Charmed baryon weak decays with decuplet baryon and $SU(3)$ flavor symmetry

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We study the branching ratios and up-down asymmetries in the charmed baryon weak decays of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ with $\mathbf{B}_{c(D)}$ antitriplet charmed (decuplet) baryon and M pseudoscalar meson states based on the flavor symmetry of $SU(3)_F$. We propose equal and physical-mass schemes for the hadronic states to deal with the large variations of the decuplet baryon momenta in the decays in order to fit with the current experimental data. We find that our fitting results of $\mathcal{B}(\mathbf{B}_c \rightarrow \mathbf{B}_D M)$ are consistent with the current experimental data in both schemes, while the up-down asymmetries in all decays are found to be sizable, consistent with the current experimental data, but different from 0 predicted in the literature. We also examine the processes of $\Xi_c^0 \rightarrow \Sigma^0 K_S/K_L$ and derive the asymmetry between the K_L/K_S modes being a constant.

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I. INTRODUCTION

There has been much interesting progress in the study of charmed baryon weak decays due to the recent measurement of the absolute branching fraction for the golden mode $\Lambda_c^+ \rightarrow p K^- \pi^+$ by the Belle Collaboration [1] with the new world average of $\mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+) = (6.23 \pm 0.33)\%$ [2] as well as other Λ_c measurements by the BESIII Collaboration [3–13]. In addition, the absolute branching ratio of $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+) = (1.80 \pm 0.52)\%$ is given by the BELLE collaboration for the first time [14]. Theoretically, the charmed baryon decays are dominated by the nonfactorizable effects, such as those associated with the nonvanishing measured branching ratios for the Cabibbo allowed decays of $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$ and $\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$ [2], which do not receive any factorizable contributions. Many efforts have been made to understand the nonfactorizable effects with different dynamical QCD models [15–23] as well as the use of the flavor symmetry of $SU(3)_F$ [24–38], which is believed to be the most reliable and simple way to examine the charmed baryon processes. In particular, it has been recently demonstrated that the results for the charmed baryon decays based on the $SU(3)_F$ approach [26–33, 35–38] are consistent with the experimental data.

However, most of the recent experimental and theoretical activities have been concentrated on the charmed baryon decays with the octet baryon in the final states, whereas there have been few studies for the decuplet modes. Note that most of the charmed baryon experiments with the decuplet baryon were all done before the millennium. In this work, we examine the two-body weak decays of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$, where $\mathbf{B}_{c(D)}$ and M represent the antitriplet charmed (decuplet) baryon and octet pseudoscalar meson states based on $SU(3)_F$. There are two important features for the decays of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$. The first one is that all factorizable amplitudes vanish, resulting in theoretically clean predictions for the nonfactorizable contributions of the decays. The other one is that the decays involve only a few $SU(3)_F$ parameters, which are able to be determined by the current experimental data.

On the other hand, the up-down asymmetries of α in $\Lambda_c^+ \rightarrow \Xi^0 K^+$ and $\Lambda_c^+ \rightarrow \Xi'^0 K^+$ have also been given recently by the BESIII Collaboration with the results of $\alpha(\Lambda_c^+ \rightarrow \Xi^0 K^+, \Xi'^0 K^+) = (0.77 \pm 0.78, -1.00 \pm 0.34)$ [11], respectively, where Ξ'^0 belongs to the decuplet baryon state with spin 3/2. Although the former experimental result is still consistent with 0, the latter one is clearly sizable. This nonvanishing large asymmetry is different from the prediction in most theoretical calculations in the literature [15–23, 34]. Recently, based on the flavor symmetry of $SU(3)_F$, we show that $\alpha(\Lambda_c^+ \rightarrow \Xi^0 K^+) = (0.94^{+0.06}_{-0.11})$ [38], which is consistent with the data, but much larger than 0. In this work, we in particular check the

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up-down asymmetry in $\Lambda_c^+ \rightarrow \Xi'^0 K^+$ to see if it agrees with the experimental nonzero result in the $SU(3)_F$ approach.

This paper is organized as follows. In Sec. II, we present the formalism. We show how the decay amplitudes are related based on $SU(3)_F$. In Sec. III, we provide the numerical results of the decay branching ratios and up-down asymmetries in $\mathbf{B}_c \rightarrow \mathbf{B}_D M$. We conclude our study in Sec. IV.

II. FORMALISM

In order to investigate the two-body decays of the antitriplet charmed baryon ($\mathbf{B}_c$) to decuplet baryon ($\mathbf{B}_D$) and octet pseudoscalar meson (M) states, we write their representations under the flavor symmetry of $SU(3)_F$ as

