Λ_c^+ transverse polarization and decay asymmetry parametersHong Chen¹ and Rong-Gang Ping^{2,3,*}¹*School of Physical Science and Technology, Southwest University, Chongqing 400715, China*²*Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918(1), Beijing 100049, China*³*University of Chinese Academy of Science, Beijing 100049, China*

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We present formulas with motivations to measure the asymmetry decay parameters of charmed baryon, Λ_c^+ , decaying into final states pK_s , $\Lambda\pi^+$, $\Sigma^+\pi^0$, $\Sigma^0\pi^+$ and $\Xi^0(1530)K^+$ by taking advantage of its possibly transverse polarization in e^+e^- collisions. Some spin observables to test and display the spin polarization effects of the charmed baryons are also derived.

DOI: [10.1103/PhysRevD.99.114027](https://doi.org/10.1103/PhysRevD.99.114027)**I. INTRODUCTION**

Studies on charmed baryon decays offer a unique laboratory for people to get better understanding of the interplay of strong and weak interactions. Over last thirty years, advancement in both theoretical and experimental studies has been relatively slow compared to the significant progress made in the arena of heavy mesons. Only a few amount of branching ratios for the Λ_c^+ decays have been established in experiments so far. Recently, precise measurements on the branching fractions for $\Lambda_c^+ \rightarrow \Sigma^0\pi^+$, $\Sigma^+\pi^0$ [1] and $\Xi^0 K^+$ [2] decays indicate that W exchange plays an essential role in charmed baryon decays. However, this nonfactorizable contribution is difficult to estimate and poorly understood from the first principle.

Experimentally, measurements on the asymmetry parameters of Λ_c^+ in nonleptonic decays provide more precise information to reveal how large this kind of nonfactorizable contribution is to its weak decays. A few Cabibbo-allowed Λ_c^+ decays into baryon and pseudoscalar meson were studied using the pole model [3–7] and their asymmetry parameters were predicted with quite different results as shown in Table I. These Λ_c^+ nonleptonic weak decays were also studied with the effective Hamiltonian [8], and the nonfactorizable contributions were described in terms of the wave function overlap parameters determined from fit to experimental data. To test these predictions, more and precise measurements on the asymmetry parameters are desirable. However, measurements are only available for

the $\Lambda_c^+ \rightarrow \Lambda\pi^+$ and $\Sigma^+\pi^0$ decays [9] with large uncertainties as given in Table I.

One measuring the decay parameters takes advantage of the fact that the Λ_c^+ weak decay, e.g., $\Lambda_c \rightarrow Y\pi$, gives rise to the spin polarization to the hyperon Y whether the Λ_c is polarized or not. The emergence of spin polarization makes the hyperon subsequent decay used as the spin analyzer. Similar to the $\Xi \rightarrow \Lambda\pi$ and $\Lambda \rightarrow p\pi$, the polarization of hyperon Y , $\mathbf{P}_Y$, is related to the polarization of Λ_c , $\mathbf{P}_c$ in its rest frame, and is given by the Lee-Yang formula [14]

$$\mathbf{P}_Y = \frac{(\alpha_{[Y\pi]}^{[\Lambda_c]} + \mathbf{P}_c \cdot \hat{\mathbf{q}}_Y) \hat{\mathbf{q}}_Y + \beta \mathbf{P}_c \times \hat{\mathbf{q}}_Y + \gamma \hat{\mathbf{q}}_Y \times (\mathbf{P}_c \times \hat{\mathbf{q}}_Y)}{1 + \alpha_{[Y\pi]}^{[\Lambda_c]} \mathbf{P}_c \cdot \hat{\mathbf{q}}_Y}, \quad (1)$$

where $\alpha_{[Y\pi]}^{[\Lambda_c]}$, β and γ are three Lee-Yang parameters, and $\hat{\mathbf{q}}_Y$ is the unit momentum vector of Y in the Λ_c rest frame. If the Λ_c is unpolarized, the above equation is reduced to $\mathbf{P}_Y = \alpha_{[Y\pi]}^{[\Lambda_c]} \hat{\mathbf{q}}_Y$, which implies that the hyperon production from the unpolarized Λ_c is longitudinally polarized by the amount of $\alpha_{[Y\pi]}^{[\Lambda_c]}$. This parameter can be extracted from the subsequent decay of angular distribution [10–13,15] with the form of $\mathcal{W}(\cos\theta_Y) \sim 1 + \alpha_{[Y\pi]}^{[\Lambda_c]} \alpha_Y \cos\theta_Y$, where α_Y is an asymmetry parameter for the hyperon decay, and θ_Y is the angle between $\hat{\mathbf{q}}_Y$ and the Λ_c momentum in its mother rest frame. The transverse polarization components are measured by the parameters β and γ .

The method to measure the asymmetry parameter for the $\Lambda_c^+ \rightarrow pK_s$ decay is different from the previous cases, since no spin analyzer of a decay is available in its subsequential decay. In this case, only if the charmed baryon is transversely or longitudinally polarized, its decay parameter can be measured from its daughter's angular distribution, namely, $\mathcal{W}(\hat{q}_p) \propto 1 + \alpha_{[pK_s]}^{[\Lambda_c^+]} \mathbf{P}_c \cdot \hat{q}_p$; here $\hat{q}_p$ is the unit

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TABLE I. Theoretical predictions and experimental measurements of asymmetry decay parameters $\alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]}$ for $\Lambda_c^+ \rightarrow \Lambda\pi^+$, $\alpha_{[\Sigma^+\pi^0]}^{[\Lambda_c^+]}$ for $\Lambda_c^+ \rightarrow \Sigma^+\pi^0$, $\alpha_{[\Xi^0 K^+]}^{[\Lambda_c^+]}$ for $\Lambda_c^+ \rightarrow \Xi^0 K^+$, and $\alpha_{[pK_S^0]}^{[\Lambda_c^+]}$ for $\Lambda_c^+ \rightarrow pK_S^0$.

