

Quantization of black hole entropy for black holes in subtracted geometry

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We carefully examine the exact analytic spectrum of quasinormal modes of general black holes in the so-called subtracted geometry of maximally supersymmetric supergravity. These black holes have the same area and surface gravity at both the outer and inner horizons as asymptotically flat Kerr black holes. We proceed to explore the relationship with conformal field theories that describe horizon physics of these black holes. As a consequence, we show that it is consistent to view the horizon area of these black holes as being quantized in units of $8\pi l_{\text{Pl}}^2$.

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Fifty years ago, Hawking [1] showed that the entropy of a stationary black hole is determined solely by the area A of its event horizon. To be precise,

$$S = \frac{kA}{4l_{\text{Pl}}^2}, \quad (1)$$

where l_{Pl} is the Planck length defined by $l_{\text{Pl}}^2 = G\hbar/c^3$. Such a formula had previously been conjectured by Bekenstein [2,3] although he was not able to find the constant of proportionality in (1). That this is a nonperturbative quantum gravity result can be seen from the appearance of a factor of l_{Pl}^2 in the denominator of (1). The Boltzmann view of entropy [4] is that it is the logarithm of the density of states of a system. Taking this to be universally true, (1) should yield information about the fundamental nature of both black holes and quantum gravity. The observation that the horizon area determines the entropy led to the idea that the quantum information about the state of a black hole is stored, perhaps holographically, in pixels on the horizon, [5–7]. In [8–11], it was speculated that these pixels should have an area

proportional to l_{Pl}^2 . For the Schwarzschild metric, some evidence in favor of this conjecture was found by Maggiore, [12]. He studied the quasinormal modes of the Schwarzschild metric and concluded that pixels had a fundamental area of $8\pi l_{\text{Pl}}^2$. Subsequently, Medved [13] studied the Kerr metric, but there the situation was less clear. One therefore wonders if it is possible to find more evidence in favor of this kind of pixelation. The problem is that for Kerr black holes, it is not possible to carry out exact calculations of the quasinormal modes owing to the well-known complexity of the Teukolsky equations. Furthermore, it is not clear what result one should be looking for.

We study the quasinormal modes of black holes in subtracted geometry. Black holes in subtracted geometry are in many ways similar to Kerr black holes. However, they have the clear advantage that the analogs of the Teukolsky equations can be solved exactly, which leads to a complete understanding of the quasinormal modes. Our analysis reveals that it is consistent with the spectrum of quasinormal modes to suppose that the area of the event horizon is quantized in units of $8\pi l_{\text{Pl}}^2$. Similar results can be obtained for the Kerr metric in the regime where the wavelength of excitations is large compared to the mass of the black hole. This is the limit in which the hidden conformal symmetry becomes manifest, [14]. Perhaps this is an indication that the kind of simple picture of area quantization that we find is not universal but rather holds only in a particular limit. Subtracted geometry arises in a particular supergravity theory and should be viewed as a

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toy model amenable to exact calculation. It would be satisfying to show that these considerations allow us to find all of the states of a black hole. However, this is not possible with the current techniques to investigate quantum gravity.

From now on, we will use natural units such that $G = c = \hbar = k = 1$. One way of exploring the spectrum of black hole states is through resonance scattering. In many ways, one can view a black hole as a gravitational analog of the atomic nucleus. In nuclear physics, information about nuclear structure can be obtained through resonance scattering. It is therefore plausible to suppose the same is true for black holes. Resonances can be found by looking at solutions of the differential equation that governs perturbations of the black hole. Physically, one is interested in waves that are ingoing at infinity and bounded on the outer horizon. It is these waves that will put the black hole into some excited state which will then decay. Waves obeying these boundary conditions are termed quasinormal modes. In general, they will form a discrete set with time dependence of the form $e^{-i\omega t}$ with ω lying in the lower half of the complex plane and where ω determines both the energy of the excited state and the timescale for its decay. Ideally, one would like a complete analysis of the quasinormal modes of the Kerr black hole. Kerr quasinormal modes have been studied extensively but are only known numerically at present. However, there is a family of black holes that in many ways resemble the Kerr black hole. These black holes are however not solutions of the vacuum field equations for general relativity but rather solutions in $N = 2$ supergravity to which has been added three vector supermultiplets. In this theory, there are black holes that, rather than living in an asymptotically flat spacetime, live in an asymptotically conical spacetime. Thus the environment of these black holes is different to those described by the Kerr metric. The spacetime of these black holes is conventionally referred to as subtracted geometry. These spacetimes were introduced in [15] and further analyzed in [16–20]. For black holes in subtracted geometry, all the quasinormal modes can be found analytically which allows us to explore the excited states these black holes. We then relate the spectrum of quasinormal modes for black holes in subtracted geometry to the conformal field theory (CFT) description of horizon physics. The properties of a black hole should not depend on its environment and so we hope our results will have general applicability.