$$\begin{aligned} \mathbf{B}_c &= (\Xi_c^0, -\Xi_c^+, \Lambda_c^+), \\ \mathbf{B}_D &= \frac{1}{\sqrt{3}} \left(\begin{pmatrix} \sqrt{3}\Delta^{++} & \Delta^+ & \Sigma'^+ \\ \Delta^+ & \Delta^0 & \frac{\Sigma^0}{\sqrt{2}} \\ \Sigma'^+ & \frac{\Sigma^0}{\sqrt{2}} & \Xi'^0 \end{pmatrix}, \begin{pmatrix} \Delta^+ & \Delta^0 & \frac{\Sigma^0}{\sqrt{2}} \\ \Delta^0 & \sqrt{3}\Delta^- & \Sigma'^- \\ \frac{\Sigma^0}{\sqrt{2}} & \Sigma'^- & \Xi'^- \end{pmatrix}, \begin{pmatrix} \Sigma'^+ & \frac{\Sigma^0}{\sqrt{2}} & \Xi'^0 \\ \frac{\Sigma^0}{\sqrt{2}} & \Sigma'^- & \Xi'^- \\ \Xi'^0 & \Xi'^- & \sqrt{3}\Omega^- \end{pmatrix} \right), \\ M &= \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \end{aligned} \quad (1)$$

Here, we have assumed that the physical meson η is solely made of the octet state [2]. The effective Hamiltonian associated with $c \rightarrow u\bar{d}s$, $c \rightarrow u\bar{q}q$ ($q = d, s$), and $c \rightarrow u\bar{s}d$ transitions is given by [27]

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= \sum_{i=+,-} \frac{G_F}{\sqrt{2}} c_i (V_{cs}^* V_{ud} O_i^{ds} + V_{cd}^* V_{ud} O_i^{qq} + V_{cd}^* V_{us} O_i^{sd}), \\ O_{\pm}^{q_1 q_2} &= \frac{1}{2} [(\bar{u}q_1)_{V-A} (\bar{q}_2 c)_{V-A} \pm (\bar{q}_2 q_1)_{V-A} (\bar{u}c)_{V-A}], \\ O_{\pm}^{qq} &\equiv O_{\pm}^{dd} - O_{\pm}^{ss}, \end{aligned} \quad (2)$$

where $(|V_{cs}^* V_{ud}|, |V_{cd}^* V_{ud}|, |V_{cd}^* V_{us}|) \simeq (1, s_c, s_c^2)$ with $s_c \equiv \sin \theta_c \approx 0.225$ [2], c_i ($i = +, -$) correspond to the Wilson coefficients, G_F is the Fermi constant, and $O_{\pm}^{q_1 q_2}$ with $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$ represent the four-quark operators.

As $O_{\pm}$ belong to $\bar{15}$ and 6 representations under $SU(3)_F$, respectively, which are symmetric and antisymmetric in flavor and color indices, we can decompose the effective Hamiltonian in the tensor forms of $H(\bar{15})$ and $H(6)$, given by

$$\begin{aligned} H(\bar{15})_{ij}^{kl} &= \left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & s_c & 1 \\ s_c & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -s_c^2 & -s_c \\ -s_c^2 & 0 & 0 \\ -s_c & 0 & 0 \end{pmatrix} \right), \\ H(6)_{ij} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & -2s_c \\ 0 & -2s_c & 2s_c^2 \end{pmatrix}, \end{aligned} \quad (3)$$

respectively, where we have used the convention of $V_{cd} = -V_{us} = s_c$.¹

The most significant feature for $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ is that the decay amplitude is essentially nonfactorizable due to the vanishing matrix element of the baryonic transition, i.e.,

$\langle \mathbf{B}_D | \bar{q} \gamma_\mu (1 - \gamma_5) c | \mathbf{B}_c \rangle = 0$. The reason is that the light quark pair in the antitriplet charmed baryon state is antisymmetric, whereas that in the decuplet one is totally symmetric. As a result, we can safely neglect $H(\bar{15})$, which only contributes to the factorizable processes [15, 39–42]. In general, the decay amplitude of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ is given by

$$\mathbf{M} = \langle \mathbf{B}_D M | H(6) | \mathbf{B}_c \rangle = i q_\mu \bar{w}_{\mathbf{B}_D}^\mu (P - D \gamma_5) u_{\mathbf{B}_c}, \quad (4)$$

¹Note that there is a sign difference between our convention and the one in Ref. [2], which does not affect our numerical results.

where q_μ is the four-momentum of the meson, $w_{\mathbf{B}_D}^\mu$ is the Rarita-Schwinger spinor vector for the spin-3/2 particle of $\mathbf{B}_D$, $P(D)$ corresponds to the $P(D)$ -wave amplitude, and $u_{\mathbf{B}_c}$ is the spin-1/2 Dirac spinor of $\mathbf{B}_c$. By assuming CP invariance, P and D can be taken to be real. Under $SU(3)_F$, the amplitudes associated with P and D are related by

$$P(\mathbf{B}_c \rightarrow \mathbf{B}_D M) = P_0 f_{\mathbf{B}_c \mathbf{B}_D M}, \quad D(\mathbf{B}_c \rightarrow \mathbf{B}_D M) = D_0 f_{\mathbf{B}_c \mathbf{B}_D M}, \quad (5)$$

respectively, where $P_0(D_0)$ is the real parameter to be determined and $f_{\mathbf{B}_c \mathbf{B}_D M}$ is the $SU(3)_F$ overlapping factor, defined by

$$f_{\mathbf{B}_c \mathbf{B}_D M} = (\mathbf{B}_D)_{ijk} (\mathbf{B}_c)_l H(6)_{om} M_q^i \epsilon^{jlo} \epsilon^{kmq}. \quad (6)$$

