

Relativistic flux-tube model predictions from charmed mesons to double-charmed baryons

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Utilizing comprehensive experimental data on charmed mesons, we systematically investigate masses of the higher radial and orbital excitations of the D and D_s meson families using the relativistic flux-tube model. Our study employs mass splitting induced by spin-dependent interactions within the j - j coupling scheme. Our predicted masses align well with the experimental measurements for the well-established D and D_s states. However, anomalous resonances such as $D_{s0}(2317)$ and $D_{s1}(2460)$ do not align with conventional meson states within our theoretical framework. We also study strong decays using an effective Lagrangian formalism that combines heavy-quark symmetry with chiral dynamics, focusing on two-body modes into a charmed meson plus a light pseudoscalar (π , K , η). Using the combined constraints from masses and decay patterns we propose spectroscopic assignments for several recently observed resonances, including $D_2(2740)^0$, $D_3^*(2750)$, $D_0(2550)^0$, $D_1^*(2600)^0$, $D_1^*(2760)^0$, $D_J^*(3000)$, $D_J(3000)$, $D_2^*(3000)$, $D_{s1}^*(2860)^\pm$, and $D_{s3}^*(2860)^\pm$. We also identify $D_{sJ}(3040)^+$ as a $2P$ excitation with $J^P = 1^+$. Extending our model, we also calculate the mass spectra of doubly charmed Ξ_{cc} and Ω_{cc} baryons within the heavy diquark–light quark picture. These theoretical predictions provide crucial guidance for ongoing and future experimental searches for higher radial and orbital excitations in the charmed meson and doubly charmed baryon sectors.

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I. INTRODUCTION

The spectroscopy of open-charm hadrons, including charmed mesons (D and D_s) and doubly charmed baryons (Ξ_{cc} and Ω_{cc}), provides a crucial window into the non-perturbative dynamics of quantum chromodynamics (QCD). These systems involve a heavy charm quark bound to light-quark constituents, and thus sit at the nexus of heavy-quark physics and light-quark dynamics.

Over the past two decades, significant experimental progress has been made in identifying numerous charmed meson states, primarily through observations by B -factories (BABAR and Belle), CLEO, and LHCb collaborations. Consequently, many D and D_s resonances are now listed by the Particle Data Group (PDG) [1], as shown in Table I. As new states are discovered, mapping out the mass spectra of D and D_s mesons becomes essential to distinguish conventional mesonic states from possible exotic configurations and to classify conventional mesonic states within the quark model framework. Furthermore, the spin-parity

quantum numbers assignments for $D_J^*(3000)^0$, $D_J(3000)^0$, and $D_{sJ}(3040)^\pm$ states have not been firmly established by experiments.

On the baryon side, the doubly charmed baryons Ξ_{cc} and Ω_{cc} (with quark content ccq and ccs , respectively) represent an important frontier in heavy hadron physics. Experimentally, evidence for the Ξ_{cc} was first claimed by the SELEX experiment in 2002, which reported a Ξ_{cc}^+ state at a mass of about 3519 MeV. However, subsequent searches by BABAR, Belle, and LHCb failed to confirm this signal. The breakthrough came in 2017 when LHCb collaboration unambiguously observed the Ξ_{cc}^{++} baryon in the $\Lambda_c^+ K^- \pi^+ \pi^+$ channel, measuring its mass to be about 3621 MeV [1]. This state is most naturally interpreted as the ground state Ξ_{cc} with quantum numbers $J^P = \frac{1}{2}^+$. LHCb has since measured the Ξ_{cc}^{++} mass with high precision [1]. To date, no Ω_{cc} (the ccs baryon) has been observed in experiments. The spin-parity of the Ξ_{cc}^{++} is yet to be directly measured—establishing J^P and discovering excited states of Ξ_{cc} and Ω_{cc} are among the next objectives for experiments like LHCb.

Over the past two decades, various theoretical models have been developed to study the spectroscopy of charmed mesons and baryons. Constituent quark models—both nonrelativistic [2–11] and relativistic [4,12–25]—remain central to this effort. These treat mesons as $q\bar{q}$ and baryons as qqq or quark-diquark systems with interquark potentials

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TABLE I. Masses and spin-parity (J^P) quantum numbers of the experimentally observed D and D_s meson and Ξ_{cc} baryon states, as reported by the PDG [1]. The states marked as UP and NP indicate mesons with unnatural parity and natural parity, respectively.

State	J^P	Mass (MeV)
$D^\pm$	0^-	1869.66 ± 0.05
D^0	0^-	1864.84 ± 0.05
$D^*(2007)^0$	1^-	2006.85 ± 0.05
$D^*(2010)^\pm$	1^-	2010.26 ± 0.05
$D_0^*(2300)$	0^+	2343 ± 10
$D_1(2420)$	1^+	2422.1 ± 0.6
$D_1(2430)^0$	1^+	2412 ± 9
$D_2^*(2460)$	2^+	2461.1 ± 0.8
$D_0(2550)^0$	0^-	2549 ± 19
$D_1^*(2600)^0$	1^-	2627 ± 10
$D_2(2740)^0$	2^-	2747 ± 6
$D_3^*(2750)$	3^-	2763.1 ± 3.2
$D_1^*(2760)^0$	1^-	2781 ± 22
$D_J^*(3000)^0$	NP	3008.1 ± 4.0
$D_J(3000)^0$	UP	2971.8 ± 8.7
$D_2^*(3000)$	2^+	$3214 \pm 29 \pm 49$
$D_s^\pm$	0^-	1968.36 ± 0.07
$D_s^{*\pm}$	1^-	2112.2 ± 0.4
$D_{s0}^*(2317)^\pm$	0^+	2317.8 ± 0.5
$D_{s1}(2460)^\pm$	1^+	2459.5 ± 0.6
$D_{s1}(2536)^\pm$	1^+	2535.11 ± 0.06
$D_{s2}^*(2573)$	2^+	2569.1 ± 0.8
$D_{s0}(2590)^+$	0^-	2591 ± 9
$D_{s1}^*(2700)^\pm$	1^-	2714 ± 5
$D_{s1}^*(2860)^\pm$	1^-	2859 ± 27
$D_{s3}^*(2860)^\pm$	3^-	2860 ± 7
$D_{sJ}(3040)^\pm$	$?^?$	3004^{+31}_{-9}
Ξ_{cc}^+	$?^?$	3518.9 ± 0.9
Ξ_{cc}^{++}	$?^?$	3621.6 ± 0.4

inspired by QCD, typically incorporating one-gluon exchange and a confining term. Relativized models such as the Godfrey-Isgur model [12] and its extensions have reliably predicted the spectra of charmed mesons, while modern relativistic quark models continue to refine the spectrum of excited D and D_s states [13,22]. In the baryon sector, the relativistic quark model by Ebert *et al.* [23] has accurately computed ground state mass of Ξ_{cc} , before its experimental confirmation.

Further, various theoretical methods have been put forward to study charmed mesons and baryons, including the Bohr-Sommerfeld quantization approach [26], QCD sum rule [27–30], 3P_0 pair-creation model [31], heavy-quark effective theory [32], the extended linear sigma model [33], a relativistic Dirac formalism [34], lattice QCD [35–41], Regge phenomenology [42], and effective field theory [43].

Although various theoretical and experimental investigations have been conducted, the spin-parity quantum

number assignments for states such as $D_J^*(3000)^0$, $D_J(3000)^0$, and $D_{sJ}(3040)^\pm$ remain unresolved, and even the ground states of doubly heavy baryons have yet to be established. The primary objective of the present study is to deliver a comprehensive theoretical account of the mass spectra of open-charm hadrons—specifically, the D and D_s mesons and the Ξ_{cc} and Ω_{cc} baryons, up to higher orbital and radial excitations within the framework of the relativistic flux-tube model.

Earlier studies employing the flux-tube model, such as Ref. [44], calculated only spin-averaged masses for the D and D_s mesons. However, their predictions could not discriminate between spin multiplets because spin-dependent interactions were not considered. Additionally, in Refs. [45,46], calculations were limited to the $1P$ and $1D$ states for the D and D_s mesons, neglecting higher radial and orbital excitations. Similarly, the authors of Ref. [47] restricted their calculations to a few low-lying states of the Ξ_{cc} and Ω_{cc} baryons.

In the current study, we employ the relativistic flux tube–derived mass formula to compute spin-averaged masses. Crucially, our methodology integrates spin-dependent interactions, enabling predictions of hyperfine structure splittings within a j - j coupling scheme suitable for heavy-light systems. We aim to identify the masses of fairly high orbital and radial excited states. This approach has been validated in our previous research, successfully reproducing mass spectra for singly heavy baryons [48]. In the present work, we extend the application of this model to open-charm mesons and doubly charmed baryons. Based on the predicted mass spectra, we further propose tentative spin-parity (J^P) assignments for several experimentally observed charmed meson states whose quantum numbers remain unconfirmed. Additionally, we provide theoretical predictions for the excitation spectrum of the doubly charmed baryons Ξ_{cc} and Ω_{cc} , which may serve as valuable input for future experimental searches aimed at discovering and classifying their excited states.

The structure of the paper is organized as follows. In Sec. II, we outline the relativistic flux-tube model, as applied to the charmed meson and doubly charmed baryon, and detail the methodology used to compute their mass spectra. In Sec. III, we present the framework used to evaluate the strong decay widths. Section IV contains our numerical results for the spectra and decay widths, along with a systematic comparison to available experimental measurements and existing theoretical studies. Finally, Sec. IV summarizes our main conclusions.

II. MASS SPECTRUM

A. Spin-averaged mass in relativistic flux-tube model

We consider the charmed meson or baryon as a two-body system consisting of a heavy component and a light component bounded by a confining color field that behaves

like a straight, rotating flux tube (string). This flux tube carries a constant tension σ (analogous to a string's energy per unit length), which accounts for the linear rising potential at large separations in QCD dynamics.

In natural units ($c = 1$), the system's dynamics can be derived from the Lagrangian of a rotating relativistic string with endpoint masses. In the rigid rotation approximation (assuming a constant angular velocity ω about the center of mass), the Lagrangian takes the form [48,49]

$$\mathcal{L} = \sum_i m_{0_i} \sqrt{1 - \omega^2 r_i^2} + T \sum_i \int_0^{r_i} dr \sqrt{1 - \omega^2 r^2}. \quad (1)$$

Here, $i = l, h$ label the light and heavy components, with mass m_{0_l} and m_{0_h} , respectively. The quantity r_i is the distance of mass m_{0_i} from the center of mass of the system.

From the Lagrangian, one can derive the total orbital angular momentum L using $L = \partial \mathcal{L} / \partial \omega$ as

$$L = \sum_i \left[\frac{m_{0_i} v_i^2}{\omega \sqrt{1 - v_i^2}} + \frac{T}{2\omega^2} \left(\sin^{-1} v_i - v_i \sqrt{1 - v_i^2} \right) \right]. \quad (2)$$

The first term inside the sum is the relativistic angular momentum of each orbiting mass. The second term is the contribution of the rotating string, obtained by integrating the differential string angular momentum density.

The total relativistic energy (Hamiltonian) of the system $H = (\omega \partial \mathcal{L} / \partial \omega) - \mathcal{L}$, which corresponds to the spin-averaged hadron mass $\bar{M}$, is derived from the Lagrangian as

$$\bar{M} = \sum_i \frac{m_{0_i}}{\sqrt{1 - v_i^2}} + \frac{T}{\omega} (\sin^{-1} v_l + \sin^{-1} v_h). \quad (3)$$

It is often convenient to define dynamical masses for the rotating light and heavy objects as $m_l = m_{0_l} / \sqrt{1 - v_l^2}$ and $m_h = m_{0_h} / \sqrt{1 - v_h^2}$, respectively. Then, we have

$$L = \frac{1}{\omega} (m_l v_l^2 + m_h v_h^2) + \frac{T}{2\omega^2} \left(\sin^{-1} v_l - v_l \sqrt{1 - v_l^2} + \sin^{-1} v_h - v_h \sqrt{1 - v_h^2} \right), \quad (4)$$

and

$$\bar{M} = m_h + m_l + \frac{T}{\omega} (\sin^{-1} v_h + \sin^{-1} v_l). \quad (5)$$

Equations (3) and (4) together define L and $\bar{M}$ as functions of the unknown angular velocity ω . To find a direct relation between $\bar{M}$ and L , we eliminate ω by using the equilibrium conditions and making a physically motivated approximation.

The equilibrium of the rotating system requires that the tension T in the string provides the necessary centripetal force on each mass. In particular, for the heavy end we have

$$T = \frac{m_h v_h \omega}{\sqrt{1 - v_h^2}} = m_h v_h \omega \left[1 + \frac{v_h^2}{2} + \frac{3v_h^4}{8} + \dots \right] \simeq m_h v_h \omega, \quad (6)$$

to leading order in v_h , which is consistent with the small- v_h limit.

For heavy-light mesons and baryons, the flux-tube model must address a highly asymmetric scenario. In this case, one end of the flux tube connects to a heavy quark, nearly static due to its large mass, while the other end attaches to a significantly lighter quark that demands a fully relativistic treatment ($m_h \gg m_l$). In this regime the light component is highly relativistic, with v_l approaching 1, while the heavy component moves more slowly ($v_h \ll 1$).

In this heavy-light limit, we expand the spin-averaged mass and angular momentum expressions, Eqs. (4) and (5), up to second order in the small parameter v_h . Subsequently, using the relation $T/\omega \approx m_h v_h$ derived from Eq. (6) to eliminate ω , we ultimately obtain a clear, Regge-like relationship between the spin-averaged mass and angular momentum [48,50],

$$(\bar{M} - m_h)^2 = \sigma \frac{L}{2} + (m_l + m_h v_h^2)^2. \quad (7)$$

The relationship between the angular speed ω of the rotating flux tube and the orbital angular momentum L can be obtained by combining Eqs. (4) and (7) as [48,50]

$$\omega = \sqrt{\frac{\sigma}{8L}}. \quad (8)$$

Using this result, we can then express the distance between the heavy and light constituents as [48,50]

$$r = \frac{v_l + v_h}{\omega} = (v_l + v_h) \sqrt{\frac{8L}{\sigma}}. \quad (9)$$

So far we have considered only the leading (lowest radial) trajectory in the (L, M^2) plane. In reality, hadrons can also be excited radially. A striking empirical fact, supported by a quantum mechanical approach [51,52], is that radial excitations of a heavy-light meson tend to lie on parallel Regge trajectories in the (L, M^2) plane (i.e., daughters roughly parallel to the leading trajectory). In the context of our semiclassical model, this can be accommodated by a simple modification of Eq. (7). Following Refs. [51,52], we introduce a phenomenological offset to L for each radial quantum number. In practice, one replaces L by $L + \lambda n_r$ in the Regge formula. Here, $n_r = n - 1$ (with $n = 1, 2, \dots$) is the radial excitation

quantum number (with $n_r = 0$ for the ground state, $n_r = 1$ for the first radial excitation, etc.), and λ is a constant offset parameter fitted to the spacing between radial trajectories. The modified relations are [48,50]

$$(\bar{M} - m_h)^2 = \frac{\sigma}{2}(L + \lambda n_r) + (m_l + m_h v_h^2)^2, \quad (10)$$

$$r = (v_l + v_h) \sqrt{\frac{8(L + \lambda n_r)}{\sigma}}. \quad (11)$$

By construction, all states with different n_r (but the same m_h, m_l) now lie on parallel lines of slope $\sigma/2$ in the $(L, (\bar{M} - m_h)^2)$ plane, separated by the constant spacing $\Delta(\bar{M} - m_h)^2 = \frac{\sigma}{2}\lambda$ between successive n_r levels. This extended formula allows one to estimate the masses of both orbital (L) and radial (n_r) excitations in the heavy-light system. The parameter λ is treated as universal for a given class of hadrons (it can be extracted by fitting the known spectrum).

B. Spin-dependent corrections

The flux-tube model described so far treats the quarks (or diquarks) as spinless, so $\bar{M}$ corresponds to a spin-averaged mass. To obtain the physical masses, we must include spin-dependent interactions that split the degeneracies among states of different total spin J (for a given L, n_r). We incorporate these effects through a perturbative Hamiltonian as

$$\Delta M = H_{so} + H_t + H_{ss}. \quad (12)$$

The spin-orbit interaction term (H_{so}) arises from the combination of short-range color magnetic forces (one-gluon exchange) and long-range Thomas precession. It can be written as an operator proportional to the orbital angular momentum ($\mathbf{L}$) dotted into the spin of the light component ($\mathbf{S}_l$) and of the heavy component ($\mathbf{S}_h$) in the form

$$H_{so} = a_1 \mathbf{L} \cdot \mathbf{S}_l + a_2 \mathbf{L} \cdot \mathbf{S}_h. \quad (13)$$

The coefficients a_1 and a_2 are

$$a_1 = \left(\frac{2\alpha}{3r^3} - \frac{b_0}{2r} \right) \frac{1}{m_l^2} + \frac{4\alpha}{3r^3} \frac{1}{m_h m_l}, \quad (14)$$

$$a_2 = \left(\frac{2\alpha}{3r^3} - \frac{b_0}{2r} \right) \frac{1}{m_h^2} + \frac{4\alpha}{3r^3} \frac{1}{m_h m_l}. \quad (15)$$

Here, α is the coupling constant of the one-gluon exchange potential, and b_0 is a parameter associated with the long-range confining potential.

