Q}} \right) e^{-\frac{m^2 \det \tilde{Q}}{2(\tilde{Q}_{11}\tilde{Q}_{33} - \tilde{Q}_{13}^2)}} \right], \quad (47)$$

$$\left\langle \frac{\mathbf{t}^2}{m^2} \right\rangle = \int \Psi(\mathbf{p}, \mathbf{q}, \mathbf{t}) \frac{\mathbf{t}^2}{m^2} \frac{d\mathbf{p}d\mathbf{q}d\mathbf{t}}{(2\pi)^9} = \sum_{I=1}^K \frac{3C_I}{\sqrt{N}} \frac{(\tilde{Q}_{11}\tilde{Q}_{22} - \tilde{Q}_{12}^2)}{\det A(I)^{\frac{3}{2}} \det \tilde{Q}(I)^{\frac{5}{2}} m^2} \times \left[\text{Erf} \left(\sqrt{\frac{m^2 \det \tilde{Q}}{2(\tilde{Q}_{11}\tilde{Q}_{22} - \tilde{Q}_{12}^2)}} \right) - \frac{\sqrt{2}}{3\sqrt{\pi}} \frac{m^3 (\det \tilde{Q})^{3/2}}{(\tilde{Q}_{11}\tilde{Q}_{22} - \tilde{Q}_{12}^2)^{3/2}} \left(1 + \frac{3(\tilde{Q}_{11}\tilde{Q}_{22} - \tilde{Q}_{12}^2)}{m^2 \det \tilde{Q}} \right) e^{-\frac{m^2 \det \tilde{Q}}{2(\tilde{Q}_{11}\tilde{Q}_{22} - \tilde{Q}_{12}^2)}} \right], \quad (48)$$

where Erf is the error function. Despite the convergence of these integrals, when calculating them in the region of large values of relative momenta, one integral is cut off at relativistic momentum m , since in the region of relativistic momenta we have poor knowledge of the wave function of the bound state.

For numerical calculations of decay width (29) with the formation of the tetraquark ($cc\bar{c}\bar{c}$), the following values of main parameters are obtained [36,41]:

$$\Psi(0,0,0) = 0.10 \text{ GeV}^{9/2}, \quad \omega_p = \left\langle \frac{\mathbf{p}^2}{m^2} \right\rangle = 0.18, \\ \omega_q = \left\langle \frac{\mathbf{q}^2}{m^2} \right\rangle = 0.18, \quad \omega_t = \left\langle \frac{\mathbf{t}^2}{m^2} \right\rangle = 0.12. \quad (49)$$

A more detailed calculation of the tetraquark wave functions and parameters is given in [36]. Other relativistic

parameters, which are determined by $\langle \frac{\mathbf{p}^2}{m^2} \rangle$, $\langle \frac{\mathbf{q}^2}{m^2} \rangle$, $\langle \frac{\mathbf{p}\mathbf{q}}{m^2} \rangle$, are omitted here, since their values are 2 orders of magnitude smaller than those considered (49). Numerical value of the decay width of $H \rightarrow T_{cc\bar{c}\bar{c}} + \gamma$ from the ZZ mechanism is presented in Table I.

B. Quark-gluon production mechanism

Another important mechanism of tetraquark production is the quark-gluon mechanism, shown in Fig. 1(a). In this case, there are ten amplitudes for the process $H \rightarrow \gamma + T_{cc\bar{c}\bar{c}}$, the construction of which is similar to previous amplitudes (17)–(20). Here, as an example, we give two decay amplitudes in which a photon is immediately emitted by a quark or an antiquark produced in the decay of the H boson,

$$\mathcal{M}_{H \rightarrow Q\bar{Q} \rightarrow T+\gamma}^{(1)} = \frac{(4\pi\alpha)(4\pi\alpha_s)m}{\sin 2\theta M_Z} \int d\mathbf{p} \int d\mathbf{q} \int d\mathbf{t} \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D^{\sigma\omega}(k_1) N(\mathbf{p}, \mathbf{t}) N(\mathbf{q}, \mathbf{t}) \\ \times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \gamma^\sigma \frac{(\frac{3}{4}\hat{T} - \hat{q} + \frac{1}{2}\hat{t} + m)}{((p_1 + p_2 + q_2)^2 - m^2)} \frac{(\frac{3}{4}\hat{T} - \hat{r} - \hat{q} + \frac{1}{2}\hat{t} + m)}{((q_1 + k)^2 - m^2)} \hat{e}_\gamma \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \gamma^\omega \right\}, \quad (50)$$

$$\mathcal{M}_{H \rightarrow Q\bar{Q} \rightarrow T+\gamma}^{(2)} = \frac{(4\pi\alpha)(4\pi\alpha_s)m}{\sin 2\theta M_Z} \int d\mathbf{p} \int d\mathbf{q} \int d\mathbf{t} \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D^{\sigma\omega}(k_1) N(\mathbf{p}, \mathbf{t}) N(\mathbf{q}, \mathbf{t}) \\ \times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \gamma^\sigma \frac{(\hat{r} - \hat{k} - \hat{q}_1 + m)}{((r - k - q_1)^2 - m^2)} \hat{e}_\gamma \frac{(\hat{r} - \hat{q}_1 + m)}{((r - q_1)^2 - m^2)} \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \gamma^\omega \right\}. \quad (51)$$

The explicit form of remaining decay amplitudes is given in Appendix A.

The color part of amplitudes (50) and (51) is allocated as a product of color factors describing pairs of quarks and antiquarks in an antisymmetric state: $\frac{1}{\sqrt{2}}\varepsilon_{ikn}\frac{1}{\sqrt{2}}\varepsilon_{jlm}T_{ij}^a$
 $T_{kl}^a = -\frac{2}{3}\delta_{nm}$.

Let us simplify the denominators of gluon $D^{\sigma\omega}(k_1)$ and quark propagators in (50) and (51) as follows:

$$\frac{1}{k_1^2} = \frac{4}{M_T^2} \left(1 - \frac{4p^2}{M_T^2} - \frac{4q^2}{M_T^2} \right), \quad \frac{1}{[(r-q_1)^2 - m^2]} = \frac{4}{3M_H^2}. \quad (52)$$

Such simplifications mean that we neglect the momenta of relative motion of quarks in denominators of propagators compared to the Higgs boson or tetraquark masses. In the numerator of amplitudes, when transforming them, we retain the second-order terms in relative momenta of heavy quarks. As a result, we can factor out the common factor from a sum of all ten amplitudes and represent the numerator of total decay amplitude as

$$\mathcal{N}_M = \frac{512}{3M_T^2 M_H^2} \left[10 + 4\frac{r_3^2}{r_2^2} + \frac{(\mathbf{p}^2 + \mathbf{q}^2)}{m^2} \left(\frac{11}{2} + 4\frac{r_3^2}{r_2^2} \right) + \frac{\mathbf{t}^2}{m^2} \left(-\frac{1}{12} + \frac{4}{3}\frac{r_3^2}{r_2^2} \right) \right] \varepsilon_{\mu\nu\lambda\sigma} v^\mu v^\nu v_\gamma^\lambda \varepsilon_\gamma^\sigma. \quad (53)$$