The value of $f_{\mathbf{B}_c \mathbf{B}_D M}$ in Eq. (6) depends on the specific mode in $\mathbf{B}_c \rightarrow \mathbf{B}_D M$, e.g.,

$$f_{\Lambda_c^+ \Delta^{++} K^-} = (\Delta^{++})_{111} (\Lambda_c^+)_{33} H(6)_{22} (K^-)_3 \epsilon^{132} \epsilon^{123} = -2. \quad (7)$$

We list the values of $f_{\mathbf{B}_c \mathbf{B}_D M}$ in the next section. We note that the factors of $f_{\mathbf{B}_c \mathbf{B}_D M}$ with M being a singlet under $SU(3)_F$ vanish so that the corresponding decays with a physical meson η' are suppressed. The reason is that $\bar{\mathbf{3}} \otimes \bar{\mathbf{6}} \otimes \mathbf{10} \otimes \mathbf{1}$ cannot form a singlet to be invariant under $SU(3)_F$, where $\bar{\mathbf{3}}$, $\bar{\mathbf{6}}$, $\mathbf{10}$ and $\mathbf{1}$ are the $SU(3)_F$ representations for the antitriplet charmed baryon, antisymmetric part of the effective Hamiltonian, decuplet baryon, and singlet meson states, respectively. In practice, since P and D share the same overlapping factor, one can alternatively combine these two real parameters into one complex parameter for convenience [26–33,35–38].

Consequently, the decay width (Γ) for $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ is given by

$$\Gamma(\mathbf{B}_c \rightarrow \mathbf{B}_D M) = \frac{|\vec{q}|}{8\pi m_{\mathbf{B}_c}^2} |\langle \overline{\mathbf{M}^2} \rangle| = \zeta (1 + \xi^2 r^2) P_0^2 f_{\mathbf{B}_c \mathbf{B}_D M}^2, \quad (8)$$

while the up-down asymmetry (α) has the form

$$\alpha(\mathbf{B}_c \rightarrow \mathbf{B}_D M) = \frac{2\xi \text{Re}(PD^*)}{|P|^2 + \xi^2 |D|^2} = \frac{2\xi r}{1 + \xi^2 r^2}, \quad (9)$$

where $|\vec{q}|$ represents the absolute value of the three-momentum of the octet pseudoscalar meson M or the decuplet baryon $\mathbf{B}_D$ in the center-mass frame, $m_{\mathbf{B}_c}$ is the mass of the charmed baryon $\mathbf{B}_c$, $|\langle \overline{\mathbf{M}^2} \rangle|$ stands for the spin average squared amplitude, $\zeta = (E_{\mathbf{B}_D} + m_{\mathbf{B}_D}) |\vec{q}|^3 m_{\mathbf{B}_c} / (6\pi m_{\mathbf{B}_D}^2)$

with $E_{\mathbf{B}_D}(m_{\mathbf{B}_D})$ representing energy (mass) of $\mathbf{B}_D$, $\xi = |\vec{q}|/(E_{\mathbf{B}_D} + m_{\mathbf{B}_D})$, and $r = D_0/P_0$.

Under the exact flavor symmetry of $SU(3)_F$, one can simply impose the equal-mass (em) conditions, given by

$$\begin{aligned} m_{\mathbf{B}_c} &\equiv m_{\Lambda_c^+} = m_{\Xi_c^0} = m_{\Xi_c^+}, \\ m_{\mathbf{B}_D} &\equiv m_{\Delta^{++}} = m_{\Delta^+} = m_{\Delta^0} = m_{\Delta^-} = m_{\Sigma^{*0}} = m_{\Sigma^{*+}} \\ &= m_{\Sigma^{*-}} = m_{\Xi^{*0}} = m_{\Xi^{*-}} = m_{\Omega^-}, \\ m_M &\equiv m_{\pi^+} = m_{\pi^0} = m_{\pi^-} = m_{K^+} = m_{K^-} = m_{K^0} \\ &= m_{\bar{K}^0} = m_\eta, \end{aligned} \quad (10)$$

leading to that both ξ and ζ are the same for all decays of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$. As a result, $\Gamma(\mathbf{B}_c \rightarrow \mathbf{B}_D M)/f_{\mathbf{B}_c \mathbf{B}_D M}^2$ and $\alpha(\mathbf{B}_c \rightarrow \mathbf{B}_D M)$ are the same for all modes of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ when the em conditions are chosen. This em scheme has been widely used in the charmed baryon decays with the octet baryon in the final states based on $SU(3)_F$ as shown in Refs. [26–33,35–38]. However, it is clear that both parameters of ξ and ζ for the decuplet modes are quite different since the three-momentum $\vec{q}$ varies largely in different decays around 0.4–0.8 GeV when the physical masses of the baryon and meson states are taken, referred to as the physical-mass (pm) scheme.

