

Charged current induced electron-proton scattering and the axial vector form factor

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We investigate the total scattering cross section (σ), the differential cross section ($\frac{d\sigma}{dQ^2}$), the longitudinal [$A_L(E_e, Q^2)$] and perpendicular [$A_P(E_e, Q^2)$] spin asymmetries of the polarized target proton, as well as the longitudinal [$P_L(E_e, Q^2)$], perpendicular [$P_P(E_e, Q^2)$], and transverse [$P_T(E_e, Q^2)$] polarization components of the final neutron, in the weak charged current induced electron-proton scattering relevant to the future experiments at the Thomas Jefferson National Accelerator Facility (JLab) and Mainz Microtron (MAMI). The analysis is performed assuming time-reversal (T) invariance as well as without assuming T invariance, allowing for a nonvanishing transverse polarization component of the final nucleon, perpendicular to the production plane. Numerical results are presented for the above mentioned observables, and their sensitivities to the various parametrizations of the axial vector form factor $g_1(Q^2)$ and a nonzero weak electric form factor $g_2(Q^2)$ are examined. We find that the cross section depends strongly on the parametrizations used for the axial vector form factor. Moreover, the dipole parametrization of $g_1(Q^2)$ with a higher value of the axial dipole mass M_A simulates the apparent enhancement in σ obtained using the nondipole parametrizations like the z -expansion and Faddeev equation form. The cross sections are found to depend only weakly on the weak electric form factor $g_2(Q^2)$, which is associated with the violation of G-invariance. On the contrary, the spin observables both $A_{L,P}(E_e, Q^2)$ and $P_{L,P}(E_e, Q^2)$ are found to be strongly dependent on $g_2(Q^2)$. This study may be useful in the analysis of the neutrino oscillation experiments to provide an alternative constrain on the parametrization of axial vector form factor, which currently has large uncertainties.

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I. INTRODUCTION

Electron scattering on the polarized proton targets constitutes a powerful and uniquely sensitive probe of the weak interactions, nucleon structure, and fundamental symmetries. High-precision electron-proton scattering from nucleon and nuclear target experiments conducted at the Thomas Jefferson National Accelerator Facility (JLab) [1] and MIT-Bates [2,3] in the United States, and Mainz Microtron (MAMI) [4] in Germany have played a pivotal role in providing stringent low-energy tests of the Standard Model, while continuously advancing the frontiers of electroweak physics. Unlike strong and electromagnetic interactions, weak interactions violate parity, thereby enabling access to observables that are otherwise inaccessible in scattering experiments where parity is conserved. In

particular, the parity-violating asymmetries measured in polarized electron-proton scattering allow the isolation of weak neutral current effects with exceptional sensitivity. In the region of low electron energies, the experiments at MAMI and MIT-Bates and the experiments at JLab in the higher energy region of electrons employing highly polarized electron beams have made significant contributions to precision measurements in low-energy electroweak physics through dedicated parity-violation studies. In recent times, efforts at JLab and MAMI have further strengthened these investigations by achieving unprecedented control over systematic uncertainties. Looking ahead, forthcoming experiments such as the upgraded facilities at JLab and the P2 experiment at MAMI promise substantial improvements in precision, offering enhanced sensitivity to possible deviations from Standard Model predictions and opening new windows for the discovery of physics beyond the Standard Model [5].

In describing the weak interactions of electrons and neutrinos with nucleon targets, nucleon form factors are fundamental quantities that are used to describe the internal structure of the nucleon and play a central role in our understanding of nonperturbative QCD dynamics. The

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vector component of the weak processes is constrained using the isovector combination of the proton and neutron electromagnetic form factors, based on the isospin symmetry and the conserved vector current (CVC) hypothesis [6], and are obtained through the experimental measurements of electric [$G_E(Q^2)$] and magnetic [$G_M(Q^2)$] Sachs' form factors. In contrast, the axial vector contribution, parametrized by the axial vector [$g_1(Q^2)$] and induced pseudoscalar [$g_3(Q^2)$] form factors, is comparatively less well determined. The isovector axial vector form factor of the nucleon occupies a particularly important position as the pseudoscalar form factor $g_3(Q^2)$ is generally expressed in terms of $g_1(Q^2)$ using the hypothesis of partially conserved axial vector current (PCAC) and pion pole dominance of the divergence of the axial vector current. The value of the axial vector form factor at zero momentum transfer defines the nucleon axial charge, which is primarily determined from neutron beta decay to be $g_1(0) = 1.267 \pm 0.0030$ [7], and is one of the most precisely measured quantities in hadronic physics [7,8]. Moreover, the momentum-transfer dependence of the axial vector form factor is of crucial importance for a wide range of weak interaction processes at higher energies (momentum transfers), most notably for the quasielastic neutrino-nucleon scattering. In recent years, several studies have emphasized that uncertainties in the measurement of the axial vector form factor directly propagate into systematic errors in the extraction of neutrino oscillation parameters, making a precise theoretical understanding of $g_1(Q^2)$ indispensable for the current and future neutrino oscillation experiments [9].

Traditionally, the axial vector form factor has been extracted using a combination of neutrino-induced charged-current quasielastic scattering on the nuclear targets including light nuclei like deuterium [10], threshold pion electroproduction data from proton [11], and weak processes such as muon capture in hydrogen [12–14]. While these methods have provided valuable insights, they suffer from experimental limitations, nuclear corrections, and model-dependent assumptions, leading to persistent uncertainties and tensions among different datasets [15]. In general, the axial vector form factor is parametrized to be of the dipole form, where the choice of the numerical value of the axial dipole mass M_A plays a crucial role in determining the neutrino-nucleon quasielastic (QE) cross section and has been a subject of intense and sustained debate within the neutrino physics community over the past several years. A remarkably wide range of values for M_A , has been reported and discussed in literature, reflecting both experimental and theoretical uncertainties as well as differences in data interpretation [6,16].

The early measurements of M_A were performed using neutrino and antineutrino beams incident on heavy nuclear targets [17]. These experiments were primarily carried out in the 1970s and 1980s and relied on bubble chambers and early tracking detectors using freon (CF_3Br), freon-propane

mixtures ($CF_3Br - C_3H_8$), iron (Fe), neon (Ne), and aluminum (Al), which were exposed to wide-band neutrino beams at CERN and other facilities. CERN experiments [18–20] provided early values around $M_A = 1.0$ GeV but suffered from large systematic errors due to limited statistics. SKAT data [21–23] analyzed primarily high- Q^2 regions to reduce nuclear uncertainties. In order to reduce the large uncertainty arising due to the nuclear medium, the experiments at ANL [24,25], BNL [26,27] using hydrogen and deuterium targets, determined the value of M_A by using a dipole form of the axial vector form factor. A reanalysis of these classic datasets by Bodek *et al.* [10] yielded a relatively precise value of $M_A = 1.014 \pm 0.014$ GeV. In contrast, Meyer *et al.* [28], analyzing the same experimental data from the ANL [24,25] and BNL [26,27] experiments, reported a broader range of values, $M_A = 1.02$ – 1.17 GeV, depending on which dataset was included in the fit. An earlier comprehensive study by Bernard *et al.* [17], which combined neutrino and antineutrino scattering data on hydrogen and deuterium with threshold pion electroproduction data from proton, obtained the best fit at $M_A = 1.026 \pm 0.021$ GeV.

With the advent of modern high-statistics experiments, a large body of new QE neutrino-nucleus scattering data have become available, covering both low- and intermediate-energy regimes and involving a variety of nuclear targets. Analyses of data from the NOMAD [29] and MINERvA [30,31] experiments, performed at relatively higher neutrino energies, continue to favor a lower value of M_A , close to 1.03 GeV, consistent with the earlier free-nucleon results. In sharp contrast, several experiments operating predominantly in the few hundreds of MeV to GeV energy range and using nuclear targets, including MiniBooNE [32], MINOS [33], K2K [34], T2K [35], and SciBooNE [36], report significantly larger effective values of M_A , typically in the range 1.2–1.35 GeV. This apparent discrepancy between low energy and high energy measurements has emerged as one of the central puzzles in the interpretation of QE neutrino scattering data. One possible explanation for the larger values of M_A extracted at lower energies is the inadequate treatment of nuclear medium effects, such as multinucleon correlations, meson-exchange currents, and final-state interactions, which can enhance the observed QE-like cross section and mimic the effect of a larger axial mass. Since the NOMAD and MINERvA experiments operate at comparatively higher energies, where such nuclear effects are expected to be less pronounced, their consistency with the lower M_A value lends support to this interpretation. This issue has been actively discussed in recent literature [6].

A major recent advance in this area has been achieved by the MINERvA collaboration [37], which performed the first high-statistics determination of $g_1(Q^2)$ from free-nucleon data using antineutrino scattering on hydrogen embedded in a plastic scintillator detector. This measurement represents a

significant milestone in the study of $g_1(Q^2)$. MINERvA measured the differential cross section as a function of the squared four-momentum transfer, Q^2 , and from these data extracted both the axial vector form factor $g_1(Q^2)$ and the axial radius r_A , which characterizes the spatial distribution of axial charge. By minimizing nuclear effects, this work provides a cleaner experimental benchmark for nucleon axial structure.

On the theoretical side, there have been many calculations for the determination of $g_1(Q^2)$ by studying the total $[\sigma(E)]$ and the differential $\left(\frac{d\sigma(E)}{dQ^2}\right)$ cross sections and their dependence on the axial dipole mass M_A in the scattering of (anti)neutrinos from the nucleons and nuclei [6,38–42]. In a recent study, Alvarez-Ruso *et al.* [43] employed a Bayesian neural-network framework to extract the nucleon axial vector form factor in a model-independent way from neutrino-deuteron ANL data. They revealed a striking sensitivity to low- Q^2 data and deuteron corrections, leading to tensions in the extracted axial radius, and underscored the need for high-precision neutrino quasielastic measurements on hydrogen and deuterium. The possibility of determining M_A by studying the various spin observables of the nucleons in the (anti)neutrino scattering from nucleons induced by the weak charged and neutral currents has been explored earlier by Block [44] and later by Bilenky and Christova [45,46]. In recent years, Tomalak [47] investigated the neutrino scattering on the polarized and unpolarized nucleons, including the recoil polarization observables, and demonstrated that the parity-violating weak interaction induces sizable spin asymmetries at the leading order. Such measurements provide an independent access to the proton axial structure, enabling, in particular, the first direct extraction of the pseudoscalar form factor from neutrino data without relying on the PCAC ansatz or pion-pole dominance assumptions. Graczyk and Kowal [48] demonstrated that single-, double-, and triple- spin asymmetries in quasielastic charged current (anti)neutrino scattering are highly sensitive probes of the nucleon axial vector form factor. In particular, the target-recoil and lepton-target-recoil asymmetries exhibit a strong dependence on the axial dipole mass, while all asymmetries, except the lepton polarization, provide observables for probing the second class current contributions.

In parallel, substantial progress has been made on the theoretical front through lattice QCD, which offers a powerful and systematically improvable framework for computing nucleon axial properties directly from the underlying theory of the strong interaction. Over the past two decades, lattice determinations of the axial charge $g_1(0)$ have improved dramatically. Recent simulations employ near-physical pion masses, large lattice volumes, and advanced analysis techniques such as multistate fits and variational methods, leading to results that are increasingly consistent with experimental determinations.

Extending beyond the axial charge, lattice QCD enables a first-principles determination of the Q^2 dependence of $g_1(Q^2)$ over a range of spacelike momentum transfers, providing direct input for neutrino-nucleon cross sections without relying on phenomenological parametrizations such as the dipole ansatz [16,49–51].

Recent studies have analyzed the axial vector form factor using both traditional dipole fits as well as using lattice QCD with the more general z -expansion [52], while the extrapolation of lattice results to the physical limit has been guided by expressions derived within the chiral perturbation theory (ChPT) [53]. Moreover, lattice QCD has contributed to the study of related axial vector parameters, including the induced pseudoscalar form factor and the pion-nucleon coupling through the axial Ward-Takahashi identity, thereby reinforcing the internal theoretical consistency of the framework [54]. Yao *et al.* [55] have considered both the pion mass and momentum transfer dependences of the nucleon axial vector form factor in baryon chiral perturbation theory (BChPT) and investigated their dependences by performing fits to the recent lattice QCD data without and with explicit $\Delta(1232)$ contribution. The synergy between high-precision experimental measurements by MINERvA and first-principles lattice QCD calculations has led to significant progress in the last one decade. In particular, combined analyses of MINERvA data and lattice QCD results have enabled meaningful and quantitative comparisons between experiment and theory, shedding new light on the momentum dependence of the nucleon axial vector form factor and the value of the axial radius [56,57]. These developments mark an important step toward a unified and precise understanding of the nucleon axial structure, with far-reaching implications for the hadron structure studies and the precision physics goals of the modern neutrino experiments.

An alternative and potentially more powerful approach for extracting detailed information on the Q^2 dependence of the axial vector form factor $g_1(Q^2)$ may be provided by the weak charged current interaction induced electron scattering from both unpolarized and polarized proton targets. Unlike neutrino induced processes, which suffer from broad energy spectra, comparatively very low event rates, etc., and thus containing large error bars, electron scattering experiments benefit from highly monochromatic beams with exceptionally high luminosity, several orders of magnitude larger than those attainable in neutrino experiments, thereby providing opportunity for the precise measurements with reduced statistical uncertainties and improved control over kinematic reconstruction. Furthermore, the availability of high-purity hydrogen targets in electron scattering experiments offers a clean experimental environment, in contrast to the neutrino experiments where the use of free hydrogen or deuterium targets is severely constrained by security related limitations. These advantages make weak charged current electron proton scattering a particularly promising alternative

for probing the nucleon axial structure. Recently, Klest [58], and independently Yang and Kumar [59], have demonstrated that at very high energies corresponding to the center of energy $\sqrt{s} = 140$ GeV, relevant to the proposed Electron Ion Collider (EIC) at BNL, and Electron Ion Collider in China (EICC), the electron scattering on a longitudinally polarized proton target induced by the weak charged currents offers an avenue to constrain the nucleon axial-vector form factor. They emphasize that target spin asymmetries in the low- Q^2 region, where the axial coupling overwhelmingly dominates, constitute a sensitive probe. However, in the region of moderately high energy corresponding to a few GeV, an experimental proposal is already under consideration at JLab [60] using polarized electron beam of energy 1.1 and 2.2 GeV to determine the axial vector form factor in the measurement of the differential cross section $\frac{d\sigma}{dQ^2}$ around $Q^2 \approx 1$ GeV², where it has maximum uncertainty. The experiment is feasible provided the background due to the electromagnetic electron-proton scattering and pion photoproduction could be kept under control [60]. Moreover, a similar experiment using polarized positron beam on the deuterium target to extract the axial vector form factor is also under consideration at JLab [61].

The present work is motivated by the aforementioned proposals [60,61] to extend our earlier works [62–68] on the study of polarization observables of the final nucleon/hyperon and lepton in the antineutrino [63,65–68] as well as the electron [62,64] induced scattering from the free nucleon and nuclear targets, to perform a comprehensive study of the weak charged current induced electron-proton scattering, focusing on a wide range of observables relevant to the ongoing and future experiments at JLab, which can also be done, in principle, at MAMI. In Refs. [62,64], we developed the formalism, following the works of Bilenky and Christova [45,46,69], to study the electron induced hyperon/nucleon production from the free nucleon target and the polarization observables of the final hyperons/nucleon, assuming T invariance as well as its violation. In this work, we have used the earlier developed formalism to study the polarization observables of the final nucleon in the weak electron-proton scattering, specifically for the kinematical observables relevant for the JLab proposals [60,61]. Furthermore, we have also studied, the spin asymmetries of the polarized target proton. The sensitivity of the cross sections, spin asymmetries of the polarized target, and the polarization observables of the final nucleon to the axial vector form factor has been studied by using the modern alternatives of the traditional dipole parametrization like the Chen-Roberts [70] Faddeev equation form and the four z -expansion fits of Meyer *et al.* [57], namely old deuterium data fit, lattice QCD (LQCD) calculation fit, MINERvA hydrogen data fit and the combined MINERvA hydrogen and LQCD fit.

Weak electron-proton scattering experiments, which can be done at MAMI and JLab would provide a high precision,

nuclear effect free, and theoretically clean avenue to study the axial structure of the proton. By directly probing the weak axial vector form factor through the electron scattering experiments, one can significantly improve our knowledge of the axial vector form factor $g_1(Q^2)$, which would play a decisive role in resolving long-standing discrepancies in neutrino cross section measurements. Specifically, we obtain the total scattering cross section (σ), differential cross section ($d\sigma/dQ^2$), the longitudinal [$A_L(E_e, Q^2)$] and perpendicular [$A_P(E_e, Q^2)$] spin asymmetries of the polarized proton target, as well as the longitudinal [$P_L(E_e, Q^2)$], perpendicular [$P_P(E_e, Q^2)$], and transverse [$P_T(E_e, Q^2)$] components of the final nucleon polarization. These polarization observables provide additional and independent sensitivity to the underlying weak form factors. Most of the calculations are done assuming time-reversal (T) invariance. However, we have also studied the effect of T violation on the polarization of the final nucleon for which the transverse component of the polarization, i.e., $P_T(E_e, Q^2)$ in a direction perpendicular to the reaction plane is nonzero, provided the weak coupling constant $g_2(0)$ is purely imaginary.

In view of the difficulty in preparing the transversely polarized proton target in lab, we have performed the numerical calculations for spin asymmetries of the polarized proton assuming T invariance. Numerical results are presented for all observables across the relevant kinematic ranges, and their sensitivities to the axial vector form factor $g_1(Q^2)$, as well as the weak electric form factor $g_2(Q^2)$ are systematically examined. Our results demonstrate that the combination of cross section measurements of the high-luminosity electron beams on the hydrogen target and polarization measurements may offer a relatively better framework, in comparison to using the wide energy band neutrino beam on moderate to heavy nuclear targets, for studying the axial structure of the nucleon and a feasible experiment using electron beam may provide a better insight to determine $g_1(Q^2)$ [6,9].