The tensor interaction term (H_t) comes from the long-range part of the color magnetic dipole-dipole interaction

between the spins. Its operator form is

$$H_t = \frac{4\alpha}{3r^3 m_h m_l} \hat{\mathbf{B}}. \quad (16)$$

Here, $\hat{\mathbf{B}}$ is a standard tensor operator with the form

$$\hat{\mathbf{B}} = \frac{3(\mathbf{S}_l \cdot \mathbf{r})(\mathbf{S}_h \cdot \mathbf{r})}{r^2} - \mathbf{S}_l \cdot \mathbf{S}_h. \quad (17)$$

The spin-spin contact interaction term (H_{ss}) is a short-range hyperfine term. We take a contact form

$$H_{ss} = c \mathbf{S}_l \cdot \mathbf{S}_h, \quad (18)$$

often modeled by a smeared delta function interaction in coordinate space. For instance, a Gaussian-smeared contact with strength $c = (32\pi\alpha\sigma_0^3)/(9\sqrt{\pi}m_h m_l)$ and width parameter σ_0 can be used. In heavy-light mesons, H_{ss} gives the well-known hyperfine splitting between (spin-weighted) spin-singlet and spin-triplet configurations. A familiar example is the mass difference between a pseudoscalar meson (total $S_{\text{tot}} = 0$) and its vector meson partner ($S_{\text{tot}} = 1$). In our model, the constants b_0 and σ_0 (as well as α) are chosen to reproduce known splittings in the charmed meson spectrum, and are kept fixed for predictions.

With these spin-dependent terms, the total mass for the heavy-light system is specified as $M = \bar{M} + \Delta M$. We adopt the j - j coupling scheme appropriate for heavy-light hadrons to calculate the expectation value of ΔM in the $|J, j\rangle$ basis. For singly charmed mesons, one typically couples the light antiquark spin $\mathbf{S}_l$ with the orbital $\mathbf{L}$ to form an intermediate $\mathbf{j} = \mathbf{L} + \mathbf{S}_l$ (sometimes called the total angular momentum of the light cloud). This $\mathbf{j}$ is then coupled with the heavy-quark spin $\mathbf{S}_h$ to give the total $\mathbf{J}$. For doubly charmed baryons in the quark-diquark picture, we do the analogous coupling: the light-quark spin $\mathbf{S}_l$ is coupled with $\mathbf{L}$ to produce an intermediate $\mathbf{j}$. Subsequently, $\mathbf{j}$ couples with the diquark spin $\mathbf{S}_h$ to yield the baryon's $\mathbf{J}$.

This scheme is particularly convenient because in the limit $m_h \rightarrow \infty$ the heavy quark's spin decouples, making j an approximate conserved quantum number (as in heavy-quark effective theory). We calculate the expectation values of the operators $\mathbf{L} \cdot \mathbf{S}_h$, $\mathbf{L} \cdot \mathbf{S}_l$, $\hat{\mathbf{B}}$, and $\mathbf{S}_h \cdot \mathbf{S}_l$ for specific (L, J, j) configurations corresponding to singly charmed mesons and doubly charmed baryons. These computed values are presented in Tables II and III, respectively. A detailed derivation of these results is provided in the Appendix for readers new to research in this field.

TABLE II. Expectation values of spin-dependent operators for various (L, J, j) configurations in heavy-light mesons.

Quantum numbers						
L	j	J	$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$	$\langle \mathbf{L} \cdot \mathbf{S}_h \rangle$	$\langle \hat{\mathbf{B}} \rangle$	$\langle \mathbf{S}_h \cdot \mathbf{S}_l \rangle$
0	1/2	0	0	0	0	-3/4
0	1/2	1	0	0	0	1/4
1	1/2	0	-1	-1	-1	1/4
1	1/2	1	-1	1/3	1/3	-1/12
1	3/2	1	1/2	-5/6	1/6	-5/12
1	3/2	2	1/2	1/2	-1/10	1/4
2	3/2	1	-3/2	-3/2	-1/2	1/4
2	3/2	2	-3/2	9/10	3/10	-3/20
2	5/2	2	1	-7/5	1/5	-7/20
2	5/2	3	1	1	-1/7	1/4
3	5/2	2	-2	-2	-2/5	1/4
3	5/2	3	-2	10/7	2/7	-5/28
3	7/2	3	3/2	-27/14	3/14	-9/28
3	5/2	4	3/2	3/2	-1/6	1/4
4	7/2	3	-5/2	-5/2	-5/14	1/4
4	7/2	4	-5/2	35/18	5/18	-7/36
4	9/2	4	2	-22/9	2/9	-11/36
4	9/2	5	2	2	-2/11	1/4
5	9/2	4	-3	-3	-1/3	1/4
5	9/2	5	-3	27/11	3/11	-9/44
5	11/2	5	5/2	-65/22	5/22	-13/44
5	11/2	6	5/2	5/2	-5/26	1/4

C. Estimation of model parameters and associated uncertainties

1. D and D_s mesons

To estimate the optimal values of the model parameters and their associated uncertainties, we utilized the MINUIT package in PYTHON [53], which is based on the MINUIT algorithm developed at CERN for robust function minimization [54]. The fitting was performed using the least-squares method by minimizing the χ^2 difference between the theoretical mass predictions by our model and the experimentally measured masses of D and D_s mesons, including both ground and excited states. The experimental measured masses, along with their respective uncertainties, of the well-established states $D^\pm$, D^0 , $D^*(2007)^0$, $D^*(2010)^\pm$, $D_0^*(2300)$, $D_1(2420)$, $D_1(2430)^0$, $D_2^*(2460)$, $D_3^*(2750)$, $D_s^\pm$, $D_s^{*\pm}$, $D_{s1}(2536)^\pm$, $D_{s2}^*(2573)$, $D_{s1}^*(2700)^\pm$, and $D_{s3}^*(2860)^\pm$, and mass of charm quark $m_{0_c} = 1273 \pm 4.6$ MeV, are used as inputs [1]. The parameter values along with their statistical uncertainties that correspond to the best agreement between the theoretical masses and the experimentally measured masses are as follows: the mass of the c quark $m_c = 1375.68 \pm 0.15$ MeV, the mass of the u or d quark $m_{u/d} = 399.95 \pm 0.02$ MeV, the mass of the s quark $m_s = 484.29 \pm 0.06$ MeV, the string tension for D meson $\sigma_D = (1.5321 \pm 0.0023) \times 10^6$ MeV², and the string tension for D_s meson $\sigma_{D_s} = (1.7882 \pm 0.0016) \times 10^6$ MeV²,

TABLE III. Expectation values of spin-dependent operators for various (L, J, j) configurations in doubly heavy baryon with axial vector diquark ($S_h = 1$).

Quantum numbers						
L	j	J	$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$	$\langle \mathbf{L} \cdot \mathbf{S}_h \rangle$	$\langle \hat{\mathbf{B}} \rangle$	$\langle \mathbf{S}_h \cdot \mathbf{S}_l \rangle$
0	1/2	1/2	0	0	0	-1
0	1/2	3/2	0	0	0	1/2
1	1/2	1/2	-1	-4/3	-4/3	1/3
1	3/2	1/2	1/2	-5/3	1/3	-5/6
1	1/2	3/2	-1	2/3	2/3	-1/6
1	3/2	3/2	1/2	-2/3	2/15	-1/3
1	3/2	5/2	1/2	1	-1/5	1/2
2	3/2	1/2	-3/2	-3	-1	1/2
2	3/2	3/2	-3/2	-6/5	-2/5	1/5
2	5/2	3/2	1	-14/5	2/5	-7/10
2	3/2	5/2	-3/2	9/5	3/5	-3/10
2	5/2	5/2	1	-4/5	4/35	-1/5
2	5/2	7/2	1	2	-2/7	1/2
3	5/2	1/2	-2	-4	-4/5	1/2
3	5/2	5/2	-2	-8/7	-8/35	1/7
3	7/2	5/2	3/2	-27/7	3/7	-9/14
3	5/2	7/2	-2	20/7	4/7	-5/14
3	7/2	7/2	3/2	-6/7	2/21	-1/7
3	7/2	9/2	3/2	3	-1/3	1/2
4	7/2	5/2	-5/2	-5	-5/7	1/2
4	7/2	7/2	-5/2	-70/63	-10/63	7/63
4	9/2	7/2	2	-44/9	4/9	-11/18
4	7/2	9/2	-5/2	35/9	5/9	-7/18
4	9/2	9/2	2	-8/9	8/99	-1/9
4	9/2	11/2	2	4	-4/11	1/2

$$\lambda = 1.479 \pm 0.015, \quad \alpha = 0.306 \pm 0.005, \quad b_0 = (36.2 \pm 1.2) \times 10^3 \text{ MeV}^2, \text{ and } \sigma_0 = 502.8 \pm 2.8 \text{ MeV}.$$

2. Ξ_{cc} and Ω_{cc} baryons with the diquark in the ground state

In Ξ_{cc} and Ω_{cc} baryons, the two c quarks form a diquark in the internal $(1S)$ ground state. By Fermi statistics (Pauli exclusion), the spatially symmetric configuration forces a symmetric spin wave function, so the diquark is an axial-vector state, denoted $\{cc\}$, with heavy-diquark spin ($S_h = 1$).

The mass of a heavy diquark ($m_{0_{\{cc\}}}$) is approximated by the sum of the individual charm quark mass: $m_{0_{\{cc\}}} = 2m_{0_c} = 2546 \pm 5.6$ MeV [1].

Due to the limited experimental data on the masses of doubly charmed baryons—aside from the well-established state Ξ_{cc}^{++} —it is necessary to rely on theoretical mass predictions for $1S$ -wave states to extract the model parameters. The study by Ebert *et al.* [23] successfully predicted the ground state mass of the Ξ_{cc} even prior to its experimental discovery, estimating it at 3620 MeV. This prediction closely matches the experimentally measured mass of 3621.55 MeV reported by LHCb collaboration [1].

Hence, we take predictions for the $1S$ -wave states for Ξ_{cc} and Ω_{cc} baryons as input from Ref. [23] to determine the dynamical mass of the diquark $m_{\{cc\}} = 2802.46$ MeV and $\alpha = 0.309$.

In the heavy-quark limit ($m_h \rightarrow \infty$), two heavy quarks inside a doubly heavy diquark behave analogously to a heavy antiquark with respect to a light quark [55]. Consequently, the light degrees of freedom in a doubly heavy baryon experience the same color field as in a heavy-light meson. Due to the identical color field structures, heavy quark-diquark symmetry at leading order implies identical string tensions (σ) for these systems. However, finite mass effects, arising when heavy-quark masses deviate from infinity, introduce small symmetry violations. Such finite mass corrections can be effectively parametrized through empirical power-law scaling relations of the string tensions in terms of the heavy-quark masses [47],

$$\frac{\sigma_D}{\sigma_B} = \left(\frac{m_c}{m_b}\right)^P, \quad \frac{\sigma_D}{\sigma_{\Xi_{cc}}} = \left(\frac{m_c}{m_{\{cc\}}}\right)^P. \quad (19)$$

Extending this phenomenological approach to strange heavy-light hadrons, we have a similar relation [47],

$$\frac{\sigma_{D_s}}{\sigma_{B_s}} = \left(\frac{m_c}{m_b}\right)^Q, \quad \frac{\sigma_{D_s}}{\sigma_{\Omega_{cc}}} = \left(\frac{m_c}{m_{\{cc\}}}\right)^Q. \quad (20)$$

Using the known string tensions for the D , B , D_s , and B_s mesons, along with the quark masses m_c , m_b , and the diquark mass $m_{\{cc\}}$ fitted from well-established experimental states, we extract the scaling parameters $P = 0.1157 \pm 0.002$ and $Q = 0.0927 \pm 0.0017$, and subsequently determine the string tensions $\sigma_{\Xi_{cc}} = (1.6636 \pm 0.0034) \times 10^6$ MeV² and $\sigma_{\Omega_{cc}} = (1.9101 \pm 0.0029) \times 10^6$ MeV².

3. Ξ_{cc} and Ω_{cc} baryons with the diquark in an excited state

In this subsection we analyze those doubly heavy configurations in which the two charm quarks inside the diquark are themselves excited. For internal excitations of the cc subsystem, the heavy diquark is required to appear in a spin-singlet state ($S_h = 0$) for the $1P$ and $2P$ waves, whereas the internal $2S$ excitation corresponds to a spin-triplet diquark ($S_h = 1$).

The masses of the Ξ_{cc} and Ω_{cc} baryons, in which the light quark is orbitally or radially excited with respect to such an internally excited heavy diquark in the $2S$, $1P$, and $2P$ waves, can be obtained from Eqs. (10), (13), (16), and (18). To make use of these relations, we first determine the masses of the heavy diquarks, $m_{h_{2S}}$, $m_{h_{1P}}$, and $m_{h_{2P}}$, corresponding to the internal $2S$, $1P$, and $2P$ excitations, respectively. As a starting point, we consider the charmonium system within the relativistic flux-tube framework. A charm quark and an anticharm quark of mass m_c , connected

by a stringlike flux tube and carrying relative orbital angular momentum L and principal quantum number $n = n_r + 1$, satisfy a Regge-type relation for the spin-averaged mass $\bar{M}$,

$$\bar{M} = 2m_c + \left(\frac{\sigma_{c\bar{c}}^2}{2\pi^2 m_c}\right)^{\frac{1}{3}} (\lambda n_r + L)^{\frac{2}{3}}, \quad (21)$$

where $\sigma_{c\bar{c}}$ denotes the charmonium string tension. From this expression, a direct relationship between the spin-averaged masses of the $1S$ and $1P$ charmonium waves can be derived,

$$\bar{M}_{1P} = \bar{M}_{1S} + \left(\frac{\sigma_{c\bar{c}}^2}{\pi^2 \bar{M}_{1S}}\right)^{\frac{1}{3}}. \quad (22)$$

We utilize the experimentally measured masses of the $1S$ and $1P$ charmonium states—specifically $\eta_c(1S)$, $J/\psi(1S)$, $\chi_{c0}(1P)$, $\chi_{c1}(1P)$, $h_c(1P)$, and $\chi_{c2}(1P)$ —to compute the corresponding spin-averaged masses for the $1S$ and $1P$ waves. This procedure allows us to extract the string tension for charmonium, yielding $\sigma_{c\bar{c}} = (1.698 \pm 0.0006) \times 10^6$ MeV².

Building upon the established charmonium framework, we extend Eq. (21) to the color antitriplet cc diquark. Denoting the principal quantum number of the diquark by $n_d = n_{rd} + 1$ and its orbital angular momentum by L_d , the analogous relation reads

$$\bar{M} = 2m_c + \left(\frac{\sigma_{cc}^2}{2\pi^2 m_c}\right)^{\frac{1}{3}} (\lambda n_{rd} + L_d)^{\frac{2}{3}}. \quad (23)$$

This leads to the following relations for the spin-averaged mass of the diquark in $1P$, $2S$, and $2P$ waves:

$$\bar{M}_{1P} = \bar{M}_{1S} + \left(\frac{\sigma_{cc}^2}{\pi^2 \bar{M}_{1S}}\right)^{\frac{1}{3}}, \quad (24)$$

$$\bar{M}_{2S} = \bar{M}_{1S} + \left(\frac{\sigma_{cc}^2}{\pi^2 \bar{M}_{1S}}\right)^{\frac{1}{3}} \lambda^{\frac{2}{3}}, \quad (25)$$

$$\bar{M}_{2P} = \bar{M}_{1S} + \left(\frac{\sigma_{cc}^2}{\pi^2 \bar{M}_{1S}}\right)^{\frac{1}{3}} (\lambda + 1)^{\frac{2}{3}}. \quad (26)$$

The physics governing the string tension σ (related to the inverse Regge slope $1/\alpha'$ by $1/\alpha' \sim \sigma$) dictates its proportionality to the square root of the color factor, $1/\alpha' \propto \sqrt{C\alpha_s}$. Here, C represents the quadratic Casimir of the color sources at the flux-tube ends [56]. For the $cc[\bar{3}_c]$ diquark (an antitriplet), $C = 2/3$, whereas for the $c\bar{c}[1_c]$ meson (a color singlet), $C = 4/3$. This implies a direct relationship between their respective string tensions,

$$\sigma_{cc} = \sqrt{\frac{2/3}{4/3}} \sigma_{c\bar{c}} = \frac{1}{\sqrt{2}} \sigma_{c\bar{c}}. \quad (27)$$

Using this relationship and our previously determined value for $\sigma_{c\bar{c}}$, we find the string tension for the cc diquark to be $\sigma_{cc} = (1.200 \pm 0.0004) \times 10^6 \text{ MeV}^2$. Subsequently, by employing this σ_{cc} value, along with the established parameters $\bar{M}_{1S} = 2546 \pm 5.6 \text{ MeV}$ and $\lambda = 1.479 \pm 0.015$, we use Eqs. (24)–(26) to extract the spin-averaged masses for the excited diquark states. The results are $\bar{M}_{2S} = 3047 \pm 6 \text{ MeV}$, $\bar{M}_{1P} = 2932 \pm 5 \text{ MeV}$, and $\bar{M}_{2P} = 3252 \pm 6 \text{ MeV}$.

As previously noted, these excited diquarks are treated as spin-singlets ($S_h = 0$) for the $1P$ and $2P$ waves and as a spin-triplet ($S_h = 1$) for the $2S$ wave. In the baryon, this diquark is connected to the light quark by a flux tube, with the system revolving about its center of mass. We extract the dynamical masses for these rotating diquarks, which have a velocity $v_h = 0.418 \pm 0.007$, obtaining $m_{h_{2S}} = 3354 \pm 14 \text{ MeV}$, $m_{h_{1P}} = 3227 \pm 13 \text{ MeV}$, and $m_{h_{2P}} = 3580 \pm 14 \text{ MeV}$.

Finally, utilizing these calculated dynamical masses for the excited diquarks, we proceed to compute the spectra for the Ξ_{cc} and Ω_{cc} baryons. This computation addresses states where the light quark is orbitally or radially excited relative to the diquark in its $2S$, $1P$, or $2P$ internal wave. The calculations rely on Eqs. (10), (13), (16), and (18), incorporating the full set of previously fitted parameters ($m_{u/d}$, m_s , λ , b_0 , σ_0 , v_h , $\sigma_{\Xi_{cc}}$, and $\sigma_{\Omega_{cc}}$).

