

Effect of the K^* meson magnetic dipole moment on the $e^+e^- \rightarrow K^+K^-\pi^0\pi^0$ cross section

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We explore the sensitivity of the $e^+e^- \rightarrow K^+K^-\pi^0\pi^0$ cross section to the magnetic dipole moment (MDM) of the K^* vector meson. We describe the $\gamma^* \rightarrow 2K2\pi$ vertex using a vector meson dominance model, including the intermediate resonant contributions relevant for energies below 2.4 GeV. Using *BABAR* data for this process, we show that this observable is indeed sensitive to the MDM of the K^* ; we obtain a central value for the MDM of $\mu_{K^*} = 4.5$ and an upper bound of $\bar{\mu}_{K^*} = 6.3$, in units of $e/2m_{K^*}$. We emphasize the need for higher precision data to provide a first data-driven determination of this parameter to confront it with theoretical predictions.

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I. INTRODUCTION

The dynamics of quarks within hadrons can be explored using global observables such as electromagnetic multipoles. The proton itself was recognized as a nonfundamental through the comparison of its measured magnetic dipole moment (MDM) with the value predicted by Dirac's theory for fundamental fermions. The MDM is usually represented by the gyromagnetic ratio, g , expressed in units of the particle magneton. The value $g = 2$ seems to characterize not only one half spin particles but also fundamental particles with higher spin [1–10].

For composite states, corrections arising from the internal dynamics are expected. Although this property has been measured for several baryons, the meson sector remains experimentally challenging, mainly due to the short lifetime of the states. A particle with spin s has $2s + 1$ electromagnetic multipole moments. For mesons, vector states are therefore the simplest systems exhibiting multipoles beyond the electric charge, namely, the magnetic dipole moment and the electric quadrupole moment. The ρ and K^* mesons are examples of such states, whose MDMs have been predicted using different approximations to QCD. Theoretical models place the value of the ρ meson MDM in the range of $\mu_\rho = [1.5, 2.7]$ in units of $e/2m_\rho$ [11–38]. In a recent work [39], a value for the MDM of the ρ meson was obtained ($\mu_\rho = 2.7 \pm 0.3$ in units of $e/2m_\rho$) considering experimental data for the $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ process,

measured by the *BABAR* experiment [40]. This provided the first data-based determination of this quantity, allowing a direct comparison with theoretical predictions. This achievement was possible due to the important role played by the ρ electromagnetic vertex, which contains information about the multipole structure of the radiating particle, in a specific energy region of the cross section measured by the *BABAR* experiment. This procedure is analogous to the one used to extract the MDM of the W gauge boson [41,42] from the process $e^+e^- \rightarrow 2jets\,l\nu$, where l and ν denote a lepton and a neutrino, respectively.

The K^* meson shares several features with the ρ meson but differs by the strangeness content, which may lead to non-negligible isospin symmetry breaking effects. The predictions for the K^* MDM, based on different approaches to QCD, are summarized in Table I. They are in the range of $\mu_{K^*} = [2, 2.68]$ in units of $e/2m_{K^*}$. However, currently no experimental information is available to compare with these predictions.

The process analogous to the one used in the ρ meson case is $e^+e^- \rightarrow K^+K^-\pi^0\pi^0$. The cross section of this process has been measured by the *BABAR* Collaboration [46], where the relevant states in the low energy region, such as the $f_0(980)$ and the K^* mesons, were identified through the reconstruction of the corresponding invariant mass distribution of their decay products. To explore the sensitivity of this process to the K^* MDM, it is necessary to study the role of the channel in which two intermediate K^* mesons decay into $K^+K^-\pi^0\pi^0$, relative to the other channels dominated by scalar resonances.

In this work, we study the effect of the K^* MDM on the $e^+e^- \rightarrow K^+K^-\pi^0\pi^0$ cross section, following the same approach used in the analysis of the ρ meson [39], profiting from the *BABAR* analysis of the intermediate states

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TABLE I. Predictions of the $K^{*\pm}$ meson MDM.

MDM ($e/2M_{K^*}$)	Ref.	Description
2.09	[43]	Relativistic Hamiltonian derived from the path integral form of the $q_1\bar{q}_2$ Green's function
2.68(1)	[44]	Lattice QCD
2.04 ± 0.4	[20]	QCD sum rules
2.23	[28]	Poincaré covariant formulation based on Dyson-Schwinger equation of QCD
2.37	[27]	Lorentz-covariant formulation based on Dyson-Schwinger equation of QCD
$2 - 0.0047$	[45]	Finite width modification

involved. In Sec. II, we establish the theoretical framework of the analysis and present the multipolar structure of the K^* radiative vertex. We then describe the $e^+e^- \rightarrow K^+K^-2\pi^0$ process, modeling the $\gamma^* \rightarrow 2K2\pi$ vertex within the vector meson dominance (VMD) approach [47–50]. There, we derive the electromagnetic gauge invariant amplitude of the process, decomposed into the different contributing channels, while enforcing Bose-Einstein symmetry and charge conjugation (C) invariance. We show that our result generalizes the isospin symmetric description obtained for the ρ [39], by introducing the isospin symmetry breaking contribution, associated with the mass difference between the kaon and the pion. In Sec. III, we compute the cross section and analyze the role of the K^* MDM. The parameters of the model are determined using the experimental information provided by the Particle Data Group (PDG) [51]. Parameters with limited experimental precision, related to the scalar mesons $f_0(980)$ and σ , as well as the K^* MDM are treated as free parameters and determined from a fit to *BABAR* experimental data. In Sec. IV, we explore the sensitivity of the cross section to the K^* MDM, using *BABAR* data. Owing to the limited precision of the available data, only an upper bound is obtained. We conclude with a discussion of the results.