I. SUBTRACTED GEOMETRY

The metric of our black holes in Boyer-Lindquist type coordinates is

$$ds^2 = -\Delta^{-\frac{1}{2}}\mathcal{G}(dt + \mathcal{A})^2 + \Delta^{\frac{1}{2}}\left(\frac{dr^2}{X} + d\theta^2 + \frac{X}{\mathcal{G}}\sin^2\theta d\phi^2\right), \quad (2)$$

where

$$\begin{aligned} \Delta &= 4m^2(2mr(\Pi_c^2 - \Pi_s^2) + 4m^2\Pi_s^2 - a^2(\Pi_c - \Pi_s)^2\cos^2\theta), \\ \mathcal{G} &= r^2 - 2mr + a^2\cos^2\theta, \quad X = r^2 - 2mr + a^2, \\ \mathcal{A} &= \frac{2mas\sin^2\theta}{\mathcal{G}}[(\Pi_c - \Pi_s)r + 2m\Pi_s]d\phi, \end{aligned}$$

and depends on four parameters, m, a, Π_c , and Π_s .¹

To be a nonextremal black hole requires $m > |a| \geq 0$ and $\Pi_c \geq \Pi_s \geq 0$. The subtracted Kerr geometry corresponds to taking $\Pi_c = 1$ and $\Pi_s = 0$ in the expressions above. The geometry of the Kerr metric differs from the subtracted geometry only by the simple replacement of Δ by Δ_{Kerr} with

$$\Delta_{\text{Kerr}} = (r^2 + a^2\cos^2\theta)^2.$$

Just like the Kerr metric, these black holes are stationary and axisymmetric, that is to say independent of both t and ϕ . Also, just as in Kerr, the zeros of X at

$$r_{\pm} = m \pm \sqrt{m^2 - a^2},$$

are the inner and outer horizons with the same value of the area and surface gravity as for the Kerr black hole.

II. THERMODYNAMICS

The subtracted black holes are completely characterized by a set of charges determined by the parameters of the solution. We give explicit formulas for these charges [20]; the black hole mass M is

$$M = \frac{1}{4}m(\Pi_c^2 + \Pi_s^2) - \frac{a^2}{8m}(\Pi_c - \Pi_s)^2,$$

its angular momentum J is

$$J = -ma(\Pi_c - \Pi_s),$$

and in addition the gauge fields yield both an electric charge $Q^{(e)}$ and a magnetic charge $Q^{(m)}$

¹Note that the corresponding asymptotically flat black holes, first found in [21,22], are parametrized by six parameters, m, a , and δ_i with $i = \{1, 2, 3, 4\}$ which specify the mass $M = \frac{m}{4}\sum_{i=1}^4 \cosh 2\delta_i$, angular momentum $J = -ma(\Pi_c - \Pi_s)$ and four charges $Q_i = \frac{m}{4}\sinh 2\delta_i$. Here $\Pi_c = \Pi_1^4 \cosh \delta_i$ and $\Pi_s = \Pi_1^4 \sinh \delta_i$. The Kerr black hole corresponds to $\delta_i = 0$ and thus $\Pi_c = 1$ and $\Pi_s = 0$. The Kerr-Newman black hole to $\delta_i = \delta$ and $\Pi_c = \cosh^4 \delta$ and $\Pi_s = \sinh^4 \delta$. The geometry of asymptotically flat black holes differs from the subtracted geometry (2) only in that it has a more complicated warp-factor Δ . The asymptotically flat black holes therefore have the same area and surface gravity on both the inner and outer horizons as those in subtracted geometry.