III. STRONG DECAY

Beyond the mass spectrum, the strong-decay pattern of the D and D_s families provides an essential test of state assignments. To analyze these decays, we employ the heavy-quark effective theory (HQET) framework combined with chiral symmetry. In this framework, charmed mesons are arranged into heavy-quark spin doublets, and their couplings to the light pseudoscalar mesons (π , K , η) are described by a leading-order chiral effective Lagrangian that preserves heavy-quark spin-flavor symmetry. Further details of the formalism and its implementation can be found in [57,58]. From this effective Lagrangian, one can derive explicit formulae for the two-body strong decay widths. These expressions depend on a set of dimensionless strong coupling constants (denoted g_{HH} , g_{SH} , g_{TH} , etc.), which parametrize the strength of transitions between specific heavy-meson multiplets and the light meson field. At leading order, the two-body strong decay widths of various charmed meson states decaying to another charmed meson states and a light pseudoscalar meson ($\mathcal{P} = \pi, \eta, K$) are given by the following expressions [57–59]:

- (1) For S -wave doublet ($J_i^P = 0^-, 1^-$) to $1S$ ground state ($J_f^P = 0^-, 1^-$) + $\mathcal{P}$:

$$\Gamma(1^- \rightarrow 0^-) = C_P \frac{g_{HH}^2 M_f}{6\pi f_\pi^2 M_i} P_P^3,$$

$$\Gamma(1^- \rightarrow 1^-) = C_P \frac{g_{HH}^2 M_f}{3\pi f_\pi^2 M_i} P_P^3,$$

$$\Gamma(0^- \rightarrow 1^-) = C_P \frac{g_{HH}^2 M_f}{2\pi f_\pi^2 M_i} P_P^3.$$

- (2) For $j = \frac{1}{2}$ P -wave doublet ($J_i^P = 0^+, 1^+$) to $1S$ ground state ($J_f^P = 0^-, 1^-$) + $\mathcal{P}$:

$$\Gamma(0^+ \rightarrow 0^-) = C_P \frac{g_{SH}^2 M_f}{2\pi f_\pi^2 M_i} [M_P^2 + P_P^2] P_P,$$

$$\Gamma(1^+ \rightarrow 1^-) = C_P \frac{g_{SH}^2 M_f}{2\pi f_\pi^2 M_i} [M_P^2 + P_P^2] P_P.$$

- (3) For $j = \frac{3}{2}$ P -wave doublet ($J_i^P = 1^+, 2^+$) to $1S$ ground state ($J_f^P = 0^-, 1^-$) + $\mathcal{P}$:

$$\Gamma(1^+ \rightarrow 1^-) = C_P \frac{2g_{TH}^2 M_f}{3\pi f_\pi^2 \Lambda^2 M_i} P_P^5,$$

$$\Gamma(2^+ \rightarrow 0^-) = C_P \frac{4g_{TH}^2 M_f}{15\pi f_\pi^2 \Lambda^2 M_i} P_P^5,$$

$$\Gamma(2^+ \rightarrow 1^-) = C_P \frac{2g_{TH}^2 M_f}{5\pi f_\pi^2 \Lambda^2 M_i} P_P^5.$$

- (4) For $j = \frac{3}{2}$ D -wave doublet ($J_i^P = 1^-, 2^-$) to $1S$ ground state ($J_f^P = 0^-, 1^-$) + $\mathcal{P}$:

$$\Gamma(1^- \rightarrow 0^-) = C_P \frac{4g_{XH}^2 M_f}{9\pi f_\pi^2 \Lambda^2 M_i} [M_P^2 + P_P^2] P_P^3,$$

$$\Gamma(1^- \rightarrow 1^-) = C_P \frac{2g_{XH}^2 M_f}{9\pi f_\pi^2 \Lambda^2 M_i} [M_P^2 + P_P^2] P_P^3,$$

$$\Gamma(2^- \rightarrow 1^-) = C_P \frac{2g_{XH}^2 M_f}{3\pi f_\pi^2 \Lambda^2 M_i} [M_P^2 + P_P^2] P_P^3.$$

- (5) For $j = \frac{5}{2}$ D -wave doublet ($J_i^P = 2^-, 3^-$) to $1S$ ground state ($J_f^P = 0^-, 1^-$) + $\mathcal{P}$:

$$\Gamma(2^- \rightarrow 1^-) = C_P \frac{4g_{YH}^2 M_f}{15\pi f_\pi^2 \Lambda^4 M_i} P_P^7,$$

$$\Gamma(3^- \rightarrow 0^-) = C_P \frac{4g_{YH}^2 M_f}{35\pi f_\pi^2 \Lambda^4 M_i} P_P^7,$$

$$\Gamma(3^- \rightarrow 1^-) = C_P \frac{16g_{YH}^2 M_f}{105\pi f_\pi^2 \Lambda^4 M_i} P_P^7.$$

- (6) For $j = \frac{5}{2}$ F -wave doublet ($J_i^P = 2^+, 3^+$) to $1S$ ground state ($J_f^P = 0^-, 1^-$) + $\mathcal{P}$:

$$\Gamma(2^+ \rightarrow 0^-) = C_{\mathcal{P}} \frac{4g_{ZH}^2}{25\pi f_\pi^2 \Lambda^4} \frac{M_f}{M_i} [M_{\mathcal{P}}^2 + P_{\mathcal{P}}^2] P_{\mathcal{P}}^5,$$

$$\Gamma(2^+ \rightarrow 1^-) = C_{\mathcal{P}} \frac{8g_{ZH}^2}{75\pi f_\pi^2 \Lambda^4} \frac{M_f}{M_i} [M_{\mathcal{P}}^2 + P_{\mathcal{P}}^2] P_{\mathcal{P}}^5,$$

$$\Gamma(3^+ \rightarrow 1^-) = C_{\mathcal{P}} \frac{4g_{ZH}^2}{15\pi f_\pi^2 \Lambda^4} \frac{M_f}{M_i} [M_{\mathcal{P}}^2 + P_{\mathcal{P}}^2] P_{\mathcal{P}}^5.$$

- (7) For $j = \frac{7}{2}$ F -wave doublet ($J_i^P = 3^+, 4^+$) to $1S$ ground state ($J_f^P = 0^-, 1^-$) + $\mathcal{P}$:

$$\Gamma(3^+ \rightarrow 1^-) = C_{\mathcal{P}} \frac{36g_{RH}^2}{35\pi f_\pi^2 \Lambda^6} \frac{M_f}{M_i} P_{\mathcal{P}}^9,$$

$$\Gamma(4^+ \rightarrow 0^-) = C_{\mathcal{P}} \frac{16g_{RH}^2}{35\pi f_\pi^2 \Lambda^6} \frac{M_f}{M_i} P_{\mathcal{P}}^9,$$

$$\Gamma(4^+ \rightarrow 1^-) = C_{\mathcal{P}} \frac{4g_{RH}^2}{7\pi f_\pi^2 \Lambda^6} \frac{M_f}{M_i} P_{\mathcal{P}}^9.$$

In the above formulas, $C_{\mathcal{P}}$ is a coefficient that depends on the identity of the emitted pseudoscalar meson (essentially an isospin factor). Specifically, $C_{\mathcal{P}} = 1$ for $\pi^\pm$, $K^\pm$, or $K^0/\bar{K}^0$, $C_{\mathcal{P}} = \frac{1}{2}$ for π^0 , and for η emission $C_{\mathcal{P}}$ takes the value $\frac{1}{6}$ if the decaying meson is a nonstrange D , or $\frac{2}{3}$ if it is a D_s . In each decay, M_i and M_f denote the masses of the initial and final mesons, respectively, while $P_{\mathcal{P}}$ is the momentum of the outgoing pseudoscalar meson (with $M_{\mathcal{P}}$ being the pseudoscalar's mass). We set the chiral symmetry-breaking scale $\Lambda = 1$ GeV and $f_\pi = 132$ MeV. We use different symbols for the strong couplings depending on the multiplets involved and the radial quantum number of the decaying meson. For decays among the lowest-lying states ($n = 1$), the couplings are written as g_{HH} , g_{SH} , g_{TH} , etc. For transitions from first excited states ($n = 2$) down to the ground state, we use tilded couplings $\tilde{g}$ (e.g., $\tilde{g}_{HH}$, $\tilde{g}_{SH}$, ...), and similarly a double tilde (such as $\tilde{\tilde{g}}_{HH}$) denotes the couplings for decays from second radially excited ($n = 3$) states to the ground state.

Using the above expressions, together with our computed masses for the initial and final D and D_s states and the PDG values for the masses of a light pseudoscalar meson [1], we calculate the strong decay widths of the relevant charmed mesons. The resulting partial decay widths are reported in terms of the corresponding strong coupling constants in Tables IV–VI.

IV. RESULTS AND DISCUSSION

A. Charmed and charmed strange mesons

In this section, we present and discuss the results obtained for the mass spectra of charmed (D) and charmed strange (D_s) mesons within our theoretical framework. To represent the state of the system, we use the notation $nL(J^P, j)$, where $n = 1, 2, 3, \dots$ denotes the radial quantum number of the light component, and $L = S, P, D, \dots$ specifies its orbital angular momentum. This is followed by the total angular momentum J and parity P of the state, and finally, the light component's total angular momentum j . The calculated masses are summarized in Tables VII and VIII, respectively, and systematically compared with experimental data from the PDG [1] as well as other theoretical predictions available in the literature [13,20–22].

1. D meson

Significant experimental progress has enabled a comprehensive understanding of the D meson spectrum. The states $D^{0,\pm}$ and $D^*(2007)^0/D^*(2010)^\pm$, firmly established as the ground-state doublet $1S(0^-, 1/2)$ and $1S(1^-, 1/2)$, respectively, are well documented experimentally [1]. Our theoretical calculations accurately reproduce their experimentally observed masses.

Regarding the $1P$ states, the PDG [1] identifies $D_0^*(2300)$, $D_1(2420)$, $D_1(2430)^0$, and $D_2^*(2460)$ as the primary candidates. The neutral $D_0^*(2300)$, with quantum numbers $J^P = 0^+$, was initially observed by Belle and subsequently confirmed by BABAR. The charged counterpart, $D_0^*(2300)^+$, observed by FOCUS, was later confirmed by LHCb. Our computed mass 2362 ± 7 MeV for the $1P(0^+, 1/2)$ state aligns well with the experimental average 2343 ± 10 MeV for $D_0^*(2300)$, consistent also with previous theoretical studies [19,20]. The predicted masses of other $1P$ states (2422 ± 6 MeV, 2429 ± 5 MeV, and 2462 ± 4 MeV) show excellent agreement with the measured values for $D_1(2420)$, $D_1(2430)^0$, and $D_2^*(2460)$.

Taking the decay width of the $D_2^*(2460)$, 47.3 ± 0.8 MeV [1], the corresponding coupling constant is fixed to $g_{TH} = 0.413 \pm 0.0035$. With this, the resulting partial-width ratio for $1P(2^+, 3/2)$ is found to be $\frac{\Gamma(D_{1P(2^+, 3/2)} \rightarrow D^+ \pi^-)}{\Gamma(D_{1P(2^+, 3/2)} \rightarrow D^{*+} \pi^-)} = 2.28$, which also lies within the experimentally measured range reported by ZEUS and CLEO [1]. For the same state, $D_2^*(2460)$, BABAR collaboration measured the ratio $\frac{\Gamma(D^{*+} \pi^-)}{\Gamma(D^+ \pi^-) + \Gamma(D^{*+} \pi^-)} = 0.62 \pm 0.03 \pm 0.02$ [1]. Our corresponding estimate yields a value of 0.69 for this ratio, indicating reasonable consistency with the BABAR result.

Several higher excitations, including the states $D(2550)^0$, $D^*(2600)^{0,+}$, $D(2750)^0$, and $D^*(2760)^{0,+}$, were first observed by BABAR and subsequently confirmed by LHCb [1]. A detailed four-body amplitude analysis of the $B^- \rightarrow D^{*+} \pi^- \pi^-$ decay by LHCb [1] categorized these

TABLE IV. Partial strong decay widths of the $1P$, $1D$, and $2S$ states of the D and D_s meson families in terms of the relevant strong couplings. The total width is the sum over all listed channels. The branching fraction (in percent) refers to the ratio of each partial width to the corresponding total width.

States $nL(J^P, j)$	D meson			D_s meson		
	Channel	Width	Branching fraction	Channel	Width	Branching fraction
$1P(2^+, 3/2)$	$D^{*+}\pi^-$	$55.35 g_{TH}^2$	19.96	$D_s^{*+}K^0$	$3.84 g_{TH}^2$	3.29
	$D^{*0}\pi^0$	$29.18 g_{TH}^2$	10.52	$D_s^{*0}K^+$	$5.01 g_{TH}^2$	4.29
	$D^{*0}\eta$	-	-	$D_s^{*+}\eta$	-	-
	$D_s^{*+}K^-$	-	-	$D_s^{*+}\pi^0$	$0.0033 g_{TH}^2$	0.0028
	$D^+\pi^-$	$126.46 g_{TH}^2$	45.61	$D_s^+K^0$	$50.74 g_{TH}^2$	43.42
	$D^0\pi^0$	$66.01 g_{TH}^2$	23.80	$D_s^0K^+$	$55.90 g_{TH}^2$	47.83
	$D^0\eta$	$0.24 g_{TH}^2$	0.08	$D_s^+\eta$	$1.37 g_{TH}^2$	1.17
	$D_s^+K^-$	-	-	$D_s^+\pi^0$	$0.0072 g_{TH}^2$	0.0062
	<i>Total</i>	$277.25 g_{TH}^2$		<i>Total</i>	$116.87 g_{TH}^2$	
$1D(1^-, 3/2)$	$D^{*+}\pi^-$	$295.29 g_{XH}^2$	10.68	$D_s^{*+}K^0$	$336.82 g_{XH}^2$	8.19
	$D^{*0}\pi^0$	$150.86 g_{XH}^2$	5.46	$D_s^{*0}K^+$	$347.61 g_{XH}^2$	8.45
	$D^{*0}\eta$	$17.72 g_{XH}^2$	0.64	$D_s^{*+}\eta$	$88.58 g_{XH}^2$	2.15
	$D_s^{*+}K^-$	$49.38 g_{XH}^2$	1.79	$D_s^{*+}\pi^0$	$0.017 g_{XH}^2$	0.00
	$D^+\pi^-$	$1160.27 g_{XH}^2$	41.98	$D_s^+K^0$	$1399.88 g_{XH}^2$	34.04
	$D^0\pi^0$	$592.77 g_{XH}^2$	21.45	$D_s^0K^+$	$1438.31 g_{XH}^2$	34.97
	$D^0\eta$	$108.72 g_{XH}^2$	3.93	$D_s^+\eta$	$501.52 g_{XH}^2$	12.19
	$D_s^+K^-$	$388.59 g_{XH}^2$	14.06	$D_s^+\pi^0$	$0.066 g_{XH}^2$	0.00
	<i>Total</i>	$2763.63 g_{XH}^2$		<i>Total</i>	$4112.81 g_{XH}^2$	
$1D(2^-, 3/2)$	$D^{*+}\pi^-$	$1017.04 g_{XH}^2$	56.30	$D_s^{*+}K^0$	$1192.26 g_{XH}^2$	43.21
	$D^{*0}\pi^0$	$519.09 g_{XH}^2$	28.74	$D_s^{*0}K^+$	$1227.68 g_{XH}^2$	44.49
	$D^{*0}\eta$	$67.80 g_{XH}^2$	3.75	$D_s^{*+}\eta$	$339.59 g_{XH}^2$	12.31
	$D_s^{*+}K^-$	$202.38 g_{XH}^2$	11.20	$D_s^{*+}\pi^0$	$0.060 g_{XH}^2$	0.00
	<i>Total</i>	$1806.32 g_{XH}^2$		<i>Total</i>	$2759.50 g_{XH}^2$	
$1D(2^-, 5/2)$	$D^{*+}\pi^-$	$128.47 g_{YH}^2$	64.65	$D_s^{*+}K^0$	$89.71 g_{YH}^2$	45.89
	$D^{*0}\pi^0$	$66.58 g_{YH}^2$	33.51	$D_s^{*0}K^+$	$95.54 g_{YH}^2$	48.87
	$D^{*0}\eta$	$1.33 g_{YH}^2$	0.67	$D_s^{*+}\eta$	$10.22 g_{YH}^2$	5.23
	$D_s^{*+}K^-$	$2.31 g_{YH}^2$	1.16	$D_s^{*+}\pi^0$	$0.009 g_{YH}^2$	0.00
	<i>Total</i>	$198.71 g_{YH}^2$		<i>Total</i>	$195.49 g_{YH}^2$	
$1D(3^-, 5/2)$	$D^{*+}\pi^-$	$84.01 g_{YH}^2$	20.80	$D_s^{*+}K^0$	$61.49 g_{YH}^2$	13.02
	$D^{*0}\pi^0$	$43.49 g_{YH}^2$	10.77	$D_s^{*0}K^+$	$65.28 g_{YH}^2$	13.83
	$D^{*0}\eta$	$1.05 g_{YH}^2$	0.26	$D_s^{*+}\eta$	$7.80 g_{YH}^2$	1.65
	$D_s^{*+}K^-$	$1.99 g_{YH}^2$	0.49	$D_s^{*+}\pi^0$	$0.0059 g_{YH}^2$	0.00
	$D^+\pi^-$	$165.855 g_{YH}^2$	41.07	$D_s^+K^0$	$148.611 g_{YH}^2$	31.48
	$D^0\pi^0$	$85.80 g_{YH}^2$	21.25	$D_s^0K^+$	$156.28 g_{YH}^2$	33.10
	$D^0\eta$	$5.56 g_{YH}^2$	1.38	$D_s^+\eta$	$32.62 g_{YH}^2$	6.91
	$D_s^+K^-$	$16.05 g_{YH}^2$	3.97	$D_s^+\pi^0$	$0.011 g_{YH}^2$	0.00
	<i>Total</i>	$403.83 g_{YH}^2$		<i>Total</i>	$472.12 g_{YH}^2$	
$2S(0^-, 1/2)$	$D^{*+}\pi^-$	$927.69 \tilde{g}_{HH}^2$	65.91	$D_s^{*+}K^0$	$536.04 \tilde{g}_{HH}^2$	46.60
	$D^{*0}\pi^0$	$472.002 \tilde{g}_{HH}^2$	33.54	$D_s^{*0}K^+$	$561.52 \tilde{g}_{HH}^2$	48.82
	$D^{*0}\eta$	$7.78 \tilde{g}_{HH}^2$	0.55	$D_s^{*+}\eta$	$52.66 \tilde{g}_{HH}^2$	4.58
	$D_s^{*+}K^-$	-	-	$D_s^{*+}\pi^0$	$0.052 \tilde{g}_{HH}^2$	0.00
	<i>Total</i>	$1407.48 \tilde{g}_{HH}^2$		<i>Total</i>	$1150.27 \tilde{g}_{HH}^2$	