In Sec. II, we present the formalism to calculate the cross section for the weak charged current interaction induced electron scattering on the proton target. Section III discusses very briefly the formalism to calculate the spin asymmetries of the polarized target proton and the polarization components of the final nucleon assuming T invariance as well as its violation and the detailed derivation is given in Appendix B. The results for the total scattering cross section and the Q^2 distribution of the differential scattering cross section $\frac{d\sigma}{dQ^2}$ at $E_e = 1.1$ GeV, which is relevant for the JLab experiment are presented and discussed in Sec. IV A. Sections IV B and IV C, respectively, present the results and discussion for the longitudinal and perpendicular spin asymmetries of the initial polarized proton, and the polarization components of the final nucleon. In Sec. IV C 2, we also discuss the transverse polarization of the final nucleon,

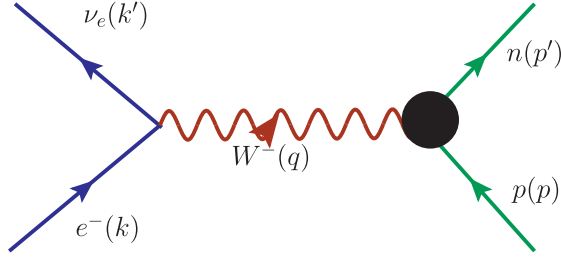


FIG. 1. Feynman diagram for the process $e^-(k) + p(p) \rightarrow \nu_e(k') + n(p')$. The quantities in the bracket represent four momenta of the corresponding particles.

which is nonzero in the case T invariance is violated. We have also performed the numerical calculations for $\frac{d\sigma}{dQ^2}$, $A_L(Q^2)$, $A_P(Q^2)$, $P_L(Q^2)$, $P_P(Q^2)$, and $P_T(Q^2)$ at $E_e = 855$ MeV, relevant for the MAMI experiment and at $E_e = 2.2$ GeV, relevant for the JLab experiment, which are qualitatively of the similar nature as the results at $E_e = 1.1$ GeV and therefore are presented in Supplemental Material [71]. Finally, in Sec. V, we conclude our findings.

II. CROSS SECTION

The general expression of the differential cross section for the scattering of unpolarized electron from unpolarized proton target, i.e.,

$$e^-(k) + p(p) \longrightarrow \nu_e(k') + n(p'), \quad (1)$$

which is diagrammatically shown in Fig. 1, in the rest frame of the initial proton, is written as

$$d\sigma = \frac{1}{(2\pi)^2} \frac{1}{4E_e M} \delta^4(k + p - k' - p') \times \frac{d^3 k'}{2E_{k'}} \frac{d^3 p'}{2E_{p'}} \overline{\sum} \sum |\mathcal{M}|^2, \quad (2)$$

where the quantities in the brackets of Eq. (1) represent the four momenta of the corresponding particles, E_e is the electron energy, and the transition matrix element squared is expressed as

$$\overline{\sum} \sum |\mathcal{M}|^2 = \frac{G_F^2 \cos^2 \theta_C}{2} \mathcal{L}_{\alpha\beta} \mathcal{J}^{\alpha\beta}, \quad (3)$$

where G_F is the Fermi coupling constant and θ_C is the Cabibbo mixing angle. The leptonic and the hadronic tensors are given by

$$\mathcal{L}^{\alpha\beta} = \frac{1}{2} \text{Tr}[\gamma^\alpha (1 - \gamma_5) \Lambda(k) \gamma^\beta (1 - \gamma_5) \Lambda(k')], \quad (4)$$

$$\mathcal{J}_{\alpha\beta} = \frac{1}{2} \text{Tr}[\Lambda(p') J_\alpha \Lambda(p) \tilde{J}_\beta], \quad (5)$$

where the factor of $\frac{1}{2}$ in the above equations comes due to the spin averaging of the initial particles (if both the beam and the target are unpolarized), $\tilde{J}_\beta = \gamma^0 J_\beta^\dagger \gamma^0$ with J_β being the weak hadronic current.

In Eqs. (4) and (5), $\Lambda(P) = (\not{P} + M_P)$ is the projection operator of an unpolarized particle with momentum P and mass M_P . To take into account the spin polarization of a particle, the projection operator $\Lambda(P)$ in Eqs. (4) and (5) is replaced by the spin projection operator $\Lambda(P, s_P)$, defined as

$$\Lambda(P, s_P) = (\not{P} + M_P) \left(\frac{1 + \gamma_5 \not{s}_P}{2} \right), \quad (6)$$

where s_P is the spin 4-vector. Moreover, for the polarized initial particle scattering, one has to drop the factor of $\frac{1}{2}$ in Eqs. (4) and (5) and use $\Lambda(P, s_P)$ instead of $\Lambda(P)$. Since the electron becomes highly relativistic at $E_e \geq 150$ MeV, therefore, for the electron energies considered in the present work, the electron is fully longitudinally polarized. In this work, we have studied the total and differential scattering cross sections of the polarized electron from the unpolarized proton target and the results are presented in Sec. IV A.

The hadronic current J_μ is expressed in terms of the vector (V_μ) and the axial vector (A_μ) currents as [72]

$$J_\mu = \bar{u}(p') (V_\mu - A_\mu) u(p) \quad (7)$$

with

$$V_\mu = \gamma_\mu f_1(Q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{2M} f_2(Q^2) + \frac{q_\mu}{M} f_3(Q^2), \quad (8)$$

$$A_\mu = \gamma_\mu \gamma_5 g_1(Q^2) + i\sigma_{\mu\nu} \gamma_5 \frac{q^\nu}{2M} g_2(Q^2) + \frac{q_\mu}{M} g_3(Q^2) \gamma_5, \quad (9)$$

where M is the mass of the nucleon. $q_\mu (= k_\mu - k'_\mu = p'_\mu - p_\mu)$ is the four momentum transfer with $Q^2 = -q^2$, $Q^2 > 0$. $f_1(Q^2)$, $f_2(Q^2)$ and $f_3(Q^2)$ are the vector, weak magnetic and scalar form factors, and $g_1(Q^2)$, $g_2(Q^2)$ and $g_3(Q^2)$ are the axial vector, weak electric (or induced tensor) and pseudoscalar form factors, respectively and are discussed briefly in Appendix A.

III. SPIN OBSERVABLES

Traditionally, the spin asymmetries are obtained using the spin projection operator formalism, for example as done by Graczyk *et al.* [73,74] and Tomalak *et al.* [75,76], in the case of neutrino scattering processes from the nucleon. We have used the covariant density matrix formalism following Bilenky and Christova [45,46,69], which has earlier been used by us [62–66] for calculating the polarization components of the target as well as the recoil nucleon. Though

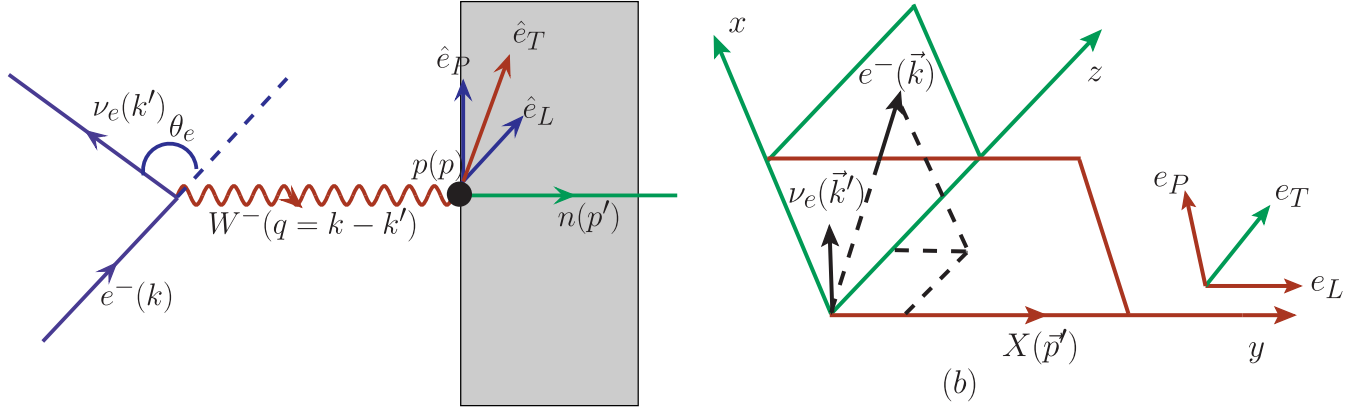


FIG. 2. Diagrammatic representation of the process $e^-(\vec{k}) + p(\vec{p} = 0) \rightarrow \nu_e(\vec{k}') + n(\vec{p}')$, and the longitudinal and perpendicular directions of the polarized proton (left panel). The longitudinal, perpendicular and transverse directions with respect to the momentum of the final nucleon (right panel).

these are two different approaches, they lead to same results for the target asymmetries of the initial polarized nucleon and the polarization observables of the final nucleon. In this work, we follow the same formalism to calculate the spin asymmetries of the polarized target and the polarization observables of the final nucleon, as developed by us earlier in Refs. [62–66]. The diagrammatic representation of the longitudinal, perpendicular, and transverse components of the polarized target proton as well as the polarized final nucleon is given in Fig. 2. In the following, we have given the explicit expressions for the spin asymmetries of the polarized target and polarization components of the final nucleon while the derivation is given in Appendix B for completeness.

A. Spin asymmetry of the polarized proton target

The polarization 4-vector (ζ^τ) of the initial nucleon is written as [69]

$$\zeta^\tau = \left(g^{\tau\sigma} - \frac{p^\tau p^\sigma}{M^2} \right) \frac{\mathcal{L}^{\alpha\beta} \text{Tr}[\gamma_\sigma \gamma_5 \Lambda(p') J_\alpha \Lambda(p) \tilde{J}_\beta]}{\mathcal{L}^{\alpha\beta} \text{Tr}[\Lambda(p') J_\alpha \Lambda(p) \tilde{J}_\beta]}. \quad (10)$$

The expressions for the longitudinal and perpendicular components of the spin asymmetry, i.e., $A_L(Q^2)$ and $A_P(Q^2)$ are obtained, using the formalism given in Appendix B, as

$$A_L(Q^2) = \frac{1}{E_e} \frac{\alpha(E_e, Q^2) |\vec{k}|^2 + \beta(E_e, Q^2) \vec{k} \cdot \vec{q}}{N(E_e, Q^2)}, \quad (11)$$

$$A_P(Q^2) = \frac{\beta(E_e, Q^2) [|\vec{q}| \sin \beta_k]}{N(E_e, Q^2)}, \quad (12)$$

where β_k is the angle between $\vec{k}$ and $\vec{q}$ and the expressions for $N(E_e, Q^2) = \mathcal{L}_{\alpha\beta} \mathcal{J}^{\alpha\beta}$, $\alpha(E_e, Q^2)$, and $\beta(E_e, Q^2)$ are given in Appendix C.

B. Polarization components of the final nucleon

Assuming T-invariance, the expressions for the longitudinal and perpendicular components of the final neutron polarization i.e., $P_L(Q^2)$ and $P_P(Q^2)$, obtained in the covariant density matrix formalism as discussed in Appendix B, are expressed as

$$P_L(Q^2) = \frac{M A(E_e, Q^2) \vec{k} \cdot \hat{p}' + B(E_e, Q^2) |\vec{p}'|}{E' N(E_e, Q^2)}, \quad (13)$$

$$P_P(Q^2) = \frac{A(E_e, Q^2) [(\vec{k} \cdot \hat{p}')^2 - |\vec{k}|^2]}{N(E_e, Q^2) |\hat{p}' \times \vec{k}|}. \quad (14)$$

The expressions for $A(E_e, Q^2)$ and $B(E_e, Q^2)$ are given in Appendix D.

In the case of T-violation, the explicit expressions for $P_L(Q^2)$ and $P_P(Q^2)$ are the same as given in Eqs. (13) and (14), except that the coefficients $A(E_e, Q^2)$ and $B(E_e, Q^2)$ are now replaced with $A'(E_e, Q^2)$ and $B'(E_e, Q^2)$. The expression for the transverse component of the neutron polarization $P_T(Q^2)$, which is nonzero in this case, is expressed as

$$P_T(Q^2) = \frac{C'(E_e, Q^2) M |\vec{p}'| [(\vec{k} \cdot \hat{p}')^2 - |\vec{k}|^2]}{N(E_e, Q^2) |\hat{p}' \times \vec{k}|}. \quad (15)$$

The expressions for $A'(E_e, Q^2)$, $B'(E_e, Q^2)$, and $C'(E_e, Q^2)$ are given in Appendix E.

IV. RESULTS AND DISCUSSION

A. Total and differential scattering cross sections

In Fig. 3, we have presented the results for the total scattering cross section $[\sigma(E_e)]$ as a function of incoming electron energy (E_e) for the longitudinally polarized electron induced scattering off the unpolarized proton target.

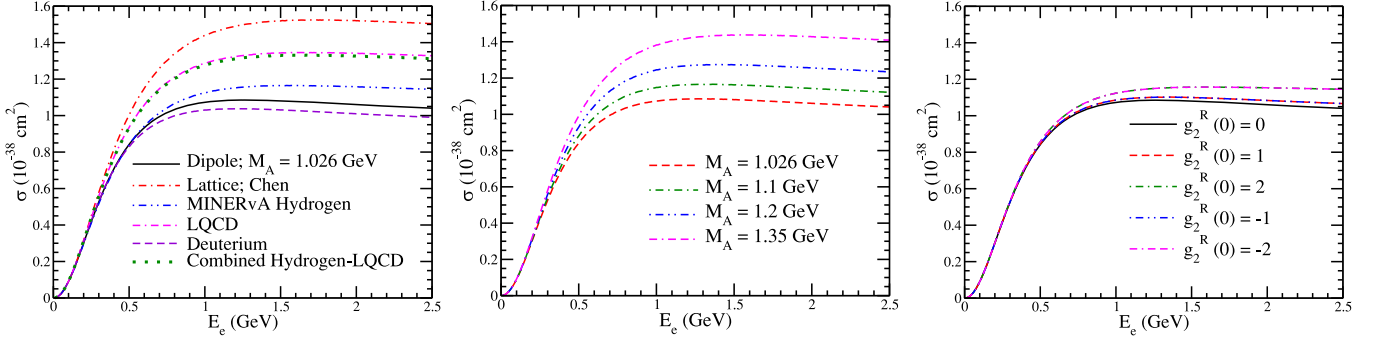


FIG. 3. Total cross section σ as a function of E_e for the process $e^- + p \rightarrow \nu_e + n$, when the initial electron is polarized. (Left panel) shows the results for the different parametrizations of $g_1(Q^2)$ viz. the dipole parametrization with $M_A = 1.026$ GeV shown by solid line, the lattice gauge parametrization of Robert and Chen [70,77] shown by the dash-dotted line, the z -expansion for the MINERvA hydrogen, LQCD, deuterium, and combined hydrogen-LQCD fits are represented by double-dot-dashed line, double dash-dotted line, dashed line and dotted line, respectively. (Middle panel) shows the results for σ using the different values of the axial dipole mass M_A viz. $M_A = 1.026$ GeV (dashed line), 1.1 GeV (dash-dotted line), 1.2 GeV (double-dot-dashed line), and 1.35 GeV (double-dash-dotted line). (Right panel) shows the results for σ obtained using the different values of $g_2^R(0)$ assuming T-invariance viz. $g_2^R(0) = 0$ (solid line), +1 (dashed line), +2 (dash-dotted line), -1 (double-dot-dashed line), and -2 (double-dash-dotted line).

The left panel of the figure shows the results obtained using the different parametrizations of the axial vector form factor $g_1(Q^2)$ available in the literature and discussed in Appendix A. The cross section is calculated using the traditional dipole parametrization for $g_1(Q^2)$ with $M_A = 1.026$ GeV and compared with the results obtained using the lattice parametrization of Chen and Roberts [70,77], and the z -expansion fit done by Meyer *et al.* [56,57] for the combined LQCD calculations of RQCD 2020 [78], NME [79], Djukanovic *et al.* [80], ETM [81], and PNDME [50], along with fitting for the experimental data of MINERvA hydrogen [37] and earlier deuterium experiments which has been recently reanalyzed [28] and for the combined LQCD and MINERvA hydrogen data [57]. It may be observed from the figure that the lattice parametrization of Chen and Roberts [70] yields the largest value of the cross section when compared with the cross sections predicted by the other parametrizations of $g_1(Q^2)$ considered in this work. This cross section is almost 27% larger than the dipole parametrization at $E_e = 700$ MeV, and increases further with increase in electron energy to become 35% and 43% at $E_e = 1$ GeV, and 2 GeV, respectively. The results obtained with the parametrization using the z -expansion fit of the deuterium data are quite consistent with the results obtained with the predictions assuming the dipole parametrization and show a small decrement of about 3%–5% in the entire range of electron energies considered in this work. In the low electron energy region, $E_e \sim 700$ MeV, the results obtained with the z -expansion fit of MINERvA hydrogen data are consistent with the dipole results, however, as the electron energy increases, the MINERvA fit yields a larger value for σ , which is about 5% at $E_e = 1$ GeV and becomes $\sim 10\%$ at $E_e = 2$ GeV. The results obtained with the z -expansion fit of the LQCD calculations and combined MINERvA hydrogen-LQCD [57] are consistent with each

other and are larger than the dipole results in the entire range of electron energy considered in this work. Quantitatively, they are larger by about 15% at $E_e = 700$ MeV, which becomes 20% and 25% at $E_e = 1$ and 2 GeV, respectively.

The middle panel of Fig. 3 shows the dependence of the total cross section on the axial dipole mass M_A . The results for $\sigma(E_e)$ are obtained for the four different values of M_A (1.026 GeV to 1.35 GeV). We find an appreciable dependence on the cross section $\sigma(E_e)$, when M_A is varied and this dependence is, quantitatively, not the same at all the incident electron energies. With the increase in M_A , $\sigma(E_e)$ increases. For example at $E_e = 1$ GeV, when M_A is changed from 1.026 GeV to 1.1 GeV, the cross section increases by about 7%, in the region of $E_e = 1$ –2 GeV, while when M_A is changed from 1.1 GeV to 1.35 GeV, the cross section increases by 20% at $E_e = 1$ GeV, and becomes 25% at $E_e = 2$ GeV.