II. DESCRIPTION OF THE $e^+e^- \rightarrow K^+K^-2\pi^0$ PROCESS

The radiative process of a vector particle (V) is illustrated in Fig. 1. We set the notation by $V(q_1, \eta_1) \rightarrow V(q_2, \eta_2)\gamma(q, \epsilon)$, where γ is the photon field, and in parentheses are the corresponding momentum and polarization vector. The electromagnetic vertex is defined through the matrix element

$$\langle V(q_2, \eta_2) | J_{\text{EM}}^\mu(0) | V(q_1, \eta_1) \rangle \equiv e\Gamma^{\mu\nu\lambda}\eta_{1\nu}\eta_{2\lambda}^*. \quad (1)$$

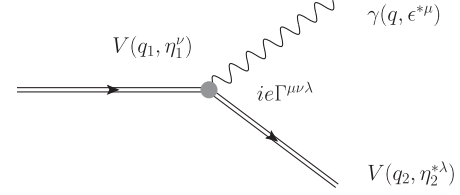


FIG. 1. $V(q_1, \eta_1) \rightarrow V(q_2, \eta_2)\gamma(q, \epsilon)$ process. The $VV\gamma$ electromagnetic vertex and momenta and polarization conventions are shown.

The C , P (parity) and CP conserving electromagnetic vertex $\Gamma^{\mu\nu\lambda}$ can be decomposed into multipolar structures as follows:

$$\Gamma^{\mu\nu\lambda} = \alpha(q^2)g^{\nu\lambda}(q_1 + q_2)^\mu + \beta(q^2)(g^{\mu\nu}q^\lambda - g^{\mu\lambda}q^\nu) - \frac{\gamma(q^2)}{M_V^2}(q_1 + q_2)^\mu q^\nu q^\lambda - q_1^\lambda g^{\mu\nu} - q_2^\nu g^{\mu\lambda}, \quad (2)$$

where $\alpha(q^2)$, $\beta(q^2)$, and $\gamma(q^2)$ correspond to the electric charge (in units of e), magnetic dipole moment (in units of $e/2M_V$), and the electric quadrupole moment (in units of e/M_V^2) form factors [52–55]. In the static limit, these quantities correspond to electromagnetic multipoles. $\alpha(0) = 1$, $\beta(0) = 2$, and $\gamma(0) = 0$ are usually taken as reference values for vector mesons. For the K^* meson, the physical values are expected to be affected by the strong interaction dynamics among quarks. Notice that the magnetic term grows as $\mathcal{O}(q)$ whereas the quadrupole grows as $\mathcal{O}(q^2)$. Thus, for a relatively small photon energy, the contribution of the MDM is dominant, and the electric quadrupole moment can be neglected ($\gamma(q^2) = 0$).

We define the notation for the process as $e^+(k_+)e^-(k_-) \rightarrow K^+(p_1)\pi^0(p_2)K^-(p_3)\pi^0(p_4)$, where the quantities in parentheses denote the corresponding particle four-momenta. The total amplitude can be written in the generic form

$$\mathcal{M} = \frac{e}{q^2} l^\mu J_\mu, \quad (3)$$

where the factor $1/q^2$ comes from the photon propagator ($q = k_+ + k_-$). The leptonic current $l^\mu \equiv \bar{v}(k_+)\gamma^\mu u(k_-)$ is common to all channels, while J_μ represents the hadronic electromagnetic current, which includes the contributions of all considered channels:

$$J^\mu = \sum_{\text{channel}} \mathcal{M}_{\text{channel}}^\mu. \quad (4)$$

This current must satisfy the Bose-Einstein symmetry, by the exchange of neutral pions, and the C invariance, by the exchange of charged kaons. The contribution of each channel to the electromagnetic current J^μ is denoted by

$\mathcal{M}_{\text{channel}}^\mu(p_1, p_2, p_3, p_4)$. Additional amplitudes corresponding to diagrams that differ only by meson exchange will be represented by the appropriate exchange of momenta in the corresponding function.

The $\gamma^* \rightarrow 2K2\pi$ vertex is described by considering the relevant hadronic degrees of freedom in the energy range of interest, guided by the *BABAR* analysis [46] and modeled within the VMD approach [47–50]. We consider the effective Lagrangian for the vector-photon and vector-pseudoscalar-pseudoscalar interaction

$$\mathcal{L} = \sum_V \frac{em_V^2}{g_V} V_\mu A^\mu + g_{VP_1P_2} V_\mu (P_1 \partial^\mu P_2 - P_2 \partial^\mu P_1), \quad (5)$$

where V and A refer to the vector meson and the photon fields, respectively, while P_i are pseudoscalar fields. The couplings are treated as free parameters that can be determined from experimental information.

The channels considered in this work are shown in Fig. 2. In all channels, the initial vector state is coupled to the virtual photon from the leptonic current. We provide the hadronic part of the amplitude of each channel excluding the common photon propagator, consistent with Eq. (3). We discuss them below.

A. Channel A

The probability amplitude of channel A is given by

$$\begin{aligned} \mathcal{M}_A^\mu(p_1, p_2, p_3, p_4) = & -e \frac{g_{\phi KK} g_{K^* K \pi}^2}{g_\phi} m_\phi^2 D_\phi^{\mu\alpha}[q] \\ & \times (q - 2p_1)_\alpha S_K[q - p_1] \\ & \times (q - p_1 + p_2)_\gamma D_{K^*}^{\gamma\eta}[s_{34}] r_{43\eta}, \end{aligned} \quad (6)$$

where we have defined $s_{ij} \equiv p_i + p_j$, $r_{ij} \equiv p_i - p_j$. $S_K[q] \equiv i/(q^2 - m_K^2)$ is the kaon propagator with mass m_K . The propagator of vector mesons, $D_V^{\mu\alpha}[p]$, with mass m_V and decay width Γ_V , is assumed to have the complex mass form