$$Q^{(e)} = \frac{2m^2 \Pi_c \Pi_s + a^2 (\Pi_c - \Pi_s)^2}{4m}, \quad Q^{(m)} = \frac{3m}{2}.$$

The entropy $S_{\pm}$ of the outer and inner horizons is given by the area formula (1). Thus,

$$S_{\pm} = 2\pi m (\Pi_c r_{\pm} + \Pi_s r_{\mp}).$$

Knowledge of the entropy motivates a complete exploration of the black hole thermodynamics. The temperatures of the outer and inner horizons are determined, as usual, by the surface gravity on the horizons and are given by

$$T_{\pm} = \pm \frac{(\Pi_c^2 - \Pi_s^2) \sqrt{m^2 - a^2}}{4\pi m (\Pi_c r_{\pm} + \Pi_s r_{\mp})}.$$

It should be noted that $T_- < 0$ which is perhaps a bit surprising.²

However, this comes about because $\frac{\partial S}{\partial M} < 0$. An explanation of why this is not paradoxical can be found in [23,24]. On each of the two horizons, there are potentials conjugate to J , $Q^{(e)}$, and $Q^{(m)}$. The angular velocities $\Omega_{\pm}$ are

$$\Omega_{\pm} = \frac{a}{4m^2} \frac{(\Pi_c - \Pi_s)(\Pi_c r_{\mp} + \Pi_s r_{\pm})}{\Pi_c r_{\pm} + \Pi_s r_{\mp}}$$

and electric and magnetic potentials on the outer and inner horizons are

$$\Phi_{\pm}^{(e)} = \frac{\Pi_c r_{\mp} + \Pi_s r_{\pm}}{\Pi_c r_{\pm} + \Pi_s r_{\mp}}$$

and

$$\begin{aligned} \Phi_{\pm}^{(m)} &= \frac{\Pi_c - \Pi_s}{8m^2} \frac{1}{(\Pi_c r_{\pm} + \Pi_s r_{\mp})} \\ &\times \left((\Pi_c^2 - \Pi_s^2)(-4m^3 + 6ma^2) \right. \\ &\mp (\Pi_c^2 + \Pi_s^2) \sqrt{m^2 - a^2} (4m^2 + 2a^2) \\ &\left. \mp \Pi_c \Pi_s \sqrt{m^2 - a^2} (8m^2 - 4a^2) \right). \end{aligned}$$

From these thermodynamic relationships, we can deduce that both the Smarr relations,

$$M = 2T_{\pm} S_{\pm} + 2\Omega_{\pm} J + \Phi_{\pm}^{(e)} Q^{(e)} + \Phi_{\pm}^{(m)} Q^{(m)}, \quad (3)$$

and the first law of thermodynamics

$$dM = T_{\pm} dS_{\pm} + \Omega_{\pm} dJ + \Phi_{\pm}^{(e)} dQ^{(e)} + \Phi_{\pm}^{(m)} dQ^{(m)}, \quad (4)$$

holds for subtracted black holes.

²Again, the entropy and temperature formulas are the same as those for the corresponding asymptotically flat black holes. The negative temperature for these geometries was first derived in [23].

