Letter

Boundary completion of vacuum persistence probability

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The vacuum persistence probability, encoded in the imaginary part of the in-out effective action, is a basic measure of vacuum instability and particle production. However, its evaluation may appear prescription-dependent: the Bogoliubov prescription and the Green's function prescription can yield different results. This discrepancy reflects the incomplete treatment of the end point vacuum states in the conventional Green's function reconstruction of the effective action. We establish a boundary-completed reconstruction that consistently incorporates the bulk Green's function and the end point vacuum contributions, thereby determining the structure of the complete in-out effective action. The resulting vacuum persistence probability is uniquely determined and agrees with the Bogoliubov result.

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Introduction. Particle production from the vacuum is a characteristic prediction of quantum field theory in external backgrounds and curved spacetime, with well-known examples including the Schwinger effect [1], Hawking radiation [2], and cosmological particle production [3,4]. Such processes can be characterized by the in-out vacuum persistence amplitude

$$Z = \langle 0, \mathrm{out}|0, \mathrm{in}\rangle \equiv e^{iW}, \tag{1}$$

which defines the in-out effective action $W = -i \log Z$ [5]. Its imaginary part determines the vacuum persistence probability, $P_{\mathrm{vac}} = |Z|^2 = e^{-2\,\mathrm{Im}\,W}$.

Two standard approaches are commonly used to determine the effective action. The Bogoliubov approach expresses it in terms of the transformation between the in- and out-mode bases,

$$W_{\mathrm{B}} = \frac{i}{2}\mathrm{Tr}\,\log\,\alpha, \tag{2}$$

up to a real phase [5–7]. The Green's function approach instead reconstructs the effective action from the coincident in-out Feynman Green's function [8,9],

$$W_{\mathrm{G}} = -\frac{1}{2}\int_{\Omega} \mathrm{d}^{d+1}x\sqrt{|g|}\int_{+\infty}^{m^2} \mathrm{d}\bar{m}^2 G_{\mathrm{F}}(x,x;\bar{m}^2), \tag{3}$$

or equivalently from the heat kernel [10]. These two constructions are commonly regarded as equivalent representations of the same in-out effective action [11] and have both been widely used in studies of particle production [12–19].

However, this presumed equivalence is challenged by several well-known examples. In de Sitter spacetime, the Bogoliubov approach [20] and the conventional Green's function approach [15,21,22] can yield different imaginary parts of the effective action [23,24]. Even in the flat space Schwinger effect, their agreement is not automatic but requires an additional identification of the longitudinal momentum integral with the interaction time [25,26]. These discrepancies raise the question of why the two constructions differ and which one corresponds to the physical vacuum persistence amplitude.

An important step toward resolving this issue was taken in Ref. [23], which analyzed the discrepancy in expanding spacetimes. It emphasized that the in-out functional integral contains not only the bulk action but also the end point wave functionals specifying the initial and final vacua, whereas the Green's function reconstruction accounts only for the bulk contribution. Including the end point contributions restores agreement with the Bogoliubov result for background fields acting over a finite time interval. For background fields that persist eternally, however, the time and momentum integrals in the bulk contribution become

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divergent, so the interchange of their order required in the finite-time analysis is no longer generally justified. Moreover, the end point contributions may also fail to admit well-defined asymptotic limits. The finite-time agreement therefore does not directly settle the eternal case.

This suggests formulating the comparison within a common regularization, before assigning separate meanings to the individual divergent contributions. In our previous analysis of de Sitter spacetime [24], we introduced explicit late-time and momentum cutoffs and found that the Green's function reconstruction in fact yields a regulator-dependent family of results. The conventional Green's function and Bogoliubov results emerge from the same regulated expression as two opposite regularization limits. That analysis, however, did not determine a unique physical result, nor did it include the end point vacuum wave functionals.

These results leave two closely related questions. What does the mass-integrated Green's function (3) actually reconstruct? And to what extent is this restoration of agreement universal once the end point vacuum wave functionals are consistently included within the same regularization?

