

Probing the diquark structure of Λ_c in a combined analysis of $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$

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The spatial distributions of $[ud]$ diquark and heavy-light diquark of the SU(3)-flavor antitriplet charmed baryons are investigated by the two singly Cabibbo-suppressed hadronic weak decays, $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$ within the nonrelativistic constituent quark model. The above two spatial distributions are reflected by the two parameters α_p and α_λ , which are the harmonic oscillator strength parameters of the charmed baryons. These two parameters obtain strong constraints from the decay widths of $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$. The larger the harmonic oscillator parameter is, the more compact the spatial distribution will become. The current α_p and α_λ indicate that neither the light diquark nor the heavy-light diquark turns out to favor a compact structure. In addition, some selection rules, including the “ Λ selection rule,” can be useful for the search of excited baryons in the heavy-flavor baryon hadronic weak decays.

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I. INTRODUCTION

Hadrons are colorless systems of (anti)quarks and gluons and have abundant structures. Those, beyond the normal meson and baryon pictures in the conventional quark model, are named as exotic hadrons. Tens of exotic candidates have been measured in experiment, which dramatically expand the hadron spectrum. Their structures are one of the essential topics to understand the underlying nonperturbative properties of quantum chromodynamics (QCD). Due to the complicated nonperturbative mechanism, many “effective objects,” e.g., constituent quarks, are introduced to reproduce the spectrum of hadrons, as well as their decay modes. The diquark is one of the most important “effective objects” and

widely used in diverse works to understand the properties of hadrons and hadronic matter [1–13].

Diquark is a two-quark system in either $\bar{3}$ or 6 color irreducible representations. Due to the Fermi-Dirac statistics, the $\bar{3}$ diquark could be spin-0 and isosinglet diquark, named as “good diquark” [1,2,7], and it has been discussed in various multiquark systems [4,14–23]. This kind of diquark is made of two light flavor quarks with color $\bar{3}$, flavor $\bar{3}$, and spin 0 [7,8]. Within such a configuration, the quark-quark interaction due to the one-gluon-exchange potential is attractive and will lower the energy of the diquark system. As a consequence, the diquark may appear as a compact effective object inside multiquark systems. Meanwhile, the spin triplet flavor symmetric diquark is named as “bad diquarks” whose color configuration is still $\bar{3}$ [1]. Within such a configuration, the quark-quark interaction becomes repulsive, and the diquark structure may not be stable.

The simplest color singlet system containing the good light diquark is the antitriplet singly heavy baryons [8]. As shown by the total wave functions (see the Appendix), the light quarks of Λ_c and Ξ_c form the scalar diquarks, while the sextet single-charm baryons contain the spin triplet

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diquark. In the light quark sector, the $u - d$ quarks of the Λ and Σ^0 are isosinglet and isotriplet, respectively. These differences account for the mass difference between Λ and Σ^0 . Since the spin-dependent interactions between s quark and ud quarks are significant [1], the light baryons are not the ideal place for studying the light diquark directly.

The heavy-light diquark is also an effective object that is widely discussed in the literature of exotic states containing the heavy quark, especially the hidden charm/bottom tetraquarks. For a review of heavy-light diquarks, one can refer to Ref. [7] and references therein. Interestingly, it is concluded by Ref. [7] that the current single bottom baryons do not support the existence of heavy-light diquarks. For the single charm baryons, the diquark of light quarks is usually concerned [24]. Although the traditional charmed baryons may not be the ideal place to study the heavy-light diquark since that c quark is much lighter than b quark [7], we are interested in how the spatial distribution of the heavy-light quark reflects their correlations in terms of the diquark structure.

For singly heavy baryons, it is not clear whether the heavy baryons behave like a three-quark system or with compact diquark degrees of freedom [25]. To be more specific, given that the $[ud]$ diquark within Λ_c be the ideal candidate for the “good diquark,” should it behave more likely to be a compact structure or still have spatial distribution? Intuitively, these two scenarios will have significant impact on the spectrum and decay properties. The decay behaviors of the single charm baryons may provide useful information about the property of diquarks. Some efforts can be found in the literature [25–35].

As the lightest singly charmed baryons, Λ_c decays only via the weak interaction. Here, we concentrate on the hadronic weak decay, which involves both the weak and strong interactions. In this work, two decay modes, $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$, are investigated in the framework of the nonrelativistic constituent quark model (NRCQM) [36,37]. An interesting difference between these two decay modes is that the constituent quarks of the final baryons are the same but with different isospins carried by the light quarks. This discrepancy provides a good opportunity to investigate the light diquark of Λ_c , which is a “good diquark,” and its evolution to the final states, where both “good” and “bad diquark” are present.

Within the framework of NRCQM, all the hadronic weak decay mechanisms, which include the direct meson emission (DME) process, color suppressed (CS) process and pole terms [37], can be described self-consistently [26]. In addition, the spatial distributions of the $[ud]$ and $[cq]$ diquarks in the charmed baryons can be described by parameters of the spatial wave functions. Thus, the effects of spatial distribution of diquarks can be conveniently investigated via the response of observables, e.g., the decay branching ratio, to the diquark parameters.

This paper is organized as follows. A brief introduction of NRQCM is presented in Sec. II. The results and discussions are given in Sec. III. The last section is a brief summary. The $SU(6) \otimes O(3)$ wave functions are listed in the Appendix.

II. FRAMEWORK

The transition diagrams for the $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$ processes are given in Fig. 1. The subfigure (a) is the DME process, where the quark pair created from the emitted W boson is hadronized into one final-state hadron. The subfigure (b) is the CS process, where the quark pair created from the emitted W boson is hadronized into different final hadrons. The subfigures (c), (d) (e), (f) are the pole terms diagrams. For the pole diagrams, there exist the intermediate states. Those four processes are categorized into two cases. The first one is denoted as WS. It means the weak (W) and strong (S) interactions happen in order. Similarly, the other one, is denoted as SW [(e) and (f)]. All transition processes are calculated in the NRCQM framework. The standard $SU(6) \otimes O(3)$ hadron wave functions [37] listed in the Appendix are employed. The relevant operators, both weak interaction and strong interaction, are shortly introduced in the following.

A. The nonrelativistic form of the weak interaction

There are two kinds of weak transition processes that contribute to the SW processes for these two channels. The first one is $cs \rightarrow su$ and the second one is $cd \rightarrow du$. For the DME and CS processes, there is only one kind of operator $c \rightarrow s(u\bar{s})$ that contributes to the weak transition vertex. In the NRCQM framework, the operators that induce these weak transition processes can be obtained with the four-fermion interaction [37,38], and the effective Hamiltonian can be expressed as

$$H_W = \frac{G_F}{\sqrt{2}} \int d\mathbf{x} \frac{1}{2} \{J^{-,\mu}(\mathbf{x}), J_{\mu}^{+}(\mathbf{x})\}, \quad (1)$$

where

$$J^{+,\mu}(\mathbf{x}) = (\bar{u} \quad \bar{c}) \gamma^{\mu} (1 - \gamma_5) \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}, \quad (2)$$

$$J^{-,\mu}(\mathbf{x}) = (\bar{d} \quad \bar{s}) \begin{pmatrix} \cos \theta_C & -\sin \theta_C \\ \sin \theta_C & \cos \theta_C \end{pmatrix} \gamma^{\mu} (1 - \gamma_5) \begin{pmatrix} u \\ c \end{pmatrix}. \quad (3)$$

θ_C is the Cabibbo angle. The weak transition operators that contribute to $cs \rightarrow su$, $cd \rightarrow du$, and $c \rightarrow s(u\bar{s})$ are written as

$$H_{W,cs \rightarrow su} = \frac{G_F}{\sqrt{2}} V_{us} V_{cs} \frac{1}{(2\pi)^3} \delta^3(\mathbf{p}_c + \mathbf{p}_s - \mathbf{p}_u - \mathbf{p}_{\bar{s}}) \bar{u}(\mathbf{p}_s) \gamma_\mu (1 - \gamma_5) u(\mathbf{p}_c) \bar{u}(\mathbf{p}_u) \gamma^\mu (1 - \gamma_5) u(\mathbf{p}_{\bar{s}}) \hat{\alpha} \hat{\beta}, \quad (4)$$

$$H_{W,cd \rightarrow du} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cd} \frac{1}{(2\pi)^3} \delta^3(\mathbf{p}_c + \mathbf{p}_d - \mathbf{p}_u - \mathbf{p}_{\bar{d}}) \bar{u}(\mathbf{p}_d) \gamma_\mu (1 - \gamma_5) u(\mathbf{p}_c) \bar{u}(\mathbf{p}_u) \gamma^\mu (1 - \gamma_5) u(\mathbf{p}_{\bar{d}}) \hat{\alpha}' \hat{\beta}', \quad (5)$$