$$D_V^{\mu\alpha}[p] = -iD_V[p] \left(g^{\mu\alpha} - \frac{p^\alpha p^\mu}{m_V^2 - im_V \Gamma_V} \right), \quad (7)$$

where $D_V[p] \equiv 1/(p^2 - m_V^2 + im_V \Gamma_V)$. This form is consistent with the inclusion of the absorptive corrections and the requirement of gauge invariance [56–59]. In some cases, the label referring to the vector meson includes the corresponding charge to facilitate the identification of the particle in the corresponding Feynman diagram. The main decay mode of the K^* meson is in a pair, $K\pi$; the energy dependence of the width is

$$\Gamma_{K^*}(s) = \Gamma_{K^*} \left(\frac{m_{K^*}}{\sqrt{s}} \right)^5 \left[\frac{\lambda(s, m_K^2, m_\pi^2)}{\lambda(m_{K^*}^2, m_K^2, m_\pi^2)} \right]^{3/2}, \quad (8)$$

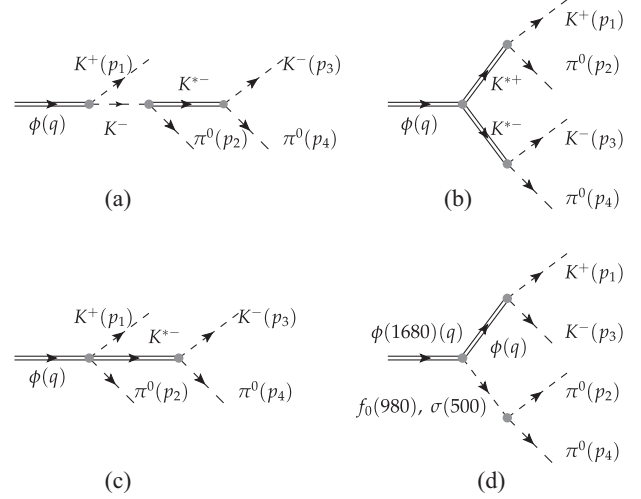


FIG. 2. Generic channels considered for the description of the $e^+e^- \rightarrow K^+K^-2\pi^0$ process. The full set of diagrams is obtained by applying Bose-Einstein symmetry and charge conjugation to each channel. Channels (a), (b), and (c) are linked by gauge invariance, while channel (d) is gauge invariant by itself.

where $\lambda(x, y, z)$ is the Källén function; and Γ_{K^*} is the decay width at $\sqrt{s} = m_{K^*}$. The amplitude, Eq. (6), can be written in the simplified form

$$\begin{aligned} \mathcal{M}_A^\mu(p_1, p_2, p_3, p_4) = & i e C_A D_{K^*}[s_{34}] \frac{x_1^\mu}{x_1 \cdot q} \\ & \times (r_{43} \cdot z_{12} - \bar{\Delta}[s_{34}] z_{12} \cdot s_{34}), \end{aligned} \quad (9)$$

where $x_1 \equiv q - 2p_1$, $z_{12} \equiv q - p_1 + p_2$ and

$$\begin{aligned} C_A & \equiv (g_{\phi KK} g_{K^* K \pi}^2 / g_\phi) m_\phi^2 D_\phi[q], \\ \Delta & \equiv m_\pi^2 - m_K^2, \\ dK^*[s_{34}] & \equiv m_{K^*}^2 - im_{K^*} \Gamma_{K^*}(s_{34}), \\ \bar{\Delta}[s_{34}] & \equiv \Delta / dK^*[s_{34}]. \end{aligned}$$

B. Channel B

In Fig. 2(b), we show the so-called K^* channel, where the ϕ meson couples to a K^* pair, through a triple vector vertex. This defines the K^* electromagnetic vertex, times a global constant $g_{\phi K^* K^*}$ that accounts for the intermediate strong interaction process. The corresponding amplitude is given by

$$\begin{aligned} \mathcal{M}_B^\mu(p_1, p_2, p_3, p_4) = & e \frac{g_{\phi K^* K^*} g_{K^* K \pi}^2}{g_\phi} m_\phi^2 D_\phi^{\mu\alpha}[q] r_{21\gamma} \\ & \times D_{K^*}^{\gamma\lambda}[s_{12}] \Gamma_{\alpha\lambda\delta}^1 D_{K^*}^{\delta\eta}[s_{34}] r_{43\eta}, \end{aligned} \quad (10)$$

where $\Gamma_{\alpha\lambda\delta}^1 \equiv (1 + i\Gamma_V/M_V) \Gamma_{\alpha\lambda\delta}$ represents the absorptive corrected vertex at one-loop level, consistent with gauge

invariance [56–59]. The tree-level vertex, Eq. (2), for this momentum configuration takes the form

$$\Gamma_{\alpha\lambda\delta} = g_{\lambda\delta} Q_{1\alpha} + \beta(q_\delta g_{\alpha\lambda} - q_\lambda g_{\delta\alpha}) + s_{12\lambda} g_{\delta\alpha} - s_{34\delta} g_{\alpha\lambda},$$

where $q = s_{12} + s_{34}$ and $Q_1 \equiv s_{34} - s_{12}$, and we have omitted the dependence on q^2 of β since the energy region to scan is small and therefore can be considered constant. The amplitude can be written in a simplified form:

$$\begin{aligned} \mathcal{M}_B^\mu(p_1, p_2, p_3, p_4) \\ = i e C_B \frac{Q_1^\mu}{q \cdot Q_1} [D_{K^*}[s_{34}](r_{21} \cdot s_{34} \bar{\Delta}[s_{34}] - r_{21} \cdot r_{43}) \\ - D_{K^*}[s_{12}](r_{43} \cdot s_{12} \bar{\Delta}[s_{12}] - r_{21} \cdot r_{43})] \\ + \mathcal{M}_B^\mu(p_1, p_2, p_3, p_4)_{GI}, \end{aligned} \quad (11)$$

where $C_B = \frac{g_{\phi K^* K^*} g_{K^* K \pi}^2}{g_\phi} m_\phi^2 D_\phi[q]$, and we define the gauge invariant tensor,

$$L^\mu(a, b) \equiv \frac{a^\mu}{a \cdot q} - \frac{b^\mu}{b \cdot q}. \quad (12)$$

$\mathcal{M}_B^\mu(p_1, p_2, p_3, p_4)_{GI}$ is the self gauge invariant part of channel B , and explicitly given in Appendix B.