The right panel of Fig. 3 shows the effect of the second class current assuming T invariance on the total cross section. The results are obtained using a nonzero value of $g_2^R(0)$ in Eq. (A6), in the range $-2 \leq g_2^R(0) \leq +2$, with a dipole mass $M_A = 1.026$ GeV in $g_1(Q^2)$. It must be pointed out that the cross section obtained using the positive as well as the negative values for $g_2^R(0)$ are exactly the same. Moreover, if one takes into account the imaginary values of the weak electric form factor, the results obtained for the cross sections are exactly the same as those obtained for $g_2^R(0)$. It is evident from the expression of $N(E_e, Q^2)$ given in Appendix C, that the cross section depends only on the square of $g_2(Q^2)$, thus, the results obtained using the positive or negative values of $g_2^R(0)$, or $g_2^I(0)$ are exactly the same. We find that the cross section increases with the increase in $g_2^R(0)$, although this increase is only marginal as compared to the increase in cross section when a larger

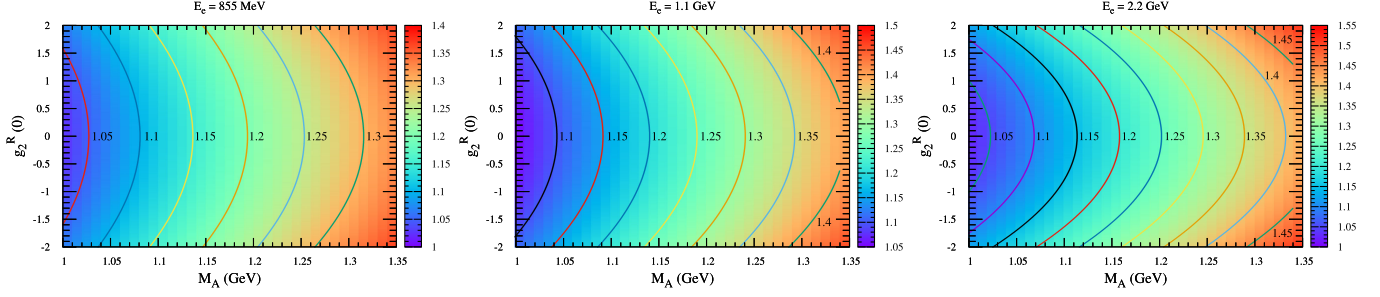


FIG. 4. $g_2^R(0) - M_A$ correlation for σ for the process $e^- + p \rightarrow \nu_e + n$, when the initial electron is polarized at $E_e = 855$ MeV (left panel), 1.1 GeV (middle panel), and 2.2 GeV (right panel). The solid contour lines are drawn for constant σ ($\times 10^{-38}$ cm ^{2}) and their values being quoted in the plot.

value of M_A is used in the parametrization of $g_1(Q^2)$. For example, at $E_e = 1$ GeV, when $g_2^R(0)$ is taken as 1 instead of zero, the increase is 1%, which becomes 2% at $E_e = 2$ GeV. And when $g_2^R(0)$ is taken as 2, instead of 1, there is a further increase of 4% at $E_e = 1$ GeV, which becomes 8% at $E_e = 2$ GeV. However, a nonzero $g_2^R(0)$ may mimic the effect of a larger value of M_A . This needs further study. Therefore, we show in Fig. 4, a correlation plot for $g_2^R(0)$ vs. M_A for the total cross section at the three different values of electron energy viz., $E_e = 855$ MeV, 1.1 GeV, and 2.2 GeV, relevant for the JLab and MAMI experiments. The figure shows a strong M_A dependence on the cross section, which is about 30% when M_A is varied in the range [0.9, 1.35] GeV. However, the cross section shows little dependence to the choice of $g_2^R(0)$ in the range $[-2, 2]$. Therefore, one may conclude, from this figure, that the presence of a nonzero value of $g_2^R(0)$, provided $|g_2^R(0)| \leq 1$, is not going to affect the determination of M_A or $g_1(Q^2)$ at JLab and MAMI experiments using the total cross section measurements.

To show the dependence of the different parametrization of $g_1(Q^2)$ on the differential scattering cross section $\frac{d\sigma}{dQ^2}$, we have presented in the left panel of Fig. 5, the results of $\frac{d\sigma}{dQ^2}$ vs Q^2 at $E_e = 1.1$ GeV, relevant for the JLab

experiment, using the different parametrizations of $g_1(Q^2)$ like the dipole parametrization with $M_A = 1.026$ GeV, z -expansion [57] for the MINERvA hydrogen data, LQCD, deuterium data, and combined hydrogen-LQCD fits, and the LQCD calculation by Chen and Roberts [70,77]. We have also studied the effect of $g_1(Q^2)$ on $\frac{d\sigma}{dQ^2}$ at other electron energies relevant for JLab and MAMI experiments viz. $E_e = 855$ MeV and 2.2 GeV, and the results are presented in Ref. [71]. It may be noticed from the left panel of Fig. 5 that the cross section is quite sensitive to the choice of $g_1(Q^2)$ at all values of E_e and Q^2 considered in the present work. Quantitatively, at $Q^2 = 1$ GeV ^{2} the results obtained with deuterium data fit are smaller than the results obtained using the dipole parametrization by about 10% at all values of electron energies. The results obtained using the z -expansion fit of the MINERvA data and combined MINERvA-LQCD are larger than the results obtained with the dipole parametrization by about 20% and 45%, respectively, at $Q^2 = 1$ GeV ^{2} . The LQCD calculations by Chen and Roberts [70,77] yield the largest value of cross section as compared to the dipole parametrization, which is about 80% at $Q^2 = 1$ GeV ^{2} .

The middle panel of Fig. 5 depicts the results for Q^2 distribution using different values of the axial dipole mass M_A in the range 1.026–1.35 GeV, in the axial vector form

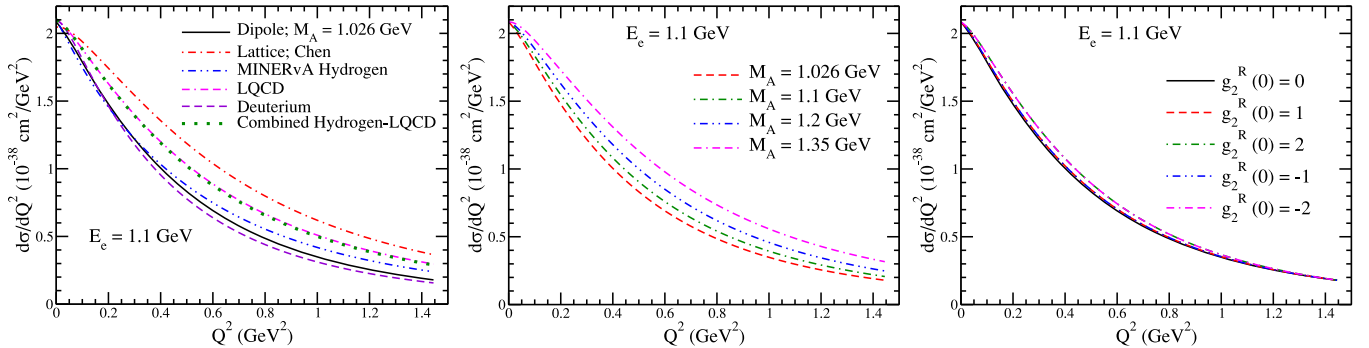


FIG. 5. $\frac{d\sigma}{dQ^2}$ as a function of Q^2 at $E_e = 1.1$ GeV for the process $e^- + p \rightarrow \nu_e + n$, when the initial electron is polarized for the different parametrizations of $g_1(Q^2)$ (left panel), different values of M_A (middle panel), and the different values of $g_2^R(0)$ (right panel). Lines and points have the same meaning as in Fig. 3.

factor [Eq. (A2)] at $E_e = 1.1$ GeV. We find a strong dependence of Q^2 on the differential cross section when M_A is varied between 1.026 and 1.35 GeV. For example, the cross section increases by about 35% at $Q^2 = 0.5$ GeV² when M_A is varied from 1.026 GeV to 1.35 GeV, which further increases with the increase in Q^2 and becomes larger by 60% at $Q^2 = 1$ GeV².

The right panel of Fig. 5 depicts the results for Q^2 distribution using the different values of $g_2^R(0)$, which is associated with the second class current form factor, in the range $[-2, 2]$ in Eq. (A6) at $E_e = 1.1$ GeV. Unlike the dependence of $g_1(Q^2)$ and M_A on the cross section, we find that, the Q^2 distribution is not much sensitive to the variation in the value of $g_2^R(0)$.

B. Spin asymmetries of the polarized proton target

In Figs. 6–10, we have presented the results for the longitudinal and perpendicular spin asymmetries for the polarized target proton in the reaction $e^- + \vec{p} \rightarrow \nu_e + n$. The effect of the different parametrizations for $g_1(Q^2)$, M_A variation, M_2 variation, and the effect of the nonzero $g_2(Q^2)$

on the Q^2 and E_e dependent spin asymmetries is studied. We show in Fig. 6 (top panel) the dependence of the different parametrizations of $g_1(Q^2)$ on the energy dependent longitudinal $[A_L(E_e)]$ and perpendicular $[A_P(E_e)]$ spin asymmetries of the initial polarized proton. It may be noticed from the figure that the spin asymmetries are not much affected by the choice of the parametrization for $g_1(Q^2)$ except for the parametrization given by Chen and Roberts [70] which predicts a smaller value ($< 2\text{--}3\%$) for $A_L(E_e)$ as compared to the results obtained using the other parametrizations, especially in the region of $E_e \geq 1.5$ GeV. In the case of $A_P(Q^2)$, the dependence on the choice of the axial vector form factor is even smaller.

In the bottom panel of Fig. 6, the results are presented for the longitudinal $[A_L(E_e)]$ and perpendicular $[A_P(E_e)]$ spin asymmetries by varying M_A in the range 1.026–1.35 GeV. Not much dependence has been observed specially at low E_e , and for $A_P(E_e)$, when M_A is varied from 1.026 GeV to 1.35 GeV. Moreover, it is worth mentioning that the spin asymmetries depend very weakly on the axial vector form factor $g_1(Q^2)$, unlike the cross section, which depends quite strongly on $g_1(Q^2)$.

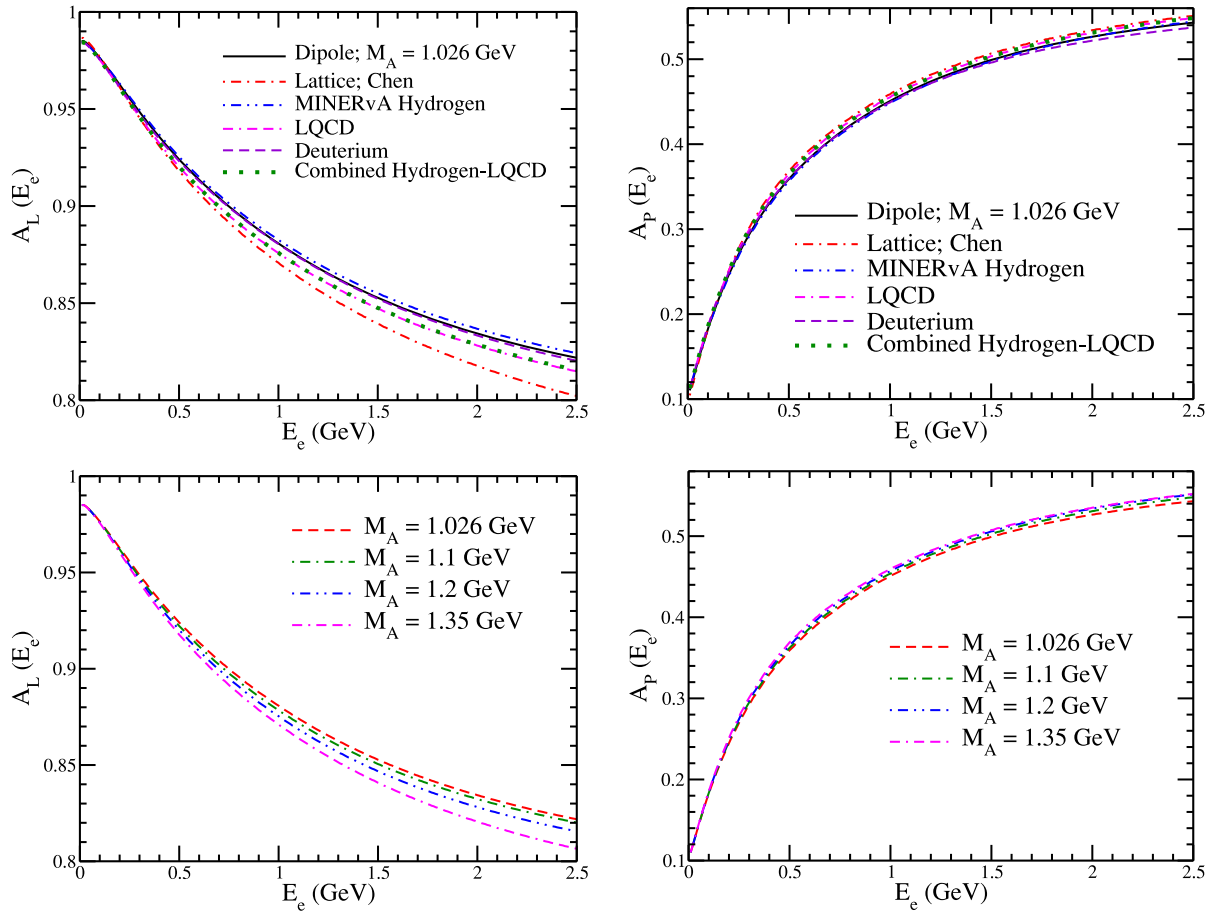


FIG. 6. $A_L(E_e)$ (left panel) and $A_P(E_e)$ (right panel) as a function of E_e for the process $e^- + p \rightarrow \nu_e + n$, when the initial proton is polarized for the different parametrizations of $g_1(Q^2)$ (top panel) and M_A (bottom panel). Lines and points have the same meaning as in Fig. 3.

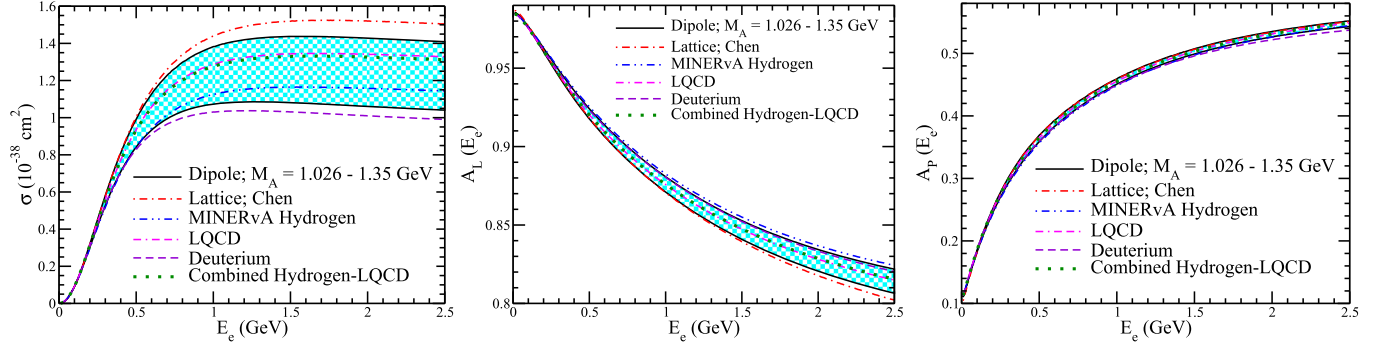


FIG. 7. σ (left panel), $A_L(E_e)$ (middle panel) and $A_P(E_e)$ (right panel) as a function of E_e for the process $e^- + p \rightarrow \nu_e + n$, when the initial proton is polarized for the different parametrizations of $g_1(Q^2)$. Lines and points have the same meaning as in Fig. 3 and the band corresponds to the dipole parametrization with M_A in the range 1.026–1.35 GeV.

It may be observed from Figs. 3 and 6 that the results obtained using a higher value of M_A for both the cross section and spin asymmetries mimic the results obtained using the nondipole $g_1(Q^2)$ parametrizations. To show this explicitly, in Fig. 7, we have presented the results for σ , $A_L(E_e)$, and $A_P(E_e)$ vs E_e using the different nondipole parametrizations of $g_1(Q^2)$ and the band corresponds to the dipole parametrization of $g_1(Q^2)$ with M_A in the range 1.026–1.35 GeV. The results for σ , $A_L(E_e)$, and $A_P(E_e)$ obtained using the lattice parametrization of Chen and Roberts [70] are higher than the dipole band while the results obtained with the z -expansion fit of the deuterium data are smaller than the dipole band. The results obtained with the z -expansion fit of the LQCD, MINERvA-hydrogen data and the combined hydrogen data and LQCD lie within the dipole band for all the three observables.

Interestingly, it may be observed from the left panel of Fig. 7 that adopting a larger axial mass, for instance $M_A = 1.4$ GeV within the conventional dipole parametrization, effectively reproduces the results obtained from the parametrization of Chen *et al.* [70] based on lattice QCD

calculations. Likewise, choosing $M_A = 1.3$ GeV in the dipole form closely mimics the outcomes derived from the z -expansion fit to the combined MINERvA hydrogen data and lattice QCD inputs [57]. This correspondence is quite revealing. It suggests that the apparent enhancement in the cross section or the associated modifications in polarization observables- can be phenomenologically interpreted, within the dipole framework, as arising from a larger effective value of M_A . In other words, the impact of more sophisticated form-factor parametrizations can, to a significant extent, be encapsulated by an upward shift in the axial dipole mass when using the traditional dipole fit.