Parameters	Predictions	Experiments	PDG
$\alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]}$	-0.70 [8], -0.67 [3] -0.95 [4], -0.95 [5] -0.99 [6], -0.99 [7]	-0.78 ± 0.25 [10] $-0.94^{+0.24}_{-0.08}$ [11] -0.96 ± 0.42 [12], $-1.1^{+0.4}_{-0.1}$ [13]	-0.91 ± 0.15
$\alpha_{[\Sigma^+\pi^0]}^{[\Lambda_c^+]}$	0.71 [8], 0.92 [3] 0.78 [4], 0.43 [5] 0.39 [6], -0.31 [7]	-0.45 ± 0.32	-0.45 ± 0.32
$\alpha_{[\Xi^0 K^+]}^{[\Lambda_c^+]}$	0 [8], 0 [3] 0 [5], 0 [6] 0 [7]	...	...
$\alpha_{[pK_S^0]}^{[\Lambda_c^+]}$	-1.0 [8], 0.51 [3] -0.49 [4], -0.97 [5] -0.66 [6], -0.99 [7]	...	...

momentum vector of proton in the Λ_c^+ rest frame. However, the production of longitudinally polarized Λ_c^+ needs the polarized beams in e^+e^- collision experiments.

Recently, an interesting observation shows that the production of $\Lambda\bar{\Lambda}$ pairs in J/ψ decays can be spontaneously polarized in transverse direction, which has turned out to be an ingenious laboratory to measure the Λ decay parameters [16], and the same polarized phenomenon was also observed in the continuum process $e^+e^- \rightarrow \Lambda\bar{\Lambda}$ [17]. Similarly, the $\Lambda_c^+\bar{\Lambda}_c^-$ production in the continuum process could be expected to acquire transverse polarization from the phase angle difference in its electric and magnetic form factors.

In this paper, we present formulas with motivations for an experimentalist to measure the asymmetry parameters for Λ_c^+ decaying into final states pK_S , $\Lambda\pi^+$, $\Sigma^+\pi^0$, $\Sigma^0\pi^+$ and $\Xi^0(1530)K^+$ by taking advantages of its possibly transverse polarization. In the last section, we perform a Monte Carlo (MC) simulation to show how to test and display the transverse polarization effects in the single tag events produced from unpolarized e^+e^- collisions.

II. JOINT ANGULAR DISTRIBUTION

A. Helicity system

The asymmetry decay parameters are supposed to be extracted from the joint angular distributions, which are formulated with helicity amplitudes. To illustrate how the helicity system is to be defined in the analysis, we take an example of the process $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$, $\Lambda_c^+ \rightarrow \Lambda\pi^+$, $\Lambda \rightarrow p\pi^-$.

For the process $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$, the momenta of Λ_c^+ and $\bar{\Lambda}_c^-$ are defined in the center of mass system of the e^+e^- pair, which is assumed to be overlapped with the laboratory system with axes (x, y, z) . (θ_0, ϕ_0) are the polar and azimuthal angles of Λ_c^+ in the (x, y, z) system. For the

helicity system describing the $\Lambda_c^+ \rightarrow \Lambda\pi^+$ decay, the momentum of Λ_c^+ is taken as the z' axis, and the x' axis is in the plane of (z, z') , with (x', y', z') axes forming the new right hand system. In this system, the (θ_1, ϕ_1) are the polar and azimuthal angles to specify the Λ momentum defined in the rest frame of Λ_c^+ . For the helicity system describing $\Lambda \rightarrow p\pi^-$, the new z'' axis is taken along the momentum of Λ , the x'' axis is in the plane of (z', z'') , and the y'' axis is normal to the (z', z'') plane. In the (x'', y'', z'') frame, the (θ_2, ϕ_2) are the polar and azimuthal angles of the proton momentum defined in the rest frame of Λ , as shown in Fig. 1.

B. Transverse polarization of the Λ_c^+ baryon

Spin polarization arising in the $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ process could be used as a tool to investigate the Λ_c^+ electromagnetic structure, in which the dynamics is modeled with electric and magnetic form factors. In the absence of the polarized beam in e^+e^- collision experiments, longitudinal polarization of $\Lambda_c^+/\bar{\Lambda}_c^-$ baryons is unavailable since the electromagnetic production process conserves the parity transformation. However, if the electric form factor acquires a relative phase angle with respect to the magnetic form factor, then the transverse polarization of baryons is induced spontaneously, with the polarization vector normal to the production plane.

FIG. 1. Definition of the helicity system for $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$, $\Lambda_c^+ \rightarrow \Lambda\pi^+$, and $\Lambda \rightarrow p\pi^-$.

To display the baryonic spin polarization effects in the process $e^+e^- \rightarrow \Lambda_c^+(\lambda_1)\bar{\Lambda}_c^-(\lambda_2)$, we formulate a spin observable using its subsequent decays, i.e., $\Lambda_c^+ \rightarrow BP$; here $B(P)$ denotes the octet baryon (pseudoscalar meson), while the $\bar{\Lambda}_c^-$ is not observed and it decays inclusively. The un-normalized spin density matrix for observation of Λ_c^+ is calculated with

$$\rho_{\lambda_1, \lambda'_1} \propto \sum_{m, \lambda_2} d_{m, \lambda_1 - \lambda_2}^1(\theta_0) d_{m, \lambda'_1 - \lambda_2}^1(\theta_0) A_{\lambda_1, \lambda_2} A_{\lambda'_1, \lambda_2}^*, \quad (2)$$

where A_{λ_1, λ_2} is the helicity amplitude for $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$; the sum runs over $\lambda_2 = \pm \frac{1}{2}$ and $m = \pm 1$; since the electron mass is negligible, and the e^+e^- annihilation conserves the helicity and CP transformation, we have $A_{\lambda_1, \lambda_2} = A_{-\lambda_1, -\lambda_2}$.

On the other hand, the spin density matrix of the spin-1/2 particle can be formulated as

$$\rho = \frac{1}{2}(P_0 I + \mathbf{P} \cdot \boldsymbol{\sigma}) = \frac{1}{2} \begin{pmatrix} P_0 + P_z & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix}, \quad (3)$$

where P_0 is an unpolarized cross section and P_x, P_y, P_z are spin polarization observables/vectors associated with the Pauli matrices σ_x, σ_y , and σ_z .