It has been widely conjectured that this data encodes the thermodynamics of a pair of two-dimensional conformal field theories, CFT_L and CFT_R , [14,15,25–30]. The relevant thermodynamic quantities are

$$\begin{aligned} T_L &= \frac{T_+ T_-}{T_+ + T_-} = \frac{\Pi_c + \Pi_s}{8\pi m}, \\ T_R &= \frac{T_- T_+}{T_- - T_+} = \frac{(\Pi_c - \Pi_s) \sqrt{m^2 - a^2}}{8\pi m^2}, \\ S_L &= \frac{1}{2} (S_+ + S_-) = 2\pi m^2 (\Pi_c + \Pi_s), \\ S_R &= \frac{1}{2} (S_+ - S_-) = 2\pi m \sqrt{m^2 - a^2} (\Pi_c - \Pi_s). \end{aligned}$$

It should be noted that unlike the horizon temperatures $T_{\pm}$, neither T_L nor T_R originate from any geometric periodicity coming from Euclideanization of the spacetime [31].

The angular momentum and charges are universal but the corresponding potentials for CFT_L are

$$\begin{aligned} \Omega_L &= -\frac{a}{8m^2} (\Pi_c - \Pi_s), \quad \Phi_L^{(e)} = -\frac{1}{2}, \\ \Phi_L^{(m)} &= -\frac{1}{4} (\Pi_c + \Pi_s)^2 - \frac{a^2}{8m^2} (\Pi_c - \Pi_s)^2, \end{aligned}$$

and for CFT_R

$$\begin{aligned} \Omega_R &= \frac{a(\Pi_c - \Pi_s)}{8m^2}, \quad \Phi_R^{(e)} = \frac{1}{2}, \\ \Phi_R^{(m)} &= (\Pi_c - \Pi_s)^2 \left(-\frac{1}{4} + \frac{3a^2}{8m^2} \right). \end{aligned}$$

Similarly to the horizons, there are Smarr-type formulas and first laws for both CFT_L and CFT_R . Let the energy of these CFTs be E_L and E_R . Then³

$$E_{L,R} = 2T_{L,R} S_{L,R} + 2\Omega_{L,R} J + \Phi_{L,R}^{(e)} Q^{(e)} + \Phi_{L,R}^{(m)} Q^{(m)}, \quad (5)$$

and

$$dE_{L,R} = T_{L,R} dS_{L,R} + \Omega_{L,R} dJ + \Phi_{L,R}^{(e)} dQ^{(e)} + \Phi_{L,R}^{(m)} dQ^{(m)}, \quad (6)$$

for both CFT_L and CFT_R . $E_L = E_R = M/2$. These formulas relate to the changes in these holographic CFTs in response to dynamical change in the black hole geometry.

³It turns out that both the Smarr formula and the first law of thermodynamics for the asymptotically flat black holes can also be cast in a similar form for both of the L and R sectors where now M is the mass of the asymptotically flat black hole and each of the four charges Q_i is accompanied by its corresponding gauge potential.

III. QUASINORMAL MODES

The wave equation for minimally coupled massless scalars that carry no charge has solutions χ of the form

$$\chi \sim e^{-i\omega t + ik\phi} P_l^k(\cos\theta) R_{lk\omega}(r).$$

For the Kerr metric, both the polar functions and radial functions are confluent Heun functions which lead to complications in their evaluation. [32–47]. For subtracted geometry, the polar functions are just the associated Legendre polynomials P_l^k and the radial functions are hypergeometric functions. The radial equation has two independent solutions. One is

$$R_{lk\omega}(r) \sim (r - r_-)^p (r - r_+)^q F\left(a, b; c; \frac{r - r_+}{r - r_-}\right),$$

where

$$p = i \frac{\omega - k\Omega_+}{4\pi T_+} - (l + 1) \quad q = -i \frac{\omega - k\Omega_+}{4\pi T_+},$$

$$a = 1 + l - i \frac{\omega}{4\pi T_R} + i \frac{k\Omega_+}{2\pi T_+}, \quad b = 1 + l - i \frac{\omega}{4\pi T_L},$$

and

$$c = 1 - i \frac{\omega}{2\pi T_+} + i \frac{k\Omega_+}{2\pi T_+},$$

and which represents waves that are ingoing on the future outer horizon. The second solution represents waves that are outgoing on the past outer horizon and are therefore not relevant to the present discussion [17,18].