The contribution to the decay width from the quark-gluon mechanism with the production of a vector tetraquark is then determined by the following expression:

$$\Gamma(H \rightarrow Q\bar{Q} \rightarrow \gamma + T) = \frac{2048\pi^3 \alpha^2 \alpha_s^2}{9M_Z^8 \sin^2 2\theta_W} \frac{\left(1 - \frac{r_3^2}{r_2^2}\right)^3}{r_3^5 r_2} |\Psi(0, 0, 0)|^2 \times \left[10 + 4\frac{r_3^2}{r_2^2} + (\omega_p + \omega_q) \left(\frac{11}{2} + 4\frac{r_3^2}{r_2^2} \right) + \omega_t \left(-\frac{1}{12} + \frac{4}{3}\frac{r_3^2}{r_2^2} \right) \right]^2. \quad (54)$$

The numerical result obtained using (54) is also presented in Table I. It follows from the calculations that this decay mechanism is dominant. Total decay amplitude is the sum of amplitudes of decays $H \rightarrow ZZ \rightarrow T + \gamma$ and $H \rightarrow Q\bar{Q} \rightarrow T + \gamma$. We examine these contributions separately to find the leading contribution. The interference terms are present in total amplitude, but their contribution is suppressed, since the contribution of the quark-gluon mechanism is dominant. Note that the branching of the decay $H \rightarrow Q\bar{Q} \rightarrow T + \gamma$ is an order of magnitude greater than the branching of the decay $H \rightarrow J/\Psi + J/\Psi$ [12].

C. Quark loop production mechanism

In the study of charmonium pair production in [12], it was shown that it is necessary to consider other decay mechanisms, including the quark-gluon loop mechanism. The order of this contribution in the interaction constants to the decay width is determined by the factor $\alpha_s^4 \alpha^2$. In this case, the dependence of the amplitude on the particle mass ratios (28) is important [12]. Figure 2 shows two different decay amplitudes that have a quark loop. In the first amplitude, a photon is emitted from internal lines of quark loop in Fig. 2(a), and in the second amplitude, a photon is emitted by quarks and antiquarks in the final state. The loop integral in the first amplitude is convergent, while the loop integral in the second amplitude diverges at large values of loop momentum.

The contribution of first amplitude is determined by two tensor functions, the first of which corresponds to the quark loop, and the second to two-gluon production of a tetraquark. If a vector tetraquark state is considered, then it arises as a result of vector addition of two moments, the spin of two quarks $\mathbf{S}_{cc}$ and the spin of two antiquarks $\mathbf{S}_{\bar{c}\bar{c}}$,

$$\varepsilon_{S_{Zcc}}^\mu \varepsilon_{S_{Z\bar{c}\bar{c}}}^\nu \langle 1, 1, S_{Zcc}, S_{Z\bar{c}\bar{c}} | 0, 0 \rangle = \frac{1}{\sqrt{3}} (g^{\mu\nu} - v^\mu v^\nu), \quad (55)$$

$$\varepsilon_{S_{Zcc}}^\mu \varepsilon_{S_{Z\bar{c}\bar{c}}}^\nu \langle 1, 1, S_{Zcc}, S_{Z\bar{c}\bar{c}} | 1, 0 \rangle = -\frac{i}{\sqrt{2}} \varepsilon^{\alpha\beta\mu\nu} v_\alpha \varepsilon_{T\beta}, \quad (56)$$

$$\varepsilon_{S_{Zcc}}^\mu \varepsilon_{S_{Z\bar{c}\bar{c}}}^\nu \langle 1, 1, S_{Zcc}, S_{Z\bar{c}\bar{c}} | 2, 0 \rangle = \varepsilon_T^{\mu\nu}. \quad (57)$$

Using (56) in second tensor function, we obtain that the contribution of the amplitude in Fig. 2(a) to the production of a vector tetraquark is equal to 0. Note also that according to Furry's theorem the contribution of this diagram is zero.

Let us now consider the calculation of a contribution of second interaction amplitude, which is shown in Fig. 2(b). This contribution contains a quark loop, the integral over which diverges and can be calculated by the usual dispersion method, given that the photon virtualities are very different: $k_1^2 \sim M_T^2$, $k_2^2 \sim M_H^2$. The tensor corresponding to the quark loop in Fig. 2(b) has the following general structure [13]:

$$T_Q^{\mu\nu} = A_Q(\tau)(g^{\mu\nu}(v_1 v_2) - v_1^\nu v_2^\mu) + B_Q(\tau) \times [v_2^\mu - v_1^\mu(v_1 v_2)][v_1^\nu - v_2^\nu(v_1 v_2)], \quad (58)$$

where $\tau = \frac{M_H^2}{4m_Q^2}$, and m_Q is the mass of the heavy quark in the loop. The structure functions $A_Q(\tau)$ and $B_Q(\tau)$ can be calculated using the explicit expression for the loop integral tensor,

$$A_Q(\tau) = \frac{T^{\mu\nu}}{2} \left[\frac{g^{\mu\nu}}{v_1 v_2} - \frac{v_1^\nu v_2^\mu}{(v_1 v_2)^2 - v_1^2 v_2^2} \right],$$

$$B_Q(\tau) = \frac{T^{\mu\nu}}{2} \left[\frac{3v_1^\nu v_2^\mu}{((v_1 v_2)^2 - v_1^2 v_2^2)^2} - \frac{g^{\mu\nu}}{v_1 v_2((v_1 v_2)^2 - v_1^2 v_2^2)} \right]. \quad (59)$$

Since the vertex function (3) is proportional to the heavy quark mass m , the loop contributions of the heavy quarks

b, t are dominant. Then, at the production of the tetraquark $T_{cc\bar{c}\bar{c}}$, another mass parameter $M_T/4m_Q \sim m/m_Q$ appears, which can be used for expansions.