C. Channel C

In Fig. 2(c), we show the process driven by a contact term ($\phi K^* K \pi$), whose amplitude can be written in the general form

$$\begin{aligned} \mathcal{M}_C^\mu(p_1, p_2, p_3, p_4) = i e \frac{g_{\phi K^* K \pi} g_{K^* K \pi}}{g_\phi} m_\phi^2 \\ \times D_\phi^{\mu\alpha}[q] \Gamma_{\alpha\gamma}^1 D_{K^*}^{\gamma\eta}[s_{34}] r_{43\eta}. \end{aligned} \quad (13)$$

Channels A , B , and C , together with their corresponding neutral and charged kaon exchanges, are linked by the gauge invariance condition. The effective coupling $g_{\phi K^* K \pi}$ and the general vertex $\Gamma_{\alpha\gamma}^1$ are fixed by such a condition. This is an important observation, since our description of channels A and B is not fully gauge invariant by itself. Here, we describe how the three channels are properly combined in order to obtain an analytically gauge invariant amplitude. To this end, we profit from the Ward-Takahashi identity, fulfilled by the $VV\gamma$ vertex, instead of introducing the counterterm as a general requirement. This procedure allows us to trace the origin of the different contributions. The details are presented in Appendix B.

In order to obtain the gauge invariant amplitude, we use the following combination of the amplitudes:

$$\begin{aligned} \mathcal{M}_{ABC_{24}}^\mu = \mathcal{M}_A^\mu(p_1, p_2, p_3, p_4) + \mathcal{M}_A^\mu(p_3, p_4, p_1, p_2) \\ + \mathcal{M}_B^\mu(p_1, p_2, p_3, p_4) + \mathcal{M}_C^\mu(p_1, p_2, p_3, p_4) \\ + \mathcal{M}_C^\mu(p_3, p_4, p_1, p_2), \end{aligned} \quad (14)$$

where we have adopted a notation that refers to the three channels and the momenta of the neutral pions. The resulting gauge invariant amplitude is

$$\begin{aligned} \mathcal{M}_{ABC_{24}}^\mu = i e C \{ L^\mu(x_1, x_3) [D_{K^*}[s_{34}](r_{43} \cdot z_{12} - z_{12} \cdot s_{34} \bar{\Delta}[s_{34}]) \\ + D_{K^*}[s_{12}](r_{21} \cdot z_{34} - z_{34} \cdot s_{12} \bar{\Delta}[s_{12}])] \\ + L^\mu(Q_1, x_3) D_{K^*}[s_{34}] (-r_{21} \cdot r_{43} + r_{21} \cdot s_{34} \bar{\Delta}[s_{34}]) \\ - L^\mu(Q_1, x_1) D_{K^*}[s_{12}] (-r_{21} \cdot r_{43} + r_{43} \cdot s_{12} \bar{\Delta}[s_{12}])] \} \\ + \mathcal{M}_B^\mu(p_1, p_2, p_3, p_4)_{GI}, \end{aligned} \quad (15)$$

where $C \equiv C_A = C_B = (g_{\phi K K} g_{K^* K \pi}^2 / g_\phi) m_\phi^2 D_\phi[q]$ are required, $x_1 \equiv q - 2p_1$ and $x_3 \equiv q - 2p_3$. A similar expression is obtained by adding the remaining amplitudes:

$$\begin{aligned} \mathcal{M}_{ABC_{42}}^\mu = \mathcal{M}_A^\mu(p_1, p_4, p_3, p_2) + \mathcal{M}_A^\mu(p_3, p_2, p_1, p_4) \\ + \mathcal{M}_B^\mu(p_1, p_4, p_3, p_2) + \mathcal{M}_C^\mu(p_1, p_4, p_3, p_2) \\ + \mathcal{M}_C^\mu(p_3, p_2, p_1, p_4), \end{aligned} \quad (16)$$

which in practice correspond to the exchange $p_2 \leftrightarrow p_4$.

D. Channel D

To model the remaining part of the electromagnetic current, following the analysis performed by *BABAR*, we consider the channel in which the photon couples the $\phi(1680)$ vector meson, which subsequently decays into the ϕ meson and the σ or $f_0(980)$ scalar meson, as shown in Fig. 2(d). The effective Lagrangian describing the VVS and PPS interactions (where V , P , and S denote vector, pseudoscalar, and scalar fields, respectively) is given by the following:

$$\begin{aligned} \mathcal{L}_S = i g_{V_1 V_2 S} (\partial_\beta V_1^\mu \partial^\beta V_{2\mu} - \partial_\beta V_1^\mu \partial_\mu V_2^\beta) S \\ + i g_{SP P} S P_1 P_2, \end{aligned} \quad (17)$$

where $g_{V_1 V_2 S}$ and $g_{SP P}$ are effective coupling constants. The simplified amplitude, considering the contribution of $f_0(980)$, is given by

$$\begin{aligned} \mathcal{M}_{D_{f_0}}^\mu(p_1, p_2, p_3, p_4) = -i e C_{f_0} D_\phi[s_{13}] D_{f_0}[s_{24}] \\ \times \frac{F^{\mu\beta}(s_{13}, q)}{q \cdot s_{13}} r_{31\beta}, \end{aligned} \quad (18)$$

where the propagator of the $f_0(980)$ meson is $D_{f_0}[s] \equiv i/(s - m_{f_0}^2 + i m_{f_0} \Gamma_{f_0})$ and $C_{f_0} \equiv (g_{f_0 \pi^0 \pi^0} g_{\phi K^+ K^-} g_{\phi f_0}) /$