Compared with the spin density matrix given by Eq. (2), one has the spin observables taken as

$$P_z = P_x = 0, P_0 = c_0(1 + \alpha \cos^2 \theta_0), \\ P_y(\alpha, \theta_0, \Delta) = c_0 \sqrt{1 - \alpha^2} \sin \theta_0 \cos \theta_0 \sin \Delta, \quad (4)$$

where Δ is the phase angle difference in the helicity amplitudes $A_{\frac{1}{2}, -\frac{1}{2}}$ and $A_{\frac{1}{2}, \frac{1}{2}}$. α is the angular distribution parameter, which is defined as $\alpha = (|A_{\frac{1}{2}, -\frac{1}{2}}|^2 - 2|A_{\frac{1}{2}, \frac{1}{2}}|^2) / (|A_{\frac{1}{2}, -\frac{1}{2}}|^2 + 2|A_{\frac{1}{2}, \frac{1}{2}}|^2)$. c_0 is a normalization factor constrained by the spin density matrix $\text{Tr} \rho = 1$, which reads

$$c_0 = \frac{3}{2(3 + \alpha)}. \quad (5)$$

Equation (4) indicates that the spin polarization vectors are vanished in the baryonic production x - z plane, formed by the $\Lambda_c^+ - e^+$ momenta, while the spin observable P_y is along the normal to the x - z plane, which is regarded as a transverse polarization vector.

Once the charmed baryon acquires transverse polarization, it transfers it to the final states of Λ_c^+ weak decays, and the spin polarization is manifested with its daughter angular distributions. We are motivated to extract its asymmetry parameters by using the transverse polarization effects, and we have to analyze the joint angular distribution of the decays, e.g., $\Lambda_c^+ \rightarrow pK_S^0, \Lambda\pi^+, \Sigma^0\pi^+$ and $\Xi(1530)^0 K^+$ as follows.

C. $\Lambda_c^+ \rightarrow pK_S^0$

For the $\Lambda_c^+ \rightarrow p(\lambda_3)K_S^0$ decay, helicity angles are defined as $\Omega_1(\theta_1, \phi_1)$, and helicity amplitudes as B_{λ_3} . The joint angular distribution is calculated with

$$\mathcal{W}_1(\theta_0, \Omega_1, \Omega_2) \propto \sum_{\lambda_1, \lambda'_1, \lambda_3} \rho_{\lambda_1, \lambda'_1} D_{\lambda_1, \lambda_3}^{\frac{1}{2}*}(\Omega_1) D_{\lambda'_1, \lambda_3}^{\frac{1}{2}}(\Omega_1) |B_{\lambda_3}|^2 \\ \propto 1 + \alpha \cos^2(\theta_0) \\ + P_y(\alpha, \theta_0, \Delta) \alpha_{[\Lambda_c^+]}^{[pK_S]} \sin(\theta_1) \sin(\phi_1), \quad (6)$$

where $\rho_{\lambda_1, \lambda'_1}$ is the Λ_c^+ spin density matrix given by Eq. (2), $D_{\lambda_1, \lambda_3}^{\frac{1}{2}}(\Omega) \equiv D_{\lambda_1, \lambda_3}^{\frac{1}{2}}(\phi, \theta, 0)$ is the Wigner- D function, $\alpha_{[\Lambda_c^+]}^{[pK_S]} \equiv (|B_{\frac{1}{2}}|^2 - |B_{-\frac{1}{2}}|^2) / (|B_{\frac{1}{2}}|^2 + |B_{-\frac{1}{2}}|^2)$ is the asymmetry parameter for $\Lambda_c^+ \rightarrow pK_S$, and we take the normalization condition $|B_{\frac{1}{2}}|^2 + |B_{-\frac{1}{2}}|^2 = 1$.

For $\bar{\Lambda}_c^- \rightarrow \bar{p}K_S^0$ decay, the joint angular distribution is the same, except for making a replacement $\Delta \rightarrow \bar{\Delta} = \Delta$, and $\alpha_{[\Lambda_c^+]}^{[pK_S]} \rightarrow \bar{\alpha}_{[\bar{\Lambda}_c^-]}^{[\bar{p}K_S]} = -\alpha_{[\Lambda_c^+]}^{[pK_S]}$.

D. $\Lambda_c^+ \rightarrow \pi^+ \Lambda, \Lambda \rightarrow p\pi^-$

Helicity angles for the decays $\Lambda_c^+ \rightarrow \pi^+ \Lambda(\lambda_3)$ and $\Lambda \rightarrow p(\lambda_4)\pi^-$ are defined by $\Omega_1(\theta_1, \phi_1)$ and $\Omega_2(\theta_2, \phi_2)$, with helicity amplitudes B_{λ_3} and C_{λ_4} , respectively. The joint angular distribution is calculated by

$$\mathcal{W}_2(\theta_0, \Omega_1, \Omega_2) \propto \sum_{\lambda_i} \rho_{\lambda_1, \lambda'_1} D_{\lambda_1, \lambda_3}^{\frac{1}{2}*}(\Omega_1) D_{\lambda'_1, \lambda_3}^{\frac{1}{2}}(\Omega_1) B_{\lambda_3} B_{\lambda'_3}^* \\ \times D_{\lambda_3, \lambda_4}^{\frac{1}{2}}(\Omega_2) D_{\lambda'_3, \lambda_4}^{\frac{1}{2}*}(\Omega_2) |C_{\lambda_3}|^2, \quad (7)$$

where the Λ_c^+ spin density matrix, $\rho_{\lambda_1, \lambda'_1}$, is given by Eq. (2); the sum runs all helicity values. The full joint angular distribution is given in the appendix.

The transverse polarization of Λ_c^+ can be observed in its daughters' angular distribution, which reads

$$\mathcal{W}_2(\theta_0, \theta_1, \theta_1) \propto 1 + \alpha \cos^2(\theta_0) \\ + P_y(\alpha, \theta_0, \Delta) \alpha_{[\Lambda_c^+]}^{[\Lambda\pi^+]} \sin(\theta_1) \sin(\phi_1). \quad (8)$$

A simple way to measure the asymmetry parameter $\alpha_{[\Lambda_c^+]}^{[\Lambda\pi^+]}$ using the $\Lambda \rightarrow p\pi^+$ as a polarimeter, and the angular distribution of proton in the Λ helicity system reads [13,18]

$$\mathcal{W}_2(\theta_2) \propto 1 + \alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]} \alpha_\Lambda \cos \theta_2, \quad (9)$$

where α_Λ is the asymmetry parameter for the decay $\Lambda \rightarrow p\pi^-$.

These formulas are also applicable to the decays $\Lambda_c^+ \rightarrow \Sigma^+\pi^0$ and $\Sigma^+ \rightarrow p\pi^0$ with replacement of asymmetry parameters $\alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]}$ and α_Λ by $\alpha_{[\Sigma^+\pi^0]}^{[\Lambda_c^+]}$ and α_Σ , respectively.