Two structure functions $A_Q(\tau)$, $B_Q(\tau)$ can be found using dispersion integrals of the form

$$A_Q(\tau) = \frac{1}{\pi} \int_1^\infty \frac{\text{Im} A_Q(\tau') d\tau'}{(\tau' - \tau + i0)}, \quad B_Q(\tau) = \frac{1}{\pi} \int_1^\infty \frac{\text{Im} B_Q(\tau') d\tau'}{(\tau' - \tau + i0)}. \quad (60)$$

The imaginary parts $\text{Im} A_Q(\tau)$ and $\text{Im} B_Q(\tau)$ can be calculated using the Mandelstam-Cutkosky rule. Since the main contribution to the Higgs boson decay width is given by the structure function $A_Q(\tau)$, we will consider this contribution below. The imaginary part of the function $A_Q(\tau)$ is equal to

$$\text{Im} A_Q = \frac{4mr_1^2}{\pi\tau^2} \left[(\tau - 2) \left(\ln \frac{\sqrt{\tau - 1} - \sqrt{\tau}}{\sqrt{\tau - 1} + \sqrt{\tau}} - i\pi \right) - 2\sqrt{\tau(\tau - 1)} \right], \quad r_1 = \frac{m}{m_Q}. \quad (61)$$

By calculating the dispersion integral (60), we obtain the following structure function $A_Q(\tau)$:

$$A_Q(\tau) = \frac{16mr_1^2}{\pi^2\tau^2} \left[(\tau - 2)(\text{ArcSin}\sqrt{\tau})^2 + 4\tau - 2\sqrt{\tau(1 - \tau)}\text{ArcSin}\sqrt{\tau} \right]. \quad (62)$$

Simplifying the denominators of the propagators in four amplitudes in Fig. 2(b), which is carried out similar to (52), allows us to represent the convolution of the tensor (58) with total numerator of amplitudes in the form

$$\begin{aligned} \mathcal{N} &= A_Q(\tau)[g^{\mu\nu}(v_1 v_2) - v_1^\nu v_2^\mu](N_1^{\mu\nu} + N_2^{\mu\nu} + N_3^{\mu\nu} + N_4^{\mu\nu}) \\ &= A_Q(\tau)\varepsilon_{\mu\nu\lambda\sigma}v^\mu v^\nu \varepsilon_T^\lambda \varepsilon_T^\sigma M_T \left[8 \frac{M_H^2}{M_T^2} + \frac{(\mathbf{p}^2 + \mathbf{q}^2)}{m^2} \left(\frac{4}{3} + \frac{14}{3} \frac{M_H^2}{M_T^2} \right) + \frac{5}{3} \frac{\mathbf{t}^2}{m^2} \frac{M_H^2}{M_T^2} \right]. \end{aligned} \quad (63)$$

As a result, the decay width for this quark-gluon loop mechanism will take the form

$$\begin{aligned} \Gamma(H \rightarrow \text{loop } Q \rightarrow T + \gamma) &= \frac{16384\pi m^4 \alpha_s^4}{M_Z^{12} \sin^2 \theta_W r_2^9 r_3^9} \left(1 - \frac{r_3^2}{r_2^2} \right)^3 |\Psi(0, 0, 0)|^2 \left[8 + \frac{14}{3}(\omega_p + \omega_q) + \frac{5}{3}\omega_t \right]^2 \\ &\times \left| \left[(\tau - 2)(\text{ArcSin}\sqrt{\tau})^2 + 4\tau - 2\sqrt{\tau(1 - \tau)}\text{ArcSin}\sqrt{\tau} \right] \right|^2. \end{aligned} \quad (64)$$

Table I presents total contribution of loops with b and t quarks to the decay width. The quark loop mechanism is expected to give a smaller contribution to the decay width compared to usual quark-gluon mechanism, which is due to the increase in powers of the constant α_s .

D. Quark-gluon production mechanism for P-wave tetraquarks

It is useful to consider the production of other vector tetraquarks within the quark-gluon mechanism, which is done further for the 1^{--} state with orbital angular momentum $L = 1$. As in the case of previous tetraquark states, we consider states in which $S_{cc} = 1$, $S_{\bar{c}\bar{c}} = 1$, but when adding these two spins together, we have a state with total spin $S_T = 0$, but with total tetraquark angular momentum $J_T = 1$. The quantum numbers of such a tetraquark are $C_T = (-1)^{S_T + L_T} = (-1)$, $P_T = (-1)^{L_T} = (-1)$. The antisymmetry of the wave function under the permutation of identical fermions is due to its color part. To estimate the decay width, we consider such an orbital excitation, which is determined by relative orbital momentum

$L_\sigma = 1$ of one pair (cc) relative to the other pair ($\bar{c}\bar{c}$). Appendix B discusses the calculation of such a wave function within the variational method.

General structure of interaction amplitudes is preserved and has the form defined by formulas (50), (51), and (A1)–(A8) of Appendix A. When adding the spins $\mathbf{S}_{cc}$ and $\mathbf{S}_{\bar{c}\bar{c}}$, the projection operator (55) is introduced in accordance with the state under consideration. When calculating the interaction amplitudes, we take into account linear terms in relative momentum t so that, for states with orbital angular momentum $L_\sigma = 1$, a characteristic integral arises,

$$\int \frac{d\mathbf{p}d\mathbf{q}d\mathbf{t}}{(2\pi)^9} t_\mu \Psi^{L_\sigma=1}(\mathbf{p}, \mathbf{q}, \mathbf{t}) = (-i)\varepsilon_\mu(L_z^\sigma) R'_\sigma(0, 0, 0), \quad (65)$$

where $\varepsilon_\mu(L_z^\sigma)$ is the polarization vector of orbital motion, $R'_\sigma(0, 0, 0)$ is the derivative of radial wave function with respect to the coordinate σ at zero. In the case of the production of P-wave states, $R'_\sigma(0, 0, 0)$ is a new parameter of the theory, which can be found within the framework of variational calculation using (B2).

By keeping in the numerator of total production amplitude the quadratic terms in relative momenta $\mathbf{p}, \mathbf{q}$, we can represent the expression in a numerator in the form

$$\begin{aligned} \mathcal{N}(\gamma + T(1^{--})) = & (\varepsilon(L_z^\sigma)\varepsilon_\gamma) \left[13 + \frac{52}{3} \frac{r_2^2}{r_3^2} + 13 \frac{r_3^2}{r_2^2} + \frac{(\mathbf{p}^2 + \mathbf{q}^2)}{m^2} \left(\frac{97}{12} + \frac{101}{9} \frac{r_2^2}{r_3^2} + \frac{119}{12} \frac{r_3^2}{r_2^2} \right) \right] \\ & + (v\varepsilon_\gamma)(v_\gamma\varepsilon(L_z^\sigma)) \left[-\frac{104}{3} + \frac{80}{3} \frac{r_3^2}{r_2^2} - \frac{(\mathbf{p}^2 + \mathbf{q}^2)}{m^2} \left(\frac{202}{9} - \frac{112}{9} \frac{r_3^2}{r_2^2} \right) \right]. \end{aligned} \quad (66)$$

As a result of all transformations, the decay width of the Higgs boson with the production of a tetraquark 1^{--} is determined by the following expression:

$$\Gamma(H \rightarrow \gamma + T(1^{--})) = \frac{32\pi^3 \alpha^2 \alpha_s^2 \left(1 - \frac{r_3^2}{r_2^2}\right)}{M_Z^8 r_3 r_2^5 \sin^2 2\theta_W} |R'_\sigma(0, 0, 0)|^2 \left[3n_2^2 - (vv_\gamma)^2 n_1^2 \right], \quad (67)$$

$$n_1 = \left[-\frac{104}{3} + \frac{80}{3} \frac{r_3^2}{r_2^2} - (\omega_p + \omega_q) \left(\frac{202}{9} - \frac{112}{9} \frac{r_3^2}{r_2^2} \right) \right], \quad (68)$$

$$n_2 = \left[13 + \frac{52}{3} \frac{r_2^2}{r_3^2} + 13 \frac{r_3^2}{r_2^2} + (\omega_p + \omega_q) \left(\frac{97}{12} + \frac{101}{9} \frac{r_2^2}{r_3^2} + \frac{119}{12} \frac{r_3^2}{r_2^2} \right) \right], \quad (69)$$

where ω_p, ω_q are relativistic parameters determined by relative momenta $\mathbf{p}, \mathbf{q}$.