In this Letter, we identify a boundary-completion structure of the in-out effective action for scalar fields governed by quadratic actions, showing how the Green's function and end point vacuum contributions combine to recover the Bogoliubov vacuum persistence probability. At the level of mass variation, the central structure is that the complete effective action contains not only the Green's function contribution but also that from the initial and final vacuum wave functionals,

$$\partial_{m^2} W = \partial_{m^2} W_{\mathrm{G}} + \partial_{m^2} W_{\Psi}. \tag{4}$$

The Green's function result differs from the Bogoliubov result by additional boundary contributions,

$$\partial_{m^2} W_{\mathrm{G}} = \partial_{m^2} W_{\mathrm{B}} - \mathcal{B}_{\mathrm{in}} - \mathcal{B}_{\mathrm{out}}, \tag{5}$$

while the end point vacuum wave functionals contribute the same boundary terms with the opposite sign,

$$\partial_{m^2} W_{\Psi} = +\mathcal{B}_{\mathrm{in}} + \mathcal{B}_{\mathrm{out}} + \partial_{m^2}\Theta, \qquad \Theta \in \mathbb{R}. \tag{6}$$

The boundary contributions therefore cancel in the complete effective action, leaving only an arbitrary real phase in addition to the Bogoliubov result. Consequently, the vacuum persistence probability is uniquely fixed by the Bogoliubov coefficients. Crucially, this cancellation occurs within the same regularization before the regulator is removed, so the separate contributions need not possess regulator-independent limits.

In-out amplitude with end point states. We begin with the path-integral representation of the vacuum persistence amplitude, keeping the boundary wave functionals explicit [27]:

$$e^{iW} = Z = \int_{\Omega} \mathcal{D}\phi \Psi^{*}_{\mathrm{out}}[\phi(\Sigma_{\mathrm{out}})] \Psi_{\mathrm{in}}[\phi(\Sigma_{\mathrm{in}})] e^{iS_m[\phi]}. \tag{7}$$

Here, we assume that Ω is globally hyperbolic, with initial and final Cauchy surfaces Σ_{in} and Σ_{out} as its temporal boundaries. The wave functionals Ψ_{in} and Ψ_{out} are the Schrödinger representations of the in- and out-vacua, evaluated on the restrictions of the integration field to the corresponding Cauchy surfaces. They specify the temporal boundary conditions of the in-out problem, while possible spatial boundary conditions are assumed to ensure sufficiently fast decay of the fields and hence yield no contributions to the integral. To balance generality and simplicity, we focus our discussion on the real scalar field of mass m, with action $S_m[\phi] = -\frac{1}{2}\int_{\Omega} \mathrm{d}^{d+1}x\sqrt{|g|}[g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + m^2\phi^2]$. The same framework also applies, with appropriate modifications, to scalar fields coupled to prescribed classical electromagnetic backgrounds, in which the theory remains quadratic in the dynamical field. A minimal $0+1$-dimensional setting based on a time-dependent harmonic oscillator is presented in the Supplemental Material Sec. III A [28] as a simpler illustration of the underlying mechanism.

The Green's function prescription reconstructs the effective action through its mass variation. For the complete amplitude (7), this variation has two origins: the explicit mass dependence of the bulk action and the mass dependence of the end point vacuum wave functionals, i.e., $\partial_{m^2} W = \langle \partial_{m^2} S_m \rangle - i\langle \partial_{m^2} \log \Psi_{\mathrm{in}} \rangle - i\langle \partial_{m^2} \log \Psi^{*}_{\mathrm{out}} \rangle$ [29]. Here, $\langle \cdots \rangle$ denotes the expectation value with respect to the in-out path-integral such that $\langle \mathcal{O} \rangle = Z^{-1}\int_{\Omega} \mathcal{D}\phi \mathcal{O} \Psi^{*}_{\mathrm{out}} \Psi_{\mathrm{in}} e^{iS_m[\phi]}$. The bulk-action variation is precisely given by the coincident in-out Feynman Green's function [22],