III. NUMERICAL RESULTS

In the em scheme, from Eq. (9) we see that there is only one combined parameter of $\bar{r} \equiv \xi r$ for $\alpha(\mathbf{B}_c \rightarrow \mathbf{B}_D M)$. By using the experimental data of $\alpha(\Lambda_c^+ \rightarrow \Xi^{*0} K^+) = -1.00 \pm 0.34$ in Ref. [11], we expect that the up-down asymmetry in every decay mode of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ should have the same value as

$$\alpha_{em}(\mathbf{B}_c \rightarrow \mathbf{B}_D M) = -1.00_{-0}^{+0.34}, \quad (11)$$

where the lower uncertainty of “0” reflects that the physical value of α cannot be less than -1 . From Eqs. (9) and (11), we obtain

$$\bar{r}_{em} = -1.00_{-1.6}^{+0.6}. \quad (12)$$

On the other hand, the decay branching ratios in Eq. (8) also depend on one unknown parameter, defined by

$$\bar{P}_0 \equiv \sqrt{\zeta(1 + \xi^2 r^2)} P_0, \quad (13)$$

which can be determined by only one experimental data point. However, there are four experimental branching ratios as shown in Table I. To obtain the most plausible value for $\bar{P}_0$ under the current experimental data, we adopt the minimal χ^2 fitting, defined by

TABLE I. Our results of the up-down asymmetries of α_{pm} and branching ratios of $\mathcal{B}_{pm}$ and $\mathcal{B}_{em}$ for the Cabibbo allowed modes of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ based on $SU(3)_F$ along with the experimental data [2,11,13,14] as well as the theoretical calculations in the literature [15,16,34].

Channel	$f_{\mathbf{B}_c \mathbf{B}_D M}$	α_{pm}	$10^3 \mathcal{B}_{pm}$	$10^3 \mathcal{B}_{em}$	$10^3 \mathcal{B}$	$10^3 \mathcal{B}$	$10^3 \mathcal{B}$	$10^3 \mathcal{B}_{ex}(\alpha_{ex})$
			our result		[15]	[16]	[34]	data
$\Lambda_c^+ \rightarrow \Delta^{++} K^-$	-2	$-0.86^{+0.44}_{-0.14}$	15.3 ± 2.4	12.4 ± 1.0	9.5	27.0	7.0 ± 4.0	10.7 ± 2.5 [2]
$\Lambda_c^+ \rightarrow \Delta^+ \bar{K}^0$	$-\frac{2\sqrt{3}}{3}$	$-0.86^{+0.44}_{-0.14}$	5.1 ± 0.8	4.1 ± 0.3	3.1	9.0	2.3 ± 1.3	
$\Lambda_c^+ \rightarrow \Sigma'^+ \pi^0$	$\frac{\sqrt{6}}{3}$	$-0.91^{+0.45}_{-0.10}$	2.2 ± 0.4	2.1 ± 0.2	2.1	5.0	4.6 ± 1.8	
$\Lambda_c^+ \rightarrow \Sigma'^+ \eta$	$\sqrt{2}$	$-0.97^{+0.43}_{-0.03}$	3.1 ± 0.6	6.2 ± 0.5	...	10.4	2.1 ± 1.1	9.1 ± 2.0 [13]
$\Lambda_c^+ \rightarrow \Sigma'^0 \pi^+$	$\frac{\sqrt{6}}{3}$	$-0.90^{+0.45}_{-0.10}$	2.2 ± 0.4	2.1 ± 0.2	2.1	5.0	4.6 ± 1.8	
$\Lambda_c^+ \rightarrow \Xi'^0 K^+$	$\frac{2\sqrt{3}}{3}$	$-1.00^{+0.34}_{-0.00}$	1.0 ± 0.2	4.1 ± 0.3	0.7	5.0	2.3 ± 0.9	5.02 ± 1.04 [11] (-1.00 ± 0.34) [11]
$\Xi_c^0 \rightarrow \Sigma'^+ K^-$	$\frac{2\sqrt{3}}{3}$	$-0.88^{+0.45}_{-0.12}$	3.1 ± 0.5	2.3 ± 0.2	1.3	4.9	1.3 ± 0.7	
$\Xi_c^0 \rightarrow \Sigma'^0 \bar{K}^0$	$\frac{\sqrt{6}}{3}$	$-0.88^{+0.45}_{-0.12}$	1.6 ± 0.2	1.2 ± 0.1	0.7	2.6	0.6 ± 0.4	
$\Xi_c^0 \rightarrow \Xi'^0 \pi^0$	$-\frac{\sqrt{6}}{3}$	$-0.91^{+0.45}_{-0.09}$	1.4 ± 0.2	1.2 ± 0.1	0.9	2.8	2.6 ± 1.0	
$\Xi_c^0 \rightarrow \Xi'^0 \eta$	$-\sqrt{2}$	$-0.97^{+0.42}_{-0.03}$	2.1 ± 0.4	3.5 ± 0.3	...	0.2	1.3 ± 0.6	
$\Xi_c^0 \rightarrow \Xi'^- \pi^+$	$-\frac{2\sqrt{3}}{3}$	$-0.91^{+0.45}_{-0.09}$	2.8 ± 0.5	2.3 ± 0.2	1.9	5.6	5.0 ± 2.0	
$\Xi_c^0 \rightarrow \Omega^- K^+$	-2	$-1.00^{+0.34}_{-0.00}$	2.3 ± 0.5	7.0 ± 0.6	1.1	3.4	4.5 ± 1.8	5.4 ± 1.6 [2,14]
$\Xi_c^+ \rightarrow \Sigma'^+ \bar{K}^0$	0	...	0	0	0	0	0	1.0 ± 0.5 ^{a,b} [2]
$\Xi_c^+ \rightarrow \Xi'^0 \pi^+$	0	...	0	0	0	0	0	< 0.10 ^b [2]

^aThe data have not been included in the data fitting.