E. $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+, \Sigma^0 \rightarrow \gamma \Lambda, \Lambda \rightarrow p \pi^-$

For decays $\Lambda_c^+ \rightarrow \Sigma^0(\lambda_3)\pi^+, \Sigma^0 \rightarrow \Lambda(\lambda_4)\gamma(\lambda_5)$, and $\Lambda \rightarrow p(\lambda_6)\pi^+$, their helicity angles are defined as $\Omega_1(\theta_1, \phi_1)$, $\Omega_2(\theta_2, \phi_2)$, and $\Omega_3(\theta_3, \phi_3)$ and amplitudes as B_{λ_3} , C_{λ_4, λ_5} , and F_{λ_6} , respectively. The joint angular distribution reads

$$\begin{aligned} \mathcal{W}_3(\theta_0, \Omega_1, \Omega_2, d\Omega_3) &\propto \sum_{\lambda_i} \rho_{\lambda_1, \lambda'_1} D_{\lambda_1, \lambda_3}^{\frac{1}{2}}(\Omega_1) D_{\lambda'_1, \lambda'_3}^{\frac{1}{2}*}(\Omega_1) B_{\lambda_3} B_{\lambda'_3}^* \\ &\times D_{\lambda_3, \lambda_4-\lambda_5}^{\frac{1}{2}}(\Omega_2) D_{\lambda'_3, \lambda'_4-\lambda'_5}^{\frac{1}{2}*}(\Omega_2) C_{\lambda_4, \lambda_5} C_{\lambda'_4, \lambda'_5}^* \\ &\times D_{\lambda_4, \lambda_6}^{\frac{1}{2}}(\Omega_3) D_{\lambda'_4, \lambda'_6}^{\frac{1}{2}*}(\Omega_3) F_{\lambda_6} F_{\lambda'_6}^*, \end{aligned} \quad (10)$$

where the Λ_c^+ spin density matrix, $\rho_{\lambda_1, \lambda'_1}$, is given by Eq. (2); the sum runs over all helicity values. The full amplitude is given in the appendix.

A simple way to measure the asymmetry parameter $\alpha_{\Sigma^0 \pi^+}^{\Lambda_c^+}$ is to look at the two-dimensional distribution $\cos \theta_2$ versus $\cos \theta_3$, which is obtained from Eq. (10) by integrating out the angles except for the θ_2 and θ_3 angles. Then one has [18]

$$\mathcal{W}_3(\theta_2, \theta_3) \propto 1 - \alpha_{\Sigma^0 \pi^+}^{\Lambda_c^+} \alpha_\Lambda \cos \theta_2 \cos \theta_3, \quad (11)$$

where α_Λ is the asymmetry parameter for $\Lambda \rightarrow p \pi^-$.

The angular distribution for $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$ is obtained by integrating out the angles Ω_2 and Ω_3 in Eq. (10); one has

$$\begin{aligned} \mathcal{W}_3(\theta_0, \theta_1, \phi_1) &\propto 1 + \alpha \cos^2(\theta_0) \\ &+ P_y(\alpha, \theta_0, \Delta) \sin(\theta_1) \sin(\phi_1) \alpha_{[\Sigma^0 \pi^+]}^{\Lambda_c^+}. \end{aligned} \quad (12)$$

F. $\Lambda_c^+ \rightarrow \Xi(1530)^0 K^+$

Helicity angles for decays $\Lambda_c^+(\lambda_1) \rightarrow \Xi(1530)^0(\lambda_3)K^+$, $\Xi(1530)^0 \rightarrow \Xi^-(\lambda_4)\pi^+$, $\Xi^- \rightarrow \Lambda(\lambda_5)\pi^-$, and $\Lambda \rightarrow p(\lambda_6)\pi^-$ are denoted by $\Omega_i(\theta_i, \phi_i)$ ($i = 1, 2, 3, 4$), and helicity amplitudes by B_{λ_3} , C_{λ_4} , F_{λ_5} , and G_{λ_6} , respectively. The joint angular distribution is calculated by

$$\begin{aligned} \mathcal{W}_4(\theta_0, \Omega_1, \Omega_2, \Omega_3, \Omega_4) &\propto \sum_{\lambda_i, \lambda'_i} \rho_{\lambda_1, \lambda'_1} D_{\lambda_1, \lambda_3}^{\frac{1}{2}}(\Omega_1) D_{\lambda'_1, \lambda'_3}^{\frac{1}{2}*}(\Omega_1) B_{\lambda_3} B_{\lambda'_3}^* \\ &\times D_{\lambda_3, \lambda_4}^{\frac{3}{2}}(\Omega_2) D_{\lambda'_3, \lambda'_4}^{\frac{3}{2}*}(\Omega_2) C_{\lambda_4} C_{\lambda'_4}^* \\ &\times D_{\lambda_4, \lambda_5}^{\frac{1}{2}}(\Omega_3) D_{\lambda'_4, \lambda'_5}^{\frac{1}{2}*}(\Omega_3) F_{\lambda_5} F_{\lambda'_5}^* \\ &\times D_{\lambda_5, \lambda_6}^{\frac{1}{2}}(\Omega_4) D_{\lambda'_5, \lambda'_6}^{\frac{1}{2}*}(\Omega_4) |G_{\lambda_6}|^2, \end{aligned} \quad (13)$$

where the Λ_c^+ spin density matrix, $\rho_{\lambda_1, \lambda'_1}$, is given by Eq. (2); the sum runs over all helicity values λ_i and λ'_i .

The joint angular distribution up to the Ξ^- decay is obtained from the above equation by integrating out the angles Ω_3 and Ω_4 ; one has

$$\begin{aligned} \mathcal{W}_4(\theta_0, \Omega_1, \Omega_2) &\propto [1 + \alpha \cos^2 \theta_0 \\ &+ P_y(\alpha, \theta_0, \Delta) \alpha_{[\Xi^0 K^+]}^{\Lambda_c^+} \sin(\theta_1) \sin(\phi_1)] \\ &\times [5 + 3 \cos(2\theta_2)]. \end{aligned} \quad (14)$$