For numerical calculations of decay width (67) with the formation of a tetraquark ($cc\bar{c}\bar{c}$) with orbital momentum 1, the value of a derivative of wave function at zero is obtained: $R'(0, 0, 0) = 0.02 \text{ GeV}^{11/2}$. When obtaining this value, it is important to know the wave function of tetraquark in the field of relativistic momenta. But since this work uses the nonrelativistic tetraquark wave function obtained by solving the Schrödinger wave equation, error determination of $R'(0, 0, 0)$ is not less than 30%.

III. CONCLUSION

In the color space, there are two colorless configurations for which there is an attraction between quarks and antiquarks and which can be considered in constructing the tetraquark $T_{cc\bar{c}\bar{c}}$: $(QQ)^3(\bar{Q}\bar{Q})^3$ and $(Q\bar{Q})^1(Q\bar{Q})^1$ (the

superscript denotes the color quantum number). In the first configuration $(QQ)^3(\bar{Q}\bar{Q})^3$, often called the diquark-antidiquark, the pair of identical particles is in a color-antisymmetric state and has spin 1 (the Pauli exclusion principle). In the second configuration $(Q\bar{Q})^1(Q\bar{Q})^1$, called the meson-meson configuration, the pair $Q, \bar{Q}$ is in a singlet color state (meson). To clarify the situation with the color for identical quarks (antiquarks) in such a configuration, it can be noted that in the color space for a pair of quarks (antiquarks) there are two types of colorless states $(QQ)^3(\bar{Q}\bar{Q})^3$ (antisymmetric in color) and $(QQ)^6(\bar{Q}\bar{Q})^6$ (symmetric in color). In addition to these color configurations, one can imagine other sets of color states: $(Q\bar{Q})^1(Q\bar{Q})^1$ (singlet state) and $(Q\bar{Q})^8(Q\bar{Q})^8$ (octet state), in which there is attraction and repulsion between Q and $\bar{Q}$. The transition from one set of basis color states to another set is determined by the expressions [42]

$$\begin{aligned}
(Q\bar{Q})^1 \otimes (Q\bar{Q})^1 &= \frac{1}{\sqrt{3}}(QQ)^{\bar{3}} \otimes (\bar{Q}\bar{Q})^3 \\
&\quad + \sqrt{\frac{2}{3}}(QQ)^6 \otimes (\bar{Q}\bar{Q})^{\bar{6}}, \\
(Q\bar{Q})^8 \otimes (Q\bar{Q})^8 &= -\sqrt{\frac{2}{3}}(QQ)^{\bar{3}} \otimes (\bar{Q}\bar{Q})^3 \\
&\quad + \frac{1}{\sqrt{3}}(QQ)^6 \otimes (\bar{Q}\bar{Q})^{\bar{6}}. \quad (70)
\end{aligned}$$

Therefore, we can say that, in the approximation when we omit the repulsive sextet states in (70), the color meson-meson state goes over to the triplet-antitriplet color state (diquark-antidiquark) with the coefficient $1/\sqrt{3}$. A detailed study of two configurations, meson-meson and diquark-antidiquark, performed in [43] shows that for a completely charmed tetraquark there are no bound states in the meson-meson configuration, while in the diquark-antidiquark configuration there can exist compact resonant states 0^+ , 1^+ , 2^+ with masses $6.2 \div 7.4$ GeV. In this paper, we limited ourselves to considering the diquark-antidiquark configuration during tetraquark production.

The production of exotic quark bound states (tetraquarks, pentaquarks, etc.) is a rapidly developing area in particle physics, starting in 2003, when the Belle Collaboration [1] discovered the first candidate to a tetraquark state. Over the past two decades, many new states have been discovered, the nature and properties of which remain to be studied and understood. Heavy tetraquarks, consisting of c and b quarks, have a special place among such states, since simple QCD-based models are believed to exist for them, allowing calculation of their observed properties. The LHCb Collaboration [2], and later CMS and ATLAS [3,8], discovered a broad structure with a mass twice the J/Ψ mass and a narrow $X(6900)$ state, which is considered to be a bound state of $(cc\bar{c}\bar{c})$.

Along with the calculation of the tetraquark mass spectrum, it is important to calculate the probabilities of tetraquark production and study various production mechanisms. The production of heavy tetraquarks has already been studied in proton-proton interaction reactions, electron-positron annihilation, and in B-meson decays [16–20,23–27]. In this paper, we estimate the width of exclusive decay of the Higgs boson with the formation of a tetraquark $T_{cc\bar{c}\bar{c}}$. Further search for tetraquarks $(cc\bar{c}\bar{c})$ can be carried out by their decays: $T \rightarrow gg + c\bar{c}$. Our calculations are based on the relativistic quark model and the variational method, within which relativistic effects connected with relative motion of quarks [12–14] are taken into account. To describe the properties of the tetraquark, we use a potential quark model with a Hamiltonian consisting of pair Coulomb interactions of quarks and antiquarks arising from the one-gluon exchange potential, as well as pair confinement terms. In the nonrelativistic approximation, these terms are supplemented by pair

hyperfine interaction potentials. When increasing the accuracy of mass spectrum calculation, we also took into account relativistic corrections. Unlike a number of other calculations, we do not use the smearing of the Dirac δ function. Unlike string quark models, in our calculations the confinement potential is related to relative distances between quarks and antiquarks. In addition, important differences from other calculations of the masses of bound states of four quarks are due to the choice of specific values of quark masses and constants of strong interaction between them. As a result, this model allows us to calculate wave functions of tetraquarks using the variational method [36] and use them to calculate the Higgs boson decay widths.