In Fig. 8, we have presented the results for the longitudinal and perpendicular spin asymmetries as a function of E_e by varying $g_2^R(0)$ in the range $[-2, 2]$. We observe a significant dependence of $g_2^R(0)$ on both $A_L(E_e)$ and $A_P(E_e)$ in the entire electron energy range considered in the present work, unlike the results for the cross sections, which depend only weakly on the variation in $g_2^R(0)$. Moreover, the positive and negative values of $g_2^R(0)$ yield different values for $A_L(E_e)$ and $A_P(E_e)$, which is evident

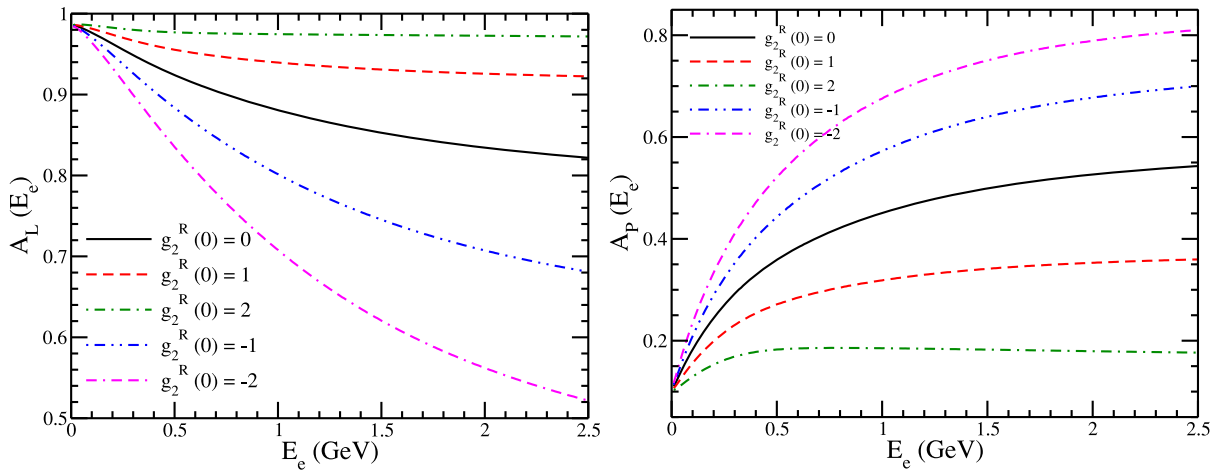


FIG. 8. $A_L(E_e)$ (left panel) and $A_P(E_e)$ (right panel) as a function of E_e for the process $e^- + p \rightarrow \nu_e + n$, when the initial proton is polarized for the different values of $g_2^R(0)$. Lines and points have the same meaning as in Fig. 3.

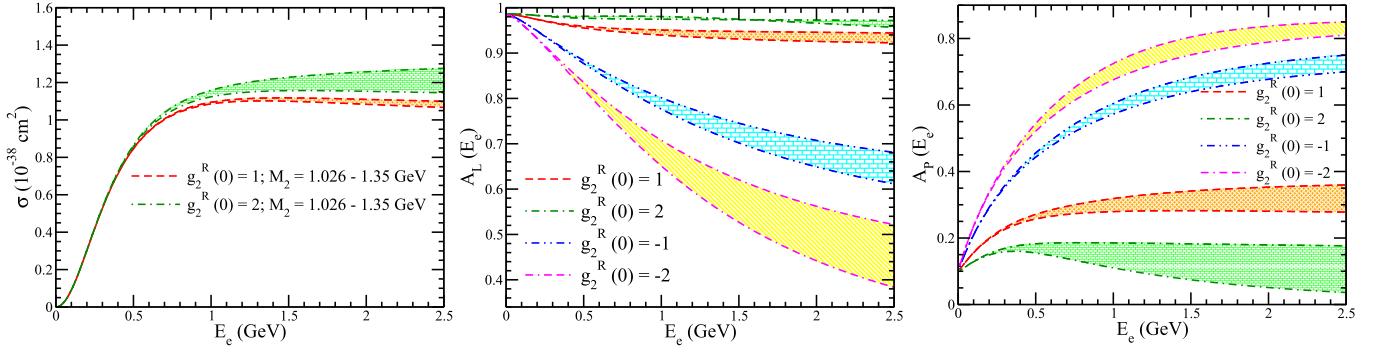


FIG. 9. σ (left panel), $A_L(E_e)$ (middle panel) and $A_P(E_e)$ (right panel) as a function of E_e for the process $e^- + p \rightarrow \nu_e + n$, when the initial proton is polarized. Lines and points have the same meaning as in Fig. 3 and the band corresponds to the M_2 variation in the range 1.026–1.35 GeV.

from the expressions of $\alpha(E_e, Q^2)$ and $\beta(E_e, Q^2)$ given in Appendix-C. However, we observe opposite trends for $A_L(E_e)$ and $A_P(E_e)$, when positive and negative values of $g_2^R(0)$ are used in the numerical calculations, i.e., $A_L(E_e)$ increases when $g_2^R(0)$ is increased from 0 to +2, while $A_P(E_e)$ decreases. Likewise, when $g_2^R(0)$ is decreased from 0 to -2, the values of $A_L(E_e)$ decreases while the values of $A_P(E_e)$ increases. Quantitatively, when $g_2^R(0)$ is varied from 0 to +2, $A_L(E_e)$ changes by about 10% at $E_e = 1$ GeV, which becomes 20% at $E_e = 2$ GeV, while when $g_2^R(0)$ is varied from 0 to -2, $A_L(E_e)$ changes by about 15% at $E_e = 1$ GeV, which becomes almost 30% at $E_e = 2$ GeV. Furthermore, the effect of $g_2^R(0)$ is more pronounced on $A_P(E_e)$, which is about 60% at $E_e = 1$ GeV, and becomes 50% at $E_e = 2$ GeV, when $g_2^R(0)$ is taken as +2 instead of 0. This enhancement further increases when negative values of $g_2^R(0)$ are considered and we find an enhancement of about 65% at $E_e = 1$ GeV, which becomes 50% at $E_e = 2$ GeV.

To study the effect of the axial dipole mass associated with the second class current form factor M_2 on σ , $A_L(E_e)$, and $A_P(E_e)$, in Fig. 9, the results are presented using a fixed value of $g_2^R(0)$ viz. $g_2^R(0) = \pm 1$ and ± 2 , while varying M_2 in the range 1.026–1.35 GeV, chosen in analogy with M_A variation in the axial vector form factor $g_1(Q^2)$. The calculations are performed using the world average value of M_A i.e. $M_A = 1.026$ GeV in $g_1(Q^2)$. It may be observed from the figure that, in the case of σ and $A_P(E_e)$, the band width of the M_2 band increases with increase in the value of $g_2^R(0)$. For example, in the case of σ , at $g_2^R(0) = 1$, we find hardly any change in the results when M_2 is varied between 1.026 and 1.35 GeV, while at $g_2^R(0) = 2$, we find some dependence of σ on M_2 , which is $\sim 3\%$ at $E_e = 1$ GeV and increase with increase in E_e to become 10% at $E_e = 2$ GeV. Moreover, in the case of $A_L(E_e)$, we find that the positive values of $g_2^R(0)$ results in broader M_2 bands as compared to the positive value of $g_2^R(0)$, which is in

contrast to $A_P(E_e)$, where we find broader bands for the positive values of $g_2^R(0)$.

To study the dependence of $g_1(Q^2)$ on the spin asymmetries of the initial nucleon, we have presented, in the left panel of Fig. 10, the results for $A_L(Q^2)$ and $A_P(Q^2)$ vs Q^2 at $E_e = 1.1$ GeV, using different parametrizations of $g_1(Q^2)$ available in the literature and discussed in Appendix A. It may be noticed from the figure that the longitudinal asymmetry shows small variation in the peak region of Q^2 when the different parametrizations for $g_1(Q^2)$ are used. The largest difference in $A_L(Q^2)$ is observed when the calculations are performed using the parametrization of Chen and Roberts [70,77], which is about 2% in the peak region of Q^2 as compared to the dipole parametrization. Moreover, the variation in $A_P(Q^2)$ is even smaller as compared to the results obtained for $A_L(Q^2)$.

The middle panel of Fig. 10 shows the results for the longitudinal and perpendicular spin asymmetries of the initial nucleon at $E_e = 1.1$ GeV, by varying M_A in the range 1.026–1.35 GeV. As observed in the case of the different parametrizations of $g_1(Q^2)$, here also the longitudinal and perpendicular spin asymmetries are not much sensitive to the choice of M_A , except in the peak region of Q^2 .

To study the dependence of $g_2^R(0)$ variation on the longitudinal and perpendicular spin asymmetries of the initial polarized proton, we have presented in the right panel of Fig. 10, the results for $A_L(Q^2)$ and $A_P(Q^2)$ vs Q^2 at $E_e = 1.1$ GeV, by taking $g_2^R(0)$ in the range $[-2, 2]$. It may be observed from the figure that the spin asymmetries show strong dependence on the choice of $g_2^R(0)$, which is larger in the case of the perpendicular asymmetry $A_P(Q^2)$ than in the case of the longitudinal asymmetry $A_L(Q^2)$. Quantitatively, in the peak region of Q^2 , $A_L(Q^2)$ increases by about 15% at $E_e \sim 1$ GeV, which further increases and becomes 20% at $E_e = 2.2$ GeV, when $g_2^R(0)$ is increased from 0 to +2, while when $g_2^R(0)$ is decreased from 0 to -2, $A_L(Q^2)$ decreases by about 25% at $E_e \sim 1$ GeV, which

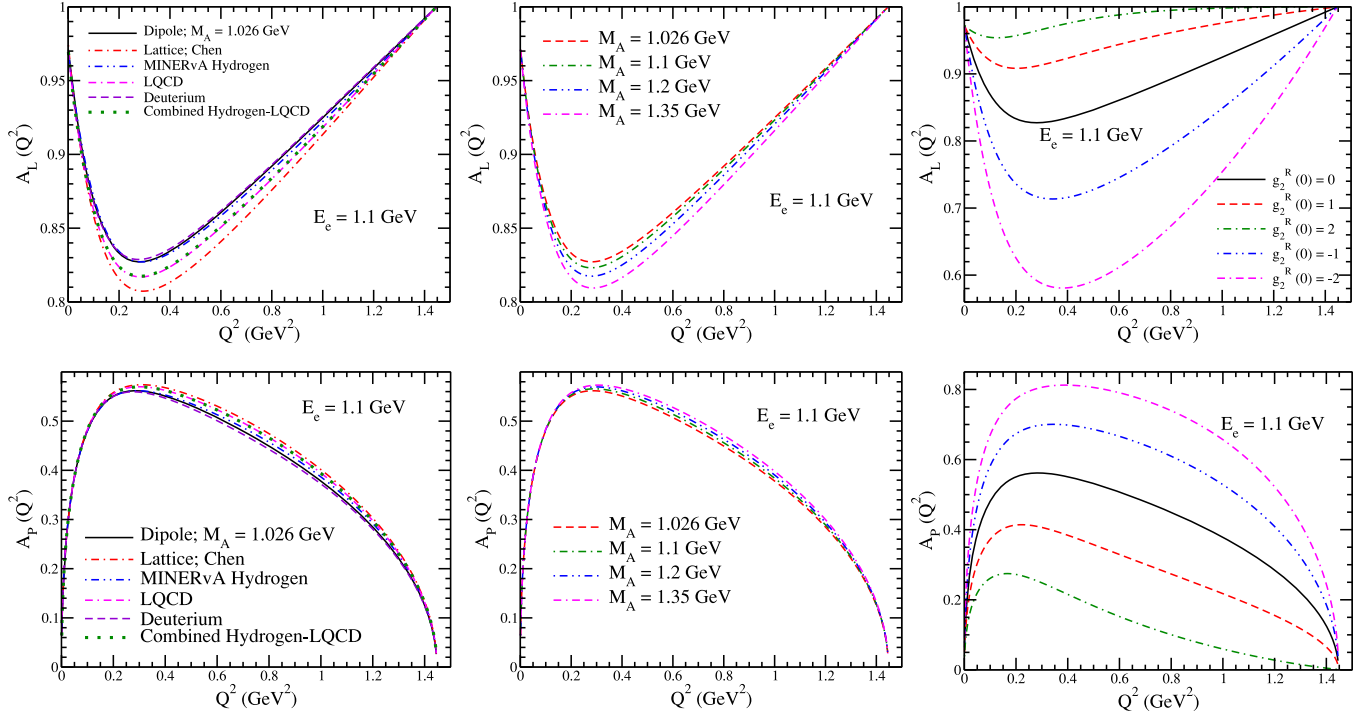


FIG. 10. $A_L(Q^2)$ (top panel) and $A_P(Q^2)$ (bottom panel) as a function of Q^2 at $E_e = 1.1$ GeV for the process $e^- + p \rightarrow \nu_e + n$, when the initial proton is polarized for the different parametrizations of $g_1(Q^2)$ (left panel), different values of M_A (middle panel), and different values of $g_2^R(0)$ (right panel). Lines and points have the same meaning as in Fig. 3.

further decreases with increase in E_e and becomes 50% at $E_e = 2.2$ GeV. In the case of perpendicular asymmetry, in the peak region of Q^2 , $A_P(Q^2)$ decreases by about 60% at $E_e = 1.1$ GeV, which becomes 66% at $E_e = 2.2$ GeV, when $g_2^R(0)$ is increased from 0 to +2, while $A_P(Q^2)$ increases by about 60% at $E_e = 1.1$ GeV, which becomes 50% at $E_e = 2.2$ GeV, when $g_2^R(0)$ is decreased from 0 to -2.

C. Polarization observables of the final nucleon

In this section, we present the results for the polarization observables of the final nucleon considering the two cases viz. T invariance and its violation. In Sec. IV C 1, the results are presented, assuming T invariance, for the polarization observables of the final nucleon in the reaction $e^- + p \rightarrow \nu_e + n$, where we have studied the effect of the different parametrizations of $g_1(Q^2)$ as well as M_A , $g_2^R(0)$, and M_2 variations on the polarization components. To study the effect of T violation on the polarization observables, in Sec IV C 2, we have presented the results for the polarization observables of the final nucleon by varying $g_2^L(0)$ in the range 0–2 and M_2 in the range 1.026–1.35 GeV.

1. Polarization observables in the case of T invariance

Figure 11 shows the results obtained using the different parametrizations of $g_1(Q^2)$, available in the literature, for

the longitudinal and perpendicular components of the neutron polarization. In the electron energy region of $E_e > 0.3$ GeV, we find some dependence of the choice of $g_1(Q^2)$ on the polarization observables, which is more pronounced in the case of $P_P(E_e)$. At $E_e = 0.5$ GeV, $P_L(E_e)$ obtained using the parametrization of Chen and Roberts [70,77] is almost 2% smaller than that obtained using the dipole parametrization with $M_A = 1.026$ GeV, and this difference further increases with the increase in electron energy, for example, it becomes 4% at $E_e = 2$ GeV. In the case of perpendicular component of the neutron polarization, at $E_e = 0.5$ GeV, the result obtained with the parametrization of Chen and Roberts [70,77] is almost 15% larger than the result obtained with the dipole parametrization with $M_A = 1.026$ GeV, and becomes 18% larger at $E_e = 2$ GeV.

To show the dependence of the polarization observables on M_A , in Fig. 12, we have presented the results for $P_L(E_e)$ and $P_P(E_e)$ using the value of M_A in the range 1.026–1.35 GeV. It may be noticed from the figure that the M_A variation is more pronounced in $P_P(E_e)$ as compared to $P_L(E_e)$. Quantitatively, we find that in the energy range of $E_e = 1$ –2 GeV, the value of $P_L(E_e)$ decreases by about 2%–3% when M_A is increased from 1.026 GeV to 1.35 GeV, while we observe an increase of about 13%–15% in the value of $P_P(E_e)$ when M_A is increased from 1.0 to 1.35 GeV for $E_e = 1$ –2 GeV.

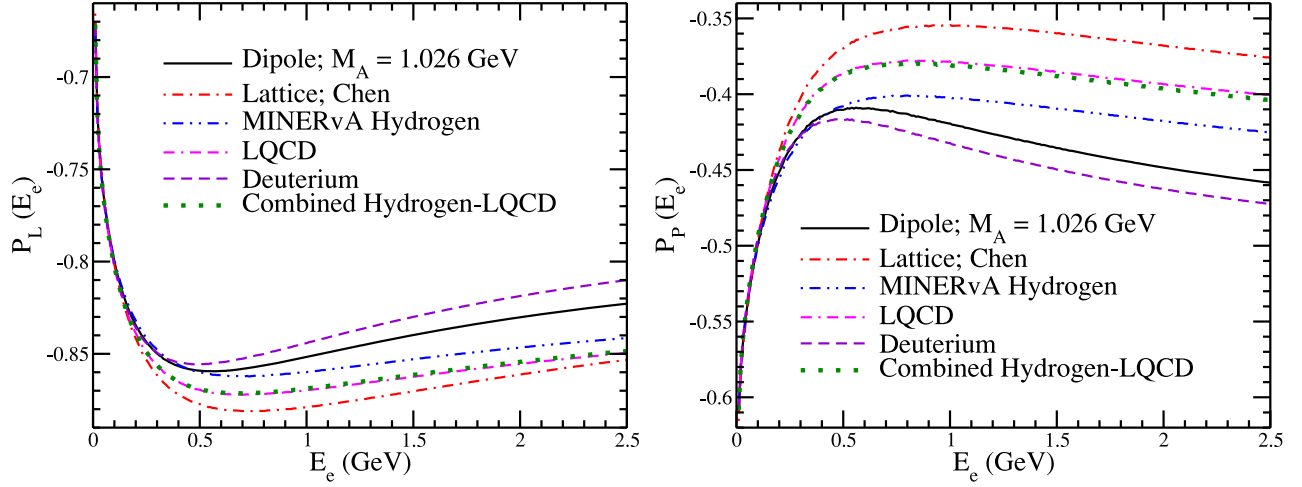


FIG. 11. $P_L(E_e)$ (left panel) and $P_P(E_e)$ (right panel) vs. E_e for the process $e^- + p \rightarrow \nu_e + n$, when the final nucleon is polarized, using the different parametrizations of the axial vector form factor. Lines and points have the same meaning as in Fig. 3.

