

Semileptonic decays of Λ_c^+ in light-front quark model with nonvalence contributions

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We investigate the exclusive semileptonic decays of $\Lambda_c^+ \rightarrow (\Lambda/n)\ell^+\nu_\ell$ ($\ell = e, \mu$) within the standard model by using the light-front quark model (LFQM). The form factor behaviors are obtained from the effective treatment of nonvalence contributions in addition to the valence ones in the Drell-Yan-West frame due to the Bethe-Salpeter formalism. Based on these form factors, we find that the decay branching ratios of $\Lambda_c^+ \rightarrow (\Lambda e^+\nu_e, \Lambda\mu^+\nu_e, ne^+\nu_e, n\mu^+\nu_e)$ are about (3.39, 3.21, 0.36, 0.35)% with the nonvalence contributions, which are consistent with the recent experimental measurements at BESIII. Furthermore, we use the experimental data to fit the β parameters in the baryonic distribution amplitudes under the LFQM, resulting in $(\beta_{\Lambda_c}, \beta_\Lambda, \beta_n) = (0.58 \pm 0.08, 0.52 \pm 0.08, 0.44 \pm 0.04)$.

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I. INTRODUCTION

The experimental and theoretical studies of semileptonic decays in both baryons and mesons have played a crucial role in building up the standard model (SM) in particle physics. Since 2015, the BESIII Collaboration has reported the measurements of the absolute branching fractions of the Λ_c semilepton decays, i.e., $B(\Lambda_c^+ \rightarrow \Lambda e^+\nu_e) = (3.63 \pm 0.38 \pm 0.20)\%$ and $B(\Lambda_c^+ \rightarrow \Lambda\mu^+\nu_\mu) = (3.49 \pm 0.46 \pm 0.27)\%$ [1–3]. Recently, the result of $B(\Lambda_c^+ \rightarrow ne^+\nu_e) = (0.357 \pm 0.034 \pm 0.014)\%$ has also been published [4]. Currently, there have been many theoretical studies related to these decays, such as those of the lattice QCD [5,6], $SU(3)_F$ flavor symmetry [7], QCD sum rule [8], quark model (QM) [9–11], light-cone sum rule [12], and light-front (LF) approach [13–15]. The results of these semileptonic decays are useful for understanding the dynamics in the charmed baryon physics.

Phenomenologically, the LF formalism offers a simple, nonperturbative, and relativistic framework for calculating the hadronic form factors. Since such a framework has been successfully applied to mesonic decay processes [16–20], it is natural to be used in the calculation of the form factors in

the baryonic decays [21–25]. The aim of this study is to evaluate the hadronic form factors and determine the decay rates of $\Lambda_c^+ \rightarrow (\Lambda/n)\ell^+\nu_\ell$ within the SM by using the LFQM based on the LF quantization. The LFQM in the present work employs a diquark picture [26,27], where the baryon contains a heavy quark along with a light diquark. This approximation makes the work similar to the meson case and reduces the computational effort considerably. The nonperturbative interaction between the two light quarks in the diquark picture can be effectively absorbed into the diquark mass. This helps us to analyze computationally the hadronic form factors in the entire (physical) timelike regime and obtain the decay rates of the semileptonic decays of Λ_c^+ .

The Drell-Yan-West framework with $q^+ = 0$ is crucial because only the valence state contribution is required, where q^μ is the momentum transfer and $q^+ = q_0 + q_z$ corresponds to its LF momentum component. This yields a form factor expressed by the overlap of the hadronic LF wave functions before and after the decay in the spacelike ($q^2 < 0$) region. Some successful phenomenological calculations of the form factors in the spacelike region in the LFQM can be found in Refs. [21,24,28,29]. However, some of the form factors in the exclusive processes may receive higher Fock state contributions, i.e., zero modes in the $q^+ = 0$ frame, within the framework of the LF quantization. Therefore, it is necessary to take into account the zero mode contributions in the spacelike region. Another option is to use the same “+” component but work in the range of $q^+ \neq 0$, that is, in the timelike ($q^2 > 0$) region. Physically, particle decay processes occur in a timelike zone, so in

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principle, form factors can be calculated using the $q^+ \neq 0$ framework in the timelike region. Similarly, those working in the timelike regime may also receive contributions from nonvalence Fock states, which require higher Fock states of the wave function, as well as valence states [30]. That is, we will inevitably encounter the nonvalence diagram, the so-called “Z diagram,” generated by the production of quark-antiquark pairs. Fortunately, effective methods for dealing with nonvalence state contributions in meson exclusion processes have been available in the literature [30–32]. The goal of our new treatment is to map this procedure directly onto the baryon semileptonic decay and to extract the value of the parameter “ β ” of the LFQM by comparison with the experimental data.

This paper is organized as follows. In Sec. II, we present the framework for the baryonic semileptonic decays. Our numerical results and discussions are given in Sec. III. We conclude in Sec. IV.

II. A FRAMEWORK FOR SEMILEPTON DECAYS OF $\mathcal{B}_i \rightarrow \mathcal{B}_f \ell \nu_\ell$

To study the exclusive spin-1/2 $\mathcal{B}_i$ to $\mathcal{B}_f$ semileptonic decays, we start with the charged current process of $Q \rightarrow q \ell^+ \nu_\ell$ at quark level, in which the effective Hamiltonian is given by

$$H_{\text{eff}}(Q \rightarrow q \ell \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{Qq} \bar{q} \gamma_\mu (1 - \gamma_5) Q \bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \ell. \quad (1)$$

From Eq. (1), we get

$$H_{\text{eff}}(\mathcal{B}_i \rightarrow \mathcal{B}_f \ell \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{Qq} \left\langle \mathcal{B}_f \left(P', S' = \frac{1}{2} \right) \right| \bar{q} \gamma_\mu (1 - \gamma_5) Q \left| \mathcal{B}_i \left(P, S = \frac{1}{2} \right) \right\rangle \times \bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \ell, \quad (2)$$

with the transition matrix elements defined as follows:

$$\begin{aligned} & \left\langle \mathcal{B}_f \left(P', S' = \frac{1}{2}, S'_z \right) \right| \bar{q} \gamma^\mu (1 - \gamma_5) Q \left| \mathcal{B}_i \left(P, S = \frac{1}{2}, S_z \right) \right\rangle \\ &= \bar{u}_\alpha(P', S'_z) \left[\gamma^\mu f_1(q^2) + i \frac{f_2(q^2)}{M_{\mathcal{B}_i}} \sigma_{\mu\nu} q^\nu + \frac{f_3(q^2)}{M_{\mathcal{B}_i}} q_\mu \right] \\ & \times u(P, S_z) - \bar{u}_\alpha(P', S'_z) \left[\gamma^\mu g_1(q^2) + i \frac{g_2(q^2)}{M_{\mathcal{B}_i}} \sigma_{\mu\nu} q^\nu \right. \\ & \left. + \frac{g_3(q^2)}{M_{\mathcal{B}_i}} q_\mu \right] \gamma_5 u(P, S_z), \end{aligned} \quad (3)$$

where $q^\mu = P^\mu - P'^\mu$ and $f_i(g_i)$ ($i = 1, 2, 3$) are the baryonic form factors.