(Table continued)

TABLE IV. (Continued)

States $nL(J^P, j)$	D meson			D_s meson		
	Channel	Width	Branching fraction	Channel	Width	Branching fraction
$2S(1^-, 1/2)$	$D^{*+}\pi^-$	625.03 $\tilde{g}_{HH}^2$	33.69	$D_s^{*+}K^0$	370.40 $\tilde{g}_{HH}^2$	21.22
	$D^{*0}\pi^0$	318.94 $\tilde{g}_{HH}^2$	17.19	$D_s^{*0}K^+$	388.71 $\tilde{g}_{HH}^2$	22.27
	$D^{*0}\eta$	5.80 $\tilde{g}_{HH}^2$	0.31	$D_s^{*+}\eta$	40.13 $\tilde{g}_{HH}^2$	2.30
	$D_s^{*+}K^-$	-	-	$D_s^{*+}\pi^0$	0.035 $\tilde{g}_{HH}^2$	0.00
	$D^+\pi^-$	520.67 $\tilde{g}_{HH}^2$	28.06	$D_s^+K^0$	405.05 $\tilde{g}_{HH}^2$	23.20
	$D^0\pi^0$	265.06 $\tilde{g}_{HH}^2$	14.29	$D_s^0K^+$	418.31 $\tilde{g}_{HH}^2$	23.96
	$D^0\eta$	26.59 $\tilde{g}_{HH}^2$	1.43	$D_s^+\eta$	123.12 $\tilde{g}_{HH}^2$	7.05
	$D_s^+K^-$	93.31 $\tilde{g}_{HH}^2$	5.03	$D_s^+\pi^0$	0.028 $\tilde{g}_{HH}^2$	0.00
	<i>Total</i>	1855.40 $\tilde{g}_{HH}^2$		<i>Total</i>	1745.79 $\tilde{g}_{HH}^2$	

$D(2550)$, $D^*(2600)$, $D(2750)$, and $D^*(2760)$ states as $J^P = 0^-, 1^-, 2^-,$ and 3^- , respectively. These determinations for $D^*(2600)$ and $D^*(2760)$ are in agreement with earlier experimental analyses [1]. The PDG labels these states $D(2550)$, $D^*(2600)$, $D(2750)$, and $D^*(2760)$ as $D_0(2550)^0$, $D_1^*(2600)^0$, $D_2(2740)^0$, and $D_3^*(2750)$, respectively [1].

Our calculated mass 2754 ± 4 MeV for $1D(3^-, 5/2)$ is in excellent agreement with experimental average mass 2763.1 ± 3.2 MeV for $D_3^*(2750)$. The experimentally measured width of the $D_3^*(2750)$, $\Gamma = 66 \pm 5$ MeV, is used to extract the corresponding strong coupling, yielding $g_{YH} = 0.404 \pm 0.015$. Further, the partial-width ratio $R_\pi(D_{1D(3^-, 5/2)}) = \frac{\Gamma(D_{1D(3^-, 5/2)} \rightarrow D\pi)}{\Gamma(D_{1D(3^-, 5/2)} \rightarrow D^*\pi)} = 1.97$ is obtained. This result exceeds the value $0.42 \pm 0.05 \pm 0.11$ reported by BABAR collaboration in 2010 [1]. This suggests that additional measurements from other experimental facilities are also needed to better establish the decay pattern of the $D_3^*(2750)$.

The resonance $D_1^*(2760)^0$, experimentally observed at 2781 ± 22 MeV with $J^P = 1^-$ by LHCb [1], closely matches our prediction for the $1D(1^-, 3/2)$ state (2739 ± 5 MeV). Although the predicted and experimental uncertainty ranges do not completely overlap, the non-overlapping region is only about 15 MeV, which remains acceptable within the typical uncertainties associated with heavy meson mass predictions. Therefore, the $D_1^*(2760)^0$ resonance may be interpreted as the $1D(1^-, 3/2)$ state. This assignment is also consistent with theoretical predictions given in Refs. [19,20,22,60,61]. Moreover, the dominant decay mode of the $1D(1^-, 3/2)$ state is predicted to be $D\pi$ [as shown in Table IV listing the decay channels of the $1D(1^-, 3/2)$ state], which aligns with the experimental observation of the $D_1^*(2760)^0$ in this channel. LHCb collaboration reports a total decay width of $177 \pm 32 \pm 21$ MeV for this state [1]. Using this measured width as input, we extract the corresponding strong coupling and obtain $g_{XH} = 0.253 \pm 0.027$, further supporting the proposed spectroscopic assignment.

The measured mass of the $D_2(2740)^0$ state, 2747 ± 6 MeV, lies within the uncertainty range of our predicted mass for the $1D(2^-, 5/2)$ configuration. In contrast, the experimental mass is located approximately 6 MeV outside the uncertainty band of the calculated mass for the $1D(2^-, 3/2)$ state. Using the previously extracted values of the couplings g_{XH} and g_{YH} , we obtain decay widths of 92 ± 23 MeV and 32.5 ± 2.5 MeV for the $1D(2^-, 3/2)$ and $1D(2^-, 5/2)$ states, respectively. Given the experimentally measured width of the $D_2(2740)^0$, $\Gamma = 88 \pm 19$ MeV [1], the assignment of this state as $1D(2^-, 3/2)$ is, therefore, favored.

Taken together, these results support the J^P assignments proposed by LHCb collaboration, identifying the $D_3^*(2750)$ and $D_2(2740)^0$ states as having quantum numbers 3^- and 2^- , respectively [1]. These assignments are also consistent with the theoretical predictions reported in Refs. [19,20,22,60].

In the case of radially excited states, the calculated masses for the $2S(0^-, 1/2)$ and $2S(1^-, 1/2)$ states (2594 ± 6 MeV and 2597 ± 6 MeV) are proximate to experimentally observed $D_0(2550)^0$ and $D_1^*(2600)^0$ resonances. Despite minor deviations of about 30–45 MeV, $D_0(2550)^0$ and $D_1^*(2600)^0$ states remain excellent candidates for radial excitations, consistent with previous theoretical analyses [2,19,60,62]. Employing our extracted value of the coupling $\tilde{g}_{HH} = 0.264 \pm 0.011$ (the details of its determination are discussed in the subsequent section for the D_s sector), we compute the decay width of the $D(2S(1^-, 1/2))$ state to be 141 ± 23 MeV. This result compares favorably with the experimentally measured width of 165 ± 24 MeV [1] for the $D_1^*(2600)^0$, lending further support to its interpretation as the $2S(1^-, 1/2)$ excitation. Using the same coupling, the decay width of the $D(2S(0^-, 1/2))$ state is predicted to be 107 ± 17 MeV. While this value is somewhat smaller than the experimental width quoted for the $D_0(2550)^0$, $\Gamma = 165 \pm 24$ MeV [1], the difference remains moderate in view of the sizable experimental uncertainty. Further insight is provided by the decay pattern: our calculation yields a dominant branching

TABLE V. The same as Table IV, but for $3S$ and $2P$ states of the D and D_s meson families.

States $nL(J^P, j)$	D meson			D_s meson		
	Channel	Width	Branching fraction	Channel	Width	Branching fraction
$3S(0^-, 1/2)$	$D^{*+}\pi^-$	3412.14 $\tilde{g}_{HH}^2$	48.18	$D_s^{*+}K^0$	3412.33 $\tilde{g}_{HH}^2$	40.26
	$D^{*0}\pi^0$	1716.02 $\tilde{g}_{HH}^2$	24.23	$D_s^{*0}K^+$	3447.71 $\tilde{g}_{HH}^2$	40.68
	$D^{*0}\eta$	333.43 $\tilde{g}_{HH}^2$	4.71	$D_s^{*+}\eta$	1614.95 $\tilde{g}_{HH}^2$	19.06
	$D_s^{*+}K^-$	1620.80 $\tilde{g}_{HH}^2$	22.88	$D_s^{*+}\pi^0$	0.19 $\tilde{g}_{HH}^2$	0.00
	<i>Total</i>	7082.40 $\tilde{g}_{HH}^2$		<i>Total</i>	8475.18 $\tilde{g}_{HH}^2$	
$3S(1^-, 1/2)$	$D^{*+}\pi^-$	2267.78 $\tilde{g}_{HH}^2$	28.67	$D_s^{*+}K^0$	2267.90 $\tilde{g}_{HH}^2$	24.13
	$D^{*0}\pi^0$	1144.01 $\tilde{g}_{HH}^2$	14.46	$D_s^{*0}K^+$	2298.47 $\tilde{g}_{HH}^2$	24.46
	$D^{*0}\eta$	222.29 $\tilde{g}_{HH}^2$	2.81	$D_s^{*+}\eta$	1076.63 $\tilde{g}_{HH}^2$	11.46
	$D_s^{*+}K^-$	1080.54 $\tilde{g}_{HH}^2$	13.66	$D_s^{*+}\pi^0$	0.13 $\tilde{g}_{HH}^2$	0.00
	$D^+\pi^-$	1455.61 $\tilde{g}_{HH}^2$	18.40	$D_s^+K^0$	1485.37 $\tilde{g}_{HH}^2$	15.80
	$D^0\pi^0$	734.23 $\tilde{g}_{HH}^2$	9.28	$D_s^0K^+$	1503.52 $\tilde{g}_{HH}^2$	16.00
	$D^0\eta$	164.34 $\tilde{g}_{HH}^2$	2.08	$D_s^+\eta$	766.51 $\tilde{g}_{HH}^2$	8.16
	$D_s^+K^-$	840.84 $\tilde{g}_{HH}^2$	10.63	$D_s^+\pi^0$	0.083 $\tilde{g}_{HH}^2$	0.00
	<i>Total</i>	7909.63 $\tilde{g}_{HH}^2$		<i>Total</i>	9398.62 $\tilde{g}_{HH}^2$	
$2P(0^+, 1/2)$	$D^+\pi^-$	3474.71 $\tilde{g}_{SH}^2$	41.11	$D_s^+K^0$	4448.32 $\tilde{g}_{SH}^2$	39.11
	$D^0\pi^0$	1752.45 $\tilde{g}_{SH}^2$	20.73	$D_s^0K^+$	4485.08 $\tilde{g}_{SH}^2$	39.43
	$D^0\eta$	551.23 $\tilde{g}_{SH}^2$	6.52	$D_s^+\eta$	2441.16 $\tilde{g}_{SH}^2$	21.46
	$D_s^+K^-$	2654.25 $\tilde{g}_{SH}^2$	31.40	$D_s^+\pi^0$	0.19 $\tilde{g}_{SH}^2$	0.00
	<i>Total</i>	8432.64 $\tilde{g}_{SH}^2$		<i>Total</i>	11374.75 $\tilde{g}_{SH}^2$	
$2P(1^+, 1/2)$	$D^{*+}\pi^-$	2692.27 $\tilde{g}_{SH}^2$	42.74	$D_s^{*+}K^0$	3541.50 $\tilde{g}_{SH}^2$	39.59
	$D^{*0}\pi^0$	1357.21 $\tilde{g}_{SH}^2$	21.55	$D_s^{*0}K^+$	3571.23 $\tilde{g}_{SH}^2$	39.92
	$D^{*0}\eta$	404.35 $\tilde{g}_{SH}^2$	6.42	$D_s^{*+}\eta$	1832.78 $\tilde{g}_{SH}^2$	20.49
	$D_s^{*+}K^-$	1845.12 $\tilde{g}_{SH}^2$	29.29	$D_s^{*+}\pi^0$	0.15 $\tilde{g}_{SH}^2$	0.00
	<i>Total</i>	6298.96 $\tilde{g}_{SH}^2$		<i>Total</i>	8945.66 $\tilde{g}_{SH}^2$	
$2P(1^+, 3/2)$	$D^{*+}\pi^-$	1721.78 $\tilde{g}_{TH}^2$	57.83	$D_s^{*+}K^0$	1608.32 $\tilde{g}_{TH}^2$	36.89
	$D^{*0}\pi^0$	875.21 $\tilde{g}_{TH}^2$	29.40	$D_s^{*0}K^+$	1653.47 $\tilde{g}_{TH}^2$	37.93
	$D^{*0}\eta$	83.31 $\tilde{g}_{TH}^2$	2.80	$D_s^{*+}\eta$	1097.90 $\tilde{g}_{TH}^2$	25.18
	$D_s^{*+}K^-$	296.97 $\tilde{g}_{TH}^2$	9.97	$D_s^{*+}\pi^0$	0.047 $\tilde{g}_{TH}^2$	0.00
	<i>Total</i>	2977.27 $\tilde{g}_{TH}^2$		<i>Total</i>	4359.74 $\tilde{g}_{TH}^2$	
$2P(2^+, 3/2)$	$D^{*+}\pi^-$	1060.50 $\tilde{g}_{TH}^2$	25.56	$D_s^{*+}K^0$	995.14 $\tilde{g}_{TH}^2$	18.84
	$D^{*0}\pi^0$	540.62 $\tilde{g}_{TH}^2$	13.03	$D_s^{*0}K^+$	1025.79 $\tilde{g}_{TH}^2$	19.42
	$D^{*0}\eta$	52.62 $\tilde{g}_{TH}^2$	1.27	$D_s^{*+}\eta$	300.13 $\tilde{g}_{TH}^2$	5.68
	$D_s^{*+}K^-$	189.25 $\tilde{g}_{TH}^2$	4.56	$D_s^{*+}\pi^0$	0.068 $\tilde{g}_{TH}^2$	0.00
	$D^+\pi^-$	1226.44 $\tilde{g}_{TH}^2$	29.56	$D_s^+K^0$	1233.50 $\tilde{g}_{TH}^2$	23.35
	$D^0\pi^0$	625.23 $\tilde{g}_{TH}^2$	15.07	$D_s^0K^+$	1267.44 $\tilde{g}_{TH}^2$	24.00
	$D^0\eta$	88.77 $\tilde{g}_{TH}^2$	2.14	$D_s^+\eta$	459.72 $\tilde{g}_{TH}^2$	8.70
	$D_s^+K^-$	365.07 $\tilde{g}_{TH}^2$	8.80	$D_s^+\pi^0$	0.075 $\tilde{g}_{TH}^2$	0.00
	<i>Total</i>	4148.50 $\tilde{g}_{TH}^2$		<i>Total</i>	5281.87 $\tilde{g}_{TH}^2$	

fraction of approximately 65.9% into the $D^*\pi^-$ channel for the $2S(0^-, 1/2)$ state. This behavior is consistent with experimental observations, as the $D_0(2550)^0$ has been reported in the $D^*\pi^-$ final state by LHCb collaborations and by *BABAR* [1]. Taken together, the mass spectrum, decay widths, and branching structure support the assignment of the $D_0(2550)^0$ as the $2S(0^-, 1/2)$ charmed meson.

LHCb collaboration has reported the observation of two higher excited charmed meson states. One of these is a

natural-parity state, denoted $D_J^*(3000)$, observed with a mass of 3008.1 ± 4.0 MeV and a total decay width of 110.5 ± 11.5 MeV, predominantly decaying into the $D\pi$ final state. In addition, an unnatural-parity resonance, labeled $D_J(3000)$, has been identified with a measured mass of 2971.8 ± 8.7 MeV. This state is observed mainly in the $D^*\pi$ channel and exhibits a comparatively larger width of 188.1 ± 44.8 MeV [1]. Our theoretical mass predictions for the $3S$ and $1F$ multiplets are found to be

TABLE VI. The same as Table IV, but for $2F$ states of the D and D_s meson families.