$$\langle \partial_{m^2} S_m \rangle = -\frac{1}{2}\int_{\Omega} \mathrm{d}^{d+1}x\sqrt{|g|} G_{\mathrm{F}}(x,x) = \partial_{m^2} W_{\mathrm{G}}, \tag{8}$$

where $G_{\mathrm{F}}(x,x') = \langle \phi(x)\phi(x') \rangle$ is the in-out Feynman Green's function in the path-integral representation. Thus, the complete mass variation of the effective action W naturally separates into a bulk part and an end point part:

$$\partial_{m^2} W = \partial_{m^2} W_{\mathrm{G}} + \partial_{m^2} W_{\Psi}, \tag{9}$$

where $\partial_{m^2} W_{\Psi} \equiv -i\langle \partial_{m^2} \log \Psi_{\mathrm{in}} \rangle - i\langle \partial_{m^2} \log \Psi^{*}_{\mathrm{out}} \rangle$ collects the end point contributions from the vacuum wave functionals. After integration over m^2, this formally gives $W(m^2) = W_{\mathrm{G}}(m^2) + W_{\Psi}(m^2) + W_0$, where W_0 is independent of m^2 and is fixed by the choice of reference condition. The Green's function prescription (3) therefore reconstructs only the bulk-action response W_{G}. Whether it

equals the full in-out effective action depends on the end point contribution W_Ψ, which will be evaluated later.

Boundary obstruction in the Green's function prescription. We next isolate what the bulk Green's function prescription computes. The result will be the Bogoliubov mass variation in addition to terms that live entirely on the boundaries. Let $\{u_k^{\rm in}, u_k^{\rm in*}\}$ and $\{u_{k'}^{\rm out}, u_{k'}^{\rm out*}\}$ be two complete bases of solutions to the scalar field equation $(-\nabla_\mu\nabla^\mu + m^2)u = 0$. The in-modes $u_k^{\rm in}$ are chosen to be positive-frequency near $\Sigma_{\rm in}$, while the out-modes $u_{k'}^{\rm out}$ are chosen to be positive-frequency near $\Sigma_{\rm out}$. These choices specify the in- and out-vacua, and the two bases are related by the Bogoliubov transformation $u_{k'}^{\rm out} = \sum_k (\alpha_{k'k}u_k^{\rm in} + \beta_{k'k}u_k^{\rm in*})$, where α and β are the Bogoliubov matrices. Throughout, we work in a consistently regularized theory, in which α is assumed to be invertible, all traces and determinants appearing below are well defined in the usual matrix sense, and the interchange of integrations and summations is justified. The regulator is removed only after all manipulations have been completed within the regularized theory.

The mode normalization is fixed by the Wronskian

$$W_\Sigma[f,g] = \int_\Sigma {\rm d}\Sigma_\mu (f\nabla^\mu g - g\nabla^\mu f), \tag{10}$$

with ${\rm d}\Sigma_\mu = {\rm d}\Sigma n_\mu$, where ${\rm d}\Sigma$ is the surface element on the Cauchy surface Σ, and n^μ is the future-directed unit normal to Σ. We use the convention $W_\Sigma[u_k^{\rm in}, u_j^{\rm in*}] = i\delta_{kj}$, $W_\Sigma[u_{k'}^{\rm out}, u_{j'}^{\rm out*}] = i\delta_{k'j'}$, with the Wronskians between two positive-frequency modes or two negative-frequency modes vanishing. Since the mode functions solve the homogeneous equation, these Wronskians are independent of the choices of the Cauchy surface. The in-out Feynman Green's function can be expressed in the in-out bases as $G_{\rm F}(x,x') = \theta(x,x')\sum_{k',k}(\alpha^{-1})_{kk'}u_{k'}^{\rm out}(x)u_k^{\rm in*}(x') + \theta(x',x)\times \sum_{k',k}(\alpha^{-1})_{kk'}u_{k'}^{\rm out}(x')u_k^{\rm in*}(x)$. Here, the step function $\theta(x,x')$ indicates that x lies in the future of x'. With the symmetric prescription $\theta(x,x) = 1/2$, the two branches contribute equally in the coincident limit, giving