$$H_{W,c \rightarrow s\bar{s}u} = \frac{G_F}{\sqrt{2}} V_{us} V_{cs} \frac{1}{(2\pi)^3} \delta^3(\mathbf{p}_c - \mathbf{p}_s - \mathbf{p}_u - \mathbf{p}_{\bar{s}}) \bar{u}(\mathbf{p}_s) \gamma_\mu (1 - \gamma_5) u(\mathbf{p}_c) \bar{u}(\mathbf{p}_u) \gamma^\mu (1 - \gamma_5) v(\mathbf{p}_{\bar{s}}) \hat{\alpha}_1 \hat{I}'_K, \quad (6)$$

where V_{cs} , V_{us} , V_{cd} and V_{ud} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. $\hat{I}'_K = b_u^\dagger b_u$ is the isospin operator for the K^+ production for the color suppressed process. $b_u^\dagger$ and b_u are the creation and annihilation operators of the u quark. $\hat{\alpha}^{(\prime)}$ and $\hat{\beta}^{(\prime)}$ are the flavor-changing operators with $\hat{\alpha}c = s$, $\hat{\beta}s = u$ and $\hat{\alpha}'c = d$, $\hat{\beta}'d = u$. In the nonrelativistic limit, the transition operator H_W for the $cs \rightarrow su$ process is rewritten as

$$H_{W,cs \rightarrow su}^{\text{PC}} = \frac{G_F}{\sqrt{2}} V_{us} V_{cs} \frac{1}{(2\pi)^3} \sum_{i \neq j} \hat{\alpha}_i \hat{\beta}_j \delta^3(\mathbf{p}'_i + \mathbf{p}'_j - \mathbf{p}_i - \mathbf{p}_j) (1 - \langle s'_{z,i} | \boldsymbol{\sigma}_i | s_{z,i} \rangle \langle s'_{z,j} | \boldsymbol{\sigma}_j | s_{z,j} \rangle), \quad (7)$$

$$H_{W,cs \rightarrow su}^{\text{PV}} = \frac{G_F}{\sqrt{2}} V_{us} V_{cs} \frac{1}{(2\pi)^3} \sum_{i \neq j} \hat{\alpha}_i \hat{\beta}_j \delta^3(\mathbf{p}'_i + \mathbf{p}'_j - \mathbf{p}_i - \mathbf{p}_j) \left\{ -(\langle s'_{z,i} | \boldsymbol{\sigma}_i | s_{z,i} \rangle - \langle s'_{z,i} | \boldsymbol{\sigma}_j | s_{z,j} \rangle) \left[\left(\frac{\mathbf{p}_i}{2m_i} - \frac{\mathbf{p}_j}{2m_j} \right) + \left(\frac{\mathbf{p}'_i}{2m'_i} - \frac{\mathbf{p}'_j}{2m'_j} \right) \right] \right. \\ \left. + i(\langle s'_{z,i} | \boldsymbol{\sigma}_i | s_{z,i} \rangle \times \langle s'_{z,i} | \boldsymbol{\sigma}_j | s_{z,j} \rangle) \left[\left(\frac{\mathbf{p}_i}{2m_i} - \frac{\mathbf{p}_j}{2m_j} \right) - \left(\frac{\mathbf{p}'_i}{2m'_i} - \frac{\mathbf{p}'_j}{2m'_j} \right) \right] \right\}, \quad (8)$$

where the superscript PC (PV) stands for the parity conserved (violated) operator $H_{W,cd \rightarrow su}^{\text{PC(PV)}}$. The subscripts i and j are used to indicate which quark acts in the operators. Similarly, we can obtain the $H_{W,cd \rightarrow du}^{\text{PC(PV)}}$ operator.

In the nonrelativistic limit, the operators for the $c \rightarrow s(u\bar{s})$ process are written as

$$H_{W,c \rightarrow s\bar{s}u}^{\text{PC}} = \frac{G_F}{\sqrt{2}} V_{us} V_{cs} \frac{\beta}{(2\pi)^3} \delta^3(\mathbf{p}_3 - \mathbf{p}'_3 - \mathbf{p}_4 - \mathbf{p}_5) \left\{ \langle s'_3 | I | s_3 \rangle \langle s_5 \bar{s}_4 | \boldsymbol{\sigma} | 0 \rangle \left(\frac{\mathbf{p}_5}{2m_5} + \frac{\mathbf{p}_4}{2m_4} \right) \right. \\ \left. - \left[\left(\frac{\mathbf{p}'_3}{2m'_3} + \frac{\mathbf{p}_3}{2m_3} \right) \langle s'_3 | I | s_3 \rangle - i \langle s'_3 | \boldsymbol{\sigma} | s_3 \rangle \times \left(\frac{\mathbf{p}_3}{2m_3} - \frac{\mathbf{p}'_3}{2m'_3} \right) \right] \langle s_5 \bar{s}_4 | \boldsymbol{\sigma} | 0 \rangle \right. \\ \left. - \langle s'_3 | \boldsymbol{\sigma} | s_3 \rangle \left[\left(\frac{\mathbf{p}_5}{2m_5} + \frac{\mathbf{p}_4}{2m_4} \right) \langle s_5 \bar{s}_4 | I | 0 \rangle - i \langle s_5 \bar{s}_4 | \boldsymbol{\sigma} | 0 \rangle \times \left(\frac{\mathbf{p}_4}{2m_4} - \frac{\mathbf{p}_5}{2m_5} \right) \right] \right. \\ \left. + \langle s'_3 | \boldsymbol{\sigma} | s_3 \rangle \left(\frac{\mathbf{p}'_3}{2m'_3} + \frac{\mathbf{p}_3}{2m_3} \right) \langle s_5 \bar{s}_4 | I | 0 \rangle \right\} \hat{\alpha}_3 \hat{I}'_K, \quad (9)$$

$$H_{W,c \rightarrow s\bar{s}u}^{\text{PV}} = \frac{G_F}{\sqrt{2}} V_{us} V_{cs} \frac{\beta}{(2\pi)^3} \delta^3(\mathbf{p}_3 - \mathbf{p}'_3 - \mathbf{p}_4 - \mathbf{p}_5) (-\langle s'_3 | I | s_3 \rangle \langle s_5 \bar{s}_4 | I | 0 \rangle + \langle s'_3 | \boldsymbol{\sigma} | s_3 \rangle \langle s_5 \bar{s}_4 | \boldsymbol{\sigma} | 0 \rangle) \hat{\alpha}_3 \hat{I}'_K, \quad (10)$$

where I is the two-dimension identity matrix. We have to specify the quark that the operator acts on, and the quark labels are illustrated by Fig. 2. β is a symmetry factor which equals to $\sqrt{3}$ for the DME process and $2/\sqrt{3}$ for the CS process. The fourth particle is an antiquark, and its spin is labeled with $\bar{s}_4$. In order to evaluate the spin matrix element such as $\langle s_5 \bar{s}_4 | I | 0 \rangle$, the particle-hole conjugation [39] is employed:

$$\langle j, -m | \rightarrow (-1)^{j+m} | j, m \rangle. \quad (11)$$

B. The kaon emission of the pole terms

The kaon emission in the pole terms is via strong interactions and can be described within the chiral quark model [31,40,41]. The Hamiltonian at leading order [31,40] reads as

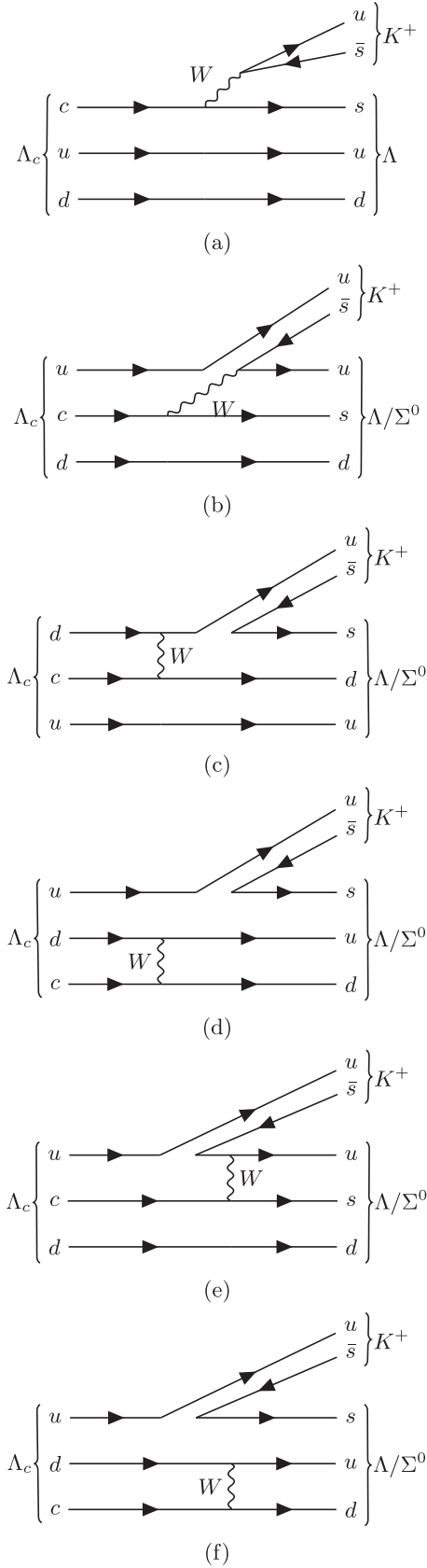


FIG. 1. The transition diagrams of the $\Lambda_c \rightarrow \Lambda/\Sigma^0 K^+$ process. (a) DME process; (b) CS process; (c)–(f) pole terms.