^bThe experimental decay branching ratios of Ξ_c^+ are measured relative to $\mathcal{B}(\Xi_c^+ \rightarrow \Xi^- 2\pi^+)$.

$$\chi_{em}^2 = \sum_{ex} \frac{(\mathcal{B}_{ex} - \mathcal{B}_{em})^2}{\sigma_{ex}^2}, \quad (14)$$

where $\mathcal{B}_{em}$ is the decay branching ratio generated by $\bar{P}_0$ in the em scheme with the experimental measured lifetime in Ref. [2] and $\mathcal{B}_{ex}(\sigma_{ex})$ corresponds to the measured branching ratio (uncertainty) in the data. By performing χ^2 fit with the minimal value of χ_{em}^2 , we obtain that

$$(\bar{P}_0)_{em} = (8.7 \pm 0.5) \times 10^{-3} G_F \text{GeV}^{\frac{5}{2}},$$

$$\chi_{em}^2/\text{d.o.f.} = 1.4, \quad (15)$$

where d.o.f. represents “degree of freedom.” The small value of $\chi_{em}^2/\text{d.o.f.}$ for the fit in Eq. (15) indicates that the em scheme is good to explain the current experimental data. Our results for the decay branching ratios of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ in the em scheme are summarized in Tables I–IV. In the tables, we also show the $SU(3)_F$ overlapping factors of $f_{\mathbf{B}_c \mathbf{B}_D M}$ for the decays of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$.

We now discuss the pm scheme. From the data of $\alpha_{ex}(\Lambda_c^+ \rightarrow \Xi'^0 K^+)$, we find that

$$r_{pm} = -6.6^{+4.1}_{-10.7}. \quad (16)$$

With the value in Eq. (16), our predictions for $\alpha_{pm}(\mathbf{B}_c \rightarrow \mathbf{B}_D M)$ are shown in Tables I–IV. To valuate the decay branching ratios, we have to refit the data, found to be

TABLE II. Results for the Cabibbo suppressed decays of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ with $SU(3)_F$.

Channel	$s_c^{-1} f_{\mathbf{B}_c \mathbf{B}_D M}$	α_{pm}	$10^4 \mathcal{B}_{pm}$	$10^4 \mathcal{B}_{em}$
$\Lambda_c^+ \rightarrow \Delta^{++} \pi^-$	-2	$-0.81^{+0.43}_{-0.19}$	12.5 ± 2.0	6.6 ± 0.6
$\Lambda_c^+ \rightarrow \Delta^+ \pi^0$	$\frac{2\sqrt{6}}{3}$	$-0.81^{+0.43}_{-0.19}$	8.3 ± 1.3	4.4 ± 0.4
$\Lambda_c^+ \rightarrow \Delta^0 \pi^+$	$\frac{2\sqrt{3}}{3}$	$-0.81^{+0.43}_{-0.19}$	4.2 ± 0.7	2.2 ± 0.2
$\Lambda_c^+ \rightarrow \Sigma'^+ K^0$	$-\frac{2\sqrt{3}}{3}$	$-0.95^{+0.44}_{-0.05}$	1.3 ± 0.2	2.2 ± 0.2
$\Lambda_c^+ \rightarrow \Sigma'^0 K^+$	$\frac{\sqrt{6}}{3}$	$-0.95^{+0.44}_{-0.05}$	0.7 ± 0.1	1.1 ± 0.1
$\Xi_c^0 \rightarrow \Delta^+ K^-$	$\frac{2\sqrt{3}}{3}$	$-0.79^{+0.43}_{-0.21}$	3.0 ± 0.5	1.2 ± 0.1
$\Xi_c^0 \rightarrow \Delta^0 \bar{K}^0$	$\frac{2\sqrt{3}}{3}$	$-0.79^{+0.43}_{-0.21}$	3.0 ± 0.5	1.2 ± 0.1
$\Xi_c^0 \rightarrow \Sigma'^+ \pi^-$	$\frac{2\sqrt{3}}{3}$	$-0.84^{+0.44}_{-0.16}$	2.5 ± 0.4	1.2 ± 0.1
$\Xi_c^0 \rightarrow \Sigma'^0 \pi^0$	$-\sqrt{3}$	$-0.84^{+0.44}_{-0.16}$	5.6 ± 0.9	2.8 ± 0.2
$\Xi_c^0 \rightarrow \Sigma'^0 \eta$	-1	$-0.89^{+0.45}_{-0.11}$	1.1 ± 0.2	0.9 ± 0.1
$\Xi_c^0 \rightarrow \Sigma'^- \pi^+$	$-\frac{4\sqrt{3}}{3}$	$-0.84^{+0.44}_{-0.16}$	9.9 ± 1.6	4.9 ± 0.4
$\Xi_c^0 \rightarrow \Xi'^0 K^0$	$\frac{2\sqrt{3}}{3}$	$-0.96^{+0.43}_{-0.04}$	0.9 ± 0.2	1.2 ± 0.1
$\Xi_c^0 \rightarrow \Xi'^- K^+$	$-\frac{4\sqrt{3}}{3}$	$-0.96^{+0.43}_{-0.04}$	3.6 ± 0.6	4.9 ± 0.4
$\Xi_c^+ \rightarrow \Delta^{++} K^-$	2	$-0.79^{+0.43}_{-0.21}$	35.0 ± 5.7	14.6 ± 1.2
$\Xi_c^+ \rightarrow \Delta^+ \bar{K}^0$	$\frac{2\sqrt{3}}{3}$	$-0.79^{+0.43}_{-0.21}$	11.7 ± 1.9	4.9 ± 0.4
$\Xi_c^+ \rightarrow \Sigma'^+ \pi^0$	$-\frac{\sqrt{6}}{3}$	$-0.84^{+0.44}_{-0.16}$	4.8 ± 0.8	2.4 ± 0.2
$\Xi_c^+ \rightarrow \Sigma'^+ \eta$	$-\sqrt{2}$	$-0.89^{+0.45}_{-0.11}$	8.7 ± 1.4	7.3 ± 0.6
$\Xi_c^+ \rightarrow \Sigma'^0 \pi^+$	$-\frac{\sqrt{6}}{3}$	$-0.84^{+0.44}_{-0.16}$	4.8 ± 0.8	2.4 ± 0.2
$\Xi_c^+ \rightarrow \Xi'^0 K^+$	$-\frac{2\sqrt{3}}{3}$	$-0.96^{+0.43}_{-0.04}$	3.5 ± 0.6	4.9 ± 0.4