The calculations show that among the studied mechanisms of vector tetraquark $(cc\bar{c}\bar{c})$ production, the main one is the quark-gluon mechanism, which gives a decay width value several orders of magnitude larger than others. Further study of various mechanisms of the tetraquark production with different quantum numbers is of interest in connection with the expansion of research in the Higgs sector [44]. It is useful to compare the results of the production of heavy tetraquarks that were obtained in different reactions [11,15,17,18,24]. Thus, in the proton-proton interaction, the cross sections of the reaction $p + p \rightarrow T_{4c} \rightarrow J/\Psi + J/\Psi$ at the energy $\sqrt{s} = 13$ TeV were obtained in [16]. Using this cross section and the collider luminosity $10^{34} \text{ cm}^{-2} \text{ c}^{-1}$, one can estimate the number of such production events at ~ 6000 . The integrated production rates for S -wave vector 1^{+-} state $T_{cc\bar{c}\bar{c}}$ was obtained in $p - p$ interaction at $\sqrt{13}$ TeV in [19]. Estimated event yield is $(0.5 \div 0.8) \times 10^9$ assuming an integrated luminosity of 3000 fb^{-1} . The predicted production of vector tetraquark 1^{+-} is suppressed relative to 0^{++} , 2^{++} by 2 orders of magnitude. In this paper, we have studied in detail only the production of a vector tetraquark (1^{+-}), but within the framework of this formalism, it is possible to study the production of other tetraquark states (0^{++} , 2^{++}). Our preliminary estimates of the decay widths with the production of tetraquarks 0^{++} , 2^{++} , for example, in the case of the ZZ mechanism, show that the relative widths for such decays are suppressed compared to the calculated one (see Table I) by several orders of magnitude: 10^{-20} (2^{++}) and 10^{-23} (0^{++}). For the 0^{++} tetraquark, the decay width in the nonrelativistic approximation vanishes, and for the 2^{++} tetraquark, the decay width suppression is determined by additional powers of small mass factors (28). A more detailed study of the decay widths for different mechanisms of production of 0^{++} , 2^{++} is planned in a separate work. The Higgs boson decay branchings obtained in this work also allow one to estimate the number of events connected with tetraquark production, if one focuses on the Higgs boson factories [FCC-hh (100 TeV, 30 ab^{-1})], where the production of up to 2.8×10^{10} Higgs bosons per year is possible [44]. Thus, the number of events producing

tetraquarks $T_{cc\bar{c}\bar{c}}$ may be about 60. Currently, it is planned to create new high-energy and luminosity electron-positron colliders, such as the International Linear Collider (ILC) [45] and Compact Linear Collider (CLIC) [46], which will provide more accurate information about the interactions of the Higgs boson. Such colliders are expected to become the basis for the study of new physics, exotic multi-quark states. Given the planned luminosity at ILC $\mathcal{L} = 2 \text{ ab}^{-1}$, corresponding to $\sqrt{s} = 250 \text{ GeV}$ and the luminosity at CLIC $\mathcal{L} = 1 \text{ ab}^{-1}$ at $\sqrt{s} = 380 \text{ GeV}$, total number of tetraquark production events $T_{cc\bar{c}\bar{c}}$ in the photoproduction reaction is estimated at approximately 9.37×10^3 and 8.43×10^4 annually [18].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [36].

APPENDIX A: PRODUCTION TETRAQUARK $T_{cc\bar{c}\bar{c}}$ AMPLITUDES IN QUARK-GLUON MECHANISM

Within the quark-gluon mechanism, there are ten $H \rightarrow Q\bar{Q} \rightarrow \gamma + T$ decay amplitudes, two of which are given above in (50) and (51). The remaining decay amplitudes are of the form

$$\begin{aligned} \mathcal{M}_{H \rightarrow Q\bar{Q} \rightarrow T+\gamma}^{(3)} &= \frac{(4\pi\alpha)(4\pi\alpha_s)m}{\sin 2\theta M_Z} \int d\mathbf{p} \int d\mathbf{q} \int dt \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D^{\sigma\omega}(k_1) N(\mathbf{p}, \mathbf{t}) N(\mathbf{q}, \mathbf{t}) \\ &\times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \gamma^\sigma \frac{\left(\frac{3}{4}\hat{T} - \hat{q} + \hat{k} + \frac{1}{2}\hat{t} + m\right)}{((r - q_1)^2 - m^2)} \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \gamma^\omega \frac{\left(\hat{r} - \frac{3}{4}\hat{T} - \hat{p} + \frac{1}{2}\hat{t} + m\right)}{((p_2 + k)^2 - m^2)} \hat{\epsilon}_\gamma \right\}, \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} \mathcal{M}_{H \rightarrow Q\bar{Q} \rightarrow T+\gamma}^{(4)} &= \frac{(4\pi\alpha)(4\pi\alpha_s)m}{\sin 2\theta M_Z} \int d\mathbf{p} \int d\mathbf{q} \int dt \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D^{\sigma\omega}(k_1) N(\mathbf{p}, \mathbf{t}) N(\mathbf{q}, \mathbf{t}) \\ &\times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \gamma^\sigma \frac{\left(\frac{3}{4}\hat{T} - \hat{q} + \hat{k} + \frac{1}{2}\hat{t} + m\right)}{((r - q_1)^2 - m^2)} \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \hat{\epsilon}_\gamma \frac{\left(\frac{3}{4}\hat{T} - \hat{r} + \hat{q} + \frac{1}{2}\hat{t} + m\right)}{((k + q_2)^2 - m^2)} \gamma^\omega \right\}, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \mathcal{M}_{H \rightarrow Q\bar{Q} \rightarrow T+\gamma}^{(5)} &= \frac{(4\pi\alpha)(4\pi\alpha_s)m}{\sin 2\theta M_Z} \int d\mathbf{p} \int d\mathbf{q} \int dt \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D^{\sigma\omega}(k_1) N(\mathbf{p}, \mathbf{t}) N(\mathbf{q}, \mathbf{t}) \\ &\times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \gamma^\sigma \frac{(\hat{r} - \hat{k} - \hat{q}_1 + m)}{((r - k - q_1)^2 - m^2)} \hat{\epsilon}_\gamma \frac{\left(\frac{3}{4}\hat{T} - \hat{q} + \frac{1}{2}\hat{t} + \hat{k} + m\right)}{((r - q_1)^2 - m^2)} \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \gamma^\omega \right\}, \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \mathcal{M}_{H \rightarrow Q\bar{Q} \rightarrow T+\gamma}^{(6)} &= \frac{(4\pi\alpha)(4\pi\alpha_s)m}{\sin 2\theta M_Z} \int d\mathbf{p} \int d\mathbf{q} \int dt \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D^{\sigma\omega}(k_1) N(\mathbf{p}, \mathbf{t}) N(\mathbf{q}, \mathbf{t}) \\ &\times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \hat{\epsilon}_\gamma \frac{\left(\frac{1}{4}\hat{T} + \hat{p} + \hat{k} + \frac{1}{2}\hat{t} + m\right)}{((p_1 + k)^2 - m^2)} \frac{\left(-\frac{3}{4}\hat{T} + \hat{p} + \frac{1}{2}\hat{t} + m\right)}{((p_2 + q_1 + q_2)^2 - m^2)} \gamma_\sigma \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \gamma^\omega \right\}, \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} \mathcal{M}_{H \rightarrow Q\bar{Q} \rightarrow T+\gamma}^{(7)} &= \frac{(4\pi\alpha)(4\pi\alpha_s)m}{\sin 2\theta M_Z} \int d\mathbf{p} \int d\mathbf{q} \int dt \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D^{\sigma\omega}(k_1) N(\mathbf{p}, \mathbf{t}) N(\mathbf{q}, \mathbf{t}) \\ &\times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \frac{\left(-\frac{3}{4}\hat{T} - \hat{k} + \hat{p} + \frac{1}{2}\hat{t} + m\right)}{((p_1 - r)^2 - m^2)} \gamma^\sigma \frac{\left(-\frac{1}{4}\hat{T} - \hat{k} - \hat{q} + \frac{1}{2}\hat{t} + m\right)}{((q_1 + k)^2 - m^2)} \hat{\epsilon}_\gamma \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \gamma^\omega \right\}, \end{aligned} \quad (\text{A5})$$