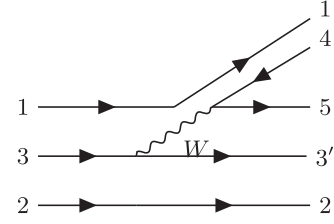


FIG. 2. The color suppressed transitions at the quark level. The charm quark is always labeled as the third particle.

$$H_\chi = \int d\mathbf{x} \frac{g_A^q}{f_m} \bar{q}(\mathbf{x}) \gamma_\mu \gamma_5 q(\mathbf{x}) \partial^\mu \phi(\mathbf{x}), \quad (12)$$

where $q(\mathbf{x})$ is the light quark field. g_A^q is the quark axial vector coupling constant. Here we can adopt that $g_A^q = 1$ [35]. f_m is the pseudoscalar meson decay constant depending on which mesons are involved in the interaction. The decay constants of K is $f_K = 1.2f_\pi$ with $f_\pi = 94$ MeV. ϕ is the pseudoscalar octet and expressed as

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \quad (13)$$

The nonrelativistic form of H_χ reads as

$$H_\chi = \frac{1}{\sqrt{(2\pi)^3 2\omega_m}} \sum_j \frac{1}{f_m} \left[\omega_m \left(\frac{\boldsymbol{\sigma} \cdot \mathbf{p}_f^j}{2m_f} + \frac{\boldsymbol{\sigma} \cdot \mathbf{p}_i^j}{2m_i} \right) - \boldsymbol{\sigma} \cdot \mathbf{k} \right] \times \hat{I}_\chi^j \delta^3(\mathbf{p}_f^j + \mathbf{k} - \mathbf{p}_i^j). \quad (14)$$

ω_m and $\mathbf{k}$ are the energy and momentum of the pseudoscalar meson, respectively. $\mathbf{p}_i^j$ and $\mathbf{p}_f^j$ are the initial and final momentum of the j th light quark, respectively. $\hat{I}_\chi^j$ is the isospin operator acting on the j th light quark and is written as

$$\hat{I}_\chi^j = \begin{cases} b_s^\dagger b_u, & \text{for } K^+, \\ b_u^\dagger b_s, & \text{for } K^-. \end{cases} \quad (15)$$

C. Amplitudes, asymmetry parameter, and decay width

The amplitudes of the DME, CS, WS, and SW processes are constructed as follows:

$$\mathcal{M}_{\text{CS,PC/PV}}^{J_f, J_f^z; J_i, J_i^z} = \langle B_f(\mathbf{P}_f; J_f, J_f^z); M(\mathbf{k}) | H_{W, c \rightarrow s\bar{s}u}^{\text{PC/PV}} | B_c(\mathbf{P}_i; J_i, J_i^z) \rangle, \quad (16)$$

$$\mathcal{M}_{\text{DME,PC/PV}}^{J_f, J_f^z; J_i, J_i^z} = \langle B_f(\mathbf{P}_f; J_f, J_f^z); M(\mathbf{k}) | H_{W,c \rightarrow s\bar{s}u}^{\text{PC/PV}} | B_c(\mathbf{P}_i; J_i, J_i^z) \rangle, \quad (17)$$

$$\mathcal{M}_{\text{WS,PC/PV}}^{J_f, J_f^z; J_i, J_i^z} = \langle B_f(\mathbf{P}_f; J_f, J_f^z); M(\mathbf{k}) | H_\chi | B_m(\mathbf{P}_i; J_i, J_i^z) \rangle \frac{i}{\not{p}_{B_m} - m_{B_m} + i\frac{\Gamma_{B_m}}{2}} \langle B_m(\mathbf{P}_i; J_i, J_i^z) | H_{W,cd \rightarrow su}^{\text{PC/PV}} | B_c(\mathbf{P}_i; J_i, J_i^z) \rangle, \quad (18)$$

$$\begin{aligned} M_{\text{SW,PC/PV}}^{J_f, J_f^z; J_i, J_i^z} &= \langle B_f(\mathbf{P}_f; J_f, J_f^z) | H_{W,cs \rightarrow su}^{\text{PC/PV}} | B'_c(\mathbf{P}_i; J_i, J_i^z) \rangle \frac{i}{\not{p}_{B'_c} - m_{B'_c} + i\frac{\Gamma_{B'_c}}{2}} \langle B'_c(\mathbf{P}_i; J_i, J_i^z); M(\mathbf{k}) | H_\chi | B_c(\mathbf{P}_i; J_i, J_i^z) \rangle \\ &+ \langle B_f(\mathbf{P}_f; J_f, J_f^z) | H_{W,cd \rightarrow du}^{\text{PC/PV}} | B'_c(\mathbf{P}_i; J_i, J_i^z) \rangle \frac{i}{\not{p}_{B'_c} - m_{B'_c} + i\frac{\Gamma_{B'_c}}{2}} \langle B'_c(\mathbf{P}_i; J_i, J_i^z); M(\mathbf{k}) | H_\chi | B_c(\mathbf{P}_i; J_i, J_i^z) \rangle, \end{aligned} \quad (19)$$

where $B_c(\mathbf{P}_i; J_i, J_i^z)$ and $B_f(\mathbf{P}_f; J_f, J_f^z)$ are the initial charmed baryon and final baryon states, respectively. $|B_m(\mathbf{P}_i; J_i, J_i^z)\rangle$ and $|B'_c(\mathbf{P}_i; J_i, J_i^z)\rangle$ denote the intermediate light baryons and charmed baryons, respectively. In this work, only the ground states and the first excited states are considered for the PC processes and PV processes, respectively, because the dominant contributions of the pole terms come from the processes that the intermediate states are approximately on-shell. $M(\mathbf{k})$ is used to label the pseudo-scalar meson.

With the transition amplitudes, we can obtain the decay width

$$\begin{aligned} \Gamma(A \rightarrow B + C) &= 8\pi^2 \frac{|k| E_B E_C}{M_A} \frac{1}{2J_A + 1} \\ &\times \sum_{\text{spin}} (|\mathcal{M}_{\text{PC}}|^2 + |\mathcal{M}_{\text{PV}}|^2), \end{aligned} \quad (20)$$

with the normalization convention

$$\langle B_f(\mathbf{P}_f) | B_i(\mathbf{P}_i) \rangle = \delta^3(\mathbf{P}_f - \mathbf{P}_i), \quad (21)$$

and J_A is the spin of the initial state. The parity asymmetry parameter is also an important observable for the hadronic weak decay, which was first introduced by Lee and Yang [42]. In our work, the parity asymmetry parameter can be obtained from the parity conserved and violated amplitudes:

$$\alpha = \frac{2\text{Re}[M_{\text{PV}}^* M_{\text{PC}}]}{|M_{\text{PC}}|^2 + |M_{\text{PV}}|^2}, \quad (22)$$

where M_{PC} and M_{PV} are the PC and the PV amplitudes with $J_i^z = J_f^z = 1/2$, respectively.

III. NUMERICAL RESULTS AND DISCUSSION

A. Parameters and inputs

The values of quark masses used in this work are $m_u = m_d = m_s = 0.38$ GeV and $m_c = 1.8$ GeV. Here, the SU(3)-flavor-symmetry is adopted for simplification. According to

Figs. 1(c)–1(f), the intermediate states of the pole terms considered in this work are as follows (also collected in Table I):

- (i) p , Ξ_c^0 , and $\Xi_c'^0$ for the PC processes, and
- (ii) $N(1535)$, $N(1650)$, $|\Xi_c^0, {}^2P_\lambda\rangle$, $|\Xi_c^0, {}^2P_\rho\rangle$, $|\Xi_c^0, {}^4P_\rho\rangle$, $|\Xi_c'^0, {}^2P_\lambda\rangle$, $|\Xi_c'^0, {}^2P_\rho\rangle$, and $|\Xi_c'^0, {}^4P_\lambda\rangle$ for the PV processes.