III. RELATIONS TO LEE-YANG VARIABLES

To describe the parity violation in the hyperon (Y) decaying into a baryon (B) and a pseudoscalar meson (M), Lee-Yang variables [19], parameters $\alpha_{[BM]}^{[Y]}$, β , and γ are introduced to measure the asymmetry angular distribution of baryons, and defined with S and P waves, such as

$$\alpha_{[BM]}^{[Y]} = \frac{2\text{Re}(S^*P)}{|S|^2 + |P|^2}, \quad \beta = \frac{2\text{Im}(S^*P)}{|S|^2 + |P|^2}, \quad \gamma = \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2}, \quad (15)$$

and with equality $\alpha_{[BM]}^{[Y]2} + \beta^2 + \gamma^2 = 1$. We relate the partial waves to the helicity amplitudes $F_{\pm\frac{1}{2}}$ as

$$S = \frac{1}{\sqrt{2}} \left(F_{\frac{1}{2}} + F_{-\frac{1}{2}} \right), \quad P = \frac{1}{\sqrt{2}} \left(F_{\frac{1}{2}} - F_{-\frac{1}{2}} \right); \quad (16)$$

then Lee-Yang variables are defined as

$$\begin{aligned} \alpha_{[BM]}^{[Y]} &= \frac{|F_{\frac{1}{2}}|^2 - |F_{-\frac{1}{2}}|^2}{|F_{\frac{1}{2}}|^2 + |F_{-\frac{1}{2}}|^2}, \quad \beta = \sqrt{1 - \alpha_{[BM]}^{[Y]2}} \sin \Delta, \\ \gamma &= \sqrt{1 - \alpha_{[BM]}^{[Y]2}} \cos \Delta, \end{aligned} \quad (17)$$

where Δ is the phase angle difference in the two helicity amplitudes. In the calculation of joint angular distribution, we choose the normalization condition $|F_{\frac{1}{2}}|^2 + |F_{-\frac{1}{2}}|^2 = 1$.

In data analysis, two variables $\alpha_{[BM]}^{[Y]}$ and Δ are chosen as free parameters, and their values are determined by fitting data, while variables β and γ are calculated from measured $\alpha_{[BM]}^{[Y]}$ and Δ using the above relations.

For the antihyperon decay $\bar{Y} \rightarrow \bar{B}M$, the asymmetry variable has a similar definition as $\alpha_{[\bar{B}M]}^{[\bar{Y}]} = (|\bar{F}_{\frac{1}{2}}|^2 - |\bar{F}_{-\frac{1}{2}}|^2) / (|\bar{F}_{\frac{1}{2}}|^2 + |\bar{F}_{-\frac{1}{2}}|^2)$, if we assume it conserves the CP transformation, and one has $F_{\pm\frac{1}{2}} = \bar{F}_{\mp\frac{1}{2}}$. So one has $\alpha_{[\bar{B}M]}^{[\bar{Y}]} = -\alpha_{[BM]}^{[Y]}$, and $\bar{\Delta} = -\Delta$.

For the Λ_c^+ decays into final states of one hyperon and one pseudoscalar meson, the relation of asymmetry parameters to the Lee-Yang variables can be obtained by

generalizing the above equations with the replacement of charmed baryon parameters.

IV. SPIN OBSERVABLE

A. Λ_c^+ transverse polarization

As shown in Eq. (4), the spin observable P_y is nonzero if the factor $\sqrt{1 - \alpha^2 \sin^2(\Delta)} \neq 0$. The observable P_y can be seen in the angular distribution of Λ_c^+ decays. From the helicity amplitude analysis, the normalized distribution function for the $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$, and $\Lambda_c^+ \rightarrow YM$ decays can be written in the general form

$$\mathcal{W}_1(\theta_0, \Omega_1) = \frac{3}{8\pi(3 + \alpha)} [1 + \alpha \cos^2(\theta_0) + P_y(\alpha, \theta_0, \Delta) \alpha_{[YP]}^{[\Lambda_c^+]} \sin(\theta_1) \sin(\phi_1)], \quad (18)$$

where α is the angular distribution parameter of charmed baryon, $\alpha_{[YP]}^{[\Lambda_c^+]}$ is the asymmetry parameter for the charmed baryon decays into hyperon (Y) and pseudoscalar meson (M). Then the distribution of expectation value $\langle \sin(\theta_1) \sin(\phi_1) \rangle_{\mathcal{W}_1}$ reads

$$\langle \sin(\theta_1) \sin(\phi_1) \rangle_{\mathcal{W}_1} = \frac{\alpha_{[YP]}^{[\Lambda_c^+]}}{2(3 + \alpha)} P_y(\alpha, \theta_0, \Delta). \quad (19)$$

For the $\Lambda_c^+ \rightarrow pK_S^0$ decay, the spin observable is taken as Eq. (19) but with the replacement of asymmetry parameter $\alpha_{[YP]}^{[\Lambda_c^+]} \rightarrow \alpha_{[pK_S^0]}^{[\Lambda_c^+]}$. The above equation tells us that the P_y distribution can be observed by filling a histogram of $\cos \theta_0$ distribution versus a weight of $\sin(\theta_1) \sin(\phi_1)$ with data events.

B. $\Lambda_c^+ \rightarrow \Lambda \pi^+$

The Λ polarizations arising from the Λ_c^+ weak decay are denoted by P_x^Λ , P_y^Λ , and P_z^Λ ; then the Λ spin density matrix ρ^Λ is parametrized as

$$\rho^\Lambda = \frac{1}{2} \begin{pmatrix} P_0^\Lambda + P_z^\Lambda & P_x^\Lambda - iP_y^\Lambda \\ P_x^\Lambda + iP_y^\Lambda & P_0^\Lambda - P_z^\Lambda \end{pmatrix}. \quad (20)$$

The $\Lambda \rightarrow p\pi^-$ decay plays the role of polarization analyzer with the decay matrix

$$\mathcal{M}_\Lambda^\dagger \mathcal{M}_\Lambda = \frac{1}{2} \begin{pmatrix} 1 + \alpha_\Lambda \cos(\theta_2) & e^{i\phi_2} \sin(\theta_2) \alpha_\Lambda \\ e^{-i\phi_2} \sin(\theta_2) \alpha_\Lambda & 1 - \alpha_\Lambda \cos(\theta_2) \end{pmatrix}, \quad (21)$$

where (θ_2, ϕ_2) are the helicity angles for the decay $\Lambda \rightarrow p\pi^-$, which has the asymmetry parameter α_Λ .