States $nL(J^P, j)$	D meson			D_s meson		
	Channel	Width	Branching fraction	Channel	Width	Branching fraction
$1F(2^+, 5/2)$	$D^{*+}\pi^-$	366.04 g_{ZH}^2	13.11	$D_s^{*+}K^0$	489.56 g_{ZH}^2	10.68
	$D^{*0}\pi^0$	186.79 g_{ZH}^2	6.69	$D_s^{*0}K^+$	502.58 g_{ZH}^2	10.96
	$D^{*0}\eta$	29.30 g_{ZH}^2	1.05	$D_s^{*+}\eta$	161.73 g_{ZH}^2	3.53
	$D_s^{*+}K^-$	101.27 g_{ZH}^2	3.63	$D_s^{*+}\pi^0$	0.02 g_{ZH}^2	0.00
	$D^+\pi^-$	1060.71 g_{ZH}^2	37.99	$D_s^+K^0$	1423.80 g_{ZH}^2	31.06
	$D^0\pi^0$	541.978 g_{ZH}^2	19.41	$D_s^0K^+$	1460.77 g_{ZH}^2	31.87
	$D^0\eta$	104.96 g_{ZH}^2	3.76	$D_s^+\eta$	545.06 g_{ZH}^2	11.89
	$D_s^+K^-$	400.72 g_{ZH}^2	14.35	$D_s^+\pi^0$	0.068 g_{ZH}^2	0.00
	<i>Total</i>	2791.79 g_{ZH}^2		<i>Total</i>	4583.61 g_{ZH}^2	
$1F(3^+, 5/2)$	$D^{*+}\pi^-$	997.69 g_{ZH}^2	53.17	$D_s^{*+}K^0$	1344.37 g_{ZH}^2	42.32
	$D^{*0}\pi^0$	508.91 g_{ZH}^2	27.12	$D_s^{*0}K^+$	1379.13 g_{ZH}^2	43.41
	$D^{*0}\eta$	82.06 g_{ZH}^2	4.37	$D_s^{*+}\eta$	453.31 g_{ZH}^2	14.27
	$D_s^{*+}K^-$	287.68 g_{ZH}^2	15.33	$D_s^{*+}\pi^0$	0.06 g_{ZH}^2	0.00
	<i>Total</i>	1876.35 g_{ZH}^2		<i>Total</i>	3176.87 g_{ZH}^2	
$1F(3^+, 7/2)$	$D^{*+}\pi^-$	1921.25 g_{RH}^2	60.87	$D_s^{*+}K^0$	2228.16 g_{RH}^2	44.18
	$D^{*0}\pi^0$	990.30 g_{RH}^2	31.38	$D_s^{*0}K^+$	2327.56 g_{RH}^2	46.15
	$D^{*0}\eta$	63.34 g_{RH}^2	2.01	$D_s^{*+}\eta$	487.34 g_{RH}^2	9.66
	$D_s^{*+}K^-$	181.25 g_{RH}^2	5.74	$D_s^{*+}\pi^0$	0.01 g_{RH}^2	0.00
	<i>Total</i>	3156.15 g_{RH}^2		<i>Total</i>	5043.08 g_{RH}^2	
$1F(4^+, 7/2)$	$D^{*+}\pi^-$	1154.12 g_{RH}^2	19.79	$D_s^{*+}K^0$	1358.00 g_{RH}^2	14.22
	$D^{*0}\pi^0$	594.61 g_{RH}^2	10.20	$D_s^{*0}K^+$	1417.35 g_{RH}^2	14.84
	$D^{*0}\eta$	39.56 g_{RH}^2	0.68	$D_s^{*+}\eta$	304.21 g_{RH}^2	3.19
	$D_s^{*+}K^-$	114.89 g_{RH}^2	1.97	$D_s^{*+}\pi^0$	0.09 g_{RH}^2	0.00
	$D^+\pi^-$	2253.29 g_{RH}^2	38.64	$D_s^+K^0$	2782.71 g_{RH}^2	29.14
	$D^0\pi^0$	1162.47 g_{RH}^2	19.94	$D_s^0K^+$	2898.58 g_{RH}^2	30.36
	$D^0\eta$	117.86 g_{RH}^2	2.02	$D_s^+\eta$	787.22 g_{RH}^2	8.24
	$D_s^+K^-$	394.36 g_{RH}^2	6.76	$D_s^+\pi^0$	0.17 g_{RH}^2	0.00
	<i>Total</i>	5831.18 g_{RH}^2		<i>Total</i>	9548.34 g_{RH}^2	

in close proximity to this mass region. So, the predicted masses suggest that the $D_J^*(3000)$ state is most plausibly associated with one of the following natural-parity states: $3S(1^-, 1/2)$, $1F(2^+, 5/2)$, or $1F(4^+, 7/2)$. For the $D_J(3000)$ state, the most probable assignments from the predicted masses include the $3S(0^-, 1/2)$, $1F(3^+, 5/2)$, and $1F(3^+, 7/2)$ configurations. As the $D_J^*(3000)$ state has been reported in the $D\pi$ channel, the ratio $R_\pi(D_{3S(1^-, 1/2)}) = \frac{\Gamma(D_{3S(1^-, 1/2)} \rightarrow D\pi)}{\Gamma(D_{3S(1^-, 1/2)} \rightarrow D^*\pi)} = 0.64$ disfavors $3S(1^-, 1/2)$ assignment for the $D_J^*(3000)$ state. Further, if $D_J(3000)$ is assigned as the $3S(0^-, 1/2)$ state, we extract the coupling $\tilde{g}_{HH} = 0.163 \pm 0.019$. The $1F(2^+, 5/2)$ and $1F(4^+, 7/2)$ states are also possible choices for $D_J^*(3000)$. Both mainly decay to $D\pi$. We find $R_\pi(D_{1F(2^+, 5/2)}) = 2.90$, and $R_\pi(D_{1F(4^+, 7/2)}) = 1.95$. Because the $1F(2^+, 5/2)$ state gives a larger preference for $D\pi$, we prefer $D_J^*(3000)$ to be $1F(2^+, 5/2)$. Using the experimental width of $D_J^*(3000)$ state, we extract $g_{ZH} = 0.199 \pm 0.010$. Overall, our results

support $D_J^*(3000)$ as most likely the $1F(2^+, 5/2)$ state, while the $1F(4^+, 7/2)$ assignment for $D_J^*(3000)^0$ cannot be ruled out. For $D_J(3000)$, the $1F(3^+, 5/2)$ and $1F(3^+, 7/2)$ assignments are still reasonable because they mostly decay to $D^*\pi$, with $\text{BR}(D^*\pi) \approx 80 - 90\%$. The decay to $D_s K$ can help to test these assignments. We predict $\text{BR}(D_s K) \simeq 14 - 15\%$ for $1F(2^+, 5/2)$ and $1F(3^+, 5/2)$, and $\text{BR}(D_s K) \simeq 6 - 7\%$ for $1F(3^+, 7/2)$ and $1F(4^+, 7/2)$. Future measurements of these modes can clarify the nature of both $D_J(3000)$ and $D_J^*(3000)$.

Lastly, the $D_2^*(3000)$ state was observed by LHCb collaboration with a measured mass of $3214 \pm 29 \pm 49$ MeV, decay width 186 ± 38 MeV, and quantum numbers $J^P = 2^+$ [1]. Our theoretical mass predictions for the $3P$ and $2D$ multiplets place lies in the same mass region. However, given that the $D_2^*(3000)$ state has $J^P = 2^+$, it cannot originate from the $2D$ multiplet. In contrast, our calculated mass for the $3P(2^+, 3/2)$ state is in excellent agreement with the experimentally measured mass of the

TABLE VII. Masses of D meson (in MeV).

$nL(J^P, j)$	MASS	PDG [1]	Ref. [22]	Ref. [13]	Ref. [20]
$1S(0^-, 1/2)$	1867 ± 9	1867.25 ± 0.035	1877	1871	1855
$1S(1^-, 1/2)$	2009 ± 8	2008.55 ± 0.035	2041	2010	2020
$2S(0^-, 1/2)$	2594 ± 6	2549 ± 19	2581	2581	2534
$2S(1^-, 1/2)$	2597 ± 6	2627 ± 10	2643	2632	2593
$3S(0^-, 1/2)$	2995 ± 8	2971.8 ± 8.7	3068	3062	2976
$3S(1^-, 1/2)$	2995 ± 8	3008.1 ± 4.0	3110	3096	3015
$4S(0^-, 1/2)$	3314 ± 9		3468	3452	
$4S(1^-, 1/2)$	3314 ± 9		3497	3482	
$5S(0^-, 1/2)$	3587 ± 11		3814	3793	
$5S(1^-, 1/2)$	3587 ± 11		3837	3822	
$6S(0^-, 1/2)$	3830 ± 12				
$6S(1^-, 1/2)$	3830 ± 12				
$1P(0^+, 1/2)$	2362 ± 7	2343 ± 10	2399	2406	2365
$1P(1^+, 1/2)$	2422 ± 6	2422.1 ± 0.6	2456	2469	2426
$1P(1^+, 3/2)$	2429 ± 5	2412 ± 9	2467	2426	2431
$1P(2^+, 3/2)$	2462 ± 4	2461.1 ± 0.8	2502	2460	2468
$2P(0^+, 1/2)$	2873 ± 6		2931	2919	2856
$2P(1^+, 1/2)$	2888 ± 5		2924	3021	2861
$2P(1^+, 3/2)$	2871 ± 5		2961	2932	2877
$2P(2^+, 3/2)$	2877 ± 5		2957	3012	2884
$3P(0^+, 1/2)$	3222 ± 7		3343	3346	
$3P(1^+, 1/2)$	3228 ± 7		3328	3461	
$3P(1^+, 3/2)$	3211 ± 7		3360	3365	
$3P(2^+, 3/2)$	3212 ± 7	$3214 \pm 29 \pm 49$	3353	3407	
$4P(0^+, 1/2)$	3510 ± 9		3697		
$4P(1^+, 1/2)$	3513 ± 9		3681		
$4P(1^+, 3/2)$	3497 ± 9		3709		
$4P(2^+, 3/2)$	3497 ± 9		3701		
$1D(1^-, 3/2)$	2739 ± 5	2781 ± 22	2817	2788	2762
$1D(2^-, 3/2)$	2763 ± 4	2747 ± 6	2816	2850	2773
$1D(2^-, 5/2)$	2738 ± 4	2747 ± 6	2845	2806	2779
$1D(3^-, 5/2)$	2754 ± 4	2763.1 ± 3.2	2833	2863	2779
$2D(1^-, 3/2)$	3122 ± 5		3231	3228	3131
$2D(2^-, 3/2)$	3131 ± 5		3212	3307	3128
$2D(2^-, 5/2)$	3102 ± 5		3248	3259	3136
$2D(3^-, 5/2)$	3107 ± 4		3226	3335	3129
$3D(1^-, 3/2)$	3427 ± 6		3588		
$3D(2^-, 3/2)$	3431 ± 6		3566		
$3D(2^-, 5/2)$	3403 ± 6		3600		
$3D(3^-, 5/2)$	3405 ± 6		3579		
$1F(2^+, 5/2)$	3013 ± 4	3008.1 ± 4.0	3132	3090	3053
$1F(3^+, 5/2)$	3028 ± 4		3108	3145	3046
$1F(3^+, 7/2)$	2987 ± 3	2971.8 ± 8.7	3143	3129	3051
$1F(4^+, 7/2)$	2997 ± 3	3008.1 ± 4.0	3113	3187	3037
$2F(2^+, 5/2)$	3338 ± 4		3490		
$2F(3^+, 5/2)$	3345 ± 4		3461		
$2F(3^+, 7/2)$	3305 ± 4		3498	3551	
$2F(4^+, 7/2)$	3308 ± 4		3466	3610	
$1G(3^-, 7/2)$	3244 ± 4		3397	3352	
$1G(4^-, 7/2)$	3255 ± 3		3364	3415	
$1G(4^-, 9/2)$	3202 ± 3		3399	3403	
$1G(5^-, 9/2)$	3209 ± 3		3362	3473	
$1H(4^+, 9/2)$	3449 ± 3				
$1H(5^+, 9/2)$	3456 ± 3				
$1H(5^+, 11/2)$	3394 ± 3				
$1H(6^+, 11/2)$	3399 ± 3				

TABLE VIII. Masses of D_s meson (in MeV).

$nL(J^P, j)$	MASS	PDG [1]	Ref. [22]	Ref. [13]	Ref. [21]
$1S(0^-, 1/2)$	1969 ± 9	1968.35 ± 0.07	1979	1969	1967
$1S(1^-, 1/2)$	2087 ± 8	2112.2 ± 0.4	2129	2111	2115
$2S(0^-, 1/2)$	2709 ± 7	2591 ± 9	2673	2688	2646
$2S(1^-, 1/2)$	2714 ± 7	2714 ± 5	2732	2731	2704
$3S(0^-, 1/2)$	3139 ± 8		3154	3219	
$3S(1^-, 1/2)$	3139 ± 8		3193	3242	
$4S(0^-, 1/2)$	3481 ± 10		3547	3652	
$4S(1^-, 1/2)$	3481 ± 10		3575	3669	
$5S(0^-, 1/2)$	3775 ± 11		3894	4033	
$5S(1^-, 1/2)$	3775 ± 11		3912	4048	
$6S(0^-, 1/2)$	4036 ± 13				
$6S(1^-, 1/2)$	4036 ± 13				
$1P(0^+, 1/2)$	2461 ± 7	2317.8 ± 0.5	2484	2509	2463
$1P(1^+, 1/2)$	2524 ± 6	2459.5 ± 0.6	2549	2574	2531
$1P(1^+, 3/2)$	2535 ± 5	2535.11 ± 0.06	2556	2536	2532
$1P(2^+, 3/2)$	2572 ± 4	2569.1 ± 0.8	2592	2571	2571
$2P(0^+, 1/2)$	3003 ± 6		3005	3054	2960
$2P(1^+, 1/2)$	3019 ± 6	3044^{+31}_{-9}	3018	3154	2979
$2P(1^+, 3/2)$	3009 ± 5		3038	3067	2988
$2P(2^+, 3/2)$	3015 ± 5		3048	3142	3004
$3P(0^+, 1/2)$	3377 ± 8		3412	3513	
$3P(1^+, 1/2)$	3384 ± 7		3416	3618	
$3P(1^+, 3/2)$	3373 ± 7		3433	3519	
$3P(2^+, 3/2)$	3374 ± 7		3439	3580	
$4P(0^+, 1/2)$	3687 ± 9		3764		
$4P(1^+, 1/2)$	3691 ± 9		3764		
$4P(1^+, 3/2)$	3680 ± 9		3778		
$4P(2^+, 3/2)$	3680 ± 9		3783		
$1D(1^-, 3/2)$	2857 ± 5	2859 ± 27	2899	2913	2865
$1D(2^-, 3/2)$	2883 ± 4		2900	2961	2877
$1D(2^-, 5/2)$	2869 ± 4		2926	2931	2882
$1D(3^-, 5/2)$	2886 ± 4	2860 ± 7	2917	2971	2883
$2D(1^-, 3/2)$	3268 ± 5		3306	3383	3244
$2D(2^-, 3/2)$	3278 ± 5		3298	3456	3247
$2D(2^-, 5/2)$	3259 ± 5		3323	3403	3252
$2D(3^-, 5/2)$	3264 ± 5		3311	3469	3251
$3D(1^-, 3/2)$	3596 ± 7		3658		
$3D(2^-, 3/2)$	3600 ± 7		3650		
$3D(2^-, 5/2)$	3581 ± 7		3672		
$3D(3^-, 5/2)$	3583 ± 7		3661		
$1F(2^+, 5/2)$	3148 ± 4		3208	3230	3159
$1F(3^+, 5/2)$	3164 ± 3		3186	3266	3151
$1F(3^+, 7/2)$	3138 ± 3		3218	3254	3157
$1F(4^+, 7/2)$	3149 ± 3		3190	3300	3143
$2F(2^+, 5/2)$	3498 ± 5		3562		
$2F(3^+, 5/2)$	3505 ± 4		3540		
$2F(3^+, 7/2)$	3478 ± 4		3569	3710	
$2F(4^+, 7/2)$	3482 ± 4		3544	3754	
$1G(3^-, 7/2)$	3394 ± 3		3469	3508	
$1G(4^-, 7/2)$	3405 ± 3		3436	3554	
$1G(4^-, 9/2)$	3370 ± 3		3469	3546	
$1G(5^-, 9/2)$	3378 ± 3		3433	3595	
$1H(4^+, 9/2)$	3613 ± 3				
$1H(5^+, 9/2)$	3620 ± 3				
$1H(5^+, 11/2)$	3578 ± 3				
$1H(6^+, 11/2)$	3583 ± 3				

$D_2^*(3000)$. Based on this correspondence, we conclude that the $D_2^*(3000)$ state is most likely the $3P(2^+, 3/2)$ state.

2. D_s meson

Turning to the D_s meson spectrum, the well-established ground-state resonances $D_s^\pm$ and $D_s^{*\pm}$ correspond well with our theoretical masses for the $1S(0^-, 1/2)$ and $1S(1^-, 1/2)$ states, respectively.

The classification of $D_{s1}(2536)^\pm$ and $D_{s2}(2573)^\pm$ resonances as the $1P(1^+, 3/2)$ and $1P(2^+, 3/2)$ states is widely accepted, and our model reproduces their masses accurately. Using the coupling $g_{TH} = 0.413 \pm 0.0035$ fixed from the D meson sector, we obtain a total decay width of 19.9 MeV for the $1P(2^+, 3/2)$ state, which is close to the PDG average value of 16.9 ± 0.7 MeV. Our predicted partial-width ratio, $\frac{\Gamma(D_{s2}^*(2573) \rightarrow D^{*0}K^+)}{\Gamma(D_{s2}^*(2573) \rightarrow D^0K^+)} = 0.09$, is also consistent with the experimental constraint $\frac{\Gamma(D_{s2}^*(2573) \rightarrow D^{*0}K^+)}{\Gamma(D_{s2}^*(2573) \rightarrow D^0K^+)} < 0.33$.

However, significant discrepancies persist for the other two $1P$ states, $D_{s0}(2317)^\pm$ and $D_{s1}(2460)^\pm$. Our calculations yield predicted masses of 2461 ± 7 MeV and 2524 ± 6 MeV for these states, respectively. These theoretical values exceed the experimentally measured masses of the $D_{s0}(2317)^\pm$ and $D_{s1}(2460)^\pm$ by approximately 144 MeV and 64 MeV, respectively. Similar deviations have been consistently observed in various theoretical works [13,21,22].

The positive-parity resonances $D_{s0}(2317)^+$ and $D_{s1}(2460)^+$, first observed by *BABAR* and *CLEO* collaborations, were initially identified as the $1P(0^+)$ and $1P(1^+)$ states of the D_s meson, respectively. However, the significantly lower experimental masses of these states, relative to predictions from conventional quark models [12,22,63], present notable theoretical challenges. Comparable discrepancies are also observed in lattice QCD calculations, which consistently predict substantially higher masses than those measured experimentally [64]. To address this discrepancy—often referred to as the “low-mass puzzle”—various nonconventional interpretations have been proposed. These include the possibility that $D_{s0}(2317)^\pm$ and $D_{s1}(2460)^\pm$ are exotic configurations, such as $D^{(*)}K$ molecular states [65–70] or compact tetraquark states [71–75]. More recent theoretical investigations suggest that these resonances may not be pure $1P$ $c\bar{s}$ mesons, but rather mixed states composed of a bare $1P$ $c\bar{s}$ core coupled dynamically to a nearby $D^{(*)}K$ threshold [76,77]. These interpretations point toward a more intricate internal structure for the $D_{s0}(2317)$ and $D_{s1}(2460)$, highlighting the necessity for further experimental measurements and refined theoretical analyses to clarify their nature.