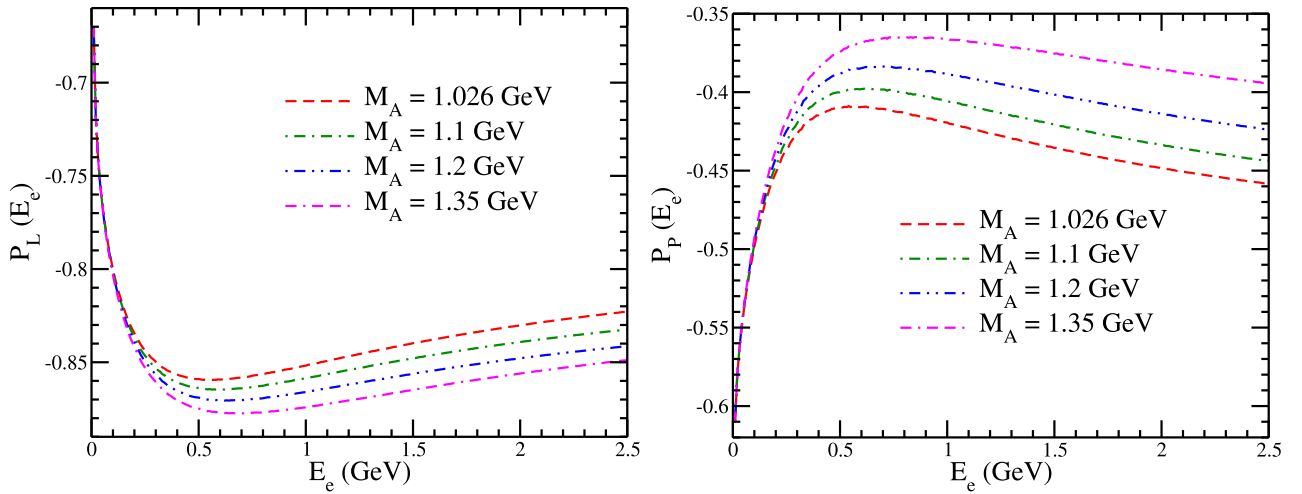


FIG. 12. $P_L(E_e)$ (left panel) and $P_P(E_e)$ (right panel) for the process $e^- + p \rightarrow \nu_e + n$, when the final nucleon is polarized, using the different values of M_A . Lines and points have the same meaning as in Fig. 3.

In Fig. 13, the results are presented for $P_L(E_e)$ and $P_P(E_e)$ using the different nondipole parametrizations of $g_1(Q^2)$ and compared with the results obtained using the dipole parametrization of $g_1(Q^2)$ with M_A being varied in the range 1.026–1.35 GeV. We find similar trends as observed in the case of σ , $A_L(E_e)$, and $A_P(E_e)$ in Fig. 9, that the results obtained with z -expansion fits except the deuterium data fit lie within the dipole band while the results obtained with the lattice parametrization of Chen *et al.* (deuterium data fit) are larger (smaller) than the dipole band for both $P_L(E_e)$ and $P_P(E_e)$.

In Fig. 14, we present the results for $P_L(E_e)$ and $P_P(E_e)$ as a function of electron energy E_e for different values of $g_2^R(0)$. It is evident that when $g_2^R(0)$ is varied from 0 to +2, $P_L(E_e)$ changes by only about 10%, whereas varying $g_2^R(0)$ from 0 to -2 leads to a much larger change of

approximately 25%. In contrast, for $P_P(E_e)$, the trend is reversed: increasing $g_2^R(0)$ from 0 to +2 results in a significant change of about 25%, while decreasing it from 0 to -2 produces a comparatively smaller change of around 10%. This clearly highlights that $P_L(E_e)$ and $P_P(E_e)$ exhibit opposite sensitivities to the variation of $g_2^R(0)$.

To study the effect of M_2 variation on $P_L(E_e)$ and $P_P(E_e)$, the results are presented in Fig. 15 for the fixed values of $g_2^R(0)$, namely $g_2^R(0) = \pm 1$ and ± 2 while M_2 being varied in the range 1.026–1.35 GeV. In the case of $P_L(E_e)$, we find that the negative values of $g_2^R(0)$ yield broader M_2 bands than those obtained for the positive values of $g_2^R(0)$. Moreover, there is almost no effect of M_2 variation for $g_2^R(0) = 1$ in $P_L(E_e)$. On the contrary, for $P_P(E_e)$, we find that the positive values of $g_2^R(0)$ lead to broader M_2 bands.

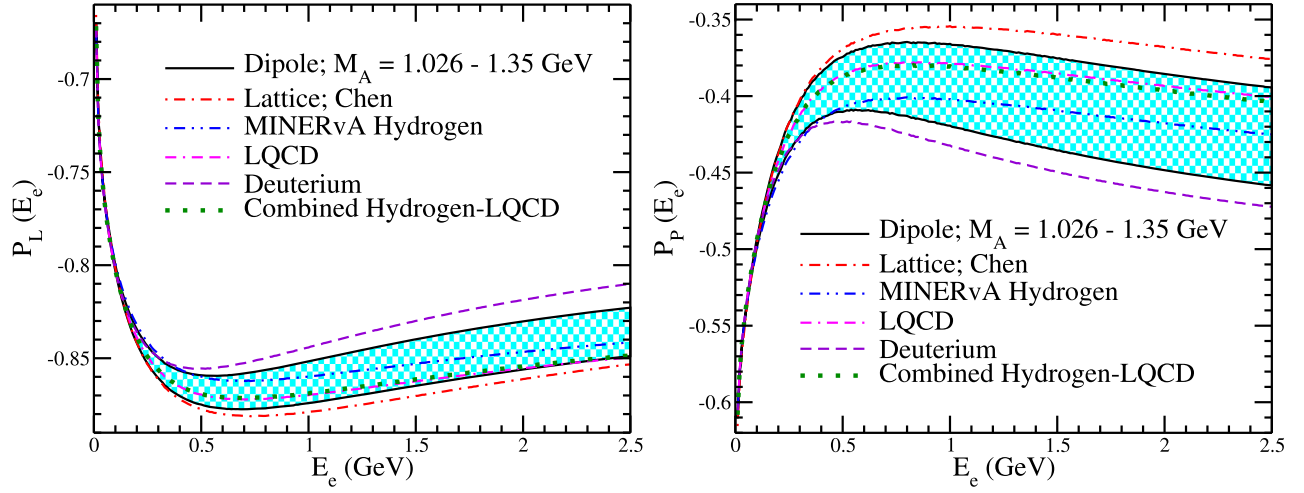


FIG. 13. $P_L(E_e)$ (left panel) and $P_P(E_e)$ (right panel) vs. E_e for the process $e^- + p \rightarrow \nu_e + n$, when the final nucleon is polarized, using the different parametrizations of the axial vector form factor. Lines and points have the same meaning as in Fig. 3 and the band corresponds to the dipole parametrization with M_A in the range 1.026–1.35 GeV.

The left panel of Fig. 16 shows the results for the neutron polarization components $P_L(Q^2)$ and $P_P(Q^2)$ vs. Q^2 at $E_e = 1.1$ GeV, by using the different parametrizations of $g_1(Q^2)$, as discussed in Appendix A. It may be observed from the figure that the neutron polarization components show almost no dependence on the choice of the axial vector form factor at all values of E_e and Q^2 considered in the present work.

To study the effect of M_A variation on the polarization components, in the middle panel of Fig. 16, we have presented the results for $P_L(Q^2)$ and $P_P(Q^2)$ at $E_e = 1.1$ GeV, by varying M_A in the range 1.026–1.35 GeV. For both $P_L(Q^2)$ and $P_P(Q^2)$, the different values of M_A yield similar results for the polarization components at all values of E_e and Q^2 .

In the right panel of Fig. 16, we present the results for $P_L(Q^2)$ and $P_P(Q^2)$ as function of Q^2 at $E_e = 1.1$ GeV, for a range of values of $g_2^R(0)$ (i.e., $-2 \leq g_2^R(0) \leq 2$). It is clearly observed that the variation is more pronounced in $P_P(Q^2)$ than in $P_L(Q^2)$. For instance, when $g_2^R(0)$ is varied from 0 to 2, in the peak region of Q^2 , $P_L(Q^2)$ changes by only about 5%, whereas varying it from 0 to -2 results in a significantly larger change of nearly 20%. This percentage change further increases with increasing E_e . Moreover, the behavior of $P_L(Q^2)$ itself is altered at high Q^2 for $g_2^R(0) = 1-2$. In contrast, for $P_P(Q^2)$, varying $g_2^R(0)$ from 0 to 2 or from 0 to -2 leads to changes in opposite directions, while the magnitude of the percentage variation remains comparable for equal values of $|g_2^R(0)|$. For example, the changes are about 10% and 15%, respectively,

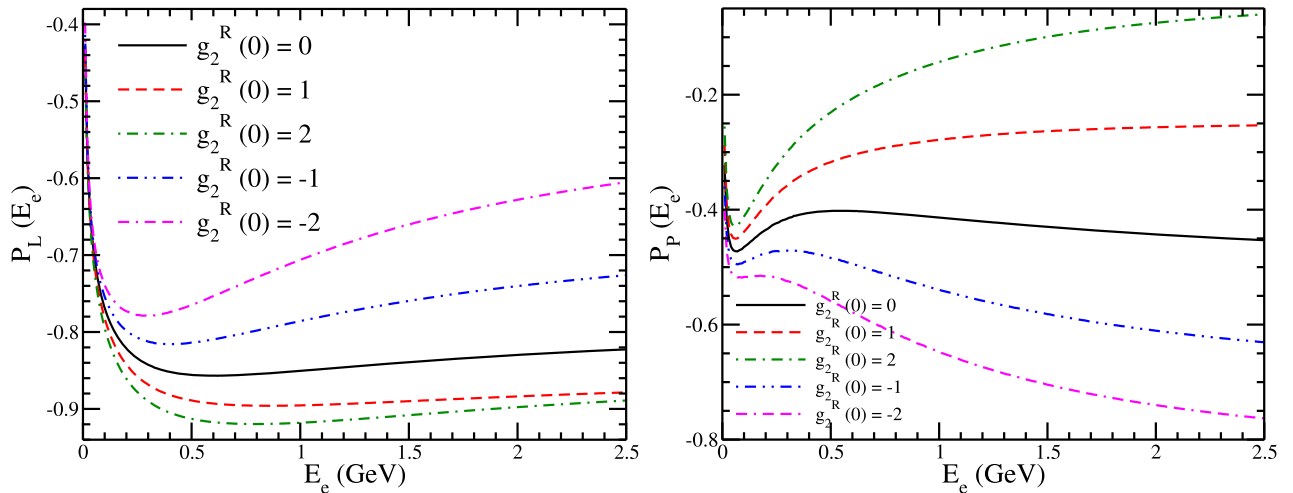


FIG. 14. $P_L(E_e)$ (left panel), $P_P(E_e)$ (middle panel), and $P_T(E_e)$ (right panel) vs. E_e for the process $e^- + p \rightarrow \nu_e + n$, when the final nucleon is polarized, using the different values of $g_2^R(0)$. Lines and points have the same meaning as in Fig. 3.

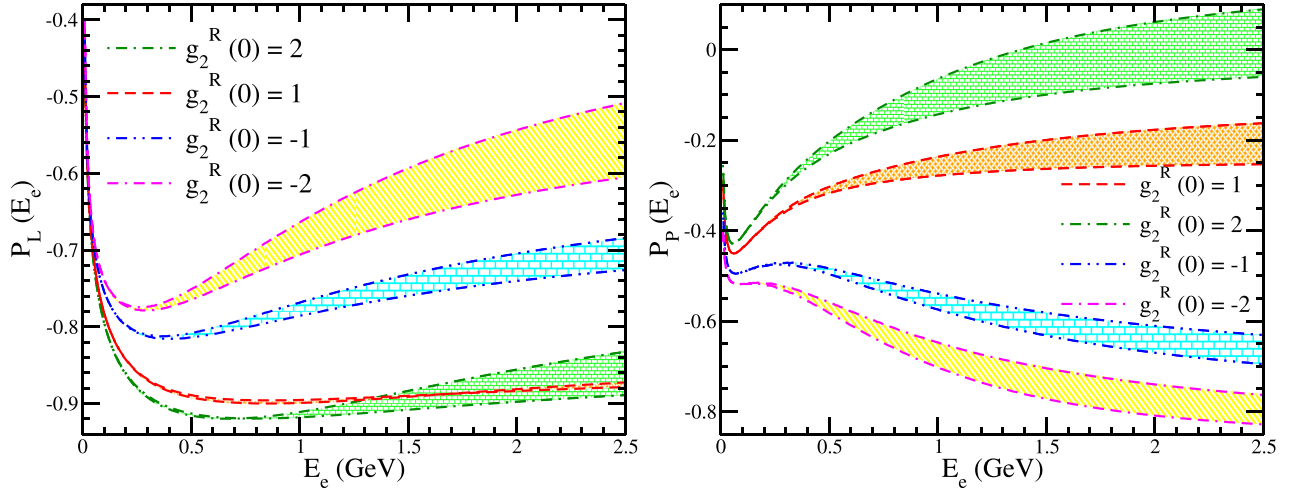


FIG. 15. $P_L(E_e)$ (left panel) and $P_P(E_e)$ (right panel) vs. E_e for the process $e^- + p \rightarrow \nu_e + n$, when the final nucleon is polarized, using the different values of $g_2^R(0)$. Lines and points have the same meaning as in Fig. 3 and the band corresponds to M_2 in the range 1.026–1.35 GeV.

in the peak region of Q^2 and remain of similar order for $g_2^R(0) = 1-2$.

2. Polarization observables in the case of T violation

In Fig. 17, we have presented the results for $P_L(E_e)$, $P_P(E_e)$, and $P_T(E_e)$ vs E_e using $g_2^I(0)$ in the range 0–2. The negative values of $g_2^I(0)$ yield the same results for $P_L(E_e)$ and $P_P(E_e)$ as obtained for positive values of

$g_2^I(0)$, while for $P_T(E_e)$, the negative values of $g_2^I(0)$ changes sign keeping the magnitude same, therefore, the results obtained using $g_2^I(0) < 0$ are not depicted in the figure. It should be noted from the figure that the longitudinal and perpendicular components of the final nucleon polarization are not much sensitive to the variation in $g_2^I(0)$, while the transverse component of polarization, which arises due to the T -violating effect, is quite sensitive to the variation in $g_2^I(0)$. The longitudinal and perpendicular

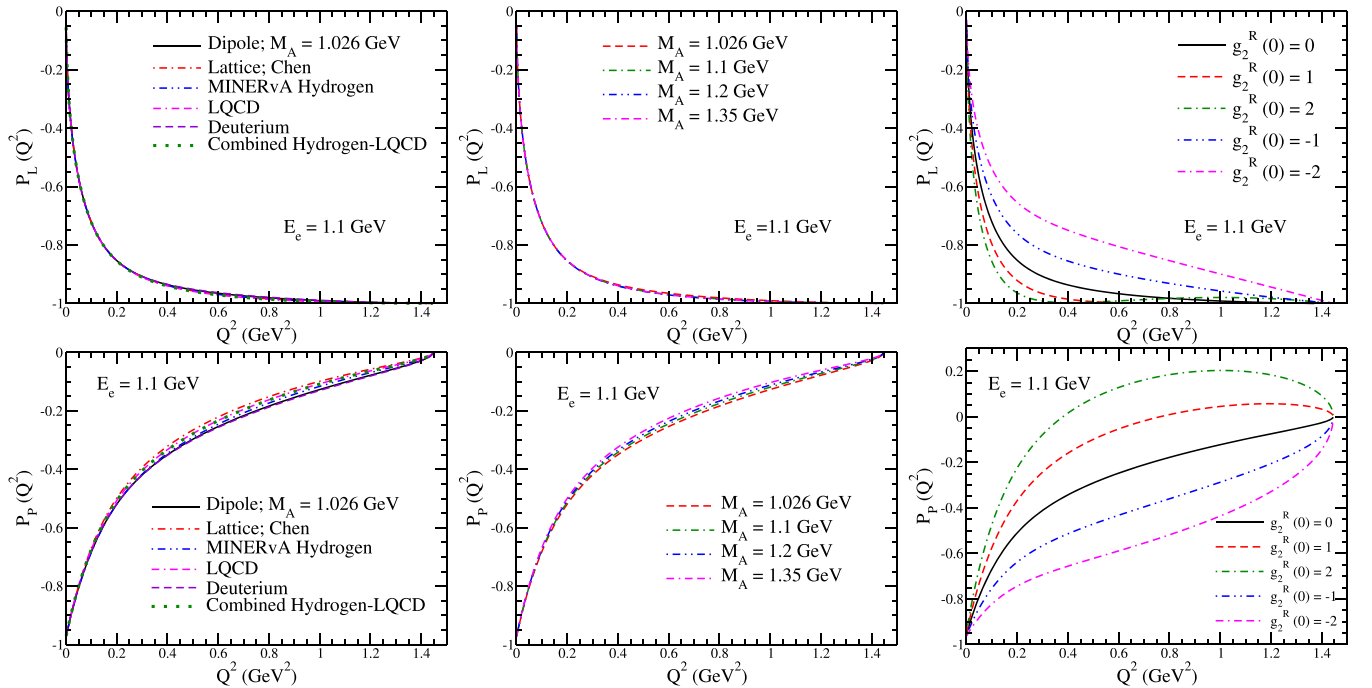


FIG. 16. $P_L(Q^2)$ (top panel) and $P_P(Q^2)$ (bottom panel) as a function of Q^2 for the process $e^- + p \rightarrow \nu_e + n$, when the final nucleon is polarized, for the different parametrizations of $g_1(Q^2)$ (left panel), different values of M_A (middle panel), and different values of $g_2^R(0)$ (right panel) at $E_e = 1.1$ GeV. Lines and points have the same meaning as in Fig. 3.