To get the Λ polarization variable, we decompose the joint angular distribution Eq. (7) into the Λ spin density matrix ρ^Λ and decay matrix as

$$\begin{aligned} \mathcal{W}_2(\theta_0, \Omega_1, \Omega_2) &\propto \text{Tr}[\rho^\Lambda \mathcal{M}_\Lambda^\dagger \mathcal{M}_\Lambda] \\ &= P_0^\Lambda + P_z^\Lambda \cos(\theta_2) \alpha_\Lambda \\ &\quad + P_x^\Lambda \cos(\phi_2) \sin(\theta_2) \alpha_\Lambda \\ &\quad - P_y^\Lambda \sin(\theta_2) \sin(\phi_2) \alpha_\Lambda, \end{aligned} \quad (22)$$

where P_0^Λ is the unpolarized cross section. The vector $(P_x^\Lambda, P_y^\Lambda, P_z^\Lambda)$ corresponds to the spin observable for Λ production. To inverse the above equation, one has

$$\begin{aligned} P_0^\Lambda &= \frac{c_0}{4\pi} [1 + \alpha \cos^2(\theta_0)] \\ &\quad + \frac{1}{4\pi} P_y(\alpha, \theta_0, \Delta) \alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]} \sin(\theta_1) \sin(\phi_1), \end{aligned} \quad (23)$$

$$\begin{aligned} P_x^\Lambda &= \frac{1}{4\pi} P_y(\alpha, \theta_0, \Delta) \sqrt{1 - \alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]^2}} \\ &\quad \times [\cos(\theta_1) \sin(\phi_1) \cos(\Delta_2) + \cos(\phi_1) \sin(\Delta_2)], \end{aligned} \quad (24)$$

$$\begin{aligned} P_y^\Lambda &= \frac{1}{4\pi} P_y(\alpha, \theta_0, \Delta) \sqrt{1 - \alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]^2}} \\ &\quad \times [\cos(\phi_1) \cos(\Delta_2) - \cos(\theta_1) \sin(\phi_1) \sin(\Delta_2)], \end{aligned} \quad (25)$$

$$\begin{aligned} P_z^\Lambda &= \frac{1}{4\pi} \alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]} [1 + \alpha \cos^2(\theta_0)] \\ &\quad + \frac{1}{4\pi} P_y(\alpha, \theta_0, \Delta) \sin(\theta_1) \sin(\phi_1), \end{aligned} \quad (26)$$

where the normalization factor c_0 is given by Eq. (5).

Thus one can see that the P_z^Λ polarization originates from two sources; one is the Λ_c^+ weak decay to endow the Λ particle with a degree of $\alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]}$ polarization, and another one is inherited from the projection of Λ_c^+ transverse polarization. Effects of Λ_c^+ weak decay can be displayed with the $\Lambda \rightarrow p\pi^-$ decay by looking at the angular distribution [18]

$$\frac{dN}{d\cos\theta_2} \propto 1 + \alpha_\Lambda \alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]} \cos\theta_2.$$

To display the $P_y(\alpha, \theta_0, \Delta)$ contribution to P_z^Λ , one can look at the expectation value $\langle \sin(\theta_1) \sin(\phi_1) \cos(\theta_2) \rangle \propto \sin(2\theta_0)$ with data events. Similarly, to display the P_x^Λ and P_y^Λ polarizations, one can look at the expectation value distributions $\langle \cos(\phi_1) \sin(\theta_2) \cos(\phi_2) \rangle$ and $\langle \cos(\phi_1) \sin(\theta_2) \sin(\phi_2) \rangle$, respectively. They are all proportional to $P_y(\alpha, \theta_0, \Delta)$.

C. $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$

In the analysis of the decay $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$, the Σ^0 particle is reconstructed with $\Sigma^0 \rightarrow \Lambda(\lambda_4) \gamma(\lambda_5)$. In the Σ^0 helicity system, the Λ helicity is constrained to take the values parallel to the photon helicity, i.e., $(\lambda_4, \lambda_5) = (1/2, 1)$ or

$(-1/2, -1)$. Thus it is interesting to note that this decay plays the role of polarization filter, the transverse Λ polarization is canceled, and only the longitudinal Λ polarization survives, i.e., $P_x^\Lambda = P_y^\Lambda = 0$. Similar to the previous case, the Λ polarization originates two sources. To display the polarization from the Λ_c^+ weak decay, one has to look at the distribution [18],

$$\frac{dN}{d\cos\theta_2 d\cos\theta_3} \propto 1 - \alpha_\Lambda \alpha_{[\Sigma^0\pi^+]}^{[\Lambda_c^+]} \cos\theta_3. \quad (27)$$

To display the Λ_c^+ transverse polarization, one can look at the expectation values $\langle \sin\theta_1 \sin\phi_1 \rangle_{\mathcal{W}_3}$ and $\langle \sin\theta_1 \sin\phi_1 \cos\theta_2 \cos\theta_3 \rangle_{\mathcal{W}_3}$, and they both distribute proportionally to $P_y(\alpha, \theta_0, \Delta)$.

Another way to display the Λ_c^+ polarization effects is by looking at the $\langle \cos\theta_i \rangle$ distribution as

$$\frac{d\langle \cos\theta_i \rangle}{d\cos\theta_j} = -\frac{1}{6} \alpha_\Lambda \alpha_{[\Sigma^0\pi^+]}^{[\Lambda_c^+]} \cos\theta_j, \quad (28)$$

with $(i, j) = (2, 3)$ or $(3, 2)$.

D. $\Lambda_c^+ \rightarrow \Xi(1530)^0 K^+$

The helicity of the Ξ particle in this decay is allowed for taking $\pm 1/2$ values due to its production from the spin-1/2 particle. The $\Lambda \rightarrow p\pi^-$ decay in the last step can be used as a polarimeter to analyze the Λ polarization. Similarly to the $\Lambda_c^+ \rightarrow \Lambda\pi^+$ case, the spin observable for P_z^Λ is chosen as

$$\langle \cos(\theta_4) \rangle_{\mathcal{W}_4} \propto \alpha_\Lambda \alpha_\Xi [1 + \alpha \cos^2(\theta_0)], \quad (29)$$

where α_Ξ is the asymmetry parameter for the $\Xi^- \rightarrow \Lambda\pi^-$ decay.

The spin observable for P_x^Λ can be taken as

$$\begin{aligned} & \langle \sin(\theta_4) \cos(\phi_4) \rangle_{\mathcal{W}_4} \\ & \propto \alpha_\Lambda \sqrt{(1 - \alpha^2)(1 - \alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]})} (1 - \alpha_\Xi^2) \\ & \quad \times \sin(\Delta_1) \sin(\Delta_2 + \Delta_3 + \Delta_4) \sin(2\theta_0), \end{aligned} \quad (30)$$

where Δ_i ($i = 1, 2, 3, 4$) are the phase angle difference in the helicity amplitude of the first four decay steps, respectively.