The matrix elements in Eq. (3) can be applied to the helicity amplitudes, given by [25,33,34]

$$H_{\lambda_{\mathcal{B}_f} \lambda_W}^{V(A)} \equiv \langle \mathcal{B}_f | (\bar{q} Q)_{V(A)} | \mathcal{B}_i \rangle \epsilon_W^\mu, \quad (4)$$

where ϵ_W^μ is the polarization of the W boson, and $\lambda_{\mathcal{B}_f(\mathcal{B}_i)} = \pm 1/2$ represent the helicity states of the final (initial) spin-1/2 baryons. Based on the helicity conservation, $\lambda_{\mathcal{B}_i} = \lambda_{\mathcal{B}_f} - \lambda_W$ is held.

The helicity amplitudes are related to the form factors through the following expressions:

$$\begin{aligned} H_{\frac{1}{2},t}^V &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left((M - M') f_1 + \frac{q^2}{M} f_3 \right), \\ H_{\frac{1}{2},0}^V &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left((M + M') f_1 - \frac{q^2}{M} f_2 \right), \\ H_{\frac{1}{2},1}^V &= i \sqrt{2Q_-} \left(-f_1 + \frac{M + M'}{M} f_2 \right), \\ H_{\frac{1}{2},t}^A &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left((M + M') f_1 - \frac{q^2}{M} g_3 \right), \\ H_{\frac{1}{2},0}^A &= -i \frac{\sqrt{Q_+}}{\sqrt{q^2}} \left((M - M') g_1 + \frac{q^2}{M} g_2 \right), \\ H_{\frac{1}{2},1}^A &= i \sqrt{2Q_+} \left(-g_1 - \frac{M - M'}{M} g_2 \right), \end{aligned} \quad (5)$$

where the subscript “ t ” denotes the helicity amplitude $H_{\frac{1}{2},t}^{V(A)}$ from the temporal component of the current of $(\bar{q} Q)_{V(A)}$. The negative helicity amplitudes are defined by

$$H_{-\lambda', -\lambda_W}^V = H_{\lambda', \lambda_W}^V \quad \text{and} \quad H_{-\lambda', -\lambda_W}^A = -H_{\lambda', \lambda_W}^A, \quad (6)$$

while the helicity ones for the left-handed current are obtained as

$$H_{\lambda', \lambda_W} = H_{\lambda', \lambda_W}^V - H_{\lambda', \lambda_W}^A. \quad (7)$$

Consequently, the differential decay width of the semileptonic process reads

$$\begin{aligned} \frac{d\Gamma_L}{dq^2} &= \frac{G_F^2 |V_{CKM}|^2 (q^2 - m_\ell^2)^2 p_{\text{cm}}}{(2\pi)^3 24M^2 q^2} \left\{ \left(1 + \frac{m_\ell^2}{2q^2} \right) \left[|H_{\frac{1}{2},0}^V|^2 \right. \right. \\ & \quad \left. \left. + |H_{-\frac{1}{2},0}^V|^2 + |H_{\frac{1}{2},1}^V|^2 + |H_{-\frac{1}{2},1}^V|^2 \right] \right. \\ & \quad \left. + \frac{3m_\ell^2}{2q^2} \left(|H_{\frac{1}{2},t}^V|^2 + |H_{-\frac{1}{2},t}^V|^2 \right) \right\}, \end{aligned} \quad (8)$$

where q^2 is the lepton pair invariant mass, $p_{\text{cm}} = \sqrt{Q_+ Q_-} / 2M$, $Q_\pm = (M \pm M')^2 - q^2$, and M (M') is the mass of the parent (daughter) baryon. It is worth noting that

the last term of Eq. (8) gives a small contribution to the decay rate as it is associated with the lepton mass. Consequently, we may ignore the calculations of $f_3(g_3)$ in this study.

In order to calculate the form factors in the LFQM, we treat the baryon as a bound state composed of three quarks q_1 , q_2 , and q_3 , where $q_{2,3}$ are combined into a diquark, expressed as $q_{[2,3]}$. Explicitly, the baryon bound state with the total momentum P and spin S can be written as [27]

$$|\mathcal{B}(P, S, S_z)\rangle = \int \{d^3 p_1\} \{d^3 p_2\} 2(2\pi)^3 \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \times \sum_{\lambda_1, \lambda_2} \Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) |q_1(p_1, \lambda_1) [q_2, q_3](p_2, \lambda_2)\rangle, \quad (9)$$

where $q_1 = c$ or s denotes the active quark corresponding to the Λ_c or Λ baryon, $[q_2, q_3]$ represents the diquark, Ψ^{SS_z} corresponds to the momentum-space wave function and $p_{1,2}$ are the on-mass-shell LF momenta, and

$$\tilde{p} = (p^+, p_\perp), \quad p_\perp = (p^1, p^2), \quad p^- = \frac{m^2 + p_\perp^2}{p^+}, \quad (10)$$

with

$$p_1^+ = x_1 P^+, \quad p_2^+ = x_2 P^+, \quad x_1 + x_2 = 1, \\ p_{1\perp} = x_1 P_\perp + k_\perp, \quad p_{2\perp} = x_2 P_\perp - k_\perp. \quad (11)$$

In Eq. (11), $(x, k_\perp)$ are the light-front relative momentum variables, and $\vec{k}_\perp$ is the component of the internal momentum $\vec{k} = (\vec{k}_\perp, k_z)$.