TABLE III. Results for the double Cabibbo suppressed decays of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ with $SU(3)_F$.

Channel	$s_c^{-2} f_{\mathbf{B}_c \mathbf{B}_D M}$	α_{pm}	$10^5 \mathcal{B}_{pm}$	$10^5 \mathcal{B}_{em}$
$\Xi_c^0 \rightarrow \Delta^+ \pi^-$	$\frac{2\sqrt{3}}{3}$	$-0.75^{+0.42}_{-0.25}$	2.2 ± 0.4	0.7 ± 0.1
$\Xi_c^0 \rightarrow \Delta^0 \pi^0$	$-\frac{2\sqrt{6}}{3}$	$-0.75^{+0.42}_{-0.25}$	4.3 ± 0.7	1.3 ± 0.1
$\Xi_c^0 \rightarrow \Delta^- \pi^+$	-2	$-0.75^{+0.42}_{-0.25}$	6.5 ± 1.1	2.0 ± 0.2
$\Xi_c^0 \rightarrow \Sigma'^0 K^0$	$\frac{\sqrt{6}}{3}$	$-0.88^{+0.45}_{-0.12}$	0.4 ± 0.1	0.3 ± 0.0
$\Xi_c^0 \rightarrow \Sigma'^- K^+$	$-\frac{2\sqrt{3}}{3}$	$-0.88^{+0.45}_{-0.12}$	0.9 ± 0.1	0.7 ± 0.1
$\Xi_c^+ \rightarrow \Delta^{++} \pi^-$	2	$-0.76^{+0.42}_{-0.24}$	25.5 ± 4.4	7.8 ± 0.7
$\Xi_c^+ \rightarrow \Delta^+ \pi^0$	$-\frac{2\sqrt{6}}{3}$	$-0.76^{+0.42}_{-0.24}$	17.0 ± 2.9	5.2 ± 0.4
$\Xi_c^+ \rightarrow \Delta^0 \pi^+$	$-\frac{2\sqrt{3}}{3}$	$-0.76^{+0.42}_{-0.24}$	8.5 ± 1.5	2.6 ± 0.2
$\Xi_c^+ \rightarrow \Sigma'^+ K^0$	$\frac{2\sqrt{3}}{3}$	$-0.88^{+0.45}_{-0.12}$	3.5 ± 0.6	2.6 ± 0.2
$\Xi_c^+ \rightarrow \Sigma'^0 K^+$	$-\frac{\sqrt{6}}{3}$	$-0.88^{+0.45}_{-0.12}$	1.7 ± 0.3	1.3 ± 0.1