$g_{\phi'} m_{\phi'}^2 D_{\phi'}[q]$, with $\phi' = \phi(1680)$ and the tensor function is given by

$$F_{\mu\alpha}(a, b) \equiv a \cdot b g_{\mu\alpha} - a_\mu b_\alpha. \quad (19)$$

Notice that this channel does not involve additional exchange of neutral or charged pions, since both pions are emitted from the same vertex. An analogous expression is obtained for the σ meson. The combined amplitude for the $f_0(980)$ and σ mesons is

$$\begin{aligned} \mathcal{M}_D^\mu(p_1, p_2, p_3, p_4) &= \mathcal{M}_{D_{f_0}}^\mu(p_1, p_2, p_3, p_4) \\ &+ e^{i\theta_1} \mathcal{M}_{D_\sigma}^\mu(p_1, p_2, p_3, p_4), \end{aligned}$$

where the two contributions are combined with a relative phase θ_1 . The decay width of the particles involved in this channel is considered as constant. This description is sufficient to describe the currently available low precision data.

Finally, the total hadronic electromagnetic current can be written as

$$J^\mu = \mathcal{M}_{ABC}^\mu + e^{i\theta_2} \mathcal{M}_D^\mu, \quad (20)$$

where $\mathcal{M}_{ABC}^\mu$ includes the contributions ABC_{24} and ABC_{42} . A relative phase θ_2 between the two amplitudes is introduced as a free parameter.

III. THE K^* MDM FROM THE TOTAL CROSS SECTION

We compute the total cross section following the PDG convention [51], neglecting the electron mass

$$d\sigma = \frac{(2\pi)^4 |\overline{\mathcal{M}}|^2}{4\sqrt{(k_+ \cdot k_-)^2}} \delta^4\left(q - \sum_{i=1}^n p_i\right) \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 2E_i}. \quad (21)$$

The spin-averaged squared amplitude $|\overline{\mathcal{M}}|^2$ is built up by the leptonic and hadronic contributions discussed above,

$$|\overline{\mathcal{M}}|^2 = \frac{e^2}{q^4} l_{\mu\nu} h^{\mu\nu}, \quad (22)$$

where $h^{\mu\nu} \equiv J^\mu J^{\nu\dagger}$ is the hadronic tensor constructed from the hadronic currents discussed above, and $l_{\mu\nu}$ is the leptonic tensor, given by

$$l_{\mu\nu} = k_{+\mu} k_{-\nu} + k_{-\mu} k_{+\nu} - \frac{q^2}{2} g_{\mu\nu}. \quad (23)$$

The kinematical variables and integration limits are chosen following Ref. [60]. The integration is performed numerically using a Fortran code and the Vegas subroutine [61].

TABLE II. Free parameters. The values were obtained from a fit to data. See text for details.

Parameter	Description
$\theta_1 = 2.9 \pm 0.29(\pi \text{ units})$	σ to $f_0(980)$ phase
$\beta = 1.5 \pm 1.2$	Magnetic dipole moment
$\theta_2 = 1.2(\pi \text{ units})$	D to A, B, C phase
$g_{\phi' f_0}/g_{\phi'} = 1.15^{+0.07}_{-0.15}$	Couplings ratio
$g_{\phi' \phi\sigma}/g_{\phi'} = 0.32 \pm 0.06$	Couplings ratio
$m_\sigma = 0.71 \pm 0.02$ (GeV)	σ -meson mass
$\Gamma_\sigma = 0.13^{+0.13}_{-0.08}$ (GeV)	σ -meson width

For the numerical analysis, we use $m_{f_0} = 0.972 \pm 0.002$ GeV and $\Gamma_{f_0} = 0.056 \pm 0.011$ GeV, obtained by BABAR [46] from the same dataset. The coupling $g_{f_0 \pi^0 \pi^0} = 1.0 \pm 0.1$ GeV is determined from the decay width of $f_0 \rightarrow \pi\pi$, where the two pion mode accounts for the total decay width. Similarly, the coupling $g_{\sigma \pi^0 \pi^0}$ is determined from the corresponding decay width of the σ meson which, together with its mass, is fixed by fitting to the cross section. See Appendix A for further details on the couplings involved.

We perform a fit to the cross section experimental data using the Minuit minimization subroutine. In Table II we list the free parameters and their numerical values obtained from the fit. The handling of the uncertainties required a separate analysis, as we describe below.

In order to illustrate the dependence on the parameter β , in Fig. 3, we show the contribution to the cross section of the $A + B + C$ channels. We consider $\beta = 1, 2$, and 3 . These results are compared with the experimental data from BABAR. We observe that the dependence on β is quadratic, causing the contribution to the cross section to increase rapidly, as does β . This feature will be important for establishing an upper bound. The plots also show that the main sensitivity to β occurs in the region between

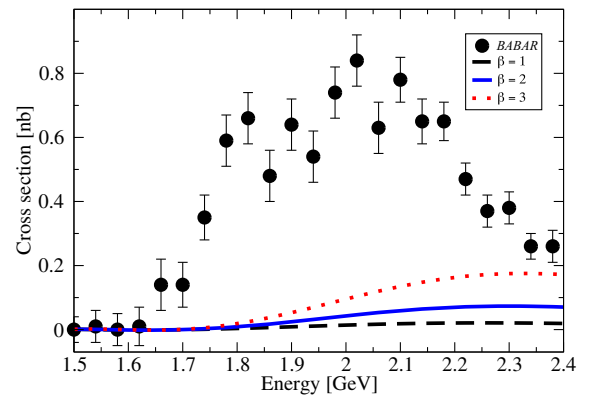


FIG. 3. β dependence on $e^+e^- \rightarrow K^+K^-2\pi^0$ cross section. BABAR data (symbols) and the contribution from A, B , and C channels (together) for $\beta = 1, 2, 3$ (dashed, solid, and dotted lines, respectively).