By the Melosh transformation [35], it is more convenient to work with the following representation of the wave function

$$\Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \frac{1}{\sqrt{2(p_1 \cdot \tilde{P} + m_1 M_0)}} \times \bar{u}(p_1, \lambda_1) \Gamma_{l,m} u(\tilde{P}, S_z) \phi(x, k_\perp), \quad (12)$$

where $\Gamma_{l,m}$ is the coupling vertex function of the decaying quark q_1 and the diquark in the baryon state. For the scalar diquark, the coupling vertex is $\Gamma_s = 1$. If the axial-vector diquark is involved, the vertex should be

$$\Gamma_A = -\frac{1}{\sqrt{3}} \gamma_5 \not{\epsilon}^*(p_2, \lambda_2). \quad (13)$$

The wave function of $\phi(x, k_\perp)$ in Eq. (12) describes the momentum distribution of the constituent quarks in the

bound state. In this work, we use the Gaussian-type function, given as

$$\phi(x, k_\perp) = 4 \left(\frac{\pi}{\beta^2} \right)^{3/4} \sqrt{\frac{dk_z}{dx}} \exp\left(\frac{-\vec{k}^2}{2\beta^2}\right), \quad (14)$$

where β is the baryon shape parameter and k_z is defined by

$$1 - x = \frac{e_1 - k_z}{e_1 + e_2}, \quad x = \frac{e_2 + k_z}{e_1 + e_2}, \quad (15)$$

with $e_i = \sqrt{m_i^2 + \vec{k}^2}$. Subsequently, we have

$$M_0 = e_1 + e_2, \quad k_z = \frac{x M_0}{2} - \frac{m_2^2 + k_\perp^2}{2x M_0}, \quad (16)$$

and

$$M_0^2 = \frac{m_1^2 + k_\perp^2}{1 - x} + \frac{m_2^2 + k_\perp^2}{x}. \quad (17)$$

In our calculation, we obtain

$$\alpha \equiv \frac{P'^+}{P^+} = \frac{M_{B_i}^2 + M_{B_f}^2 - q^2 + \sqrt{(M_{B_i}^2 + M_{B_f}^2 - q^2)^2 - 4M_{B_i}^2 M_{B_f}^2}}{2M_{B_i}^2}. \quad (18)$$

Using the bound states of $|\mathcal{B}_i(P, S, S_z)\rangle$ and $|\mathcal{B}_f(P', S', S'_z)\rangle$ in Eq. (9) and the above identities, we derive the matrix elements of the baryonic transition in the LF frame. By considering the $\mu = +$ component, the transition matrix elements are given by

$$\langle \mathcal{B}_f(P', S', S'_z) | \bar{q} \gamma^\mu (1 - \gamma_5) Q | \mathcal{B}_i(P, S, S_z) \rangle = N_{fs} \int \{d^3 p_2\} \frac{\Psi_i(x, \mathbf{k}_\perp) I^+ \Psi_f(x', \mathbf{k}'_\perp)}{(1-x)(1-x')}, \quad (19)$$

where $I^+ = \sum_{\lambda_2} \bar{u}(\tilde{P}', S'_z) [\bar{\Gamma}_{S(A)}' (\not{p}'_1 + m'_1) \gamma^+ (1 - \gamma_5) \times (\not{p}_1 + m_1) \Gamma_{S(A)}] u(\tilde{P}, S_z)$, $\bar{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0$, and the flavor spin factor N_{fs} is the overlap factor of particle transformations, which are given by different processes. The trace term I^+ in Eq. (19) can be written as the sum of the valence I_V^+ and nonvalence I_{NV}^+ parts, as shown in Fig. 1(a). In the region of $0 < x < \alpha$ with $0 < p^+ < P'^+$ and $p_{[qq]}^- = p_{[qq]on}^- = (m_{[qq]}^2 + k_\perp^2)/p_{[qq]}^+$, in which the subscript “on” indicates on the mass shell, called the valence region as seen in Fig. 1(b), the effective contribution of the LF valence amplitude is given by

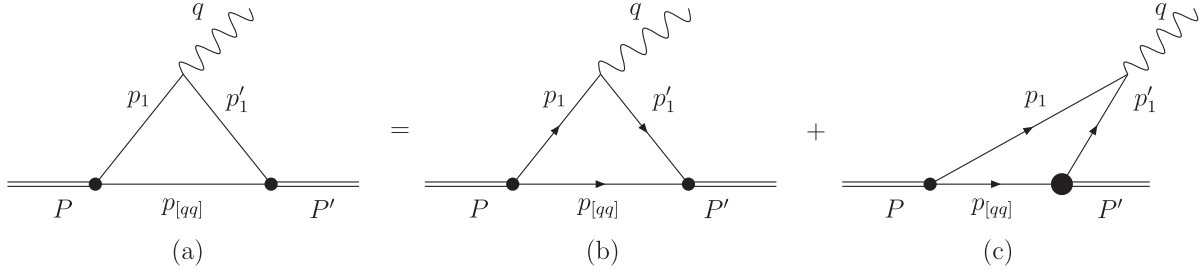


FIG. 1. Effective treatment of the LF amplitude (a) can be displayed into the LF valence part (b) in $0 < x < \alpha$ and the nonvalence one (c) in $\alpha < x < 1$, where the small and big dots of the mediator-quark vertices in (b) and (c) represent the LF ordinary and nonvalence wavefunction vertices, respectively.

$$\mathcal{M}_{val} = \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \frac{\Psi_i(x, \mathbf{k}_\perp) I_V^+ \Psi_f(x', \mathbf{k}'_\perp)}{(1-x)(1-x')},$$

$$I_V^+ = \bar{u}(\bar{P}', S'_z) \bar{\Gamma}'(\not{p}'_1 + m'_1) \gamma^+ (1 - \gamma_5) (\not{p}_1 + m_1) \Gamma u(\bar{P}, S_z). \quad (20)$$

Following Refs. [36–39], we obtain the transition form factors:

$$\begin{aligned} f_{1\nu}(q^2) &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \frac{\Psi_i(x, \mathbf{k}_\perp) \Psi_f(x', \mathbf{k}'_\perp)}{8(1-x)(1-x') P^+ P'^+} \text{Tr}[(\not{P} + M_0) \gamma^+ (\not{P}' + M'_0) (\not{p}'_1 + m'_1) \gamma^+ (\not{p}_1 + m_1)] \\ &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \frac{\Psi_i(x, \mathbf{k}_\perp) \Psi_f(x', \mathbf{k}'_\perp)}{(1-x)(1-x')} [k_\perp \cdot k'_\perp + (x_1 M_0 + m_1)(x'_1 M'_0 + m'_1)], \\ g_{1\nu}(q^2) &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \frac{\Psi_i(x, \mathbf{k}_\perp) \Psi_f(x', \mathbf{k}'_\perp)}{8(1-x)(1-x') P^+ P'^+} \text{Tr}[(\not{P} + M_0) \gamma^+ \gamma_5 (\not{P}' + M'_0) (\not{p}'_1 + m'_1) \gamma^+ \gamma_5 (\not{p}_1 + m_1)] \\ &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \frac{\Psi_i(x, \mathbf{k}_\perp) \Psi_f(x', \mathbf{k}'_\perp)}{(1-x)(1-x')} [-k_\perp \cdot k'_\perp + (x_1 M_0 + m_1)(x'_1 M'_0 + m'_1)], \\ \frac{f_{2\nu}(q^2)}{M} &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \frac{\Psi_i(x, \mathbf{k}_\perp) \Psi_f(x', \mathbf{k}'_\perp)}{8(1-x)(1-x') P^+ P'^+ q_\perp^+} \text{Tr}[(\not{P} + M_0) \sigma^{\nu+} (\not{P}' + M'_0) (\not{p}'_1 + m'_1) \gamma^+ (\not{p}_1 + m_1)] \\ &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \frac{\Psi_i(x, \mathbf{k}_\perp) \Psi_f(x', \mathbf{k}'_\perp)}{(1-x)(1-x') q_\perp^2} [-(m_1 + x_1 M_0) k'_\perp \cdot q_\perp + (m'_1 + x'_1 M'_0) k_\perp \cdot q_\perp], \\ \frac{g_{2\nu}(q^2)}{M} &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \frac{\Psi_i(x, \mathbf{k}_\perp) \Psi_f(x', \mathbf{k}'_\perp)}{8(1-x)(1-x') P^+ P'^+ q_\perp^+} \text{Tr}[(\not{P} + M_0) \sigma^{\nu+} \gamma_5 (\not{P}' + M'_0) (\not{p}'_1 + m'_1) \gamma^+ \gamma_5 (\not{p}_1 + m_1)] \\ &= \frac{N_{fs}}{16\pi^3} \int_0^\alpha dx \int d^2\mathbf{k}_\perp \frac{\Psi_i(x, \mathbf{k}_\perp) \Psi_f(x', \mathbf{k}'_\perp)}{(1-x)(1-x') q_\perp^2} [-(m_1 + x_1 M_0) k'_\perp \cdot q_\perp - (m'_1 + x'_1 M'_0) k_\perp \cdot q_\perp], \end{aligned} \quad (21)$$

where $\nu = 1, 2$ and the variables of $(x', k'_\perp)$ are the light-front relative momentum variables of $\mathcal{B}_f(P', S', S'_z)$ with the definitions given by replacing $x \rightarrow x'$ in Eq. (11). In this manuscript, we will not calculate $f_3(g_3)$ for the reasons explained in the calculations of helicity amplitudes earlier.

In the nonvalence region of $\alpha < x < 1$, $P'^+ < p^+ < P^+$, and $p_1^- = p_{1on}^- = (m_1^2 + k_{1\perp}^2)/p_1^+$, as shown in Fig. 1(c), the trace term in Eq. (19) can be separated into the on-shell propagating and instantaneous parts of I_{on}^μ and I_{inst}^μ via

$$\not{p} + m = (\not{p}_{on} + m) + \frac{1}{2} \gamma^+ (p^- - p_{on}^-), \quad (22)$$

respectively. The effective contribution can be found as

$$\begin{aligned} \mathcal{M}_{non-val} &= \frac{N_{fs}}{16\pi^3} \int_\alpha^1 dx \int d^2\mathbf{k}_\perp \frac{\Gamma_g(x, \mathbf{k}_\perp) I_{NV}^+ \Psi_i(x, \mathbf{k}_\perp)}{(1-x)(x'-1)} \\ &\quad \times \int \frac{dy}{y(1-y)} \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \Psi_f(y, \mathbf{l}_\perp), \end{aligned} \quad (23)$$

where I_{NV}^+ is the trace term in the nonvalence region. Substituting Eq. (22) into I^+ , we can obtain $I_{NV}^+ = I_V^+(p_i^- = p_{ion}^- = (m_i^2 + k_{i\perp}^2)/p_i^+) + I_{inst}^+$. The LF vertex function of a gauge boson Γ_g in Eq. (23) corresponds to

the LF energy denominator with its explicit form given by [29,30,32]

$$\Gamma_g^{-1}(x, \mathbf{k}_\perp) = \alpha \left[\frac{q^2}{1-\alpha} - \left(\frac{\mathbf{k}_\perp^2 + m_1^2}{1-x} + \frac{\mathbf{k}'_\perp^2 + m_1'^2}{\alpha-x} \right) \right]. \quad (24)$$

The trace terms in Eqs. (20) and (23), both corresponding to the products of the initial and final LF spin wave functions, can be obtained by off-shell Meloche transformations. The form factors related to the nonvalence diagram I_{NV}^+ are given by

$$\begin{aligned} f_{1_{NV}}(q^2) &= \frac{N_{fs}}{16\pi^3} \int_\alpha^1 dx \int d^2\mathbf{k}_\perp \Gamma_g(x, \mathbf{k}_\perp) \Psi_i(x, \mathbf{k}_\perp) \\ &\quad \times \frac{[k_\perp \cdot k'_\perp + ((1-x_1)M_0 + m_1)((1-x'_1)M' + m'_1)] + I_{\text{inst}}^+}{(1-x)(x'-1)} \\ &\quad \times \int \frac{dy}{y(1-y)} \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \Psi_f(y, \mathbf{l}_\perp), \\ g_{1_{NV}}(q^2) &= \frac{N_{fs}}{16\pi^3} \int_\alpha^1 dx \int d^2\mathbf{k}_\perp \Gamma_g(x, \mathbf{k}_\perp) \Psi_i(x, \mathbf{k}_\perp) \\ &\quad \times \frac{[-k_\perp \cdot k'_\perp + ((1-x_1)M_0 + m_1)((1-x'_1)M' + m'_1)] + I_{\text{inst}}^+}{(1-x)(x'-1)} \\ &\quad \times \int \frac{dy}{y(1-y)} \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \Psi_f(y, \mathbf{l}_\perp), \\ \frac{f_{2_{NV}}(q^2)}{M} &= \frac{N_{fs}}{16\pi^3} \int_\alpha^1 dx \int d^2\mathbf{k}_\perp \Gamma_g(x, \mathbf{k}_\perp) \Psi_i(x, \mathbf{k}_\perp) \\ &\quad \times \frac{[(m_1 + (1-x_1)M_0)k'_\perp \cdot q_\perp - (m'_1 + (1-x'_1)M')k_\perp \cdot q_\perp] + I_{\text{inst}}^+}{(1-x)(x'-1)q_\perp^2} \\ &\quad \times \int \frac{dy}{y(1-y)} \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \Psi_f(y, \mathbf{l}_\perp), \\ \frac{g_{2_{NV}}(q^2)}{M} &= \frac{N_{fs}}{16\pi^3} \int_\alpha^1 dx \int d^2\mathbf{k}_\perp \Gamma_g(x, \mathbf{k}_\perp) \Psi_i(x, \mathbf{k}_\perp) \\ &\quad \times \frac{[-(m_1 + (1-x_1)M_0)k'_\perp \cdot q_\perp - (m'_1 + (1-x'_1)M')k_\perp \cdot q_\perp] + I_{\text{inst}}^+}{(1-x)(x'-1)q_\perp^2} \\ &\quad \times \int \frac{dy}{y(1-y)} \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \Psi_f(y, \mathbf{l}_\perp), \end{aligned} \quad (25)$$