$$G_{\rm F}(x,x) = \sum_{k',k}(\alpha^{-1})_{kk'}u_{k'}^{\rm out}(x)u_k^{\rm in*}(x). \tag{11}$$

A detailed derivation of the in-out mode representation of $G_{\rm F}$ is provided in the Supplemental Material Sec. II A [28].

The bulk trace is therefore controlled by the integral of the form $\int_\Omega {\rm d}^{d+1}x\sqrt{|g|}u_{k'}^{\rm out}u_k^{\rm in*}$. Since the modes satisfy the homogeneous equation, the Green-Lagrange identity converts this product into the divergence of a Wronskian current $\nabla_\mu(u_{k'}^{\rm out}\nabla^\mu\partial_{m^2}u_k^{\rm in*} - \partial_{m^2}u_k^{\rm in*}\nabla^\mu u_{k'}^{\rm out}) = u_{k'}^{\rm out}u_k^{\rm in*}$ [30]. Applying Gauss's theorem over Ω, and assuming that the contribution from any lateral boundary vanishes, the integral becomes,

$$\int_\Omega {\rm d}^{d+1}x\sqrt{|g|}u_{k'}^{\rm out}u_k^{\rm in*} = W_{\Sigma_{\rm in}}[u_{k'}^{\rm out}, \partial_{m^2}u_k^{\rm in*}] - W_{\Sigma_{\rm out}}[u_{k'}^{\rm out}, \partial_{m^2}u_k^{\rm in*}]. \tag{12}$$

in which only contributions from the initial and final Cauchy surfaces remain. The end point Wronskians can be reorganized using the overlap relation $W_\Sigma[u_{k'}^{\rm out}, u_k^{\rm in*}] = i\alpha_{k'k}$. This allows the variation of the Bogoliubov coefficient $\alpha_{k'k}$ to be separated from the remaining end point contributions. The detailed reduction of the Green's function spacetime integral to a boundary Wronskian form is given in the Supplemental Material Sec. II A [28]. Finally, the mass variation reconstructed from the Green's function prescription becomes

$$\begin{aligned}\partial_{m^2}W_{\rm G} = {}& \frac{i}{2}\sum_{k,k'}(\alpha^{-1})_{kk'}\partial_{m^2}\alpha_{k'k} \\ & - \frac{1}{2}\sum_{k,k'}(\alpha^{-1})_{kk'}W_{\Sigma_{\rm in}}[u_{k'}^{\rm out}, \partial_{m^2}u_k^{\rm in*}] \\ & - \frac{1}{2}\sum_{k,k'}(\alpha^{-1})_{kk'}W_{\Sigma_{\rm out}}[\partial_{m^2}u_{k'}^{\rm out}, u_k^{\rm in*}]\end{aligned} \tag{13}$$

The first term on the right is exactly the mass variation of the Bogoliubov expression (2), while the remaining two terms involve the end point Wronskians. Denoting them by $\mathcal{B}_{\rm in}$ and $\mathcal{B}_{\rm out}$, we obtain

$$\partial_{m^2}W_G = \partial_{m^2}W_B - \mathcal{B}_{\rm in} - \mathcal{B}_{\rm out}. \tag{14}$$

The Green's function prescription therefore does not by itself reproduce the Bogoliubov variation: the remaining end point Wronskian terms constitute a boundary obstruction.

Boundary completion by vacuum wave functionals. We now evaluate the end point contribution from the vacuum wave functionals in the complete variation formula (9). This contribution is absent from the Green's function prescription and will be shown to cancel the boundary obstruction in (14).