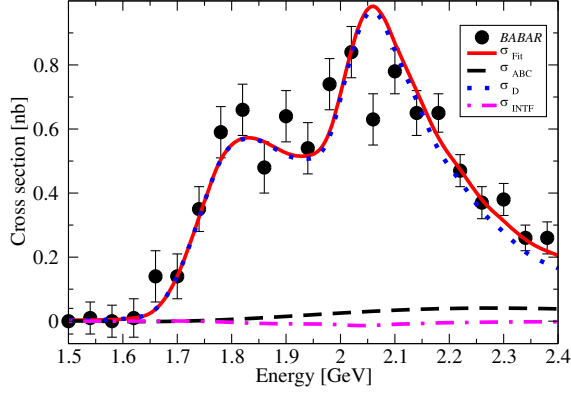


FIG. 4. $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$ cross section. *BABAR* data (symbols) and the fit to the total cross section (solid line). We also show the contribution of each channel to the total cross section: A , B , and C channels grouped together (dashed line), D (dotted line), and the interference between them (dot-dashed line).

1.8 GeV and 2.4 GeV. For energies above 2.4 GeV additional structures appear that are not captured by the present description, and therefore we limit our fit to energies below 2.4 GeV.

In Fig. 4, we show the total cross section data from *BABAR* [46] (symbols) and the corresponding fit from this work (solid line). In the same figure, we show the contribution of each channel: A , B , and C channels grouped together (dashed line), D (dotted line), as well as the interference between channels ABC and D . Error bars are not displayed for the sake of clarity. We observe that the interference between the ABC and D channels is very small. For this reason, the relative phase (θ_2) between these channels cannot be determined with sufficient precision. Therefore, in Table II, we quote only a central value favored by the fit, which does not significantly modify the final result.

The fit favors $\beta = 1.5 + 1.2$ with $\chi^2/\text{dof} = 1.6$. The upper value was determined by identifying the increase in χ^2/dof by one unit. No lower bound is quoted, as the experimental data are unable to constrain this parameter. In order to properly obtain the static limit value of the MDM, we consider the static limit of the electric charge form factor,

$$|F_{K^*}(0)| = \lim_{q^2 \rightarrow 0} \left| \frac{g_{\phi KK} m_\phi^2}{g_\phi} D_\phi[q^2] \right| = 1.$$

Using the value of the parameters, we obtain $|F_{K^*}(0)| = 0.33 \pm 0.005$, where the uncertainty arises mainly from the uncertainty in g_ϕ . In order to properly define the MDM extracted, we normalize β to this value. Thus, the central value of the MDM of the K^* meson, extracted from the *BABAR* data, is

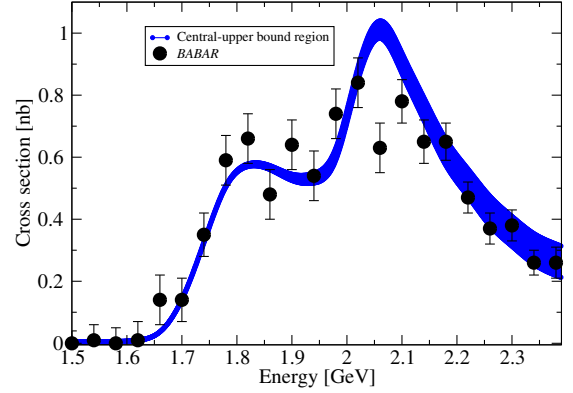


FIG. 5. *BABAR* data for the $e^+e^- \rightarrow K^+K^-2\pi^0$ total cross section (symbols with error bars). The shadowed region is delimited by the central value and the upper bound of the K^* MDM.

$$\mu_{K^*} = 4.5 \quad \text{in units of } (e/2m_{K^*}), \quad (24)$$

with an upper bound of $\bar{\mu}_{K^*} = 6.3$ in units of $(e/2m_{K^*})$.

In Fig. 5, we show the experimental cross section as measured by *BABAR* (symbols with error bars). The shadowed region is delimited by the central value and the upper bound of the K^* MDM.

IV. CONCLUSIONS

We have performed an analysis of the $e^+e^- \rightarrow K^+K^-2\pi^0$ cross section measured by the *BABAR* Collaboration [40], to determine its sensitivity to the MDM of the K^* meson. We modeled the process in the VMD approach, obtaining explicit gauge invariant expressions for the corresponding amplitudes, in particular for the A , B , and C channels. This result generalizes the analogous result obtained for the ρ meson [39], incorporating the isospin symmetry breaking contribution.

We have shown that the cross section is sensitive to the MDM of the K^* and that it may provide the possibility to determine a first value based on experimental data. The outcome of the numerical analysis favors the central value $\mu_{K^*} = 4.5$ with an upper bound of $\mu_{K^*} < 6.3$ in units of $(e/2m_{K^*})$.

This large result may be affected by the simplifications adopted in the modeling of the process and the limited precision of the available data. For the modeling part, in our approach, we neglected the electric quadrupole moment contribution (which must also be incorporated in a gauge-invariant manner). In order to quantify this approximation, we have performed a calculation of the cross section associated with diagrams A , B , and C including the electric quadrupole contribution. We find that the central value of the MDM may be reduced by approximately 10% for a quadrupole strength of $\gamma = 0.1$ [see the corresponding factor in Eq. (2)]. This value was chosen so that the resulting cross section remains below the experimental data

in the energy region under study. Additional contributions from missing resonances are also possible. However, given the current precision of the available data, no specific states can be identified without introducing further assumptions. For this reason, we restricted our analysis to a limited energy region, despite the existence of experimental data at higher energies. Regarding the effect of improved experimental data, we find that the value of the upper uncertainty of the MDM decreases by approximately 35%, assuming a reduction of the experimental uncertainties by a factor of 2. This is a nontrivial result, since the dependence of the cross section on the MDM is not linear.