where $I_{\text{inst}}^+ = (1-x)(M^2 - M_0^2)(m'_1 + (1-x')M')$ and $I_{\text{inst}}'^+ = -(1-x)(M^2 - M_0^2)k'_\perp \cdot q_\perp$. The complete form factors are $f(g)_j = f(g)_{j_V} + f(g)_{j_{NV}}$ ($j = 1, 2$). It is worth noting that the instantaneous contribution exists in the nonvalence diagram only when the “+” component is used. The vertex of the nonvalence wave function is usually obtainable from the Bethe-Salpeter (BS) amplitude in the BS theory [30,40]. The LF BS equation is expressed as [30,41,42]

$$\begin{aligned} &(M^2 - M_0^2) \Psi'_f(x_i, k_{i\perp}) \\ &= \int [dy] [d^2\mathbf{l}_\perp] \mathcal{K}(x_i, \mathbf{k}_{i\perp}; y, \mathbf{l}_\perp) \Psi_f(y, \mathbf{l}_\perp). \end{aligned} \quad (26)$$

Both valence and nonvalence BS amplitudes can be regarded as solutions of Eq. (26). The normal and nonvalence BS amplitudes correspond to $x > \alpha$ and $x < \alpha$, respectively. In Fig. 1(c), the nonvalence BS amplitude is the analytic continuation of the valence BS amplitude. In the LFQM, the relationship between the BS amplitudes of two regions is given in Refs. [30–32]. However, for the integral equation of Eq. (26) it is necessary to use the nonperturbative QCD method to obtain the kernel.

The relevant operator $\mathcal{K}$ in Eq. (26) is the BS core, which in principle contains contributions from high Fock states. It is the high Fock component of the bound state that is related to the lowest Fock component with this kernel and

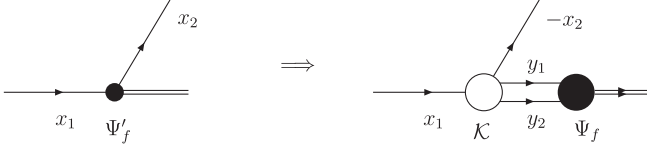


FIG. 2. Nonvalence vertex (small black dot) linked to an ordinary light front wave function (big black dot) through a relevant operator $\mathcal{K}$ (big white dot).

describes the connection of nonvalence vertices (small black dots) to ordinary light front wave functions (large black dots) through the correlation operator $\mathcal{K}$ (large white dots), as shown in Fig. 2, where $\Psi'(x_i, k_{i\perp})$ represents the nonvalent BS amplitude. We can imagine that the correlation operator $\mathcal{K}$ is a bridge between the nonvalent vertices and the ordinary light front wave function. So the kernel $\mathcal{K}$ will be a function of all internal momenta ($x; k; y; l$). We define that $G_{B_i B_f} \equiv \int [dy][d^2\mathbf{l}_\perp] \mathcal{K}(x_i, \mathbf{k}_{i\perp}; y, \mathbf{l}_\perp) \Psi_f(y, \mathbf{l}_\perp)$, which depends only on x and $\mathbf{k}_\perp$. The range of the momentum fraction x relies on the external momenta for the embedded states.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Form factors

To numerically evaluate the exclusive transition form factors in the LFQM, we choose to work directly in the timelike region. Instead of assuming $q_\perp \neq 0$, we first introduce $q_\perp = 0$ in the final numerical calculation process, although we could also get the results in the timelike region by directly taking $q_\perp^2 \rightarrow -q^2$ for those in the spacelike one. In our calculation, we use [27]

$$\begin{aligned} m_{u,d} &= 0.25, & m_s &= 0.38, \\ m_{[qq]} &= 0.5, & m_c &= 1.2 \text{ GeV}, \end{aligned} \quad (27)$$

where the quark masses are the constituent masses used in the quark model. In Eq. (23), $G_{B_i B_f}$ is treated as a constant in the range of 1.0–6.0, which was previously tested in some exclusively semileptonic mesonic decay processes and shown to be a good approximation for processes with a small momentum transfer [30–32]. Explicitly, in our numerical evaluation we choose $G_{B_i B_f} = 5(2)$ and the flavor-spin factors N_{fs} are $\frac{1}{\sqrt{3}}(\frac{1}{\sqrt{2}})$ for $\Lambda_c \rightarrow \Lambda(n)$ transitions [13], respectively.

To describe the momentum q^2 behaviors, we parametrize the form factors in the double-pole forms of

$$F(q^2) = \frac{F(0)}{1 + a(q^2/m_p^2) + b(q^4/m_p^4)}, \quad (28)$$

where $m_p = 6.0 \text{ GeV}$ is the M_B scale for the modes of $\Lambda_c \rightarrow \Lambda(n)$, and $(F(0), a, b)$ can be determined in the

TABLE I. Form factors of the $\Lambda_c \rightarrow (\Lambda/n)$ transitions.

$\Lambda_c \rightarrow \Lambda$	$F(0)$	a	b
f_1	$0.546^{+0.030}_{-0.039}$	-19.02	179.03
f_2	$-0.43^{+0.039}_{-0.015}$	-22.07	183.29
g_1	$0.441^{+0.004}_{-0.009}$	-10.86	59.74
g_2	$0.131^{+0.016}_{-0.018}$	-7.84	-372.00
$\Lambda_c \rightarrow n$	$F(0)$	a	b
f_1	$0.595^{+0.033}_{-0.040}$	-16.14	142.204
f_2	$-0.478^{+0.032}_{-0.037}$	-15.76	129.07
g_1	$0.490^{+0.008}_{-0.025}$	-8.51	23.22
g_2	$0.057^{+0.0118}_{-0.0128}$	1.002	-369.925

numerical analysis. Our results for the form factors are given in Table I. In our results, the errors come from uncertainties in the shape parameters of β .