The mass of p , Ξ_c^0 , $\Xi_c'^0$, $N(1535)$ and $N(1650)$ are taken from Particle Data Group (PDG) [43]. As for $N(1535)$ and $N(1650)$, we take two cases to describe them. The first one is that $N(1535)$ and $N(1650)$ are treated as the states of $[70, {}^2_8]$ and $[70, {}^4_8]$, respectively. The second one is that $N(1535)$ and $N(1650)$ are treated as the mixing states of $[70, {}^2_8]$ and $[70, {}^4_8]$ and are expressed as

$$\begin{pmatrix} N(1535) \\ N(1650) \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} [70, {}^2_8, 1, 1, \frac{1}{2}] \\ [70, {}^4_8, 1, 1, \frac{1}{2}] \end{pmatrix}. \quad (23)$$

θ is the mixing angle that ranges from 24° to 32° [44]. In this work, we take $\theta = 30^\circ$. The masses of $N(1535)$ and $N(1650)$ are also taken from PDG [43] for both cases.

Due to lack of experimental data of the mass of the first excited single charmed baryons, the mass of the first excited states of Ξ_c are taken from Ref. [25] in a constituent quark model. In this literature, there two different models, and the masses based on three-quark model are employed in our calculation. More theoretical results about the mass of the first excited single charmed baryons can be find in the Table V of Ref. [45] and used as a comparison. The width of the first excited charmed baryons is also not determined in experiments. As the result, the estimated value 5×10^{-4} GeV is taken from Ref. [46].

The values of harmonic oscillator strengths for the baryons are listed in Table II. Only the values of α_ρ are listed. Because the strong interaction is flavor independent, the strength constants α_λ and α_ρ are related with

$$\frac{\alpha_\lambda}{\alpha_\rho} = \frac{1}{\sqrt{2}} \left[\frac{3(m_1 + m_2)m_3}{(m_1 + m_2 + m_3)m_1 m_2} \right]^{1/4}. \quad (24)$$

TABLE I. The hadron masses and widths (in unit of GeV). The ‘...’ means the width is very small, which can be neglected or unrelated to this work. The width of first excited charmed baryons are estimated values.

States	p	Λ	Σ^0	Λ_c	K^+	Ξ_c^0	$\Xi_c'^0$	$N(1535)$	$N(1650)$
Mass	0.938	1.116	1.193	2.286	0.494	2.47	2.579	≈ 1.510	≈ 1.665
Width	...	...	...	...	...	...	...	0.11	0.135
States	Ξ_c^0 [25]				$\Xi_c'^0$ [25]				
	$ ^2P_\lambda\rangle$	$ ^2P_\rho\rangle$	$ ^4P_\rho\rangle$	$ ^2P_\lambda\rangle$	$ ^2P_\rho\rangle$	$ ^4P_\lambda\rangle$			
Mass	2.788	2.935	2.977	2.893	3.040	2.935			
Width	5×10^{-4}	5×10^{-4}	5×10^{-4}	5×10^{-4}	5×10^{-4}	5×10^{-4}			

TABLE II. The values of harmonic oscillator strengths (in unit of GeV). θ is the mixing angle.

θ	0°	30°
$\alpha_\rho(S=0, C=0)$	0.49	0.45
$\alpha_\rho(S=1, C=0)$	0.49	0.4
$\alpha_\rho(S=0, C=1)$	0.4	0.55
$\alpha_\rho(S=1, C=1)$	0.4	0.5

More details can be find in the Appendix. Considering the flavor symmetry breaking, the value of α_ρ are different for the charmed and light baryons. Note that the value of α_ρ are taken empirically as in our previous work [26,46]. This is reasonable as this work is a qualitative analysis of the diquark structures in the Λ_c .

B. The numerical results and the selection rules

The Λ and the Σ^0 have the same constituent quarks, so the $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$ processes share the same decay mechanisms as shown by Fig. 1. While the branching ratio of the $\Lambda_c \rightarrow \Lambda K^+$ process is $(6.42 \pm 0.31)\%$, which is about 1.7 times as that, i.e., $(3.70 \pm 0.31)\%$ [43], of the $\Lambda_c \rightarrow \Sigma^0 K^+$ process. The only apparent difference between these two channels is that the isospin of the final baryons. The Λ (Σ^0) is an isospin singlet (triplet) state. This implies that the key to understand the different branching ratios is the configuration of u and d quarks. All the values of transition amplitudes are listed in Table IV. One should note that some transition amplitudes are vanishing, as either the strong transition processes or the weak transition processes are forbidden by selection rules, such as the “ Λ selection rule” making that the $N(1650)$ has no contributions to the pole terms of $\Lambda_c \rightarrow \Lambda K^+$.

Firstly, we note that the DME process has no contribution to the $\Lambda_c \rightarrow \Sigma^0 K^+$. This is because the u and d quarks are spectators in the DME process, as illustrated by Fig. 1(a). The isospin of the u and d quarks pair should not be changed. Λ_c belongs to the $SU(3)$ flavor $\bar{3}$ representation, leaving the

isospin of the $[ud]$ quarks 0. As the Λ (Σ^0) is an isospin singlet (triplet) state, thus, the DPE process of the $\Lambda_c \rightarrow \Lambda K^+$ ($\Lambda_c \rightarrow \Sigma^0 K^+$) process is allowed (forbidden).

Secondly, as shown in Table IV, several transition amplitudes of the pole terms are also 0. When $\theta = 0^\circ$, $N(1650)$ has no contribution to the WS process of the $\Lambda_c \rightarrow \Lambda K^+$ process because of the “ Λ selection rule” [47] that leads to the vanishing transition matrix element between N^* of $[70,^4 8]$ and $[56,^2 8]$ in $N^* \rightarrow \Lambda K/K^*$. This follows because, to keep the total wave function of Λ completely antisymmetric, the total spin of the $[ud]$ in the Λ has to be $S_{[ud]} = 0$. The $N(1650)$ is a $[70,^4 8]$ state. So, the total spin of $[ud]$ in the $N(1650)$ is 1 as illustrated by the wave function of $N(1650)$ given in the Appendix. With the spectator approximation, the strangeness emissions in $N(1650) \rightarrow \Lambda K$, the spin of $[ud]$ must be persevered. As a consequence, this leads to a correlated vanishing transition matrix element $N(1650) \rightarrow \Lambda K^+$.

For the SW processes, c quark is not involved in the strong interaction and can be treated as a spectator, and the spin of c is not changed. So, only the light degrees of freedom need to be dealt with at the strong interaction vertex. In other words, the heavy quark spin symmetry (HQSS) is applicable for this kinds of processes. The J^P of the light degrees of freedom of the charmed baryons are listed in Table III.

For the PC part of SW process, the strong transition processes are $B_{\bar{3}} \rightarrow B_{\bar{3}} K^+$ and $B_{\bar{3}} \rightarrow B_6 K^+$. The first process can be presented as $0^+ \rightarrow 0^+ 0^-$. In order to keep parity conservation, only odd orbital angular momentum is allowed. However, only the S wave interaction is allowed because of the total angular momentum conservation. Thus, the strong transition amplitude $B_{\bar{3}} \rightarrow B_{\bar{3}} K^+$ is vanishing. As for $B_{\bar{3}} \rightarrow B_6 K^+$, e.g., $0^+ \rightarrow 1^+ 0^-$, the P wave interaction is allowed. So, as shown in Table IV, Ξ_c^0 has no contribution, but $\Xi_c'^0$ can contribute to the PC parts of the SW processes in $\Lambda_c \rightarrow \Lambda/\Sigma^0 K^+$.

Table IV also shows that the first orbital excitation state of $|\Xi_c^{0,2} P_\lambda\rangle$ and $|\Xi_c'^{0,2} P_\rho\rangle$ do not contribute to the PV parts of the SW processes. Here, we also focus on the light degrees of freedom. Note that the total spin of the light

TABLE III. The quantum number of the light degrees of freedom for charmed baryons.

States	J^P
$ ^2S, \frac{1}{2}^+\rangle_{\bar{3}}$	0^+
$ ^2P_{\lambda}, \frac{1}{2}^-\rangle_{\bar{3}}$	1^-
$ ^2P_{\rho}, \frac{1}{2}^-\rangle_{\bar{3}}$	0^- or 1^-
$ ^4P_{\rho}, \frac{1}{2}^-\rangle_{\bar{3}}$	0^- or 1^-
$ ^2S, \frac{1}{2}^+\rangle_6$	1^+
$ ^2P_{\lambda}, \frac{1}{2}^-\rangle_6$	0^- or 1^-
$ ^2P_{\rho}, \frac{1}{2}^-\rangle_6$	1^-
$ ^4P_{\lambda}, \frac{1}{2}^-\rangle_6$	0^- or 1^-

quarks in $|\Xi_c^{0,2}P_{\lambda}\rangle$ and $|\Xi_c^{0,2}P_{\rho}\rangle$ is 0. Thus, the J^P of the light degrees of freedom for these two states are 1^- . The transition of $0^+ \rightarrow 1^-0^-$ is not allowed because of the momentum conservation and parity conservation. At the quark level, one can easily find that

$$\langle \chi_{\frac{1}{2}, -\frac{1}{2}}^{\rho} | \sigma_{x,y,z}^{(1)} | \chi_{\frac{1}{2}, -\frac{1}{2}}^{\rho} \rangle = 0, \quad (25)$$

which leads to that $|\Xi_c^{0,2}P_{\lambda}\rangle$ and $|\Xi_c^{0,2}P_{\rho}\rangle$ do not contribute to the strong interaction matrix elements of the SW processes.