$$\mathcal{M}_{H \rightarrow Q\bar{Q} \rightarrow T+\gamma}^{(8)} = \frac{(4\pi\alpha)(4\pi\alpha_s)m}{\sin 2\theta M_Z} \int d\mathbf{p} \int d\mathbf{q} \int d\mathbf{t} \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D^{\sigma\omega}(k_1) N(\mathbf{p}, \mathbf{t}) N(\mathbf{q}, \mathbf{t}) \\ \times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \frac{\left(-\frac{3}{4}\hat{T} + \hat{p} - \hat{k} + \frac{1}{2}\hat{t} + m\right)}{((r-p_1)^2 - m^2)} \gamma_\sigma \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \gamma^\omega \frac{\hat{r} - \left(\frac{3}{4}\hat{T} - \hat{p} + \frac{1}{2}\hat{t} + m\right)}{((k+p_2)^2 - m^2)} \hat{\varepsilon}_\gamma \right\}, \quad (\text{A6})$$

$$\mathcal{M}_{H \rightarrow Q\bar{Q} \rightarrow T+\gamma}^{(9)} = \frac{(4\pi\alpha)(4\pi\alpha_s)m}{\sin 2\theta M_Z} \int d\mathbf{p} \int d\mathbf{q} \int d\mathbf{t} \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D^{\sigma\omega}(k_1) N(\mathbf{p}, \mathbf{t}) N(\mathbf{q}, \mathbf{t}) \\ \times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \frac{\left(-\frac{3}{4}\hat{T} - \hat{k} + \hat{p} + \frac{1}{2}\hat{t} + m\right)}{((p_1-r)^2 - m^2)} \gamma^\sigma \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \hat{\varepsilon}_\gamma \frac{\left(\frac{3}{4}\hat{T} - \hat{r} + \hat{q} + \frac{1}{2}\hat{t} + m\right)}{((q_2+k)^2 - m^2)} \hat{\varepsilon}_\gamma \gamma^\omega \right\}, \quad (\text{A7})$$

$$\mathcal{M}_{H \rightarrow Q\bar{Q} \rightarrow T+\gamma}^{(10)} = \frac{(4\pi\alpha)(4\pi\alpha_s)m}{\sin 2\theta M_Z} \int d\mathbf{p} \int d\mathbf{q} \int d\mathbf{t} \bar{\Psi}_0(\mathbf{p}, \mathbf{q}, \mathbf{t}) D^{\sigma\omega}(k_1) N(\mathbf{p}, \mathbf{t}) N(\mathbf{q}, \mathbf{t}) \\ \times \text{Tr} \left\{ \hat{\Pi}_1^V(\mathbf{p}, \mathbf{t}) \frac{\left(-\frac{3}{4}\hat{T} + \hat{p} - \hat{k} + \frac{1}{2}\hat{t} + m\right)}{((r-p_1)^2 - m^2)} \hat{\varepsilon}_\gamma \frac{\left(-\frac{3}{4}\hat{T} + \hat{p} + \frac{1}{2}\hat{t} + m\right)}{((p_1+k-r)^2 - m^2)} \gamma_\sigma \hat{\Pi}_2^V(\mathbf{q}, \mathbf{t}) \gamma^\omega \right\}. \quad (\text{A8})$$

The total decay amplitude of the Higgs boson from the quark-gluon mechanism gives the decay width (54).

as the sum of three terms related to the relative coordinates ρ, λ, σ ,

APPENDIX B: TETRAQUARK $T_{cc\bar{c}\bar{c}}$ WAVE FUNCTION FOR STATES WITH ORBITAL MOMENTUM $L=1$

To calculate the decay width of the Higgs boson with the production of a tetraquark in the excited state $L=1$, it is necessary to know the corresponding wave function. Various orbital excitations are possible in a system of four particles, so that the orbital momentum can be represented

$$\mathbf{L} = \mathbf{L}_\rho + \mathbf{L}_\lambda + \mathbf{L}_\sigma. \quad (\text{B1})$$

We restrict ourselves further to the consideration of orbital excitations in the variable σ , which are connected with the orbital moment of one pair of particles (cc) relative to another pair of particles ($\bar{c}\bar{c}$). Therefore, we will look for the wave function of the tetraquark with $L_\sigma = 1$ in tensor form,

$$\Psi^{L=1}(\rho, \lambda, \sigma) = \sum_{I=1}^K C_I(\sigma \boldsymbol{\varepsilon}) e^{-\frac{1}{2} [A_{11}(I)\rho^2 + 2A_{12}(I)\rho\lambda + A_{22}(I)\lambda^2 + 2A_{13}(I)\rho\sigma + 2A_{23}(I)\lambda\sigma + A_{33}(I)\sigma^2]}, \quad (\text{B2})$$

where $\boldsymbol{\varepsilon}$ is the polarization vector of orbital motion.