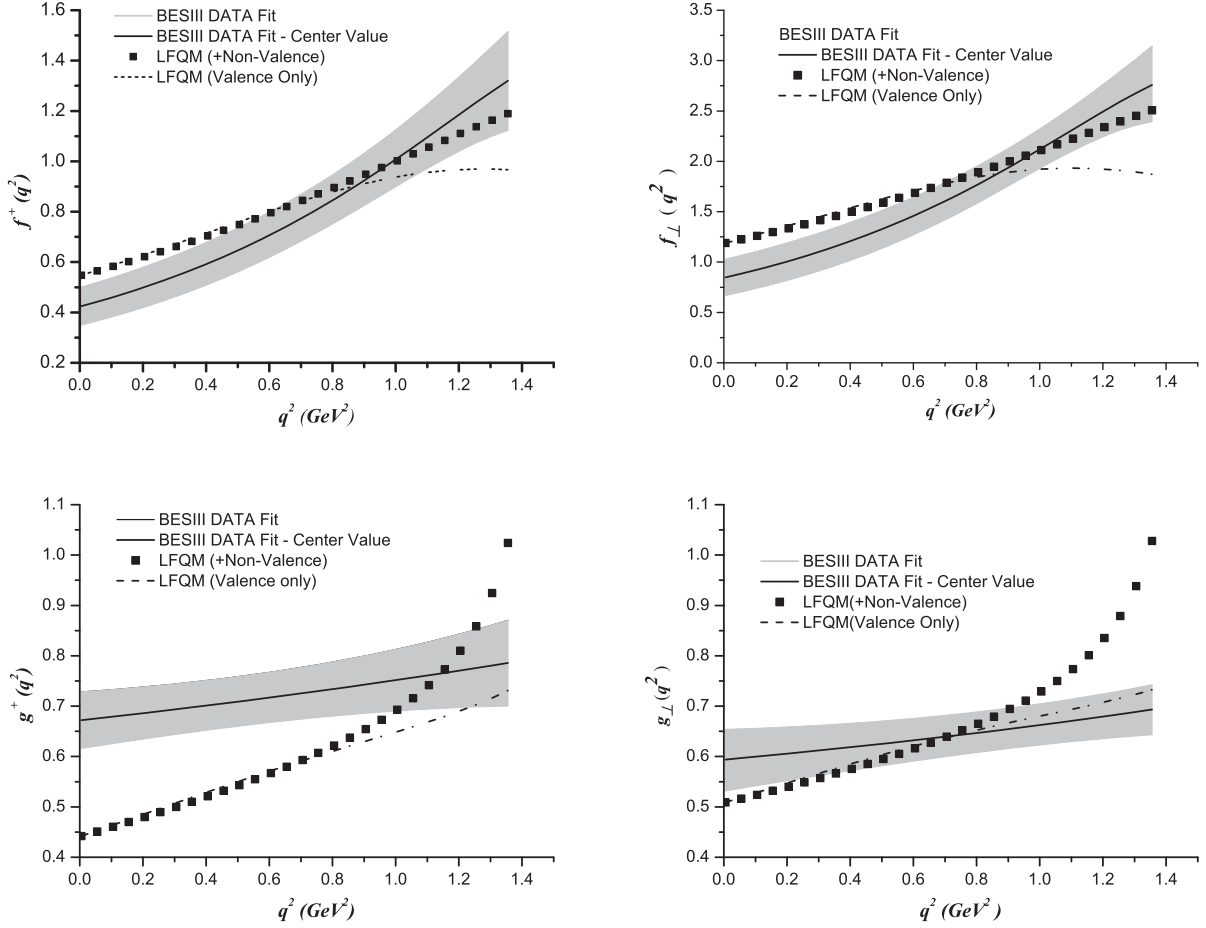
To find the value of β , we first used the possible ranges of $\beta_{c[qq]} = 0.46 \sim 0.76$, $\beta_{s[qq]} = 0.4 \sim 0.6$ in Refs. [13,14] as the initial values to calculate the decay branching ratios, and then took the mean values of all branching ratios with one-standard-deviations as their errors. Consequently, we compare the experimental results in Fig. 3 with the form factors corresponding to the mean values of the obtained branching ratios to determine the most suitable combinations of β values. Based on our results, the β values for Λ_c , Λ , and n in the LFQM are determined to be

$$\begin{aligned} \beta_{c[qq]} &= 0.58 \pm 0.08, & \beta_{s[qq]} &= 0.52 \pm 0.08, \\ \beta_{u[qq]} &= 0.40 \pm 0.04, \end{aligned} \quad (29)$$

where $(\beta_{c[qq]} - \beta_{s[qq]}) \sim 0.08$ and $(\beta_{c[qq]} - \beta_{u[qq]}) \sim 0.18$, so that the form factors corresponding to the changes in the q^2 spectra clearly have better behaviors to compare with the experimental results. We note that the ratios of the uncertainties for the form factors in Table I are not the same due to the different values of β_Λ and β_n in the $\Lambda_c \rightarrow \Lambda$ and $\Lambda_c \rightarrow n$ transitions. According to Ref. [43], we can rewrite the four form factors as

$$\begin{aligned} f^+(q^2) &= \frac{1}{M + M'} \left((M + M') f_1 - \frac{q^2}{M} f_2 \right), \\ f_\perp(q^2) &= \left(f_1 - \frac{M + M'}{M} f_2 \right), \\ g^+(q^2) &= \frac{1}{M - M'} \left((M - M') g_1 + \frac{q^2}{M} g_2 \right), \\ g_\perp(q^2) &= \left(g_1 + \frac{M - M'}{M} g_2 \right), \end{aligned} \quad (30)$$

which correspond to the four helicity amplitudes, respectively. In Fig. 3, we present our evaluations of the form

FIG. 3. Form factors of $\Lambda_c \rightarrow \Lambda$, where the gray bands represent the uncertainties.

factors as functions of q^2 for the $\Lambda_c \rightarrow \Lambda$ transition, where the BESIII data fits are also given. Considering Eq. (30) and the results of BESIII in Fig. 3, $f^+ = f_1 \sim 0.41$ at $q^2 = 0$. Comparing to $f_\perp$ in Fig. 3, f_1 and f_2 must have opposite signs. Our predictions are very close to those of the BESIII values for f^+ and $f_\perp$ at $q^2 = 0$, but significantly smaller than the BESIII one for $g^+(0)$. This leads to a branching ratio smaller than that of BESIII. It is also worth mentioning that our calculations as well as the LQCD one indicate that $f^+(0) > g^+(0)$, but the opposite result is given from BESIII.