The $D_{sJ}(2860)^+$ resonance was first reported by *BABAR* collaboration with a measured mass of $M_{\text{exp}} = 2856.6 \pm$

6.5 MeV [1]. Its existence was subsequently confirmed by LHCb collaboration using proton-proton collision data. In 2014, a detailed amplitude analysis by LHCb revealed that the structure observed near this mass consists of two overlapping resonances: $D_{s1}^*(2860)^-$, with quantum numbers $J^P = 1^-$, and $D_{s3}^*(2860)^-$, with $J^P = 3^-$. The $D_{s3}^*(2860)^+$ resonance was later observed in the $D^{*+}K_s^0$ decay channel, with resonance parameters consistent with $D_{s3}^*(2860)^-$ [1].

Our theoretical mass prediction for the $1D(1^-, 3/2)$ state is 2857 ± 5 MeV, which is in excellent agreement with the experimentally measured mass of the $D_{s1}^*(2860)^\pm$ resonance. Using previously extracted coupling, $g_{XH} = 0.253 \pm 0.027$, we obtain a total width of 263 ± 57 MeV for the $1D(1^-, 3/2)$ state. Within uncertainties, this is compatible with the experimental width $\Gamma = 160 \pm 80$ MeV quoted for the $D_{s1}^*(2860)^\pm$. Taken together, the mass proximity and the width systematics favor the $D_{s1}^*(2860)^\pm$ state as the $1D(1^-, 3/2)$ state. Further, for the $D_{s3}^*(2860)^\pm$, the observed mass is consistent with our prediction for the $1D(3^-, 5/2)$ configuration. Employing $g_{YH} = 0.404 \pm 0.015$, we find $\Gamma[1D(3^-, 5/2)] = 77 \pm 6$ MeV, close to the experimental value 53 ± 10 MeV for the $D_{s3}^*(2860)^\pm$. The combined agreement in mass and width supports the assignment of $D_{s3}^*(2860)^\pm$ to the $1D(3^-, 5/2)$ state. These identifications place $D_{s1}^*(2860)^\pm$ and $D_{s3}^*(2860)^\pm$ as the $J^P = 1^-$ and $J^P = 3^-$ members of the $1D$ $c\bar{s}$ multiplet, consistent with earlier analyses [22,78,79].

For the first radial excitations, the $D_{s1}^*(2700)^\pm$ resonance is generally considered to be the $D_s(2S(1^-, 1/2))$ state. This resonance was first observed by *BABAR*, and later it was confirmed by Belle, and its spin-parity numbers were determined to be $J^P = 1^-$ [1]. Later, the existence of $D_{s1}^*(2700)^+$ was further confirmed using pp collision data at LHCb. We observe that the measured mass of $D_{s1}^*(2700)^\pm$ resonance is well reproduced in our work. For the $D_s(2S(1^-, 1/2))$ state we obtain $\frac{\Gamma(D_{s1}^*(2700) \rightarrow D^{*0}K)}{\Gamma(D_{s1}^*(2700) \rightarrow D^0K)} = 0.92$, which matches well the *BABAR* measurement $\frac{\Gamma(D_{s1}^*(2700) \rightarrow D^{*0}K)}{\Gamma(D_{s1}^*(2700) \rightarrow D^0K)} = 0.91 \pm 0.13 \pm 0.12$ [1]. Motivated by this agreement, we use the measured total width of $D_{s1}^*(2700)^\pm$ as input to fix the corresponding coupling $\tilde{g}_{HH} = 0.264 \pm 0.011$.

LHCb collaboration recently reported the observation of a new excited D_s meson, $D_{s0}(2590)^+$, with quantum numbers $J^P = 0^-$. This state has been proposed as a candidate for the first radial excitation, corresponding to the $2S$ level. In our model, the $2S(0^-, 1/2)$ state is predicted to lie at 2709 ± 7 MeV, which is consistent with the mass range reported in other theoretical studies [22,23]. However, our predicted mass is approximately 100 MeV higher than the experimentally measured mass of the $D_{s0}(2590)^+$ resonance. Using the coupling $\tilde{g}_{HH} = 0.264 \pm 0.011$, we obtain

TABLE IX. Masses of Ξ_{cc} baryon (in MeV).

$n_d L_d n L(J^P, j)$	MASS	PDG [1]	[80]	[23]	[42]	[8]	[24]	[7]	[6]
$1S1s(1/2^+, 1/2)$	3622 ± 16	3621.6 ± 0.4	3615 ± 33	3620	3581	3676	3685	3547	3579
$1S1s(3/2^+, 1/2)$	3727 ± 16		3703 ± 33	3727	3726	3753	3754	3719	3656
$1S2s(1/2^+, 1/2)$	4222 ± 11				3925	4029	4079	4183	3910
$1S2s(3/2^+, 1/2)$	4225 ± 11				3988	4042	4114	4282	4027
$1S1p(1/2^-, 1/2)$	4027 ± 13		3930 ± 20	4053		3910	3947		3880
$1S1p(1/2^-, 3/2)$	4064 ± 12		3971 ± 22	4136		4074	4135		
$1S1p(3/2^-, 1/2)$	4073 ± 12		4009 ± 31	4101	3926	3921	3949		
$1S1p(3/2^-, 3/2)$	4077 ± 11			4196		4078	4137		
$1S1p(5/2^-, 3/2)$	4099 ± 11			4155	4074	4092	4163		
$1P1s(1/2^-, 1/2)$	4130 ± 24		4246 ± 193	3838			4149		
$1P1s(3/2^-, 1/2)$	4221 ± 24			3959			4159		
$2S1s(1/2^+, 1/2)$	4281 ± 26			3910			4159		
$2S1s(3/2^+, 1/2)$	4369 ± 25			4027			4131		
$1S2p(1/2^-, 1/2)$	4493 ± 10								4018
$1S2p(1/2^-, 3/2)$	4483 ± 10								
$1S2p(3/2^-, 1/2)$	4505 ± 10				4144		4137		
$1S2p(3/2^-, 3/2)$	4486 ± 9								
$1S2p(5/2^-, 3/2)$	4490 ± 9				4183		4488		
$1S1d(1/2^+, 3/2)$	4362 ± 10								
$1S1d(3/2^+, 3/2)$	4372 ± 10								
$1S1d(3/2^+, 5/2)$	4353 ± 9								
$1S1d(5/2^+, 3/2)$	4389 ± 10				4243	4047	4115		
$1S1d(5/2^+, 5/2)$	4360 ± 9					4091	4164		
$1S1d(7/2^+, 5/2)$	4370 ± 9				4394	4097			
$2P1s(1/2^-, 1/2)$	4550 ± 27			4085					
$2P1s(3/2^-, 1/2)$	4633 ± 27			4197					
$1S2d(1/2^+, 3/2)$	4732 ± 9								
$1S2d(3/2^+, 3/2)$	4736 ± 9								
$1S2d(3/2^+, 5/2)$	4707 ± 8								
$1S2d(5/2^+, 3/2)$	4743 ± 8				4299				
$1S2d(5/2^+, 5/2)$	4710 ± 8								
$1S2d(7/2^+, 5/2)$	4714 ± 8								
$1S1f(3/2^-, 5/2)$	4626 ± 9								
$1S1f(5/2^-, 5/2)$	4633 ± 8						4534		
$1S1f(5/2^-, 7/2)$	4593 ± 8								
$1S1f(7/2^-, 5/2)$	4644 ± 8				4538				
$1S1f(7/2^-, 7/2)$	4598 ± 8								
$1S1f(9/2^-, 7/2)$	4605 ± 8				4692				
$1S1g(5/2^+, 7/2)$	4853 ± 8								
$1S1g(7/2^+, 7/2)$	4859 ± 8								
$1S1g(7/2^+, 9/2)$	4804 ± 8								
$1S1g(9/2^+, 7/2)$	4866 ± 7				4815				
$1S1g(9/2^+, 9/2)$	4808 ± 7								
$1S1g(11/2^+, 9/2)$	4814 ± 7				4973				

$\Gamma[D_s(2S(0^-, 1/2))] = 80 \pm 7$ MeV, which is compatible with the experimental width 89 ± 20 MeV for $D_{s0}(2590)^+$ [1]. Even so, the large difference in mass raises questions regarding the identification of the $D_{s0}(2590)^+$ as a conventional $2S(0^-, 1/2)$ state. Therefore, it may be too early to call $D_{s0}(2590)^+$ a standard radial excitation, and more experimental studies are needed.

Finally, the $D_{sJ}(3040)^+$ resonance was first observed by BABAR collaboration in the D^*K decay channel. Subsequently, LHCb collaboration reported weak evidence

for the $D_{sJ}(3040)^+$, indicating that it is consistent with an unnatural-parity assignment [1]. The measured mass of the $D_{sJ}(3040)^+$ is 3044^{+31}_{-9} MeV, which aligns well with our theoretical predictions for the $2P$ multiplet. In particular, this multiplet includes the unnatural-parity states $2P(1^+, 1/2)$ and $2P(1^+, 3/2)$. Given the agreement between the experimental mass and our calculated values, we suggest that the $D_{sJ}(3040)^+$ resonance may correspond to a $2P$ excitation of the D_s meson family with quantum numbers $J^P = 1^+$. This interpretation is also supported by

TABLE X. Masses of Ω_{cc} baryon (in MeV).

$n_d L_d n L(J^P, j)$	MASS	[80]	[23]	[42]	[8]	[24]	[6]	[7]
1S1s(1/2 ⁺ , 1/2)	3718 ± 16	3733 ± 13	3778	3719	3815	3832	3697	3648
1S1s(3/2 ⁺ , 1/2)	3805 ± 16	3793 ± 30	3872	3847	3876	3883	3769	3770
1S2s(1/2 ⁺ , 1/2)	4337 ± 12			4044	4180	4227	4112	4268
1S2s(3/2 ⁺ , 1/2)	4340 ± 11			4096	4188	4263	4160	4334
1S1p(1/2 ⁻ , 1/2)	4129 ± 13	4041 ± 15	4208		4046	4086	4009	
1S1p(1/2 ⁻ , 3/2)	4170 ± 12	4063 ± 13	4271		4135	4199		
1S1p(3/2 ⁻ , 1/2)	4176 ± 12	4115 ± 70	4252	3947	4052	4086		
1S1p(3/2 ⁻ , 3/2)	4184 ± 11		4325		4140	4201		
1S1p(5/2 ⁻ , 3/2)	4207 ± 11		4303	4153	4152	4220		
1P1s(1/2 ⁻ , 1/2)	4225 ± 24	4395 ± 41	4002			4210		
1P1s(3/2 ⁻ , 1/2)	4300 ± 24		4102					
2S1s(1/2 ⁺ , 1/2)	4376 ± 26		4075			4295		
2S1s(3/2 ⁺ , 1/2)	4449 ± 25		4174			4265		
1S2p(1/2 ⁻ , 1/2)	4620 ± 10			4135			4101	
1S2p(1/2 ⁻ , 3/2)	4617 ± 10							
1S2p(3/2 ⁻ , 1/2)	4632 ± 10			4259				
1S2p(3/2 ⁻ , 3/2)	4620 ± 10							
1S2p(5/2 ⁻ , 3/2)	4624 ± 10			4247		4555		
1S1d(1/2 ⁺ , 3/2)	4479 ± 10							
1S1d(3/2 ⁺ , 3/2)	4489 ± 10							
1S1d(3/2 ⁺ , 5/2)	4481 ± 10							
1S1d(5/2 ⁺ , 3/2)	4506 ± 10			4162	4202	4264		
1S1d(5/2 ⁺ , 5/2)	4488 ± 9				4232	4299		
1S1d(7/2 ⁺ , 5/2)	4499 ± 9			4438	4230			
2P1s(1/2 ⁻ , 1/2)	4644 ± 27		4251					
2P1s(3/2 ⁻ , 1/2)	4713 ± 27		4345					
1S2d(1/2 ⁺ , 3/2)	4873 ± 9							
1S2d(3/2 ⁺ , 3/2)	4877 ± 9							
1S2d(3/2 ⁺ , 5/2)	4858 ± 9							
1S2d(5/2 ⁺ , 3/2)	4883 ± 9			4407				
1S2d(5/2 ⁺ , 5/2)	4861 ± 9							
1S2d(7/2 ⁺ , 5/2)	4865 ± 8							
1S1f(3/2 ⁻ , 5/2)	4757 ± 9							
1S1f(5/2 ⁻ , 5/2)	4764 ± 8							
1S1f(5/2 ⁻ , 7/2)	4739 ± 8					4600		
1S1f(7/2 ⁻ , 5/2)	4775 ± 8			4367				
1S1f(7/2 ⁻ , 7/2)	4745 ± 8							
1S1f(9/2 ⁻ , 7/2)	4752 ± 8			4706				
1S1g(5/2 ⁺ , 7/2)	4997 ± 8							
1S1g(7/2 ⁺ , 7/2)	5003 ± 8							
1S1g(7/2 ⁺ , 9/2)	4966 ± 8							
1S1g(9/2 ⁺ , 7/2)	5010 ± 7			4562				
1S1g(9/2 ⁺ , 9/2)	4971 ± 7							
1S1g(11/2 ⁺ , 9/2)	4976 ± 7			4960				

previous theoretical studies [21,22,45]. These comprehensive predictions thus provide critical benchmarks for ongoing and future experimental explorations.

B. Doubly charmed baryons

The calculated masses for the Ξ_{cc} and Ω_{cc} baryons are shown in Tables IX and X, respectively. The states listed in Tables IX and X are labeled by $n_d L_d n L(J^P, j)$. Here,

$n_d = 1, 2, 3, \dots$ counts the radial excitation inside the cc diquark, i.e., the excitation of one c quark relative to the other, while $n = 1, 2, 3, \dots$ specifies the radial excitation of the light quark with respect to the diquark as a whole. The symbol $L_d = S, P, D, \dots$ denotes the internal orbital angular momentum of the two c quarks within the diquark, whereas $L = s, p, d, \dots$ refers to the orbital angular momentum associated with the light-quark motion relative to the diquark. The baryon's total angular momentum and

parity are indicated by J and P , and j represents the total angular momentum carried by the light degrees of freedom relative to the cc diquark.

The only experimentally confirmed baryon containing two heavy quarks is the ground state of the doubly charmed Ξ_{cc} baryon. The PDG lists two isospin partners in this sector: Ξ_{cc}^+ and Ξ_{cc}^{++} . The Ξ_{cc}^+ was initially reported by SELEX collaboration in the $\Lambda_c^+ K^- \pi^+$ decay channel, with a measured mass of approximately 3519 MeV [1]. However, subsequent searches by BABAR, Belle, and LHCb failed to confirm the existence of this state, leading the PDG to classify it with a one-star rating, indicating weak experimental evidence. In contrast, the Ξ_{cc}^{++} was unambiguously observed by LHCb collaboration through multiple decay channels, with a precisely measured mass of 3621 MeV [1]. This state is assigned a three-star rating in the PDG and is most naturally interpreted as the ground-state Ξ_{cc} baryon, with quantum numbers $J^P = \frac{1}{2}^+$. Our theoretical calculations well reproduce its observed mass. To date, no Ω_{cc} has been observed experimentally. However, they have been studied using various theoretical methods. We list these predictions in Tables IX and X for comparison.

Ebert *et al.* [23] analyzed the masses of the $1S$ and $1P$ states of doubly heavy baryons utilizing a relativistic quark model. In contrast, Oudichhya *et al.* [42] derived mass spectra for doubly heavy baryons employing Regge phenomenology, specifically targeting higher orbital and radial excitations. Nonrelativistic quark models were adopted in the studies by Roberts *et al.* [8], Yoshida *et al.* [24], and Valcarce *et al.* [6] to determine mass spectra for both single and doubly heavy baryons. Additionally, Giannuzzi [7] investigated the $1S$ to $3S$ states within a semirelativistic quark model framework, leveraging potentials inspired by the AdS/QCD correspondence.

Our computed masses for the $1P$ states of the Ξ_{cc} and Ω_{cc} baryons fall comfortably within the range of values reported by these varied theoretical approaches, demonstrating a notable agreement with predictions from Refs. [8,23,24,42]. However, we note that our results exceed those presented by Valcarce *et al.* [6]. For the $1D$ state masses, our calculations align closely with those obtained by Oudichhya *et al.* [42], while the nonrelativistic quark model predictions [8,24] appear slightly underestimated. Regarding the $2S$ state masses, our values match well with the predictions by Refs. [7,24] but are found to be a bit larger than that predicted by Refs. [6,8,42]. Further, our predictions for higher orbital excitations ($1F$ and $1G$ states) exhibit good agreement with the Regge phenomenological calculations reported by Oudichhya *et al.* [42]. Overall, our results are mostly consistent with existing theoretical models, although some differences arise due to the specific assumptions used in each approach.

Moreover, a comparison with the lattice QCD results of Bahtiyar *et al.* [80] shows that our predictions for the lowest

Ξ_{cc} and Ω_{cc} levels are broadly consistent with first-principles calculations. In particular, Ref. [80] quotes $1S$ ground-state masses ($J^P = \frac{1}{2}^+$) of $M(\Xi_{cc}) = 3615 \pm 33$ MeV and $M(\Omega_{cc}) = 3733 \pm 13$ MeV, both of which lie very close to our central values. For the negative-parity sector, their lowest $1P$ excitations with $J^P = \frac{1}{2}^-$ appear at 3930 ± 20 MeV for Ξ_{cc} and 4041 ± 15 MeV for Ω_{cc} , again compatible with our spectrum. Reference [80] further reports a second $\frac{1}{2}^-$ state nearly degenerate with the first, at about 3971 MeV for Ξ_{cc} and about 4063 MeV for Ω_{cc} . When we track these excitations in our framework, we also obtain a set of closely spaced low-lying $\frac{1}{2}^-$ states, although their absolute positions can differ from the lattice values by roughly 60–80 MeV. Overall, the agreement for the ground state and the first excitation is particularly strong, while the remaining differences for higher levels remain moderate, supporting the view that our model reproduces the main spectral features of doubly charmed baryons.