The normalization condition for the wave function will take the form

$$\langle \Psi^{L=1} | \Psi^{L=1} \rangle = \sum_{I,J=1}^K C_I C_J \frac{12\sqrt{2}\pi^{7/2}(B_{11}B_{22} - B_{12}^2)}{(\det B)^{5/2}}. \quad (\text{B3})$$

To find the wave functions on the basis of the variational method, we calculate the matrix elements of kinetic energy operator, which for a system of four particles consists of three terms,

$$\hat{T} = \frac{\mathbf{p}_\rho^2}{2\mu_1} + \frac{\mathbf{p}_\lambda^2}{2\mu_2} + \frac{\mathbf{p}_\sigma^2}{2\mu_3}, \quad (\text{B4})$$

where the reduced masses of particles μ_1, μ_2, μ_3 are introduced,

$$\mu_1 = \frac{m_1 m_2}{m_{12}}, \quad \mu_2 = \frac{m_3 m_4}{m_{34}}, \quad \mu_3 = \frac{m_{12} m_{34}}{m_{1234}}. \quad (\text{B5})$$

As a result of analytical calculation, we obtain the following expressions for matrix elements of the Laplace operators:

$$\begin{aligned} \langle \Delta_\rho \rangle = \sum_{I,J=1}^K C_I C_J \frac{12\sqrt{2}\pi^{7/2}}{(\det B)^{7/2}} \bigg\{ & (A_{11}^I)^2 [3B_{11}B_{22}B_{33} - 3B_{11}B_{22}B_{23}^2 - 3B_{12}^2B_{22}B_{33} + 5B_{12}^2B_{23}^2 \\ & - 4B_{12}B_{13}B_{22}B_{23} + 2B_{13}^2B_{22}^2] + A_{11}^I A_{12}^I [B_{11}B_{22}(10B_{13}B_{23} - 6B_{12}B_{33}) - 4B_{11}B_{12}B_{23}^2 + 6B_{12}^3B_{33} \\ & - 2B_{12}^2B_{13}B_{23} - 4B_{12}B_{13}^2B_{22}] + 10A_{11}^I A_{13}^I [B_{11}B_{22}(B_{12}B_{23} - B_{13}B_{22}) - B_{12}^3B_{23} + B_{12}^2B_{13}B_{22}] \\ & + A_{11}^I [3B_{11}^2B_{22}B_{23} + 6B_{11}B_{12}^2B_{22}B_{33} - 3B_{11}B_{12}^2B_{23}^2 - 6B_{11}B_{12}B_{13}B_{22}B_{23} + 3B_{11}B_{22}(B_{13}^2B_{22} \\ & - B_{11}B_{22}B_{33}) - 3B_{12}^4B_{33} + 6B_{12}^3B_{13}B_{23} - 3B_{12}^2B_{13}^2B_{22}] + (A_{12}^I)^2 [3B_{11}^2B_{22}B_{33} + 2B_{11}^2B_{23}^2 \\ & + B_{12}^2(5B_{13}^2 - 3B_{11}B_{33}) - 4B_{11}B_{12}B_{13}B_{23} - 3B_{11}B_{13}^2B_{22}] + 10A_{12}^I A_{13}^I [B_{11}B_{12}^2B_{23} + B_{11}B_{22}(B_{12}B_{13} \\ & - B_{11}B_{23}) - B_{12}^3B_{13}] + 5(A_{13}^I)^2 (B_{11}B_{22} - B_{12}^2)^2 \bigg\}, \quad (\text{B6}) \end{aligned}$$

$$\begin{aligned} \langle \Delta_\lambda \rangle = \sum_{I,J=1}^K C_I C_J \frac{12\sqrt{2}\pi^{7/2}}{(\det B)^{7/2}} \bigg\{ & (A_{12}^I)^2 [3B_{11}B_{22}B_{33} - 3B_{11}B_{22}B_{23}^2 - 3B_{12}^2B_{22}B_{33} + 5B_{12}^2B_{23}^2 \\ & - 4B_{12}B_{13}B_{22}B_{23} + 2B_{13}^2B_{22}^2] + A_{12}^I A_{22}^I [-6B_{11}B_{12}B_{22}B_{33} - 4B_{11}B_{12}B_{23}^2 + 10B_{11}B_{13}B_{22}B_{23} \\ & - 4B_{12}B_{13}^2B_{22} - 2B_{12}^2B_{13}B_{23} + 6B_{12}^3B_{33}] + 10A_{12}^I A_{23}^I [(B_{11}B_{22} - B_{12}^2)(B_{12}B_{23} - B_{13}B_{22})] \\ & + (A_{22}^I)^2 [3B_{11}^2B_{22}B_{33} + 2B_{11}^2B_{23}^2 + B_{12}^2(5B_{13}^2 - 3B_{11}B_{33}) - 4B_{11}B_{12}B_{13}B_{23} - 3B_{11}B_{13}^2B_{22}] \\ & + 10A_{22}^I A_{23}^I [-B_{11}^2B_{22}B_{23} + B_{11}B_{12}^2B_{23} + B_{11}B_{12}B_{13}B_{22} - B_{12}^3B_{13}] + 5(A_{23}^I)^2 (B_{11}B_{22} - B_{12}^2)^2 \\ & + A_{22}^I [-3B_{11}^2B_{22}B_{33} + 3B_{11}^2B_{22}B_{23}^2 + 6B_{11}B_{12}^2B_{22}B_{33} - 3B_{11}B_{12}^2B_{23}^2 - 6B_{11}B_{12}B_{13}B_{22}B_{23} \\ & - 3B_{12}^4B_{33} + 6B_{12}^3B_{13}B_{23} - 3B_{12}^2B_{13}^2B_{22} + 3B_{11}B_{13}^2B_{22}] \bigg\}, \quad (\text{B7}) \end{aligned}$$

$$\begin{aligned} \langle \Delta_\sigma \rangle = \sum_{I,J=1}^K C_I C_J \frac{12\sqrt{2}\pi^{7/2}}{(\det B)^{7/2}} \bigg\{ & (A_{13}^I)^2 [3B_{11}B_{22}(B_{22}B_{33} - B_{23}^2) + B_{12}^2(5B_{23}^2 - 3B_{22}B_{33}) \\ & - 4B_{12}B_{13}B_{22}B_{23} + 2B_{13}^2B_{22}^2] + 2A_{13}^I A_{23}^I [-B_{12}(3B_{11}B_{22}B_{33} + 2B_{11}B_{23}^2 + 2B_{13}^2B_{22}) + 3B_{12}^3B_{33} \\ & + 5B_{11}B_{13}B_{22}B_{23} - B_{12}^2B_{13}B_{23}] + 10A_{13}^I A_{33}^I [(B_{13}B_{22} - B_{12}B_{23})(B_{12}^2 - B_{11}B_{22})] \\ & - 2A_{13}^I (B_{13}B_{22} - B_{12}B_{23}) [B_{11}(B_{23}^2 - B_{22}B_{33}) + B_{12}^2B_{33} - 2B_{12}B_{13}B_{23} + B_{13}^2B_{22}] \\ & + (A_{23}^I)^2 [B_{12}^2(5B_{13}^2 - 3B_{11}B_{33}) - 4B_{11}B_{12}B_{13}B_{23} + B_{11}(3B_{11}B_{22}B_{33} + 2B_{11}B_{23}^2 - 3B_{13}^2B_{22})] \\ & + 10A_{23}^I A_{33}^I (B_{11}B_{23} - B_{12}B_{13})(B_{12}^2 - B_{11}B_{22}) - 2A_{23}^I (B_{12}B_{13} - B_{11}B_{23}) \det B \\ & + 5(A_{33}^I)^2 (B_{12}^2 - B_{11}B_{22})^2 + 5A_{33}^I (B_{12}^2 - B_{11}B_{22}) \det B \bigg\}. \quad (\text{B8}) \end{aligned}$$