$$(P_0, D_0)_{pm} = (3.2 \pm 0.4, -5.1 \pm 2.5) 10^{-2} G_F \text{GeV},$$

$$R_{P_0 D_0} = 0.70,$$

$$\chi^2_{pm}/\text{d.o.f.} = 11, \quad (17)$$

where $R_{P_0 D_0}$ stands for the correlation between the two fitted parameters of P_0 and D_0 and the data point of $\alpha_{ex}(\Lambda_c^+ \rightarrow \Xi'^0 K^+)$ has also been included in the fit. Our results of $\mathcal{B}(\mathbf{B}_c \rightarrow \mathbf{B}_D M)$ are listed in Tables I–IV. We note that unlike the cases in the em scheme, ζ and ξ vary from $(0.02 \sim 0.11) \text{ GeV}^3$ and $(0.2 \sim 0.3)$ for the different modes of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ in the pm scheme, respectively, resulting in the main differences for the two schemes. In the pm scheme, it is clear that the $SU(3)_F$ flavor symmetry is broken by the mass difference in the kinematic part, but still kept in the $P(D)$ -wave amplitude. In contrast, $SU(3)_F$ is exact both kinematically and dynamically in the em scheme. The larger value of χ^2_{pm} compared to that of χ^2_{em} may result from the improper handling of the $P(D)$ -wave amplitude. The $SU(3)_F$ breaking effect in the amplitude would compensate that from the kinematic part. However, such an effect can be considered within the $SU(3)_F$ approach only when more experimental data points are available in the future.

In Table I for the Cabibbo allowed modes of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ based on $SU(3)_F$, we also show the experimental data [2, 11, 13, 14] as well as the theoretical calculations in the literature [15, 16, 34]. In particular, Xu and Kamal in

Ref. [15] consider the baryon pole term as the nonfactorizable amplitude, Korner and Kramer in Ref. [16] take account of the heavy quark symmetry and covariant quark model for the baryon wave function, and Sharma and Verma in Ref. [34] study the branching ratios with $SU(3)_F$ based on the old experimental data. Note that our result of $\mathcal{B}_{em}(\Lambda_c^+ \rightarrow \Xi'^0 K^+) = (4.1 \pm 0.3) \times 10^{-3}$ is smaller than, but still consistent with, the current experimental value of $(5.02 \pm 1.04) \times 10^{-3}$. However, it fits well with the previous experimental result of $\mathcal{B}(\Lambda_c^+ \rightarrow \Xi'^0 K^+) = (4.0 \pm 1.0) \times 10^{-3}$ as shown in Table 1 of Ref. [11]. However, our result of $\mathcal{B}_{pm}(\Lambda_c^+ \rightarrow \Xi'^0 K^+) = (1.0 \pm 0.2) \times 10^{-3}$ is inconsistent with the data. It is interesting to point out that the up-down asymmetries for all decays are expected to be 0 by theoretical studies in Refs. [15, 16, 34] due to the vanishing D-wave amplitudes, which are different from our nonzero results and inconsistent with the current experimental result of $\alpha_{ex}(\Lambda_c^+ \rightarrow \Xi'^0 K^+) = -1.00 \pm 0.34$ [11]. We recommend measuring $\alpha(\Lambda_c^+ \rightarrow \Delta^{++} K^-)$ in the future experiment as this decay channel has the largest decay branching rate, which is a clean justification of the $SU(3)_F$ approach. In addition, the authors in Ref. [34] use $SU(3)_F$ without neglecting $H(\overline{15})$ but treat the D-wave amplitude as 0. Nonetheless, they still arrive at the conclusion that $H(\overline{15})$ is negligible compared to $H(6)$. However, our results are somewhat different from those in Ref. [34].

There are some common features between our results and those in Refs. [15, 16, 34]. The most important one is that the vanishing amplitudes in the Cabibbo allowed decays of $\Xi_c^+ \rightarrow \Sigma'^+ \bar{K}^0$ and $\Xi_c^+ \rightarrow \Xi'^0 \pi^+$. It is clear that the current experimental data of $\mathcal{B}(\Xi_c^+ \rightarrow \Sigma'^+ \bar{K}^0)/\mathcal{B}(\Xi_c^+ \rightarrow \Xi'^0 \pi^+) = (1.0 \pm 0.5)$ and $\mathcal{B}(\Xi_c^+ \rightarrow \Xi'^0 \pi^+)/\mathcal{B}(\Xi_c^+ \rightarrow \Xi'^0 \pi^+) < 0.1$ [2] are insufficient to rule out this feature yet. It is interesting to note that the decay branching ratios given in the various theoretical calculations may not obey the flavor symmetry of $SU(3)_F$ in general, but they all preserve the isospin symmetry. In particular, the isospin relations in the Cabibbo allowed decays can be summarized as follows:

$$\begin{aligned}
\mathcal{B}(\Lambda_c^+ \rightarrow \Delta^{++} K^-) &= 3\mathcal{B}(\Lambda_c^+ \rightarrow \Delta^+ \bar{K}^0), \\
\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma'^+ \pi^0) &= \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma'^0 \pi^+), \\
\mathcal{B}(\Xi_c^0 \rightarrow \Sigma'^+ K^-) &= 2\mathcal{B}(\Xi_c^0 \rightarrow \Sigma'^0 \bar{K}^0), \\
\mathcal{B}(\Xi_c^0 \rightarrow \Xi'^0 \pi^0) &= \frac{1}{2} \mathcal{B}(\Xi_c^0 \rightarrow \Xi'^- \pi^+).
\end{aligned} \quad (18)$$

Similar relations in the singly and doubly Cabibbo suppressed decays are also expected.