To evaluate the end point wave functional contributions, we introduce the boundary data of the mode functions. For either the in- or out-basis, denoted collectively by $\sigma = \{{\rm in}, {\rm out}\}$, define on a Cauchy surface Σ_σ, $q_k^\sigma(\boldsymbol{x}) \equiv u_k^\sigma|_{\Sigma_\sigma}(\boldsymbol{x})$, $p_k^\sigma(\boldsymbol{x}) \equiv n^\mu\nabla_\mu u_k^\sigma|_{\Sigma_\sigma}(\boldsymbol{x})$, where $\boldsymbol{x}\in\Sigma_\sigma$. The annihilation conditions defining the in- and out-vacua fix the end point wave functionals to be Gaussian [27,31,32]. We write them in the compact form as

$$\Psi_\sigma[\varphi^\sigma] = \mathcal{N}_\sigma \exp\left[\frac{i}{2}\varphi^\sigma \cdot K^\sigma \cdot \varphi^\sigma\right],$$
$$K^\sigma(\boldsymbol{x},\boldsymbol{y}) = \sum_k (q^{-1})^{\sigma*}_k(\boldsymbol{x}) p^{\sigma*}_k(\boldsymbol{y}), \qquad \boldsymbol{x},\boldsymbol{y}\in\Sigma_\sigma. \quad (15)$$

Here φ^σ denotes a boundary field configuration on Σ_σ, i.e. the restriction of the integration field ϕ to that end point. The dot denotes integration over the end point surface, for instance, $X^\sigma \cdot Y^\sigma = \int_{\Sigma_\sigma} \mathrm{d}\Sigma_{\boldsymbol{x}} X^\sigma(\boldsymbol{x}) Y^\sigma(\boldsymbol{x})$. The quantity q^{-1} denotes the dual basis to the boundary mode functions, satisfying $(q^{-1})^\sigma_k \cdot q^\sigma_j = \delta_{kj}$ and $\sum_k q^\sigma_k(\boldsymbol{x})(q^{-1})^\sigma_k(\boldsymbol{y}) = \delta_{\Sigma_\sigma}(\boldsymbol{x},\boldsymbol{y})$. With this notation the kernel satisfies $K^\sigma \cdot q^{\sigma*}_j = p^{\sigma*}_j$, showing that the end point Gaussian is fixed by the choice of positive-frequency boundary data.

The normalization factor $\mathcal{N}_\sigma$ is part of the end point wave functional and therefore must be varied together with the Gaussian kernel. Its form follows from the normalization of the vacuum state. To express this factor in terms of a well-defined determinant, we work temporarily with a finite N-site regularization of the boundary field space. The normalization factor then takes the form

$$\mathcal{N}_\sigma = (2\pi)^{-\frac{N}{4}}\left[\det_{\Sigma_\sigma} Q^\sigma\right]^{-\frac{1}{4}} e^{i\theta^\sigma},$$
$$Q^\sigma(\boldsymbol{x},\boldsymbol{y}) = \sum_k q^\sigma_k(\boldsymbol{x}) q^{\sigma*}_k(\boldsymbol{y}), \qquad \boldsymbol{x},\boldsymbol{y}\in\Sigma_\sigma. \quad (16)$$

The determinant is taken over the regulated boundary field space. The normalization condition fixes only $|\mathcal{N}_\sigma|$, leaving an arbitrary real phase θ^σ associated with the end point-state convention. Accordingly, $\langle\partial_{m^2}\log\Psi_\sigma\rangle = \partial_{m^2}\log\mathcal{N}_\sigma + \frac{i}{2}\langle\varphi^\sigma\cdot\partial_{m^2}K^\sigma\cdot\varphi^\sigma\rangle$. The first term on the right is independent of the boundary field configuration, whereas the second term probes the boundary two-point function of the integration field.