To find the form of $R_{lk\omega}(r)$ for large r , we can use the connection formula,

$$\begin{aligned} & F\left(a, b; c; \frac{r - r_+}{r - r_-}\right) \\ &= \frac{\Gamma(c)\Gamma(c - a - b)}{\Gamma(c - a)\Gamma(c - b)} F\left(a, b; a + b - c + 1; \frac{r_+ - r_-}{r - r_-}\right) \\ &+ \left\{ \left(\frac{r_+ - r_-}{r - r_-} \right)^{c-a-b} \frac{\Gamma(c)\Gamma(a + b - c)}{\Gamma(a)\Gamma(b)} \right. \\ &\quad \left. \times F\left(c - a, c - b; c - a - b + 1; \frac{r_+ - r_-}{r - r_-}\right) \right\}. \end{aligned} \quad (7)$$

As $r \rightarrow \infty$ the first term in (7) results in a contribution to $R_{lk\omega}(r)$ that scales like r^{-l-1} whereas the second term scales like r^l . Requiring $R_{lk\omega}(r)$ to be bounded as $r \rightarrow \infty$ requires the second term to have vanishing coefficient. It is this condition that determines the complex frequencies of the quasinormal modes. Since the Γ -function has poles at negative integers and zero, we require either a or b to be a negative integer or zero. There are then two families of

quasinormal modes remarkably corresponding precisely to the two CFTs. We term these two families of quasinormal modes the L family and the R family. For the left family, we find⁴

$$\frac{\omega_L}{T_L} = -4\pi i(n + l), \quad (8)$$

and for the right

$$\frac{\omega_R}{T_R} = -4\pi i(n + l) + \frac{k\Omega_+}{T_+}, \quad (9)$$

where $n \in \mathbb{Z}^+$. Note that in both cases, the imaginary part of ω is negative as one would expect for a mode that is damped. It represents the width or lifetime of the state that is being excited.

The cross section for absorption by the black hole contains a factor of

$$|\Gamma(a)\Gamma(b)|^2. \quad (10)$$

This last term should be compared to a corresponding term in the absorption cross section for a $\text{CFT}_L \otimes \text{CFT}_R$ where the waves have frequencies ω_L and ω_R and the CFT's have temperatures of T_L and T_R ,

$$\left| \Gamma\left(1 + \frac{i\omega_L}{4\pi T_L}\right) \Gamma\left(1 + \frac{i\omega_R}{4\pi T_R}\right) \right|^2. \quad (11)$$

We see now that there is a precise match up between the absorption cross section of the black hole and the absorption cross section of a pair of conformal field theories at temperatures T_L and T_R provided that no angular momentum is added so that $k = 0$. Our relations show that at fixed $J, Q^{(e)}$, and $Q^{(m)}$ the energy levels of the black hole, or equivalently CFT_L and CFT_R , are evenly spaced. Elsewhere, we will explain how to remove the constraint of fixed angular momentum without changing our results. Also, we will explain how to extend these ideas to five-dimensional black holes. Our results reveal a remarkable correspondence in that they give considerable weight to the idea that these conformal field theories are holographic avatars of the true degrees of freedom of a black hole..

IV. OSCILLATORS

It may seem strange that the L family of quasinormal modes has a frequency that is purely imaginary. One should not find this to be a matter of concern for black hole physics as was first clearly explained by Maggiore [12,13]. Consider a classical damped oscillator that we take to be a model of the states of the black hole. Such a system is

⁴These results agree with the calculation in [18]. In [17] the results are unfortunately missing a factor of 2.