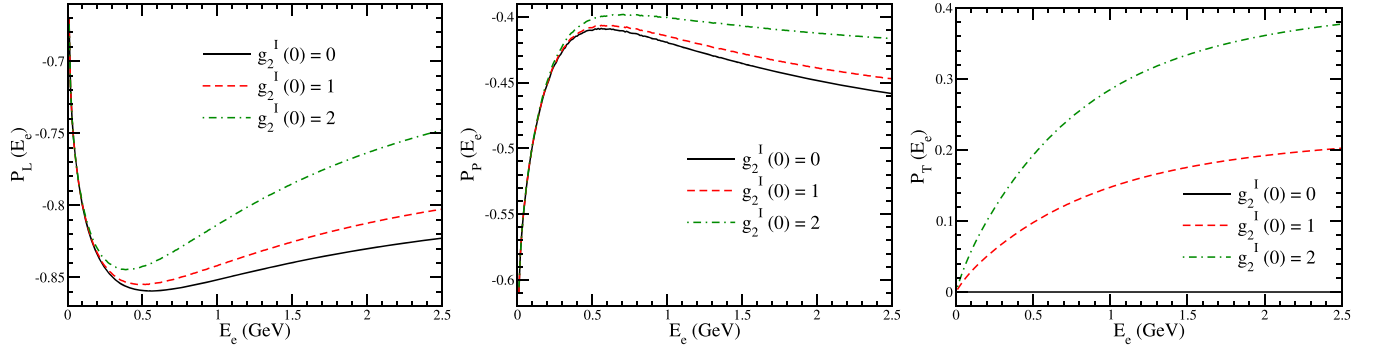


FIG. 17. $P_L(E_e)$ (left panel), $P_P(E_e)$ (middle panel), and $P_T(E_e)$ (right panel) vs. E_e for the process $e^- + p \rightarrow \nu_e + n$, when the final nucleon is polarized, using the different values of $g_2^l(0)$ viz., $g_2^l(0) = 0$ (solid line), 1 (dashed line), and 2 (dash-dotted line).

components of the neutron polarization show an increment of about 2% when $g_2^l(0)$ is increased from 0 to 2, at $E_e = 0.5$ GeV, which increases with increasing electron energy and becomes 4% and 8%, respectively, at $E_e = 1$ and 2 GeV. While $P_T(E_e)$ shows an increment of about 20% at $E_e = 0.5$ GeV, which becomes 30% and 35%, respectively, at $E_e = 1$ and 2 GeV.

To study the effect of M_2 variation on $P_L(E_e)$, $P_P(E_e)$, and $P_T(E_e)$, the results are presented in Fig. 18 for fixed values of $g_2^l(0) = 1$ and 2, while M_2 is varied in the range 1.026–1.35 GeV. The polarization observables of the final nucleon are quite sensitive to variation in M_2 , especially in the energy region $E_e > 0.5$ GeV. Moreover, for $P_L(E_e)$ and $P_P(E_e)$, the width of the band increases with increasing $g_2^l(0)$, whereas for $P_T(E_e)$, the width of the M_2 band remains almost unchanged as $g_2^l(0)$ increases.

Figure 19 shows the results for the neutron polarization components $P_L(Q^2)$, $P_P(Q^2)$, and $P_T(Q^2)$ as a function of Q^2 at $E_e = 1.1$ GeV, obtained using different values of $g_2^l(0)$ in the range 0–2 in Eq. (A6). It may be observed from the figure that $P_L(Q^2)$ exhibits some dependence on the choice of $g_2^l(0)$, especially when $g_2^l(0) = 2$ is used in the numerical calculations. This dependence increases with increasing electron energy. However, there is almost no

effect of the different choice of $g_2^l(0)$ ($0 \leq g_2^l(0) \leq 2$) on $P_P(Q^2)$ for all values of Q^2 and E_e considered in the present work. In contrast, $P_T(Q^2)$ shows a significant dependence on $g_2^l(0)$, particularly in the peak region of Q^2 . The transverse polarization component $P_T(Q^2)$ increases by about 30% at $E_e = 855$ MeV, when $g_2^l(0)$ is increased from 0 to 2. This increase becomes more pronounced with increasing electron energy, reaching approximately 35% and 45% at $E_e = 1.1$ GeV and 2.2 GeV, respectively. Therefore, studying $P_T(Q^2)$ for a polarized neutron produced in the scattering of electron from the free proton target could be a promising alternative to investigate the second class current.

V. SUMMARY AND CONCLUSION

In this work, we have investigated the total scattering cross section (σ), the differential cross section ($d\sigma/dQ^2$), the longitudinal [$A_L(E_e, Q^2)$] and perpendicular [$A_P(E_e, Q^2)$] spin asymmetries of the initial polarized proton as well as the longitudinal [$P_L(E_e, Q^2)$], perpendicular [$P_P(E_e, Q^2)$], and transverse [$P_T(E_e, Q^2)$] polarization components of the final neutron, in the weak charged current induced electron-proton scattering. This

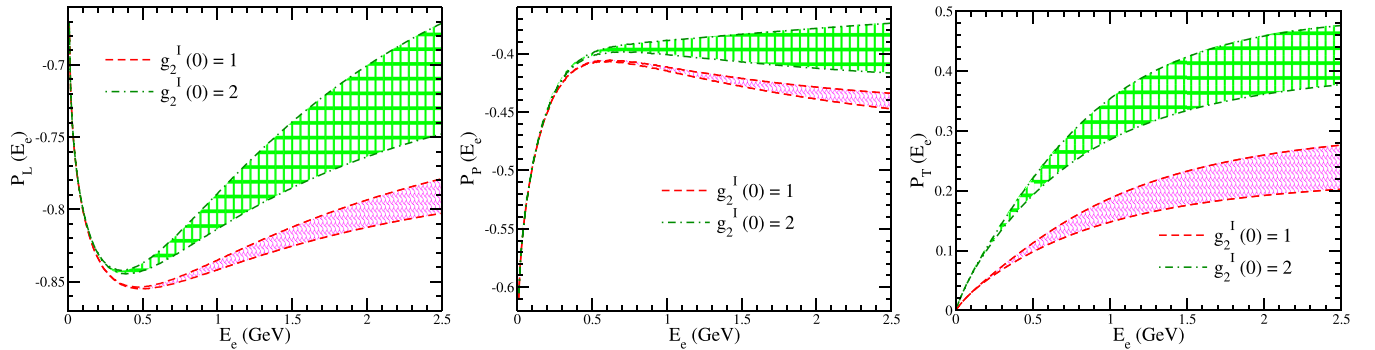


FIG. 18. $P_L(E_e)$ (left panel), $P_P(E_e)$ (middle panel), and $P_T(E_e)$ (right panel) vs. E_e for the process $e^- + p \rightarrow \nu_e + n$, when the final nucleon is polarized, using the different values of $g_2^l(0)$. Lines and points have the same meaning as in Fig. 17 and the band corresponds to M_2 in the range 1.026–1.35 GeV.

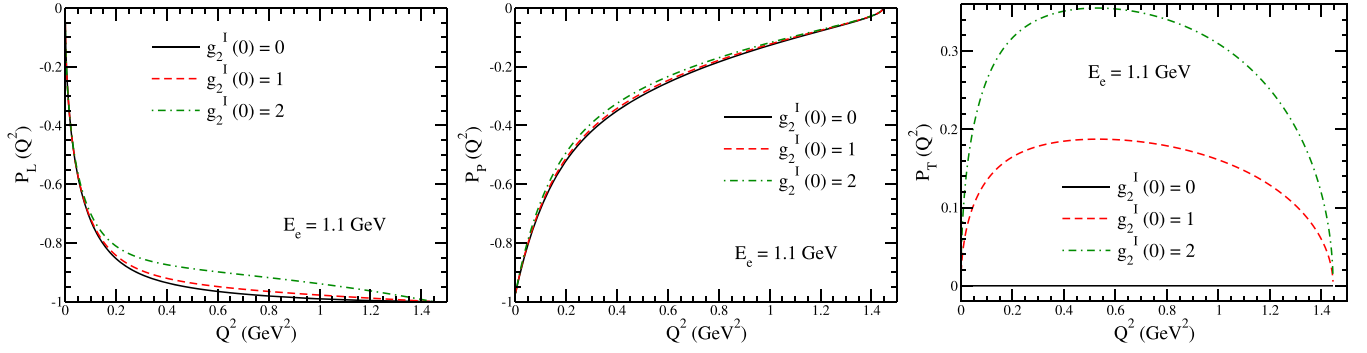


FIG. 19. $P_L(Q^2)$ (left panel), $P_P(Q^2)$ (middle panel), and $P_T(Q^2)$ (right panel) as a function of Q^2 for the process $e^- + p \rightarrow \nu_e + n$, when the final nucleon is polarized, for the different values of $g_2^1(0)$ at $E_e = 1.1$ GeV. Lines and points have the same meaning as in Fig. 17.

study is motivated by experiments being planned at the Thomas Jefferson National Accelerator Facility (JLab). The calculations have been performed both under the assumption of time-reversal (T) invariance and without imposing T invariance. Numerical results are presented for all the above mentioned observables, and their sensitivities to the axial vector and weak electric form factors are examined.

Our main observations are as follows:

(1) Total and differential scattering cross section

- (a) The choice of $g_1(Q^2)$ used in the literature results in substantial change in the total cross section (σ). Specifically, the combined hydrogen-LQCD fit and the lattice fit by Chen *et al.* [70] result in an increase of about 40% and 50%, respectively, in the cross section from the results obtained using the dipole fit with the world average value of M_A .
- (b) The recommendation for the different values of M_A obtained in the recent experiments like MINERvA, MicroBooNE, MiniBooNE, K2K, T2K, NOMAD, etc., varying in the range 1.026 GeV to 1.35 GeV, results in an increase in the total cross section by about 40% ($M_A = 1.026$ GeV vs. 1.35 GeV).
- (c) A nonzero value of the weak electric form factor $g_2(Q^2)$, associated with the second class currents, irrespective of whether $g_2(0)$ is purely real or purely imaginary, results in an increase in the total cross section, which is about 10% for $g_2(0) \approx 2$ and is small around 2% for $g_2(0) \approx 1$ for electron energy above 1 GeV, considered in this work.
- (d) In the case of the Q^2 dependence of the differential scattering cross section $\frac{d\sigma}{dQ^2}$, we observe similar results as in the case of σ for the different parametrizations of $g_1(Q^2)$ as well as for the choice of the value of M_A and $g_2(0)$.

(2) Spin asymmetry of the initial nucleon

- (a) The choice of the different parametrizations for $g_1(Q^2)$ to obtain $A_L(E_e)$ and $A_P(E_e)$ results in not very significant change ($< 2-3\%$) in these observables.
 - (b) When the different values of M_A are considered to obtain $A_L(E_e)$ and $A_P(E_e)$, we observe hardly any variation except for $M_A = 1.35$ GeV vs. $M_A = 1.026$ GeV, which is also $< 2\%$.
 - (c) For a nonzero $g_2^R(0)$, we find strong dependence for both $A_L(E_e)$ and $A_P(E_e)$, specially at high electron energies considered in this work.
 - (d) The Q^2 dependent spin asymmetries of the initial nucleon show very little dependence on the choice of $g_1(Q^2)$ parametrization or on the value of M_A considered in this work.
 - (e) When a nonzero value of $g_2^R(0)$ is taken into account, both $A_L(Q^2)$ and $A_P(Q^2)$ show strong dependence on the choice of $g_2^R(0)$ at all values of Q^2 and E_e considered in the present work.
- (3) Polarization observables of the final nucleon
- (a) In the case of the perpendicular polarization $P_P(E_e)$ vs. E_e , we find some dependence on the choice of $g_1(Q^2)$, like $P_P(E_e)$ increases by 15%–18% when LQCD prescription of Chen *et al.* [70] is used in comparison to the results obtained using the dipole parametrization, while this dependence is small around 2%–4% in the case of the longitudinal polarization $P_L(E_e)$.
 - (b) When the different values for M_A are used to obtain $P_L(E_e)$ and $P_P(E_e)$, we find that the results in the case of $P_P(E_e)$ change by about 13%–15%, whereas in the case of $P_L(E_e)$, the results change by about 2%–3% when M_A is changed to $M_A = 1.35$ GeV from $M_A = 1.026$ GeV.
 - (c) In the case of $g_2^R(0)$ variation assuming T invariance, $P_L(E_e)$ and $P_P(E_e)$ show a strong dependence on the second class current form factor, when a nonzero value of $g_2^R(0)$ is taken in the numerical calculations.

- (d) In the case of $g_2^I(0)$ variation assuming T violation, $P_L(E_e)$ and $P_P(E_e)$ show a mild dependence, which is about 4%–8% in the electron energy region considered in the present work, while the transverse component of polarization $P_T(E_e)$ shows a considerable enhancement, which is about 30%–35% for $g_2^I(0) = 2$ vs. 0 at $E_e = 1 - 2$ GeV.
- (e) In the case of M_2 variation in $g_2^R(Q^2)$, we find both $P_L(E_e)$ and $P_P(E_e)$ to be sensitive to the choice of M_2 .
- (f) In the case of M_2 variation in $g_2^I(Q^2)$, we find that in the case of $P_L(E_e)$ and $P_P(E_e)$, the width of the M_2 band increases with increase in $g_2^I(0)$, while $P_T(E_e)$ shows constant bands for the different $g_2^I(0)$ values.
- (g) When $P_L(Q^2)$ and $P_P(Q^2)$ are obtained at different electron energies, using the different parametrizations for $g_1(Q^2)$ or using the different values for M_A , we hardly find any change in the numerical results.
- (h) We find a strong dependence of both $P_L(Q^2)$ and $P_P(Q^2)$ on $g_2^R(0)$ variation at all values of Q^2 and E_e considered in the present work.
- (i) When a nonzero value of $g_2^I(0)$ is considered, we find small change in $P_L(Q^2)$, which becomes almost negligible for $P_P(Q^2)$, whereas in the case of $P_T(Q^2)$, there is a substantial dependence on the choice of $g_2^I(0)$, which is about 30%–40% in the peak region of Q^2 ($g_2^I(0) = 2$ vs. 0).

Thus, to conclude the electron beam facilities at Jefferson Lab (JLab), Mainz Microtron (MAMI), and the upcoming Electron-Ion Collider (EIC) offer a uniquely powerful opportunity to explore the weak nucleon form factors. With their monochromatic energy beams, precisely controlled kinematics, and exceptionally high luminosity, these facilities can deliver clean and systematically robust measurements that are difficult to achieve in neutrino experiments. Ultimately, these advances would directly empower the neutrino physics community to extract oscillation parameters with substantially improved precision and reliability, strengthening the overall scientific reach of next-generation neutrino experiments.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data are not publicly available. The data are available upon reasonable request from the authors.

APPENDIX A: WEAK FORM FACTORS

The weak vector and axial vector form factors are constrained by well-established symmetry considerations of the vector and axial vector weak currents like T-invariance, G-invariance, CVC and PCAC hypotheses, etc. [44,72,82–86]. In the following, we give the explicit expressions of the vector and axial vector form factors:

- (i) The vector form factors are expressed as

$$f_i(Q^2) = f_i^p(Q^2) - f_i^n(Q^2), \quad i = 1, 2 \quad (\text{A1})$$

where $f_i^{p,n}(Q^2)$ are the electromagnetic form factors of the nucleon, which, in turn, are expressed in terms of the Sachs' electric and magnetic form factors $G_E^{p,n}(Q^2)$ and $G_M^{p,n}(Q^2)$ of the nucleons. For $G_E^{p,n}(Q^2)$ and $G_M^{p,n}(Q^2)$ various parametrizations are available in the literature and in our numerical calculations, we have used the parametrization given by Bradford *et al.* [87], known in the literature as BBBA05.

- (ii) For the axial vector form factor $g_1(Q^2)$, traditionally a dipole parametrization has been used

$$g_1(Q^2) = g_A(0) \left(1 + \frac{Q^2}{M_A^2} \right)^{-2}, \quad (\text{A2})$$

where M_A is the axial dipole mass and $g_A(0)$ is the axial charge. For the numerical calculations, we have used the world average value of $M_A = 1.026$ GeV [17] unless stated, and the value of $g_A(0) = 1.267$ is determined from the neutron beta decays [7].

- (iii) Recently, in the literature, the axial vector form factor is determined using the z -expansion [52], which provides a conformal mapping that transforms Q^2 into a small expansion variable z across the entire kinematic region relevant to quasielastic scattering. The conformal variable z is defined as

$$z = \frac{\sqrt{t_c + Q^2} - \sqrt{t_c - t_0}}{\sqrt{t_c + Q^2} + \sqrt{t_c - t_0}}, \quad (\text{A3})$$

where the parameter $t_c \leq (3m_\pi)^2$, with m_π being the mass of the pion, is fixed by the particle production threshold for the axial vector current interaction, and the variable $t_0 = -0.5$ GeV² fixes the value of Q^2 for which $z = 0$ is satisfied.

Within this framework, the axial vector form factor is expressed as a power series in the variable z

$$g_1(z) = \sum_{k=0}^{\infty} a_k z^k. \quad (\text{A4})$$

In general, the sum over k is truncated at a finite order $k_{\max}$, which is determined by the fit range and data

TABLE I. Values of the parameters a_k ; ($k = 0-6$) as obtained in Ref. [57].

	MINERvA hydrogen fit	LQCD fit	deuterium fit	Combined hydrogen-LQCD fit
a_0	0.61490770	0.71742019	0.54264533	0.71070233
a_1	-1.64778080	-1.72089706	-2.08493637	-1.74307738
a_2	0.94181417	0.30982708	1.89831616	0.37944565
a_3	0.41239729	1.62125837	2.40319245	1.69894456
a_4	0.36611559	-0.27506993	-5.88979056	-0.60326876
a_5	-1.18722194	-1.25297945	4.14554900	-0.95690585
a_6	0.49976799	0.60044079	-1.01497601	0.51415945

accuracy. In this work, we have followed Ref. [56,57] for this fitting, where the authors concluded that the best fit is obtained with $k_{\max} = 6$ [57].

The values of the parameters a_k ; ($k = 0-6$) for the MINERvA hydrogen, LQCD, previous deuterium, and combined MINERvA hydrogen and LQCD fits as obtained in Ref. [57] are tabulated in Table I.

- (iv) Chen and Roberts [70] used lattice gauge formalism to determine the axial vector form factor and parametrized it as a function of x , where $x = Q^2/m_N^2$, with $m_N = 1.18$ GeV

$$g_1(x) = \frac{1.248 + 0.039x}{1 + 1.417x + 0.318x^2 + 0.071x^3}. \quad (\text{A5})$$

To study the dependence of the parametrizations of the axial vector form factor on Q^2 , in Fig. 20, we show the results of $g_1(Q^2)$ as a function of Q^2 using (i) the dipole parametrization with two values of M_A viz. $M_A = 1.026$ and 1.35 GeV, (ii) the z -expansion parametrization [57] using (a) MINERvA hydrogen data, (b) LQCD, (c) deuterium, and (d) combined hydrogen-LQCD fits, and (iii) the lattice gauge parametrization given by Chen and Roberts [70].