The variation of the normalization prefactor is immediate:

$$\partial_{m^2}\log\mathcal{N}_\sigma = -\frac{1}{2}\sum_k \mathrm{Re}[(q^{-1})^\sigma_k\cdot\partial_{m^2}q^\sigma_k] + i\partial_{m^2}\theta^\sigma. \quad (17)$$

The explicit $(2\pi)^{-N/4}$ factor is independent of m^2 and drops out. For the complex-conjugate normalization factor at the out-end point, the modulus gives the same contribution, while the phase term changes sign.

It remains to compute the Gaussian-kernel contribution. This requires the boundary two-point function of the path-integral field. This is obtained by restricting the in-out Feynman Green's function to the end point surface. Using $\theta(x,x) = 1/2$, one finds

$$\langle\varphi^\sigma(\boldsymbol{x})\varphi^\sigma(\boldsymbol{y})\rangle = \frac{1}{2}\sum_{k,k'}(\alpha^{-1})_{kk'}[u^{\mathrm{out}}_{k'}(\boldsymbol{x})u^{\mathrm{in}*}_k(\boldsymbol{y}) + u^{\mathrm{out}}_{k'}(\boldsymbol{y})u^{\mathrm{in}*}_k(\boldsymbol{x})], \qquad \boldsymbol{x},\boldsymbol{y}\in\Sigma_\sigma. \quad (18)$$

We can now evaluate the end point wave functional contributions explicitly. Here we only sketch the calculation, with the details given in the Supplemental Material Sec. II B [28]. On the Cauchy surface Σ_{in}, $u^{\mathrm{in}*}_k|_{\Sigma_{\mathrm{in}}} = q^{\mathrm{in}*}_k$. The contraction of $\partial_{m^2}K^{\mathrm{in}}$ with the boundary two-point function is therefore determined by the in-mode Cauchy data, $q^{\mathrm{in}*}_k$ and $p^{\mathrm{in}*}_k$, together with their mass variations and the values of the out-modes on Σ_{in}. The in-out Wronskian relation $W_{\Sigma_{\mathrm{in}}}[u^{\mathrm{out}}_{k'}, u^{\mathrm{in}*}_j] = i\alpha_{k'j}$ further allows the out-mode boundary values to be related to their normal derivatives. With these relations, the Gaussian-kernel contribution can be organized into the Wronskian $W_{\Sigma_{\mathrm{in}}}[u^{\mathrm{out}}_{k'}, \partial_{m^2}u^{\mathrm{in}*}_k]$, together with a remaining boundary trace:

$$\langle\varphi^{\mathrm{in}}\cdot\partial_{m^2}K^{\mathrm{in}}\cdot\varphi^{\mathrm{in}}\rangle = \sum_{kk'}(\alpha^{-1})_{kk'}W_{\Sigma_{\mathrm{in}}}[u^{\mathrm{out}}_{k'}, \partial_{m^2}u^{\mathrm{in}*}_k] - i\sum_k[(q^{-1})^{\mathrm{in}*}_k\cdot\partial_{m^2}q^{\mathrm{in}*}_k]. \quad (19)$$

Adding the prefactor variation to this kernel contribution gives the full initial wave functional variation:

$$\langle\partial_{m^2}\log\Psi_{\mathrm{in}}\rangle = \frac{i}{2}\sum_{kk'}(\alpha^{-1})_{kk'}W_{\Sigma_{\mathrm{in}}}[u^{\mathrm{out}}_{k'}, \partial_{m^2}u^{\mathrm{in}*}_k] + \frac{i}{2}\sum_k \mathrm{Im}\,[(q^{-1})^{\mathrm{in}*}_k\cdot\partial_{m^2}q^{\mathrm{in}*}_k] + i\partial_{m^2}\theta^{\mathrm{in}}. \quad (20)$$