governed by a familiar second-order ordinary differential equation. Modeling the black hole in a way similar to the liquid drop model of the nucleus, the amplitude of its oscillation will be $f(t)$ and governed by

$$\ddot{f} + 2\gamma\dot{f} + \omega_o^2 f = \text{forcing term.} \quad (12)$$

Suppose the system is initially stationary with $f(t) = 0$ for $t < 0$ and the forcing term is just $\delta(t)$. The solution is then,

$$f(t) = \frac{i}{\omega_+ - \omega_-} [e^{-i\omega_+ t} - e^{-i\omega_- t}], \quad (13)$$

where

$$\omega_{\pm} = \pm \sqrt{\omega_o^2 - \gamma^2} - i\gamma. \quad (14)$$

Thus we see the damping factor is just γ but the frequency of oscillation is $\sqrt{\omega_o^2 - \gamma^2}$. We therefore see that an apparently purely damped quasinormal mode does indeed correspond to a natural frequency of oscillation. We conclude that the spectrum of quasinormal modes, even if resulting in a purely imaginary ω , does indeed reflect the modes of excitation of the black hole and these in turn reflect the spectrum of excitations of $\text{CFT}_L \otimes \text{CFT}_R$.

V. PERTURBATIONS

Suppose we now add to the black hole a quantum of energy δM whilst keeping the angular momentum J , and the charges $Q^{(e)}$ and $Q^{(m)}$ constant. One should note that since our quasinormal modes do not vanish on the horizon, these modes will backreact on the black hole geometry, inducing a physical change in the state of the black hole. A transformation of this kind could be the result of the absorption of a quantum by either the left family or the right family of quasinormal modes. If from the right family, then we are looking at the case where $k = 0$. The energy added will be either $|\omega_L|$ or $|\omega_R|$ as can be seen from Eq. (14). In either case, one can use the first law for the left family or the right family as appropriate to see that the change in S_L or S_R is $2\pi \times (\text{some integer})$. Since $S_+ = S_L + S_R$, we conclude that it is consistent for the area of the outer horizon is quantized in units of $8\pi l_{\text{Pl}}^2$. A similar observation holds for the inner horizon.

At this point we need to resolve a puzzle. In [23] it was shown that in the Smarr law for CFT_L and CFT_R , $E_L = E_R = M/2$. To deduce horizon quantization, we used the first law for CFT_L and CFT_R and took $dE_L = dM = |\omega_L|$ or $dE_R = dM = |\omega_R|$. This is a reasonable thing to do since the first law describes infinitesimal changes, something that is consistent with adding a single

quantum in perturbation theory. However, there is now tension with the Smarr formulas. However, the Smarr formulas, constitutive relations, relate to macroscopic quantities and one expects them to be true only on average after some degree of equilibrium has been achieved. Of course, if the black hole absorbs a random collection of quanta over an extended period, then at each step, we expect the first law to hold. However, we expect the Smarr formulas to hold only on average over the extended period.⁵

It also possible that beyond perturbation theory more complicated things happen, but it is remarkable that such simplicity as we have found appears. Elsewhere, we will explain how to remove the constraint of fixed angular momentum without changing our results. Also, we will explain how to extend these ideas to five-dimensional black holes. Our results reveal a remarkable correspondence. They give considerable weight to the idea that these conformal field theories are holographic avatars of the true degrees of freedom of a black hole. We also intend to carefully examine the cases of Kerr and Kerr-Newman to investigate possible similar correspondences. Of course, we do not know if our approach can account for all of the black hole degrees of freedom. Nevertheless, we believe our results do provide some illumination of the information paradox [48]. In particular, although supergravity theories have prescribed matter content, we do not seem to see any progress towards an understanding of the species problem.

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DATA AVAILABILITY

No data were created or analyzed in this study.

⁵The results are true for the whole family of charged subtracted geometries. Of course they reduce to the Kerr one for $\Pi_c = 1$ and $\Pi_s = 0$ and for Kerr-Newman with $\Pi_c = \cosh^4 \delta$ and $\Pi_s = \sinh^4 \delta$.