Furthermore, in our predicted mass spectra for Ξ_{cc} and Ω_{cc} baryons, we observe that for states having identical n , L , and J , the mass of the state with higher j is greater than that of the state with lower j for $1S$, $2S$, and $1P$ multiplets, while for higher multiplets we observe that for states having identical n , L , and J , the mass of the state with higher j is lower than that of the state with higher j . For the $1P$ wave, this feature is also reflected in the mass spectra predicted by [23]. The distinct characteristics observed in mass spectra from various theoretical models indicate the necessity for additional experimental measurements to effectively differentiate between them.

V. CONCLUSION

We have carried out a comprehensive study of the mass spectra of singly charmed mesons (D and D_s) and doubly charmed baryons (Ξ_{cc} and Ω_{cc}) within the relativistic flux-tube model framework. In this model, a charm quark and a light antiquark (for D and D_s mesons) or a tightly bound $\{cc\}$ diquark and a light quark (for Ξ_{cc} and Ω_{cc} baryons) are connected by a rotating color flux tube that encapsulates the QCD string tension. We incorporate spin-dependent interactions in the j - j coupling scheme to account for hyperfine-structure splitting in the spectra. This approach successfully reproduces the mass of the known D and D_s meson states, and it predicts a rich spectrum of excited states. To complement the spectroscopy, we also study strong decays of the D and D_s mesons within the HQET framework. Using the measured total widths of well-established resonances, we extract the relevant strong couplings, and we find that the resulting couplings provide a consistent description of the decay widths of other experimentally observed charmed meson states.

For the charmed mesons, the calculated spectrum and strong decay width allows us to classify experimentally observed resonances. Notably, the $D_2(2740)^0$ is interpreted

as a $1D$ excitation with $J^P = 2^-$ (with the light-quark total angular momentum $j = 5/2$ or $3/2$), and the $D_3^*(2750)$ is identified as the companion $1D(3^-, 5/2)$ state. The lower-mass resonances $D(2550)^0$ and $D^*(2600)^0$ are assigned to the first radial excitations, $2S(0^-, 1/2)$ and $2S(1^-, 1/2)$, respectively, while the neutral $D_1^*(2760)^0$ fits well as the remaining $1D(1^-, 3/2)$ state. Furthermore, our model suggests that the broad structures labeled $D_J(3000)$ and $D_J^*(3000)$ correspond to either second radial excitations ($3S$) or members of the first F -wave family—specifically, $D_J^*(3000)$ is assigned to $1F, (2^+, 5/2)$ or $1F, (4^+, 7/2)$, whereas $D_J(3000)$ is consistent with $3S, (1^-, 1/2)$, $1F, (2^+, 5/2)$, or $1F, (4^+, 7/2)$. In addition, the newly reported $D_2^*(3000)$ is naturally assigned to the $3P(2^+, j = 3/2)$ level. In the charmed strange meson sector, the observed masses of $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$ cannot be explained by pure $1P$ -wave $c\bar{s}$ states, indicating possible exotic or coupled-channel structures. The $D_{s1}^*(2860)^\pm$ and $D_{s3}^*(2860)^\pm$ are definitively identified as second orbital excitations $1D, (1^-, 3/2)$ and $1D, (3^-, 5/2)$, respectively, while the $D_{sJ}(3040)^+$ is assigned as a candidate for the $2P(1^+)$ excitation (with $j = 1/2$ or $3/2$).

Based on a good description of the well-established states of the charmed meson sector, we have extended the same model to explore the spectrum of doubly charmed baryons. Treating the dicharm (cc) pair as an effective heavy axial-vector diquark in the ground state, attached to a light quark by a flux tube, we predict the masses of orbital and radial excited states of Ξ_{cc} and Ω_{cc} baryons that have yet to be observed. The mass estimates and quantum number assignments presented here can serve as reference points for upcoming heavy-flavor experiments—especially at LHCb, but also at facilities like Belle II and any future charm factories—in their efforts to discover and identify higher excited charmed mesons and double charmed baryon resonances.

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DATA AVAILABILITY

The data supporting this study's findings are available within the article.

APPENDIX: SPIN-DEPENDENT OPERATORS IN HEAVY-LIGHT MESONS AND DOUBLY HEAVY BARYONS

Here, we detail the construction of the spin-dependent mass-splitting operators and their expectation values for the cases of heavy-light mesons and doubly heavy baryons. We

consider the S -wave (ground state) and the first two orbitally excited states (P -wave and D -wave) for demonstration. We start with the L - S coupling scheme to construct the basis and then transform it to the $|J, j\rangle$ coupling basis. Here, j denotes the total angular momentum of the light degrees of freedom, which is particularly relevant in the heavy-quark limit.

The L - S basis states can be constructed using standard Clebsch-Gordan coefficients as follows [48]:

$$|^{2S+1}L_J; J_3\rangle = \sum_{S_{l_3}, S_{h_3}, L_3, S_3} C_{S_{l_3} S_{h_3} S_3}^{S_l S_h S} C_{S_3 L_3 J_3}^{S L J} |S_{l_3}, S_{h_3}, L_3\rangle, \quad (A1)$$

where S_{h_3} , S_{l_3} , S_3 , L_3 , and J_3 are the projection quantum numbers corresponding to the quantum numbers S_h , S_l , S , L , and J , respectively. The state $|S_{h_3}, S_{l_3}, L_3\rangle$ denotes the product of the uncoupled states $|S_h S_{h_3}\rangle$, $|S_l S_{l_3}\rangle$, and $|L L_3\rangle$.

The spin-orbit interaction operator $\mathbf{L} \cdot \mathbf{S}_i$ (for $i = h$ or l) can be expressed in terms of ladder operators, as described in Ref. [81],

$$\mathbf{L} \cdot \mathbf{S}_i = \frac{1}{2}(L_+ S_{i-} + L_- S_{i+}) + L_3 S_{i3}, \quad (A2)$$

which is useful to calculate the expectation value of $\mathbf{L} \cdot \mathbf{S}_i$ by evaluating matrix elements between the $|^{2S+1}L_J\rangle$ basis.

Utilizing these expectation values of $\mathbf{L} \cdot \mathbf{S}_i$ in $|^{2S+1}L_J\rangle$ basis states, we can further calculate the expectation value of the tensor interaction operator using the following relation [48,81]:

$$\langle \hat{B} \rangle = -\frac{3}{(2L-1)(2L+3)} \left[(\mathbf{L} \cdot \mathbf{S}_h)(\mathbf{L} \cdot \mathbf{S}_l) + (\mathbf{L} \cdot \mathbf{S}_l)(\mathbf{L} \cdot \mathbf{S}_h) - \frac{2}{3}L(L+1)\mathbf{S}_h \cdot \mathbf{S}_l \right]. \quad (A3)$$

Additionally, the expectation value of the spin-spin operator in the L - S coupling scheme can be obtained using the identity $\mathbf{S} = \mathbf{S}_h + \mathbf{S}_l$ and $\mathbf{S}^2 = S(S+1)$, and one finds

$$\langle \mathbf{S}_h \cdot \mathbf{S}_l \rangle = \frac{1}{2}[S(S+1) - S_h(S_h+1) - S_l(S_l+1)]. \quad (A4)$$

1. Heavy-light mesons

a. The S -wave. There is no orbital excitation for $L = 0$ (S -wave). Only the spin-spin contact hyperfine interaction contributes to the first order. In this case, the total spin S of the meson is $S_h + S_l$, which can be 0 (singlet) or 1 (triplet). The spin-orbit and tensor terms vanish for $L = 0$. The expectation value of the spin-spin operator is the same in either coupling scheme (L - S or J - j) with value $\langle \mathbf{S}_h \cdot \mathbf{S}_l \rangle = -\frac{3}{4}$ for the spin-singlet ($S = 0$) state and $\langle \mathbf{S}_h \cdot \mathbf{S}_l \rangle = \frac{1}{4}$ for

the spin-triplet ($S = 1$) state. These reproduce the hyperfine splitting between, e.g., the D meson ($J^P = 0^-, S = 0$) and D^* meson ($J^P = 1^-, S = 1$).

b. The P-wave. For $L = 1$ (P-wave), the heavy quark ($S_h = \frac{1}{2}$) and light antiquark ($S_l = \frac{1}{2}$) spins can couple to total spin $S = 0$ or $S = 1$. These combine with $L = 1$ to yield total J quantum numbers $J = 0, 1, 2$ for the meson. In the L - S coupling basis, the corresponding states can be labeled as $|^1P_1\rangle$ (with $S = 0, J = 1$) and $|^3P_J\rangle$ (with $S = 1, J = 0, 1, 2$). In particular, there are two distinct $J = 1$ basis states, $|^1P_1\rangle$ and $|^3P_1\rangle$, which will mix under spin-dependent interactions, while $J = 0$ ($|^3P_0\rangle$) and $J = 2$ ($|^3P_2\rangle$) are each unique.

The L - S basis can be constructed in terms of $|S_{h3}, S_{l3}, L_3\rangle$ using Eq. (A1) as

$$|^3P_0; 0\rangle = \frac{1}{\sqrt{3}} \left| -\frac{1}{2}, -\frac{1}{2}, 1 \right\rangle - \frac{1}{\sqrt{6}} \left| \frac{1}{2}, -\frac{1}{2}, 0 \right\rangle - \frac{1}{\sqrt{6}} \left| -\frac{1}{2}, \frac{1}{2}, 0 \right\rangle + \frac{1}{\sqrt{3}} \left| \frac{1}{2}, \frac{1}{2}, -1 \right\rangle, \quad (\text{A5})$$

$$|^1P_1; 1\rangle = \frac{1}{\sqrt{2}} \left| \frac{1}{2}, -\frac{1}{2}, 1 \right\rangle - \frac{1}{\sqrt{2}} \left| -\frac{1}{2}, \frac{1}{2}, 1 \right\rangle, \quad (\text{A6})$$

$$|^3P_1; 1\rangle = -\frac{1}{2} \left| \frac{1}{2}, -\frac{1}{2}, 1 \right\rangle - \frac{1}{2} \left| -\frac{1}{2}, \frac{1}{2}, 1 \right\rangle + \frac{1}{\sqrt{2}} \left| \frac{1}{2}, \frac{1}{2}, 0 \right\rangle, \quad (\text{A7})$$

$$|^3P_2; 2\rangle = \left| \frac{1}{2}, \frac{1}{2}, 1 \right\rangle. \quad (\text{A8})$$

Using this L - S coupling basis, we compute the expectation values of the operators given in Eqs. (A2)–(A4) within the $[^2P_J, ^4P_J]$ basis for all allowed values of J , as listed below:

For $J = 0$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = -1, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = -1, \quad \langle \hat{\mathbf{B}} \rangle = -1, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{4}. \quad (\text{A9})$$

For $J = 1$,

$$\begin{aligned} \langle \mathbf{L} \cdot \mathbf{S}_l \rangle &= \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} \end{bmatrix}, & \langle \mathbf{L} \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} \end{bmatrix}, \\ \langle \hat{\mathbf{B}} \rangle &= \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, & \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} -\frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}. \end{aligned} \quad (\text{A10})$$

For $J = 2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = \frac{1}{2}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = \frac{1}{2}, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{1}{10}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{4}. \quad (\text{A11})$$

In the heavy-quark limit ($m_h \gg m_l$), the spin-orbit interaction term involving $\mathbf{L} \cdot \mathbf{S}_l$ dominates over the other interaction terms. Consequently, it is necessary to adopt a basis in which $\mathbf{L} \cdot \mathbf{S}_l$ is diagonal. This condition is satisfied in the $|J, j\rangle$ basis, corresponding to the j - j coupling scheme, where the orbital angular momentum $\mathbf{L}$ is first coupled with the light-quark spin $\mathbf{S}_l$ to form $j = L + S_l$. For $L = 1$ and $S_l = \frac{1}{2}$, the possible values of j are $\frac{1}{2}$ and $\frac{3}{2}$. In this limit, meson states can thus be classified by bases $|J, j\rangle$ in which the $\mathbf{L} \cdot \mathbf{S}_l$ matrix is diagonal, while the remaining interaction terms are treated as perturbations.

Diagonalizing the $\mathbf{L} \cdot \mathbf{S}_l$ operator within the $J = 1$ subspace, for example, yields two orthogonal linear combinations of $|^1P_1\rangle$ and $|^3P_1\rangle$ corresponding to definite j . The eigenvalues are $\lambda = -1$ for $j = \frac{1}{2}$ and $\lambda = +\frac{1}{2}$ for $j = \frac{3}{2}$ [these values come from $\frac{1}{2}j(j+1) - L(L+1) - S_l(S_l+1)$ with $L = 1, S_l = 1/2$]. The corresponding eigenstates can be written as

$$\lambda = -1: \left| J = 1, j = \frac{1}{2} \right\rangle = \frac{1}{\sqrt{3}} |^1P_1\rangle + \sqrt{\frac{2}{3}} |^3P_1\rangle, \quad (\text{A12})$$

$$\lambda = \frac{1}{2}: \left| J = 1, j = \frac{3}{2} \right\rangle = -\sqrt{\frac{2}{3}} |^1P_1\rangle + \frac{1}{\sqrt{3}} |^3P_1\rangle. \quad (\text{A13})$$

Similarly, for $J = 0$ we have only the $j = \frac{1}{2}$ state $|^3P_0\rangle$, and for $J = 2$ we have only the $j = \frac{3}{2}$ state $|^3P_2\rangle$. These are already eigenstates of $\mathbf{L} \cdot \mathbf{S}_l$ with eigenvalue $+\frac{1}{2}$,

$$\left| J = 0, j = \frac{1}{2} \right\rangle = |^3P_0\rangle, \quad (\text{A14})$$

$$\left| J = 2, j = \frac{3}{2} \right\rangle = |^3P_2\rangle. \quad (\text{A15})$$

Having established the $|J, j\rangle$ basis (in which $\mathbf{L} \cdot \mathbf{S}_l$ is diagonal), we can now evaluate the expectation values of the various mass-splitting operators. Table II summarizes the expectation values of $\mathbf{L} \cdot \mathbf{S}_h, \mathbf{L} \cdot \mathbf{S}_l, \hat{\mathbf{B}}$, and $\mathbf{S}_h \cdot \mathbf{S}_l$ in the j - j basis.

c. The D-wave. For $L = 2$ (D-wave), a similar analysis can be carried out. In the heavy-light meson case $S_h = S_l = \frac{1}{2}$, so again $S = 0$ or 1 . Coupling S with $L = 2$ yields possible J values $1, 2, 3$. The L - S basis states can be labeled as $|^1D_2\rangle$ ($S = 0, J = 2$) and $|^3D_J\rangle$ ($S = 1, J = 1, 2, 3$). Thus, $J = 2$ occurs for both $S = 0$ and 1 (two basis states $|^1D_2\rangle$ and $|^3D_2\rangle$ that can mix), while $J = 1$ ($|^3D_1\rangle$) and $J = 3$ ($|^3D_3\rangle$) are pure triplet states with no partner.

We begin by using Eq. (A1) to build the L - S basis, and the results are

$$|{}^3D_1; 1\rangle = \sqrt{\frac{3}{5}} \left| -\frac{1}{2}, -\frac{1}{2}, 2 \right\rangle - \frac{1}{2} \sqrt{\frac{3}{5}} \left| \frac{1}{2}, -\frac{1}{2}, 1 \right\rangle - \frac{1}{2} \sqrt{\frac{3}{5}} \left| -\frac{1}{2}, \frac{1}{2}, 1 \right\rangle + \frac{1}{\sqrt{10}} \left| \frac{1}{2}, \frac{1}{2}, 0 \right\rangle, \quad (\text{A16})$$

$$|{}^1D_2; 2\rangle = \frac{1}{\sqrt{2}} \left| \frac{1}{2}, -\frac{1}{2}, 2 \right\rangle - \frac{1}{\sqrt{2}} \left| -\frac{1}{2}, \frac{1}{2}, 2 \right\rangle, \quad (\text{A17})$$

$$|{}^3D_2; 2\rangle = -\frac{1}{\sqrt{3}} \left| \frac{1}{2}, -\frac{1}{2}, 2 \right\rangle - \frac{1}{\sqrt{3}} \left| -\frac{1}{2}, \frac{1}{2}, 2 \right\rangle + \frac{1}{\sqrt{3}} \left| \frac{1}{2}, \frac{1}{2}, 1 \right\rangle, \quad (\text{A18})$$

$$|{}^3D_3; 3\rangle = \left| \frac{1}{2}, \frac{1}{2}, 2 \right\rangle. \quad (\text{A19})$$

Using these L - S coupling bases, we compute the expectation values of the operators given in Eqs. (A2)–(A4) within the $[{}^2P_J, {}^4P_J]$ basis for all allowed values of J , as listed below.

Subsequently, we compute the expectation values of the mass-splitting operators within the L - S . These values, for specific values of J , are presented in the list below:

For $J = 1$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = -\frac{3}{2}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = -\frac{3}{2}, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{1}{2}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{4}. \quad (\text{A20})$$

For $J = 2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = \begin{bmatrix} 0 & -\sqrt{\frac{3}{2}} \\ -\sqrt{\frac{3}{2}} & -\frac{1}{2} \end{bmatrix}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = \begin{bmatrix} 0 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & -\frac{1}{2} \end{bmatrix},$$

$$\langle \hat{\mathbf{B}} \rangle = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \begin{bmatrix} -\frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}. \quad (\text{A21})$$

For $J = 3$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = 1, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = 1, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{1}{7}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{4}. \quad (\text{A22})$$

Following the same steps as the P-wave, we first diagonalize the dominant light-quark spin-orbit term $\mathbf{L} \cdot \mathbf{S}_l$. With $L = 2$ and $S_l = \frac{1}{2}$, the total light-quark angular momentum $j = L + S_l$ can be $j = \frac{3}{2}$ or $\frac{5}{2}$. The eigenvalues of $\mathbf{L} \cdot \mathbf{S}_l$ are $\lambda = -\frac{3}{2}$ for $j = \frac{3}{2}$ and $\lambda = +1$ for $j = \frac{5}{2}$. In the

$J = 2$ subspace, one finds the corresponding eigenvectors as mixtures of $|{}^1D_2\rangle$ and $|{}^3D_2\rangle$,

$$\lambda = -\frac{3}{2}: \left| J=2, j=\frac{3}{2} \right\rangle = \sqrt{\frac{2}{5}} |{}^1D_2\rangle + \sqrt{\frac{3}{5}} |{}^3D_2\rangle, \quad (\text{A23})$$

$$\lambda = 1: \left| J=2, j=\frac{5}{2} \right\rangle = -\sqrt{\frac{3}{5}} |{}^1D_2\rangle + \sqrt{\frac{2}{5}} |{}^3D_2\rangle. \quad (\text{A24})$$

For $J = 1$ and $J = 3$, only the $|{}^3D_1\rangle$ and $|{}^3D_3\rangle$ states (with $j = \frac{3}{2}$ and $j = \frac{5}{2}$, respectively) exist, which are already eigenstates of $\mathbf{L} \cdot \mathbf{S}_l$ with $\lambda = -\frac{3}{2}$ for $J = 1$ and $\lambda = +1$ for $J = 3$. Hence, the following relations hold:

$$\left| J=1, j=\frac{3}{2} \right\rangle = |{}^3D_1\rangle, \quad (\text{A25})$$

$$\left| J=3, j=\frac{5}{2} \right\rangle = |{}^3D_3\rangle. \quad (\text{A26})$$

Using these $|j, J\rangle$ bases, the expectation values of the operators in the D-wave can be computed in a similar manner. These computed values then contribute to the spin-dependent splitting of the heavy-light meson spectra of the D-wave.