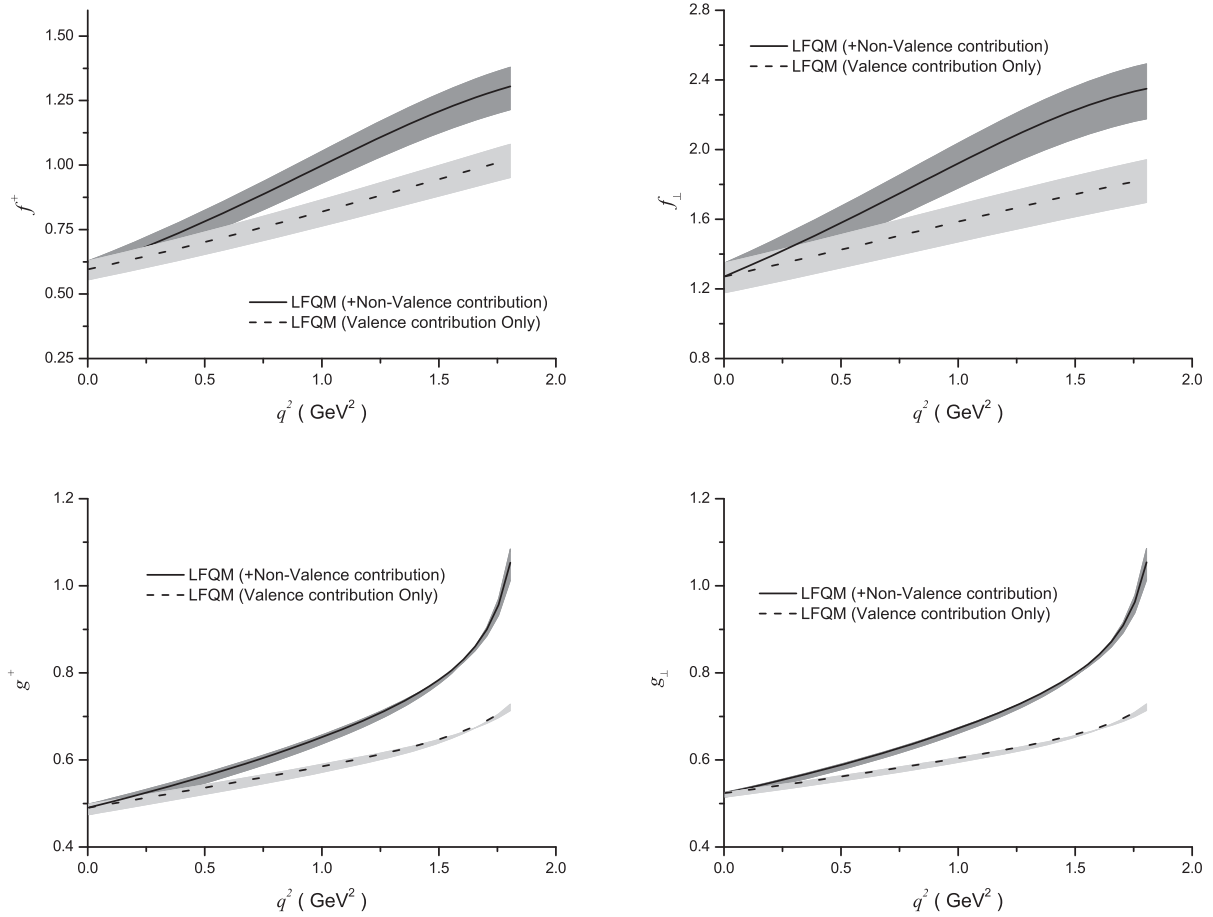
Figure 3 shows that the four form factors exhibit similar behaviors throughout the q^2 spectra. Among them, f^+ and $f_\perp$ are more significant and closer to the BESIII values. However, for g^+ and $g_\perp$, especially in the high q^2 region, the LFQM calculation yields steeper slopes. Obviously, our LFQM results with the nonvalence contributions are consistent with the BESIII data as well as those of the LQCD [5,43]. Additionally, Fig. 4 displays the form factors for the $\Lambda_c \rightarrow n$ transition, similar to those for the $\Lambda_c \rightarrow \Lambda$ transition.

B. Decay branching ratios

The amplitudes of $\Lambda_c \rightarrow (\Lambda/n)\ell\nu_\ell$ contain some independent mixtures of helicity components, described by $h_{\lambda,\lambda_p}^{V(A)}$, where λ and λ_p represent the helicity components of the final baryon and W propagator, respectively. From Eq. (8), we can easily separate the integrals of the longitudinal and transverse polarization asymmetries. The differential branching ratios are given by

$$\begin{aligned} \frac{d\Gamma(\Lambda_c \rightarrow (\Lambda/n)\ell\nu_\ell)}{\Gamma(\Lambda_c)dq^2} &= \frac{G_F^2 V_{cq}^2 (q^2 - m_\ell^2)^2 p_{\text{cm}}}{(2\pi)^3 24M^2 q^2 \Gamma(\Lambda_c)} \\ &\times \left\{ \left(1 + \frac{m_\ell^2}{2q^2} \right) \left[|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right] \right. \\ &\left. + \frac{3m_\ell^2}{2q^2} \left(|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},1}|^2 \right) \right\}. \end{aligned} \quad (31)$$

In Table II, we show the decay branching ratios of $\Lambda_c \rightarrow (\Lambda/n)\ell\nu_\ell$ ($\ell = e, \mu$) in various LFQMs along with the BESIII data, where I and II represent our predictions with

FIG. 4. Form factors of $\Lambda_c \rightarrow n$, where the gray bands show the uncertainties.

and without nonvalence contributions, respectively. It can be seen that the contribution including the nonvalence part is about 10% more than that without it. Note that the excess mainly comes from regions with higher q^2 . Compared with the other LFQM calculations, our results agree with those in Ref. [14] well and Ref. [15] within the allowed errors. In particular, Ref. [12], which works directly with three-quark state wave functions, gives almost the same results as ours. This suggests that using the diquark picture in the LFQM is a reasonable assumption. However, the values in Ref. [13] are about only half of ours. Our predictions are also consistent with the BESIII data. We note that the errors in our results are only taken from the uncertainties in the

β factors. Clearly, if the uncertainties of other parameters, such as the quark masses and different shape parameters, are added, these errors will increase.

The differential decay rates of $\Lambda_c \rightarrow (\Lambda, n)\ell\nu_\ell$ ($\ell = e, \mu$) as functions of q^2 are displayed in Figs. 5 and 6, respectively. As shown in Fig. 5, we see that a considerable consistency across the q^2 region between our results and those from BESIII for the differential decay rates.

Finally, we discuss the sensitivities of the input parameters to our results. In Table II, the central values of our results for the decay branching ratios correspond to the central ones of $\beta_{c[qq]}$, $\beta_{s[qq]}$ and $\beta_{u[qq]}$ in Eq. (29), while those of upper (lower) uncertainties are calculated by taking

TABLE II. Decay branching ratios of $\Lambda_c \rightarrow (\Lambda/n)\ell\nu_\ell$ ($\ell = e, \mu$).