- [1] S. W. Hawking, Particle creation by black holes, *Commun. Math. Phys.* **43**, 199 (1975); **46**, 206(E) (1976).
- [2] J. D. Bekenstein, Black holes and the second law, *Lett. Nuovo Cimento* **4**, 737 (1972).
- [3] J. D. Bekenstein, Black holes and entropy, *Phys. Rev. D* **7**, 2333 (1973).
- [4] L. Boltzmann, Über die Beziehung eines allgemeine mechanischen Satzes zum zweiten Hauptsatz der Wärmetheorie, *Sitzungber. Kais. Akad. Wiss. Math. Naturwiss. Cl. Abt II* **67**, 373 (1877).
- [5] J. A. Wheeler, Information, physics, quantum: The search for links, in *Proceedings of the Third International Symposium on the Foundations of Quantum Mechanics*, edited by S.-I. Kobayashi (Physical Society of Japan, Tokyo, 1990).
- [6] G. 't Hooft, The black hole interpretation of string theory, *Nucl. Phys.* **B335**, 138 (1990).
- [7] L. Susskind, The world as a hologram, *J. Math. Phys. (N.Y.)* **36**, 6377 (1995).
- [8] J. D. Bekenstein, The quantum mass spectrum of the Kerr black hole, *Lett. Nuovo Cimento* **11**, 467 (1974).
- [9] V. F. Mukhanov, Are black holes quantized?, *JETP Lett.* **44**, 63 (1986).
- [10] J. D. Bekenstein and V. F. Mukhanov, Spectroscopy of the quantum black hole, *Phys. Lett. B* **360**, 7 (1995).
- [11] S. Hod, Bohr's correspondence principle and the area spectrum of quantum black holes, *Phys. Rev. Lett.* **81**, 4293 (1998).
- [12] M. Maggiore, The physical interpretation of the spectrum of black hole quasinormal modes, *Phys. Rev. Lett.* **100**, 141301 (2008).
- [13] A. J. M. Medved, On the Kerr quantum area spectrum, *Classical Quantum Gravity* **25**, 205014 (2008).
- [14] A. Castro, A. Maloney, and A. Strominger, Hidden conformal symmetry of the Kerr black hole, *Phys. Rev. D* **82**, 024008 (2010).
- [15] M. Cvetič and F. Larsen, Conformal symmetry for black holes in four dimensions, *J. High Energy Phys.* **09** (2012) 076.
- [16] M. Cvetič and G. W. Gibbons, Conformal symmetry of a black hole as a scaling limit: A black hole in an asymptotically conical box, *J. High Energy Phys.* **07** (2012) 014.
- [17] M. Cvetič and G. W. Gibbons, Exact quasinormal modes for the near horizon Kerr metric, *Phys. Rev. D* **89**, 064057 (2014).
- [18] M. Cvetič, G. W. Gibbons, and Z. H. Saleem, Quasinormal modes for subtracted rotating and magnetized geometries, *Phys. Rev. D* **90**, 124046 (2014).
- [19] M. Cvetič, G. W. Gibbons, and Z. H. Saleem, Thermodynamics of asymptotically conical geometries, *Phys. Rev. Lett.* **114**, 231301 (2015).
- [20] O. S. An, M. Cvetič, and I. Papadimitriou, Black hole thermodynamics from a variational principle: Asymptotically conical backgrounds, *J. High Energy Phys.* **03** (2016) 086.
- [21] M. Cvetič and D. Youm, Entropy of nonextreme charged rotating black holes in string theory, *Phys. Rev. D* **54**, 2612 (1996).
- [22] Z. W. Chong, M. Cvetič, H. Lu, and C. N. Pope, Charged rotating black holes in four-dimensional gauged and ungauged supergravities, *Nucl. Phys.* **B717**, 246 (2005).
- [23] M. Cvetič, G. W. Gibbons, H. Lü, and C. N. Pope, Killing horizons: Negative temperatures and entropy super-additivity, *Phys. Rev. D* **98**, 106015 (2018).
- [24] D. Frenkel and P. B. Warren, Gibbs, Boltzmann, and negative temperatures, *Am. J. Phys.* **83**, 163 (2015).
- [25] M. Cvetič and F. Larsen, Grey body factors for rotating black holes in four-dimensions, *Nucl. Phys.* **B506**, 107 (1997).