2. Doubly heavy baryons

a. The S-wave. We now turn to doubly heavy baryons containing a heavy QQ' diquark and a light quark. We take the diquark to be in the spin-1 (vector) configuration, $S_h = 1$, as appropriate for a symmetric QQ' state in a color $\bar{\mathbf{3}}$ (antisymmetric) combination. The light quark has $S_l = \frac{1}{2}$. For the ground-state baryons with $L = 0$, only the spin-spin contact interaction between the heavy diquark and light quark is relevant. Analogous to Eq. (A4), one finds

$$\langle \mathbf{S}_h \cdot \mathbf{S}_l \rangle = \frac{1}{2} [S(S+1) - S_h(S_h+1) - S_l(S_l+1)],$$

with $S = |S_h - S_l|, \dots, S_h + S_l$. For $S_h = 1$ and $S_l = \frac{1}{2}$, the possible total spin of the baryon's two-body subsystem is $S = \frac{1}{2}$ or $\frac{3}{2}$. In the $L = 0$ ground state, the light quark and diquark are predominantly in the relative S-wave, so the total J^P of the baryon is $J^P = S^+$. The spin-spin operator $\langle \mathbf{S}_h \cdot \mathbf{S}_l \rangle$ yields -1 for the $S = \frac{1}{2}$ configuration and $+\frac{1}{2}$ for $S = \frac{3}{2}$, indicating that the heavier-spin state is elevated.

b. The P-wave. For $L = 1$ orbital excitation of a doubly heavy baryon, we have three angular momenta: the orbital $L = 1$, the heavy diquark spin $S_h = 1$, and the light-quark spin $S_l = \frac{1}{2}$. In the L - S coupling scheme, S_h and S_l first combine to total spin S , which can be $\frac{1}{2}$ or $\frac{3}{2}$. Coupling S with $L = 1$ yields the baryon total J . The L - S basis states

can be denoted $|^2P_J\rangle$ for $S = \frac{1}{2}$ and $|^4P_J\rangle$ for $S = \frac{3}{2}$, with J taking the values $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. They can be constructed in terms of $|S_{h3}, S_{l3}, L_3\rangle$ using Eq. (A1) as

$$|^2P_{1/2}; 1/2\rangle = \frac{\sqrt{2}}{3} \left| -\frac{1}{2}, 0, 1 \right\rangle - \frac{\sqrt{2}}{3} \left| -\frac{1}{2}, 1, 0 \right\rangle - \frac{2}{3} \left| \frac{1}{2}, -1, 1 \right\rangle + \frac{1}{3} \left| \frac{1}{2}, 0, 0 \right\rangle, \quad (\text{A27})$$

$$|^4P_{1/2}; 1/2\rangle = \frac{1}{3} \left| -\frac{1}{2}, 0, 1 \right\rangle - \frac{1}{3} \left| -\frac{1}{2}, 1, 0 \right\rangle + \frac{1}{3\sqrt{2}} \left| \frac{1}{2}, -1, 1 \right\rangle - \frac{\sqrt{2}}{3} \left| \frac{1}{2}, 0, 0 \right\rangle + \frac{1}{\sqrt{2}} \left| \frac{1}{2}, 1, -1 \right\rangle, \quad (\text{A28})$$

$$|^2P_{3/2}; 3/2\rangle = -\sqrt{\frac{2}{3}} \left| -\frac{1}{2}, 1, 1 \right\rangle + \frac{1}{\sqrt{3}} \left| \frac{1}{2}, 0, 1 \right\rangle, \quad (\text{A29})$$

$$|^4P_{3/2}; 3/2\rangle = -\sqrt{\frac{2}{15}} \left| -\frac{1}{2}, 1, 1 \right\rangle - \frac{2}{\sqrt{15}} \left| \frac{1}{2}, 0, 1 \right\rangle + \sqrt{\frac{3}{5}} \left| \frac{1}{2}, 1, 0 \right\rangle, \quad (\text{A30})$$

$$|^4P_{5/2}; 5/2\rangle = \left| \frac{1}{2}, 1, 1 \right\rangle. \quad (\text{A31})$$

In particular, there are two $J = \frac{1}{2}$ states ($|^2P_{\frac{1}{2}}\rangle$ and $|^4P_{\frac{1}{2}}\rangle$) and two $J = \frac{3}{2}$ states ($|^2P_{\frac{3}{2}}\rangle$ and $|^4P_{\frac{3}{2}}\rangle$) that can mix, while $J = \frac{5}{2}$ has only the $|^4P_{\frac{5}{2}}\rangle$ state (with $S = \frac{3}{2}$).

The expressions for spin-dependent operators [Eqs. (A2)–(A4)] are analogous to the mesonic case, with the only difference being that the $\mathbf{S}_h$ now represents the spin of the heavy diquark and the $\mathbf{S}_l$ represents the spin of the light quark.

Using these expressions, we can calculate the expectation value for the spin-dependent operators within the $[^2P_J, ^4P_J]$ basis for all allowed values of J , as listed below: For $J = 1/2$,

$$\begin{aligned} \langle \mathbf{L} \cdot \mathbf{S}_l \rangle &= \begin{bmatrix} \frac{1}{3} & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & -\frac{5}{6} \end{bmatrix}, & \langle \mathbf{L} \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} -\frac{4}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & -\frac{5}{3} \end{bmatrix}, \\ \langle \hat{\mathbf{B}} \rangle &= \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -1 \end{bmatrix}, & \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \end{aligned} \quad (\text{A32})$$

For $J = 3/2$,

$$\begin{aligned} \langle \mathbf{L} \cdot \mathbf{S}_l \rangle &= \begin{bmatrix} -\frac{1}{6} & -\frac{\sqrt{5}}{3} \\ -\frac{\sqrt{5}}{3} & -\frac{1}{3} \end{bmatrix}, & \langle \mathbf{L} \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} \frac{2}{3} & \frac{\sqrt{5}}{3} \\ \frac{\sqrt{5}}{3} & -\frac{2}{3} \end{bmatrix}, \\ \langle \hat{\mathbf{B}} \rangle &= \begin{bmatrix} 0 & \frac{1}{2\sqrt{5}} \\ \frac{1}{2\sqrt{5}} & \frac{4}{5} \end{bmatrix}, & \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \end{aligned} \quad (\text{A33})$$

For $J = 5/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = \frac{1}{2}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = 1, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{1}{5}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{2}. \quad (\text{A34})$$

The heavy QQ' diquark, which is much more massive, is analogous to a heavy antiquark in a meson. Thus, in the heavy-diquark limit ($M_{QQ} \gg m_q$), the $\mathbf{L} \cdot \mathbf{S}_l$ term associated with the light quark dominates the spin-orbit splitting. We, therefore, employ a J - j coupling scheme where we first couple $\mathbf{L}$ with $\mathbf{S}_l$ to form $j = L + S_l$. For $L = 1$ and $S_l = \frac{1}{2}$, one gets $j = \frac{1}{2}$ or $\frac{3}{2}$. The baryon states can then be labeled $|J, j\rangle$, and one can treat the remaining $\mathbf{L} \cdot \mathbf{S}_h$ and tensor interactions (involving the heavy diquark's spin) as perturbations. Diagonalizing $\mathbf{L} \cdot \mathbf{S}_l$ within a given J subspace yields eigenvalues $\lambda = -1$ for $j = \frac{1}{2}$ and $\lambda = +\frac{1}{2}$ for $j = \frac{3}{2}$. For example, the two physical $J = \frac{1}{2}$ states can be expressed as

$$\lambda = -1: \left| J = \frac{1}{2}, j = 1/2 \right\rangle = \frac{1}{3} |^2P_{1/2}\rangle + \frac{2\sqrt{2}}{3} |^4P_{1/2}\rangle, \quad (\text{A35})$$

$$\lambda = \frac{1}{2}: \left| J = \frac{1}{2}, j = 3/2 \right\rangle = -\frac{2\sqrt{2}}{3} |^2P_{1/2}\rangle + \frac{1}{3} |^4P_{1/2}\rangle, \quad (\text{A36})$$

in terms of the L - S basis $\{|^2P_{1/2}\rangle, |^4P_{1/2}\rangle\}$. These correspond to $j = \frac{1}{2}$ and $j = \frac{3}{2}$ eigenstates of $\mathbf{L} \cdot \mathbf{S}_l$, with eigenvalues -1 and $+\frac{1}{2}$, respectively.

Likewise, for the $J = \frac{3}{2}$ sector we get

$$\lambda = -1: \left| J = \frac{3}{2}, j = 1/2 \right\rangle = \frac{2}{3} |^2P_{3/2}\rangle + \frac{\sqrt{5}}{3} |^4P_{3/2}\rangle, \quad (\text{A37})$$

$$\lambda = \frac{1}{2}: \left| J = \frac{3}{2}, j = 3/2 \right\rangle = -\frac{\sqrt{5}}{3} |^2P_{3/2}\rangle + \frac{2}{3} |^4P_{3/2}\rangle, \quad (\text{A38})$$

while the $J = \frac{5}{2}$ state remains purely $|^4P_{5/2}\rangle$ with $j = \frac{3}{2}$,

$$\left| J = \frac{5}{2}, j = 3/2 \right\rangle = |^4P_{5/2}\rangle. \quad (\text{A39})$$

Using these states, we determine the expectation values of the operators in the $J, j\rangle$ basis and list the results in Table III, which can be used to calculate the P-wave mass splittings.

c. The D-wave. Finally, we consider the $L = 2$ (D-wave) excitations of doubly heavy baryons. The heavy diquark has $S_h = 1$ and the light quark $S_l = \frac{1}{2}$, as before. In the L - S scheme, these combine to $S = \frac{1}{2}$ or $\frac{3}{2}$, which then couple with $L = 2$ to yield possible total J values $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$. The corresponding L - S basis for D -wave states can be formed using Eq. (A1) as

$$|^4D_{1/2}; 1/2\rangle = -\sqrt{\frac{2}{5}} \left| -\frac{1}{2}, -1, 2 \right\rangle + \frac{1}{\sqrt{5}} \left| -\frac{1}{2}, 0, 1 \right\rangle - \frac{1}{\sqrt{15}} \left| -\frac{1}{2}, 1, 0 \right\rangle + \frac{1}{\sqrt{10}} \left| \frac{1}{2}, -1, 1 \right\rangle - \sqrt{\frac{2}{15}} \left| \frac{1}{2}, 0, 0 \right\rangle + \frac{1}{\sqrt{10}} \left| \frac{1}{2}, 1, -1 \right\rangle, \quad (\text{A40})$$

$$|^2D_{3/2}; 3/2\rangle = \frac{2}{\sqrt{15}} \left| -\frac{1}{2}, 0, 2 \right\rangle - \sqrt{\frac{2}{15}} \left| -\frac{1}{2}, 1, 1 \right\rangle - 2\sqrt{\frac{2}{15}} \left| \frac{1}{2}, -1, 2 \right\rangle + \frac{1}{\sqrt{15}} \left| \frac{1}{2}, 0, 1 \right\rangle, \quad (\text{A41})$$

$$|^4D_{3/2}; 3/2\rangle = \frac{2}{\sqrt{15}} \left| -\frac{1}{2}, 0, 2 \right\rangle - \sqrt{\frac{2}{15}} \left| -\frac{1}{2}, 1, 1 \right\rangle + \sqrt{\frac{2}{15}} \left| \frac{1}{2}, -1, 2 \right\rangle - \frac{2}{\sqrt{15}} \left| \frac{1}{2}, 0, 1 \right\rangle + \frac{1}{\sqrt{5}} \left| \frac{1}{2}, 1, 0 \right\rangle, \quad (\text{A42})$$

$$|^2D_{5/2}; 5/2\rangle = -\sqrt{\frac{2}{3}} \left| -\frac{1}{2}, 1, 2 \right\rangle + \frac{1}{\sqrt{3}} \left| \frac{1}{2}, 0, 2 \right\rangle, \quad (\text{A43})$$

$$|^4D_{5/2}; 5/2\rangle = -\frac{2}{\sqrt{21}} \left| -\frac{1}{2}, 1, 2 \right\rangle - 2\sqrt{\frac{2}{21}} \left| \frac{1}{2}, 0, 2 \right\rangle + \sqrt{\frac{3}{7}} \left| \frac{1}{2}, 1, 1 \right\rangle, \quad (\text{A44})$$

$$|^4D_{7/2}; 7/2\rangle = \left| \frac{1}{2}, 1, 2 \right\rangle. \quad (\text{A45})$$

Then, the expectations of the mass-splitting operators given in Eqs. (A2)–(A4), in the basis $[|^2D_J\rangle, |^4D_J\rangle]$, are evaluated, and the results are as follows:

For $J = 1/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = -\frac{3}{2}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = -3, \quad \langle \hat{\mathbf{B}} \rangle = -1, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{2}. \quad (\text{A46})$$

For $J = 3/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = \begin{bmatrix} \frac{1}{2} & -1 \\ -1 & -1 \end{bmatrix}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}, \quad \langle \hat{\mathbf{B}} \rangle = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \quad (\text{A47})$$

For $J = 5/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = \begin{bmatrix} -\frac{1}{3} & -\frac{\sqrt{14}}{3} \\ -\frac{\sqrt{14}}{3} & -\frac{1}{6} \end{bmatrix}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = \begin{bmatrix} \frac{4}{3} & \frac{\sqrt{14}}{3} \\ \frac{\sqrt{14}}{3} & -\frac{1}{3} \end{bmatrix}, \quad \langle \hat{\mathbf{B}} \rangle = \begin{bmatrix} 0 & \frac{1}{\sqrt{14}} \\ \frac{1}{\sqrt{14}} & \frac{5}{7} \end{bmatrix}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \quad (\text{A48})$$

For $J = 7/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = 1, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = 2, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{2}{7}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{2}. \quad (\text{A49})$$

As before, we now switch to the J - j coupling by diagonalizing the $\mathbf{L} \cdot \mathbf{S}_l$ matrix. This yields $j = \frac{3}{2}$ (with $\lambda = -\frac{3}{2}$) and $j = \frac{5}{2}$ (with $\lambda = +1$) as the light-quark total angular momenta. Consequently, the two $J = \frac{3}{2}$ basis states ($|^2D_{3/2}\rangle$ and $|^4D_{3/2}\rangle$) mix to form $|J = \frac{3}{2}, j = \frac{3}{2}\rangle$ and $|J = \frac{3}{2}, j = \frac{5}{2}\rangle$ eigenstates of $\mathbf{L} \cdot \mathbf{S}_l$,

$$\lambda = -\frac{3}{2}: \left| J = \frac{3}{2}, j = 3/2 \right\rangle = \frac{1}{\sqrt{5}}|^2D_{3/2}\rangle + \frac{2}{\sqrt{5}}|^4D_{3/2}\rangle, \quad (\text{A50})$$

$$\lambda = 1: \left| J = \frac{3}{2}, j = 5/2 \right\rangle = -\frac{2}{\sqrt{5}}|^2D_{3/2}\rangle + \frac{1}{\sqrt{5}}|^4D_{3/2}\rangle. \quad (\text{A51})$$

Similarly, the $J = \frac{5}{2}$ basis states ($|^2D_{5/2}\rangle$ and $|^4D_{5/2}\rangle$) mix to form $|J = \frac{5}{2}, j = \frac{3}{2}\rangle$ and $|J = \frac{5}{2}, j = \frac{5}{2}\rangle$ eigenstates of $\mathbf{L} \cdot \mathbf{S}_l$, as below:

$$\lambda = -\frac{3}{2}: \left| J = \frac{5}{2}, j = 3/2 \right\rangle = 2\sqrt{\frac{2}{15}}|^2D_{5/2}\rangle + \sqrt{\frac{7}{15}}|^4D_{5/2}\rangle, \quad (\text{A52})$$

$$\lambda = 1: \left| J = \frac{5}{2}, j = 5/2 \right\rangle = -\sqrt{\frac{7}{15}}|^2D_{5/2}\rangle + 2\sqrt{\frac{2}{15}}|^4D_{5/2}\rangle. \quad (\text{A53})$$

The extreme J values, $J = \frac{1}{2}$ and $J = \frac{7}{2}$, correspond uniquely to $|^4D_{1/2}\rangle$ and $|^4D_{7/2}\rangle$,

$$\left| J = \frac{1}{2}, j = 3/2 \right\rangle = |^4D_{1/2}\rangle, \quad (\text{A54})$$

$$\left| J = \frac{7}{2}, j = 5/2 \right\rangle = |^4D_{7/2}\rangle. \quad (\text{A55})$$

Ultimately, we calculate the expectation values of the mass-splitting operators within the $|J, j\rangle$ basis and compile the findings in Table III for detailed analysis and reference.

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