The spin observable for P_y^Λ can be taken as

$$\begin{aligned} & \langle \sin(\theta_4) \sin(\phi_4) \rangle_{\mathcal{W}_4} \\ & \propto \alpha_\Lambda \sqrt{(1 - \alpha^2)(1 - \alpha_{[\Lambda\pi^+]}^{[\Lambda_c^+]})} (1 - \alpha_\Xi^2) \\ & \quad \times \sin(\Delta_1) \cos(\Delta_2 + \Delta_3 + \Delta_4) \sin(2\theta_0). \end{aligned} \quad (31)$$

V. TOY MC STUDY

We show how to estimate the statistical significance for observing the Λ_c^+ transverse polarization in the single tag events. The estimation is based on the statistics observed in the Λ_c^+ decays into the pK_S^0 , $\Lambda\pi^+$, and $\Sigma^0\pi^+$ modes, which correspond to the data set accumulated at the energy point $\sqrt{s} = 4.6$ GeV [1]. The asymmetry parameters are available only for the last two modes as given in Table II, and theoretical predictions for the pK_S^0 decay are quite uncertain, most with negative value in the range $[-1, -0.5]$; we take $\alpha_{[pK_S^0]}^{[\Lambda_c^+]} = -0.75$ in the MC simulation. We naively set the asymmetry parameter as $\alpha_{[BP]}^{[\Lambda_c^+]} = -0.70$.

Using the sampling of the accept-reject MC technique, we produce an ensemble of pseudo data sets at truth level incorporating the Λ_c^+ transverse polarization effects according to Eq. (18) by naively setting $\Delta = \pi/4$. An alternative sample is produced without transverse polarization by setting $\Delta = 0$. The angular distribution parameter is taken as $\alpha = -0.2$ [1]. Each data set contains the total 2741 events with each mode as given in Table II.

Estimation of statistical significance to observe the Λ_c^+ transverse polarization is done with a similar procedure as done in Ref. [20]. We define a likelihood ratio $t \equiv -2\ln(\mathcal{L}_o/\mathcal{L}_w)$ to discriminate hypotheses between with and without (w/o) the Λ_c^+ transverse polarization effects, here the likelihood values of $\mathcal{L}_o$ and $\mathcal{L}_w$ are calculated with the formula Eq. (18) with $\Delta = 0$ and $\pi/4$, respectively.

The t distribution for the $\Delta = \pi/4$ simulated experiments is shown in red points in Fig. 2, compared with the histogram for the $\Delta = 0$ simulated experiments. The t distributions imply that the Λ_c^+ transverse polarization is significantly discriminated from the alternative hypothesis. The significance of rejecting alternative hypothesis is approximately calculated by $\frac{\langle t_w \rangle - \langle t_o \rangle}{\sigma(t_o)}$; here $\langle t_o \rangle$ and $\sigma(t_o)$ denote the mean and root mean square of the $\Delta = 0$ simulated experiments, respectively, and the significance is determined to be 5.4σ . It is noted that the significance is strongly dependent on the $\alpha_{[BP]}^{[\Lambda_c^+]}$ value. If we take $\alpha_{[BP]}^{[\Lambda_c^+]} = -0.1$, the significance decreases to 0.4σ .

Figure 3 shows the expectation value $\langle \sin(\theta_1) \sin(\phi_1) \rangle$ distribution in terms of $\cos(\theta_0)$ for the $\Delta = \pi/4$ simulated sample, which contains 10,000 events. The distribution is

TABLE II. Branching fraction (Br), asymmetry parameters ($\alpha_{[BM]}^{[\Lambda_c^+]}$), and observed events (N^{obs}) for the Λ_c^+ decay into $\Lambda_c^+ \rightarrow pK_S^0$, $\Lambda\pi^+$ and $\Sigma^0\pi^+$

Decays	Br [9]	$\alpha_{[BM]}^{[\Lambda_c^+]}$	N^{obs} [1]
$\Lambda_c^+ \rightarrow pK_S^0$	$(1.58 \pm 0.08)\%$	-0.75	1243
$\Lambda_c^+ \rightarrow \Lambda\pi^+$	$(1.29 \pm 0.07)\%$	-0.91 ± 0.15 [9]	706
$\Lambda_c^+ \rightarrow \Sigma^0\pi^+$	$(1.28 \pm 0.07)\%$	-0.45 ± 0.32 [9]	522

FIG. 2. Distributions of the test statistic $t \equiv -2 \ln(\mathcal{L}_o/\mathcal{L}_w)$ for the $\Delta = \pi/4$ simulated experiments in red points, and the $\Delta = 0$ simulated experiments in the histogram.

FIG. 3. Distribution of the expectation value $\langle \sin(\theta_1) \sin(\phi_1) \rangle$ in terms of $\cos(\theta_0)$; the points with error bars are the $\Delta = \pi/4$ simulated experiments, and the curve is the predicted distribution given by Eq. (19).

well described with the predicted spin observables as given by Eq. (19). The unflat distribution implies the significant observation of the Λ_c^+ transverse polarization.

VI. SUMMARY AND DISCUSSION

We formulate the Λ_c^+ decays into $K_S^0 \pi^+$, $\Lambda \pi^+$, $\Sigma^+ \pi^0$, $\Sigma^0 \pi^+$, and $\Xi^0(1530) K^+$ for measuring their asymmetry decay parameters. Especially, we deal with the single tag events of the continuum process $e^+ e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$, where $\bar{\Lambda}_c^-$ candidates can be tagged with its exclusive decays in experiments. This method allows for the event selection to enjoy a high detection efficiency compared with the double tag of charmed baryon events. The results show that the asymmetry parameters can be observed in its subsequent decays into hyperons due to the fact that the weak decays of hyperons are spin self-analyzer.