Finally, we explore the decay processes of $\Xi_c^0 \rightarrow \Sigma'^0 K_S/K_L$, which contain both Cabibbo allowed and doubly suppressed contributions as shown in Table IV, resulting in an asymmetry due to the interference between the two contributions. Explicitly, the $K_S - K_L$ asymmetry is found to be

TABLE IV. Results for $\Xi_c^0 \rightarrow \Sigma'^0 K_S/K_L$ with $SU(3)_F$.

Channel	$f_{\mathbf{B}_c \mathbf{B}_D M}$	α_{pm}	$10^3 \mathcal{B}_{pm}$	$10^3 \mathcal{B}_{em}$
$\Xi_c^0 \rightarrow \Sigma'^0 K_S$	$-\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3} s_c^2$	$-0.88^{+0.45}_{-0.12}$	0.70 ± 0.11	0.52 ± 0.04
$\Xi_c^0 \rightarrow \Sigma'^0 K_L$	$\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3} s_c^2$	$-0.88^{+0.45}_{-0.12}$	0.87 ± 0.14	0.64 ± 0.05

$$\begin{aligned} \mathbf{R} &\equiv \frac{\Gamma(\Xi_c^0 \rightarrow \Sigma'^0 K_S^0) - \Gamma(\Xi_c^0 \rightarrow \Sigma'^0 K_L^0)}{\Gamma(\Xi_c^0 \rightarrow \Sigma'^0 K_S^0) + \Gamma(\Xi_c^0 \rightarrow \Sigma'^0 K_L^0)} \\ &= \frac{(1 - s_c^2)^2 - (1 + s_c^2)^2}{(1 - s_c^2)^2 + (1 + s_c^2)^2} = -0.106, \end{aligned} \quad (19)$$

which is independent of the fitting. As a consequence, the asymmetry in Eq. (19) provides a clean prediction in the $SU(3)_F$ approach for the charmed baryon decays, which can be tested by the experiments in BELLE and BESIII.

IV. CONCLUSIONS

We have studied the decay branching ratios and up-down asymmetries in the charmed baryon weak decays of $\mathbf{B}_c \rightarrow \mathbf{B}_D M$ based on the flavor symmetry of $SU(3)_F$. It is interesting to emphasize that these $\mathbf{B}_c$ decays with the decuplet spin-3/2 baryon receive only nonfactorizable contributions. We have shown that our fitting results for $\mathcal{B}(\mathbf{B}_c \rightarrow \mathbf{B}_D M)$ are consistent with the current experimental data in both pm and em schemes. In particular, the em scheme leads to a much smaller number for the χ^2 fit than the pm one, resulting in that the predicted values of $\mathcal{B}(\mathbf{B}_c \rightarrow \mathbf{B}_D M)$ in the em scheme contain many fewer uncertainties than those in the pm one. To reduce the large uncertainties in the pm scheme, the $SU(3)_F$ breaking effect should be included in the amplitude as well when more precision measurements of $\mathcal{B}(\mathbf{B}_c \rightarrow \mathbf{B}_D M)$ are available. We have demonstrated that the isospin relations for the decay branching ratios in Eq. (18) are scheme and model independent. It is also interesting to note that the vanishing

rates for the Cabibbo allowed decays of $\Xi_c^+ \rightarrow \Sigma'^+ \bar{K}^0$ and $\Xi_c^+ \rightarrow \Xi'^0 \pi^+$ have not been supported by the experimental data yet.

For the up-down asymmetries, we have found that they are sizable, which is different from the prediction of 0 due to the vanishing D-wave contributions in the literature. In particular, we have obtained that $\alpha(\mathbf{B}_c \rightarrow \mathbf{B}_D M) = -1.00_{-0}^{+0.34}$ for all decay modes in the em scheme, while they range from -1 to -0.42 at the 1σ level in the pm scheme, consistent with the current only available data of $\alpha_{ex}(\Lambda_c^+ \rightarrow \Xi'^0 K^+) = -1.00 \pm 0.34$ [11] for the up-down asymmetry. To justify the $SU(3)_F$ approach, we have proposed searching for $\alpha(\Lambda_c^+ \rightarrow \Delta^{++} K^-)$, which is predicted to be $-0.86_{-0.14}^{+0.44}$, in the future experiments, as the decay has the largest branching rate among $\mathbf{B}_c \rightarrow \mathbf{B}_D M$.

In addition, we have examined the processes of $\Xi_c^0 \rightarrow \Sigma'^0 K_S/K_L$, which contain both Cabibbo allowed and doubly suppressed contributions. We have predicted the $K_L - K_S$ asymmetry of $\mathbf{R}(\Xi_c^0 \rightarrow \Sigma'^0 K_S/K_L)$ is -0.106 , which depends on neither model/scheme nor the data fitting. Clearly, this asymmetry is a clean result in the $SU(3)_F$ approach, which should be tested by the experiments.

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