The final end point has similar structure, with the complex conjugation Ψ^*_{out}:

$$\langle\partial_{m^2}\log\Psi^*_{\mathrm{out}}\rangle = \frac{i}{2}\sum_{kk'}(\alpha^{-1})_{kk'}W_{\Sigma_{\mathrm{out}}}[\partial_{m^2}u^{\mathrm{out}}_{k'}, u^{\mathrm{in}*}_k] + \frac{i}{2}\sum_{k'}\mathrm{Im}\,[(q^{-1})^{\mathrm{out}}_{k'}\cdot\partial_{m^2}q^{\mathrm{out}}_{k'}] - i\partial_{m^2}\theta^{\mathrm{out}}. \quad (21)$$

In both end point variations, the right side in the first line is a Wronskian contribution, while the remaining terms in the second line are purely imaginary. After multiplication by $-i$ in W_Ψ, the latter contribute only to the real part of the effective action. The two Wronskian terms are exactly the end point structures denoted by $\mathcal{B}_{\mathrm{in}}$ and $\mathcal{B}_{\mathrm{out}}$ in (14). Therefore, the end point wave functionals contribute

$$\partial_{m^2}W_\Psi = +\mathcal{B}_{\mathrm{in}} + \mathcal{B}_{\mathrm{out}} + \partial_{m^2}\Theta. \quad (22)$$

This is the boundary completion: the vacuum wave functionals reproduce the end point Wronskian terms arising from the Green's function prescription (14), but with the opposite signs, thereby exactly cancel the boundary obstruction. The remaining term Θ is real and collects the non-Wronskian end point contributions. Inserting this end point contribution into the complete variation formula (9), together with the bulk relation (14), it gives

$$\partial_{m^2} W = \partial_{m^2} W_{\mathrm{B}} + \partial_{m^2}\Theta. \tag{23}$$

Thus, the complete in-out variation equals the Bogoliubov variation up to a real phase convention. And, the Green's function prescription becomes equivalent to the Bogoliubov prescription only after the end point wave functionals are included.

Integrating over m^2, the real term Θ can affect only the real part of W, while an m^2-independent constant W_0 may remain. We fix W_0 by taking the large-mass limit as the reference condition. In this limit particle production is suppressed, $\alpha\alpha^\dagger \to I$, and hence $\mathrm{Im}\, W_0 = 0$. The imaginary part of the complete effective action is therefore

$$\mathrm{Im}\, W = \frac{1}{4}\mathrm{Tr}\, \log\,[\alpha\alpha^\dagger]. \tag{24}$$

This is precisely the Bogoliubov result: once the end point wave functionals are included consistently, the imaginary part of the complete in-out effective action is given by the Bogoliubov expression. Therefore, the result (24) obtained from the boundary completion of the Green's function prescription resolves the previous discrepancy without relying on a particular spacetime background.

Discussion. The immediate consequence of (24) is that the vacuum persistence probability is not prescription dependent: $P_{\mathrm{vac}} = e^{-2\,\mathrm{Im}\, W} = e^{-2\,\mathrm{Im}\, W_{\mathrm{B}}}$. The residual end point phase affects only the real part of W, leaving the vacuum persistence probability unchanged. Without an additional phase convention or renormalization condition, however, ReW is not uniquely defined because it can be shifted by the normalization phase of the end point wave functionals.

Our analysis also clarifies the prescription dependence found in de Sitter particle production [23,24]. In that de Sitter calculation [24], the effective action is reconstructed solely from the mode-function representation of the coincident Green's function and is sensitive to how the late-time and momentum cutoffs are correlated [24]. From the present viewpoint, this sensitivity reflects how the end point Wronskian term in (14) is treated. The vacuum wave functional contribution carries the corresponding prescription dependence with the opposite sign. When the two contributions are combined within the same regularization scheme, the prescription dependence cancels as demonstrated explicitly in the Supplemental Material Sec. III B [28]. The cutoff analysis therefore diagnoses a genuine boundary sensitivity of the bulk Green's function prescription, rather than an intrinsic ambiguity of the complete vacuum persistence probability.