Effects of possible transverse polarization are taken into consideration when formulating the charmed baryons produced from the $e^+ e^-$ annihilation. It is analogous to the case of hyperon pairs production, e.g., $\Lambda \bar{\Lambda}$ in J/ψ decays [16] and the continuum process [17]. Since the

mean transverse polarization is 0 when integrating over the full detector coverage space, it cannot be observed by looking at the distribution of angle between the transverse polarization vector and the Λ_c^+ daughter particle flying direction [13]. The spin observable of transverse polarization is given by Eq. (19), and it is manifested by looking at the first moment of $\sin \theta_1 \sin \phi_1$ for the charmed baryon subsequent decay as done in Ref. [16]; and its significance in the $e^+ e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$ at $\sqrt{s} = 4.6$ GeV is estimated with MC simulation. The results show that the significance is strongly dependent on the magnitude of the transverse polarization, and it is consistent with the expectation that the larger magnitude the charmed baryon acquired, the higher significance we achieved in a sample with limited statistics.

If the Λ_c^+ transverse polarization is significantly observed, it plays an important role in measuring the asymmetry parameters when both transverse and longitudinal polarization information is used in experiments. The asymmetry parameters, and the phase angle difference associated with the charmed baryon decays, together with the Λ_c^+ angular parameter α and the phase difference Δ parameters, can be extracted from a simultaneous fit to Λ_c^+ decays into $K_S^0 \pi^+$, $\Lambda \pi^+$, $\Sigma^+ \pi^0$, $\Sigma^0 \pi^+$, and $\Xi(1530)^0 \pi^+$ final states. A bad situation is that if the produced Λ_c^+ baryons are unpolarized, the asymmetry parameter for the $\Lambda_c^+ \rightarrow K_S^0 p$ decay cannot be measured in the signal tag events. In this case, the asymmetry parameter can only be measured in the double tag events using spin correlation of charmed baryons in the exclusive decays of $e^+ e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$ with $\Lambda_c^+ \rightarrow K_S^0 p$ and $\bar{\Lambda}_c^- \rightarrow K_S^0 \bar{p}$.

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APPENDIX A: $\Lambda_c^+ \rightarrow \Lambda \pi^+, \Lambda \rightarrow p \pi^+$

Helicity angles are defined as described in the text, and symbols are given in Table III.

TABLE III. Definition of helicity angles and parameters for each decay.

Decay	Helicity angle	Parameters
$e^+ e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$	(θ_0, ϕ_0)	Angular: α
$\Lambda_c^+ \rightarrow \Lambda \pi^+$	(θ_1, ϕ_1)	Asymmetry: α_{Λ_c}
$\Lambda \rightarrow p \pi^+$	(θ_2, ϕ_2)	Asymmetry: α_{Λ}

The joint angular distribution reads

$$\begin{aligned}
\frac{d\Gamma}{d\cos\theta_0 d\Omega_1 d\Omega_2} \propto & \sqrt{1-\alpha^2}\alpha_\Lambda \sin(\Delta_1) \sin(2\theta_0) \sin(\theta_1) \cos(\theta_2) \sin(\phi_1) \\
& + \sqrt{1-\alpha^2}\alpha_\Lambda \sin(\Delta_1) \sin(2\theta_0) \cos(\theta_1) \sin(\theta_2) \sin(\phi_1) \sqrt{1-\alpha_{\Lambda_c}^2} \cos(\Delta_2 + \phi_2) \\
& + \sqrt{1-\alpha^2}\alpha_\Lambda \sin(\Delta_1) \sin(2\theta_0) \sin(\theta_2) \cos(\phi_1) \sqrt{1-\alpha_{\Lambda_c}^2} \sin(\Delta_2 + \phi_2) \\
& + \sqrt{1-\alpha^2} \sin(\Delta_1) \sin(2\theta_0) \sin(\theta_1) \sin(\phi_1) \alpha_{\Lambda_c} + 2\alpha\alpha_\Lambda \cos^2(\theta_0) \cos(\theta_2) \alpha_{\Lambda_c} + 2\alpha_\Lambda \cos(\theta_2) \alpha_{\Lambda_c} \\
& + 2\alpha \cos^2(\theta_0) + 2,
\end{aligned} \tag{A1}$$

where Δ_1 and Δ_2 are the phase angle differences between the two independent helicity amplitudes for the $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$ and $\Lambda_c^+ \rightarrow \Lambda \pi^+$, respectively.

APPENDIX B: $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+, \Sigma^0 \rightarrow \gamma \Lambda, \Lambda \rightarrow p \pi^+$

Helicity angles are defined in the same way as described in text, and shown in Table IV.

The joint angular distribution reads

$$\begin{aligned}
\frac{d\Gamma}{d\cos(\theta_0) d\Omega_1 d\Omega_2 d\Omega_3} \propto & -\sqrt{1-\alpha^2}\alpha_\Lambda \sin(\Delta_1) \sin(2\theta_0) \sin(\theta_1) \cos(\theta_2) \cos(\theta_3) \sin(\phi_1) + 2\alpha \cos^2(\theta_0) \\
& - \sqrt{1-\alpha^2}\alpha_\Lambda \sin(\Delta_1) \sin(2\theta_0) \cos(\theta_1) \sin(\theta_2) \cos(\theta_3) \sin(\phi_1) \sqrt{1-\alpha_{\Lambda_c}^2} \cos(\Delta_2 + \phi_2) \\
& + \sqrt{1-\alpha^2} \sin(\Delta_1) \sin(2\theta_0) \sin(\theta_1) \sin(\phi_1) \alpha_{\Lambda_c} \\
& - \sqrt{1-\alpha^2}\alpha_\Lambda \sin(\Delta_1) \sin(2\theta_0) \sin(\theta_2) \cos(\theta_3) \cos(\phi_1) \sqrt{1-\alpha_{\Lambda_c}^2} \sin(\Delta_2 + \phi_2) \\
& - 2\alpha\alpha_\Lambda \cos^2(\theta_0) \cos(\theta_2) \cos(\theta_3) \alpha_{\Lambda_c} - 2\alpha_\Lambda \cos(\theta_2) \cos(\theta_3) \alpha_{\Lambda_c} + 2,
\end{aligned} \tag{B1}$$

where Δ_1 and Δ_2 are the phase angle differences between the two independent helicity amplitudes for the $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$ and $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$, respectively.

TABLE IV. Definition of helicity angles and parameters for each decay.

Decay	Helicity angles	Parameters
$e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$	(θ_0, ϕ_0)	Angular: α
$\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$	$\Omega_1(\phi_1, \theta_1)$	Asymmetry: α_{Λ_c}
$\Sigma^0 \rightarrow \Lambda \gamma$	$\Omega_2(\phi_2, \theta_2)$	...
$\Lambda \rightarrow p \pi^+$	$\Omega_3(\phi_3, \theta_3)$	Asymmetry: α_Λ

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