Thus, we consider it promising that a new measurement of the $e^+e^- \rightarrow K^+K^-2\pi^0$ cross section may shed light on the K^* MDM and contribute to a deeper understanding of the QCD structure of hadrons.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: COUPLINGS

For consistency, we provide the general expressions used to evaluate the couplings involved in this work. These couplings have been discussed in more detail in Ref. [62] within the same approach. The values of the parameters involved are taken from the PDG [51], unless otherwise stated.

1. VP_1P_2 coupling

The coupling constant associated with the vertex VP_1P_2 in the decay $V(q) \rightarrow P_1(p_1)P_2(p_2)$ is given by

$$g_{VP_1P_2} = \left(\frac{48\pi m_V^5 \Gamma(V \rightarrow P_1P_2)}{\lambda^{3/2}(m_V^2, m_{P_1}^2, m_{P_2}^2)} \right)^{1/2}. \quad (\text{A1})$$

For the decay $\phi \rightarrow K^+K^-$, we obtain $g_{\phi KK} = 4.47 \pm 0.03$.

For the decay $K^{*+} \rightarrow K^+\pi^0$, we obtain $g_{K^*\pi\pi} = 3.05 \pm 0.05$, where we make use of the isospin relation,

$$\Gamma(K^{*+} \rightarrow K^+\pi^0) = \frac{1}{3}\Gamma(K^{*+} \rightarrow K\pi). \quad (\text{A2})$$

2. $V\gamma$ coupling

This coupling is determined from the leptonic decay $V \rightarrow l^+l^-$, within the VMD approach,

$$g_V = \sqrt{\frac{4\pi\alpha^2(2m_l^2 + m_V^2)\sqrt{m_V^2 - 4m_l^2}}{3m_V^2\Gamma(V \rightarrow l^+l^-)}}. \quad (\text{A3})$$

In the case of interest for this work, the process is $\phi \rightarrow e^+e^-$. We obtain

$$g_\phi = 13.40 \pm 0.10.$$

3. SPP coupling

The coupling constant g_{SPP} is calculated using

$$g_{SPP} = \sqrt{\frac{16\pi M^2\Gamma(S \rightarrow PP)}{\sqrt{M^2 - 4m^2}}}, \quad (\text{A4})$$

where M (m) denotes the mass of the scalar (pseudoscalar) particle, and Γ is the total decay width.

In this work, we have two processes of interest: $f_0 \rightarrow \pi^0\pi^0$ and $\sigma \rightarrow \pi^0\pi^0$. For the f_0 meson, we use $M_{f_0} = 0.972 \pm 0.002$ and $\Gamma_{f_0} = 0.056 \pm 0.011$ GeV, as obtained from the BABAR Collaboration from the same dataset under consideration [46]. The total and partial decay widths are related by

$$\Gamma(f_0 \rightarrow \pi^0\pi^0) = \frac{1}{3}\Gamma(f_0 \rightarrow \pi\pi). \quad (\text{A5})$$

Hence, the value of the coupling constant $g_{f_0\pi^0\pi^0}$ is

$$g_{f_0\pi^0\pi^0} = 1.0 \pm 0.1 \text{ GeV}. \quad (\text{A6})$$

For the decay $\sigma \rightarrow \pi^0\pi^0$ we use the same relations, with the mass and width of the σ meson determined from the fit to the cross section and

$$g_{\sigma\pi^0\pi^0} = 1.3 \pm 0.5 \text{ GeV}. \quad (\text{A7})$$

APPENDIX B: CHANNELS A, B, AND C GAUGE INVARIANT AMPLITUDE INCLUDING ISOSPIN SYMMETRY BREAKING

We derive the fully gauge invariant amplitude for channels A, B, and C. The condition of gauge invariance requires

$$q_\mu(\mathcal{M}_A^\mu + \mathcal{M}_B^\mu) = -q_\mu\mathcal{M}_C^\mu. \quad (\text{B1})$$

The amplitude $\mathcal{M}_A^\mu(p_1, p_2, p_3, p_4)$, Eq. (11), upon contraction with q_μ , becomes

$$q_\mu \mathcal{M}_A^\mu(p_1, p_2, p_3, p_4) = i e C_A D_{K^*}[s_{34}][z_{12} \cdot r_{43} - z_{12} \cdot s_{34} \bar{\Delta}[s_{34}]], \quad (\text{B2})$$

where $x_1 \equiv q - 2p_1$, $z_{12} \equiv q - p_1 + p_2$ and $q = s_{21} + s_{43}$.

In a similar way, the amplitude $\mathcal{M}_B^\mu(p_1, p_2, p_3, p_4)$, Eq. (9), upon contraction with q_μ , becomes

$$q_\mu \mathcal{M}_B^\mu(p_1, p_2, p_3, p_4) = i e C_B [D_{K^*}[s_{34}](r_{21} \cdot s_{34} \bar{\Delta}[s_{34}] - r_{21} \cdot r_{43}) - D_{K^*}[s_{12}](r_{43} \cdot s_{12} \bar{\Delta}[s_{12}] - r_{21} \cdot r_{43})], \quad (\text{B3})$$