This framework may also be viewed as a generalization of the cosmological analysis presented in Ref. [23]. We extend the analysis to general globally hyperbolic spacetimes and identify the boundary-completion mechanism that reconciles the Green's function and Bogoliubov approaches. Since the completion holds at the regulated level, the case of a background field acting over an infinite time interval can be treated as limits of regularized problems while preserving an unambiguous vacuum persistence probability. More generally, boundary completion does not require asymptotic regions: the in- and out-vacua may instead be specified on arbitrary Cauchy surfaces through their wave functionals. Thus, the lack of asymptotic regions does not preclude the application of the in-out formalism; what is required is a complete and consistent specification of the boundary states.

The boundary-completion framework admits an extension to more general quadratic theories. The key ingredients underlying our derivation—the linear mode equations and the Gaussian structure of the vacuum wave functionals—remain available in such theories. For example, in the presence of a background electromagnetic field, this involves replacing the relevant derivative operators with their gauge-covariant counterparts. By contrast, for theories with nonlinear interactions, the equations of motion are no longer linear and the vacuum wave functionals are generally non-Gaussian. Our present results therefore do not apply directly to such theories, and their extension requires further investigation. A potentially viable approach would be to treat the interactions perturbatively. Nevertheless, the underlying structural point remains: the vacuum wave functionals are part of the in-out path integral and should be included consistently.

It is also instructive to contrast the present discussion with the Schwinger-Keldysh formalism. The in-out formalism describes transition amplitudes between specified states, whereas the Schwinger-Keldysh formalism computes expectation values in a given initial state [33]. In the latter, unitarity implies $Z_{\mathrm{SK}}[J,J] = 1$ and hence $W_{\mathrm{SK}}[J,J] = 0$. The initial state nevertheless remains explicit in the closed-time-path integral through the initial density matrix. If the initial state depends on the parameter being varied, its variation must therefore be included together with that of the bulk action; otherwise, the resulting variation need not preserve the unitarity relation $Z_{\mathrm{SK}}[J,J] = 1$. In this sense, the need for a consistent treatment of boundary-state data persists also in the Schwinger-Keldysh formalism, although there is no independent final-state contribution.

It is worth noting that a possible relation between the real and imaginary parts of the effective action has been suggested within the in-out formalism. In particular, studies

based on Bogoliubov coefficients have conjectured a one-to-one correspondence between vacuum polarization and vacuum persistence, supported by a number of exactly solvable backgrounds [34–36]. Our present analysis makes explicit the structure of the full in-out effective action: in addition to the Bogoliubov-dependent part, its real part contains contributions from the end point wave functionals, while its imaginary part remains fixed by the Bogoliubov coefficients. This structure provides a setting in which the proposed correspondence can be further tested and its possible extension to the full boundary-completed effective action investigated. However, such a test necessarily requires additional input, since the phase convention for the end point states and the renormalization prescription must be specified to uniquely determine the real part of the effective action.

A further extension concerns systems with nontrivial spatial boundary conditions [37,38], such as fields in AdS spacetime [39–41] or background electric fields described in a static gauge [42,43]. In such cases, Eq. (14) generally acquires additional Green-Lagrange terms from spatial boundaries. Moreover, if the spatial boundary conditions depend on the parameter being varied, their variation may provide additional contributions to the effective action. This suggests a possible spatial analog of the boundary-completion mechanism identified here, whose precise formulation is left for future work.

Conclusion. We have established a boundary-completed reconstruction of the in-out effective action by consistently incorporating the contributions from bulk Green's function and the end point vacuum states. The resulting vacuum persistence probability agrees with the Bogoliubov result. The construction is independent of the particular spacetime background and does not require asymptotic in- and out-regions, with the vacuum states specified instead on general Cauchy surfaces. Our result therefore provides a framework for reconstructing the complete in-out effective action.

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