In addition to the above situations, we also find that the weak interaction operator $H_{cd \rightarrow du}$ does not contribute to the $\Lambda_c \rightarrow \Sigma^0 K^+$ process. Instead it contributes to $\Lambda_c \rightarrow \Lambda K^+$ process via the SW diagrams with the $\Xi_c |^2P_{\rho}\rangle, \Xi_c |^4P_{\rho}\rangle, \Xi_c' |^2P_{\lambda}\rangle$, and $\Xi_c' |^4P_{\lambda}\rangle$ charmed baryons as the intermediate states. For some specific cases, the transition amplitudes equal zero. Taking $\langle \Sigma^0 | H_W | \Xi_c^{0,2}P_{\lambda} \rangle$, for example, the flavor-spin part makes $\langle \Sigma^0 | H_W^{\text{PC}} | \Xi_c^{0,2}P_{\lambda} \rangle = 0$. The difference of the wave function between Λ and Σ^0 lies in the flavor wave function. Consider that both the initial c quark and the final d quark are labeled as the third ones. As the result, only the final states with d quark as the third quark need to be considered, i.e., the relevant part of flavor wave function

$$\phi_{\Lambda}^{\lambda} = -\frac{1}{2}(su + us)d, \quad \phi_{\Lambda}^{\rho} = \frac{1}{2\sqrt{3}}(us - su)d, \quad (26)$$

$$\phi_{\Sigma^0}^{\lambda} = \frac{1}{2\sqrt{3}}(su + us)d, \quad \phi_{\Sigma^0}^{\rho} = \frac{1}{2}(su - us)d. \quad (27)$$

This clearly shows that the differences in $\langle \Lambda | H_W | B_m \rangle$ and $\langle \Sigma^0 | H_W | B_m \rangle$ arise solely from their different flavor coefficients.

As shown by Table IV, the amplitudes of the nonfactorizable processes could have the same order of magnitude as

TABLE IV. The values of the $\Lambda_c \rightarrow \Lambda/\Sigma^0 K^+$ amplitudes (in unit of $10^{-9} \text{ GeV}^{-1/2}$). For the SW processes, there are two kinds of weak transitions processes, i.e., $cs \rightarrow su$ and $cd \rightarrow ud$, and the relevant results are labeled with (a, b) . Here, a is the amplitude for $cs \rightarrow su$, and b is the amplitudes for $cd \rightarrow ud$. The imaginary part of the amplitudes less than 10^{-11} GeV is set as zero.

Parity	Processes	States	$\Lambda_c \rightarrow \Lambda K^+$		$\Lambda_c \rightarrow \Sigma^0 K^+$	
			$\theta = 0^\circ$	$\theta = 30^\circ$	$\theta = 0^\circ$	$\theta = 30^\circ$
PC	DPE	...	7.78	5.96	0	0
	CS	...	0.82	0.50	-1.42	-0.85
	WS	p	-4.46	-4.88	0.85	0.94
	SW	Ξ_c^0	(0, 0)	(0, 0)	(0, 0)	(0, 0)
		$\Xi_c'^0$	(1.69, 3.22)	(2.28, 4.32)	(-3.19, 0)	(-4.09, 0)
	Total	...	9.05	8.18	-3.76	-4.00
PV	DPE	...	-4.71	-4.11	0	0
	CS	...	-1.19	-0.80	2.19	1.43
	WS	$N(1535)$	$4.11 - 0.23i$	$2.22 - 0.13i$	$0.92 - 0.052i$	$-0.85 + 0.098i$
		$N(1650)$	0	$0.59 - 0.033i$	$8.91 - 0.82i$	$5.46 - 0.49i$
	SW	$\Xi_c^0 ^2P_{\rho}\rangle$	(-0.57, 1.10)	(-1.11, 2.11)	(1.10, 0)	(2.04, 0)
		$\Xi_c^0 ^2P_{\lambda}\rangle$	(0, 0)	(0, 0)	(0, 0)	(0, 0)
		$\Xi_c^0 ^4P_{\rho}\rangle$	(2.27, -4.31)	(4.35, -8.26)	(-4.30, 0)	(-8.01, 0)
		$\Xi_c^0 ^2P_{\rho}\rangle$	(0, 0)	(0, 0)	(0, 0)	(0, 0)
		$\Xi_c^0 ^2P_{\lambda}\rangle$	(-0.56, -1.06)	(-1.08, -2.06)	(1.04, 0)	(1.98, 0)
		$\Xi_c^0 ^4P_{\lambda}\rangle$	(2.17, 4.14)	(4.26, 8.08)	(-4.09, 0)	(-7.77, 0)
	Total	...	$1.38 - 0.23i$	$4.17 - 0.16i$	$5.78 - 0.87i$	$-5.72 - 0.40i$

TABLE V. The branching ratio ($\times 10^{-4}$) and the asymmetry parameter of $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$.

Channel	Experimental value	Branching ratio		α	
		$\theta=0^\circ$	$\theta=30^\circ$	$\theta=0^\circ$	$\theta=30^\circ$
$\Lambda_c \rightarrow \Lambda K^+$	6.42 ± 0.31	6.51	6.55	-0.30	-0.81
$\Lambda_c \rightarrow \Sigma^0 K^+$	3.70 ± 0.31	3.47	3.51	0.90	-0.94

that of the factorizable ones. It means that the nonfactorizable processes are important and need to be treated carefully considering the interference among these terms. Meanwhile, one should note that the contributions of p , $N(1535)$, and $N(1650)$ are very different for these two channels because of the “ Λ selection rule” [47]. As a consequence, the mixing of $[70,^2 8]$ and $[70,^4 8]$ has noticeable impact on the total parity violating amplitudes, which is reflected in the asymmetry parameters as shown by Table V.

C. The distribution of the diquark

In the NRCQM framework, the spatial distribution of the constituent quarks is described by the spatial wave function $\psi(\boldsymbol{\rho}, \boldsymbol{\lambda})$. The average value of the square of displacement between quarks (i.e., kind of mean square radii) $\mathbf{r}_{ij}^2 = (\mathbf{r}_i - \mathbf{r}_j)^2$ can be obtained by

$$\langle \mathbf{r}_{ij}^2 \rangle = \langle \psi(\boldsymbol{\rho}, \boldsymbol{\lambda}) | \mathbf{r}_{ij}^2 | \psi(\boldsymbol{\rho}, \boldsymbol{\lambda}) \rangle, \quad (28)$$

where $\psi(\boldsymbol{\rho}, \boldsymbol{\lambda})$ is the spatial wave function in the coordinate space. $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ are the Jacobi coordinates, and the details are listed in the Appendix. For the ground states, we have

$$\langle \boldsymbol{\rho}^2 \rangle = \frac{3}{2} \frac{1}{\alpha_\rho^2}, \quad \langle \boldsymbol{\lambda}^2 \rangle = \frac{3}{2} \frac{1}{\alpha_\lambda^2}. \quad (29)$$

Then, $\sqrt{\langle \boldsymbol{\rho}^2 \rangle}$, which is the average distance between the u and d quark, can serve as a useful quantity describing the size of the $[ud]$ diquark of the ground states of the singly charmed baryons. Thus, we can see that the larger value of α_ρ means that the $[ud]$ diquark is more compact. For the ground states of the singly charmed baryons, similarly, the size of the heavy-light diquark can be defined as

$$\begin{aligned} \langle \mathbf{r}_{cq}^2 \rangle &= \langle \psi(\boldsymbol{\rho}, \boldsymbol{\lambda}) | (\mathbf{r}_3 - \mathbf{r}_q)^2 | \psi(\boldsymbol{\rho}, \boldsymbol{\lambda}) \rangle \\ &= \langle \psi(\boldsymbol{\rho}, \boldsymbol{\lambda}) | \frac{3}{2} \boldsymbol{\lambda}^2 + \frac{1}{2} \boldsymbol{\rho}^2 | \psi(\boldsymbol{\rho}, \boldsymbol{\lambda}) \rangle \\ &= \frac{9}{4} \frac{1}{\alpha_\lambda^2} + \frac{3}{4} \frac{1}{\alpha_\rho^2}. \end{aligned} \quad (30)$$

This result indicates that the larger value of α_ρ or α_λ , the more compact of the heavy-light diquark and that the α_λ plays a leading role.