It may be observed from the figure that the parametrization of Chen and Roberts [70] yields the largest value of $g_1(Q^2)$, while the results for the z -expansion of deuterium data yields the smallest value of $g_1(Q^2)$ at all values of Q^2 . The results of the MINERvA hydrogen data for $g_1(Q^2)$ are consistent with the dipole parametrization with $M_A = 1.026$ GeV in the region of $Q^2 \leq 0.4$ GeV², however, with the increase in Q^2 , the MINERvA data shows a larger value for $g_1(Q^2)$ as compared to the dipole parametrization with $M_A = 1.026$ GeV. The results obtained with z expansion of the LQCD calculations and combined MINERvA and LQCD calculations overlap in the entire range of Q^2 considered in this work, and these results are comparable with the results obtained with the dipole parametrization of $g_1(Q^2)$ with $M_A = 1.35$ GeV.

Thus, the results obtained with the z -expansion of combined LQCD and MINERvA data are almost 30% larger than the results obtained with the traditional dipole parametrization with $M_A = 1.026$ GeV in the entire region of Q^2 considered in the present work.

- (v) The weak electric form factor $g_2(Q^2)$ is taken to be of dipole form, i.e.,

$$g_2(Q^2) = g_2(0) \left(1 + \frac{Q^2}{M_2^2}\right)^{-2}, \quad (\text{A6})$$

where M_2 is the axial dipole mass corresponding to the weak electric form factor $g_2(Q^2)$. For simplicity, in the numerical calculations, we have taken $M_2 = M_A = 1.026$ GeV [65]. Moreover, throughout this paper, whenever the variation in M_A is studied, we

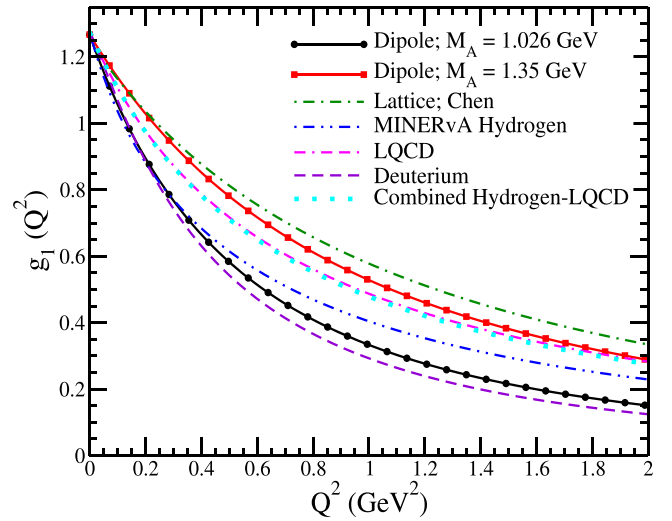


FIG. 20. $g_1(Q^2)$ vs. Q^2 using the dipole parametrization with $M_A = 1.026(1.35)$ GeV shown by solid lines with circle (square). The results obtained using the lattice gauge parametrization of Chen and Roberts [70] are shown by the dash-dotted line. The results obtained using the z -expansion [56,57] for the MINERvA hydrogen, LQCD, deuterium, and combined hydrogen-LQCD fits are represented by double-dot-dashed line, double dash-dotted line, dashed line and dotted line, respectively.

have fixed M_2 to be 1.026 GeV and used the different values for M_A in the range 1.026–1.35 GeV. Similarly, to study M_2 variation, we fixed M_A to be 1.026 GeV and varied M_2 in the range 1.026–1.35 GeV.

There exists limited experimental information on $g_2(0)$, which has been extracted from the analyses of nuclear β decays [88–91], muon capture [92] and the (anti)neutrino quasielastic scattering performed at BNL [26,93] and SKAT [21]. While the nuclear β decay and muon capture experiments obtained a small value of $g_2(0)$ namely $g_2(0) = (0.504 \pm 1.134)$ [92] and $g_2(0) = 0 \pm 0.075$ [88–91,94], the neutrino experiments reported limits on $g_2(0)$, which are quite large. For example, the neutrino induced quasielastic scattering using bubble chamber detector [26] and antineutrino quasielastic scattering using scintillator detector [93] recommended, respectively, the real values of $g_2(0)$ to be -3.69 ± 1.26 and -1.008 . Furthermore, Belikov *et al.* [21] at SKAT used neutrino and antineutrino quasielastic scattering data to estimate the upper limit for $g_2(0)$ to be -1.63 at 90% CL. Some of the earlier theoretical calculations for the (anti)neutrino-nucleon/nucleus scattering were performed assuming real as well as imaginary values of $g_2(0)$. For example; Fujii and Yamaguchi [95,96] considered both the real as well as imaginary values of $g_2(0)$ taken as 1.92, while Berman and Veltman [97] used $\text{Im } g_2(0)$ to be 3.7 and 6. The best fit value for the transverse polarization calculated by Rujula and Rafael [98] corresponds to the range $2 \leq g_2(0) \leq 3$. Day and McFarland [94] have used $\text{Re } g_2(0)$ taken as ~ 0.2 in their calculations. Therefore, following our earlier works [62,65,66], we have used $g_2(0)$ in the range $[-2, 2]$ both for real and imaginary values, corresponding to T invariance and its violation, respectively.

APPENDIX B: SPIN OBSERVABLES

1. Spin asymmetry of the polarized target nucleon

In the rest frame of the initial nucleon, the nucleon spin vector $\zeta^\tau = (0, \vec{\zeta})$, where $\vec{\zeta}$ is expressed in terms of the orthogonal vectors $\hat{e}_i (i = L, P, T)$, i.e.,

$$\vec{\zeta} = \zeta_L \hat{e}_L + \zeta_P \hat{e}_P + \zeta_T \hat{e}_T, \quad (\text{B1})$$

where $\hat{e}_L$, $\hat{e}_P$, and $\hat{e}_T$ are chosen to be the set of orthogonal unit vectors corresponding to the longitudinal, perpendicular, and transverse directions with respect to the momentum of the initial electron, shown in Fig. 2, and are written as [74]

$$\hat{e}_L = \frac{\vec{k}}{|\vec{k}|}, \quad \hat{e}_T = \frac{\vec{q} \times \vec{k}}{|\vec{q} \times \vec{k}|}, \quad \hat{e}_P = \frac{\vec{k}}{|\vec{k}|} \times \frac{\vec{q} \times \vec{k}}{|\vec{q} \times \vec{k}|}. \quad (\text{B2})$$

Since the transverse component lies perpendicular to the reaction plane and vanishes when T invariance is assumed, therefore, the spin vector ζ^τ is expressed only in terms of $\hat{e}_L$ and $\hat{e}_P$.

The longitudinal and perpendicular components of the polarization vector $\vec{\zeta}_{L,P}(Q^2)$ using Eqs. (B1) and (B2) may be written as

$$\zeta_{L,P}(Q^2) = \vec{\zeta} \cdot \hat{e}_{L,P}. \quad (\text{B3})$$

In the rest frame of the initial nucleon, the polarization vector $\vec{\zeta}$ is expressed as

$$\vec{\zeta} = \alpha(E_e, Q^2) \vec{k} + \beta(E_e, Q^2) \vec{q}, \quad (\text{B4})$$

and is explicitly obtained using Eq. (10). The expressions for the coefficients $\alpha(E_e, Q^2)$, and $\beta(E_e, Q^2)$ are given in the Appendix C.

The longitudinal $[A_L(Q^2)]$ and perpendicular $[A_P(Q^2)]$ spin asymmetries of the initial nucleon are written as

$$A_L(Q^2) = \zeta_L(Q^2), \quad A_P(Q^2) = \zeta_P(Q^2). \quad (\text{B5})$$

The expressions for $A_L(Q^2)$ and $A_P(Q^2)$ are calculated using Eqs. (B3) and (B4) in Eq. (B5) and the explicit expressions are given in Eqs. (11) and (12).

2. Polarization components of the final nucleon

Using the covariant density matrix formalism, the polarization 4-vector (ξ^τ) of the final nucleon produced in the reaction, given in Eq. (1) is written as [69]

$$\xi^\tau = \left(g^{\tau\sigma} - \frac{p'^\tau p'^\sigma}{M^2} \right) \frac{\mathcal{L}^{\alpha\beta} \text{Tr}[\gamma_\sigma \gamma_5 \Lambda(p') J_\alpha \Lambda(p) \tilde{J}_\beta]}{\mathcal{L}^{\alpha\beta} \text{Tr}[\Lambda(p') J_\alpha \Lambda(p) \tilde{J}_\beta]}. \quad (\text{B6})$$

One may write the polarization vector $\vec{\xi}$ in terms of the three orthogonal vectors $\hat{e}_i (i = L, P, T)$, i.e.,

$$\vec{\xi} = \xi_L \hat{e}_L + \xi_P \hat{e}_P + \xi_T \hat{e}_T, \quad (\text{B7})$$

where $\hat{e}_L$, $\hat{e}_P$ and $\hat{e}_T$ are chosen to be the set of orthogonal unit vectors corresponding to the longitudinal, perpendicular and transverse directions with respect to the momentum of the final nucleon, shown in Fig. 2(b), and are written as

$$\hat{e}_L = \frac{\vec{p}'}{|\vec{p}'|}, \quad \hat{e}_P = \hat{e}_L \times \hat{e}_T, \quad \hat{e}_T = \frac{\vec{p}' \times \vec{k}}{|\vec{p}' \times \vec{k}|}. \quad (\text{B8})$$

The longitudinal, perpendicular and transverse components of the polarization vector $\vec{\xi}_{L,P,T}(Q^2)$ using Eqs. (B7) and (B8) may be written as

$$\xi_{L,P,T}(Q^2) = \vec{\xi} \cdot \hat{e}_{L,P,T}. \quad (\text{B9})$$

In the case of final nucleon polarization, we have considered two cases: (i) when time reversal invariance is assumed, and (ii) when time reversal is violated. As already discussed in Appendix A, the assumption of T invariance implies all the form factors to be real, therefore, in the first case we have taken into account purely real values of $g_2(0)$ and represent it by $g_2^R(0)$. Moreover, in the second case, when time reversal is violated, the numerical calculations are performed by taking into account purely imaginary values for $g_2(0)$ and represent it by $g_2^I(0)$.

a. T invariance

In the rest frame of the initial nucleon, assuming T invariance, the polarization vector $\vec{\xi}$ is expressed as

$$\vec{\xi} = A(E_e, Q^2)\vec{k} + B(E_e, Q^2)\vec{p}' \quad (\text{B10})$$

and is explicitly calculated using Eq. (B6). The expressions for the coefficients $A(E_e, Q^2)$ and $B(E_e, Q^2)$, obtained using the real values of the weak electric form factor, associated with the second class currents, i.e., $g_2(Q^2) = g_2^R(Q^2)$, are given in Appendix D.

The longitudinal $[P_L(Q^2)]$ and perpendicular $[P_P(Q^2)]$ components of the polarization vector in the rest frame of the final nucleon is then obtained by performing a Lorentz boost and is written as [65]

$$P_L(Q^2) = \frac{M}{E'} \xi_L(Q^2), \quad P_P(Q^2) = \xi_P(Q^2), \quad (\text{B11})$$

where E' is the energy of the outgoing nucleon. The expressions for $P_L(Q^2)$ and $P_P(Q^2)$ are then obtained using Eqs. (B8), (B9) and (B10) in Eq. (B11) and are given in Eqs. (13) and (14).

b. T violation

To study the effect of T violation on the polarization observables of the final nucleon, the polarization vector $\vec{\xi}$, in the rest frame of the initial nucleon, is expressed as

$$\vec{\xi} = A'(E_e, Q^2)\vec{k} + B'(E_e, Q^2)\vec{p}' + C'(E_e, Q^2)M(\vec{k} \times \vec{p}') \quad (\text{B12})$$

and is explicitly calculated using Eq. (B6). The expressions for the coefficients $A'(E_e, Q^2)$, $B'(E_e, Q^2)$ and $C'(E_e, Q^2)$ are given in Appendix E.

The longitudinal $[P_L(Q^2)]$ and perpendicular $[P_P(Q^2)]$ components of the polarization vector in the rest frame of the final nucleon are given in Eq. (B11) and the transverse $[P_T(Q^2)]$ component of polarization is written as [65]

$$P_T(Q^2) = \xi_T(Q^2). \quad (\text{B13})$$

In the case of T violation, the explicit expressions for $P_L(Q^2)$ and $P_P(Q^2)$ are the same as given in Eqs. (13) and (14), except that the coefficients $A(E_e, Q^2)$ and $B(E_e, Q^2)$ are now replaced with $A'(E_e, Q^2)$ and $B'(E_e, Q^2)$. The expression for $P_T(Q^2)$ is given in Eq. (15).

APPENDIX C: EXPRESSION FOR $N(E_e, Q^2)$, $\alpha(E_e, Q^2)$, AND $\beta(E_e, Q^2)$

The expressions for $N(E_e, Q^2)$, $\alpha(E_e, Q^2)$, and $\beta(E_e, Q^2)$ are given in terms of the four form factors; $f_{1,2}(Q^2)$ and $g_{1,2}(Q^2)$ as

$$\begin{aligned} N(E_e, Q^2) = & 2f_1^2(Q^2)[8E_e^2M^2 - 4E_eMQ^2 - 2M^2Q^2 + Q^4] + f_2^2(Q^2)\left[4E_e^2Q^2 - \frac{2E_eQ^4}{M} + Q^4\right] \\ & + 2g_1^2(Q^2)[8E_e^2M^2 - 4E_eMQ^2 + 2M^2Q^2 + Q^4] + |g_2(Q^2)|^2\left[4E_e^2Q^2 - \frac{2E_eQ^4}{M} - Q^4\right] \\ & + 4f_1(Q^2)f_2(Q^2)Q^4 + 4f_1(Q^2)g_1(Q^2)[4E_eMQ^2 - Q^4] + 4f_2(Q^2)g_1(Q^2)[4E_eMQ^2 - Q^4] \end{aligned} \quad (\text{C1})$$

$$\begin{aligned} \alpha(E_e, Q^2) = & 8f_1^2(Q^2)MQ^2 - \frac{2Q^4}{M}f_2^2(Q^2) + 2f_1(Q^2)f_2(Q^2)\left[4MQ^2 - \frac{Q^4}{M}\right] + 8f_1(Q^2)g_1(Q^2)M[4E_eM - Q^2] \\ & + 2f_1(Q^2)g_2^R(Q^2)\left[4E_eQ^2 - \frac{Q^4}{M}\right] + 2f_2(Q^2)g_1(Q^2)\left[\frac{Q^4}{M} - 4E_eQ^2\right] \\ & - 2f_2(Q^2)g_2^R(Q^2)\left[\frac{Q^4}{M} - 4E_eQ^2\right] + 2g_1(Q^2)g_2^R(Q^2)\left[\frac{Q^4}{M} + 4MQ^2\right] \end{aligned} \quad (\text{C2})$$

$$\begin{aligned}
\beta(E_e, Q^2) = & -4f_1^2(Q^2)MQ^2 + 4f_2^2(Q^2)E_eQ^2 + 4g_1^2(Q^2)M(4E_eM - Q^2) + 4f_1(Q^2)f_2(Q^2)Q^2[E_e - M] \\
& + 8f_1(Q^2)g_1(Q^2)M(Q^2 - 2E_eM) + 4f_1(Q^2)g_2^R(Q^2)[-4E_e^2M + E_eQ^2 + MQ^2] \\
& + 4f_2(Q^2)g_1(Q^2)[4E_e^2M - E_eQ^2 + MQ^2] - 4f_2(Q^2)g_2^R(Q^2)E_eQ^2 \\
& - 4g_1(Q^2)g_2^R(Q^2)Q^2(E_e + M)
\end{aligned} \tag{C3}$$

APPENDIX D: EXPRESSIONS FOR $A(E_e, Q^2)$ AND $B(E_e, Q^2)$

Assuming T invariance, the expressions for the coefficients $A(E_e, Q^2)$ and $B(E_e, Q^2)$ are given by

$$\begin{aligned}
A(E_e, Q^2) = & 8f_1^2(Q^2)MQ^2 - 2f_2^2(Q^2)\frac{Q^4}{M} + 2f_1(Q^2)f_2(Q^2)\left[4MQ^2 - \frac{Q^4}{M}\right] \\
& + 8f_1(Q^2)g_1(Q^2)M[4E_eM - Q^2] + f_1(Q^2)g_2^R(Q^2)\left[\frac{2Q^4}{M} - 8E_eQ^2\right] \\
& + 2f_2(Q^2)g_1(Q^2)\left[\frac{2Q^4}{M} - 8E_eQ^2\right] + 2f_2(Q^2)g_2^R(Q^2)\left[\frac{2Q^4}{M} - 8E_eQ^2\right] \\
& - g_1(Q^2)g_2^R(Q^2)\left[\frac{2Q^4}{M} + 8MQ^2\right]
\end{aligned} \tag{D1}$$

$$\begin{aligned}
B(E_e, Q^2) = & 2f_1^2(Q^2)\left[\frac{Q^2(-4E_eM - 2M^2 + Q^2)}{M}\right] + f_2^2(Q^2)\left[\frac{2Q^4}{M} - 4E_eQ^2\right] \\
& - 2g_1^2(Q^2)\left[\frac{(2M^2 + Q^2)(4E_eM - Q^2)}{M}\right] - 4f_1(Q^2)f_2(Q^2)\left[\frac{Q^2(3E_eM + M^2 - Q^2)}{M}\right] \\
& - 4f_1(Q^2)g_1(Q^2)\left[\frac{(8E_e^2M^2 + 4E_e(M^3 - MQ^2) + Q^4)}{M}\right] \\
& + 4f_1(Q^2)g_2^R(Q^2)[-4E_e^2M + 3E_eQ^2 + MQ^2] \\
& + 4f_2(Q^2)g_1(Q^2)\left[-4E_e^2M + 3E_eQ^2 - \frac{4Q^2(M^2 + Q^2)}{M}\right] \\
& + 2f_2(Q^2)g_2^R(Q^2)\left[\frac{Q^2(4E_e^2M + 2E_e(M^2 - Q^2) - MQ^2)}{M^2}\right] + 4Q^2g_1(Q^2)g_2^R(Q^2)[E_e + M]
\end{aligned} \tag{D2}$$