where the gauge invariant part of channel B is

$$\begin{aligned} \mathcal{M}_B^\mu(p_1, p_2, p_3, p_4)_{GI} = & i e C_B (1 + i\gamma) D_{K^*}[s_{12}] D_{K^*}[s_{34}] \left[\Delta \left\{ \left(\frac{s_{12}^2}{dK^*[s_{12}]} - 1 \right) \right. \right. \\ & \times [q \cdot r_{43} L^\mu(Q_1, r_{43}) - \bar{\Delta}[s_{34}] q \cdot s_{34} L^\mu(Q_1, s_{34})] - \left(\frac{s_{34}^2}{dK^*[s_{34}]} - 1 \right) \\ & \times [q \cdot r_{21} L^\mu(Q_1, r_{21}) - \bar{\Delta}[s_{12}] q \cdot s_{12} L^\mu(Q_1, s_{12})] \left. \right\} \\ & + \beta(q^2) \{ q \cdot r_{43} q \cdot r_{21} L^\mu(r_{21}, r_{43}) - \bar{\Delta}[s_{34}][q \cdot r_{21} q \cdot s_{34} L^\mu(r_{21}, s_{34})] \\ & + \bar{\Delta}[s_{12}][q \cdot s_{12} q \cdot r_{43} L^\mu(r_{43}, s_{12})] + \bar{\Delta}[s_{12}] \bar{\Delta}[s_{34}] q \cdot s_{34} q \cdot s_{12} L^\mu(s_{12}, s_{34}) \} \left. \right]. \quad (\text{B4}) \end{aligned}$$

For channel C , Eq. (13), upon contraction with q_μ becomes

$$q_\mu \mathcal{M}_C^\mu(p_1, p_2, p_3, p_4) = -i e C_C q^\alpha \Gamma_{\alpha\gamma}^1 D_{K^*}[s_{34}] \times (r_{43}^\gamma - s_{34}^\gamma \bar{\Delta}[s_{34}]). \quad (\text{B5})$$

Therefore, by choosing the following combination of amplitudes, of A and B channels, we obtain

$$\begin{aligned} & q_\mu [\mathcal{M}_A^\mu(p_1, p_2, p_3, p_4) + \mathcal{M}_A^\mu(p_3, p_4, p_1, p_2) \\ & + \mathcal{M}_B^\mu(p_1, p_2, p_3, p_4)] \\ & = i e C [D_{K^*}[s_{34}](z_{12\gamma} - r_{21\gamma})(r_{43}^\gamma - s_{34}^\gamma \bar{\Delta}[s_{34}]) \\ & \times D_{K^*}[s_{12}](z_{34\gamma} - r_{43\gamma})(r_{21}^\gamma - s_{12}^\gamma \bar{\Delta}[s_{12}])], \quad (\text{B6}) \end{aligned}$$

where $C_A = C_B \equiv C$ and $g_{\phi K^* K^*} = g_{\phi K K}$.

Choosing the following combination of amplitudes for C channel, we obtain

$$\begin{aligned} & -q_\mu [\mathcal{M}_C^\mu(p_1, p_2, p_3, p_4) + \mathcal{M}_C^\mu(p_3, p_4, p_1, p_2)] \\ & = i e C_C q^\alpha [\Gamma_{\alpha\gamma}^1 D_{K^*}[s_{34}](r_{43}^\gamma - s_{34}^\gamma \bar{\Delta}[s_{34}]) \\ & - \Gamma_{\alpha\gamma}^4 D_{K^*}[s_{12}](r_{21}^\gamma - s_{12}^\gamma \bar{\Delta}[s_{12}])]. \quad (\text{B7}) \end{aligned}$$

Comparing Eqs. (B6) and (B7), we identify $C_C = C$ and

$$g_{\phi K^* K \pi} = g_{\phi K K} \cdot g_{K^* K \pi}, \quad (\text{B8})$$

and the corresponding structures for the vertices satisfy

$$\begin{aligned} q^\alpha \Gamma_{\alpha\gamma}^1 &= z_{12\gamma} - r_{21\gamma}, \\ q^\alpha \Gamma_{\alpha\gamma}^4 &= z_{34\gamma} - r_{43\gamma}. \quad (\text{B9}) \end{aligned}$$

Factorizing q^α , the vertices $\Gamma_{\alpha\gamma}^i$ are constructed as

$$\begin{aligned} \Gamma_{\alpha\gamma}^1 &= \frac{x_{3\alpha}}{x_3 \cdot q} (z_{12\gamma} - r_{21\gamma}), \\ & \text{and} \\ \Gamma_{\alpha\gamma}^4 &= \frac{x_{1\alpha}}{x_1 \cdot q} (z_{34\gamma} - r_{43\gamma}). \quad (\text{B10}) \end{aligned}$$

Thus, the final expression for the selected amplitudes of channel C is

$$\begin{aligned} & \mathcal{M}_C^\mu(p_1, p_2, p_3, p_4) \\ & = -i e C \frac{x_3^\mu}{x_3 \cdot q} D_{K^*}[s_{34}][z_{12} \cdot r_{43} - r_{21} \cdot r_{43} \\ & - \bar{\Delta}[s_{34}](z_{12} \cdot s_{34} - r_{21} \cdot s_{34})]. \quad (\text{B11}) \end{aligned}$$

Thus, in order to obtain the gauge invariant amplitude, we selected a particular set of amplitudes to combine, namely,

$$\begin{aligned} \mathcal{M}_{ABC_{24}}^\mu &= \mathcal{M}_A^\mu(p_1, p_2, p_3, p_4) + \mathcal{M}_A^\mu(p_3, p_4, p_1, p_2) \\ & + \mathcal{M}_B^\mu(p_1, p_2, p_3, p_4) + \mathcal{M}_C^\mu(p_1, p_2, p_3, p_4) \\ & + \mathcal{M}_C^\mu(p_3, p_4, p_1, p_2). \quad (\text{B12}) \end{aligned}$$

The final gauge invariant amplitude for this set of diagrams is given by Eq. (15). A similar expression is

obtained for the exchange $p_2 \leftrightarrow p_4$, by adding the proper amplitudes [Eq. (16)] and replacing $Q_1 \equiv s_{34} - s_{12}$ with $Q_2 \equiv s_{23} - s_{41}$.

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