The matrix elements of the potential energy of pairwise Coulomb interactions are obtained in the form

$$\begin{aligned} \left\langle \frac{1}{\rho} \right\rangle = \sum_{I,J=1}^K C_I C_J \frac{8\pi^3}{(\det B)^2 (B_{22}B_{33} - B_{23}^2)^{3/2}} \bigg[& 3B_{11}B_{22}(B_{22}B_{33} - B_{23}^2) \\ & - B_{12}^2(3B_{22}B_{33} - 2B_{23}^2) + 2B_{12}B_{13}B_{22}B_{23} - B_{13}^2B_{22}^2 \bigg], \quad (\text{B9}) \end{aligned}$$

$$\begin{aligned} \left\langle \frac{1}{\lambda} \right\rangle = \sum_{I,J=1}^K C_I C_J \frac{8\pi^3}{(\det B)^2 (B_{22}B_{33} - B_{23}^2)^{3/2}} \bigg[& 3B_{11}B_{22}(B_{11}B_{33} - B_{13}^2) \\ & - B_{12}^2(3B_{11}B_{33} - 2B_{13}^2) + 2B_{11}B_{12}B_{13}B_{23} - B_{11}^2B_{23}^2 \bigg], \quad (\text{B10}) \end{aligned}$$

$$\begin{aligned}
\left\langle \frac{1}{|\mathbf{r}_1 - \mathbf{r}_3|} \right\rangle = & \sum_{I,J=1}^K C_I C_J \frac{2\pi^3}{(\det B)^2 (F_1 F_2 - F_3^2)^{3/2}} \left\{ B_{11}^2 (8B_{22}^2 - 8B_{22}B_{23} + 3B_{22}B_{33} - B_{23}^2) \right. \\
& + B_{12}^2 [2(4B_{23}(B_{11} + B_{13}) - 8B_{11}B_{22} + B_{13}^2 - 4B_{13}B_{22} + B_{23}^2) - 3B_{33}(B_{11} + B_{22})] \\
& + 2B_{12}(B_{13} - B_{23})[B_{11}(4B_{22} + B_{23}) - B_{13}B_{22}] + 6B_{11}B_{12}B_{22}B_{33} + B_{11}B_{22}(-3B_{13}^2 + 8B_{13}B_{22} \\
& \left. - 4B_{13}B_{23} + 3B_{22}B_{33} - 3B_{23}^2) + 8B_{12}^4 + B_{12}^3(-8B_{13} + 8B_{23} - 6B_{33}) - B_{13}^2B_{22}^2 \right\}, \quad (B11)
\end{aligned}$$

where auxiliary functions are introduced,

$$F_1 = B_{11} + \frac{1}{4}B_{33} + B_{13}, \quad (B12)$$

$$F_2 = B_{22} + \frac{1}{4}B_{33} - B_{23}, \quad (B13)$$

$$F_3 = B_{12} + \frac{1}{2}B_{23} - \frac{1}{4}B_{33} - \frac{1}{2}B_{13}. \quad (B14)$$

The matrix elements $\langle \frac{1}{|\mathbf{r}_1 - \mathbf{r}_4|} \rangle$, $\langle \frac{1}{|\mathbf{r}_2 - \mathbf{r}_3|} \rangle$, $\langle \frac{1}{|\mathbf{r}_2 - \mathbf{r}_4|} \rangle$ are obtained from (B11) using the substitutions: $B_{12} \rightarrow -B_{12}$, $B_{23} \rightarrow -B_{23}$; $B_{12} \rightarrow -B_{12}$, $B_{13} \rightarrow -B_{13}$; $B_{13} \rightarrow -B_{13}$, $B_{23} \rightarrow -B_{23}$.

The matrix elements of the potential energy of paired confinement interactions are obtained in the form

$$\begin{aligned}
\langle \rho \rangle = & \sum_{I,J=1}^K C_I C_J \frac{16\pi^3}{(\det B)^3 \sqrt{(B_{22}B_{33} - B_{23}^2)}} \left[3B_{11}B_{22}(B_{22}B_{33} - B_{23}^2) \right. \\
& \left. - B_{12}^2(3B_{22}B_{33} - 4B_{23}^2) - 2B_{12}B_{13}B_{22}B_{23} + B_{13}^2B_{22}^2 \right], \quad (B15)
\end{aligned}$$

$$\begin{aligned}
\langle \lambda \rangle = & \sum_{I,J=1}^K C_I C_J \frac{16\pi^3}{(\det B)^3 \sqrt{(B_{11}B_{33} - B_{13}^2)}} \left[B_{12}^2(4B_{13}^2 - 3B_{11}B_{33}) \right. \\
& \left. - 2B_{11}B_{12}B_{13}B_{23} + B_{11}^2(3B_{22}B_{33} + B_{23}^2) - 3B_{11}B_{22}B_{13}^2 \right], \quad (B16)
\end{aligned}$$

$$\begin{aligned}
\langle |\mathbf{r}_1 - \mathbf{r}_3| \rangle = & \sum_{I,J=1}^K C_I C_J \frac{4\pi^3}{(\det B)^3 \sqrt{(F_1 F_2 - F_3^2)}} \left\{ B_{11}^2 (16B_{22}^2 - 16B_{22}B_{23} + 3B_{22}B_{33} + B_{23}^2) \right. \\
& + B_{12}^2 [4(-8B_{11}B_{22} + 4B_{11}B_{23} + B_{13}^2 + B_{13}(B_{23} - 4B_{22}) + B_{23}^2) - 3B_{33}(B_{11} + B_{22})] \\
& + 2B_{12}(B_{13} - B_{23})(8B_{11}B_{22} - B_{11}B_{23} + B_{13}B_{22}) + 6B_{11}B_{12}B_{22}B_{33} + B_{11}B_{22}(-3B_{13}^2 + 16B_{13}B_{22} \\
& \left. - 8B_{13}B_{23} + 3B_{22}B_{33} - 3B_{23}^2) + 16B_{12}^4 - 2B_{12}^3(8B_{13} - 8B_{23} + 3B_{33}) + B_{13}^2B_{22}^2 \right\}. \quad (B17)
\end{aligned}$$

The matrix elements $\langle |\mathbf{r}_1 - \mathbf{r}_4| \rangle$, $\langle |\mathbf{r}_2 - \mathbf{r}_3| \rangle$, $\langle |\mathbf{r}_2 - \mathbf{r}_4| \rangle$ are obtained from (B17) using the substitutions $B_{12} \rightarrow -B_{12}$, $B_{23} \rightarrow -B_{23}$; $B_{12} \rightarrow -B_{12}$, $B_{13} \rightarrow -B_{13}$; $B_{13} \rightarrow -B_{13}$, $B_{23} \rightarrow -B_{23}$.

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