- [26] A. Strominger, Black hole entropy from near horizon microstates, *J. High Energy Phys.* **02** (1998) 009.
- [27] M. Cvetič and F. Larsen, Greybody factors and charges in Kerr/CFT, *J. High Energy Phys.* **09** (2009) 088.
- [28] A. Castro, J. M. Lapan, A. Maloney, and M. J. Rodriguez, Black hole scattering from monodromy, *Classical Quantum Gravity* **30**, 165005 (2013).
- [29] A. Castro, J. M. Lapan, A. Maloney, and M. J. Rodriguez, Black hole monodromy and conformal field theory, *Phys. Rev. D* **88**, 044003 (2013).
- [30] S. Haco, S. W. Hawking, M. J. Perry, and A. Strominger, Black hole entropy and soft hair, *J. High Energy Phys.* **12** (2018) 098.
- [31] G. W. Gibbons and M. J. Perry, Black holes and thermal Green's functions, *Proc. R. Soc. A* **358**, 467 (1978).
- [32] C. V. Vishveshwara, Scattering of gravitational radiation by a Schwarzschild black-hole, *Nature (London)* **227**, 936 (1970).
- [33] S. Chandrasekhar and S. L. Detweiler, The quasi-normal modes of the Schwarzschild black hole, *Proc. R. Soc. A* **344**, 441 (1975).
- [34] E. W. Leaver, An analytic representation for the quasi normal modes of Kerr black holes, *Proc. R. Soc. A* **402**, 285 (1985).
- [35] E. W. Leaver, Spectral decomposition of the perturbation response of the Schwarzschild geometry, *Phys. Rev. D* **34**, 384 (1986).
- [36] H.-P. Nollert and B. G. Schmidt, Quasinormal modes of Schwarzschild black holes: Defined and calculated via Laplace transformation, *Phys. Rev. D* **45**, 2617 (1992).
- [37] H.-P. Nollert, Quasinormal modes of Schwarzschild black holes: The determination of quasinormal frequencies with very large imaginary parts, *Phys. Rev. D* **47**, 5253 (1993).
- [38] H.-P. Nollert, Quasinormal modes: The characteristic "sound" of black holes and neutron stars, *Classical Quantum Gravity* **16**, R159 (1999).
- [39] K. D. Kokkotas and B. G. Schmidt, Quasinormal modes of stars and black holes, *Living Rev. Relativity* **2**, 2 (1999).
- [40] L. Motl and A. Neitzke, Asymptotic black hole quasinormal frequencies, *Adv. Theor. Math. Phys.* **7**, 307 (2003).
- [41] V. Ferrari and L. Gualtieri, Quasi-normal modes and gravitational wave astronomy, *Gen. Relativ. Gravit.* **40**, 945 (2008).
- [42] E. Berti, V. Cardoso, and A. O. Starinets, Quasinormal modes of black holes and black branes, *Classical Quantum Gravity* **26**, 163001 (2009).
- [43] R. A. Konoplya and A. Zhidenko, Quasinormal modes of black holes: From astrophysics to string theory, *Rev. Mod. Phys.* **83**, 793 (2011).
- [44] B. Carneiro da Cunha and F. Novaes, Kerr scattering coefficients via isomonodromy, *J. High Energy Phys.* **11** (2015) 144.

- [45] B. Carneiro da Cunha and J.P. Cavalcante, Confluent conformal blocks and the Teukolsky master equation, [Phys. Rev. D **102**, 105013 \(2020\)](#).
- [46] G. Aminov, A. Grassi, and Y. Hatsuda, Black hole quasinormal modes and Seiberg–Witten theory, [Ann. Henri Poincaré **23**, 1951 \(2022\)](#).
- [47] G. Bonelli, C. Iossa, D.P. Lichtig, and A. Tanzini, Exact solution of Kerr black hole perturbations via CFT2 and instanton counting: Greybody factor, quasinormal modes, and Love numbers, [Phys. Rev. D **105**, 044047 \(2022\)](#).
- [48] S. W. Hawking, Breakdown of predictability in gravitational collapse, [Phys. Rev. D **14**, 2460 \(1976\)](#).