In the NRCQM framework, the parameters α_ρ and α_λ are not independent, which are determined by the quark mass and the spring constant K as shown by Eq. (A37). Also, α_ρ and α_λ determine the baryon spectrum. In this part, we assume that α_ρ and α_λ are independent of each other and regard them as free parameters. Strictly speaking, the baryon spectrum changes with the variation of α_ρ and α_λ as shown by Eq. (A41). Those two variables can be treated as the parameter of form factors, which can be defined by matching the quark-level wave function convolution to the hadronic coupling for a hadron interaction vertex (see, e.g., [48,49]). In this sense, the variation of harmonic oscillator strengths could be treated as the variation of the form factors. With the charmed baryons masses fixed by the kinematic variables, the changes of the harmonic oscillator strengths will expose the correlation between the diquark structure and the calculated observables. Then, the physical values of α_ρ and α_λ can be constrained by the relevant decay widths.

In this work, only the distribution of the good light diquark for the singly charmed baryons is concerned. While keeping the ratio of the parameters α_ρ and α_λ fixed in the Λ_c and Ξ_c wave functions, and holding the other parameters constant, the allowed α_ρ and α_λ parameter regions are determined by the $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$ partial decay widths. For example, the allowed $\alpha_\rho - \alpha_\lambda$ parameter regions for the $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$ decay widths are shown by the orange and blue regions in Fig. 3, respectively. The red line is obtained with Eq. (24). Figure 3 indicates that the allowed region of α_ρ and α_λ is restricted and consistent with the traditional values of NRCQM, no matter if the mixing of $N(1535)$ and $N(1650)$ is considered or not. As both of these two parameters favor small values, neither the $[ud]$ quark pair nor the heavy-light diquark in the singly charmed baryons is a pointlike structure, which is consistent with the conclusion of Ref. [26].

We also notice that if the distribution of the $[ud]$ diquark of Λ_c and Λ or Σ^0 are very different from each other, the contribution of DME will be suppressed. The reason is that the overlap of the spatial wave function between the ρ mode of Λ_c and the ρ mode of Λ will be suppressed with the increase of α_ρ of the antitriplet charmed baryons. This leads to the fact that the decay width of these two channels will be saturated by the contribution of the CS and pole terms. It means that the ratio of the decay width R_Γ , which is defined as

$$R_\Gamma = \frac{\text{Br}(\Lambda_c \rightarrow \Lambda K^+)}{\text{Br}(\Lambda_c \rightarrow \Sigma^0 K^+)}, \quad (31)$$

should be close to a constant. It is determined by the flavor symmetry, with the increase of α_ρ .

For the $cs \rightarrow su$, $cd \rightarrow du$ or $c \rightarrow s(u\bar{s})$ processes, the total isospin and the third component of the isospin of the

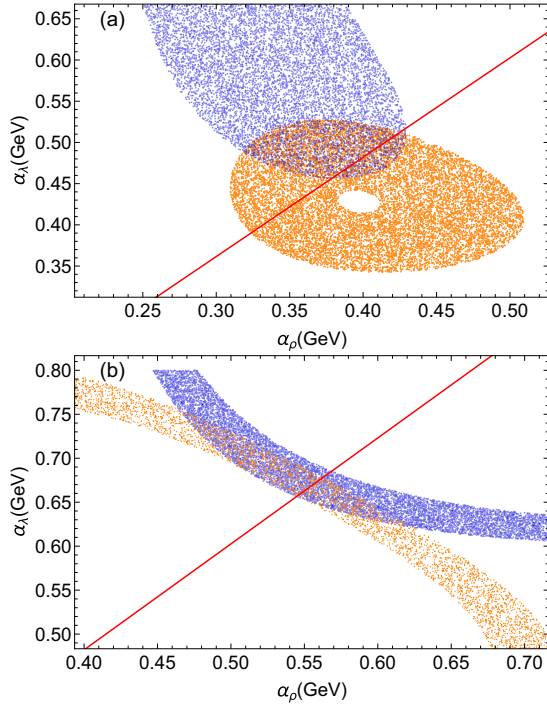


FIG. 3. The parameter ranges of α_ρ and α_λ constrained by the $\Lambda_c \rightarrow \Lambda K^+$ (orange region) and $\Lambda_c \rightarrow \Sigma^0 K^+$ (blue region) decay widths. The red solid line is determined within quark model, i.e., via Eq. (24). Subfigure (a) and (b) are for the mixing angle $\theta = 0^\circ$ and $\theta = 30^\circ$, respectively.

system are all changed as $\Delta I = 1/2$ and $\Delta I_3 = 1/2$. Using the weak isospin analysis and ignoring the phase space difference, we can obtain that

$$R_\Gamma \approx \frac{|\langle \Lambda K^+ | H_W | \Lambda_c \rangle|^2}{|\langle \Sigma^0 K^+ | H_W | \Lambda_c \rangle|^2} = \frac{|\langle 0, 0; \frac{1}{2}, \frac{1}{2} | 0, 0; \frac{1}{2}, \frac{1}{2} \rangle|^2}{|\langle 1, 0; \frac{1}{2}, \frac{1}{2} | 0, 0; \frac{1}{2}, \frac{1}{2} \rangle|^2} = 3. \quad (32)$$

However, the above formula holds only if the two processes share identical single-diagram contributions. In our framework (Fig. 1), the schematic diagrams are not identical. Consequently, a ratio of 3 is not realistic for asymptotically large values of α_ρ . For instance, Fig. 4(a) clearly shows that R_Γ is close to 3 at infinity large value of α_ρ when the mixing between $[70, {}^2_8]$ and $[70, {}^4_8]$ is not considered. When the mixing between $[70, {}^2_8]$ and $[70, {}^4_8]$ is considered, R_Γ is close to 0. This is because the parameters $\alpha_{\rho/\lambda}$ take the different values for strange and light baryons, within the antitriplet singly charmed states. The amplitude of $\Lambda_c \rightarrow \Sigma^0 K^+$ decreases much slower than that of $\Lambda_c \rightarrow \Lambda K^+$ with the increasing α_ρ . To some extent, this reveals the complication of the mechanism of hadronic weak decay and the importance of the interference among various of processes. In either case, Fig. 4 shows that the value of α_ρ should not be an infinitely large value, thereby confirming that the diquark configuration is not a realistic description.

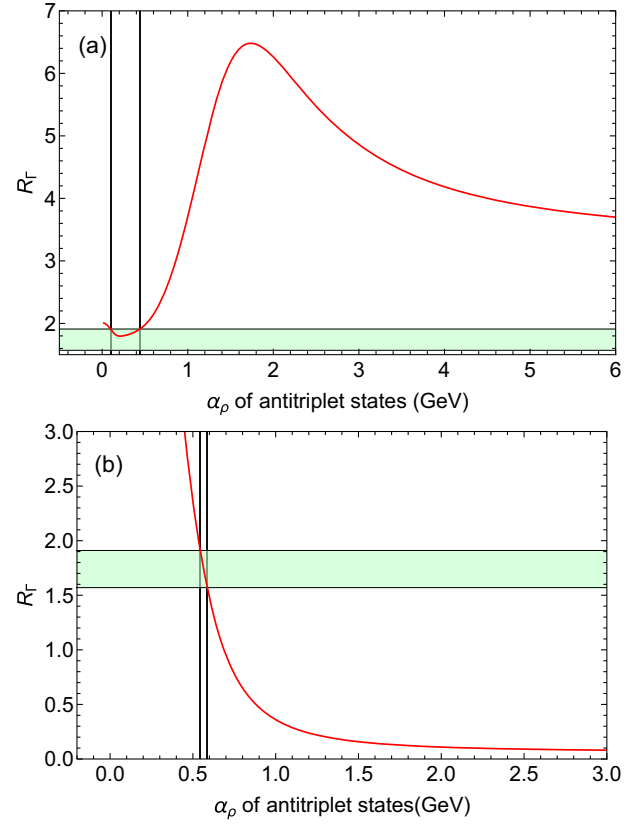


FIG. 4. The ratio R_Γ varies with α_ρ of the antitriplet states. The parameter α_λ of the antitriplet states is fixed. The cyan band is the range of the experimental data. The two vertical lines indicate the boundary of α_ρ of Λ_c . The left panel (a) is for the case with $\theta = 0^\circ$ and the right panel (b) is for the case with $\theta = 30^\circ$.

IV. SUMMARY AND OUTLOOK

Two singly Cabibbo-suppressed hadronic weak decays of Λ_c , i.e., $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$, are investigated within the NRCQM frame work. The final baryons have different isospins, which provide an opportunity to discuss diquarks of the charmed baryons. The spatial distribution of the diquark is reflected by the harmonic oscillator parameters. The larger the harmonic oscillator parameter is, the more compact the spatial distribution is. Using the $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$ processes, we find that the allowed value region of α_ρ and α_λ of the antitriplet singly charmed baryons is small and close to the commonly adopted value of the constituent quark model. Neither the $[ud]$ diquark nor the heavy-light one turns out to favor a compact structure.