APPENDIX E: EXPRESSIONS FOR $A'(E_e, Q^2)$, $B'(E_e, Q^2)$, AND $C'(E_e, Q^2)$

In the case of T violation, the expressions for the coefficients $A'(E_e, Q^2)$, $B'(E_e, Q^2)$, and $C'(E_e, Q^2)$ are given by

$$\begin{aligned}
A'(E_e, Q^2) = & 8f_1^2(Q^2)MQ^2 - 2f_2^2(Q^2)\frac{Q^4}{M} + 2f_1(Q^2)f_2(Q^2)\left[4MQ^2 - \frac{Q^4}{M}\right] \\
& + 8f_1(Q^2)g_1(Q^2)M[4E_eM - Q^2] + f_2(Q^2)g_1(Q^2)\left[\frac{2Q^4}{M} - 8E_eQ^2\right]
\end{aligned} \tag{E1}$$

$$\begin{aligned}
B'(E_e, Q^2) = & 2f_1^2(Q^2) \left[\frac{Q^2(-4E_e M - 2M^2 + Q^2)}{M} \right] + f_2^2(Q^2) \left[\frac{2Q^4}{M} - 4E_e Q^2 \right] \\
& - 2g_1^2(Q^2) \left[\frac{(2M^2 + Q^2)(4E_e M - Q^2)}{M} \right] - 4f_1(Q^2)f_2(Q^2) \left[\frac{Q^2(3E_e M + M^2 - Q^2)}{M} \right] \\
& - 4f_1(Q^2)g_1(Q^2) \left[\frac{(8E_e^2 M^2 + 4E_e(M^3 - MQ^2) + Q^4)}{M} \right] + 4f_2(Q^2)g_1(Q^2) \left[-4E_e^2 M + Q^2(3E_e - M) - \frac{Q^4}{M} \right]
\end{aligned} \tag{E2}$$

$$C'(E_e, Q^2) = -\frac{4Q^2}{M} f_1(Q^2)g_2^l(Q^2) - \frac{4Q^2}{M} f_2(Q^2)g_2^l(Q^2) + 4g_1(Q^2)g_2^l(Q^2) \left[\frac{(Q^2 - 4E_e M)}{M} \right] \tag{E3}$$

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- [1] P. A. Adderley, D. Bullard, Y. C. Chao, C. H. Garcia, J. Grames, J. Hansknecht, A. Hofler, R. Kazimi, J. Musson, C. Palatchi *et al.*, *Nucl. Instrum. Methods Phys. Res., Sect. A* **1046**, 167710 (2023).
- [2] E. J. Beise, M. L. Pitt, and D. T. Spayde, *Prog. Part. Nucl. Phys.* **54**, 289 (2005).
- [3] B. D. Milbrath *et al.* (Bates FPP Collaboration), *Phys. Rev. Lett.* **80**, 452 (1998); **82**, 2221(E) (1999).
- [4] F. E. Maas, *Eur. Phys. J. A* **17**, 339 (2003).
- [5] K. S. Kumar, S. Mantry, W. J. Marciano, and P. A. Souder, *Annu. Rev. Nucl. Part. Sci.* **63**, 237 (2013).
- [6] M. Sajjad Athar, A. Fatima, and S. K. Singh, *Prog. Part. Nucl. Phys.* **129**, 104019 (2023).
- [7] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [8] M. González-Alonso, O. Naviliat-Cuncic, and N. Severijns, *Prog. Part. Nucl. Phys.* **104**, 165 (2019).
- [9] A. M. Ankowski, A. Ashkenazi, S. Bacca, J. L. Barrow, M. Betancourt, A. Bodek, M. E. Christy, L. D. S. Dytman, A. Friedland, O. Hen *et al.*, *J. Phys. G* **50**, 120501 (2023).
- [10] A. Bodek, S. Avvakumov, R. Bradford, and H. S. Budd, *J. Phys. Conf. Ser.* **110**, 082004 (2008).
- [11] U. Muller and Milan Potokar (A1 Collaboration), *Nucl. Phys. A* **721**, C425 (2003).
- [12] V. A. Andreev *et al.* (MuCap Collaboration), *Phys. Rev. Lett.* **110**, 012504 (2013).
- [13] V. A. Andreev *et al.* (MuCap Collaboration), *Phys. Rev. C* **91**, 055502 (2015).
- [14] A. Czarnecki, W. J. Marciano, and A. Sirlin, *Phys. Rev. Lett.* **99**, 032003 (2007).
- [15] L. Alvarez-Ruso, J. L. Barrow, L. Bellantoni, M. Betancourt, A. Bross, L. Cremonesi, K. Duffy, S. Dytman, L. Fields, T. Fukuda *et al.*, *arXiv:2203.11298*.
- [16] O. Tomalak, A. S. Meyer, C. Wret, T. Cai, R. J. Hill, and K. S. McFarland, *Phys. Rev. D* **113**, 073004 (2026).
- [17] V. Bernard, L. Elouadrhiri, and U. G. Meissner, *J. Phys. G* **28**, R1 (2002).
- [18] S. Bonetti, G. Carnesecchi, D. Cavalli, P. Negri, A. Pullia, M. Rollier, F. Romano, and R. Schira, *Nuovo Cimento Soc. Ital. Fis.* **38A**, 260 (1977).
- [19] M. Pohl *et al.* (GARGAMELLE NEUTRINO PROPANE Collaboration), *Lett. Nuovo Cimento* **26**, 332 (1979).
- [20] N. Armenise, O. Erriquez, M. T. Fogli Muciaccia, S. Nuzzo, F. Ruggieri, A. Halsteinslid, K. Myklebost, A. Rognebakke, O. Skjeggstad, S. Bonetti *et al.*, *Nucl. Phys. B* **152**, 365 (1979).
- [21] S. V. Belikov, A. P. Bugorsky, V. S. Epshtein, N. V. Kalganov, L. A. Klimenko, V. I. Kochetkov, Y. G. Kornelyuk, V. I. Kurbakov, A. I. Mukhin, V. F. Perelygin *et al.*, *Z. Phys. A* **320**, 625 (1985).
- [22] A. E. Asratian, P. A. Gorichev, V. I. Efremenko, G. K. Kliger, V. Z. Kolganov, S. P. Kruchinin, M. A. Kubantsev, I. V. Makhlyueva, A. V. Fedotov, V. G. Shevchenko *et al.*, *Yad. Fiz.* **39**, 619 (1984).
- [23] H. J. Grabosch, K. K. Kaufmann, U. Krecker, R. Nahnauer, Z. Nowak, S. S. Schlenstedt, V. V. Ammosov, A. A. Ivanilov, P. V. Ivanov, V. I. Konyushko *et al.*, *Sov. J. Nucl. Phys.* **47**, 1032 (1988); [*Yad. Fiz.* **47**, 1630 (1988)].
- [24] S. J. Barish, J. Campbell, G. Charlton, Y. Cho, M. Derrick, R. Engelmann, L. G. Hyman, D. Jankowski, A. Mann, B. Musgrave *et al.*, *Phys. Rev. D* **16**, 3103 (1977).
- [25] G. M. Radecky, V. E. Barnes, D. D. Carmony, A. F. Garfinkel, M. Derrick, E. Fernandez, L. Hyman, G. Levman, D. Koetke, B. Musgrave *et al.*, *Phys. Rev. D* **25**, 1161 (1982); **26**, 3297(E) (1982).
- [26] N. J. Baker, A. M. Cnops, P. L. Connolly, S. A. Kahn, H. G. Kirk, M. J. Murtagh, R. B. Palmer, N. P. Samios, and M. Tanaka, *Phys. Rev. D* **23**, 2499 (1981).
- [27] T. Kitagaki, S. Tanaka, H. Yuta, K. Abe, K. Hasegawa, A. Yamaguchi, K. Tamai, T. Hayashino, Y. Otani, H. Hayano *et al.*, *Phys. Rev. D* **28**, 436 (1983).
- [28] A. S. Meyer, M. Betancourt, R. Gran, and R. J. Hill, *Phys. Rev. D* **93**, 113015 (2016).
- [29] V. Lyubushkin *et al.* (NOMAD Collaboration), *Eur. Phys. J. C* **63**, 355 (2009).

- [30] D. Ruterbories *et al.* (MINERvA Collaboration), *Phys. Rev. D* **99**, 012004 (2019).
- [31] G. A. Fiorentini *et al.* (MINERvA Collaboration), *Phys. Rev. Lett.* **111**, 022502 (2013).
- [32] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), *Phys. Rev. Lett.* **100**, 032301 (2008).
- [33] P. Adamson *et al.* (MINOS Collaboration), *Phys. Rev. D* **91**, 012005 (2015).
- [34] R. Gran *et al.* (K2K Collaboration), *Phys. Rev. D* **74**, 052002 (2006).
- [35] K. Abe *et al.* (T2K Collaboration), *Phys. Rev. D* **92**, 112003 (2015).
- [36] J. L. Alcaraz-Aunión *et al.* (SciBooNE Collaboration), *AIP Conf. Proc.* **1189**, 145 (2009).
- [37] T. Cai *et al.* (MINERvA Collaboration), *Nature (London)* **614**, 48 (2023).
- [38] F. Akbar, M. Rafi Alam, M. Sajjad Athar, S. Chauhan, S. K. Singh, and F. Zaidi, *Int. J. Mod. Phys. E* **24**, 1550079 (2015).
- [39] O. Benhar and D. Meloni, *Phys. Rev. D* **80**, 073003 (2009).
- [40] O. Benhar, P. Coletti, and D. Meloni, *Phys. Rev. Lett.* **105**, 132301 (2010).
- [41] L. Alvarez-Ruso *et al.* (NuSTEC Collaboration), *Prog. Part. Nucl. Phys.* **100**, 1 (2018).
- [42] K. S. Kuzmin, V. V. Lyubushkin, and V. A. Naumov, *Eur. Phys. J. C* **54**, 517 (2008).
- [43] L. Alvarez-Ruso, K. M. Graczyk, and E. Saul-Sala, *Phys. Rev. C* **99**, 025204 (2019).
- [44] M. M. Block *et al.*, *Phys. Rev. Lett.* **12**, 262 (1964).
- [45] S. M. Bilenky and E. Christova, *J. Phys. G* **40**, 075004 (2013).
- [46] S. M. Bilenky and E. Christova, *Phys. Part. Nucl. Lett.* **10**, 651 (2013).
- [47] O. Tomalak, *Phys. Rev. D* **103**, 013006 (2021).
- [48] K. M. Graczyk and B. E. Kowal, *Phys. Rev. D* **101**, 073002 (2020).
- [49] R. Gupta, *Universe* **10**, 135 (2024).
- [50] Y. C. Jang *et al.* (Precision Neutron Decay Matrix Elements (PNDME) Collaboration), *Phys. Rev. D* **109**, 014503 (2024).
- [51] R. Petti, R. J. Hill, and O. Tomalak, *Phys. Rev. D* **109**, L051301 (2024).
- [52] R. J. Hill and G. Paz, *Phys. Rev. D* **82**, 113005 (2010).
- [53] L. Alvarez-Ruso, A. M. Ankowski, A. Ashkenazi, J. Barrow, M. Betancourt, K. Borah, M. Sajjad Athar, E. Catano-Mur, P. Coloma, P. Dunne *et al.*, [arXiv:2503.23556](https://arxiv.org/abs/2503.23556).
- [54] Y. Aoki, K.-I. Ishikawa, Y. Kuramashi, S. Sasaki, K. Sato, E. Shintani, R. Tsuji, H. Watanabe, and T. Yamazaki (PACS Collaboration), *Phys. Rev. D* **112**, 074510 (2025).
- [55] D. L. Yao, L. Alvarez-Ruso, and M. J. Vicente-Vacas, *Phys. Rev. D* **96**, 116022 (2017).
- [56] A. S. Meyer, [arXiv:2601.02676](https://arxiv.org/abs/2601.02676).
- [57] A. S. Meyer *et al.* (MINERvA Collaboration), [arXiv:2512.14097](https://arxiv.org/abs/2512.14097).
- [58] H. T. Klest, *Phys. Rev. D* **113**, 033002 (2026).
- [59] G. Yang and P. Kumar, [arXiv:2603.00703](https://arxiv.org/abs/2603.00703).
- [60] T. Averett, J. Napolitano, B. Wojtsekhowski, and W. Xiong, *The Nucleon Axial-Vector Form Factor from the $p(\bar{\nu}_e, n)\nu_e$ Reaction*, Proposal PR12-25-009 (Thomas Jefferson National Accelerator Facility, Newport News, VA, 2025), proposal to JLab PAC 53.
- [61] D. Androic, D. Armstrong *et al.*, *The Axial Form Factor of the Nucleon from Weak Capture of Positrons*, Letter of Intent LOI12-23-002 (Thomas Jefferson National Accelerator Facility, Newport News, VA, 2025), Letter of Intent to PAC 51.
- [62] A. Fatima, M. Sajjad Athar, and S. K. Singh, *Eur. Phys. J. A* **54**, 95 (2018).
- [63] F. Akbar, M. Rafi Alam, M. Sajjad Athar, and S. K. Singh, *Phys. Rev. D* **94**, 114031 (2016).
- [64] F. Akbar, M. Sajjad Athar, A. Fatima, and S. K. Singh, *Eur. Phys. J. A* **53**, 154 (2017).
- [65] A. Fatima, M. Sajjad Athar, and S. K. Singh, *Phys. Rev. D* **98**, 033005 (2018).
- [66] A. Fatima, M. S. Athar, and S. K. Singh, *Front. Phys.* **7**, 13 (2019).
- [67] A. Fatima, M. Sajjad Athar, and S. K. Singh, *Phys. Rev. D* **102**, 113009 (2020).
- [68] A. Fatima, M. S. Athar, and S. K. Singh, *Phys. Rev. D* **105**, 073004 (2022).
- [69] S. M. Bilenky, *Basics of Introduction to Feynman Diagrams and Electroweak Interactions Physics* (Editions Frontières, 1994).
- [70] C. Chen and C. D. Roberts, *Eur. Phys. J. A* **58**, 206 (2022).
- [71] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/bffn-fzgx> for the Q^2 distribution of the cross section, the spin asymmetries of the polarized nucleon target, and the polarization observables of the final nucleon, at $E_e = 855$ MeV and 2.2 GeV which includes Refs. [57,70,77].
- [72] C. H. Llewellyn Smith, *Phys. Rep.* **3**, 261 (1972).
- [73] K. M. Graczyk and B. E. Kowal, *Phys. Rev. D* **97**, 013001 (2018).
- [74] K. M. Graczyk and B. E. Kowal, *Phys. Rev. D* **108**, 093002 (2023).
- [75] O. Tomalak, R. Gupta, and T. Bhattacharya, *Phys. Rev. D* **108**, 074514 (2023).
- [76] K. Borah, M. Betancourt, R. J. Hill, T. Junk, and O. Tomalak, *Phys. Rev. D* **110**, 013004 (2024).
- [77] C. Chen, C. S. Fischer, C. D. Roberts, and J. Segovia, *Phys. Rev. D* **105**, 094022 (2022).
- [78] G. S. Bali, Sara Collins, Simon Heybrock, Marius Löffler, Rudolf Rödl, Wolfgang Söldner, and Simon Weishäupl (RQCD Collaboration), *Phys. Rev. D* **108**, 034512 (2023).
- [79] S. Park *et al.* (Nucleon Matrix Elements (NME) Collaboration), *Phys. Rev. D* **105**, 054505 (2022).
- [80] D. Djukanovic, G. von Hippel, J. Koponen, H. B. Meyer, K. Ottnad, T. Schulz, and H. Wittig, *Phys. Rev. D* **106**, 074503 (2022).
- [81] C. Alexandrou, S. Bacchio, M. Constantinou, J. Finkenrath, R. Frezzotti, B. Kostrzewa, G. Koutsou, G. Spanoudes, and C. Urbach (Extended Twisted Mass Collaboration), *Phys. Rev. D* **109**, 034503 (2024).
- [82] A. Pais, *Ann. Phys. (N.Y.)* **63**, 361 (1971).
- [83] R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969).
- [84] S. Weinberg, *Phys. Rev.* **112**, 1375 (1958).

- [85] M. L. Goldberger and S. B. Treiman, *Phys. Rev.* **111**, 354 (1958).
- [86] J. Bernstein, *Elementary Particles and their Currents* (W. H. Freeman and Company, San Francisco, 1968).
- [87] R. Bradford, A. Bodek, H. Budd, and J. Arrington, *Nucl. Phys. B, Proc. Suppl.* **159**, 127 (2006).
- [88] M. Oka and K. Kubodera, *Phys. Lett.* **90B**, 45 (1980).
- [89] K. Minamisono, K. Matsuta, T. Minamisono, T. Yamaguchi, T. Sumikama, T. Nagatomo, M. Ogura, T. Iwakoshi, M. Fukuda, M. Mihara *et al.*, *Phys. Rev. C* **65**, 015501 (2002).
- [90] K. Minamisono, T. Nagatomo, K. Matsuta, C. D. P. Levy, Y. Tagishi, M. Ogura, M. Yamaguchi, H. Ota, J. A. Behr, K. P. Jackson *et al.*, *Phys. Rev. C* **84**, 055501 (2011).
- [91] D. H. Wilkinson, *Eur. Phys. J. A* **7**, 307 (2000).
- [92] E. D. Commins and P. H. Bucksbaum, *Weak Interactions of Leptons and Quarks* (Cambridge University Press, Cambridge, England, 1983).
- [93] L. A. Ahrens *et al.*, *Phys. Lett. B* **202**, 284 (1988).
- [94] M. Day and K. S. McFarland, *Phys. Rev. D* **86**, 053003 (2012).
- [95] A. Fujii and Y. Yamaguchi, *Nuovo Cimento* **43**, 325 (1966).
- [96] A. Fujii and Y. Yamaguchi, *Prog. Theor. Phys.* **33**, 552 (1965).
- [97] S. M. Berman and M. Veltman, *Phys. Lett.* **12**, 275 (1964).
- [98] A. De Rujula and E. De Rafael, *Phys. Lett.* **32B**, 495 (1970).