Result	$\mathcal{B}(\Lambda_c \rightarrow \Lambda e \nu_e)$	$\mathcal{B}(\Lambda_c \rightarrow \Lambda \mu \nu_\mu)$	$\mathcal{B}(\Lambda_c \rightarrow n e \nu_e)$	$\mathcal{B}(\Lambda_c \rightarrow n \mu \nu_\mu)$
I	$(3.39^{+0.26}_{-0.32})\%$	$(3.214^{+0.245}_{-0.326})\%$	$(0.361^{+0.02}_{-0.029})\%$	$(0.345^{+0.019}_{-0.029})\%$
II	$(3.18^{+0.26}_{-0.32})\%$	$(3.004^{+0.222}_{-0.298})\%$	$(0.282^{+0.016}_{-0.023})\%$	$(0.267^{+0.016}_{-0.021})\%$
[13]	1.63%	...	0.201%	...
[14]	$(3.36 \pm 0.87)\%$	$(3.21 \pm 0.85)\%$	$(0.36 \pm 0.15)\%$	$(0.34 \pm 0.14)\%$
[15]	$(4.04 \pm 0.75)\%$	$(3.90 \pm 0.73)\%$	...	...
BESIII [1–3]	$(3.63 \pm 0.38 \pm 0.20)\%$	$(3.49 \pm 0.46 \pm 0.27)\%$	$(0.357 \pm 0.034 \pm 0.014)\%$	...

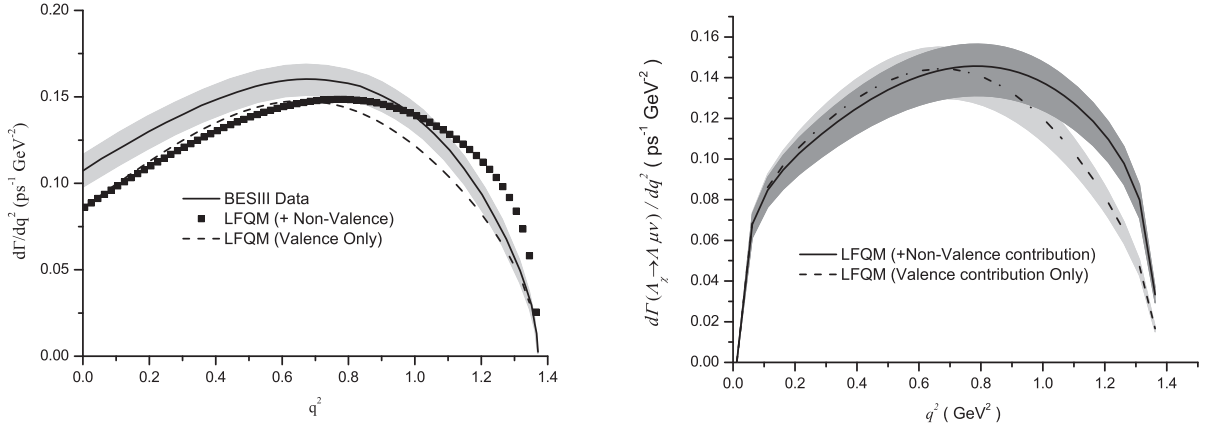


FIG. 5. Differential decay widths of $\Lambda_c \rightarrow \Lambda \ell \nu_\ell (\ell = e, \mu)$, where the gray bands show the total uncertainties.

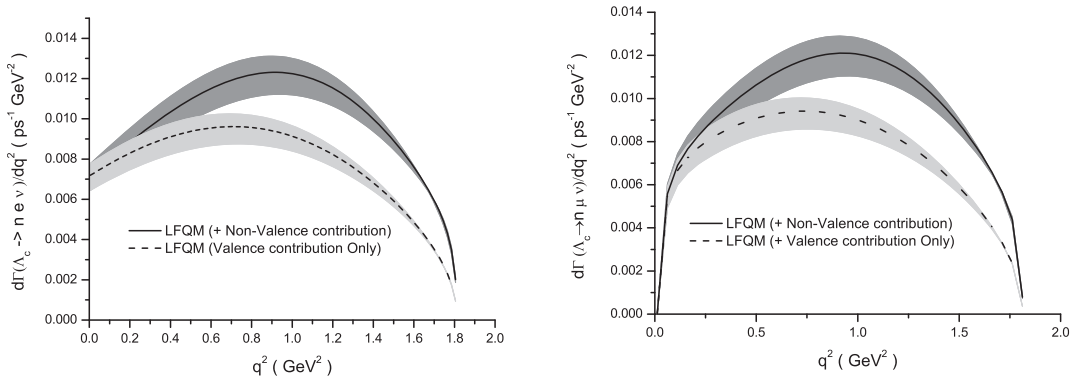


FIG. 6. Differential decay widths of $\Lambda_c \rightarrow n \ell \nu_\ell (\ell = e, \mu)$, where the gray bands show the total uncertainties.

the for the upper (lower) values of β , respectively. Note that our results are based on fixed values of m_q , and $m_{[qq]}$ in Eq. (27). Clearly, different choices of these parameters also affect the outcome. For instance, for $\Lambda_c^+ \rightarrow \Lambda \ell^+ \nu_\ell$, if the diquark mass $m_{[qq]}$, which ranges from 0.4 to 0.7 GeV [27], increases 20% without altering the baryonic β values, the decay branching ratio will also increase about 5%. However, increasing the c-quark mass in Λ_c by 0.2 GeV ($\sim 15\%$) would decrease the branching ratio by about 15%. As for the s-quark in Λ , the change of m_s has a negligible effect on our results.

IV. CONCLUSION

We have studied the exclusive semilepton decays of $\Lambda_c^+ \rightarrow (\Lambda, n) \ell^+ \nu_\ell$ in the SM by using the LFQM. In our calculations, we have analyzed the form factors of the baryonic transitions based on the BS formalism in $q^+ > 0$, efficiently dealing with nonvalence contributions. The q^2 behaviors of the form factors are consistent with the BESIII data fits. As shown in Table II, our results with including

nonvalence contributions agree with the existing experimental data of BESIII as well as other theoretical evaluations. In addition, we have also extracted the parameters of possible β values in the LFQM for Λ_c , Λ and n , respectively. Our phenomenological predictions for the semileptonic decays of Λ_c can be tested by the experiments at various charmed baryon facilities. Finally, applications to other similar baryonic decay processes will be considered elsewhere.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [13,43].

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