In this work, the final-state interactions are not taken into account in the calculation of the $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$ branching ratios. Reference [50,51] and the references therein show that some coupled-channel effects may become crucial. Usually, the final-state interaction becomes important when the interaction is an S wave and near threshold. Considering that the mass difference between Λ_c and $K\Lambda(K\Sigma)$ is far from the $K\Lambda(\Sigma)$ threshold, the effects of

final-state interactions should be not crucial. This fact is reflected by the numerical results of the $\Lambda_c \rightarrow \Lambda K^+$ and $\Lambda_c \rightarrow \Sigma^0 K^+$ branching ratios, which are substantially consistent with the experimental results. Meanwhile, the consequence of the final-state interactions will be included for future studies.

We also find that the “ Λ selection rule” plays an essential role for understanding why the contributions of the $N(1535)$ and the $N(1650)$ are different. Such selection rules can be useful for the search of excited baryon states or charmed baryons in the heavy-flavor baryon hadronic weak decays.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [25,43].

APPENDIX: THE WAVE FUNCTIONS OF HADRONS

The standard $SU(6) \otimes O(3)$ wave functions are used in this work, and the explicit baryons and pseudoscalar mesons are presented in this section. The spin-flavor wave functions can be expressed as $|N_6, {}^{2S+1}N_3\rangle$, where S stands for the total spin. $N_6 (= 56, 70 \text{ or } 20)$ and $N_3 (= 8, 10, \text{ or } 1)$ denote the dimension of the $SU(6)$ and $SU(3)$ representations, respectively. The $SU(6) \otimes O(3)$ wave function is denoted with $|N_6, {}^{2S+1}N_3, N, L, J\rangle$ and follows the convention of Refs. [52,53]. N is the principal quantum number, L is the total orbital angular momentum, and J is the total angular momentum of the hadron.

1. The spin wave functions

The spin wave functions for the three-quark system are

$$\chi_{\frac{1}{2}, \frac{1}{2}}^\rho = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad (\text{A1})$$

$$\chi_{\frac{1}{2}, \frac{1}{2}}^\lambda = -\frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow), \quad (\text{A2})$$

$$\chi_{\frac{1}{2}, -\frac{1}{2}}^\rho = \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow), \quad (\text{A3})$$

$$\chi_{\frac{1}{2}, -\frac{1}{2}}^\lambda = \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow), \quad (\text{A4})$$

$$\chi_{\frac{3}{2}, \frac{3}{2}}^s = \uparrow\uparrow\uparrow, \quad (\text{A5})$$

$$\chi_{\frac{3}{2}, -\frac{3}{2}}^s = \downarrow\downarrow\downarrow, \quad (\text{A6})$$

$$\chi_{\frac{3}{2}, \frac{1}{2}}^s = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow), \quad (\text{A7})$$

$$\chi_{\frac{3}{2}, -\frac{1}{2}}^s = \frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow). \quad (\text{A8})$$

The superscripts, $\sigma = \rho, \lambda, s, a$, are used to label the symmetry type of the spin wave functions. s stands for the spin wave function that is completely symmetric, and a stands for the wave function that is completely antisymmetric. ρ and λ are used to label the mixed symmetry. The spin wave functions for the quark-antiquark system are

$$\begin{aligned} \chi_{1,1}^s &= \uparrow\uparrow, & \chi_{1,0}^s &= \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), & \chi_{1,-1}^s &= \downarrow\downarrow, \\ \chi_{0,0}^a &= \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow). \end{aligned} \quad (\text{A9})$$

2. The flavor wave functions

The flavor wave functions for the $SU(3)$ flavor octet baryons, are written as [37]

$$\phi_\Lambda^\lambda = -\frac{1}{2}(sud + usd - sdu - dsu), \quad (\text{A10})$$

$$\phi_\Lambda^\rho = \frac{1}{2\sqrt{3}}(usd + sdu - sud - dsu - 2dus + 2uds), \quad (\text{A11})$$

$$\phi_{\Sigma^+}^\lambda = \frac{1}{\sqrt{6}}(2uus - suu - usu), \quad (\text{A12})$$

$$\phi_{\Sigma^+}^\rho = \frac{1}{\sqrt{2}}(suu - usu), \quad (\text{A13})$$

$$\phi_{\Sigma^0}^\lambda = \frac{1}{2\sqrt{3}}(sdu + sud + usd + dsu - 2uds - 2dus), \quad (\text{A14})$$

$$\phi_{\Sigma^0}^\rho = \frac{1}{2}(sud + sdu - usd - dsu), \quad (\text{A15})$$

$$\phi_{\Sigma^-}^\lambda = \frac{1}{\sqrt{6}}(sdd + dsd - 2dds), \quad (\text{A16})$$

$$\phi_{\Sigma^-}^\rho = \frac{1}{\sqrt{2}}(sdd - dsd), \quad (\text{A17})$$

$$\phi_{\Xi^0}^\lambda = \frac{1}{\sqrt{6}}(2ssu - sus - uss), \quad (\text{A18})$$

$$\phi_{\Xi^0}^\rho = \frac{1}{\sqrt{2}}(sus - uss), \quad (\text{A19})$$

$$\phi_{\Xi^-}^\lambda = \frac{1}{\sqrt{6}}(2ssd - sds - dss), \quad (\text{A20})$$

$$\phi_{\Xi^-}^\rho = \frac{1}{\sqrt{2}}(sds - dss). \quad (\text{A21})$$

For the charmed baryons, the flavor wave functions are written as [31,54,55]

$$\phi_c^{\bar{3}} = \begin{cases} \frac{1}{\sqrt{2}}(ud - du)c, & \text{for } \Lambda_c^+, \\ \frac{1}{\sqrt{2}}(us - su)c, & \text{for } \Xi_c^+, \\ \frac{1}{\sqrt{2}}(ds - sd)c, & \text{for } \Xi_c^0, \end{cases} \quad (\text{A22})$$

$$\phi_c^6 = \begin{cases} uuc, & \text{for } \Sigma_c^{++}, \\ \frac{1}{\sqrt{2}}(ud + du)c, & \text{for } \Sigma_c^+, \\ ddc, & \text{for } \Sigma_c^0, \\ \frac{1}{\sqrt{2}}(us + su)c, & \text{for } \Xi_c^{+}, \\ \frac{1}{\sqrt{2}}(ds + sd)c, & \text{for } \Xi_c^0, \\ ssc, & \text{for } \Omega_c^0. \end{cases} \quad (\text{A23})$$

The flavor wave functions of π and K are written as [37]

$$\phi_{\pi^+} = u\bar{d}, \quad \phi_{\pi^-} = -d\bar{u}, \quad \phi_{\pi^0} = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad (\text{A24})$$

$$\phi_{K^+} = u\bar{s}, \quad \phi_{K^-} = -s\bar{u}. \quad (\text{A25})$$

3. The spatial wave functions of hadrons

Ignoring the hyperfine interaction, the Hamiltonian for the three-quark system [37,53] is expressed as

$$H = \sum_{i=1}^3 \frac{\mathbf{p}_i^2}{2m_i^2} + \sum_{i \neq j} \frac{1}{2} K |\mathbf{r}_i - \mathbf{r}_j|^2. \quad (\text{A26})$$

$\mathbf{r}_i$ and $\mathbf{p}_i$ are the coordinate and momentum for the i th quark in the baryon rest frame. K is the spring constant that describes the strength of the interaction between constituent quarks. With the Jacobi coordinate, the Hamiltonian can be reduced to

$$H = \frac{\mathbf{P}^2}{2M} + \frac{\mathbf{p}_\rho^2}{2m_\rho} + \frac{\mathbf{p}_\lambda^2}{2m_\lambda} + \frac{1}{2} m_\rho \omega_\rho^2 \rho^2 + \frac{1}{2} m_\lambda \omega_\lambda^2 \lambda^2. \quad (\text{A27})$$

where ρ and λ are defined as

$$\rho = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \quad (\text{A28})$$

$$\lambda = \sqrt{\frac{2}{3}} \left(\frac{m_1}{m_1 + m_2} \mathbf{r}_1 + \frac{m_2}{m_1 + m_2} \mathbf{r}_2 - \mathbf{r}_3 \right). \quad (\text{A29})$$

$\mathbf{P}$ is the total momentum of the baryon. $\mathbf{p}_\rho$ and $\mathbf{p}_\lambda$ are defined as

$$\mathbf{p}_\rho = m_\rho \dot{\rho} = \sqrt{2} \left(\frac{m_2}{m_1 + m_2} \mathbf{p}_1 - \frac{m_1}{m_1 + m_2} \mathbf{p}_2 \right), \quad (\text{A30})$$

$$\mathbf{p}_\lambda = m_\lambda \dot{\lambda} = \sqrt{\frac{3}{2}} \left(\frac{m_3}{M} \mathbf{p}_1 + \frac{m_3}{M} \mathbf{p}_2 - \frac{m_1 + m_2}{M} \mathbf{p}_3 \right). \quad (\text{A31})$$

$M = m_1 + m_2 + m_3$ is the total mass of the system, m_ρ and m_λ are the reduced mass and expressed as

$$m_\rho = 2 \frac{m_1 m_2}{m_1 + m_2}, \quad m_\lambda = \frac{3}{2} \frac{m_3 (m_1 + m_2)}{M}. \quad (\text{A32})$$

In the momentum space, the spatial wave function of the three baryon system can be easily obtained and reads as

$$\Psi_{NLL_z}(\mathbf{P}, \mathbf{p}_\rho, \mathbf{p}_\lambda) = \delta^3(\mathbf{P} - \mathbf{P}_c) \sum_m \langle l_\rho, m; l_\lambda, L_z - m | L, L_z \rangle \times \psi_{n_\rho l_\rho m}^{\alpha_\rho}(\mathbf{p}_\rho) \psi_{n_\lambda l_\lambda L_z - m}^{\alpha_\lambda}(\mathbf{p}_\lambda), \quad (\text{A33})$$

where

$$\psi_{nlm}^\alpha(\mathbf{p}) = (i)^l (-1)^n \left[\frac{2n!}{(n+l+1/2)!} \right]^{\frac{1}{2}} \frac{1}{\alpha^{l+3/2}} \exp\left(-\frac{\mathbf{p}^2}{2\alpha^2}\right) \times L_n^{l+1/2}(\mathbf{p}^2/\alpha^2) \mathcal{Y}_{lm}(\mathbf{p}). \quad (\text{A34})$$

In the coordinate space, the spatial wave function is expressed as

$$\Psi_{NLL_z}(\mathbf{R}_c, \rho, \lambda) = \frac{1}{(2\pi)^{3/2}} \exp(-i\mathbf{P} \cdot \mathbf{R}_c) \times \sum_m \langle l_\rho, m; l_\lambda, L_z - m | L, L_z \rangle \psi_{n_\rho l_\rho m}^{\alpha_\rho}(\rho) \times \psi_{n_\lambda l_\lambda L_z - m}^{\alpha_\lambda}(\lambda), \quad (\text{A35})$$

where

$$\psi_{nlm}^\alpha(\mathbf{r}) = \left[\frac{2n!}{(n+l+1/2)!} \right]^{1/2} \alpha^{l+3/2} \exp\left(-\frac{\alpha^2 \mathbf{r}^2}{2}\right) \times L_n^{l+1/2}(\alpha^2 \mathbf{r}^2) \mathcal{Y}_{lm}(\mathbf{r}). \quad (\text{A36})$$

α_ρ and α_λ are the harmonic oscillator strengths. They are expressed as

$$\alpha_\rho^2 = m_\rho \omega_\rho, \quad \alpha_\lambda^2 = m_\lambda \omega_\lambda, \quad (\text{A37})$$

with the frequencies of the ρ mode and λ mode

$$\omega_\rho^2 = \frac{3K}{m_\rho}, \quad \omega_\lambda^2 = \frac{3K}{m_\lambda}. \quad (\text{A38})$$

With the standard notation, the total principal quantum number N is

$$N = N_\rho + N_\lambda, \quad (\text{A39})$$

where $N_\rho = (2n_\rho + l_\rho)$ and $N_\lambda = (2n_\lambda + l_\lambda)$ are the principal quantum numbers of the ρ mode and λ mode, respectively. $n_{\rho(\lambda)}$ and $l_{\rho(\lambda)}$ are the radial excitation quantum number and the orbital angular momenta of the $\rho(\lambda)$ mode. The total orbital angular-momentum L of a state is obtained by

$$\mathbf{L} = \mathbf{l}_\rho + \mathbf{l}_\lambda. \quad (\text{A40})$$

Then, eigenvalues of Eq. (A27) can be expressed as [25,31]

$$E_N = (N_\rho + 3/2)\omega_\rho + (N_\lambda + 3/2)\omega_\lambda. \quad (\text{A41})$$

With Eq. (A37), it is easy to find that the energy of the baryons is a quadratic function of $\alpha_{\rho(\lambda)}$ with the $n_{\rho(\lambda)}$ and $l_{\rho(\lambda)}$ fixed.

For the light mesons, only the spatial wave function of the ground state is involved and expressed as

$$\Psi'_{000}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{\pi^{3/4} R^{3/2}} \exp\left[-\frac{(\mathbf{p}_1 - \mathbf{p}_2)^2}{8R^2}\right], \quad (\text{A42})$$

where R characterizes the size of mesons.

4. The total wave function of hadrons

With the above spin, flavor, and spatial wave functions, we can construct the total wave functions of the light baryons, which are denoted by $|N_6, {}^{2S+1}N_3, N, L, J\rangle$. We also abbreviate $|N_6, {}^{2S+1}N_3, N, L, J\rangle$ to $[N_6, {}^{2S+1}N_3]$ without causing ambiguity. The light baryons wave functions involved in this work read as

$$|56, {}^2 8, 0, 0, \frac{1}{2}\rangle = \frac{1}{\sqrt{2}} (\phi_B^\rho \chi_{S, S_z}^\rho + \phi_B^\lambda \chi_{S, S_z}^\lambda) \Psi_{0,0,0}, \quad (\text{A43})$$

$$|70, {}^2 8, 1, 1, J\rangle = \sum_{L_z + S_z = J_z} \left\langle 1, L_z; \frac{1}{2}, S_z | JJ_z \right\rangle \times \frac{1}{2} [(\phi_B^\rho \chi_{S, S_z}^\rho + \phi_B^\lambda \chi_{S, S_z}^\lambda) \Psi_{1,1,L_z}^\rho + (\phi_B^\rho \chi_{S, S_z}^\rho - \phi_B^\lambda \chi_{S, S_z}^\lambda) \Psi_{1,1,L_z}^\lambda], \quad (\text{A44})$$

$$|70, {}^4 8, 1, 1, J\rangle = \sum_{L_z + S_z = J_z} \left\langle 1, L_z; \frac{3}{2}, S_z | JJ_z \right\rangle \times \frac{1}{\sqrt{2}} [\phi_B^\rho \chi_{S, S_z}^\rho \Psi_{1,1,L_z}^\rho + \phi_B^\lambda \chi_{S, S_z}^\lambda \Psi_{1,1,L_z}^\lambda]. \quad (\text{A45})$$

The total wave functions of the ground states and the first orbital excitation single charmed baryons are expressed as

$$\Psi_{\bar{3}} = \begin{cases} |^2S, \frac{1}{2}^+\rangle = \Psi_{000}^s \chi_{S_z}^\rho \phi_c^{\bar{3}} \\ |^2P_\lambda, \frac{1}{2}^-\rangle = \Psi_{11L_z}^\lambda \chi_{S_z}^\rho \phi_c^{\bar{3}} \\ |^2P_\rho, \frac{1}{2}^-\rangle = \Psi_{11L_z}^\rho \chi_{S_z}^\lambda \phi_c^{\bar{3}} \\ |^4P_\rho, \frac{1}{2}^-\rangle = \Psi_{11L_z}^\rho \chi_{S_z}^s \phi_c^{\bar{3}} \end{cases}, \quad (\text{A46})$$

$$\Psi_6 = \begin{cases} |^2S, \frac{1}{2}^+\rangle = \Psi_{000}^s \chi_{S_z}^\lambda \phi_c^6 \\ |^2P_\lambda, \frac{1}{2}^-\rangle = \Psi_{11L_z}^\lambda \chi_{S_z}^\lambda \phi_c^6 \\ |^2P_\rho, \frac{1}{2}^-\rangle = \Psi_{11L_z}^\rho \chi_{S_z}^\rho \phi_c^6 \\ |^4P_\lambda, \frac{1}{2}^-\rangle = \Psi_{11L_z}^\lambda \chi_{S_z}^s \phi_c^6 \end{cases}. \quad (\text{A47})$$

The Clebsch-Gordan series for the spin and angular-momentum addition

$$|^{2S+1}L_\sigma, J^P\rangle = \sum_{L_z + S_z = J_z} \langle LL_z, SS_z | JJ_z \rangle \Psi_{NLL_z}^\sigma \chi_{S_z} \phi_c \quad (\text{A48})$$

has been omitted for the orbital excitation states, and $\sigma = \lambda, \rho, s$, which is also used to label the symmetry type of the wave function. And the total wave function of pseudoscalar mesons is written as

$$\Phi_{0,0,0}(\mathbf{p}_1, \mathbf{p}_2) = \delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{P}) \phi_p \chi_{0,0}^a \Psi'_{000}(\mathbf{p}_1, \mathbf{p}_2). \quad (\text{A49})$$

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